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A review

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Technology Evaluation and Uncertainty-Based Design Optimization: A Review

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Evaluation and assessment of novel technologies for aerospace applications is essential for business strategy and decision making regarding development efforts. Since technology is evaluated in the conceptual design phase and little is known about the technology, large uncertainty is present. This uncertainty needs to be accurately assessed and managed. To investigate the research efforts that have been performed to perform technology evaluation under uncertainty, a literature review was conducted, focusing on methods and modeling approaches to assign and quantify these uncertainties. It is found that probability theory is still the most popular theory for representing uncertainty. Polynomial Chaos Expansions and Stochastic Collocation methods are gaining popularity for propagating uncertainty through a modeling environment, but Monte Carlo Simulations are still widely used. Commonly, surrogate models are used to reduce computational effort. Other efforts focus on the use of multifidelity approaches to reduce computational effort when high-fidelity methods are required. Four issues that may need to be addressed in future research were identified.

I. Introduction

In the design of complex systems, such as aircraft, the systems engineering (SE) approach is usually employed. It evolved during the 1950's to fulfill the need for system performance and project management.[1] In systems engineering, a system is defined as an integrated set of elements to accomplish a defined objective and comprises of elements, subsystems, assemblies, subassemblies, components and parts. The SE process is iterative in nature, moving from a mission definition, to requirement specification to concept definition and analysis to design, production and operation. SE encompasses the entire cycle-of-life (COL). However, early in this lifecycle, i.e. at the conceptual design phase, important decisions need to be made with respect to system configuration and technology selection, which impact the final system performance and risk. Figure 1 shows a typical relation between the freedom in design, commitment made to a certain design (and committed funds) and the knowledge available on the design and its performance.[2] From this relation, it follows that these important decisions during the conceptual and preliminary design phases are based on relatively little knowledge, or in other words, with large uncertainty.

Multidisciplinary design analysis (MDA) and multidisciplinary design optimization (MDO) techniques have been developed to take into account the interactions in the design of complex systems and make sure the optimal design meets key performance indicators set during the requirements definition.[3–5] However, especially when new technologies are involved, uncertainty exists in terms of the effect of these technologies and parameters needed to model them.[6] As such, the assumptions made are carried through the MDO process and are reflected in the resulting design. To reduce the risk associated with these assumptions, uncertainty based design can be employed which is concerned with improving robustness and/or reliability of the resulting design.[7]

Needs and opportunities for uncertainty based design (UBD) are provided by Zang et al.[8] They also summarize the potential benefits of UBD:

- Increase in confidence in design tools

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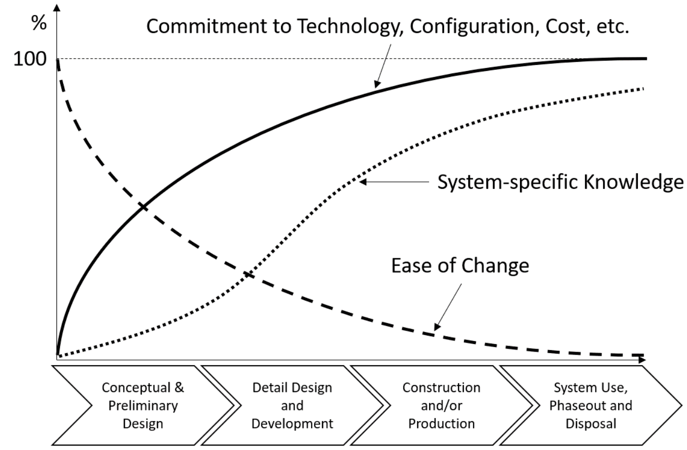

Figure 1. Commitment, freedom and knowledge throughout the system design life cycle. Adapted from Cooper[2]

- Reduction of design cycle time, cost and risk
- Increase in system performance while meeting reliability requirements
- More robust designs
- Assessment of systems at off-nominal conditions
- Increase in use of composite structures (although this is a rather aerospace related benefit, it could be generalized into stating that use of any innovative technology will increase).

Despite these benefits, uncertainty based design is still not widely applied, mainly due to the difficulty of proving these benefits[7] and due to the computational cost associated with UBD methodologies.[8]

Applications of technology evaluation and selection in aircraft conceptual design are ubiquitous.[3,9–11] This paper seeks answer to the question how technology evaluation and assessment is performed in the current state-of-the-art. Some critical remarks are provided that reflect the shortcomings of current state-of-the-art and possible areas of improvement.

II. Technology Evaluation and Selection

As outlined in the introduction, during the conceptual design phase important and irreversible decisions are made regarding a design, including selection of technologies to be developed and implemented. The aim of any technology selection methodology is a structured, repeatable and traceable way to exploit all data and information available on applicable technologies and extracting useful information to reduce subjectivity in the technology selection process.[6] Addressing the benefit of possible novel and immature technologies during the conceptual design phase to understand impact on design and top-level requirements is important.

In general, technology evaluation and selection is paired with uncertainty and as such this should be incorporated in the analysis.[12] Figure 2 shows an overview of the generic process for technology selection under uncertainty. First, the variables that are uncertain need to be defined, including Technology Readiness Level (TRL) and the Technology Impact Matrix (TIM). Then, since not all of these variables will be statistically independent, their dependency structure needs to be established. These dependencies depend partly on the second order effects between different technologies (synergistic or discordant) and the compatibility between any two technologies. Then, the actual probability distributions and the correlation parameters are estimated. All of this work, as will become evident from the following review, is commonly performed by Subject Matter Experts (SMEs). Section III-B elaborates on these points. Finally, the Uncertainty Quantification (UQ) can be performed, resulting in uncertainty distributions over Quantities of Interest (QoI) and selection of the most promising technologies can be conducted, using, for example, TOPSIS (Technique for

Order Preference based on Similarity to Ideal Solution). Throughout this review paper, we will touch upon each of these constituent processes.

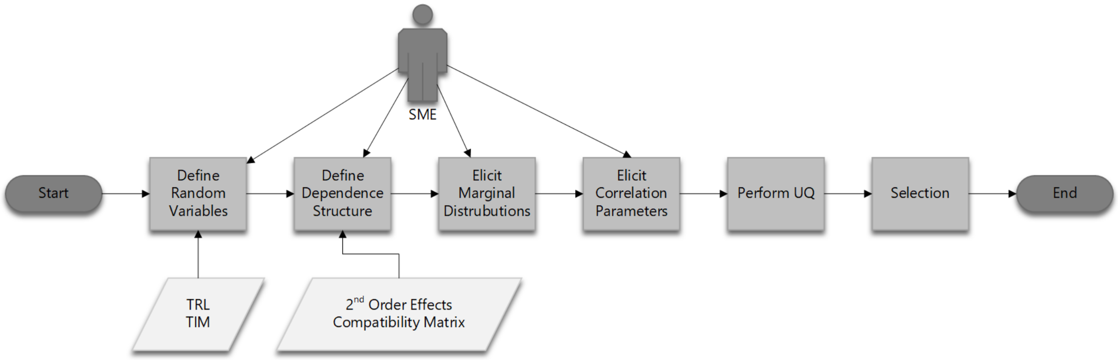

Figure 2. Technology evaluation and selection under uncertainty

Firstly, we look at the definition of the random variables. These include the technology impacts (and possibly design variables) and the technology readiness levels. A common technique to account for the impact of technologies is through assigning a difference to a certain parameter that is present in the aircraft system. For example, the effect of an entire flap system can be represented by a change in $C_{L_{\max}}$ and subsystem mass. The TIES (Technology Identification, Evaluation and Selection)[13–16] methodology, developed at Georgia Tech, works this way. A probabilistic approach towards uncertainty is adopted and technology selection is done based on the highest probability of meeting objectives. The impacts of technologies are called k-factors. The advantage of using k-factors is that key disciplinary metrics are taken into account and no commitment needs to be made to model a technology.[12] It is therefore a straightforward approach, and can quickly and easily identify the impact of certain technologies. However, subtle characteristics specific to a certain technology may not be taken into account and therefore certain effects may be overlooked.

Impact factors can be dependent on the state-of-the-art assumptions for a certain technology. Epistemic uncertainty with regard to subsystem assessment and state-of-the-art (SOTA) assumptions can be dealt with using k-factors. K-factors are applied to sources of epistemic uncertainty and consecutively modified in a sensitivity study.[17–19] In addition to SOTA, business strategy and value of a project (and hence, technologies) should be taken into account when performing technology assessment and selection.[20]

Technology maturity level needs to be addressed to keep overall acquisition cost in check. A methodology is shown in Figure 3 that combines the Technology Readiness Level (TRL), compatibility matrix, Integration Readiness Level (IRL), sensitivity analysis and System Readiness Level (SRL). Uncertainty associated with new technology can usually be derived based on the TRL.[21] To associate TRL with uncertainty, the work of Kirby and Mavris[13] may be used. SRL can be seen as a more sophisticated measure than TRL, but the difficulties in assigning IRL are exposed, as Jimenez et al.[22] pointed out. UQ can be used to determine how to progress the maturity level of technologies.[23]

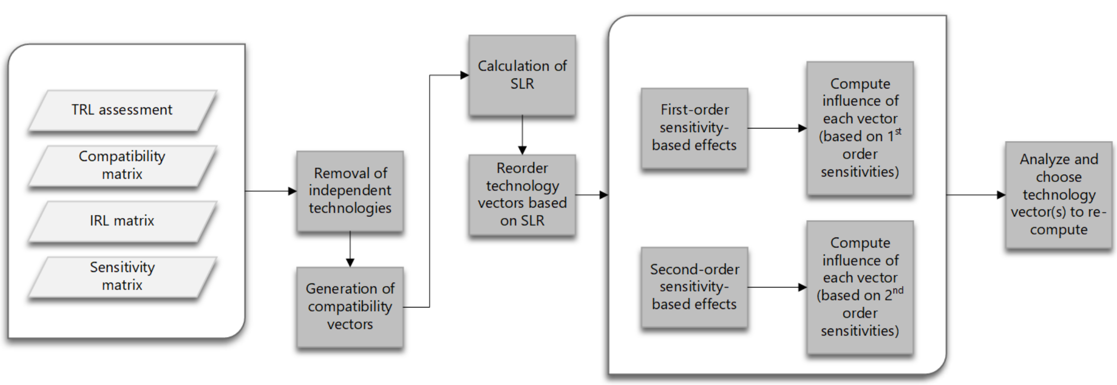

Figure 3. Flow diagram of technology investigation method to assess impact on aircraft measures of effectiveness. Adapted from Amadori[6]

An elaborate characterization of technology readiness levels and technology integration is made by Jimenez et al.,[22] who conclude that technology integration is part of technology readiness and should be accounted for as such. Two models to illustrate technology integration are the Vee model and the four-level hierarchy proposed by NASA (Figure 4). There usually is a gap in between level 1 and 3 in Figure 4, since it is difficult to create a method or tool that relates fundamental physics to technology specific responses. Additionally, for novel technologies, level 1 is also lacking.

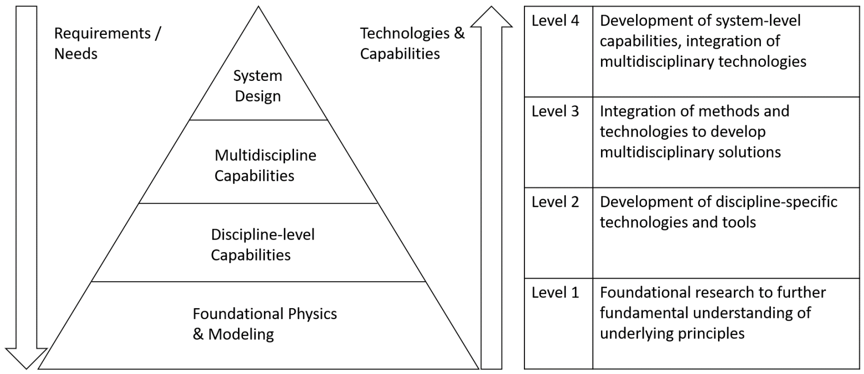

Figure 4. Integration through technology research and development. Adapted from Jimenez[22]

We now skip to the "Perform UQ" block from Figure 2. Technology assessment (and uncertainty quantification) of aerospace vehicles requires an integrated, multidisciplinary platform, with parametric geometry and sensitivity analysis. The multidisciplinary analysis platform should be able to capture those performance aspects that are relevant for decision making. Additionally, the higher the fidelity of analyses used, the more accurate the predictions become and the lower the associated uncertainty is. There is a trend towards using physics-based analyses, which are more expensive than semi-empirical models.[6] An example of such physics-based analyses is the framework developed at Linköping University.[24,25]

Two major limitations of current conceptual design environments were identified by Lu et al.:[26]

1. The application of design tools is limited to a specific vehicle type, because the sizing method is fixed

2. Flexibility and scalability of disciplinary analysis tools is lacking

Both of these limitations should be overcome for effective evaluation of novel concepts and technologies. However, creating a flexible and modular design environment is challenging due to the coupling of operational and systems capabilities.[27] Another important issue for designing unconventional aircraft is geometry representation. As of now, no suitable geometry definition tool or CAD package is available that allows geometry to be reused from conceptual design into detailed design.[28] Some efforts were made to apply knowledge-based engineering to carry over geometry among different design stages.[29] Another tool, GENAIR, was developed to create unconventional geometries using NURBS surfaces.[30] Veley et al.[4] state that an adaptive class structure and a unified part model are necessary.

Despite these challenges, several efforts have been made to arrive at a conceptual aircraft design framework, some including uncertainty quantification.[31,32] A generalized methodology for sizing unconventional aircraft and unconventional propulsion was proposed by Bucsan et al.,[33] building on previous work.[17,34,35] The work by Nam[34] does include uncertainty quantification in the form of PASM (Probabilistic Aircraft Sizing Method) and recommends inclusion of evidence theory or the Bayesian approach into the method. It is remarked that sizing depends greatly on component model accuracy, which would be an area where uncertainty quantification can help. An agile decision support system for aircraft design was proposed that integrates M&S technology, data mining and artificial intelligence.[36] Another probabilistic approach focuses on integration of system-level and component-level requirements in a multidisciplinary optimization.[37–40] It includes probabilistic margins and optimizes components separately, while satisfying consistency on an integrated (system) level, which is supposedly more efficient.

Consideration of the entire life cycle of an aircraft system, including evolution of the design (upgrades, redesigns, etc.), is desirable since performance degradation may be prevented.[41] The Evaluation of Lifelong

Vehicle Evolution (EvoLVE) framework was proposed, which relies on a Stochastic Programming with Recourse (SPR) technique to account for uncertainties associated with future requirements. It is complemented with a Risk-Averse Strategy Selection (RASS) technique to gauge risk. In addition, a study on fleet level showed that more efficient aircraft do not necessarily lead to a global reduction in emissions.[42] This indicates that it may be necessary to not only model aircraft technology, but also include air travel growth and airline operations into the future.[43]

NASA has developed several physics-based analysis tools for different aspects, which were used in an aircraft design study for the Next Generation Air Transportation System.[44] For engine thermodynamic analysis, NPSS (Numerical Propulsion System Simulation) is used, engine component weights are estimated using WATE (Weight Analysis of Turbine Engines), aircraft weights and performance are computed using FLOPS (FLight OPtimization System), aircraft noise levels using ANOPP (Aircraft Noise Prediction Program), wave drag using AWAVE, induced drag with WINGDES, skin friction drag with BDAP and low speed aerodynamics with AERO2S. Hosted Simulation is a technique that can be applied when different simulation tools use different models.[45]

To tackle the issue of flexibility in MDA (and flexible synthesis/representation of systems), an interesting technique is functional decomposition (FD).[46] It is commonly used for design synthesis, since it allows systems to be represented by their intended functions and actual behaviours. Hirtz et al.[47] have constructed a functional basis of generic physical functions, through reconciling previous efforts. Avoiding ambiguity is the aim of such a basis, while maintaining representational flexibility. This basis is used by Yuan[48] to set up an automated functional decomposition framework. Functional decomposition is starting to be used in aircraft conceptual design,[46,49,50] and it is expected to gain popularity. Several different approaches to functional decomposition exist (well documented in multiple review papers[51,52]), including Structure-Behaviour-Function (SBF) decomposition[53] and SAPPhIRE.[54] Principles for specific cognitively oriented AI methodology for functional modeling are presented by Goel et al.,[55] and the evolution of SBF is analyzed. Functional decomposition is closely related to knowledge based engineering (KBE). A comprehensive review of KBE identifies several shortcomings, of which most relevant to technology evaluation are: case-based, ad hoc development of KBE applications; a tendency toward development of black-box applications; and, a lack of knowledge re-use.[56]

Technologies can be analyzed individually or as combinations: technology portfolios. Technology selection then becomes portfolio selection. Technologies are first evaluated individually, followed by a global sensitivity analysis to identify the most influential k-factors. UQ is used to propagate the impacts and uncertainties to system level and consecutively identify technology portfolio performance. When displaying the results, two metrics can be used for comparison: probability of success (POS) and signal-to-noise ratio (S/N). S/N represents both performance and uncertainty and therefore offers a dimensionality reduction.[57] TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is used to create a ranking of portfolios.

It is recognized that most modern UQ techniques only take into account the variance of quantities of interest, while a distinction should be made between positive and negative uncertainty, which should be characterized by an asymmetric distribution. Moreover, a multi-objective methodology should be used for trade-offs.[20] In that case, the objective of a design optimization is a weighted function of multiple objectives and the weighting is fixed throughout the search. Therefore, to obtain a change in solution depending on the weighting, the optimization has to be repeated for different weights. This results in a *Pareto front* of solutions. However, such a method is computationally expensive, which is why a Niched-Pareto approach was developed.[58] This method uses *a posteriori articulation* of preferences, where the Multi-Objective Decision Making (MODM) process is run to obtain Pareto-optimal solutions. A multi-objective genetic algorithm is used by Jimenez et al.[59] to generate a set of Pareto-optimal technology combinations and the results show promising technologies to reach aircraft environmental goals in a decade.

III. Uncertainty Quantification

The process of uncertainty quantification has the goal to propagate the uncertainty distributions on the input variables to quantities of interest. Additionally, it used to identify where sources of uncertainty are largest, such that these can be reduced (e.g. through experiments). It is not necessarily the purpose to estimate the discrepancy with reality, but to ascertain the validity of decisions based on model evaluations, assuming the model has some resemblance of reality.[60]

Aside from computational effort associated with uncertainty quantification methods, there are two major issues:

1. Identification and specification of a dependency structure between random variables
2. Validation of expert judgment for estimation of input distributions

As indicated earlier, the dependencies of random variables are often neglected in uncertainty quantification endeavors, although they are important to take into account to fully reflect the integration effects of technologies on a complete system. A powerful technique to specify a dependency structure between two random variables is the copula.[61] The joint distribution defined by a copula can take on any desired form while its univariate marginal distributions remain uniform.[62] They are used for technology evaluation to reduce the overall uncertainty of the analysis.[62–64] Copula based sampling for a Bayesian Network is employed by Liang[65,66] for stochastic multidisciplinary analysis with high-dimensional coupling. Bayesian Networks[67] are ubiquitously applied for modeling dependency structures of systems and combined with copulas to specify the joint distribution functions. Vines, and vine copulas, can be used to model high-dimensional distributions.[68–70] An application of Bayesian Belief Networks (BBN) to design space exploration leads to a new technique called sculpting.[71] The BBN computes probabilistic measures of merit and includes dependencies between design, uncertain and margin variables. Cobweb plots are used to down-select the most feasible regions of the design space. When uncertain variables are dependent and the joint distribution does not follow a specific type, mixtures are an effective method to represent these joint distributions. Additionally, they are well capable of representing multimodality or tail characteristics, such as leptokurtosis or skewness.[72]

Although several mathematical techniques exist to represent dependencies, the actual structure and correlation parameters commonly are assumed by experts. That leads to the second issue with UQ. As shown in Figure 2, the estimation of distributions of random variables, dependencies and correlations are performed by experts. However, as Cooke[71] points out, experts suffer from overconfidence, inaccuracy and lack of informativeness. In order to mitigate these problems, cross-validation of expert estimates can be performed.[73] Another (not mutually exclusive) technique is to update elicited expert judgments using Bayesian calibration.[74,75]

A. Uncertainty Definition and Taxonomies

Regardless of how the dependencies and probability distributions are assigned, uncertainty quantification can be performed in a multitude of techniques. However, first, a definition of uncertainty is required. Secondly, uncertainty can be mathematically defined with different theories. Finally, several propagation techniques exist that compute the uncertainty on quantities of interest.

Uncertainty can be defined as the incompleteness in knowledge and the inherent variability of the system and its environment.[7] The most common distinction is the division into two classes:

1. **Aleatory** (or statistical) **uncertainty**, which can be seen as the inherent variation in variables. Other ways to define aleatory uncertainty is as type A or stochastic uncertainty. It is an inevitable, irreducible and uncontrollable form of uncertainty, but well identifiable.
2. **Epistemic uncertainty**, i.e. uncertainty due to lack of knowledge. It is also known as cognitive, type B, reducible or subjective uncertainty.

As is done in the works by Oberkampf *et al.*,[76–78] it is convenient to use the following three characterizations: aleatory and epistemic uncertainty and (numerical) error, where the latter is not actually a form of uncertainty, but does affect the accuracy of the solution. Error is seen as purely numerical noise due to floating point arithmetic.

An important form of epistemic uncertainty is model (form) uncertainty and model discrepancy. Model discrepancy is the uncertainty in the prediction from a certain model. Model form uncertainty is the uncertainty related to selection of the appropriate model for a certain phenomenon.[79] It is related to model discrepancy in the sense that the model with least discrepancy to the real world is most likely the most appropriate model. Obviously, complete certainty regarding the real world is not possible, even through experiments, since there always exists measurement uncertainty and experiments often ignore certain parameters.

Current V& V and UQ applications require a lot of accurate information, which limits their use in practice. A framework for early model validation with limited information is presented by Carlsson.[80,81] By reducing the amount of uncertain parameters the uncertainty quantification issue is solved. This reduction comes from only modeling at component output level, rather than at component input.

Model discrepancy can be modeled as:

$$y(x,t) = f(x,t,\phi) + \epsilon(x,t,\phi) + \epsilon_y \tag{1}$$

where the response y is modeled by the model f depending on the input vector x, time t and to-be-calibrated variables ϕ (more on this later in the discussion of Bayesian model calibration). The model discrepancy is modeled using the function ϵ and ϵ_y is the measurement error in the response data. Many other researchers lump the last two parts together:[79,82–84]

$$y(x) = f(x) + \delta(x) \tag{2}$$

Model discrepancy quantification is concerned with estimating either $\epsilon(x,t,\phi)$ or $\delta(x)$.

Bayesian calibration is a method that is often used to quantify model form uncertainty[85] and the foundations were laid by Kennedy and O'Hagan.[86] A model can be represnted as a Bayesian Network (or Dynamic Bayesian Network for time-dependent models). Such a network connects responses through probabilistic paths. The Bayesian Network can be calibrated using experimental data using Bayes' rule. DeCarlo et al.[87–89] use it for increasing confidence of a coupled aerothermoelastic analysis.

In a coupled system, which often occurs in aircraft design (e.g. aero-structural coupling) model discrepancy propagates in a loop, and therefore, rigorous quantification of this uncertainty is essential. Especially for multifidelity approaches this is important, since different levels of fidelity may be used simultaneously. The coupled system can be modeled as a *Markov Chain* (essentially, a Dynamic Bayesian Network can be represented as a Markov Chain), in which uncertainty is propagated using *Gibbs sampling* (which is a Markov Chain Monte Carlo (MCMC) method).[83] Important is that consistency is reached between coupling variables, i.e. a detailed balance condition is applied to ensure that stopping at any point in the Markov Chain results in a consistent answer. An optimization routine finds the statistics of the sampling results. Normal distributions were assumed for the model discrepancy, but later work modeled these as a Gaussian process with a squared exponential covariance function.[84] A pattern search optimization ensures the marginal and conditional distributions of each variable are approximately equal, to satisfy detailed balance.

Extension of Bayesian calibration to high-dimensional, spatially varying model parameters is researched by Nath et al.[90] An approach is presented that uses singular value decomposition (SVD) to reduce dimensionality for surrogate models build using the Kriging approach. Bayesian calibration using Gaussian processes is shown to successfully estimate parameter uncertainty, surrogate modeling uncertainty and characterized and uncharacterized observation and modeling errors.[91] Further work is recommended to extend surrogate modeling capabilities and the transfer of posterior uncertainty to new predictions.

Bayesian model averaging is a methodology that handles both model discrepancy (or model form uncertainty) and parametric uncertainty, by integrating model predictions weighted by model probabilities.[79,82,92–94] The adjustment factor approach builds upon the model averaging technique and works by adjusting the prediction of the best model by a factor that is computed using the predictions of all models. The adjustment factor can either be additive or multiplicative. Riley et al.[92] point out that the adjustment factor approach is dependent on expert opinion rather than experiments. Additionally, they state Bayesian model averaging requires many experiments, which often are unavailable and hence propose a framework that determines where additional experiments are required.

A novel approach to model form uncertainty is to apply game theory and have processes play adversarial games with respect to missing information.[95] As such, partial information is taken into account and reduced order models can be constructed.

B. Uncertainty Modeling

Probability theory is a commonly used technique for representation of uncertainty, since it is relatively easy to implement and is well understood by engineers. A probability density function (PDF) is assigned to uncertain variables, which assigns a probability to each value the variable can attain. With sufficient data available, a PDF can easily be fitted. The PDF model can be chosen depending on uncertainty characteristics and its parameters can be estimated using the method of moments or maximum likelihood method. However, usually during conceptual design little information is available and the probability model has to be assumed by engineers (commonly uniform[96]), which adds to the uncertainty of the design results.[97]

Aleatory uncertainty is often modeled using probability theory, but representing epistemic uncertainty using probability is questionable since there is no reason to prefer one probability distribution over another.[98] All other theories discussed here are capable of representing both aleatory and epistemic uncertainty.

Bayesian theory (BT) is an extension of probability theory that includes evidence to support some hypothesis. Essentially, the theory revolves around Bayes' rule which is stated as follows:

$$P(b|a) = \frac{P(a|b)P(b)}{P(a)} \tag{3}$$

where $P(\cdot)$ is the probability of the contained statement, a and b are some variables and the operator $|$ should be interpreted as *"given"* ($P(\cdot|\cdot)$ is called a conditional probability). The $P(b)$ is called the prior, i.e. an estimation of the probability of b without evidence, while $P(b|a)$ is called the posterior, i.e. the updated probability of b now that some evidence has become available.

A more general form of Bayes' rule is:

$$\boldsymbol{P}(Y|X, \boldsymbol{e}) = \frac{\boldsymbol{P}(X|Y, \boldsymbol{e})\boldsymbol{P}(Y|\boldsymbol{e})}{\boldsymbol{P}(X|\boldsymbol{e})} \tag{4}$$

where $\boldsymbol{e}$ is some background evidence. Using this formulation it is clear that Bayesian theory allows for revision of the probability of some condition when new evidence becomes available.

Although the Bayesian (subjectivist) view of probability has its merits over the classical (frequentist) probability theory, one of its largest drawbacks is through the "Principle of Insufficient Reason".[99] Basically, the Bayesian approach relies on a complete probabilistic model of the domain, or in other words, a frame of discernment. This frame of discernment sometimes has to be chosen arbitrarily, while it has a major impact on the resulting probabilities. Therefore, the Bayesian approach needs to distinguish between uncertainty and ignorance.[99] A similar argument is given by Soundappan et al.:[100] if the evidence is imprecise, assumptions need be made to estimate the likelihood of the evidence. The posterior probability can be sensitive to these assumptions.

Possibility theory can be seen as an extension of classical probability theory with two measures: necessity and possibility. It was developed by Zadeh[101] and builds upon his theory of fuzzy sets. When using probability (not possibility!) theory, one has to assume a certain distribution, which is incorrect if not enough data is available to make a good decision on the type of distribution. Therefore, taking upper and lower bounds on the probability of uncertain variables is useful, which is what interval analysis and fuzzy set theory aim to do.[102]

A review of possibilistic approaches to reliability analysis and optimization in engineering design is presented by He et al.[103] Most research effort on possibility theory is focused on the integration of reliability-based and possibility-based design optimization and the improvement of efficient numerical techniques for computing the metrics.

Interval analysis only takes an upper and a lower bound on the uncertain variables. Hence, no uncertainty structure is imposed on the uncertain variables. These minima and maxima are propagated to the output, usually through algebraic procedures.[104,105] Interval analysis can be seen as a special case of possibility theory for crisp sets.[76] However, two major drawbacks are associated with this theory: the computational cost is prohibitive for large number of inputs and the output range is only valid for monotonic functions. In other words, if the output features local extrema, the interval analysis is usually wrong.[102]

Dempster-Schafer (D-S) theory (also called theory of belief functions or evidence theory) is a more general uncertainty theory than either classical probability theory or possibility theory.[76] It was developed by Shafer,[106] who built on the work by Dempster.[107] The theory applies to both aleatory and epistemic uncertainty. It concerns two complementary measures: belief and plausibility, which are both fuzzy measures. D-S theory can be used when there is conflicting evidence, otherwise possibility theory is advised.[102] Additionally, it can be used when only a small amount of information is available. Oberkampf et al.[76] argue it could be an effective path forward in engineering applications due to its ability to deal with well characterized uncertainty as well as situations of near-total ignorance. An even more general method than D-S theory is Dezer-Smarandache (DSm) theory, which uses new models for the FD and new rules of combination to take into account paradoxical and uncertain information.[108]

Akram and Mavris use D-S theory for a technology valuation process / technology portfolio analysis,[109–113] and conclude it is better capable of providing insight into epistemic uncertainties, as compared to deterministic or probability theory methods. Additionally they state: "It reduces the number of assumptions during the elicitation process, when experts are forced to assign probability distributions to their opinions without sufficient knowledge." In a comparison with Monte Carlo approaches it results that both methods scale similarly, but D-S theory is an order of magnitude quicker.[110] Furthermore, D-S theory can be used for the quantification of model form uncertainty and model discrepancy.[93,114]

Comparing the theories presented, it can be concluded that probability theory provides the least conservative results.[106,115] D-S theory is less conservative, and will converge to the probability theory result with enough information.[104] Probability theory might provide a false sense of exactness,[104] while D-S theory and possibility theory may lead to wide bounds, on which no decisions can be based.[100] Using D-S theory, the analyst does not need to make assumptions. However, it is recommended to use both BT and D-S theory for decision-making, since D-S theory leads to worse decisions in the long run.[100] Despite the promising conclusions drawn by practitioners of D-S theory, there are compelling arguments against its use. These revolve around the lack of operational meaning of the metrics in theories other than (subjective) probability theory.[116,117]

A literature review on uncertainty quantification metrics for whole product life cycle cost in aerospace innovation is presented by Schwabe et al.,[118] who indicate that the probability density function is still the most used metric. Additionally, quantification of uncertainty is still largely subjective (i.e. expert judgment of uncertainty) and no commonly accepted cost estimation methodologies exist for research and development projects. Key future concepts are: entropy, complex adaptive systems, uncertainty treshold responses and deep uncertainty, rather than the current approaches based on the Central Limit Theorem (or law of large numbers). A framework is developed based on this premise, i.e. one that does not rely on the Central Limit Theorem, but instead uses spatial geometry and the notion of symmetry to estimate cost variance over time.[119]

C. Uncertainty Propagation

Different methods are available to propagate uncertainties of input and (modeling) parameters to the output quantities. The characterizations and management of uncertainties are required at both the discipline level and integrated system level, such that also the relationship between uncertainties affecting input and those affecting output are involved.[8] Computational efficiency is the main challenge that is to be tackled by these methods. Most methods suffer from the so-called *curse of dimensionality*, i.e. with an increase in uncertain variables, the algorithmic expense grows exponentially. Additionally, the propagation method can introduce an error in the estimated uncertainty, for example by using too few samples, or simply because it assumes a certain function for the uncertainty. An entire framework for uncertainty quantification, encompassing all forms of uncertainty and error, has been introduced.[120] An example of industrial interest in uncertainty quantification for the evaluation of complex physical systems is the DARPA EQUiPS (Enabling Quantification of Uncertainty in Physical Systems) program,[121] which aims to develop a rigorous framework for the propagation and management of uncertainty in modeling and design of complex engineering systems.

Monte Carlo Simulations (MCS) remain a popular method to propagate uncertainty through a system, especially when a black-box system is used. The main reasons are the ease of implementation and the insensitivity to the dimensionality of the problem. The main disadvantage of MCS is the large sample size required to provide an accurate estimate, due to the Central Limit Theorem. To improve the accuracy of the estimation, i.e. reducing its variance, many variance reduction techniques were invented: antithetic variates, control variates, importance sampling, conditional Monte Carlo sampling and stratified sampling, for example.[60] A review of improved Monte Carlo methods in UMDO for aerospace vehicles is provided by Hu et al.[122]

Polynomial Chaos (PC) is a technique for probabilistic analysis of uncertainties by means of solving stochastic differential equations. Its cost is several orders of magnitude greater than for the deterministic solution of a partial differential equation. However, it is still much cheaper than sampling methods such as Monte Carlo analysis.[8] Uncertain variables are transformed to independent standard random variables for which, ideally, known orthogonal polynomials exist with respect to the probability distribution of these variables.[123] Then, the coefficients of these polynomials are to be found, using one of two methods: spectral projection (Galerkin projection) or linear regression (point collocation or stochastic response method). The integrals in Galerkin projection are usually computed using quadrature methods, which suffer from the curse of dimensionality.[124]

Methods for computation of higher-order statistical moments of the response distributions obtained using either intrusive or non-intrusive polynomial chaos expansions are detailed by Savin et al.[125] These methods are useful when more properties, other than mean and variance, of the response are required. Polynomial chaos expansions are frequently used in recent research on uncertainty quantification in aerospace applications. This shows the potential it has to replace the common Monte Carlo method, as is supported by Shimoyama and Inoue.[126]

Stochastic collocation (SC)[127–129] is similar to PC, except that the interpolation functions to a known set of coefficients are to be found. These interpolation functions are usually Lagrange polynomials and the coefficients are the response values at the collocation points. Stochastic collocation has very similar performance to PC, but is shown to be slightly more efficient.[123] Another advantage is it only is dependent on the collocation points, and not on the synchronized definition of expansion formulations and coefficient estimation.[123] Both polynomial chaos and stochastic collocation result in a functional description of response uncertainty, such that statistical features can easily be deduced and consecutively used for design under uncertainty.[130]

Beside the three aforementioned approaches, many other techniques for uncertainty propagation exist, such as First-Order Reliability Model (FORM) or Second-Order Reliability Model (SORM),[131] Fast Probability Integration (FPI),[12,13] among others. However, these approaches do not appear often in literature and therefore it may be concluded their applicability and benefits are limited. Another interesting perspective is that of analytical propagation of uncertainty.[132,133] Such an approach avoids sampling error and provides a functional form of response uncertainty, rather than some fixed number. Propagation techniques for the D-S theory metrics (belief and plausibility) are different from those used by probability theory. Nonetheless, several methods have been proposed.[98,106,115,128]

IV. Sensitivity Analysis

Sensitivity analysis provides design insight on a local level, or method insight on a global level. In a deterministic setting, local sensitivity analysis is usually performed to find the impact of a change in input variables on the output variables.[133] This results in partial derivatives of the output. Several techniques can be used to compute these derivatives, which are discussed below. Gradient-based optimizers make use of these gradients to guide a design solution to a local optimum.

In the context of design under uncertainty, input variables have a distribution of possible values, and global sensitivity analysis (GSA) is performed to investigate the effect on output variables with respect to the entire range of input values.[133] GSA can either be used before design to *screen out* those variables that have little influence (i.e. neglect uncertainty in these variables, to reduce dimensionality in the context of uncertainty quantification) and investigate the interaction between design and noise variables. Otherwise, it can be applied after design has finished to determine where efforts should be made to reduce uncertainty.

A. Local Sensitivity Analysis

Different techniques exist for computing gradients, i.e. derivatives, of functions (or equivalently, systems). Symbolic differentiation (which is very similar to differential calculus) produces an exact derivative. However, for even moderately complex functions, the symbolic derivative can amount to several pages of expressions. Nonetheless it has successfully been applied in an aerodynamic shape optimization algorithm.[134] Numerical differentiation, i.e. finite differences or divided differences, is an approximate differentiation technique that is well known and can easily be applied to any function, no matter how complex. However, it only produces an estimate of the derivative and for functions with many inputs and outputs, the computational cost of this technique is considerable, if not prohibitive.

Lastly, automatic differentiation was introduced as a method that has neither of these problems: it produces an exact result at only a small computational cost. Automatic differentiation is in principle the application of the chain rule to computer programs.[135] The technique can either be applied in a forward-mode or reverse-mode fashion (or a hybrid combination thereof). The reverse-mode is closely related to adjoint differential equations.[135]

Several papers discuss the principles of automatic differentiation.[136–139] A robust optimization of an aircraft concept using automatic differentiation is presented by Su et al.[140] These studies were all performed in the last century and recent research makes no use of automatic differentiation, which may lead to the conclusion that its drawbacks are more significant than its merits. A likely explanation is the issues involved with applying automatic differentiation to existing software, i.e. black-box systems.

B. Global Sensitivity Analysis

Global Sensitivity Analysis (GSA) can be performed using various methods, but in general, a trade-off is made: accuracy of the solution versus computational efficiency. In order to take the entire probability distribution into account, which provides an accurate description of results, GSA becomes computationally inefficient. In fact, for a large number of variables it becomes an intractable problem. On the other hand, simplifying the analysis by only considering certain statistical parameters such as variance, GSA can be performed, at a loss of accuracy and a high computational burden. Another important observation is that most GSA techniques assume independence of input variables, which, in many cases, is warranted. If not, different approaches should be used, such as copulas.[133]

ANOVA (analysis of variance) is a commonly used method for GSA. It decomposes a function into its contributing components, for which the effects and variances can be determined.[133] From the variance, so called sensitivity indexes can be computed: the *main sensitivity index* (MSI), which describes the effect of one variable, and the *total sensitivity index* (TSI), which is the effect of a variable and all its interaction effects combined. ANOVA is used in a study by Bae et al.,[141] where it is used to investigate the effect of component confidence intervals on the system confidence interval of Bayesian networks.

The advantage of variance-based sensitivity analysis is that it is easy to implement and interpret, but it can not sufficiently describe uncertainty of systems with highly skewed or multimodal responses.[85] An alternative to ANOVA, but still a variance-based method, is FAST (Fourier amplitude sensitivity test).[142] Alternatively, Sobol' indices[143] are anoter variance-based GSA technique. GSA based on Sobol' indices is used by DeCarlo et al.[144] on an aerothermal problem, where an importance sampling-based kernel method is developed to estimate the indices. It allows for time-dependent multidisciplinary analysis of the sensitivity. A study by Chen et al.[133] develops an analytical variance based GSA method, by observing that many surrogate models are in the form of multivariate tensor-product basis functions, for which analytical solutions exist of the integrals needed to compute the sensitivity indices. It is shown that the method performs faster than Monte Carlo simulation and avoids sampling error. The Multidisciplinary Statistical Sensitivity Analysis (MSSA)[85] method is a relative-entropy-based SA technique proposed by Jiang et al.,[85] which captures the entire distribution of a QoI and is therefore especially suitable for reliability-based design optimization. Another method, based on Kullback-Leibler entropy, leads to the same conclusion.[145] High-dimensional model representation (HDMR) theory combined with ANOVA was introduced by Opgenoord and Willcox[146] to efficiently compute sensitivities of computationally costly models. The sensitivities are used to update risk and uncertainty budgets, based on which a design can be evaluated. Polynomial Dimensional Decomposition (PDD) can also be employed for GSA and UQ of stochastic systems and a sparse representation can be obtained that results in fewer model calls.[147]

C. Reduced Order Modeling

An alternative to sensitivity analysis to reduce a problem's dimensionality is the use of active subspaces. Active subspaces are defined on the coordinates of the design space where variability is largest.[148] It is shown that for problems where only few variables contribute to variability, the active subspace method is more efficient than local sensitivity techniques. For developing design insights, the active subspace method also proves useful.[149] It can be used to identify active dimensionless parameters, and, even more interestingly, to detect important missing variables. In a study by Del Rosario et al.,[149] dependence of lift on the camber of an airfoil was discovered algorithmically using active subspaces. Similar to active subspaces, but more general, is principal component analysis (PCA), also known as proper orthogonal decmoposition (POD).[150] It can also be used for data reconstruction.

V. Uncertainty-based Multidisciplinary Design and Optimization

When evaluating a technology or technology portfolio, either a cycled or an uncycled design analysis can be performed. The latter would mean that a technology is simply added or replacing an existing system and the effect on measures of merit is evaluated. The former case would require an update of the design, including the technology, and hence require an optimization strategy to minimize some objective function. This section deals with design optimization, and particularly design optimization under uncertainty.

Traditionally, aircraft design incorporates uncertainty through safety factors, which are set arbitrarily. These factors, or margins, lead to oversized aircraft that are very likely to meet requirements. However, this approach is inefficient, hence, a probabilistic method to set these margins was developed.[151] The strategy would not provide the same confidence in the probability of success as the full probabilistic analysis, but would provide much more confidence than a conventional deterministic process.

UMDO is a relatively new trend of MDO and tries to enhance systems design by exploiting synergistic effects and properly accounting for uncertainty, reducing conservative solutions.[7] Traditional design optimization does not take into account uncertainty and is therefore also referred to as *deterministic design optimization*.

Nondeterministic (or uncertainty-based) design can be divided into two objectives, namely design for robustness and design for reliability.[8] A robust system is one that has low sensitivity to variations in the system itself and its environment. A reliable system is one that has a high likelihood of performing its function without failure under stated (severe) operating conditions. In relation to probability, robustness and reliability can be seen as shown in Figure 5, where robustness is associated with the mean of the PDF and reliability with the tail. Evidently, it is more difficult to characterize the tail of the distribution. Nonetheless, methods for robust design are less well-developed than reliability-based design procedures.[8]

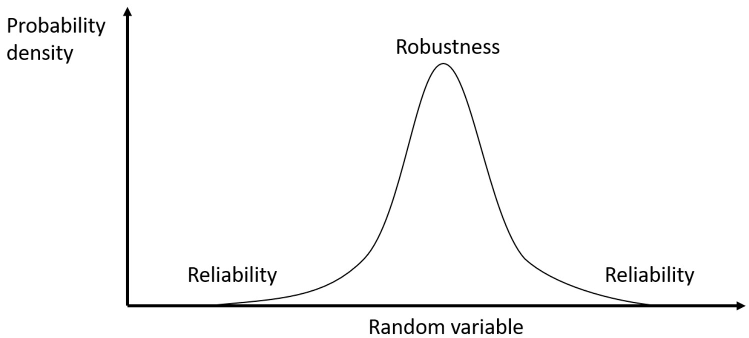

Figure 5. Reliability versus robustness in terms of the probability density function. Adapted from Zang[8]

Robust design optimization is a methodology where a design is optimized for insensitivity to various variations. Robust design to uncertainty is more likely to meet performance requirements as incorporated technologies mature over the aircraft design process, avoiding expensive redesigns.[152] Different applications to aircraft design problems have been demonstrated.[21,153] However, only very conceptual design is considered there, with around 10 design variables. An extensive discussion of the strengths and weaknesses of different robust optimization techniques is presented by Beyer and Sendhoff[154] and support the statement that most design problems in literature are low-dimensional. They indicate that evolutionary algorithms are suitable direct optimization techniques that work well on noisy problems (such as highly-coupled aircraft design). However, these techniques still should be matured, i.e. knowledge on the expected performance indices must be extended. Additionally, the relation between performance (of a system) and robustness should be more closely examined. An approach based on Bayesian calibration, including both aleatory and epistemic uncertainty has also been proposed for robustness-based design optimization.[65,155–157]

Reliability-based design optimization (RBDO) optimizes a design to have a small chance of failure. Several methods are proposed that aim to improve the computational efficiency of RBDO, which classically is performed using a bi-level strategy.[158,159] Additionally, dealing with epistemic (particularly model discrepancy) appears challenging in reliability analysis,[160] but solutions for this problem have also been proposed.[159,161]

A combination of robust and reliability-based design optimization is also possible, where the objective function of robust design optimization and the constraint function of reliability-based design optimization are combined into a single formulation. An example of a method using both reliability and robustness for conceptual aircraft MDO is presented by Jaeger et al.[162] They consider both model and design variable uncertainty, but model the former using normal distributions.

Regardless of the type of optimization under uncertainty, it is also possible to optimize to match a desired response distribution as closely as possible, which is what a density-matching approach was developed for.[163] As such, skewness in the response may be obtained, rather than only the first two statistical moments that conventional RBDO obtains. An alternative is horsetail matching, which has been shown to be superior to density matching in terms of computational efficiency while producing satisfactory designs.[164] Horsetail matching allows to optimize for different targets. However, when many uncertainties are involved, the employed surrogate model effectiveness deteriorates and it has to be investigated how to deal with higher dimensionality. For optimization with expensive objective functions, Bayesian optimization has become a popular approach.[165]

Decoupling of a multidisciplinary environment allows for parallel execution of analyses, which leads to shorter computation times.[166–168] Different coupling approaches exist, such as Concurrent Subspace Optimization (CSSO)[169] and Analytical Target Cascading (ATC), Colloborative Optimization (CO) and Bilevel System Integrated System Synthesis (BLISS). Some works address the issues that consequently arise regarding uncertainty quantification.[167,168]

Different models can be used for analysis and modeling of the system under investigation: low-fidelity models offer low accuracy estimates of quantities of interest at low computational cost, while high-fidelity models offer required accuracy at high computational cost. Outer loop applications, such as optimization, inference and uncertainty quantification require many model evaluations and the cost of high-fidelity models consequently becomes prohibitive.[60] Multifidelity methods can be employed to reduce computational time while accuracy is maintained, as several applications show.[170–173] The multifidelity approach can be used both for the actual objective function (M& S environment) or the uncertainty quantification method.[130]

Two important properties of multi-fidelity methods are to leverage low-fidelity models to speedup computations, and to have recourse to high-fidelity models to maintain accuracy and/or convergence. The two ingredients necessary to accomplish this are: low-fidelity models that usefully approximate the response of the high-fidelity models, and a model management strategy that guarantees accuracy and convergence through distributing work among models in an efficient manner. An important observation is that models can be seen as information sources and as such can be used in conjunction with e.g. expert opinions, experimental data and historical data.[60]

All these topics come together in the form of multifidelity optimization under uncertainty, where the objective function usually includes one or more statistics that depend on the underlying uncertainty. Therefore, each optimization iteration embeds an uncertainty quantification loop. Different researchers have used different approaches to this end, but, as Peherstorfer et al.[60] note: "optimization under uncertainty is an important target area for future multi-fidelity methods. It is a computationally demanding process, but one with critical importance to many areas, including engineering design.". They conclude with the important observation that multi-fidelity methods rely on the *assumption* that the high-fidelity models resemble some truth and in principle all models are inadequate. The way forward for multi-fidelity methods is to move beyond models solely and include other information sources as well. This in turn requires new model management strategies that have to decide *when* and *where* to evaluate the information sources.

Many techniques exist for multi-fidelity optimization, even targeted to aircraft design. These include an approach relying on an information reuse estimator,[152,174,175] statistical surrogate modeling for non-hierarchical information sources,[176] a Multilevel Monte Carlo (MLMC) method,[177] a regular Monte Carlo method[27] and gradient-optional multi-fidelity methods.[178]

Coupled multidisciplinary systems are usually solved through fixed point iteration (FPI), in order to arrive at a consistent solution for all disciplines. However, a new method is proposed that leverages adaptive surrogates for uncertainty analysis in black-box systems.[179] Other methods have solved the feedback-coupling problem by decoupling the system. However, when sensitivity to coupling variables is high, such approaches might produce poor results.

Surrogate models of the modeling and simulation environment are often employed to reduce the computational time required for uncertainty quantification and optimization under uncertainty. The problems and challenges still present in the creation and use of metamodels are extensively reviewed by Viana et al.[180] The main challenges are the curse of dimensionality, computational complexity, numerical noise, mixed discrete and continuous variables and validation of metamodels and underlying models. Fuzzy clustering analysis can be used to decrease the amount of sample points for construction of the surrogates.[181] In order to enhance multi-fidelity optimization and uncertainty quantification, a multi-fidelity locally optimized surrogate that is more efficient than global single-fidelity ones is proposed.[182] To represent both epistemic and aleatory uncertainty a surrogate modeling approach based on non-deterministic Kriging is proposed.[183] Another surrogate modeling approach was developed for propagating uncertainty and for global sensitivity analysis, balancing between computational time and uncertainty in QoI estimation.[184]

VI. Conclusions

A literature review was conducted on technology evaluation and selection, including uncertainty quantification and design under uncertainty for aerospace applications specifically. While many works have focused on these topics, it remains a challenging task to accurately predict performance of novel technologies or aircraft designs in the conceptual design phase. Moreover, effective quantification of uncertainty remains a challenge. The workload of uncertainty quantification is estimated twice the development effort for only model development and verification and validation. To accurately predict the impact of novel technologies, a generalized sizing and assessment method is deemed appropriate. Unfortunately such a method does not exist, yet. This is partly due to the challenge of parameterizing geometry and using generic analysis methods. Another issue is the modeling at technology level. Usually, this step is avoided and replaced by introducing factors that account for a technology's impact on system parameters. Such approaches require expert knowledge as input, which is is not necessarily a problem. However, subjectivity, conservatism or overconfidence may result, not even mentioning the common lack of experts. Regarding uncertainty quantification, dependencies should be taken into account and assumptions regarding probability distributions should be reduced. Both these problems still need to be tackled. Model form uncertainty remains difficult to quantify and most efforts focus on correction strategies using Bayesian calibration or multi-fidelity approaches. Both require high fidelity data, which may not be available or requires significant computational effort. High dimensionality of practical engineering problems is an often mentioned issue. Techniques to reduce a problem's dimensionality are available, but are not effective enough to tackle this issue entirely.

Summarizing, we believe technology evaluation including uncertainty has come a long way in the past couple of decades. It is effectively applied in practice, but several practical issues remain. Therefore, additional research efforts that focus on the issues identified here are deemed necessary.

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