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Robustness Assessment of Link Capacity Reduction for Complex Networks: Application for Public Transport Systems

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Highlights:

- Quantifying network transmission losses under planned capacity reductions
- Integrating capacity degradation results per link into network robustness assessment
- Formulating measures of link criticality and rapidity of network deterioration
- An application to the urban public transport network of Amsterdam
- Small and long capacity degradations should be preferred over large and short ones

Abstract

Network robustness refers to as the capacity to absorb disturbances with a minimal impact on system performance. Notwithstanding, network robustness assessment has been mostly confined to the analysis of complete link breakdown based on topological metrics. We propose reliability indicators that encompass changes in network performance with respect to the entire range of possible capacity reductions. Link criticality and degradation rapidity are measured by constructing network degradation curve that describe the relation between local capacity reduction and global change in network performance. We develop a public transport robustness assessment model which computes passenger flow distribution and network performance metrics under planned capacity reductions. The model is applied to the urban rail-bound network of Amsterdam. Link criticality and degradation rapidity are studied by performing a full-scan impact analysis which demonstrates how the robustness indicators introduced in this paper contribute to a more complete assessment of network robustness.

Keywords: Robustness; Link Criticality; Vulnerability; Public Transport; Network Assignment; Complex Networks

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1. Introduction

Social-technical systems are subject to man-made, technical and natural disruptions. Systems are considered robust based on their capacity to absorb disruptions with a minimal impact on system performance. The performance can be measured in terms of the worst affected component, total transmission costs or the extent to which the system has disintegrated (e.g. size of the largest sub-network that remains connected). Even though robustness of critical infrastructures such as mass public transport networks (PTN) is high on the planning and policy agenda (Homeland Security 2010), there is lack of knowledge on how to assess and quantify network robustness towards a range of possible disruptions. While disruptions of critical infrastructure are often limited to a partial reduction in link transmission capacity, most research has focused on complete link breakdown (i.e. link removal) with few noticeable exceptions (Sullivan et al. 2010 and Cats and Jenelius 2016). The analysis performed in the latter does not allow for a network robustness analysis because it either did not propose a method to integrate information from various link-level capacity reduction scenarios or did not perform a full-scan network analysis.

A system can be considered robust with respect to disruptions on a certain component if minor disturbances of its performance do not have severe consequences on the overall system performance, i.e., when overall travel time reliability remains constant under minor capacity reductions. Hence, robustness is not merely based on the magnitude of the ramifications of a complete breakdown but should also refer to the trajectory that describes how different disruption severity influence severity in network degradation. The latter provides information on the overall range of values and the sensitivity of network performance to alterations in the performance of individual network elements. The commonly asked question who is the weakest link becomes thus multi-dimensional and requires expanding our toolset for quantifying network robustness.

The primary objective of the current study is to propose reliability indicators that encompass changes in network performance with respect to the entire range of possible capacity reductions and can be used in a wide range of domains. To this end, two robustness indicators are conceptualized and formulated by constructing performance curves to allow quantifying the absolute change and the first moment of the degradation in network performance as a function of link capacity reductions. This study contributes to the state-of-the-art of network robustness by enabling the quantification of network robustness in terms of network transmission losses for the range of possible capacity reductions. The analysis of these indicators allows identifying the extent and the relation between capacity reduction and performance reductions and thus support infrastructure management and capacity allocation. The indicators are demonstrates in the context of PTN, where network performance under disruptions is modelled using a passenger load (re-)distribution model. The impacts of disruptions on the robustness of PTN has long been an understudied topic, in particular in the context of partial degradations.

Assessing the impact of different capacity reduction scenarios on the robustness of the PTN is not only important for efficiency reasons, but also for reliability and safety considerations. Various capacity-related definitions of network reliability were proposed in the literature for road networks (Chen et al. 1999, 2002, Al-Deek & Emam 2006, Scott et al. 2006, Sullivan et al. 2010). Some of these definitions can be transferred to the public transport domain when measuring day-to-day travel time variations. For example, planners can monitor whether during planned maintenance periods the reliability of individual lines or indeed the whole PTN stays within a certain margin in relation to normal operations. Alternatively, a probabilistic notion of network reliability performance can refer to the probability that the network can accommodate a certain traffic demand at a pre-defined desired service level.

Most related work on safety assessment of (public) transport systems focuses on system-level analysis (Evans 1994, Stoop & Thissen 1997, Mohan & Tiwari 1999, Macchi et al. 2012). Only limited work has been done on the relation between (link) capacity reduction and safety. It is expected that different maintenance plans will have a different impact on the reliability, safety and associated costs for a public transport network. It is for example expected that a full closure of a link will be safer for maintenance crews, but will have a larger effect (per time unit) on the workings of the rest of the network. Another aspect that might be relevant in this context is that of vehicle operators: slower moving metros and trams, i.e., with reducing the capacity on a line, can be expected to lead to fewer errors (White 2002) as operators have more time to react to unexpected events.

Even though public transport constitutes critical infrastructure in many urban and regional transport systems, only little is known about the determinants of its vulnerability and methods and techniques to analyse and mitigate the impacts of disruptions. The vast majority of previous studies focused on road networks (Reggiani 2013, Berdica 2012, Chen et al. 2002), the degradations of its physical infrastructure and its evaluation (Scott et al. 2006, Jenelius and Mattsson 2012, Taylor and Susilawatii 2012). While these studies provide some relevant conceptual foundations, the vulnerability analysis techniques have only limited transferability to public transport. PTN are characterized by greater complexity due to the relation between the infrastructure and service layers. In the context of PTN, the nonlinear properties of network effects and probabilistic flow distribution may result in non-trivial relations between the magnitude of the failure and its consequences.

The ability of PTN to maintain their function under circumstances which strongly deviate from plan is essential to their robustness. PTN are prone to recurrent disruptions, ranging from mechanical and technical problems (e.g. vehicle breakdown, switch failure) to traffic accidents and suicide attempts (Cats et al. 2016). Many of these causes, in addition to planned construction and maintenance works, result with limited traffic capacity. The performance of the rail-bound Amsterdam network in the case of planned capacity reductions is thereof selected to demonstrate the proposed indicators and the insights gained by their measurement.

Different kinds of planned capacity reductions are performed by system managers in urban public transport networks. These reductions can vary from a reduced speed or frequency on a link to a full link closure due to maintenance routines or in conjunction with construction works. In some cases, project manager can trade-off between the extent of capacity reduction and its duration. However, there is lack of knowledge on the impacts of such decisions on passengers' travel costs, including the consideration of rerouting possibilities and disconnected passengers.

The remaining of this paper is organized as follows. The following section reviews related work on alternative approaches to measuring network performance and distinguishing between four types of network robustness measures. Two robustness indicators, *Link criticality* and *Degrading Rapidity*, are proposed, formulated and illustrated in Section 3. A general network robustness assessment procedure and its specification for an application to public transport systems is presented in Section 4 along with the passenger flow distribution and the calculation of network performance metrics. Section 5 details the application of this model to the urban rail network in Amsterdam including the experiment set-up and the statistical and spatial analysis of the proposed robustness indicators. The paper concludes with a discussion of study implications and directions for future research.

2. Related Work

There is a large body of related work on network robustness. This section provides an overview of the main approaches found in the literature, starting by considering network performance indicators followed by examining network robustness indicators.

2.1 Measuring Network Performance

Network performance indicators aim at quantifying the functioning of a network, allowing the comparison of different conditions or configurations of the same network, or different networks altogether. Network performance indicators can be categorized into three main groups of studies, with an increasing level of detail: infrastructures studies, service network studies and flow distribution studies.

The network performance indicators used in infrastructure studies are, in general, topological indicators based on graph theory. The network performance in these studies is, for example, expressed in terms of global connectivity expressed as the minimum number of links that needs to be removed to disconnect the remaining nodes from each other (Berche et al. 2009, von Ferber et al. 2012), the largest connected component of the network (Wang et al. 2015) or changes in accessibility (Kermanshah and Derrible 2016). These indicators are referred by Faturechi and Miller-Hooks (2014a) as topological measures of effectiveness (MOE). Such purely topological infrastructure studies and corresponding indicators neglect the state of the networks in terms of saturation and flow distribution. The advantage of such indicators is that they can be applied to a wide variety of networks. The main disadvantage is that they only consider topological indicators and thus cannot, for example, differentiate between a network with congestion and the same network without congestion. When considering PT networks, infrastructure indicators discard the notion of lines and corresponding service network characteristics for which the same disruptions can have different effects on the network performance. For example, using a topological approach, a link breakdown results with remaining links operating normally although in practice upstream and downstream links of the same line will be affected.

When the state of the network is considered, more detailed analysis of the performance of the network is possible. Public transport studies that include the service network define different transport lines which are superimposed on top of the physical infrastructure. The representation of both topological and service layers allow to consider transfers, link travel times and line waiting times. The network performance can then be measured as the mean travel time between all the OD-pairs or the mean number of transfers over all the OD-pairs. Berche et al. (2009) for example represented the largest metro networks in the world using both an infrastructure representation as well as a service network representation and examined the effect of random attacks on the performance of the network. They expressed the effect in terms of the change in the size of the largest cluster and the average inverse mean shortest path length. Another example by Ellens et al. (2011) considers the effective resistance between all pairs of vertices in a power grid, leading to the notion of effective graph resistance, which allows the analysis of robustness of a power grid without looking at the actual power flow.

The most detailed network indicators also consider flow and link capacity (Simonsen 2008), making it possible, for example, to study flow distribution and congestion in networks. Such indicators always need to take the physical infrastructure into account, which makes them more detailed but also less widely applicable. Examples of networks, other than transport networks, that have been studied using more specific performance indicators include: power grids (Koç 2013), water distribution networks (Yazdani 2011a, 2011b), gas distribution networks (Carvalho 2009) and virus spread (Youssef 2011). Studies that consider flow and capacity can reflect functional, topological and economic values. The latter are based on a monetization that is performed by taking the product of the functional MOE with values of time (VOT). Examples of studies in which the network performance is expressed in economic MOE are from Cats and Jenelius (2014). The effect of a disruption on the network performance can also be analysed using non-monetarized network performance indicators, for example, the share of cut-off, affected or delayed passengers and the corresponding delays (Cats 2016).

2.2 Measuring Network Robustness

Four types of network robustness measures can be distinguished in the literature: threshold values, local slopes, ratios and areas under curves. Most of these measures are thoroughly discusses by Hosseini et al. (2015) and Faturechi and Miller-Hooks (2014b).

A first category of measures which are applied in robustness studies (Berche et al. 2009) is the measurement of network performance at a certain threshold of the capacity reduction. An example of which is a measure that examines the effect on the network performance indicator in the event of a certain percentage-wise reduction in the total network capacity. Larger negative effects indicate a less robust network. This indicator conveys at what percentage of capacity reduction the decrease in network performance becomes unacceptable.

The second category of measures is the measurement of the slopes of the relationships at some specific point. This was for example carried out in a study on freight resilience measures (Adams et al. 2012). While the application of this type of indicator can give an indication on the form of the curve and thus on the sensitivity of the link to different percentage of capacity reduction, it is not able to indicate the total effect of a link on the network robustness assessing a whole range of disruption scenarios.

A third category of measures are ratios of performance values for different situations or scenarios. Example of the use of this category of measures can be found in studies on system resilience. Applications in these studies are performed by measuring the difference between system performance before and after disruption (Faturechi, R., & Miller-Hooks 2014a, 2014b, Faturechi et al. 2014, Nair et al. 2010, Sullivan et al. 2010). Another variant of this measure is the ratio between the system performance after disruption and the worst-case degraded performance level (Orwin et al. 2004, Cox et al. 2011).

Finally, the fourth category is able to capture the accumulated effect of the total range of capacity reduction on the network performance by measuring the area under the graph. In some reviewed studies this is done in a simple way by calculating the triangle between the network performance for a 100% capacity reduction and the network performance for a base case scenario (Zhang et al. 2009, Zobel 2011) and in a more sophisticated way by taking the integral of the relationship (Faturechi and Miller-Hooks 2014a, Bruneau et al. 2003, Omer et al. 2011, Rose 2007, Vugrin et al. 2011, Cats and Jenelius 2016). By calculating the area as a triangle it is possible to measure the accumulated effect of the total range of capacity reductions but only under the assumption that the curve is linear. By taking the integral of the curves it is possible to measure the accumulated effect of the total range of capacity reductions without making an assumption on the form of the curve.

The next section introduces two new robustness indicators. Both indicators require constructing a Network Degradation curve as explained in the following section. The *Link Criticality* indicator quantifies the accumulated effect of the whole range of capacity reduction on a given link on the network performance and the *Degrading Rapidity* indicator measures the relative change in link criticality by combining properties of the second and fourth abovementioned categories.

3. Robustness Indicators

A metric indicating link robustness should reflect the degradation in network performance induced by the entire range of possible capacity reductions.

To this end, we propose an indicator of *Link Criticality* which quantifies the accumulated effect of the whole range of capacity reduction on a given link on the network performance. While this indicator captures the overall performance loss, it does not convey information on the relation between capacity reduction and network degradation. We therefore propose a second robustness indicator denominated *Degrading Rapidity* to indicate the relative criticality for low capacity reduction relative to high capacity reductions. Before turning into the formulation of these two novel indicators, we first present in the following sub-section how the network degradation curve, upon which both rely, is constructed. Unlike the network robustness indexes proposed by Sullivan et al (2010) to compare capacity reduction scenarios, the indicators proposed in this paper integrate information from the range of capacity reductions into a link-specific value that characterizes its impact and significance to network robustness.

3.1 Network degradation curves

A directed graph $G(S, E)$ represents the network with a set of nodes S and the link set $E \subseteq S \times S$ for direct connections between nodes. The capacity of each link $e \in E$ under normal (i.e. undisrupted) conditions is denoted by κ_e^0. Capacity is defined in terms of the number of units that can be transmitted within a given time unit (e.g. megabytes per seconds, kilowatts per minute, vehicles per hour). The (residual) capacity $\kappa_e(x)$ on link e during scenario x can then be defined as:

$$\kappa_e(x) = \kappa_e^0 \cdot (1 - x) \quad \forall e \in E, \forall x \in [0,1] \qquad (1)$$

Where x is the percentage-wise reduction in capacity on link e. The range of capacity reduction is bounded by $x \in [0,1]$. The lower bound of $x = 0$ corresponds to the original full capacity situation whereas the upper bound value, $x = 1$, implies a complete breakdown of the link, i.e. link closure.

For each link, a curve is established by obtaining for each capacity reduction the corresponding network transmission costs. Every link capacity reduction case results in a certain network performance depending on network structure, demand and the mechanisms governing flow distribution. Let us denote by $y(x)$ the network transmission costs that are incurred under scenario x. Depending on the context, costs can be expressed in terms of energy consumed or transmission times. The change in network transmission costs induced by a certain scenario, $\Delta y(x)$, are then calculated as the difference between the costs occurring under the disruption $y(x)$ and those in case of normal operations, $y(0)$.

Figure 1 illustrates possible network degradation curves for two links in a certain network by plotting the change in network transmission costs under various capacity reduction scenarios, ranging from normal operations to complete link failures. Under most circumstances, it is reasonable to assume that network transmission costs increase in the event of a capacity reduction on a network element since this implies a more binding constraint that is expected to result with an inferior solution. Hence, the network degradation curve is expected to increase monotonically with x as the network performance remains unaffected or worsens for increasing capacity reductions. While the performance under normal conditions is the same for both links since it corresponds to the same scenario, the extent to which network performance deteriorates when each of the links breakdown may vary. Furthermore, the curve may take different forms (e.g. convex or concave as illustrated in Figure 1) depending on the extent to which the network deteriorates in response to the same capacity reduction when already subject to a partial disruption. Hence, the form of the curve contains information on how susceptible network performance is towards marginal increases in capacity reductions to different initial reductions (Cats and Jenelius 2016).

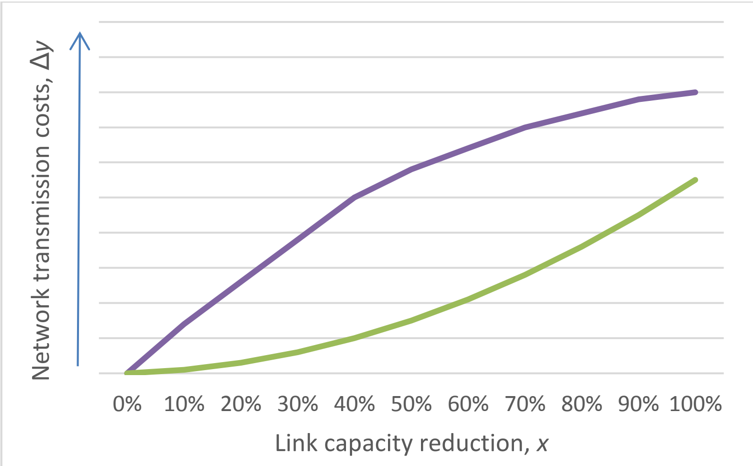

Figure 1 Illustration of network degradation curves for two network links

3.2 Link Criticality

We propose a *Link Criticality* indicator that encompasses network degradation for any possible capacity reduction. The criticality of link e is calculated as

$$c_e = \int_0^1 [y_e(x) - y_e(0)]\, dx = \sum_{x \in X} [y_e(x) - y_e(0)] \;\; \forall e \in E \qquad (2)$$

This formulation corresponds graphically to the area under the network degradation curve for the respective link. The area is calculated as the integral over the function $\Delta y(x)$ or the summation of the those values observed for a given set of scenarios, X. Figure 2 illustrates how the link criticality indicator is obtained for the two links displayed in Figure 1. The area under the solid purple curve in Figure 1 is significantly larger than the one under the green stripped curve. If information concerning the probabilities concerning the exposure of different capacity reductions occurring on a certain link is available, the probabilities can be embedded into Eq. 2 by assigning weights for each corresponding scenario x. Network exposure to a certain disruption incorporates information on the failure probability as well as the duration of the disruption, see Cats et al. (2016). *Link Criticality* is defined within the context of a certain network as it measures the degradation of this network in the event of capacity reductions on a given link. The impact of the disruption depends on network structure and the availability of alternative routing.

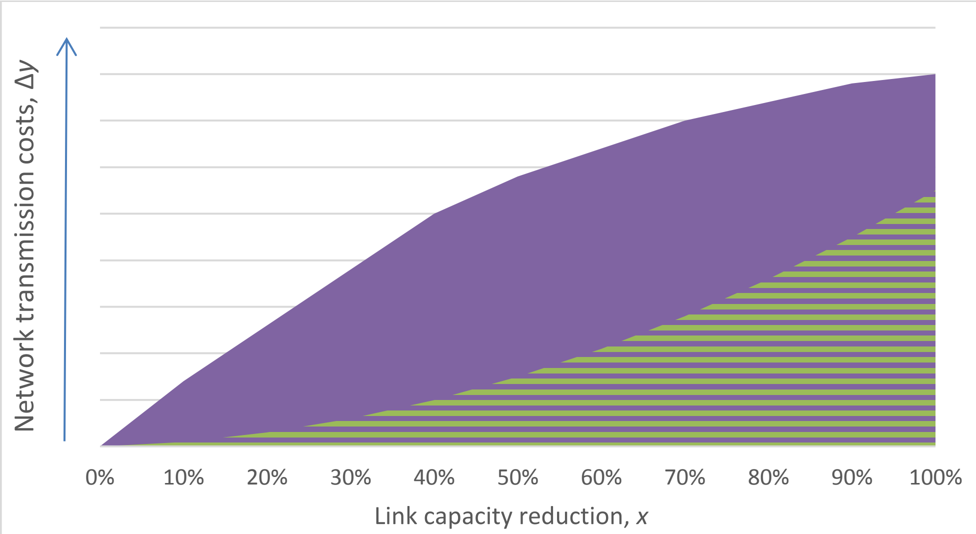

Figure 2 Illustration of Link Criticality for two network links

While the *Link Criticality* indicator provides an absolute measure of network robustness to capacity reductions, it does not capture the trend that the network degradation exercises. Different links may be equally critical but manifest strikingly different trends as was found by Cats and Jenelius (2016). For example, one link may be very vulnerable to even the slightest capacity reduction, inducing immediately adverse effects but then further reductions may not result in worsening consequences. Conversely, the network might be able to withstand capacity reductions on another link for low and medium capacity reductions but then beyond a certain tipping point result with severe implications. Decline in network performance may be proportional to the capacity reduction in other cases. We thus propose next an additional indicator that aims to distinguish these trends.

3.3 Degrading Rapidity

Degrading Rapidity measures the relative change in link criticality. In order to allow comparing different links when investigating the relative change, the network degradation curve is first scaled against the worse degradation:

$$\hat{y}_e(x) = \frac{y_e(x) - y_e(0)}{y_e(x_e^{max}) - y_e(0)} \qquad (3)$$

$$x_e^{max} = \arg\max_x \{y_e(x) - y_e(0)\} \qquad (4)$$

Under most circumstances $x_e^{max} = 1$. The *Degrading Rapidity* of link e can now be calculated as follows:

$$r_e = \int_0^1 \hat{y}_e(x)\, dx = \sum_{x \in X} \hat{y}_e(x) \; \forall e \in E \qquad (5)$$

The indicator, $r_e \in [0,1]$, corresponds to the area under the scaled curve, or the summation over a discrete number of scenarios. The scaled curve for the same two links for which the curves were plotted in Figures 1 and 2 is presented in Figure 3. The area under the solid purple curve yield $r_e > 0.5$ implying a super-linear degradation trend where the same marginal decrease in link capacity leads to higher losses for low initial levels than for higher initial levels. In contrast, the link with the scaled curve displayed in striped green follows a sub-linear trend with $r_e < 0.5$, implying that marginal increases in network transmission costs are greater when occurring under already reduced capacity conditions.

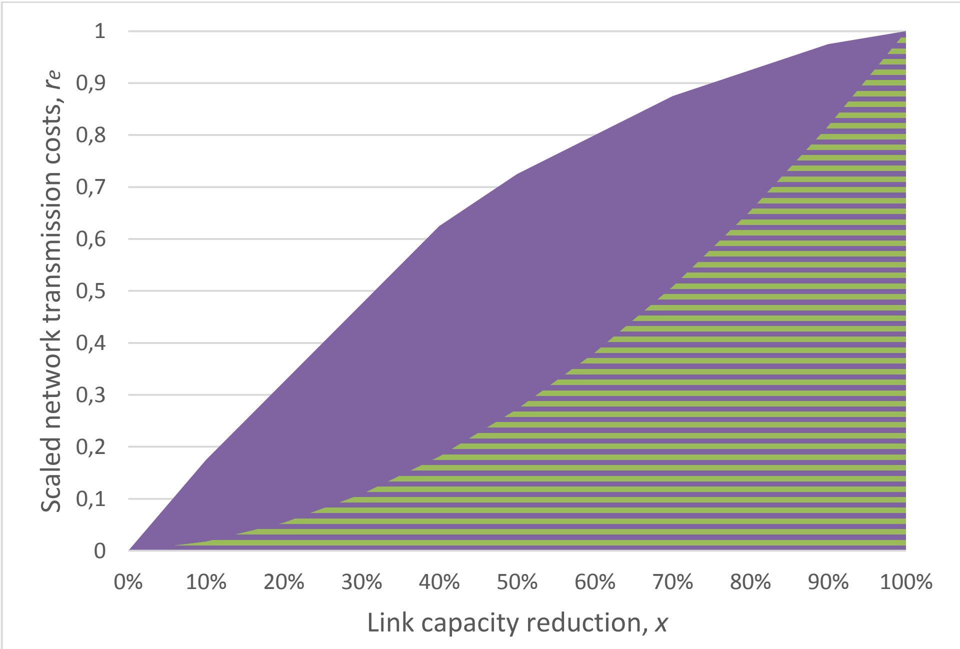

Figure 3 Illustration of Link Degrading Rapidity for two network links

4. Network Robustness Assessment Model

We hereby describe the general process that is to be undertaken to perform the proposed robustness assessment (4.1), provide an introduction to the public transport network modelling context (4.2), after which we detail each of its steps for the case of analysing public transport systems (4.3-4.6).

4.1 Assessment procedure and modelling approach

The combination of *Link Criticality* and *Degrading Rapidity* allows quantifying the accumulated effect of capacity reductions as well as capturing the relative change in criticality (i.e. first momentum) of the scaled criticality to allow direct comparison among network links.

Figure 4 presents the overall model framework. The base (i.e. undisrupted) network configuration and the flow to be assigned to the network are given as input to the model. The assessment procedure devised in this study consists of four components (highlighted in Figure 4), three of which compose the main loop which executes all the disruption scenarios as follows:

(i) Network representation – the network used in each scenario is generated by modifying the basic network configuration based on the details of the disruption under consideration

(ii) Flow distribution model – a model is used to distribute the flows over the network based on cost minimization principles

(iii) Network performance indicator – the performance of the network is assessed by calculating network transmission costs

This sequence of steps is repeated for each scenario. After all relevant scenarios – a full network-scan of capacity-reductions per link – have been performed, this loop terminates and the fourth post-processing module takes place:

(iv) Robustness indicator – encompassing network performance under all capacity reductions into aggregate indicators which reflect the magnitude and trend of network robustness per link

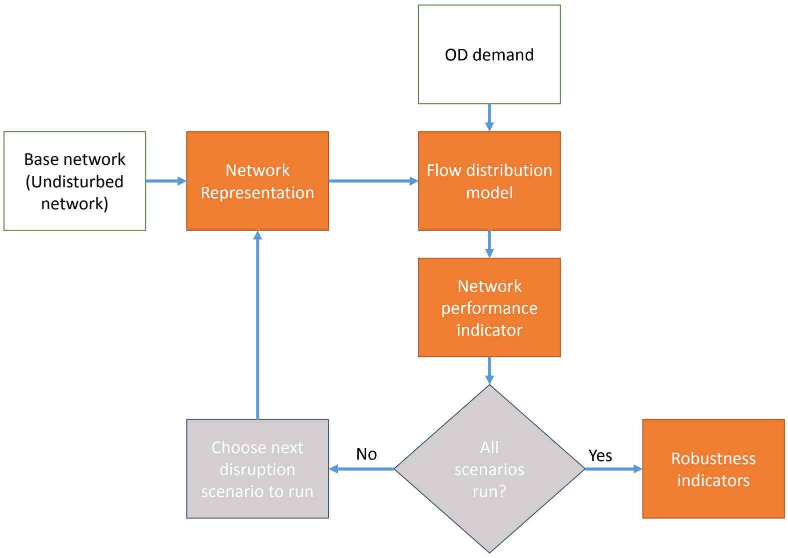

Figure 4 Robustness assessment modelling framework

4.2 Modelling public transport networks

The robustness assessment of link capacity reductions is applied to the analysis of public transport networks. Such an analysis can be performed as part of network design and project evaluation in the strategic planning phase or when considering capacity allocation and periodical timetables in the tactical planning stage. In contrast to many other networks, including car traffic networks, public transport operations involve a service network layer that is superimposed on the physical (typically road or rail) infrastructure layer. As an additional input, travel demand is given in the form of an origin-destination matrix with the corresponding number of trips per time interval. It is essential to model the notion of public transport lines in order to adequately capture the impact of capacity reductions on line performance and thereof on passengers flow distribution. The latter requires a behavioural model which consists of an initial step in which a choice-set comprising a limited number of attractive route alternatives is composed and thereafter choice probabilities are calculated. As passengers differ in their knowledge of the network and their travel preferences, a probabilistic route choice model is used to assign trips to the network based on trade-offs between path attributes. Since we are interested in a strategic network assessment, passengers are distribute over network the network in terms of lines and links without considering their time-dependency. Hence, the approach taken in this study is to perform a probabilistic frequency-based assignment model while explicitly modelling individual public transport lines. For more information on PTN representation and assignment model, the reader is referred to Gentile et al. (2016). Assignment outputs are then processed to calculate network performance indicators. This process is repeated for each capacity reduction scenario for a given link, after which the link robustness indicators can be derived. In the following, the sequence of steps undertaken in this process are detailed.

4.3 Network Representation

4.3.1 Public transport graph

The PTN is represented in two spaces commonly used in complex network theory. First, the physical PTN, i.e. road and rail infrastructure, is represented using L-space. Each station or stop is represented by a node and two nodes are connected if the corresponding stops are consecutive stops on at least one service line (Barthélemy, 2011), while removing duplicates caused by common corridors (Derrible & Kennedy, 2011). Link labels are the respective travel times. The L-space is used to calculate in-vehicle travel times and model the consequences of disruptions and mitigation measures. Second, the PTN is represented in P-space which is instrumental in modelling service lines and their respective frequencies as well as passenger path choices and transfer possibilities. While nodes still correspond to stops, nodes are connected if there is at least one line serving both stops (Barthélemy, 2011). Link labels are the respective line frequencies. Note that in contrast to L-space, the network represented in P-space is not spatial. The data on infrastructure and service networks are processed to generate an operational network representation that is then subject to a disruption scenario as described in the following sub-section.

4.3.2 Disruption scenarios

The effects of bi-directional link-based disruption scenarios are investigated in this study. Capacity reductions are conceived in this study in terms of speed reductions. Each link $e \in E$ is assigned with a baseline mean speed v_e^0 under normal operations. Similarly to Eq. 2, the mean speed v_e on link e during disruption scenario x can then be described as

$$v_e(x) = v_e^0 \cdot (1 - x) \qquad (6)$$

The lower bound of x corresponds to normal operations (i.e. $v_e(x) = v_e^0$) whereas the upper bound value implies a complete breakdown of the link (i.e. $v_e(x) = 0$, the link is dysfunctional).

4.4 Flow distribution model

4.4.1 Route choice set generation

A route choice set is generated in an initialization phase by performing a k-shortest path for each pair of nodes. The choice-set generation algorithm applies the following filtering rules:

1. The number of transfers (i.e. number of links in P-space) may equal or exceed by one the minimum number of transfers
2. Only paths without loops are retained in the route choice set
3. Dominated paths i.e. paths that are not better than another alternative in the choice set in terms of the number of transfers, total in-vehicle time and total waiting time and are worse off with respect to at least one of these aspects, are removed from the choice-set

This process results with a consideration choice-set $K_{i,j}$ for passengers travelling between $i \in S$ and $j \in S$ for each pair of stops in the network. Each path $k \in K_{i,j}$ is defined by a sequence of links $k = (e_{k,1}, e_{k,2}, \dots, e_{k,|k|})$. We let $e \in k$ mean that link e is part of the definition of path k.

4.4.2 Passenger assignment

Passenger demand is distributed over the network by applying a probabilistic route choice model. A static passenger assignment is performed by considering the generalized travel cost associated with each route. Since the robustness assessment performed in this study is concerned with planned capacity reductions, it is assumed that a-priori knowledge of network conditions is available. The generalized travel cost is composed of the in-vehicle time t_k^t , number of transfers n_k and waiting time

t_k^w. The generalized travel cost is conceived in terms of the utility v_k of path k with a random component ε_k and is calculated as follows:

$$v_k = t_k^t + \beta^w * t_k^w + \beta^n * n_k + \varepsilon_k \quad \forall k \in K_{i,j};\ \forall i,j \in S \qquad (7)$$

The passenger demand between origin i and destination j is distributed over the different routes in the route choice set by using a multinomial logit model, i.e. assuming that the random component is distributed Gumbel. The probability of choosing route k when travelling between origin i and destination j is thus calculated as follows:

$$p_k = \frac{\exp(v_k)}{\sum_{k \in K_{i,j}} \exp(v_k)} \quad \forall k \in K_{i,j};\ \forall i,j \in S \qquad (8)$$

Finally, the passenger flow assigned to link e is

$$q_e = \sum_{i \in S} \sum_{j \in S} \sum_{k \in K_{i,j}} \left[\pi_{ij} \cdot p_k \cdot \delta_{k,e}\right] \quad \forall e \in E \quad (9)$$

Where π_{ij} is the passenger demand between origin i and destination j within the time period under consideration and $\delta_{k,e}$ is a dummy that takes the value one if $e \in k$ and zero otherwise. Correlations among route alternatives are not accounted for in the multinomial logit model, potentially leading to an overestimation of passenger flows on overlapping routes. This modelling drawback is partially counteracted by the consolidation of route alternatives with common lines or transfer stops in the route choice-set generation phase.

4.5 Network performance

Network performance is measured using Network Transmission Costs, y, defined as summation of experienced generalized travel costs, calculated as follows:

$$y = \sum_{i \in S} \sum_{j \in S} \sum_{k \in K_{i,j}} \left[\pi_{ij} \cdot p_k \cdot v_k\right] \qquad (10)$$

4.6 Robustness indicators

The network performance value calculated by Eq. 10 can be obtained for each scenario to establish the network degrading curve. Eq. 2-5 can be then used for calculating Link Criticality and Degrading Rapidity indicators per link.

5. Case Study: The Public Transport System of Amsterdam

The assessment model described in the previous section and the two robustness measures proposed in Section 3 are applied to an urban public transport system: the metro and tram systems of Amsterdam, the Netherlands.

5.1 Network description and experimental set-up

The public transport system of the city of Amsterdam consists of tram, metro, bus and train services. The scope of the case study discussed in this section is limited to the urban rail-bound networks, metro and tram. The case study corresponds to the situation in 2020 when the North-South metro line is operational. The network consists of 4 metro lines and 14 tram lines. As shown in Figure 5, the case study network is dense in the city-centre characterized by the historical canal ring system with radial lines extending to the outer districts. The system includes 216 stops of which 195 are served by tramlines and 29 are served by metro lines. The total rail network length is 270 km, of which 190 km is used by the tram and 80 km is used by the metro. The average number of travellers on the urban rail-bound public transport network during the afternoon peak period (16:00-18:00) is approximately 107,000 passengers per hour.

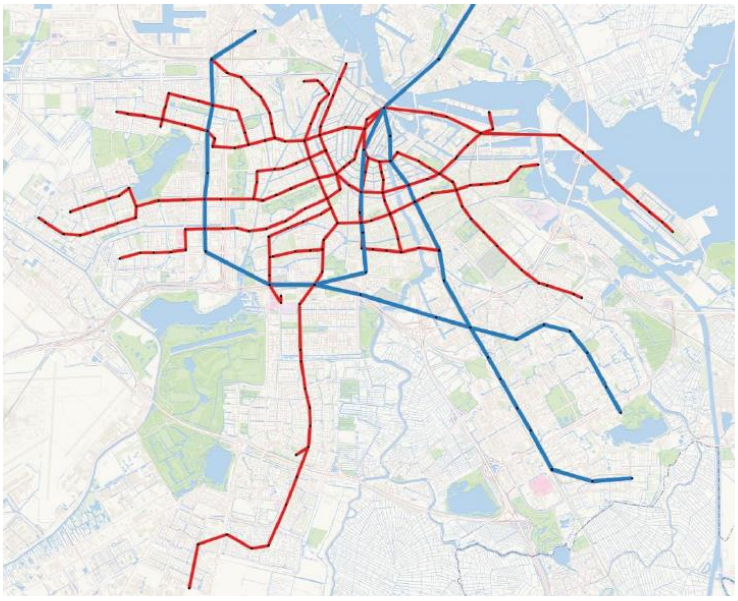

Figure 5 Metro (blue) and tram (red) lines in of the public transport system of the city of Amsterdam in 2020.

The performance of the network was tested under capacity reductions on each single link in the public transport system of Amsterdam. Incremental steps of 10%, i.e. $x = \{0.1, 0.2, \dots, 1.0\}$, were applied. This scenario design resulted with 10 capacity reduction scenarios for each of the 246 bi-directional network links, hence, a total of 2460 disruption scenarios. These scenarios were translated into modifications in the network representation step of the assessment procedure described in the previous section. Speed reductions were applied in partial capacity reduction scenarios, whereas partial lines were assumed to operate on the remaining functional sections in the scenarios that correspond to complete breakdowns $(x = 1)$. In the event of the latter, parts of the network may be disconnected. The transmission cost associated with unsatisfied demand is assumed in this study to be equivalent to the longest delay experienced across the network (i.e. maximum delay over all origin-destination pairs). Note that a disruption on a link is applied to both directions. In addition, a base case scenario of normal operations $(x = 0)$ is used as a benchmark.

The network with its 216 nodes (i.e. stops) was represented in two forms: (a) a physical infrastructure representation in L-space, and; (b) a service network representation in P-space. The nodes are linked by 492 directed links in L-space, whereas these nodes and linked by 6746 directed links in P-space. The initial step of the route choice-set generation, the network-specific initialization phase of the flow distribution model (see section 4.4.1 and Figure 4) is implemented in JAVA while all subsequent modelling steps – passenger assignment (4.4.2), network performance (4.5) and robustness indicators (4.6) - are executed in MATLAB.

5.2 Results and analysis

The application of the robustness measures introduced in the previous section results in a quantitative indication per link for criticality and degrading rapidity. Systems that operate close to capacity and have very limited redundancy as is the case for urban public transport systems are especially vulnerable to capacity reductions of links in the network as is illustrated in the following sections.

5.3.1 Link Criticality

Link Criticality expresses the accumulated effect of the whole range of capacity reductions on a link. The network degradation curves were first constructed for each link and then summarized to obtain information on link criticality. The Link Criticality indicator can thus be used to study in which geographical region in the network the consequences of link capacity reduction are worst, as displayed on the network map in Figure 6. The most critical links are clearly found on the high-capacity branches stretching beyond the network core. These links are served by the metro system with the most critical link can be found in the South-East part of the metro network. Large passenger flows traverse these links and no alternative routes are available. In contrast, the network is robust to disruptions in the central high density parts of the network which have low consequences.

Figure 6 Link Criticality indicator (10-100%) for the links in the Amsterdam urban rail bound public transport network (thicker and more saturated red links indicate higher values)

As can be observed in Figure 6, Link Criticality values vary significantly among network links since the impact of disruptions depend on the number of passengers directly affected as well as the availability of alternative routes that passengers can use in case of disruptions. The average Link Criticality is 146 332 minutes of weighted total passenger travel time, or approximately 1.36 additional minutes per traveller (an increase of 5.6% compared to the undisrupted case) . The distribution of the Link Criticality indicator for the Amsterdam PT network can be seen in Figure 7. While most Link Criticality values do not exceed 5 minutes of extra generalized travel time per traveller, the value of the most critical link exceeds 1 million minutes, or 17 minutes per traveller.

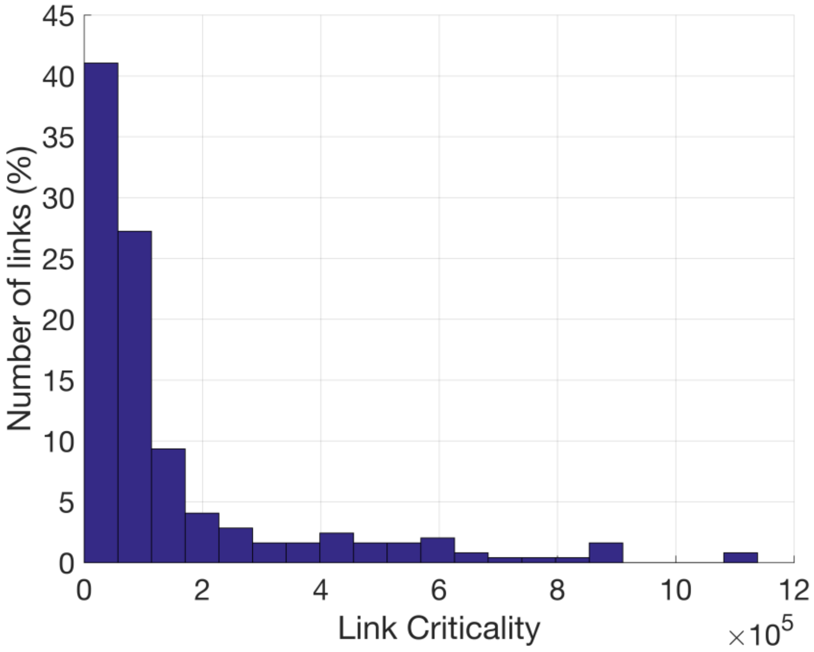

Figure 7 Link criticality histogram

The results of the application of the Link Criticality indicator on the Amsterdam public transport network shows that the indicator is able to express the accumulated effect of the whole range of capacity reductions on a link rather than merely a full breakdown. An evaluation of link criticality based on full breakdown only was found to result with distinctively different results because of the abrupt effect associated with a full breakdown as opposed to partial capacity reductions (more on this in the next section). Furthermore, the indicator is not purely based on total traversing passenger flows, but also the number and quality of the alternative routes. The passenger flows in the centre of the network are quite high but the consequences are mitigated by the high quality alternatives. These results are in line with the analysis of network preparedness to disruptions for a wide range of generic network structures (Zhang et al. 2015). The real-world case study of Amsterdam offers a significantly larger and more complex network which includes a combination of elements from diverse network structure prototypes: grid (within the core), radial (leading to the core), diverging tails (in the outskirts) and random features. Arguably, real-world networks cannot be represented as direct extrapolation of generic elements and exhibit idiosyncratic demand distribution patterns. The latter were often neglected in previous studies which were limited to a topological perspective (e.g. Derrible and Kennedy 2010, Zhang et al. 2015). Consequently, the spatial distribution of link criticality is a signature of network utilization whereas the overall link criticality distribution exercises general patterns characterized by a long right tail.

5.3.2 Degradation Rapidity

The Degrading Rapidity expresses the relative change in link criticality, i.e., it expresses the form of the curves and thus the relative criticality for minor capacity reductions relative to major capacity reductions. The results of the application of the Degrading Rapidity measure for the Amsterdam public transport network are visualized in Figure 8. As can be clearly observed, network degradation measured in terms of change in network transmission costs, worsens comparably rapidly for the most central high density parts of the network. This is especially noticeable for links for which the new North-South metro line offers an attractive alternative. In contrast, links outside of these areas are characterized by limited or no alternative routes which cause a severe degradation in network performance in case of a full breakdown and thus comparably low degradation rate when compared against the corresponding worst scenario. When determining the most important links for network robustness, strikingly different results are attained when using the Link Criticality and Degradation Rapidity indicators (contrast Figures 6 and 8). Previous research found that the most detrimental links for network robustness vary depending on which performance indicator – average travel time, share of disconnected demand or share of delayed passengers – was used (Cats 2016). The findings of this study provide therefore further support to the need to develop a multifaceted and multi-criteria approach in robustness assessment.

Figure 8 Degrading Rapidity indicator (10-100%) for the links in the Amsterdam urban rail bound public transport network (Thicker and more saturated green links indicate a higher score)

The average Degradation Rapidity is 0.26 and the standard deviation is 0.18, resulting with most links following a sub-linear relation ($r_e < 0.5$) where marginal increased in network transmission costs are greater when occurring under already reduced capacity conditions. As can be seen in Figure 9, most links have a low degrading rapidity score between 0.1 and 0.25 indicating that the degrading rapidness to full link breakdown network performance is relatively low.

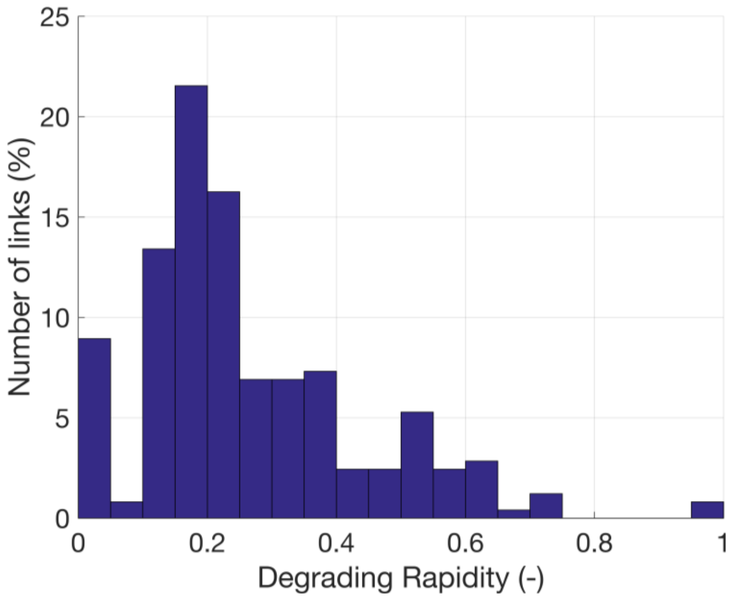

Figure 9 Degrading Rapidity histogram

Figure 10 shows capacity reduction vs. normalized change in generalized travel cost curves for each link in the case study network. It can be observed that the low Degradation Rapidity values reported are partially due to the drastic and abrupt degradation in network performance in the transition from a 90% reduction in link capacity to a full breakdown. This 10% reduction results with a qualitatively different scenario which leads to a disproportional increase in network transmission costs which amounts to 10 times greater consequences than induced by the previous increment. The vast majority of the curves exercise a convex-like shape, again indicating that the degrading rapidness to full link breakdown network performance is relatively low. One noticeable exception is a central link which in the event of a small reduction in capacity becomes obsolete due to superior travel alternatives, resulting with a degradation rapidity value close to one.

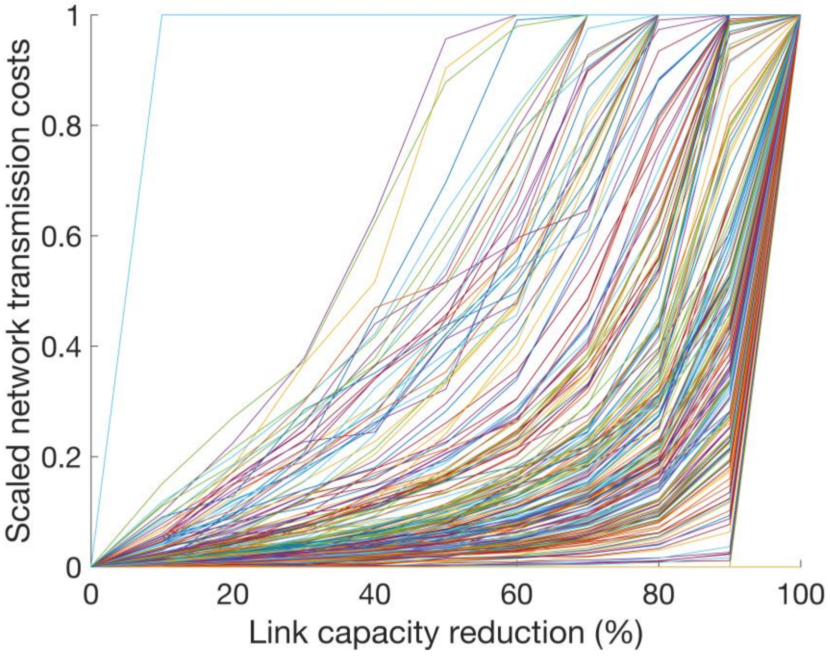

Figure 10 Scaled network degradation curve

The results of the application of the Degrading Rapidity indicator for the Amsterdam public network shows that the indicator is able to express the relative criticality when encountering minor capacity reductions as compared to major capacity reductions on the same link. It clearly shows that it is able to express the mitigating effect of quality and number of route alternatives. The indicator can thus be instrumental in scheduling of planned local capacity reductions and prioritize mitigation measures to reduce the effect of unplanned local capacity reductions. While previous research has investigated the optimal scheduling of maintenance works, especially in the context of railway operations (e.g. Budai et al. 2006), to the best of the authors knowledge, this is the first research to examine potential trade-offs between the duration and extent of planned capacity reductions. For example, the convex relation indicates that it is advisable to plan for long and small rather than short and large capacity reductions.

6. Conclusion

Network robustness depends on its ability to withstand a range of disruptions ranging from small disturbances to the complete failure of some of its elements. The results of this study point to the importance of considering link criticality based on the network-wide flow distribution resulting from local reductions in link transmission capacity. As demonstrated in the case study of the urban rail-bound network of Amsterdam, link critically can be used to identify for which geographical region in the network the consequences of link capacity reduction are most severe, i.e., indicating for which geographical regions the robustness with respect to capacity reduction is lowest.

As a complementary to link criticality, the degradation rapidity indicator is used to quantify the relative sensitivity of minor capacity reductions on a link relative to major capacity reductions, where a higher degradation rapidity indicates that a small capacity reduction can quickly escalate into a full link breakdown. Thus parts of the network with a high degradation rapidity can be considered as less robust with respect to small capacity reductions. Certain disturbances which might be considered minor in terms of the link capacity reduction they entail, induce a disproportionally severe reductions in network performance. One can thus not speak about the 'robustness' of the metro and tram public transport systems of the city of Amsterdam as a whole, but rather that multiple robustness indicators, studying different properties, need to be used to get a more complete understanding of the relatively vulnerable parts of the network. The robustness indicators introduced in this paper contribute to conceptualizing, measuring and mapping a more complete overview of the robustness of networks.

PTNs are subject to many temporary changes due to large-scale events, maintenance and construction works. The results for the Amsterdam case study suggest that the impacts of local capacity reductions on global network performance varies greatly among network links. The distribution of link criticality indicates that while many links have a relatively limited consequences and the average time loss per passenger is 1.36 min, few highly critical links inflict a severe loss of up to 17 min per passenger which implies a significant economic loss. While the high-capacity branches stretching beyond the network core were found to be the most critical links in the network, links situated in the central and dense part of the network degrade most rapidly in response to partial capacity reductions. This analysis can thus support the prioritization of measures aimed at improved network robustness as well as infrastructure and capacity management.

The results of this study can support network planners in strategic, tactical and real-time operation decisions. At the strategic level, planners can identify the most critical links in their network and consider potential investments in increasing the capacity of these links or of links that offer alternative routing possibilities (Cats and Jenelius 2015). Decisions made by network managers at the tactical level such as planning construction and maintenance works can be based on the vulnerability curve. Our findings suggest that from network performance perspective, small capacity reductions that last a long period of time should be preferred over larger reductions over a short period of time. Network managers can also prioritize where to install switches that allow for rerouting by examining their benefits to network robustness. Real-time resource allocation in response to disruptions should prioritize services where the marginal contribution of these measures will be the largest. Case study results indicate that this is not only link-dependent but also depends on the capacity reduction level where for most links network performance degrades more rapidly at higher levels of link capacity reductions. The extent to which the findings of this study depend on network structure and demand distribution remain unknown. By applying the proposed method for various network and demand patterns, future research may examine the extent to which the results can be transferred to other contexts.

The robustness indicators proposed in this study can be applied in a range of contexts. These includes transport networks such as shipping lines and road traffic, as well as power grid, water supply and telecommunication networks. More generally, the indicators can be applied to any labelled graph. Both indicators require information on the values of a certain metric of network performance under a range of capacity reduction in link transmission capabilities. These values can be either attained by means of analysing empirical data or model outputs. The latter requires a network model that describes how flow is distributed over the network in the event of a disruption.

The analysis performed in this study is limited to single-link disruptions. The method as well as the tool used in the application can be directly used to analyse disruptions that imply node capacity reductions. In the context of rail transport this could be caused by platform blockage. Furthermore, the impacts of disruptions on multiple network links can be modelled and analysed using the proposed indicators. This can be especially relevant for connected links that are prone to be disrupted simultaneously, such as in the case of a sequence of rail tracks subject to maintenance works. Another direction for further research is the inclusion of the disruption duration. The relation between disruption duration and its impact may vary among network elements and could be established by adding it as a third dimension to the network degradation curve presented in this paper. Finally, another direction for future work is a structured and detailed analysis on the relation between capacity reduction in public transport networks and reliability and safety aspects thereof.

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