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Energy Management for Building Climate Comfort in Uncertain Smart Thermal Grids with Aquifer Thermal Energy Storage ⋆

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Abstract: In this paper, we present an energy management framework for building climate comfort systems that are interconnected in a grid via aquifer thermal energy storage (ATES) systems in the presence of two types of uncertainty, namely private and common uncertainty sources. The ATES system is considered as a large-scale storage system that can be a heat source or sink, or a storage for thermal energy. While the private uncertainty source refers to uncertain thermal energy demand of individual buildings, the common uncertainty source describes the uncertain common resource pool (ATES) between neighbors. To this end, we develop a large-scale uncertain coupled dynamical model to predict the thermal energy imbalance in a network of interconnected building climate comfort systems together with mutual interactions between the local ATES systems. A finite-horizon mixed-integer quadratic optimization problem with multiple chance constraints is formulated at each sampling time, which is in general a non-convex problem and hard to solve. We then provide a computationally tractable framework based on an extension to the so-called robust randomized approach which offers a less conservative solution for a problem with multiple chance constraints. A simulation study is provided to compare two different configurations, namely: completely decoupled, and centralized solutions.

Keywords: ATES, Smart Grids, Multiple Chance Constraints, Robust Randomized.

1. INTRODUCTION

Aquifer thermal energy storage (ATES) is a less well-known sustainable storage system that can be used to store large quantities of thermal energy in underground aquifers. It is especially suitable for climate comfort systems of large buildings such as offices, hospitals, universities, musea and greenhouses. Most buildings in moderate climates have a heat shortage in winter and a heat surplus in summer. Where aquifers exist, this temporal discrepancy can be overcome by storing and extracting thermal energy into and out of the subsurface, enabling the reduction of energy usage and CO_2 emissions of climate comfort systems in buildings (Jaxa-Rozen et al., 2016).

Smart Thermal Grids (STGs) have been studied implicitly in the context of micro combined heat and power systems (Ummenhofer et al., 2017), building with a dynamical storage tank (Van Vliet, 2013), termocline thermal energy storage systems (Powell and Edgar, 2013), or general smart grids, e.g., see Larsen et al. (2013), Larsen et al. (2014) and the references therein. A deterministic view on STGs was studied by a few researchers (Rivarolo et al., 2013), (Lund et al., 2014), (Sameti and Haghighat, 2017). STGs with uncertain thermal energy demands have been considered in Farahani et al. (2016), where a model predictive control (MPC) strategy was employed with a heuristic Monte Carlo sampling approach to make the solution robust. A dynamical model of thermal energy imbalance in STGs with a probabilistic view on uncertain thermal energy demands was established in Rostampour and Keviczky (2016), where a stochastic MPC with a theoretical guarantee on the feasibility of the obtained solution was developed.

ATES as a seasonal storage system has not, to the best of our knowledge, been considered in STGs. In Rostampour et al. (2016a) and Rostampour et al. (2016b), a dynamical model for an ATES system integrated in a building climate comfort system has been developed. Following these studies, the first results toward developing an optimal operational framework to control ATES systems in STGs is presented here. In this framework, uncertain thermal energy demands are considered along with the possible mutual interactions between ATES systems, which may cause limited performance and reduced energy savings. The main contributions of this paper are twofold: 1) We develop a novel large-scale stochastic hybrid dynamical model to predict the dynamics of thermal energy imbalance in STGs consisting of building climate comfort systems with hourly-based operation and ATES as a seasonal energy storage system. Using an MPC paradigm, we formulate a finite-horizon mixed-integer quadratic optimization problem with multiple chance constraints at each sampling time leading to a non-convex problem, which is difficult to solve. 2) We develop a computationally tractable framework to approximate a solution for our proposed formulation based on our previous work in Ros-

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tampour and Keviczky (2016). In particularly, we extend the framework in Rostampour and Keviczky (2016) to cope with multiple chance constraints which provides a less conservative solution compared to the so-called robust randomized approach in Margellos et al. (2013). Our framework is closely related to, albeit different from the approach of Schildbach et al. (2013). In Schildbach et al. (2013) the problem formulation consists of an objective function with multiple chance constraints, in which the terms in objective and constraints are univariate. In contrast the objective function in our problem formulation consists of separable additive components.

2. SYSTEM DYNAMICS MODELING

2.1 Seasonal Storage Systems

We consider an ATES system consisting of warm and cold wells to store warm water during warm season and cold water during cold season, respectively. Each well can be described as a single thermal energy storage where the amount of stored energy is proportional to the temperature difference between stored water and aquifer ambient water. Stored thermal energy from the last season is going to be used for the current season and so forth. Depending on the season, the operating mode (heating or cooling) of an ATES system changes, by reversing the direction of water between wells as it is shown in Fig. 1.

We first define the volume of stored water, $\mathrm{V}^h_{\mathrm{a},k}, \mathrm{V}^c_{\mathrm{a},k}[\mathrm{m}^3]$, and the thermal energy content, $\mathrm{S}^h_{\mathrm{a},k}, \mathrm{S}^c_{\mathrm{a},k}[\mathrm{W}]$, of warm and cold wells to be the state variables of an ATES system, and then, propose the following model dynamics:

$$\mathrm{V}^h_{\mathrm{a},k+1} = \mathrm{V}^h_{\mathrm{a},k} - (u^h_{\mathrm{a},k} - u^c_{\mathrm{a},k}), \tag{1a}$$
$$\mathrm{V}^c_{\mathrm{a},k+1} = \mathrm{V}^c_{\mathrm{a},k} + (u^h_{\mathrm{a},k} - u^c_{\mathrm{a},k}), \tag{1b}$$
$$\mathrm{S}^h_{\mathrm{a},k+1} = \eta_{\mathrm{a}}\, \mathrm{S}^h_{\mathrm{a},k} - (h^h_{\mathrm{a},k} - h^c_{\mathrm{a},k}), \tag{1c}$$
$$\mathrm{S}^c_{\mathrm{a},k+1} = \eta_{\mathrm{a}}\, \mathrm{S}^c_{\mathrm{a},k} + (c^h_{\mathrm{a},k} - c^c_{\mathrm{a},k}), \tag{1d}$$

where $\eta_{\mathrm{a}} \in (0,1)$ is a lumped coefficient of losses, $u^h_{\mathrm{a},k}\,[\mathrm{m}^3\mathrm{h}^{-1}]$, and $u^c_{\mathrm{a},k}\,[\mathrm{m}^3\mathrm{h}^{-1}]$ are control variables corresponding to the pump flow rate of ATES system during heating and cooling modes at each sampling time k, respectively. $h^h_{a,k}\,[\mathrm{W}], c^h_{a,k}\,[\mathrm{W}]$ denote the amount of thermal energy that is extracted from warm well and injected into cold well of ATES system during heating mode, respectively. $c^c_{a,k}\,[\mathrm{W}], h^c_{a,k}\,[\mathrm{W}]$ are the amount of thermal energy that is extracted from cold well and injected into warm well of ATES system during cooling mode, respectively. We also define $h_{a,k}\,[\mathrm{W}]$ and $c_{a,k}\,[\mathrm{W}]$ to be the amount of thermal energy that can be delivered to the building during heating and cooling modes, respectively. They are determined using the following relations:

$$\begin{cases} h_{a,k} = \alpha\, u^h_{a,k} \\ c_{a,k} = \alpha\, u^c_{a,k} \end{cases}, \begin{cases} h^h_{a,k} = \alpha_h\, u^h_{a,k} \\ c^c_{a,k} = \alpha_c\, u^c_{a,k} \end{cases}, \begin{cases} h^c_{a,k} = \alpha_h\, u^c_{a,k} \\ c^h_{a,k} = \alpha_c\, u^h_{a,k} \end{cases},$$

where $\alpha_h = \rho_w\, c_{pw}\, (\mathrm{T}^h_{\mathrm{a},k} - \mathrm{T}^{\mathrm{amb}}_{\mathrm{a},k})$, and $\alpha_c = \rho_w\, c_{pw}\, (\mathrm{T}^{\mathrm{amb}}_{\mathrm{a},k} - \mathrm{T}^c_{\mathrm{a},k})$ are the thermal energy coefficients of warm and cold wells, respectively. $\alpha = \alpha_h + \alpha_c$ is the total thermal energy coefficient, ρ_w, and c_{pw} are density and specific heat capacity of water, respectively. $\mathrm{T}^h_{\mathrm{a},k}$ [°C], $\mathrm{T}^c_{\mathrm{a},k}$ [°C] and $\mathrm{T}^{\mathrm{amb}}_{\mathrm{aq},k}$ [°C] denote the water temperature of warm well, cold well and aquifer ambient, respectively.

Remark 1. There is always only one operating mode active in ATES systems, which leads to: $u^h_{\mathrm{a},k}\, u^c_{\mathrm{a},k} = 0, \forall k$.

2.2 Building Climate Comfort Systems

Thermal energy demand, $\mathrm{Q}^{\mathrm{B}}_{\mathrm{d},k}$ [W], of a building climate comfort system at each sampling time k is determined by using our developments in Rostampour et al. (2017b) via

$$\mathrm{Q}^{\mathrm{B}}_{\mathrm{d},k} = f_{\mathrm{B}}(p^{\mathrm{B}}_s, \mathrm{T}^{\mathrm{B}}_{\mathrm{des},k}, \vartheta_k), \tag{2}$$

where p^{B}_s, $\mathrm{T}^{\mathrm{B}}_{\mathrm{des},k}$ [°C] denote a parameter vector and a desired indoor air temperature of building, respectively. $\vartheta_k = [\mathrm{T}^{\mathrm{B}}_{o,k}, \mathrm{I}_{o,k}, \mathrm{v}_{o,k}, \mathrm{Q}_{p,k}, \mathrm{Q}_{e,k}] \in \mathbb{R}^5$ is a vector of uncertain variables that contains outside air temperature, solar radiation, wind velocity, the thermal energy produced due to occupancy by people and total electrical devices/lighting installation inside the building.

Remark 2. We are interested in capturing the variation of thermal energy demand w.r.t. the outside air temperature $\mathrm{T}^{\mathrm{B}}_{o,k}$. Therefore, the uncertain variable in (2), ϑ_k, is assigned to $\mathrm{T}^{\mathrm{B}}_{o,k}$, and the rest of the variables are fixed to their nominal (forecast) values at each sampling time k. From (2), it follows that the mapping from the uncertain variable ϑ_k to the thermal energy demand $\mathrm{Q}^{\mathrm{B}}_{\mathrm{d},k}$ is measurable, so that $\mathrm{Q}^{\mathrm{B}}_{\mathrm{d},k}$ can be viewed as a random variable on the same probability space as ϑ_k.

Remark 3. The operating modes (heating or cooling) of building climate comfort system are determined based on the sign of $\mathrm{Q}^{\mathrm{B}}_{\mathrm{d},k}$ at each sampling time k. $\mathrm{Q}^{\mathrm{B}}_{\mathrm{d},k}$ with positive and negative signs, represents the thermal energy demand during heating mode and the building surplus thermal energy during cooling mode, respectively. $\mathrm{Q}^{\mathrm{B}}_{\mathrm{d},k} = 0$, is related to the comfort mode of building, and thus, no heating or cooling is requested. We also distinguish between the thermal energy demand of building during heating mode $h_{\mathrm{d},k}$, and cooling mode $c_{\mathrm{d},k}$, using the relation: $\mathrm{Q}^{\mathrm{B}}_{\mathrm{d},k} = h_{\mathrm{d},k} - c_{\mathrm{d},k}$. Moreover, the thermal energy demand can be only either for heating $h_{\mathrm{d},k}$, or cooling $c_{\mathrm{d},k}$ modes, which leads to: $h_{\mathrm{d},k}\, c_{\mathrm{d},k} = 0, \forall k$.

3. ENERGY MANAGEMENT PROBLEM

3.1 Energy Balance in Single Agent System

Consider a single agent (i.e. building) $i \in \{1, \cdots, N\}$ that is facilitated with a boiler, a heat pump, a storage tank for the heating mode, and a chiller, a storage tank for the cooling mode together with an ATES system that is available for both operating modes (see Fig. 1). For a day-ahead planning problem of each agent, we consider a finite-horizon N_h with hourly steps, and introduce the subscript t in our notation to characterize the value of the quantities for a given time instance $t \in \{k, k+1, \cdots, N_h + k\}$.

For each agent i one can rewrite the proposed dynamics of ATES system in (1) in a more compact format:

$$x^{\mathrm{a}}_{i,t+1} = a^{\mathrm{a}}_i x^{\mathrm{a}}_{i,t} + b^{\mathrm{a}}_i u^{\mathrm{a}}_{i,t}, \tag{3}$$

where $x^{\mathrm{a}}_{i,t} = \begin{bmatrix} \mathrm{V}^h_{\mathrm{a},t} & \mathrm{V}^c_{\mathrm{a},t} & \mathrm{S}^h_{\mathrm{a},t} & \mathrm{S}^c_{\mathrm{a},t} \end{bmatrix}^\top \in \mathbb{R}^4$ denotes the state vector, $u^{\mathrm{a}}_{i,t} = \begin{bmatrix} u^h_{\mathrm{a},t} & u^c_{\mathrm{a},t} \end{bmatrix}^\top \in \mathbb{R}^2$ is the control vector, and $a^{\mathrm{a}}_i, b^{\mathrm{a}}_i$ can be obtained via (1). An important operational

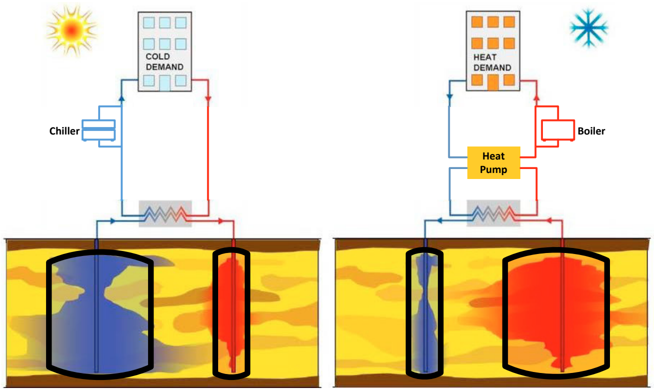

Fig. 1. Heating and cooling operating modes of building climate comfort system with an ATES system during warm (left) and cold (right) seasons.

limitation of ATES systems is that the sum of injected and extracted thermal energy over a specific period of time (typically a year), N_y, has to be zero:

$$\sum\nolimits_{t=k}^{N_y} (h^c_{\mathrm{a},t} - c^c_{\mathrm{a},t}) = 0 \ , \ \sum\nolimits_{t=k}^{N_y} (c^h_{\mathrm{a},t} - h^h_{\mathrm{a},t}) = 0 \ . \quad (4)$$

This restriction imposed by the government to prevent any long-term changes/effects in the aquifer ambient temperature and to make the ATES system sustainable Bloemendal et al. (2014). These constraints should be satisfied within one calendar year or longer periods of time (once in each five years). To handle such a constraint, one can use shrinking-horizon dynamic programming Skaf et al. (2010). In our proposed model (1), the amount of thermal energy content in each well is defined to be the state variable of an ATES system. This yields an advantage to reformulate (4) as follows:

$$\mathrm{S}^h_{\mathrm{a},t} + \mathrm{S}^c_{\mathrm{a},t} \leq \bar{\mathrm{S}}_\mathrm{a} + e_{i,t} \ , \ \mathrm{S}^h_{\mathrm{a},t} + \mathrm{S}^c_{\mathrm{a},t} \geq \bar{\mathrm{S}}_\mathrm{a} - e_{i,t} \ , \quad (5)$$

where $\bar{\mathrm{S}}_\mathrm{a}$ corresponds to the initial amount of thermal energy in the wells of ATES system, $e_{i,t}$ is introduced as an auxiliary control variable for each agent i at each sampling time t to soften the formulated constraint (5). It is important to mention that the proposed reformulation (5) is not meant to be an equivalent constraint as (4). This is due to the fact that (4) has to be satisfied within a longer period of time, whereas (5) is imposed along the prediction horizon. We however state here that (5) may be equivalent with (4), whenever the prediction horizon is long enough (a year) and it is imposed only at the final step.

Define $u^h_{i,t} = \left[h_{\mathrm{boi},t} \ h_{\mathrm{im},t}\right]^\top \in \mathbb{R}^2$, to be the vector of control variables during heating, $u^c_{i,t} = \left[c_{\mathrm{chi},t} \ c_{\mathrm{im},t}\right]^\top \in \mathbb{R}^2$ to be the vector of control variables during cooling mode in each agent i at each sampling time t, respectively. $h_{\mathrm{boi},t}$, $c_{\mathrm{chi},t}$, $h_{\mathrm{im},t}$, and $c_{\mathrm{im},t}$ denote the production of boiler, chiller, the imported energies from external parties during heating and cooling modes, respectively. We also consider to have freedom to decide about the on-off status of boiler and chiller by $v_{i,t} = \left[v_{\mathrm{boi},t} \ v_{\mathrm{chi},t}\right]^\top \in \{0,1\}^2$. Moreover, the startup cost of boiler and chiller are taken into account by $c^{\mathrm{su}}_{i,t} = \left[c^{\mathrm{su}}_{\mathrm{boi},t} \ c^{\mathrm{su}}_{\mathrm{chi},t}\right]^\top \in \mathbb{R}^2$ for each agent i at each time step t. Consider $x^h_{i,t} = h_{\mathrm{s},t} - h^{\mathrm{f}}_{\mathrm{d},t}$, and $x^c_{i,t} = c_{\mathrm{s},t} - c^{\mathrm{f}}_{\mathrm{d},t}$ to be the imbalance errors of heating and cooling modes in each agent i at time step t. $h^{\mathrm{f}}_{\mathrm{d},t}$, $c^{\mathrm{f}}_{\mathrm{d},t}$, $h_{\mathrm{s},t}$, and $c_{\mathrm{s},t}$ represent the forecast of thermal energy demand, the level of storage tank during heating and cooling modes, respectively. $h_{\mathrm{s},t}$, and $c_{\mathrm{s},t}$ have the following dynamics:

$$h_{\mathrm{s},t+1} = \eta^h_\mathrm{s} x^h_{i,t} + \eta^h_\mathrm{s} \left(h_{\mathrm{boi},t} + h_{\mathrm{im},t} + \alpha_{\mathrm{hp}} h_{\mathrm{a},t}\right),$$
$$c_{\mathrm{s},t+1} = \eta^c_\mathrm{s} x^c_{i,t} + \eta^c_\mathrm{s} \left(c_{\mathrm{chi},t} + c_{\mathrm{im},t} + c_{\mathrm{a},t}\right),$$

where η^h_s, $\eta^c_\mathrm{s} \in (0,1)$ are the thermal loss coefficients, and $\alpha_{\mathrm{hp}} = \mathrm{COP}(\mathrm{COP}-1)^{-1}$ represents the effect of heat pump. COP stands for the coefficient of performance of heat pump. Substituting $h_{\mathrm{s},t}$, $c_{\mathrm{s},t}$ into $x^h_{i,t}$, $x^c_{i,t}$, we derive the dynamical behavior of imbalance errors:

$$x^h_{i,t+1} = a^h_i x^h_{i,t} + b^h_i u^h_{i,t} + b^h_{i,\mathrm{a}} u^{\mathrm{a}}_{i,t} + c^h_i w^h_{i,t} \ , \quad (6a)$$
$$x^c_{i,t+1} = a^c_i x^c_{i,t} + b^c_i u^c_{i,t} + b^c_{i,\mathrm{a}} u^{\mathrm{a}}_{i,t} + c^c_i w^c_{i,t} \ , \quad (6b)$$

where $a^h_i = \eta^h_\mathrm{s}$, $a^c_i = \eta^c_\mathrm{s}$, $b^h_i = \left[\eta^h_\mathrm{s} \ \eta^h_\mathrm{s}\right] \in \mathbb{R}^{1\times 2}$, $b^c_i = \left[\eta^c_\mathrm{s} \ \eta^c_\mathrm{s}\right] \in \mathbb{R}^{1\times 2}$, $b^h_{i,\mathrm{a}} = \left[\eta^h_\mathrm{s}\alpha_{\mathrm{hp}}\,\alpha \ 0\right] \in \mathbb{R}^{1\times 2}$, $b^h_{i,\mathrm{a}} = \left[\eta^c_\mathrm{s}\,\alpha \ 0\right] \in \mathbb{R}^{1\times 2}$, $c^h_i = -1$, and $c^c_i = -1$. The variables $w^h_{i,t} = h^{\mathrm{f}}_{\mathrm{d},t+1}$ and $w^c_{i,t} = c^{\mathrm{f}}_{\mathrm{d},t+1}$ refer to the forecast of thermal energy demand during heating and cooling modes in the next time step, respectively. The only uncertain variable in each agent i is considered to be the deviation of actual thermal energy demand from its forecast value, and therefore, $w^h_{i,t}$ and $w^c_{i,t}$ represent uncertain parameters. For each agent i the system dynamics can be written as:

$$x_{i,t+1} = a_i x_{i,t} + b_i u_{i,t} + c_i w_{i,t} \ , \quad (7)$$

where $x_{i,t} = \left[x^{h\top}_{i,t} \ x^{c\top}_{i,t} \ x^{\mathrm{a}\top}_{i,t}\right]^\top \in \mathbb{R}^6$ denotes the state vector, $u_{i,t} = \left[u^{h\top}_{i,t} \ u^{c\top}_{i,t} \ u^{\mathrm{a}\top}_{i,t} \ c^{\mathrm{su}\top}_{i,t} \ e_{i,t}\right]^\top \in \mathbb{R}^9$ is the control vector, and $w_{i,t} = \left[w^h_{i,t} \ w^c_{i,t}\right]^\top \in \mathbb{R}^2$ is the uncertainty vector. The system parameters a_i, b_i, c_i, can be readily derived from their definitions and we omit them in the interest of space.

We are now in a position to formulate an optimization problem for each agent i at each sampling time t. We however refer the interested reader to the formulation in (Rostampour and Keviczky, 2016, Problem 3) for the detailed representation of constraints, such as the status change of production units (boiler, chiller), limitations on the production capacity (box constraints), together with the constraints in (5). We here associate a quadratic cost function with each agent i at each sampling time t as follows:

$$J_i(x_{i,t}, u_{i,t}) = x^\top_{i,t}\, Q_i\, x_{i,t} + u^\top_{i,t}\, R_i\, u_{i,t} \ , \quad (8)$$

where $Q_i = \mathrm{diag}\left(\left[q^h_i \ q^c_i \ \mathbf{0}_{1\times 4}\right]\right) \in \mathbb{R}^{6\times 6}$, $R_i = \mathrm{diag}\left(r_i\right) \in \mathbb{R}^{9\times 9}$ denote diagonal matrices with the weighting coefficients of imbalance errors, and the cost vector $r_i = \left[r_{\mathrm{boi}} \ r^h_{\mathrm{im}} \ r_{\mathrm{chi}} \ r^c_{\mathrm{im}} \ r^h_\mathrm{a} \ r^c_\mathrm{a} \ 1 \ 1 \ 1\right]^\top \in \mathbb{R}^9$, on their diagonals, respectively. r_{boi} (r_{chi}) relates to the cost of natural gas that is used by boiler (chiller), r^h_{im} (r^c_{im}) denotes to the cost of imported thermal energy from an external party during heating (cooling) mode, and r^h_a (r^c_a) corresponds to the electricity cost of pump of ATES system to extract the required thermal energy during heating (cooling) modes. Consider $\boldsymbol{x}_i \in \mathbb{R}^{6N_h = n_x}$, $\boldsymbol{u}_i \in \mathbb{R}^{9N_h = n_u}$, $\boldsymbol{v}_i \in \mathbb{R}^{2N_h = n_v}$, and $\boldsymbol{w}_i \in \mathbb{R}^{2N_h = n_w}$ to be the concatenated vectors of state, control input, binary variables, and uncertain variables along the prediction horizon of each agent i, respectively. Note that $\boldsymbol{w}_i$ is a possible realization (scenario) of the uncertainty for agent i throughout a finite-horizon. The total cost function $\mathcal{J}_i(\boldsymbol{x}_i, \boldsymbol{u}_i)$ for the full prediction horizon at each sampling time t is given by

$$\mathcal{J}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) = \boldsymbol{x}_i^\top \boldsymbol{Q}_i \boldsymbol{x}_i + \boldsymbol{u}_i^\top \boldsymbol{R}_i \boldsymbol{u}_i \ ,$$

where $\boldsymbol{Q}_i$ and $\boldsymbol{R}_i$ are two block diagonal matrices with Q_i and R_i on the diagonal for each agent i. We are now able to formulate a finite-horizon chance-constrained mixed-integer quadratic optimization problem for each agent $i = 1, \cdots, N$, in a condensed format:

$$\min_{\boldsymbol{u}_i, \boldsymbol{v}_i} \quad \mathcal{V}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) = \mathbb{E}_{\boldsymbol{w}_i}\big[\mathcal{J}_i(\boldsymbol{x}_i, \boldsymbol{u}_i)\big] \tag{9a}$$

$$\text{s.t.} \quad E_i \boldsymbol{u}_i + F_i \boldsymbol{v}_i + P_i \leq 0, \tag{9b}$$

$$\mathbb{P}_{\boldsymbol{w}_i}\big[A_i x_{i,k} + B_i \boldsymbol{u}_i + C_i \boldsymbol{w}_i \geq 0\big] \geq 1 - \varepsilon_i \ , \tag{9c}$$

$$\forall \boldsymbol{w}_i \in \mathcal{W}_i \ , \tag{9d}$$

where E_i , F_i , P_i are matrices that are built by concatenating all constraints, and $\varepsilon_i \in (0, 1)$ is the admissible constraint violation parameter. Note that given an initial state vector $x_{i,k}$ at each sampling time t, we eliminate the state variables from the dynamics (7), and obtain the system dynamics as in (9c). The exact form of A_i, B_i and C_i matrices are omitted in the interest of space and can be found in (Borrelli et al., 2011, Section 9.5).

Assumption 4. Following Remark 2, $\boldsymbol{w}_i$, is defined on some probability space $(\mathcal{W}_i, \mathfrak{B}(\mathcal{W}_i), \mathbb{P}_{\boldsymbol{w}_i})$, where $\mathcal{W}_i \subseteq \mathbb{R}^{n_w}$, $\mathfrak{B}(\cdot)$ denotes a Borel σ-algebra, and $\mathbb{P}_{\boldsymbol{w}_i}$ is a probability measure defined over $\mathcal{W}_i$.

Remark 5. $\mathcal{J}_i(\cdot)$ is a random variable, and thus, we consider $\mathbb{E}_{\boldsymbol{w}_i}[\mathcal{J}_i(\cdot)]$ to obtain a deterministic cost function.

Remark 6. The index of $\mathbb{E}_{\boldsymbol{w}_i}, \mathbb{P}_{\boldsymbol{w}_i}$ denotes the dependency of the state trajectory $\boldsymbol{x}_i$ on the string of random scenarios $\boldsymbol{w}_i$ for each agent i. It is worth to mention that for our study we only need a finite number of instances of $\boldsymbol{w}_i$, and we do not require the probability space $\mathcal{W}_i$ and the probability measure $\mathbb{P}_{\boldsymbol{w}_i}$ to be known explicitly. The availability of the number of scenarios from the sample space $\mathcal{W}_i$ is enough which can for instance be obtained from historical data.

We refer to the proposed optimization problem (9) as a single agent optimization problem, and whenever all agents solve this problem separately in a receding horizon fashion without any coupling constraints, it is referred to as the *decoupled solution* (DS) in the subsequent parts. It is important to notice that the proposed problem (9) is in general a non-convex problem and hard to solve. In the following section, we will develop a tractable framework to obtain an ε_i-feasible solution for each agent i.

3.2 ATES in Smart Thermal Grids

Consider a regional thermal grid consisting of N agents with heterogeneous parameters as it was developed in the previous part. Such a STG setting however can lead to unwanted mutual interactions between ATES systems as it is illustrated in Fig. 2. We therefore need to introduce a proper coupling constraint between neighboring agents, that makes use of the following assumption.

Assumption 7. Each well of an ATES system is considered as a growing reservoir with respect to the horizontal axis (Fig. 1, black solid line). We therefore assume to have a cylindrical reservoir with a fixed height ℓ[m] (filter screen length) and a growing radius $r^h_{\mathrm{a},t}$, $r^c_{\mathrm{a},t}$[m] (thermal radius) for each well of an ATES system.

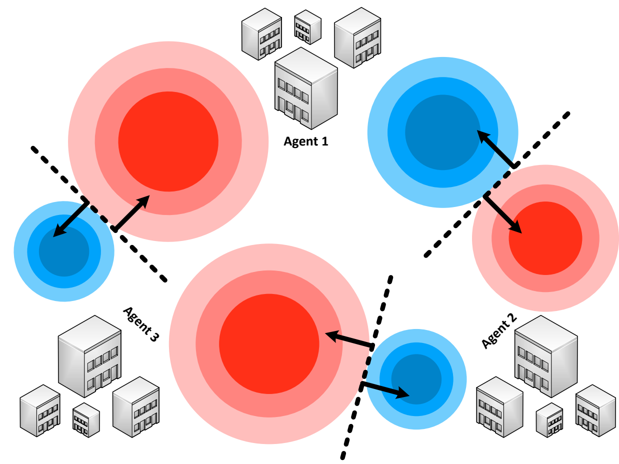

Fig. 2. Three-agent ATES system in a STG. Each agent has a single ATES system which consists of a warm and a cold well. Horizontal cross sections of warm and cold wells are shown with red and blue circles. The black dashed lines represent the unwanted mutual interactions between neighboring ATES systems.

Using the volume of stored water in each well of ATES system, one can determine the thermal radius using:

$$r^h_{\mathrm{a},t} = \left(\frac{c_{pw}\,\mathrm{V}^h_{\mathrm{a},t}}{c_{aq}\,\pi\ell}\right)^{0.5} \ , \ r^c_{\mathrm{a},t} = \left(\frac{c_{pw}\,\mathrm{V}^c_{\mathrm{a},t}}{c_{aq}\,\pi\ell}\right)^{0.5} \ , \tag{10}$$

where $c_{aq} = (1 - n_p)c_{\text{sand}} + n_p c_{pw}$ is the aquifer heat capacity. c_{sand} $[\mathrm{Jkg}^{-1}\mathrm{K}^{-1}]$ relates to the sand specific heat capacity, and n_p $[-]$ is the porosity of aquifer. Let us now define the set of neighbors of agent i by $\mathcal{N}_i \subseteq \{1, 2, \cdots, N\} \setminus \{i\}$. We impose a limit on the thermal radius of warm well $r^h_{\mathrm{a},t}$ and cold well $r^c_{\mathrm{a},t}$ of ATES system in each agent i, based on the corresponding wells of its neighbor $j \in \mathcal{N}_i$:

$$(r^h_{\mathrm{a},t})_i + (r^c_{\mathrm{a},t})_j \leq d_{ij} \ , \ j \in \mathcal{N}_i \ , \tag{11}$$

where d_{ij} is a given distance between agent i and its neighbor $j \in \mathcal{N}_i$. This constraint prevents overlapping between the growing domains of warm and cold wells of ATES systems in a STG setting. Due to the nonlinear transformation in (10), we propose the following reformulation of this constraint to simplify the problem:

$$(\mathrm{V}^h_{\mathrm{a},t})_i + (\mathrm{V}^c_{\mathrm{a},t})_j \leq \mathrm{V}_{ij} - \bar{\delta}_{ij,t} \ , \tag{12}$$

where $\mathrm{V}_{ij} = c_{aq}\pi\ell\,(d_{ij})^2/c_{pw}$ denotes the total volume of common resource pool between agent i and its neighbor $j \in \mathcal{N}_i$. $\bar{\delta}_{ij,t} = 2c_{aq}\pi\ell\,(\bar{r}^h_{\mathrm{a},t})_i\,(\bar{r}^c_{\mathrm{a},t})_j/c_{pw}$ represents a time-varying parameter that captures the mismatch between the linear and nonlinear constraint relations. The following corollary is a direct result of the above reformulation.

Corollary 8. If $(\bar{r}^h_{\mathrm{a},t})_i$ and $(\bar{r}^c_{\mathrm{a},t})_j$ represent the current thermal radius of warm and cold wells of ATES system in agent i and j, respectively, then constraints (11) and (12) are equivalent.

Proof. The proof is straightforward by substituting the corresponding relationships. Notice that the thermal radius is always greater than or equal to zero $(r^h_{\mathrm{a},t})_i \geq 0$, $\forall i \in \{1, \cdots, N\}$ and thus, $\bar{\delta}_{ij,t} \geq 0$. □

Definition 9. We define $\delta_{ij,t}$ to be a common uncertainty source between each agent i and its neighboring agent $j \in \mathcal{N}_i$, using the following model:

$$\delta_{ij,t} := \bar{\delta}_{ij,t}\,(1 \pm 0.1\,\zeta)\,, \tag{13}$$

where ζ is a random variable defined on some probability space, $\bar{\delta}_{ij,t}$ is constructed by using two given possible $(\bar{r}^h_{\mathrm{a},t})_i\,, (\bar{r}^c_{\mathrm{a},t})_j$ realizations that can be obtained using historical data in the DS framework. Since the mapping (13) from ζ to $\delta_{ij,t}$ is measurable, one can view $\delta_{ij,t}$ as a random variable on the same probability space as ζ.

3.3 Problem Formulation in Multi-Agent Network

We now formulate the energy management problem for ATES systems in STGs as follows:

$$\min_{\{\boldsymbol{u}_i,\boldsymbol{v}_i\}_{i=1}^N} \sum_{i=1}^{N} \mathcal{V}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \tag{14a}$$

$$\text{s.t.} \quad E_i\boldsymbol{u}_i + F_i\boldsymbol{v}_i + P_i \le 0\,, \tag{14b}$$

$$\mathbb{P}_{\boldsymbol{w}_i}\Big[A_i x_{i,k} + B_i\boldsymbol{u}_i + C_i\boldsymbol{w}_i \ge 0\Big] \ge 1-\varepsilon_i\,, \tag{14c}$$

$$\mathbb{P}_{\boldsymbol{\delta}_{ij}}\Big[H_i\boldsymbol{x}_i + H_j\boldsymbol{x}_j \le \bar{V}_{ij} - \boldsymbol{\delta}_{ij}\Big] \ge 1-\bar{\varepsilon}_{ij}\,, \tag{14d}$$

$$\forall \boldsymbol{w}_i \in \mathcal{W}_i\,, \forall \boldsymbol{\delta}_{ij} \in \Delta_{ij}\,, \forall j \in \mathcal{N}_i\,, \quad \forall i \in \{1,2,\cdots,N\}\,, \tag{14e}$$

where H_i, H_j are matrices of appropriate dimensions, $\bar{V}_{ij} \in \mathbb{R}^{N_h}$ is the upper-bound on the total common resource pool, $\boldsymbol{\delta}_{ij}$ is a vector of common uncertainty variables, and $\bar{\varepsilon}_{ij} \in (0,1)$ denotes the level of admissible coupling constraint violation, for each agent i and $\forall j \in \mathcal{N}_i$. $\bar{V}_{ij}$ can be expressed as $\bar{V}_{ij} = \mathbf{1}^{N_h} \bigotimes V_{ij}$, using the Kronecker product. It is important to notice that the index of $\mathbb{P}_{\boldsymbol{\delta}_{ij}}$ denotes the dependency of the state trajectories on the string of random common scenarios $\boldsymbol{\delta}_{ij} = [\delta_{ij,k}, \delta_{ij,k+1}, \cdots, \delta_{ij,k+N_h}] \subseteq \mathbb{R}^{N_h = n_\delta}$.

Assumption 10. Following Definition 9, $\boldsymbol{\delta}_{ij}$ is defined on some probability space $(\Delta_{ij}, \mathfrak{B}(\Delta_{ij}), \mathbb{P}_{\boldsymbol{\delta}_{ij}})$, where $\Delta_{ij} \subseteq \mathbb{R}^{n_\delta}$, $\mathfrak{B}(\cdot)$ denotes a Borel σ-algebra, and $\mathbb{P}_{\boldsymbol{\delta}_{ij}}$ is a probability measure defined over Δ_{ij}.

Assumption 11. $\boldsymbol{w}_i \in \mathbb{R}^{n_w}$ and $\boldsymbol{\delta}_{ij} \in \mathbb{R}^{n_\delta}$ are two independent string of random scenarios from two disjoint probability space $\mathcal{W}_i$ and Δ_{ij}, respectively.

We refer to the proposed optimization problem (14) as a multi-agent network problem, and whenever the proposed problem (14) is solved in a receding horizon fashion, it is mentioned as the *centralized solution* (CS) in the following parts. However, the feasible set in (14) is in general non-convex and hard to determine explicitly due to the presence of chance constraints (14c), (14d). In what follows, we develop a tractable framework to obtain probabilistically feasible solutions for all agents.

4. COMPUTATIONALLY TRACTABLE FRAMEWORK

Consider $\boldsymbol{y}_i = (\boldsymbol{u}_i, \boldsymbol{v}_i) \in \mathbb{R}^{(n_u+n_v)=n_y}$, $\boldsymbol{y} = \mathrm{col}(\boldsymbol{y}_i)_{i=1}^N$, where $\mathrm{col}(\cdot)$ is an operator to stack elements. Define $\boldsymbol{w} = \mathrm{col}(\boldsymbol{w}_i)_{i=1}^N \subseteq \mathcal{W}$ to be the private uncertainty sources for a network of agents, $\boldsymbol{\delta}_i = \mathrm{col}(\boldsymbol{\delta}_j)_{j\in\mathcal{N}_i} \subseteq \Delta_i$ to be the common uncertainty sources for each agent, and $\boldsymbol{\delta} = \mathrm{col}(\boldsymbol{\delta}_i)_{i=1}^N \subseteq \Delta$ to be the common uncertainty sources for a multi-agent network, where

$$\mathcal{W} := \prod_{i=1}^{N} \mathcal{W}_i\ ,\ \Delta_i := \prod_{j\in\mathcal{N}_i} \Delta_{ij}\ ,\ \Delta := \prod_{i=1}^{N} \Delta_i\ .$$

Consider now the proposed optimization problem in (14) in a more compact format:

$$\min_{\boldsymbol{y}} \quad \sum\nolimits_{i=1}^{N} \mathcal{V}_i(\boldsymbol{x}_i,\boldsymbol{u}_i) \tag{15a}$$

$$\text{s.t.} \quad \mathbb{P}_{\boldsymbol{w}}\left[\boldsymbol{y} \in \prod_{i=1}^{N} \mathcal{Y}_i(\boldsymbol{w}_i)\right] \ge 1-\varepsilon\,, \forall \boldsymbol{w} \in \mathcal{W} \tag{15b}$$

$$\mathbb{P}_{\boldsymbol{\delta}}\left[\boldsymbol{y} \in \prod_{i=1}^{N} \bigcap_{j\in\mathcal{N}_i} \breve{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij})\right] \ge 1-\bar{\varepsilon}\,, \forall \boldsymbol{\delta} \in \Delta \tag{15c}$$

where $\varepsilon := \sum_{i=1}^N \varepsilon_i \in (0,1)$, $\bar{\varepsilon} := \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \bar{\varepsilon}_{ij} \in (0,1)$. $\mathcal{Y}_i(\boldsymbol{w}_i) \in \mathbb{R}^{n_y}$ is the local feasible set of each agent i and can be described by constraints (14b) and (14c). $\mathcal{Y}_{ij}(\boldsymbol{\delta}_{ij}) \in \mathbb{R}^{2n_y}$ is the common feasible set between each agent i and its neighbor $j \in \mathcal{N}_i$ and can be determined via (14d). It is important to notice that $\breve{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) \in \mathbb{R}^{2n_y N_i}$ represents the cylindrical extension[1] of $\mathcal{Y}_{ij}(\boldsymbol{\delta}_{ij})$. In the subsequent parts, we refer to the constraint (15b) as the agents' private chance constraints, and to the constraint (15c) as the agents' common chance constraints.

The proposed formulation (15) is a mixed-integer quadratic optimization problem with multiple chance constraints, due to the binary variables $\{\boldsymbol{v}_i\}_{i=1}^N$, and the chance constraints (15b), (15c). It is worth to mention that the index of $\mathbb{P}_{\boldsymbol{w}}$ and $\mathbb{P}_{\boldsymbol{\delta}}$ denote the dependency on the string of random scenarios $\boldsymbol{w} \in \mathcal{W}$ and $\boldsymbol{\delta} \in \Delta$, respectively.

Building upon our previous work in Rostampour and Keviczky (2016), we extend the so-called robust randomized approach in Margellos et al. (2013) to be more applicable to handle a problem with multiple chance constraints. The proposed optimization problem (15) is a stochastic program with multiple chance constraints, where $\mathbb{P}_{\boldsymbol{w}}$ and $\mathbb{P}_{\boldsymbol{\delta}}$ denote two different probability measures for private uncertainty and common uncertainty sources, respectively. In summary, the formulation in (Margellos et al., 2014, Proposition 1) considered a worst-case chance constraint defined by

$$\max_{k\in N_{\mathrm{MCP}}} \mathbb{P}\left[f_k(\boldsymbol{y},\cdot)\right] \ge 1-\tilde{\varepsilon}\ , \tag{16}$$

where $\tilde{\varepsilon} = \min_{k\in\mathcal{N}_{\mathrm{MCP}}}\{\varepsilon_k\}$, $f_k(\boldsymbol{y},\cdot)$ denotes the k-th chance constraint function, and $\mathcal{N}_{\mathrm{MCP}}$ is the set of indices of chance constraint functions formulated in the proposed optimization problem (15). However, this procedure leads to a considerable amount of conservatism, due to the fact that it requires the solution to satisfy all constraints with the highest probability $1-\tilde{\varepsilon}$. We instead employ the robust randomized approach for each chance constraint function $f_k(\boldsymbol{y},\cdot), k \in \mathcal{N}_{\mathrm{MCP}}$, separately. Our framework is closely related to, albeit different from the approach of Schildbach et al. (2013), since the feasible set in (15) is non-convex. Moreover, the problem formulation in Schildbach et al. (2013) consists of an objective function with multiple chance constraints, in which the terms in objective and

[1] Cylindrical extension simply replicates the membership degrees from the existing dimensions into the new dimensions.

constraints are univariate w.r.t. the decision variables. In contrast the objective function in our problem formulation (15) consists of separable additive components and constraint functions are also separable w.r.t. (15b), (15c) between each agent $i = 1, \cdots, N$ and $\forall j \in \mathcal{N}_i$.

We define $\mathcal{B}_i$, $\bar{\mathcal{B}}_{ij}$ to be two bounded sets for each agent i, respectively. $\mathcal{B}_i$, $\bar{\mathcal{B}}_{ij}$ are assumed to be axis-aligned hyper-rectangular sets. This is not restrictive and any convex set with convex volume could have been chosen instead as in Rostampour et al. (2017a). We parametrize $\mathcal{B}_i(\boldsymbol{\gamma}) := [\overline{\gamma}, \underline{\gamma}]$ by $\boldsymbol{\gamma} = (\overline{\gamma}, \underline{\gamma}) \in \mathbb{R}^{2n_w}$, and $\bar{\mathcal{B}}_{ij}(\boldsymbol{\lambda}) := [\overline{\boldsymbol{\lambda}}, \underline{\boldsymbol{\lambda}}]$ by $\boldsymbol{\lambda} = (\overline{\boldsymbol{\lambda}}, \underline{\boldsymbol{\lambda}}) \in \mathbb{R}^{2n_\delta}$, and formulate two chance-constrained problems similarly to (Rostampour and Keviczky, 2016, Problem 8). Following the so-called scenario approach in Calafiore and Campi (2006), one can determine the number of required uncertainty scenarios to formulate a tractable problem, (Rostampour and Keviczky, 2016, Problem 9), using $N_s = \frac{2}{\epsilon}(\xi + \ln \frac{1}{\nu})$, where ξ is the dimension of decision vector, ϵ, ν are the level of violation, and the confidence level, respectively. The optimal solutions $(\boldsymbol{\gamma}^*, \boldsymbol{\lambda}^*)$ of the proposed tractable problem are probabilistically feasible for the chance-constrained problems, (Campi and Garatti, 2008, Theorem 1). Moreover, $\boldsymbol{\gamma}^*$, and $\boldsymbol{\lambda}^*$ also characterize our desired probabilistic bounded sets $\mathcal{B}_i^*$ and $\bar{\mathcal{B}}_{ij}^*$, respectively.

Assumption 12. $\mathcal{S}_i$ and $\bar{\mathcal{S}}_{ij}$ are two collections of random scenarios that are i.i.d.

After determining $\mathcal{B}_i^*$ and $\bar{\mathcal{B}}_{ij}^*$ for all agents $i \in \{1, \cdots, N\}$, we are now able to reformulate the robust counterpart of the original problem (15) via:

$$\min_{\boldsymbol{y}} \quad \sum_{i=1}^{N} \mathcal{V}_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \tag{17a}$$

$$\text{s.t.} \quad \boldsymbol{y} \in \prod_{i=1}^{N} \bigcap_{\boldsymbol{w}_i \in \{\mathcal{B}_i^* \bigcap \mathcal{W}_i\}} \mathcal{Y}_i(\boldsymbol{w}_i) \,, \tag{17b}$$

$$\boldsymbol{y} \in \prod_{i=1}^{N} \bigcap_{j \in \mathcal{N}_i} \bigcap_{\boldsymbol{\delta}_{ij} \in \{\bar{\mathcal{B}}_{ij}^* \bigcap \Delta_{ij}\}} \breve{\mathcal{Y}}_{ij}(\boldsymbol{\delta}_{ij}) \,. \tag{17c}$$

Note that the aforementioned problem is not a randomized program, and instead, the constraints have to be satisfied for all values of the private uncertainty in $\{\mathcal{B}_i^* \bigcap \mathcal{W}_i\}$, and common uncertainty in $\{\bar{\mathcal{B}}_{ij}^* \bigcap \Delta_{ij}\}$. The proposed problem (17) is a robust mixed-integer quadratic program. In Bertsimas and Sim (2006), it was shown that the robust problems are tractable and remain in the same class as the original problems, e.g. robust mixed-integer programs remain mixed-integer programs, for a certain class of uncertainty sets, such as in our problem (17), the uncertainty is bounded in a convex set. The following theorem quantifies the robustness of solution obtained by (17) w.r.t. the initial problem (15).

Theorem 13. Let $\varepsilon_i, \bar{\varepsilon}_i, \bar{\varepsilon}_{ij}, \varepsilon, \bar{\varepsilon}, \beta_i, \bar{\beta}_i, \bar{\beta}_{ij}, \beta, \bar{\beta} \in (0,1)$, $\forall j \in \mathcal{N}_i, i = 1, 2, \cdots, N$ be chosen such that $\varepsilon = \sum_{i=1}^{N} \varepsilon_i$, $\beta = \sum_{i=1}^{N} \beta_i$, $\bar{\varepsilon}_i = \sum_{j \in \mathcal{N}_i} \bar{\varepsilon}_{ij}$, $\bar{\beta}_i = \sum_{j \in \mathcal{N}_i} \bar{\beta}_{ij}$, and $\bar{\varepsilon} = \sum_{i=1}^{N} \bar{\varepsilon}_i$, $\bar{\beta} = \sum_{i=1}^{N} \bar{\beta}_i$. If $\boldsymbol{y}_s^*$ is a feasible solution of the problem (17), then $\boldsymbol{y}_s^*$ is also a feasible solution for the chance constraints (15b) and (15c), with the confidence levels of $1 - \beta$ and $1 - \bar{\beta}$, respectively.

Due to space limitation the proof is omitted, the reader is referred to Rostampour and Keviczky (2017).

Remark 14. Following the approach in Rostampour et al. (2015), we approximate the objective function empirically for each agent i. $\mathbb{E}_{\boldsymbol{w}_i}[\mathcal{J}_i(\cdot)]$ can be approximated by averaging the value of its argument for some number of different scenarios, which plays a tuning parameter role. To improve the objective value of our proposed formulation, one can employ scenario removal algorithms, leading a tradeoff between feasibility and optimality, see e.g. Mohajerin Esfahani et al. (2015); Campi and Garatti (2011).

Remark 15. A tractable formulation for DS framework in (9), can be achieved by removing the robust coupling constraint (17c) from the tractable problem (17). Notice that, since there is no longer a coupling constraint, each agent i can solve its problem, separately.

Remark 16. The solution of (17) is the optimal planned input sequence $\{u_{i,k}^*, v_{i,k}^*, \cdots, u_{i,k+N_h}^*, v_{i,k+N_h}^*\}_{i=1}^N$. Based on an MPC paradigm, the current input is implemented as $\{u_{i,t}, v_{i,t}\}_{i=1}^N := \{u_{i,k}^*, v_{i,k}^*\}_{i=1}^N$ and we proceed in a receding horizon fashion. This means (17) is solved at each step t by using the current measurement of the state $\{x_{i,k}\}_{i=1}^N$.

5. NUMERICAL STUDY

5.1 Simulation Setup

We simulate three problem formulations, namely: DS, and CS, using the tractable framework (17). The simulation time is one year with hourly-based sampling time. The prediction horizon for DS and CS is considered to be a day-ahead (24 hours). For comparison purposes, we also simulate a deterministic DS (DDS), where we fixed the uncertain elements $(\boldsymbol{w}_i)$ to their forecast value for each agent $i = 1, 2, 3$.

In order to generate random scenarios from the private uncertainty sources, we use a discrete normal stochastic process, where the thermal energy demand of each building varies within 10% of its actual value at each sampling time. A similar technique is used for the common uncertainty sources. The simulation environment was MATLAB with YALMIP as the interface Löfberg (2004) and Gurobi as the solver.

5.2 Simulation Results

Fig. 3 depicts an a-posteriori feasibility validation of the obtained results via DDS, DS, and CS, formulations for the three-agent ATES-STG example. Fig. 3 (a)-(b) present the results of thermal energy imbalance during heating mode in agent 1, whereas Fig. 3 (c) shows the feasibility of the coupling constraint between agent 1 and 2. Fig. 3 (a) shows the obtained results for the last five days in March 2011, and Fig. 3 (b) shows the results for one year simulation from June 2010 until June 2011. In Fig. 3 (a)-(b) the "red" color denotes the solution of DDS, and "black" color shows the solution of CS.

Fig. 3 (a) focuses on a five-day period to allow a better comparison between the results of DDS, and CS. It is

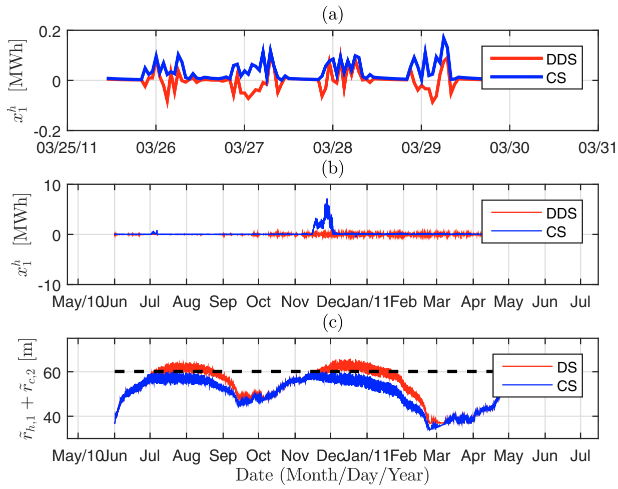

Fig. 3. A-posteriori feasibility validation of the obtained results via DDS, DS, and CS, formulations for the three-agent ATES-STG example.

clearly shown that the obtained results via CS provide a feasible (nonnegative) trajectory of the thermal energy imbalance error during heating mode, whereas the solution of DDS, leads to some violations throughout the simulation time. In Fig. 3 (b), the complete one year results of DDS, and CS, are shown. Two important observations are as follows: the obtained results of CS have very small number of violations, much less than our desired level of violations, throughout the simulation time. This yields a less conservative approach compared to the classical robust control approach (see (Borrelli et al., 2011, Ch.14)). As the second observation, in the results of CS one can see a large non-zero imbalance error, which is expected. By taking into account the coupling constraints between agents, the solutions of agents are going to extract the stored thermal energy from their ATES systems to prevent the mutual interactions between their ATES systems as in Fig. 3 (c). Fig. 3 (c) shows the evaluation of our proposed reformulation in (12). We plot the obtained $\tilde{r}_{h,1}+\tilde{r}_{c,2}$ using DS, and CS formulations. As it is clearly shown DS results are violating the coupling constraint which leads to overlap between the stored water in warm well of ATES system in agent 1 and the stored water in cold well of ATES system in agent 2.

It is worth to mention that Fig. 3 illustrates our two main contributions: 1) having a probabilistic feasible solution for each agent w.r.t. the private uncertainty sources as it is encoded via (15b), and 2) respecting almost surely the common resource pool between neighboring agents in STGs as it is formulated in (15c); the first and second outcomes are the direct results of our theoretical guarantee in Theorem 13. An important observation is that one can use a longer prediction horizon which yields an anticipatory control decision to improve the operation of an ATES system. This is a subject of our current research direction to improve our proposed control strategies.

Fig. 4 shows the results of a simulation study using a more realistic aquifer simulation environment (Harbaugh, 2005, MODFLOW) to validate our developed framework. The impact of our control strategy, DS (red) and CS (blue), on average thermal energy efficiency in each building is

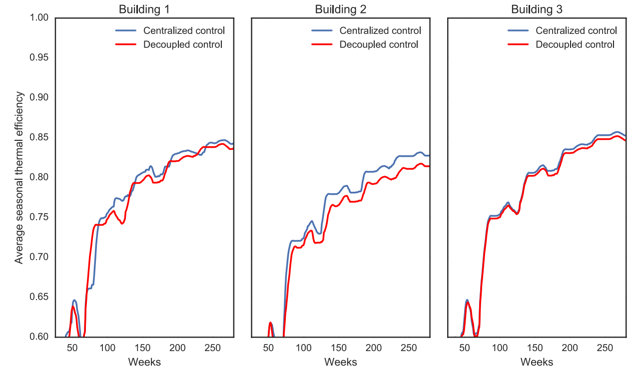

Fig. 4. Impact of DS and CS on average thermal efficiency.

shown in Fig. 4, which illustrates that we can store and retrieve the same amount of thermal energy in ATES systems, in a more efficient way using the results of CS formulation compared to DS. This is due to the fact that the mutual interactions between wells lead to the loss of stored thermal energy, which can be prevented using the CS formulation.

6. CONCLUSIONS

This paper proposed a stochastic MPC framework for an energy management problem in STGs consisting of ATES systems integrated into building climate comfort systems. We developed a large-scale stochastic hybrid model to capture thermal energy imbalance errors in an ATES-STG. In such a framework, we formalized two important practical concerns, namely: 1) the balance between extraction and injection of energy from and into the aquifers within a certain period of time; 2) the unwanted mutual interaction between ATES systems in STGs. Using our developed model, we formulated a finite-horizon mixed-integer quadratic optimization problem with multiple chance constraints. To solve such a problem, we proposed a tractable formulation based on the so-called robust randomized approach. In particular, we extended this approach to handle a problem with multiple chance constraints. We simulated our proposed framework using a three-agent ATES-STG example which confirmed the expected performance improvements.

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