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Publication date
2010

Document Version
Final published version

Published in
International Journal of Exergy

Citation (APA)
Jansen, S., & Woudstra, N. (2010). Understanding the exergy of cold: theory and practical examples. *International Journal of Exergy*, *7*(6), 693.

Understanding the exergy of cold: theory and practical examples

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Abstract: Exergy analysis is used to evaluate the thermodynamic performance of processes, including energy conversion and supply systems. This often involves the calculation of the exergy of heat, at a temperature either above or below the environmental temperature (T_0). The exergy of 'cold', i.e., heat at $T < T_0$, is less used and therefore sometimes also less understood. This paper broadens the understanding of the exergy of cold by discussing the theory and giving two useful examples illustrating the added value of exergy when considering cold: The regasification of Liquefied Natural Gas (LNG) and the exergy demand of cooling in buildings.

Keywords: exergy of cold; cool exergy; exergy analysis; cool exergy examples; exergy factor; cool exergy potential; exergy demand of cooling processes.

Reference to this paper should be made as follows: Jansen, S. and Woudstra, N. (2010) 'Understanding the exergy of cold: theory and practical examples', *Int. J. Exergy*, Vol. 7, No. 6, pp.693–713.

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1 Introduction

The application of exergy analyses for the assessment and improvement of energy conversion systems was first introduced in the 1950s. It is used for the analysis of energy conversion and industrial processes, and in the last decades the concept has also been introduced for analysis of energy supply systems in the built environment, although it is still mainly used in academic circles. Exergy analyses of many energy conversion systems as well as energy supply systems for heating and cooling purposes involve the calculation of the exergy of heat. Heat refers to the transfer of energy resulting from a temperature difference between two systems; it can take place at temperatures above or below the environmental temperature (T_0). In this paper, the word cold is used for heat at $T < T_0$.

The exergy of cold is less straightforward to understand than the exergy of heat. Therefore, in addition to the existing literature on the exergy of cold, of which many important publications are discussed in Section 3, this paper tries to broaden the understanding of the exergy of cold and its application. It will try to do so by:

- Explaining the theory: On the basis of the definition of exergy, the relevant equations for the exergy of cold are derived and their meaning is explained by comprehensible figures and schemes
- Discussing some basic issues to clarify the theory:
  - Why the exergy of cold at T_1 in an environment at T_2 is greater than the exergy of heat at T_2 in an environment at T_1 ($T_1 < T_2$)
  - Why the amount of exergy can be greater than the amount of cold available, even though the Carnot efficiency is never greater than unity
  - How the direction of the heat transfer relates to the 'direction' of the exergy.
- Discussing two examples that illustrate the importance and relevance of the exergy of cold:
  - A simplified analysis of the regasification of LNG
  - A detailed analysis of the exergy of cooling in buildings, illustrating two different cooling situations.

2 Definitions

In general, *exergy* can be defined as the maximum theoretical work that can be obtained from a quantity of energy or matter by bringing this energy or matter into equilibrium with a reference environment. The maximum theoretical work will be obtained if the considered energy or matter is converted in a system in which only reversible processes take place, in such a way that finally equilibrium with the environment is achieved.

Heat is the transfer of energy between two systems as a result of a difference in temperature. This energy is not related to matter. When analysing one system, heat is the transfer of energy across the system boundary, taking place at the temperature of the system boundary. In this paper, the word *cold* is used to refer to heat at temperatures below the environmental temperature T_0. In spontaneous processes, the transfer of heat will take place from the warmer to the cooler system, due to the driving force of the difference in temperature. If there is this driving force, it can be said that heat or cold is available. If the heat transfer is required to take place in the opposite direction, it cannot take place spontaneously, which means heat or cold has to be produced.

The *exergy of heat* is the theoretical maximum work that can be obtained by bringing the heat in thermal equilibrium with the environment using a reversible process. The general equation for the exergy of heat is given in equation (1). Its origin and meaning will be explained in Section 4 of this paper.

$$Ex_Q = Q \cdot \left(1 - \frac{T_0}{T}\right). \tag{1}$$

The ratio between the exergy (Ex) and energy of the heat transferred (Q) is defined as the exergy factor f_{ex}. In literature, the exergy factor is also called 'quality factor' or 'exergetic quality factor' (Dincer and Rosen, 2007). The exergy factor illustrates the work potential per unit heat and thereby it indicates its quality.

The sign convention used in this paper is according to most textbooks on thermodynamics (Bejan et al., 1996; Moran and Shapiro, 2004; Dincer and Rosen, 2007):

- $Q > 0$ = Heat transfer to a system; $Q < 0$ = heat transfer from a system (universal definition)
- $W > 0$ = Work done by a system; $W < 0$ = work done on a system (not a universal definition).

The sign of the heat or work is thus dependent on the defined system, which shows the importance of defining the system under consideration and its boundaries.

Exergy is never supplied to or removed from a system as exergy; it is always accompanying other forms of energy transfer. The sign convention for exergy accompanying heat is:

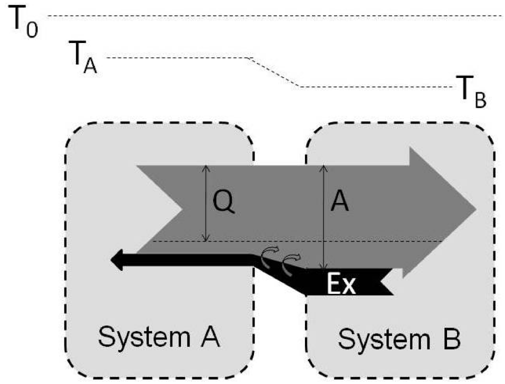
- $Ex_Q > 0$ = Exergy transferred to the system; $Ex_Q < 0$ = exergy transferred from the system.

3 Short literature review of the exergy of cold

Rant (1956) first introduced the word exergy as the word for 'technische Arbeitsfähigkeit' (technical available work). He also used the word anergy, referring to the part of the energy that is not convertible to technical work (Rant, 1964). Energy is defined as the sum of exergy and anergy ($E = Ex + A$). The exergy of heat is calculated according to equation (1), which is shown in the previous section. In all real processes, exergy is (partly) converted to anergy, while the amount of energy stays constant, in accordance with the first law of thermodynamics.

Baehr used the concepts of exergy and anergy to explain the process of exergy destruction in energy conversion and heat transfer processes (Baehr and Bergmann, 1965). For the analysis of heat transfer at $T < T_{0,}$ the same approach is followed (Baehr, 1978). Regarding two systems A at T_A and B at T_B, ($T_B < T_A < T_0$), heat transfer takes place from system A to system B (see Figure 1). Regarding this heat transfer as positive and using equation (1) and equation '$E = Ex + A$', Baehr finds that the exergy becomes negative, which means it is flowing in the opposite direction of the energy flow. The anergy is flowing in the same direction as the energy flow and is increased during the process. The direction of all flows is shown using exergy flow figures: these show that when a system at $T < T_0$ is cooled (in this case system A), energy and anergy are flowing out of a cooled space and exergy is flowing in. It is also shown that the anergy of the energy that is transferred at $T < T_0$ is greater than the energy itself. The exergy of system B is decreased during the process.

Figure 1 Diagram according to Baehr (1978), illustrating energy, anergy and exergy flows between two systems at $T < T_0$



Grassmann (1965) defined anergy as $\Delta A = \Delta E + Ex$ for processes at $T < T_0$. By using Δ anergy and Δ energy, the signs of the values are all positive, which causes the need to slightly change the equation as defined by Baehr. When applying the sign conventions as used in this paper, cooling a system at $T < T_0$ (like system A in Figure 1) would result in a negative value for the heat (out of the system), a lower (more negative) value for the anergy and a positive value for the exergy. The anergy produced in a process equals the exergy destroyed in a process.

In Obert and Gaggioli (1963), the exergy is referred to as availability and the availability of heat is also discussed. It is mentioned that

> "the available energy transferred with heat is called the availability of heat and is the maximum work obtainable when […] and equation (2) the heat rejection process of the reversible cycle is made at T_0."

It is also mentioned that "a sink at a temperature less than T_0 requires work" (Obert and Gaggioli, 1963). This work shows the traditional way of looking at the exergy of cold as a requirement for exergy, neglecting the fact that also cold can be available.

A study was done by Gaggioli (1981) on second-law analysis of building systems, including the analysis of a cooling demand. The results show that the cooling demand is regarded as an energy output from the building, while this cooling requires an exergy input. The exergy for a cooling case at $T < T_0$ is thus going in the opposite direction of the energy flow. Gaggioli also mentions that occasionally part of the cooling demand

> "is available energy supplied at 37°C (98°F) with heat from the lights; supplied, inasmuch as the heat is at a temperature greater than $T_0 = 35.3$°C (95°F). This heat could be collected with outdoor air."

This is an important note that mentions the fact that if cooling of a system (or building) takes place at $T > T_0$, this cooling can be achieved by 'free' outdoor air and thus does not require an exergy input. This will be further discussed in Section 6.2.

In Dincer and Rosen (2007), a brief general explanation of the exergy of cold is given, stating that "for heat transfer at sub-environmental temperatures τ [the exergy factor] < 0, implying that exergy and energy flow in opposite directions in such cases". They also note "that the magnitude of the exergy flow exceeds that of the energy flow when $\tau < -1$, which corresponds to $T < T_0/2$". This last statement is also further explained in this paper.

Wall and Gong (2001) also discuss the exergy of heat transferred at $T < T_0$. In this work, it is stated that

> "When $T < T_0$, there is a lack of energy in the system, i.e., coldness, which is sometimes expressed as negative energy. However, the exergy is still positive. By definition, exergy can only be positive, because there is no such thing as negative work."

Wall and Gong place the exergy factor between absolute brackets (see equation (9)), obtaining a positive exergy factor for cooling.

The research done on the use of exergy analysis in the built environment (Annex37, 2009; Shukuya and Hammache, 2002; Schmidt, 2004) is mostly focused on the exergy of heating. A clear explanation on the exergy of cold in the indoor air of a room however is given by Shukuya (2009). Here, it is explained that exergy is also contained by a volume of air at $T < T_0$, which can be clarified by imagining a Carnot cycle between the environment and the cold source (the volume of air), receiving heat from the environment and rejecting it to the cold source (acting as the sink). The exergy of the cold source is referred to as 'cool exergy' and cool exergy has to be supplied (and is consumed) to keep an indoor space at a temperature below T_0.

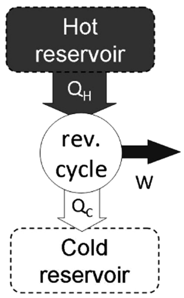
The literature review shows that the exergy of cold is considered by several authors, some of whom are mentioned earlier. However, the general discussions are often quite brief and the conclusions and comments do not always concur. This might cause confusion for researchers whose main field of science is not thermodynamics but who wish to use the concept of exergy for the analysis of certain systems within their field, such as for example the built environment. Therefore, this paper attempts to broaden the understanding of the topic by giving a clear derivation of the exergy of cold using the fundamentals of exergy, explaining some basic issues and discussing two useful applied examples.

4 Deriving the exergy factor of cold

4.1 A reversible thermal power cycle

The exergy of heat is based on a reversible thermal power cycle (e.g. Carnot cycle) operating between a hot and a cold reservoir, see Figure 2. Heat is transferred to the system from the hot reservoir. From the second law, it is known that not all heat to the system can be converted into work, but a certain amount must be rejected to the cold reservoir.

Figure 2 Scheme of a reversible thermal power cycle (Carnot cycle)



The energy balance of the system can be determined using the first law of thermodynamics:

$$W = Q_{H(\text{hot})} + Q_{C(\text{cold})}. \tag{2}$$

Since the heat transferred to the cold reservoir is transferred *from* the system, Q_C will have a negative value.

The definition of the thermodynamic temperature has resulted from the second law of thermodynamics. The thermodynamic temperature is defined in such a way that in the case of a reversible system:

$$\frac{-Q_C}{Q_H} = \frac{T_C}{T_H}. \tag{3}$$

From these equations, the maximum work obtainable from a given Q_H at given temperatures can be calculated, using equation (4). The factor $(1 - T_C/T_H)$ is called the Carnot efficiency (η_{Carnot}):

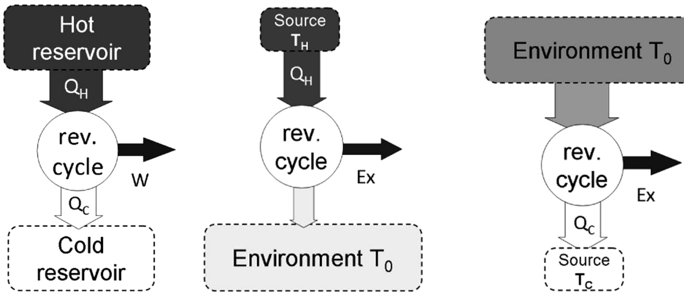
$$W_{\text{rev}} = Q_H + Q_C = Q_H - Q_H \cdot \frac{T_C}{T_H} = Q_H \cdot \left(1 - \frac{T_C}{T_H}\right) \tag{4}$$

$$\eta_{\text{Carnot}} = \frac{W}{Q_H} = \left(1 - \frac{T_C}{T_H}\right). \tag{5}$$

4.2 From a reversible thermal power cycle to the exergy of heat

Referring to the definition given in Section 2, the calculation of the exergy of heat is based on the reversible cycle described above, but assumes the environment (T_0) to be one of the reservoirs. Whether T_0 acts as the hot or the cold reservoir depends on the temperature of the heat available relative to the temperature of the environment ($T > T_0$ or $T < T_0$). This is illustrated in Figure 3 (where for simplicity the source containing hot or cold thermal energy is considered to have a constant temperature while heat is being transferred).

Figure 3 From a reversible thermal power cycle to exergy: (a) a reversible cycle between two infinite reservoirs; (b) the exergy of heat at $T > T_0$; (c) the exergy of cold at $T < T_0$; based on a reversible cycle between the environment and one finite source for heat transfer



(a) (b) (c)

4.3 The exergy of heat ($T > T_0$)

If heat is available at $T > T_0$, this heat is in fact Q_H at T_H and the environment acts as T_C. The work obtainable from this cycle equals the exergy of the heat (Q_H) and can be calculated based on equation (4) by replacing T_C by T_0, which results in the following equation:

$$Ex_{Q(H)} = W_{rev} = Q_H \cdot \left(1 - \frac{T_0}{T_H}\right). \tag{6}$$

4.4 The exergy of 'cold' ($T < T_0$)

The exergy of cold can also be calculated using a reversible thermal power cycle. However, in this case the cold is available at $T_C < T_0$, which means the 'cold' is in fact a possible disposal of heat Q_C at T_C. The environment then supplies the heat $Q_0 = Q_H$. The equation for the exergy of cold can be derived from the fundamental equations (2) and (3), similar to the derivation as shown in equation (4). This results in equation (7):

$$Ex_{Q(C)} = W_{rev} = Q_{0(=H)} + Q_C = -Q_C \cdot \frac{T_0}{T_C} + Q_C = Q_C \cdot \left(1 - \frac{T_0}{T_C}\right) \tag{7}$$

where Q_C has a negative value; but also the term within the brackets is negative. Thus, for the exergy of the cold ($Ex_{Q(C)}$), a positive value will be obtained.

4.5 General equation for exergy accompanying heat

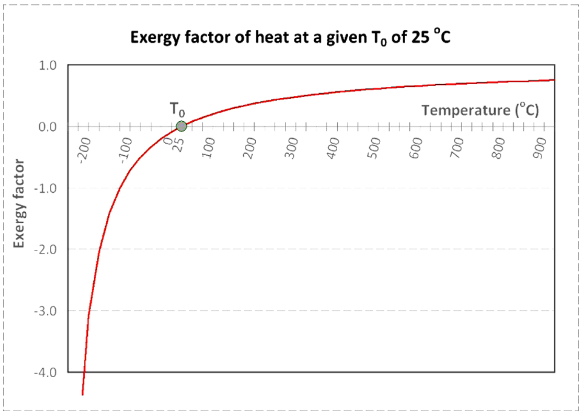
Equations (6) and (7) have shown that the exergy of heat transfer for both $T > T_0$ and $T < T_0$ is calculated by multiplying the heat transfer Q with the same factor $(1 - T_0/T)$. The exergy of heat can, therefore, generally be calculated using equation (8), which was already shown in equation (1) and is also used in several textbooks (Bejan et al., 1996; Moran and Shapiro, 2004). This equation is thus valid for both heat transfer at $T > T_0$ and heat transfer at $T < T_0$, bearing in mind that in the last case Q refers to the heat rejected and is then a negative value.

$$Ex_Q = Q \cdot \left(1 - \frac{T_0}{T}\right). \tag{8}$$

4.6 The exergy factor

Often, the quality of a form of energy is expressed using the exergy factor, which is defined as the exergy of a system divided by the energy. The factor $(1-T_0/T)$ in equation (8) is the exergy factor of heat (both at $T > T_0$ and $T < T_0$). In Figure 4, it is shown for a given environmental temperature of 25°C.

Figure 4 Exergy factor of heat (see online version for colours)



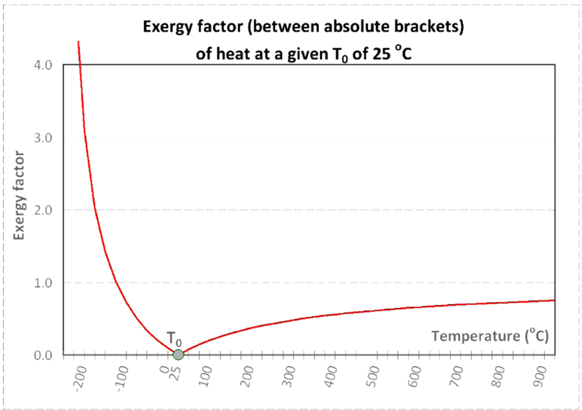
From this graph, it can be seen that when T approaches infinity, the exergy factor becomes unity, which means heat at very high temperatures can theoretically be totally converted to work. When $T < T_0$, the quality factor is a negative value. A negative exergy factor means the exergy has the opposite value of the heat (see equation (8)). Taking into account that Q_C has a negative value this means the exergy is positive, or in other words: When there is cold available, it has a potential to do work. This can also be seen in Figure 3.

Since many (building) professionals are used to regarding cold as a *positive* value, sometimes the exergy factor is placed between absolute brackets (Wall and Gong, 2001), resulting in equation (9). This way also a positive exergy value for heat transfer at $T < T_0$ is obtained, leading to Figure 5.

$$\frac{Ex_Q}{Q} = \left| 1 - \frac{T_0}{T} \right|. \tag{9}$$

Figure 5 leads to an easier comparison of the exergy of heat and cold. However, it is only valid when regarding both Q_H (at $T > T_0$) and Q_C (at $T < T_0$) as positive values. Also, since the exergy factor between absolute brackets can only result in positive values, it does not express when the exergy has the opposite value of the heat. This means the algebraic values for heat and exergy do not demonstrate whether they are inputs or outputs to a system under consideration, so that this must be additionally determined by logical reasoning. By using the sign conventions as explained in Section 2 and equation (8) (the exergy factor without absolute brackets), this additional reasoning is not needed, since the signs of results will express whether heat and exergy are inputs or outputs.

Figure 5 Exergy factor of heat, placed between absolute brackets (see online version for colours)

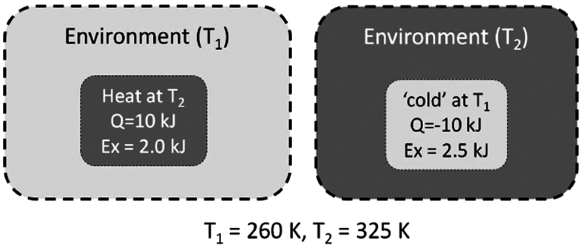


5 Basic issues to clarify the theory

5.1 Why the exergy of cold is greater than the exergy of heat (given two temperatures and equal 'energy' content)

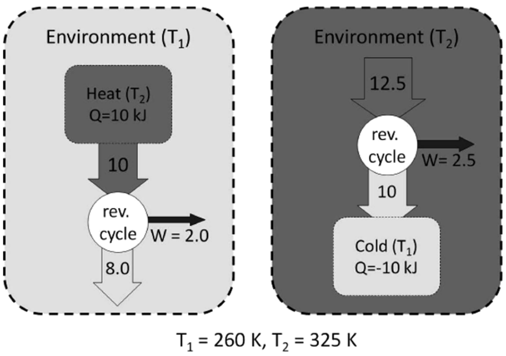
Consider Figure 6, where a theoretical comparison between available heat (at constant T_2) in a colder environment at T_1 and available cold (at constant T_1) in a warmer environment at T_2 ($T_1 < T_2$) is illustrated. The amount of heat is assumed to be equal to the amount of cold (e.g. (–)10 kJ). Assuming T_1 to be 260 K and T_2 to be 325 K, the exergy of the heat (left figure) is 2.0 kJ while the exergy of the cold (right figure) is 2.5 kJ. It will be shown that given two temperatures T_1 and T_2, the exergy of the cold will always be greater than the exergy of the heat.

Figure 6 Two theoretical cases: a hot source (T_2) in a colder environment (T_1) and a cold source (T_1) in a warmer environment (T_2)



If spontaneous heat transfer would take place (meaning without a reversible cycle), the heat transferred *to* or *from* the environment would be equal to the energy of the heat or cold available (in this example 10 kJ in both cases). However, by imagining a reversible thermal power cycle between the heat or cold and their environments, the heat or cold available remains the same as without this cycle, but the heat transfer to or from the *environment* would adapt according to the temperatures, in such a way that $Q_H/T_H = -Q_C/T_C$. This is shown in Figure 7.

Figure 7 Two theoretical cases: a hot source (at constant T_2) in a colder environment (T_1) and a cold source (at constant T_1) in a warmer environment (T_2). The exergy is explained with an imaginary reversible (Carnot) cycle between the source and the environment



The essential difference between available heat and available cold is thus:

- The hot source limits the heat supply (=Q_H, the heat transferred *to* the reversible thermal power cycle). The heat rejected to the environment adjusts to a smaller amount, according to the temperatures.
- The cold source limits the heat sink (=Q_C, the heat transferred *from* the reversible thermal power cycle), while the heat supply by the environment is unlimited and thus adjusts to a larger amount.

For this example, this means that the hot source limits Q_H at 10 kJ, whereas the cold source of –10 kJ gives the opportunity to the environment to transfer heat (Q_H) to the reversible thermal power cycle at a value >10 kJ. It is noted that the greater the temperature difference between T_1 ad T_2, the greater the difference between the exergy of heat and cold, as can also be seen in Figure 5.

5.2 Why the amount of exergy can be greater than the amount of cold available

The amount of (theoretically) obtainable work can be greater than the amount of available cold, which is the case if $f_{ex} < -1$, see Figure 4 (or, when regarding cold as a positive value, if $f_{ex;absolute} > 1$, see Figure 5).

Similar to the explanation given before, this is due to the fact that the cold limits the heat to be rejected, while the heat supply by the environment is unlimited and adjusts in such a way that $Q_H/{-Q_C} = T_H/T_C$. Therefore:

- as soon as $T_H > 2\ T_C$ this means $Q_H > 2\ (-Q_C)$
- since $W = Q_H + Q_C$ this means $W > (-Q_C)$.

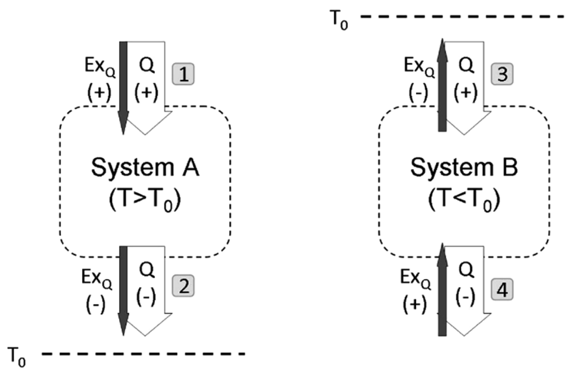
The exergy of cold is the work obtainable from Q_C. On the contrary, the Carnot efficiency of equation (5) is the work obtainable from heat Q_H (at a temperature higher than the environmental temperature); the Carnot efficiency can therefore never be greater than unity, which means that the amount of work obtainable from heat (at $T < T_0$) is always smaller than the amount of heat available.

(It is well known from thermodynamics that the production of cold at temperatures close to 0 K will require very large amounts of power, also if a reversible refrigeration cycle is used).

5.3 The direction of exergy for heating and cooling processes: exergy input or output?

In the previous paragraph, the focus is on the exergy of *available* heat and cold. However, it depends on the direction of the heat transfer *and* to the temperatures of the system and T_0, whether heat has the ability to produce work or requires the input of work. In Figure 8, two systems *A* ($T > T_0$) and *B* ($T < T_0$) are illustrated with for each system a heat input (heating) and heat output (cooling), with the accompanying exergy (these involve arbitrary systems, not necessarily thermal power cycles). The direction of the accompanying exergy can be determined using equation (8).

Figure 8 Exergy accompanying heat transfer, depending on the temperatures T and T_0



From these figures, it can be seen whether the heating or cooling process involves an exergy transfer *to* or *from* the system, implicating whether exergy is required or is available, as is outlined in Table 1:

Table 1 Implication of heating and cooling processes on the exergy transfer to or from the system involved

Process	*Heat transferred*	*Exergy transferred*	*the process*
Heating a system (A) at $T > T_0$	*to* the system A	*to* the system A	requires exergy
Cooling a system (A) at $T > T_0$	*from* the system A	*from* the system A	has exergy available
Heating a system (B) at $T < T_0$	*to* the system B	*from* the system B	has exergy available
Cooling a system (B) at $T < T_0$	*from* the system B	*to* the system B	requires exergy

In general, these rules are valid:

- Heat transfer bringing a system into equilibrium with the environment (and thus *closer to* T_0) can theoretically *produce work*. Or in other words: heat transfer that would also take place spontaneously can produce work.
- Heat transfer *bringing a system further from* T_0 *requires work* (all non-spontaneous heat transfer requires work).
- For heat transfer at $T > T_0$, the accompanying exergy is always in the same direction as the heat transfer.
- For heat transfer at $T < T_0$, the accompanying exergy is always in the opposite direction of the heat transfer.

Furthermore:

- When using the sign conventions as explained in Section 2 and the general equation for the exergy of heat without the absolute brackets (8), the signs of the results will reveal the direction of the heat transfer and the accompanying exergy.
- It is important to clearly define the system under consideration and its boundaries, since this is needed to determine the direction of the heat transferred and the accompanying exergy.
- It is noted that the negative value for the exergy should not be understood as 'negative work', but as an indication of the direction of the exergy *from* the system.

The previous part describes the exergy available from heat transfer or required to obtain heat transfer. The thermal energy of matter ($T > T_0$), or the lack of thermal energy in case the matter is at $T < T_0$, always *contains* exergy, since there is no negative work (Wall and Gong, 2001). This means the air in a room, be it at $T > T_0$ or at $T < T_0$, contains hot or cool exergy, as is also explained by Shukuya (2009). However, to obtain air or any matter at $T \neq T_0$, exergy is required.

6 Practical examples

6.1 Example 1: 'Cool exergy' of Liquefied Natural Gas (LNG)

An example of a situation where there is cold available is at LNG terminals where the regasification of LNG takes place. The gas is transported by sea ships in a liquid phase (to reduce its specific volume) at a temperature of 110 K. This cold contains a considerable amount of exergy. (Of course, the liquefaction process that has taken place before has also *cost* a considerable amount of exergy.)

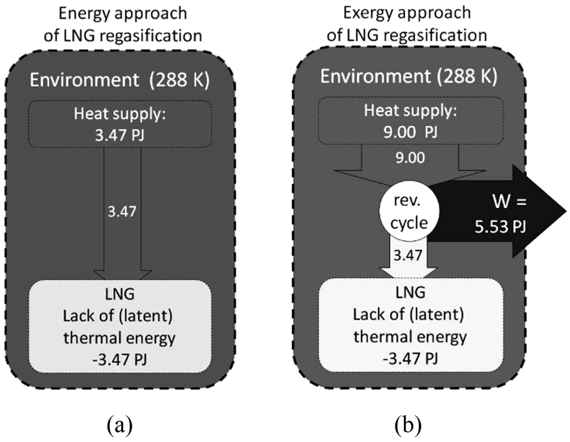
Given a certain amount of LNG, the actual available exergy will not only depend on the temperatures, but also on the pressures, thus the correct way of calculating the exergy is by using the enthalpy and the entropy of the state of arrival and the state at which the gas send-out into the transmission grids takes place. Several studies on the exergy of LNG can be found, both on the liquefaction process and on the regasification process (Kanoglu et al., 2007; Szargut and Szczygiel, 2009). This paper does not try to give a full representation of the regasification of LNG, but it will only use the LNG regasification to provide a useful example of the benefits of exergy analysis compared with energy analysis, especially when cold is concerned. For increased understanding, the example is highly simplified.

Assume an LNG regasification terminal with a capacity of 10 BCM (billion cubic metres, based on the vapour phase) per year. For simplicity, only the latent energy is regarded, and the vapourisation energy for methane of 510 kJ/kg is used (LNG contains over 95% of methane). The total latent heat at 110 K then amounts to 10×10^9 [m^3] 0.68 [kg/m^3]·510 [kJ/kg] = 3.47 PJ/year.

Using a traditional energy analysis, the cold of the LNG is regarded as a lack of energy, and to revapourise the gas an energy demand of 3.47 PJ is defined. In practice this heat is often supplied by burning part of the gas, by waste heat from electricity production plants or by seawater. The 3.47 PJ of heat 'required' is only around 1% of the chemical energy of the 10 BCM gas, but in absolute value it is a considerable amount. Of course, the last two options are already better than the first, but still the energy approach totally lacks the insight that would be gained from an exergy analysis: The lack of thermal energy in the LNG should not be considered as a demand, but it should be considered as (work) potential that ought to be harvested as much as possible!

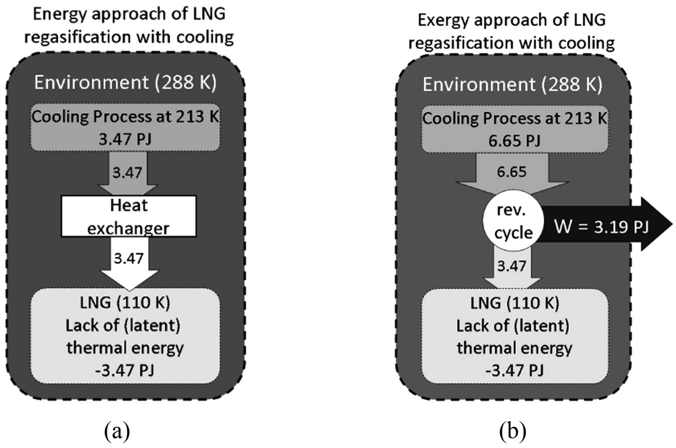
The exergy of the latent heat of the LNG can be calculated using equation (8), which for an environmental temperature of 15 C (288 K) results in an amount of 3.47 [PJ/year]·(1–288/110) = 5.53 PJ/year. Of course, the exergy is the theoretical maximum work obtainable, but even with an exergetic efficiency of 50% this would mean 2.77 PJ of electricity per year could be gained, which equals the electricity consumption of almost 230,000 average households in the Netherlands (Senternovem, 2007). This electricity could be harvested with no additional emissions of CO_2 gases, since the heat supply can be environmental heat. Of course, LNG itself is not a CO_2-free fuel, but the emissions are related to the chemical energy of the fuel, not to the 'cold' energy. In Figure 9, the difference between the energy and the exergy approach is illustrated.

Figure 9 Schemes showing the energy approach (a) and the exergy approach (b) for the analysis of the lack of thermal energy in Liquefied Natural Gas (LNG)



Since all energy conversions in reality consume exergy, a good option is also to use the cold of the LNG for freezing processes, if they are present near the terminal. However, if the process of the cooling demand at the nearest temperature level has a significantly higher temperature than 110 K, for example an industrial freezing process of 213 K (–60 C), it is not advisable to just transfer the cold using a heat exchanger. Using a heat exchanger, a maximum energy efficiency of 100% can be obtained, which means the same amount of heat extracted from the industrial cooling will be supplied to the LNG. Also, in this case, a better solution will be to withdraw the heat from the freezer using a heat engine, and use the cold of the LNG as a heat sink. This principle is shown in Figure 10. It results both in a larger amount of available cooling at 213 K and in the production of work, which makes the situation on the right of Figure 10(b) much more attractive. Of course, all depends on the technical and economical feasibilities, which are not treated here, but at least the potential should be evaluated.

Figure 10 Schemes showing the energy approach (a) and the exergy approach (b) when analysing the lack of thermal energy in Liquefied Natural Gas (LNG) to be used for cooling purposes



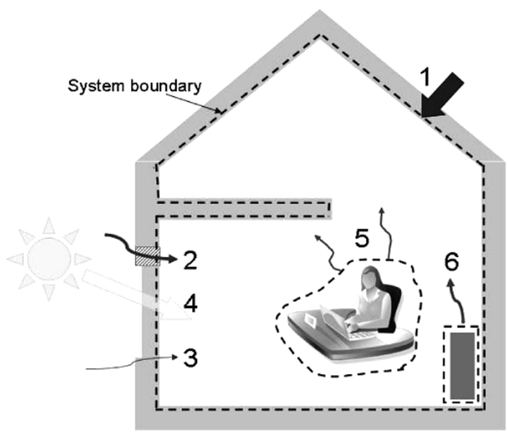
From this example, the importance of performing an exergy analysis when regarding cold is demonstrated. An exergy analysis does not only show the 'lack of energy', but also the potential that could be harvested. In the example, this means that even though heat is required for the regasification of LNG, no *exergy* is required for it and in fact work can be obtained in this process.

6.2 Example 2: Cooling in the built environment

The second example involves the cooling demand in buildings. The heating and cooling demand of a building can be defined as the amount of heat or cold required to keep the indoor environment within the comfort ranges required by its users. The heating and cooling demand depends on building characteristics, user behaviour and weather data. It is independent from the technical installations providing this heating or cooling and therefore different from the final energy input to the installations of the building by electricity or gas. The exergy demand is then defined as the exergy of the heating or cooling demand, or in other words the *minimum* amount of work required to produce the required heat or cold.

The exergy demand calculation is based on the energy demand, since it cannot be based on an exergy balance owing to the unknown component of the exergy that is destroyed within the building zone. The heating or cooling demand of a building can be calculated using energy balance equations. In Figure 11, all the relevant flows of energy and matter into and out of the building are shown:

Figure 11 Energy flows across the boundaries of a building zone



1 Transmission (heat transfer through closed surfaces)*
2 Ventilation (controlled airflow into and out of the building through devices designed for this purpose)
3 Infiltration (uncontrolled airflow into and out of the building, through cracks)
4 Solar heat gains
5 Internal heat gains (lighting, appliances and people)
6 Active energy input (+ = heating, – = cooling)

*with dynamic simulations this includes heat coming from energy stored in the walls)

From the energy balance, the energy demand can be calculated, resulting in a positive value in case of a heating demand (heat transfer into the building), and a negative value in case of a cooling demand (heat transfer out of the building).

In this example, the exergy demand for heating and cooling in a standard office space in the Dutch climate is analysed. The characteristics of this office space are shown in Table 2. Three steady-state situations are analysed, as outlined in Table 3. The results of these steady-state energy balance calculations are shown in Figure 12.

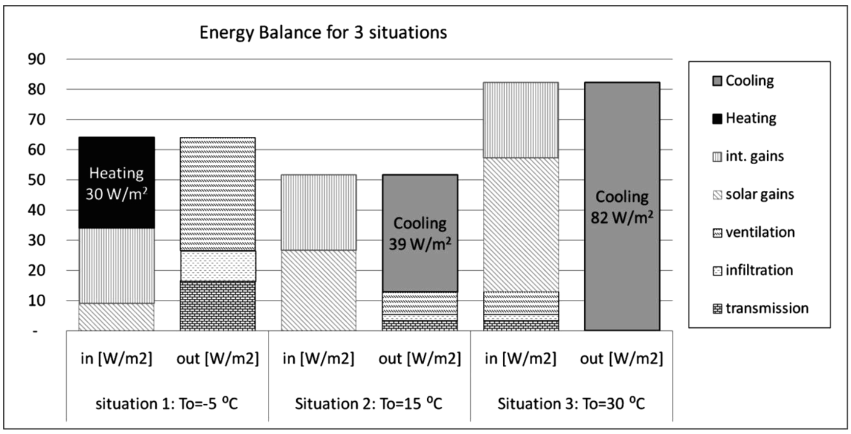
Table 2 Office space characteristics

Volume (length × width × height = 5.4 × 5.4 × 3.0 m)	87.5 (m^3)
Floor surface	29.2 (m^2)
External façade	
(all other boundaries are assumed to be adiabatic)	16.2 (m^2)
Glass percentage (including window frame 15%)	40 (%)
U-value glazing (including window frame)	2.4 (W/m^2K)
U-value closed part of the external façade	0.37 (W/m^2K)
g-value (solar transmittance)	0.4 (–)
Ventilation (air change rate in zone volume per hour)	1.5 (Vol/h)
Infiltration (air change rate in zone volume per hour)	0.4 (Vol/h)
Internal heat load (people, light and appliances)	25 (W/m^2)

Table 3 Description of three steady-state situations

	T_i (°C)	T_0 (°C)	*Solar radiation on façade* (W/m^2)
Situation 1	20	–5	100
Situation 2	25	15	300
Situation 3	25	30	500

Figure 12 Energy balances for three steady-state situations, resulting in heating (situation 1), or cooling (situation 2 and 3)



As can be seen in Figure 12, both situations 2 and 3 represent a cooling demand, which means there is a need for heat transfer out of the system (the building zone) in both cases. However, in exergy terms, the two situations are essentially different: In situation 3, the outdoor air temperature is higher than the indoor temperature, whereas in situation 2 the indoor temperature is the higher of the two. Consequently, the cooling demand can either represent a required *exergy input* or an *exergy output*, depending on the indoor and outdoor temperatures.

In situation 2, there is a cooling demand even though the environmental temperature is *lower* than the indoor temperature, which can occur due to high internal and solar gains. In this case, the aim is to bring the temperature of the system *closer* to the environmental temperature. This theoretically means that work could be obtained, meaning there is no exergy *demand* but theoretically there is (warm) exergy *available* (This is possible since buildings are not built with the aim to have the highest exergy content, but with the aim to provide comfortable thermal conditions.). This cooling demand corresponds to the heat transfer (2) in Figure 8.

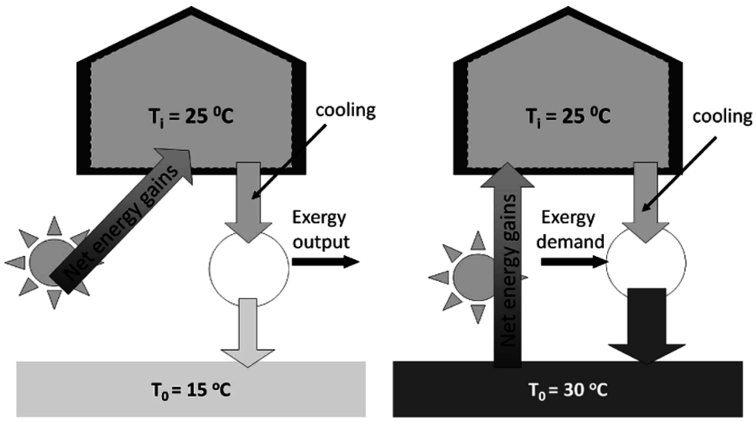
In situation 3, the environmental temperature is higher than the indoor temperature, which means the cooling demand represents a required *exergy input*, because obtaining or maintaining any state different from the environmental state requires an exergy input, be it warmer (warm exergy) or cooler (cool exergy), as is also explained by Shukuya (2009). This cooling demand corresponds to the heat transfer (4) in Figure 8.

The three situations that can occur in buildings are outlined in Table 4. The two cooling situations are illustrated in Figure 13, clarifying the results with an imaginary reversible (Carnot) cycle.

Table 4 Heating and cooling demand situations

Situation			*Energy*	*Exergy*
1	$T_0 < T_i$	Losses > Gains	Heating (heat *to* the building)	*to* the building/exergy required
2	$T_0 < T_i$	Losses < Gains	Cooling (heat *from* the building)	*from* the building/exergy available
3	$T_0 > T_i$	Losses < Gains	Cooling (heat *from* the building)	*to* the building/exergy required

Figure 13 Exergy demand related to cooling at $T > T_0$ (left) and $T < T_0$ (right)



The algebraic value of the exergy of the cooling and heating demand can be calculated using equation (8), assuming all heating and cooling demand to be supplied or removed as heat at indoor air temperature (Schmidt, 2004). The outdoor air temperature is taken as the reference temperature. The results of this analysis are listed in Table 5.

Table 5 Results of the exergy demand calculations for the three situations

Situation		T_i °C (K)	T_0°C (K)	*Energy demand* [W/m²]	*Exergy demand* [W/m²]	
1	$T_0 < T_i$	20 (293)	–5 (268)	30	= 30·(1 – 268/293) =	2.6
2	$T_0 < T_i$	25 (298)	15 (288)	–39	= –39·(1 – 288/298) =	–1.3
3	$T_0 > T_i$	25 (298)	30 (303)	–82	= –82·(1 – 303/298) =	1.4

In Table 5, the positive exergy demand indicates that there is a 'real' demand; that is, there is an exergy input required. The negative exergy 'demand' of situation 2 indicates that there is an exergy output of the building, which means ideally no exergy is required for the cooling in this situation but exergy is available, as explained earlier. It is also shown that when using the sign conventions explained in Section 2 and equation (8) (the exergy factor without the absolute brackets), the signs of the results immediately confirm the direction of the exergy. The equation with absolute brackets should be handled with care in this case.

It can also be seen from these results that the exergy demand is extremely low compared with the energy demand. This means theoretically very little work has to be supplied to provide the required heating and cooling. In practice, the exergy input to the building with its installations by high-quality sources like gas and electricity is much higher. This means that often there is a great improvement potential for these systems, but this is not further treated in this paper.

The Dutch climate and many others are such that situation 2 ($T_i > T_0$) happens much more often than situation 3 ($T_i < T_0$). Additionally, in situation 3, the temperature difference between T_i and T_0 is usually very small since temperatures above 30°C rarely occur. This means the annual exergy demand for cooling is often extremely low and it confirms that passive systems should be optimised especially for cooling. Thus, in addition to energy analysis, an exergy gives insight in the difference between cooling demand at $T < T_0$ (required cool exergy input) and cooling demand at $T > T_0$ (theoretical warm exergy output).

7 Conclusions

This paper has tried to broaden the understanding of the exergy of 'cold', i.e., heat transfer at $T < T_0$. Since the exergy of heat (both at $T > T_0$ and $T < T_0$) is based on the theoretical maximum work obtainable using a reversible thermal power cycle (Carnot cycle), it is important to understand the cold as a potential sink for this cycle, where the heat supplied comes from the environment. Cold (Q_C) has a negative value since it is a transfer of heat out of the system. However, the exergy factor is also negative, resulting in a positive value for the exergy of cold. Thus, work can be obtained if there is cold available.

For the exergy of heat and cold, there is an essential difference between available heat and available cold: While the available heat and cold are limited, the heat transferred to or from the environment is unlimited and adapts in such a way that $Q_H/–Q_C = T_H/T_C$, according to the second law of thermodynamics. This means:

- A hot source limits the heat supply (=Q_H, the heat transferred *to* the reversible thermal power cycle). The heat rejected to the environment ($Q_0 = Q_C$) adjusts to a smaller amount, according to the temperatures.
- A cold source limits the heat sink (=Q_C, the heat transferred *from* the reversible thermal power cycle), while the heat supply by the environment ($Q_0 = Q_H$) is unlimited and thus adjusts to a larger amount.

This means cold at T_1 in an environment at T_2 contains more exergy than heat at T_2 in an environment at T_1 ($T_1 < T_2$). It also means that as soon as $T_0 > 2T_C$, the amount of (theoretically) obtainable work is larger than the amount of available cold. Therefore, cold at low temperatures is very valuable.

The paper has also shown that it depends on the direction of the heat or cold and to the temperatures of the system and T_0, whether there is an ability to produce work or the input of work is required. In general, the following rules are valid:

- Heat transfer bringing a system into equilibrium with the environment (and thus *closer to* T_0) can theoretically *produce work*. This means heat transfer that would also take place spontaneously can produce work.
- Heat transfer *bringing a system further from* T_0 *requires work* (All non-spontaneous heat transfer requires work).
- For heat transfer at $T > T_0$, the accompanying exergy is always in the same direction as the heat transfer.
- For heat transfer at $T < T_0$, the accompanying exergy is always in the opposite direction of the heat transfer.
- Negative values for exergy should not be understood as negative work, but as a sign showing the direction of exergy (as *out of* the system according to the sign conventions used in this paper).

Two useful examples have been explained: A simplified analysis of the gasification of LNG and a discussion on the exergy of cooling in buildings. The analysis of the regasification of LNG has demonstrated the importance of performing an exergy analysis when regarding cold thermal energy. An exergy analysis does not only show the 'lack of energy' of cold, but shows the (work) potential that could be harvested.

The discussion on the exergy of the cooling demand of buildings has shown that there are two different situations to be distinguished: a cooling demand at $T_i < T_0$, and a cooling demand at $T_i > T_0$, which can occur due to high solar and internal gains. In the first case, the cooling demand represents a required *exergy input*, because obtaining or maintaining any state different from the environmental state requires an exergy input. In the second case, the cooling demand represents an *exergy output* (there is no exergy demand but theoretically there is exergy available), since the aim is to bring the temperature of the system *closer* to the environmental temperature.

It is concluded that even though the exergy of cold might be less intuitive to understand, 'cold' can be a very valuable and when regarding cold an exergy analysis has much added value compared with an analysis regarding only the first law of thermodynamics.

References

Annex37 (2009) *LowExNet – Network of International Society for Low Exergy Systems in Buildings*, Retrieved 20-09-2009, from www.lowex.org

Baehr, H.D. (1978) *THERMODYNAMIK: eine Einführung in die Grundlagen und ihre technischen Anwendungen*, Springer, Berlin.

Baehr, H.D. (1965) 'Energie, exergie, anergie', in Baehr, H.D. and Bergmann, E. (Eds.): *Energie und Exergie Die Anwendung des Exergiebegriffs in der Energietechnik*, VDI-Fachgruppe Energietechnik, VDI-Verlag, Düsseldorf, Germany, pp.5–20 (in German).

Bejan, A., Tsatsaronis, G. and Moran, M. (1996) *Thermal Design and Optimization*, Wiley, New York.

Dincer, I. and Rosen, M.A. (2007) *Exergy Energy, Environment and Sustainable Development*, Elsevier, Amsterdam.

Gaggioli, R.A. and Wepfer, W.J. (1981) 'Second law analysis of building systems', *Energy Conversion and Management*, Vol. 21, No. 1, pp.65–75.

Grassmann, P. (1965) 'Anwendungen von Exergiebetrachtungen in der Kältetechnik', in Baehr, H.D. and Bergmann, E. (Eds.): *Energie und Exergie Die Anwendung des Exergiebegriffs in der Energietechnik*, VDI-Fachgruppe Energietechnik, VDI-Verlag, Düsseldorf, Germany, pp.21–31, (in German).

Kanoglu, M., Dincer, I. and Rosen, M.A. (2007) 'Performance analysis of gas liquefaction cycles', *Int. J. Energy Res.*, Vol. 32, pp.35–43, Published online 5 June 2007 in Wiley InterScience 2008.

Moran, M.J. and Shapiro, H.N. (2004) *Fundamentals of Engineering Thermodynamics*, John Wiley & Sons Inc., New York.

Obert, E.F. and Gaggioli, R.A. (1963) *Thermodynamics*, 2nd ed., McGraw-Hill, New York.

Rant, Z. (1956) 'Exergie, ein neues Wort für technische Arbeitsfähigkeit', *Forschung Ing.Wesens*, Vol. 22, No. 1, pp.36–37.

Rant, Z. (1964) 'Thermodynamische Bewertung der Verluste bei technischen Energieumwandlungen', *BWK*, Vol. 16, No. 9, Braunschweig, Germany, pp.453–457 (in German).

Schmidt, D. (2004) 'Design of low exergy buildings- method and a pre-design tool', *The International Journal of Low Energy and Sustainable Buildings*, Vol. 3, pp.1–47.

Senternovem (2007) *Cijfers en Tabellen (in Dutch)*, Senternovem, Utrecht, The Netherlands.

Shukuya, M. (2009) 'Exergy concept and its application to the built environment', *Building and Environment*, Vol. 44, No. 7, pp.1545–1550.

Shukuya, M. and Hammache, A. (2002) *Introduction to the Concept of Exergy – for a Better Understanding of Low-Temperature-Heating and High-Temperature-Cooling Systems*, Espoo, Finland, VTT Technical Research Centre of Finland.

Szargut, J. and Szczygiel, I. (2009) 'Utilization of the cryogenic exergy of liquid natural gas (LNG) for the production of electricity', *Energy*, Vol. 34, No. 7, pp.827–837.

Wall, G. and Gong, M. (2001) 'On exergy and sustainable development – Part 1: conditions and concepts', *Exergy, An International Journal*, Vol. 1, No. 3, pp.128–145.

Nomenclature

A	Anergy (J)
Ex	Exergy (J)
f_{ex}	Exergy factor
LNG	Liquefied Natural Gas
Q	Heat (J)
T	Temperature (K, unless stated differently)
W	Work (J)
Greek letters	
η	Efficiency
τ	Exergetic temperature factor
Subscripts	
0	Reference environment
C	Cold reservoir
Carnot	Carnot
Ex	Exergy
H	Hot reservoir
Q	Associated with heat transfer