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Mathematical Models for Air Traffic Conflict and Collision Probability Estimation

Mihaela Mitici and Henk A.P. Blom (Fellow IEEE)

***Abstract*—Increasing traffic demands and technological developments provide novel design opportunities for future Air Traffic Management (ATM). In order to evaluate current air traffic operations and future designs, over the past decades several mathematical models have been proposed for air traffic conflict and collision probability estimation. However, few comparative evaluations of these models with respect to their mathematical core exist. Such comparative evaluations are particularly difficult since different authors employ different model definitions, notations and assumptions, even when using the same modeling techniques. The aim of this paper is: i) to present the mathematical core of the existing approaches for air traffic conflict and collision probability estimation using the same body of notations and definitions; ii) to outline the advances in estimating the probability of air traffic conflict and collision using a unified mathematical framework; iii) to various air traffic applications and their use of directed mathematical models for air traffic conflict and collision probability estimation; and iv) to provide insight into the capabilities and restrictions of the mathematical models in the evaluation of future ATM designs.**

***Index Terms*—air traffic, collision, conflict, aircraft, risk, simulation**

I. Introduction

AIR Traffic Management (ATM) involves a Cyber-Physical-Social System (CPSS) that consists of many complex technical systems as well as well-trained human (e.g. pilots, controllers). Through decades of evolutionary development this CPSS system has become very safe. Estimating the probability of air traffic conflict and collision has always played an important role in this development, and is further growing in importance with the development of future, intelligent air transportation. In literature, several models for air traffic conflict and collision probability modeling and estimation have been proposed. Typically, each proposes its own aircraft conflict and collision definitions, notations and assumptions. This makes the comparison between models challenging. In this light, the aim of this paper is: i) to present the mathematical core of the existing approaches for air traffic conflict and collision probability estimation models using the same body of notations and definitions; ii) to outline the advances in estimating the probability of air traffic conflict and collision using a unified mathematical framework; iii) to review various ATM applications on their use of directed air traffic conflict and collision probability estimation models and generic risk models; and iv) to provide insight into the capabilities and restrictions of the mathematical models in the evaluation of future ATM designs.

This paper goes significantly beyond existing surveys on aircraft conflict and collision. [1] gives a comprehensive survey of air traffic conflict detection and resolution systems, whose design relies on the quantification of aircraft conflict and collision probability. [2] provides a high level outline of the main directions in safety risk analysis in aviation, though without modeling details. [3] provides a broad database of safety methods in various safety critical industries, including civil aviation, also without mathematical details. [4] forms a significant exception by giving a mathematical survey of collision risk models that are internationally accepted for safety verification of changes in separation minima. However, this overview does not address more recent safety methods. Moreover, it does not address safety risk modeling of architectural changes in ATM design, for instance moving human responsibilities from ground to air or from human to automation [5]. The aim of this paper is to give a broader perspective in aircraft conflict and collision probability modeling and estimation within a unified mathematical framework.

In the systems control domain the safety risk analysis problem has been characterized in terms of estimating reach probabilities for a stochastic hybrid system and verifying that it is smaller than the applicable safety criterion (e.g., [6]). Both for systems control design and for future ATM design, if analysis shows that a safety critical reach probability is too high for the proposed design, this does not simply imply that the complex design has failed. In most cases, feeding back the safety risk analysis outcomes to a complex design triggers valuable design improvements. A convincing illustration in systems control is recently given by [7] where a safety reachability analysis forms an integral part of optimizing an air traffic conflict detection and resolution algorithm.

Commercial aviation shows such good safety statistics that costly investments in the development of a future ATM design are typically not driven by safety objectives, though by other objectives, such as airspace capacity, flight efficiency and controller productivity. Hence the key challenge in developing a novel ATM design is to realize the non-safety objectives jointly with the safety objectives. This asks for evaluation of a novel ATM design on all key performance areas, including safety risk. Moreover, in contrast to current ATM practice [8], [9], the latter should already start in the early design phase. In the early design phase feedback of the insight gained from safety risk analysis can relative easily trigger design improvements that would be extremely costly when they had to be done in a later design phase [10]. The objective of this paper is to review established and advanced air traffic conflict

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and collision probability estimation models that may be of potential use for safety risk analysis of future ATM designs from the early design phase on.

From a safety modeling and estimation perspective, an ATM design poses various complementary challenges. One challenge is that it is a continuous-time stochastic hybrid system, i.e. one in which interactions happen on a continuous time-line between discrete-valued processes and continuous-valued processes. [11] has shown that this already applies at the level of aircraft control through its Flight Management System (FMS). In this model the continuous-valued process includes for instance the evolution of aircraft position and velocity over time under the influence of a wind field that varies in space and time. The discrete-valued process includes the current FMS mode setting. The evolution of aircraft positions define when two aircraft are in conflict, i.e. their separation falls below minimum separation standards. Subsequent evolution of aircraft positions define when the two aircraft collide, i.e. their physical shapes touch each other. However these evolutions depend on an interaction with FMS mode switching. For instance, upon reaching an FMS desired flight level, the FMS will switch climbing mode to level flying mode. This mode switching subsequently changes the evolution of aircraft position. On top of this hybrid system model by [11] other aircraft systems and the aircraft crew interact with the FMS and the flight evolution. Similarly systems and air traffic controllers on the ground interact with aircraft systems and flight crew. Each time such an extra control layer, with interactions, is modelled, the stochastic hybrid system model of the ATM design grows in complexity.

This paper is organized as follows. Section II provides mathematical definitions of collision and conflict events. Section III reviews models aiming for the estimation of collision and conflict events that happen in the top layer of the ATM design. Subsequently, Section IV outlines models to represent the other layers in the ATM design, including interactions between layers. In doing so, we make use of an extended version of the modeling power hierarchy from [12]. Since the modeling of multiple ATM design layers is complex, in Section V we provide an overview of complementary techniques in managing a structured model development and validation approach. Next, in Section VI we give an outline of various ATM applications of collision and conflict probability estimation models. For each application we identify the specific models used from sections III, IV and V. Section VII summarizes the results and draws conclusions.

II. Definition of conflict and collision events

In ATM, conflicts between aircraft are expressed in terms of overlaps between volumes that are defined around aircraft or, equivalently, in terms of a reduction in the distance between aircraft. A collision is the event when the actual shapes of aircraft touch. In the following, we formally define aircraft conflicts and collision. We also relate these definitions to internationally accepted standards.

A. Aircraft in-crossing and out-crossing of a static volume

In this section we consider in-crossing, conflict and collision events between a moving aircraft and a static volume. Assume a flat Earth geometry where position and velocity vectors are given in Cartesian coordinates. Let $s_t = [s_{1,t}, s_{2,t}, s_{3,t}]$ be the vector of positions of the aircraft at time t in the x, y and z dimensions. Let $v_t = [v_{1,t}, v_{2,t}, v_{3,t}]$ be the vector of velocities of the aircraft at time t in the x, y and z dimensions. We assume that the position vector satisfies the ordinary differential equation $ds_t = v_t dt$, where $\{v_t\}$ is pathwise continuous. Let $V \in \mathbb{R}^3$ denote a finite volume that is an open subset of $\mathbb{R}^3$ and $\bar{V}$ its closure.

Definition 1. *We say that an in-crossing between an aircraft and volume V occurs at a (random) time τ if at time $\tau - \epsilon$, $s_{\tau-\epsilon} \notin V$ and at time τ, $s_\tau \in V$, with $\epsilon \downarrow 0$.*

Definition 2. *We say that an out-crossing between an aircraft and volume V occurs at a (random) time $\tilde{\tau}$ if at time $\tilde{\tau} - \epsilon$, $s_{\tilde{\tau}-\epsilon} \in V$ and at time $\tilde{\tau}$, $s_{\tilde{\tau}} \notin V$, with $\epsilon \downarrow 0$.*

Since in-crossings and out-crossings of a volume V can occur multiple times, we define an increasing sequence of in-crossing and out-crossing times τ_k and $\tilde{\tau}_k$, respectively, $k = 1, 2, \ldots$, as

$$\tau_k = \inf_t \{t > \tilde{\tau}_{k-1} : s_t \in V\} \quad (1)$$

$$\tilde{\tau}_k = \inf_t \{t > \tau_k : s_t \notin V\}, \quad (2)$$

with $\tilde{\tau}_0 = 0$ if $s_0 \notin V$ and $\tau_1 = 0$ if $s_0 \in V$.

When V corresponds to the volume of a static object, then at time τ_1, which is the first in-crossing of V, we say that a *collision* occurs between the aircraft and the object. When V is a defined, protective volume (around a static object), τ_k is the moment of the k-th *conflict* with this protective volume. In the following, we define in-crossing, conflict and collision events between two moving aircraft. In doing so, we consider general aircraft shapes and orientation.

B. In-crossing and conflict events between two aircraft

Let $\{X_t^i, \theta_t^i\}$ be a hybrid-state process related to aircraft i, where X_t^i is an Euclidean valued component that assumes values from $\mathbb{R}^n$ and θ_t^i is a discrete valued component assuming values from a finite set. Some elements of the process $\{X_t^i\}$ are mapped into the position and velocity of the center of aircraft i, i.e., $s_t^i = M_s(X_t^i)$ and $v_t^i = M_v(X_t^i)$, $s_t^i, v_t^i \in \mathbb{R}^3$. Also, $s_t^i = (s_{1,t}^i, s_{2,t}^i, s_{3,t}^i)$, where $s_{1,t}^i$, $s_{2,t}^i$ and $s_{3,t}^i$ denote the position of aircraft i at time t in the three dimensions. Similarly, $v_t^i = (v_{1,t}^i, v_{2,t}^i, v_{3,t}^i)$, where $v_{1,t}^i$, $v_{2,t}^i$ and $v_{3,t}^i$ denote the velocity of aircraft i at time t in the three dimensions, respectively. Let $V(X_t^i)$ denote a 3D open volume around aircraft i at moment t, where $V(X_t^i)$ takes into account the orientation of aircraft i at moment t. Let $s_t^{ij} = s_t^i - s_t^j$ and $v_t^{ij} = v_t^i - v_t^j$ denote the relative position and velocity between aircraft i and j at time t. Let τ_k^{ij} and $\tilde{\tau}_k^{ij}$, $k = 1, 2, \ldots$, denote increasing sequences of in-crossings and out-crossings between $V(X_t^i)$ and $V(X_t^j)$, respectively, i.e.,

$$\tau_k^{ij} = \inf_t \{t > \tilde{\tau}_{k-1}^{ij} : V(X_t^i) \cap V(X_t^j) \neq \emptyset\}, \quad (3a)$$

$$\tilde{\tau}_k^{ij} = \inf_t \{t > \tau_k^{ij} : V(X_t^i) \cap V(X_t^j) = \emptyset\}, \quad (3b)$$

with $\tilde{\tau}_0^{ij} = 0$ if $V(X_0^i) \cap V(X_0^j) = \emptyset$ and $\tau_0^{ij} = 0$ if $V(X_0^i) \cap V(X_0^j) \neq \emptyset$. By definition, $\tau_k^{ij} = \infty$ if the set in (4a) is empty. Similarly, $\tilde{\tau}_k^{ij} = \infty$ if the set in (4b) is empty.

Definition 3. *We say that the k-th in-crossing of volumes around aircraft i and j occurs at a (random) time* τ_k^{ij} *if* $\tilde{\tau}_0^{ij} = 0$ *or, when* $k \geq 2$*, there exists* $\tilde{\tau}_{k-1}^{ij} = \inf_t \{t > \tau_{k-1}^{ij} : V(X_t^i) \cap V(X_t^j) = \emptyset\}$*, and* $\tau_k^{ij} = \inf_t \{t > \tilde{\tau}_{k-1}^{ij} : V(X_t^i) \cap V(X_t^j) \neq \emptyset\}$.

Definition 4. *We say that the k-th out-crossing of volumes around aircraft i and j occurs at a (random) time* $\tilde{\tau}_k^{ij}$ *if* $\tau_1^{ij} = 0$ *or, when* $k \geq 2$*, there exists* $\tau_k^{ij} = \inf_t \{t > \tilde{\tau}_{k-1}^{ij} : V(X_t^i) \cap V(X_t^j) \neq \emptyset\}$*, and* $\tilde{\tau}_k^{ij} = \inf_t \{t > \tau_k^{ij} : V(X_t^i) \cap V(X_t^j) = \emptyset\}$.

Definition 5. *We say that the k-th overlap of volumes* $V(X_t^i)$ *and* $V(X_t^j)$ *starts and stops at (random) times* τ_k^{ij} *and* $\tilde{\tau}_k^{ij}$*, where* τ_k^{ij} *is the time of the k-th in-crossing, as defined in Definition 3, and* $\tilde{\tau}_k^{ij}$ *is the time of the k-th out-crossing, as defined in Definition 4.*

In ATM there are several lateral and vertical criteria in use for the distance between the centers of two aircraft. For the minimum radar separation distance the criteria are $d_\perp^{MS}$ for minimum vertical separation and d_-^{MS} for minimum horizontal separation. If two aircraft centers come closer to each other than both these minimum radar separation distances, then the aircraft are said to be in conflict with each other. The values used depend of the type of airspace and air traffic control capabilities [13]. Typical en-route values are $d_\perp^{MS} = 1000ft$ and $d_-^{MS} = 5NM$(Nautical Mile). When a conflict is expected to occur within the next $\Delta > 0$ time step, then we refer to a *predicted conflict*. For a conflict to be called a serious conflict, lower distance criteria $d_\perp^{SC}$ and d_-^{SC} apply. For example, UK applies a serious conflict distance criteria $d_\perp^{SC} = \frac{2}{3} d_\perp^{MS}$ and $d_-^{SC} = \frac{2}{3} d_-^{MS}$ [14]. For a conflict to be a near mid-air collision (NMAC), the distance criteria commonly used are $d_\perp^{MNAC} = 100ft$ and $d_-^{MNAC} = 500ft$, e.g. [15]. Hence, Definition 5 defines starts and stops of periods of a conflict, of a serious conflict and of an NMAC. Let $V(x^i), x^i \in \mathbb{R}^n$, denote an open cylindrical volume where,

$$V(x^i) = \{(s_1, s_2, s_3) \in \mathbb{R}^3 : (s_1^i, s_2^i, s_3^i) = M_s(x^i), \left[\sqrt{(s_1 - s_1^i)^2 + (s_2 - s_2^i)^2} < \frac{1}{2} d_-\right] \cap \left[|s_3 - s_3^i| < \frac{1}{2} d_\perp\right]\}; \tag{4}$$

then $V(X_t^i)$ is the conflict volume around X_t^i for $d_\perp = d_\perp^{MS}$ and $d_- = d_-^{MS}$. Else, $V(X_t^i)$ is the serious conflict volume around X_t^i for $d_\perp = d_\perp^{SC}$ and $d_- = d_-^{SC}$. Else, $V(X_t^i)$ is the NMAC volume around X_t^i for $d_\perp = d_\perp^{NMAC}$ and $d_- = d_-^{NMAC}$.

Proposition 1: *If* $V(.)$ *is invariant to aircraft orientation, then* $V(X_t^i) \cap V(X_t^j) = \emptyset$ *if and only if* $s^{ij} \notin V(0)$.

C. Collision event

Definition 6. *When* $V(X_t^i)$ *and* $V(X_t^j)$ *are shapes of aircraft i and j, respectively, we say that a collision occurs between aircraft i and j at time* τ_1^{ij} *if aircraft i and j are collision free at the beginning, i.e.,* $\tilde{\tau}_0^{ij} = 0$ *and* τ_1^{ij} *is the first in-crossing of shapes of aircraft i and j, i.e.,*

$$\tau_1^{ij} = \inf_t \{t > \tilde{\tau}_0^{ij} = 0 : V(X_t^i) \cap V(X_t^j) \neq \emptyset\}.$$

Remark 1: When τ_1^{ij} is reached while aircraft i and j are airborne, then we say that a mid-air collision (MAC) occurs at time τ_1^{ij}.

Remark 2: If one or none of the aircraft i and j are airborne when τ_1^{ij} is reached, then we say that an on-ground collision occurs at time τ_1^{ij}.

Example 1: Cylindric aircraft shapes [4]

Let $V(X_t^i)$ to be a vertical cylinder shape of aircraft i, with λ_-^i the diameter of the cylinder and $\lambda_\perp^i$ the height of the cylinder. We do not account for the orientation of the aircraft, i.e.,

$$V(x^i) = \{(s_1, s_2, s_3) \in \mathbb{R}^3 : (s_1^i, s_2^i, s_3^i) = M_s(x^i), \left[\sqrt{(s_1 - s_1^i)^2 + (s_2 - s_2^i)^2} < \lambda_-^i\right] \cap \left[|s_3 - s_3^i| < \lambda_\perp^i\right]\}.$$

We say that the k-th in-crossing occurs at a (random) time τ_k^{ij} between cylindrical shapes of aircraft i and j if $\tau_0^{ij} = 0$ or, when $k \geq 2$, there exists $\tilde{\tau}_{k-1}^{ij} = \inf_t \{t > \tau_{k-1}^{ij} : V(X_t^i) \cap V(X_t^j) = 0\}$, and $\tau_k^{ij} = \inf_t \{t > \tilde{\tau}_{k-1}^{ij} : V(X_t^i) \cap V(X_t^j) \neq 0\}$.

Example 2: Parallelepiped aircraft shape [4]

Let $V(X_t^i)$ to be a $\lambda_{/\!/}^i \times \lambda_{\vdash}^i \times \lambda_\perp^i$ parallelepiped that envelopes the real shape of aircraft i, with $\lambda_{/\!/}^i, \lambda_{\vdash}^i$ and $\lambda_\perp^i$ the length, the width and the height of the volume, respectively. We consider that aircraft fly parallel to the x-axis, i.e.,

$$V(x^i) = \{(s_1, s_2, s_3) \in \mathbb{R}^3 : (s_1^i, s_2^i, s_3^i) = M_s(x^i), [|s_1 - s_1^i| < \lambda_{/\!/}^i] \cap [|s_2 - s_2^i| < \lambda_{\vdash}^i] \cap [|s_3 - s_3^i| < \lambda_\perp^i]\}.$$

We say that the k-th in-crossing occurs at a (random) time τ_k^{ij} between parallelepiped shapes of aircraft i and j if $\tau_0^{ij} = 0$ or, when $k \geq 2$, there exists $\tilde{\tau}_{k-1}^{ij} = \inf_t \{t > \tau_{k-1}^{ij} : V(X_t^i) \cap V(X_t^j) = 0\}$, and $\tau_k^{ij} = \inf_t \{t > \tilde{\tau}_{k-1}^{ij} : V(X_t^i) \cap V(X_t^j) \neq 0\}$. Following Definition 6, in both examples only the first in-crossing is a collision.

D. Probability of in-crossing/collision between two aircraft

Having defined the (random) time of in-crossing and collision events between two volumes around or shapes of two aircraft, we next define the probability of such an event occurring in a time period $[0, T]$.

Definition 7. *The probability that the k-th in-crossing of* $V(X_t^i)$ *and* $V(X_t^j)$ *of aircraft i and j occurs in a time period* $[0, T]$ *is* $P(\tau_k^{ij} \in [0, T])$*, with* τ_k^{ij} *defined in Definition 3.*

Definition 8. *The probability that an in-crossing of* $V(X_t^i)$ *and* $V(X_t^j)$ *of aircraft i and j occurs in a time period* $[0, T]$ *is* $P(\exists\, \tau_k^{ij} \in [0, T], k \geq 1)$*, with* τ_k^{ij} *defined in Definition 3.*

Definition 9. *The probability that the k-th overlap between volumes* $V(X_t^i)$ *and* $V(X_t^j)$ *of aircraft i and j occurs in a time* *period $[0,T]$ is $P(\tau_k^{ij} \in [0,T])$, with τ_k^{ij} defined in Definition 5.*

Definition 10. *The probability that an overlap between volumes $V(X_t^i)$ and $V(X_t^j)$ of aircraft i and j occurs in a time period $[0,T]$ is $P(\exists\, \tau_k^{ij} \in [0,T], k \geq 1)$, with τ_k^{ij} defined in Definition 5.*

Definition 11. *The probability that a collision between shapes of aircraft i and j occurs in a time period $[0,T]$, given that $\tilde{\tau}_0 = 0$ (i.e., the aircraft are collision free at the beginning), is $P(\tau_1^{ij} \in [0,T])$, with τ_1^{ij} defined in Definition 6.*

Definition 12. *The in-crossing rate of volumes around aircraft i and j, denoted by $\varphi^{ij}(t)$, is defined as, $\varphi^{ij}(t) = \lim_{\Delta \downarrow 0} \frac{P(\exists\, \tau_k^{ij} \in [t, t+\Delta], k \geq 1)}{\Delta}$, with τ_k^{ij} defined in Definition 5.*

Definition 13. *The probability of overlap between volumes around aircraft i and j at time t, denoted by $P_O^{ij}(t)$, is defined as $P_O^{ij}(t) = P(V(X_t^i) \cap V(X_t^j) \neq 0)$.*

Definition 14. *Consider N aircraft in a volume of airspace. The expected number of in-crossings of volumes around (or shapes of) aircraft i with any of the other aircraft in a time period $[0,T]$ is $\Phi^i([0,T]) = \int_0^T \sum_{\substack{j=1 \\ j \neq i}}^N \varphi^{ij}(t)dt$.*

E. Internationally agreed metrics regarding aircraft conflict and collision

[16] defines an "aircraft accident" as "an occurrence associated with the operation of an aircraft which takes place between the time any person boards the aircraft with the intention of flight until such time as all such persons have disembarked, in which a person is fatally, or seriously injured or the aircraft sustains damage or structural failure, or the aircraft is missing or is completely inaccessible". A "Fatal aircraft accident is an aircraft accident involving one or more on-board fatalities". [17] (chapter 6) defines the relation between collision and fatal accident as follows: "a collision between two aircraft represents two fatal accidents". Hence, Definition 6 for the collision between two aircraft corresponds to two fatal accidents, i.e. one fatal accident for each of the aircraft involved with the collision.

[16] defines an "aircraft incident" as "an occurrence, other than an accident, associated with the operation of an aircraft which affects or could affect the safety of operation". Thus Definition 5 for a (serious) conflict between two aircraft corresponds to two (serious) conflict incidents, i.e. one (serious) conflict incident for each of the aircraft involved in the conflict. Same applies for an NMAC.

[17] (chapter 6) presents an internationally agreed minimum criterion for the risk of collision between aircraft in designing future ATM. For en-route the TLS value specified is " 5×10^{-9} fatal accidents per flight hour due to collision between two aircraft"; this TLS value applies in each of the three geometry directions of a possible collision, i.e. top-bottom, head-tail, and head/side-side. [17] also explains that this TLS value should hold true to a new design also when the safety risk reducing effect of Traffic Collision Avoidance System (TCAS) is not taken into account [17] (chapter 3). This means that the true target collision risk is significantly better than this TLS value. In order to be in line with ICAO's TLS unit, for Definition 12 this means that the in-crossing rate unit is: "Expected number of fatal accidents by aircraft i per hour flying by aircraft i, that are due to collisions with other aircraft".

	Number of aircraft	Model aimed for collision risk	Model determines in-crossing rate	Model allows non-stationary processes	Model allows non- Gaussian processes	Model allows dependency between position & velocity	Type of volume considered	Dimensions of volumes considered	Aircraft evolution is relative to aircraft flight plan
Gas law model [20]	N	✓	✓	-	✓	-	c	$\mathbb{R}^3$	-
Paielli&Erzberger model [21]	2	-	-	-	-	✓	c	$\mathbb{R}^3$	✓
Rice theory 1 [22]	-	-	✓	-	-	✓	-	$\mathbb{R}$	-
Rice theory 2 [23]	-	-	✓	✓	✓	✓	-	$\mathbb{R}$	-
Rice theory 3 [24]	-	-	✓	✓	✓	✓	s	$\mathbb{R}^n$	-
Reich model [25], [26], [27]	2	✓	✓	-	✓	-	c, b	$\mathbb{R}^3$	✓
Generalized Reich model [28]	2	✓	✓	✓	✓	✓	b	$\mathbb{R}^3$	✓
Markov chain approximation [29]	2	-	✓	-	✓	✓	e	$\mathbb{R}^3$	✓
Monte Carlo simulation [18]	N	-	✓	✓	✓	✓	a	$\mathbb{R}^3$	✓
Rare event Monte Carlo simulation [19]	N	✓	✓	✓	✓	✓	a	$\mathbb{R}^3$	✓

TABLE I
COMPARISON OF AIRCRAFT IN-CROSSING, CONFLICT AND COLLISION PROBABILITY MODELS WITH RESPECT TO MODEL GENERALITY, a= ANY SHAPE OF VOLUME, c=CYLINDER, e=ELLIPSOID, b=PARALLELEPIPED, s= VOLUME WITH SMOOTH SURFACE, "-"=NOT-APPLICABLE.

III. AIR TRAFFIC IN-CROSSING, CONFLICT AND COLLISION PROBABILITY MODELS

For the top layer of the ATM design, several aircraft in-crossing, conflict and collision probability estimation models have been developed. At their core, these models aim to model and estimate the probability and/or rate of an in-crossing, a conflict or a collision event occurring for one aircraft flight evolution with any other aircraft flight evolution. Some of these models are derived under rather restrictive conditions, while others accommodate more general settings. Table I shows a classification of existing aircraft in-crossing, conflict and collision models based on the generality of their underlying model assumptions. One restrictive condition is the limitation to two aircraft as well as the shape of an aircraft. For modeling purposes, an aircraft is represented as a parallelepiped (l), a cylinder (c) or an ellipsoid (e), rather than a specific volume of an aircraft. Many models also assume that aircraft position and velocity are independent, although, in reality, they are correlated over time. Often the stochastic processes according to which aircraft motion is modeled has to be stationary and Gaussian, neither of which holds true in practice. The least restrictive are Monte Carlo simulation [18] and rare event simulation [19]; they allowing the modeling and evaluation of non-stationary, non-Gaussian processes, any dependencies, and scenarios involving more than 2 aircraft.

A. Gas law model

The gas law model [20] is an in-crossing model where aircraft volumes are represented by gas molecules in a confined 3D space. The gas model is an in-crossing rate model (see Definition 12). The 3D gas model assumes that there are N aircraft in a 3D airspace of volume B. Aircraft i is represented as a vertical cylinder with volume $V(X_t^i)$ with diameter d_- and height $d_\perp$, i.e., invariant to aircraft orientation. Aircraft are uniformly and independently distributed in B. Let $\bar{v}_\perp = \mathbb{E}[|v_\perp^{ij}|]$ and $\bar{v}_- = \mathbb{E}[|v_-^{ij}|]$ denote the expected relative vertical and horizontal velocity, respectively, between aircraft i and j, $i \neq j, i, j \in \{1, 2, \ldots, N\}$. Overlap between volumes around aircraft i and j means $s_t^{ij} \in V(0)$ or, equivalently, $V(X_t^i) \cap V(X_t^j) \neq \emptyset$ (see Proposition 1). An in-crossing of $V(0)$ occurs every time the process $\{s_t^{ij}\}$ hits $V(0)$. The in-crossing rate $\varphi^{ij}(t)$ of $V(0)$ by s_t^{ij} is:

$$\varphi^{ij}(t) = \frac{1}{B}(\frac{1}{4}\pi d_-^2 \bar{v}_\perp + d_- d_\perp \bar{v}_-).$$

If there are N aircraft in volume B, then the expected number of in-crossings of aircraft i with any other aircraft in the time period $[0, T]$ is (see Definition 14):

$$\Phi^i([0,T]) = T\frac{N-1}{B}(\frac{1}{4}\pi d_-^2 \bar{v}_\perp + d_- d_\perp \bar{v}_-).$$

B. Paielli & Erzberger model

Paielli & Erzberger [30], [21] proposed an aircraft conflict probability model where aircraft fly in straight lines at constant velocities. This model is a conflict probability model, i.e. it aims to estimate the conflict incident probability during the encounter period of one aircraft with another aircraft (Definition 10). The deviations of the aircraft from the aircraft flight plans are assumed to be normally distributed. The conflict volume around an aircraft i has the shape of a vertical cylinder with volume $V(x^i)$ with diameter d_-^{MS}, height $d_\perp^{MS}$ and aircraft center s_t^i, invariant to aircraft heading. Let $t^* = \underset{t}{\operatorname{argmin}}\{\mathbb{E}[|s_t^i - s_t^j|]\}$ denote the moment of expected miss distance, i.e., the time the expected distance between the centers of aircraft i and j is at a minimum. The probability of a conflict at time t^* during an encounter between aircraft i and j is defined as the probability that, at time t^*, $V(X_t^i)$ and $V(X_t^j)$ overlap, which, using Definition 13, is:

$$\begin{aligned} P_O^{ij}(t^*) &= P(V(X_{t^*}^i) \cap V(X_{t^*}^j) \neq \emptyset) = P(s_{t^*}^{ij} \in V(0)) \\ &= \int_{\mathbb{R}^2} \mathbf{1}(\sqrt{s_1^2 + s_2^2} < d_-) \int_{-d_\perp}^{d_\perp} p_{s_{t^*}^{ij}}(s) ds, \end{aligned} \tag{5}$$

with $p_{a_t}(.)$ denoting the probability density function (pdf) of a time-dependent random variable a_t. Upon employing orthonormal transformations of the relative position and velocity between aircraft i and j (see [21]), equation (5) becomes

$$P_O^{ij}(t^*) = \int_{-\infty}^{\infty} \int_{-y_0}^{y_1} \int_{-z_0}^{z_1} p_{\bar{s}_{1,t^*}^{ij}}(s_1) p_{\bar{s}_{2,t^*}^{ij}}(s_2) p_{\bar{s}_{3,t^*}^{ij}}(s_3) ds_1 ds_2 ds_3, \tag{6}$$

where $\bar{s}_{1,t^*}^{ij} = \mathcal{T}(s_{1,t^*}^{ij})$, $\mathcal{T}$ an orthonormal transformation and (y_0, y_1) and (z_0, z_1) the integration bounds resulting from this transformation. For level flights, (6) is exact. For non-level flights, (6) is an upper bound for $P_O^{ij}(t^*)$.

C. In-crossing models based on Rice theory

Rice [22] developed a model for the up-crossing rate (Definition 12) of a level by a one-dimensional Gaussian process. This has subsequently been extended to a non-Gaussian and non-stationary $\mathbb{R}^n$-valued stochastic process.

a) Rice theory 1: stationary, scalar-valued, Gaussian stochastic processes [22]. Let X_t be an ergodic, stationary, Gaussian stochastic process that is $\mathbb{R}$-valued. Let U_t be the derivative of X_t. Let $x_L > 0$ be an up-crossing level. Then, the up-crossing rate of level x_L is:

$$\varphi(t) = \int_0^\infty u p_{X_t,U_t}(x_L, u) du, \tag{7}$$

where $p_{X_t,U_t}(.)$ is the joint pdf of X_t and U_t.

b) Rice Theory 2 [23] extends Rice eq. (7) to non-stationary, $\mathbb{R}$-valued, non-Gaussian stochastic processes. Applying Rice theory 2 for aircraft in-crossing of Example 2, we consider the processes $s_t^{ij} \in \mathbb{R}^3$ and its derivative $v_t^{ij} \in \mathbb{R}^3$. We assume that the relative distances and velocities in the three dimensions are independent and, thus, the joint probability density $p_{s_t^{ij}, v_t^{ij}}(.) = \prod_{d=1}^3 p_{s_{d,t}^{ij}, v_{d,t}^{ij}}(.)$. Let $V_p = \prod_{d=1}^3 [-\lambda_d, \lambda_d]$, $d \in \{1, 2, 3\}$ denote the in-crossing volume in three level dimensions between parallelepiped shaped aircraft i and j. The in-crossing rate of V_p by process $\{s_t^{ij}\}$ in the 3 dimensions is,

$$\begin{aligned} \varphi^{ij}(t) = \sum_{d=1}^3 \varphi_d^{ij}(t) = \sum_{d=1}^3 P(s_{\underline{d},t}^{ij} \in V_{p,\underline{d}})[\int_0^\infty u\, p_{s_{d,t}^{ij}, v_{d,t}^{ij}}(-\lambda_d, u)\, du \\ - \int_{-\infty}^0 u\, p_{s_{d,t}^{ij}, v_{d,t}^{ij}}(\lambda_d, u)\, du]. \end{aligned} \tag{8}$$

c) Rice theory 3 [24] is a further extension of the Rice theory to non-stationary, $\mathbb{R}^n$-valued, non-Gaussian stochastic processes and a volume with a smooth surface. Let X_t be a non-stationary, $\mathbb{R}^n$-valued, non-Gaussian stochastic process. Let U_t be the derivative of X_t. Let $S_x \in \mathbb{R}^n$ be a *smooth* in-crossing surface. Then, the in-crossing rate of the volume with smooth surface S_x is:

$$\varphi(t) = \int_{\mathbb{R}^n} \oint_{S_x} (\vec{\nu}(x) \cdot \vec{u})^+ p_{X_t,U_t}(x, u)\, dx\, du,$$

where we define $(u)^+ = \max\{u, 0\}$, $\vec{\nu}(x)$ is an inward normal unit vector at x, and the dot product is defined for any two vectors $\vec{u}_1$ and $\vec{u}_2$ as $\vec{u_1} \cdot \vec{u_2} = ||\vec{u_1}||||\vec{u_2}||cos\theta$, with θ the angle between $\vec{u_1}$ and $\vec{u_2}$.

The smooth surface condition of Rice 3 allows to apply it for ellipsoid shaped aircraft i and j. Let ∂V_e be the boundary of an ellipsoid open subset V_e between aircraft i and j. Then the expected number of in-crossings that occur in a time period $[0, T]$, is:

$$\Phi^{ij}(0,T) = \int_0^T \varphi^{ij}(t)\, dt.$$

$$\varphi^{ij}(t) = \oint_{\partial V_e} \int_{\mathbb{R}^3} (\vec{\nu}(x) \cdot \vec{u})^+ p_{X_t,U_t}(x, u)\; du\, dx$$

with $\vec{\nu}(x)$ the normal vector of the surface at $x \in \partial V_e$. Conditioning yields:

$$\begin{aligned}\varphi^{ij}(t) &= \oint_{\partial V_e} p_{X_t}(x) \left(\int_{\mathbb{R}^3} (\vec{\nu}(x) \cdot \vec{u})^+ p_{U_t|X_t}(u|x)\, du \right) dx \\ &= \oint_{\partial V_e} p_{s_t^{ij}}(x) \mathbb{E}[\vec{\nu}(x) \cdot \vec{v}_t^{ij} | s_t^{ij} = x]\, dx, \qquad (9)\end{aligned}$$

with s_t^{ij} and v_t^{ij} the relative position and velocity of aircraft i and j.

D. Reich model

The Reich model is an expected number of in-crossing model (Definition 14) that aims to estimate the rate of collisions that one aircraft is expected to have with any other aircraft (for cylindrical and parallelepiped aircraft shapes). The typical unit is "Expected number of fatal accidents per flight hour flying, due to collisions with other aircraft". The Reich model [25], [26], [27] for parallelepiped aircraft shapes follow from Rice 2 theory based eq. (8) under the following additional assumptions:

A1) v_t^{ij} is independent of s_t^{ij}, i.e., $p_{s_t^{ij}, v_t^{ij}}(.) = p_{s_t^{ij}}(.) p_{v_t^{ij}}(.)$.
A2) $p_{s_t^{ij}}(s) = p_{s_t^{ij}}(0)$ for every $s \in [-\lambda^i_{/\!/}, \lambda^i_{/\!/}] \times [-\lambda^i_{\vdash}, \lambda^i_{\vdash}] \times [-\lambda^i_{\perp}, \lambda^i_{\perp}]$.

With these assumptions and parallelepipedic aircraft shape (Example 2), eq. (8) simplifies to:

$$\begin{aligned}\phi^{ij}(t) &= \sum_{d=/\!/, \vdash, \perp} \left[P(s^{ij}_{\underline{d},t} \in V_{p,\underline{d}}) p_{s^{ij}_{d,t}}(0) \mathbb{E}[|v^{ij}_{d,t}|] \right] \\ &= \sum_{d=/\!/, \vdash, \perp} \left[4 \frac{\lambda_{/\!/} \lambda_{\vdash} \lambda_{\perp}}{\lambda_d} p_{s^{ij}_{/\!/,t}}(0) p_{s^{ij}_{\vdash,t}}(0) p_{s^{ij}_{\perp,t}}(0) \mathbb{E}[|v^{ij}_{d,t}|] \right] \qquad (10)\end{aligned}$$

This basic model can be used to assess in-crossing risk of aircraft i with various other aircraft j. For various elaborations of (10) we refer to [4]; here we illustrate this approach for $N-1$ aircraft flying on a parallel lane at the same flight level of aircraft i. In the time period $[0, T]$, the expected number of in-crossings of parallelepiped shaped aircraft i with any of the other parallelepiped shaped aircraft $j, j \neq i$, that are flying at a mean distance L on another lane and in opposite direction of aircraft i, then becomes: The expected number of in-crossings between aircraft i and any of the other aircraft during time interval $[0, T]$ is:

$$\begin{aligned}\Phi^i[(0,T)] &= \int_0^T \sum_{\substack{j=1 \\ j \neq i}}^N \varphi^{ij}(t) dt \\ &= 4 \lambda_{/\!/} \lambda_{\vdash} \lambda_{\perp} \frac{T}{L} p_{s^{ij*}_{\vdash,t}}(0) p_{s^{ij*}_{\perp,t}}(0) \sum_{d=/\!/, \vdash, \perp} \frac{\mathbb{E}[|v_d^{ij^*}|]}{\lambda_d}, \qquad (11)\end{aligned}$$

with j^* the aircraft nearest to aircraft i.

The expected velocity terms in (11) satisfy:

$$\begin{aligned}\mathbb{E}[|v^{ij^*}_{/\!/}|] &= \mathbb{E}[|v^{ij^*}_{/\!/}|] + \mathbb{E}[|v^{j^*}_{/\!/}|] \\ \mathbb{E}[|v^{ij^*}_{\vdash}|] &= \left[\mathbb{E}[|v^{ij^*}_{\vdash}|] + \mathbb{E}[|v^{j^*}_{\vdash}|] \right] / \sqrt{2} \\ \mathbb{E}[|v^{ij^*}_{\perp}|] &= \left[\mathbb{E}[|v^{ij^*}_{\perp}|] + \mathbb{E}[|v^{ij^*}_{\perp}|] \right] / \sqrt{2}\end{aligned}$$

Quantification of these expected velocity differences as well as $p_{s^{ij*}_{\vdash}}(0)$ and $p_{s^{ij*}_{\perp}}(0)$ is often accomplished through collection and analysis of large sets of real air traffic flight data.

In the Reich model, $\Phi^{ij}[(0, T)]$ is considered to be the aircraft collision risk. However, similar to the Rice model, multiple in-crossings between aircraft i and j may occur in a time interval $[0, T]$, which means that $\Phi^{ij}[(0, T)]$ is an upper bound of the aircraft collision risk [31].

E. Generalized Reich model

The Generalized Reich model is an expected number of in-crossings model that aims to estimate the rate of collisions that one aircraft is expected to have with any other aircraft (Definition 14) with parallelepiped aircraft shapes. The typical unit is: "Expected number of fatal accidents per flight hour, due to collisions with other aircraft". Following [28], the volume $V(X_t^i)$ of an aircraft i is assumed to be a parallelepiped $\lambda^i_{/\!/} \times \lambda^i_{\vdash} \times \lambda^i_{\perp}$, with $\lambda^i_{/\!/}$ the along-track length, $\lambda^i_{\vdash}$ the across-track width and $\lambda^i_{\perp}$ the height of an aircraft. Moreover, the aircraft are assumed to fly parallel or opposite. The Generalized Reich model essentially adopts some technical assumptions regarding a sufficiently smooth behavior of the process $\{s_t^{ij}, v_t^{ij}\}$ near the edges of the parallelepiped.

Under these assumptions, the in-crossing rate between parallelepiped shaped aircraft i and j through the ceiling and floor follows from Rice 3 theory, eq. (8):

$$\begin{aligned}\varphi^{ij}_{\perp}(t) = \int_{-\lambda_{/\!/}}^{\lambda_{/\!/}} \int_{-\lambda_{\vdash}}^{\lambda_{\vdash}} \Bigg[&\int_0^\infty u p_{s^{ij}_{/\!/,t} s^{ij}_{2_{\vdash},t} s^{ij}_{\perp,t} v^{ij}_{\perp,t}}(x, y, -\lambda_{\perp}, u) du \\ &+ \int_{-\infty}^0 -u p_{s^{ij}_{/\!/,t} s^{ij}_{\vdash,t} s^{ij}_{\perp,t} v^{ij}_{\perp,t}}(x, y, \lambda_{\perp}, u) du \Bigg] dx dy. \qquad (12a)\end{aligned}$$

Similar equations apply for $\varphi^{ij}_{/\!/}(t)$and $\varphi^{ij}_{\vdash}(t)$. Summing over all the three directions, the total in-crossing rate is:

$$\varphi^{ij}(t) = \sum_{d=/\!/, \vdash, \perp} \varphi^{ij}_d(t). \qquad (12b)$$

To avoid over-estimation of the probability of collision, as in the case of the Rice model, where multiple in-crossings that may occur in a time period are counted, [32] develop an equation to compensate for this. Approximation of $p_{s_t^{ij}, v_t^{ij}}$ can be obtained through Monte Carlo simulation of the underlying processes $\{X_t^i, X_t^j\}$.

Consider an airspace volume with N aircraft. Then the expected number of in-crossings between parallelepiped shaped aircraft i and any other aircraft during time period $[0, T]$ is:

$$\Phi^i([0,T]) = \int_0^T \sum_{j=1, j\neq 1}^{N} \varphi^{ij}(t)dt, \quad (13)$$

with $\varphi^{ij}(t)$ satisfying (12a) and (12b).

F. Markov chain approximation

Markov chain approximation in [33], [29] aims to estimate the conflict probability of one aircraft during the encounter period with another aircraft (Definition 10).

In [33], [29] an in-crossing model based on a Markov chain framework is proposed. The motion of aircraft is defined as the solution of a stochastic differential equation (SDE). The solution of the SDE for the relative position of two aircraft is approximated by a space discretization approach that results in a discrete-state Markov chain for the relative position of the aircraft. The overlap area between aircraft i and j is defined as $V(X_t^i) \cap V(X_t^j)$. To determine the probability of a conflict between aircraft i and j, an open domain $U \in R^{2n}$ that contains the overlap is considered. Then, the conflict probability between aircraft i and j in time period $[0,T]$ is,

$$P(\exists \tau_k^{ij} \in [0,T], k \geq 1) = P(\exists t \in [0,T] : \Big[V(X_t^i) \cap V(X_t^j) \neq \emptyset\Big] \cap \big[V(X_s^i) \cap V(X_s^j) = \emptyset, \forall s < t\big]). \quad (14)$$

The initial condition is:

$$V(X_0^i) \cap V(X_0^j) = \emptyset.$$

To evaluate (14), the transition probabilities of the approximated discrete-state Markov chain are propagated backwards in time starting from $V(X_T^i) \cap V(X_T^j) \neq \emptyset$ at time T.

G. Monte Carlo simulation

Monte Carlo (MC) simulation can be used to estimate in a multi-aircraft scenario of given time duration: i) the probability of (serious) conflict with another aircraft (Definition 10); and/or ii) the probability of a fatal accident due to a collision with another aircraft (Definition 11). By dividing such estimated probabilities by the time duration of the simulated scenario, this yields (serious) conflict rate and/or collision rate. MC simulation also allows to use models of the physical aircraft shapes [15].

Applying MC simulation for air traffic conflict and collision probability estimation, stochastic dynamic equations of motion of aircraft i are used to generate R sample paths of the process $\{X_t^i\}, i \in \{1, \ldots, N\}$ within a time horizon $[0,T]$, where $t \in [0,T]$ and N the total number of aircraft considered. For each simulation run $r, 1 \leq r \leq R$, it is counted whether an aircraft collision/in-crossing/conflict occurs between aircraft i and j. [34] introduce a computational effective method in simulating the moment of in-crossing between volumes around aircraft. The probability of collision between aircraft i and j within the time period $[0,T]$ (Definition 11) is estimated as,

$$\hat{P}(\tau_1^{ij} \in [0,T]) = \frac{\sum_{r=1}^{R} \mathbf{1}_{\tau_1^{ij,r} \in [0,T]}}{R},$$

where $\tau_1^{ij,r}$ is the moment of collision (see Definition 6) between aircraft i and j in the simulation run r.

The estimated probability of a fatal accident per aircraft, due to collision with other aircraft in time period $[0,T]$ is,

$$\sum_{j=1, j\neq i}^{N} \hat{P}(\tau_1^{ij} \in [0,T]).$$

Similarly, the estimated probability of a serious conflict occurring within time period $[0,T]$ (Definition 10) is,

$$\hat{P}(\exists \tau_k^{ij} \in [0,T], k \geq 1) = \frac{\sum_{r=1}^{R} \mathbf{1}_{\exists \tau_k^{ij,r} \in [0,T], k \geq 1}}{R}, \quad (15)$$

where τ_k^{ij} is from Definition 5 and $\tau_k^{ij,r}$ is the moment of the k-th in-crossing in simulation run r.

The estimated probability of a serious conflict incident per aircraft i in time period $[0,T]$ is:

$$\sum_{j=1, j\neq i}^{N} \hat{P}(\exists\, \tau_k^{ij} \in [0,T], k \geq 1).$$

MC simulation of aircraft implies that only a confined volume of airspace can be considered. By using a proper Periodic Boundary Condition (PBC) around such confined volume of airspace, it is possible to virtually simulate an infinite volume of airspace [35]. Applying PBC [36] requires the definition of an infinite, space-filling array of identical copies of a simulation region. As a result, an object that leaves the simulation region through a specific boundary face immediately re-enters the region through the opposite face. Moreover, the objects in the simulation region have to interact with the objects in adjacent copies of the simulation region as if these copied objects are other aircraft. When using PBC, one should avoid a simulation region being so small that a simulated aircraft can interfere with one of its copies in an adjacent region.

H. Importance Sampling and Splitting

Application of straightforward MC simulation may lead to computer runs that are very costly in computer time. This is typically the case for simulating collisions in a multi-aircraft scenario with a model of a realistic ATM design. The aim of rare event simulation is to accelerate the MC simulation by making use of mathematically-based methods such as Importance Sampling [37] and Importance Splitting MC simulation [38], [39].

Importance Sampling MC simulation: Importance Sampling is a variance-reduction method based on changing the reference probability such that the probability that rare events occur becomes larger.

Applying Importance Sampling MC simulation for aircraft collision, the original stochastic dynamic equations of motion of aircraft are used to generate R sample paths from a process $\{\tilde{X}_t^i\}, i \in \{1, \ldots, N\}$ that differs from $\{X_t^i\}, i \in \{1, \ldots, N\}$ only through using initial samples from an initial density $p_{\tilde{X}_0}(x)$ instead of $p_{X_0}(x)$. The moment of collision of the modified aircraft states $\{\tilde{X}_t^i\}$ and $\{\tilde{X}_t^j\}$ is then represented by $\tilde{\tau}_1^{ij}$. Moreover, for each of the R samples this leads to a weighting factor $w_r = \frac{p_{X_0}(\tilde{x}_r)}{p_{\tilde{X}_0}(\tilde{x}_r)}$, which compensates for the fact that sample $\tilde{x}_r$ comes from $p_{\tilde{X}_0}(x)$ instead of $p_{X_0}(x)$. Then the probability of collision between aircraft i and j in time period $[0,T]$ is estimated as:

$$\hat{P}(\tau_1^{ij} \in [0,T]) = \frac{1}{R}\sum_{r=1}^{R} w_r \mathbf{1}_{\tilde{\tau}^{ij,r}\in[0,T]}.$$

Importance Splitting MC simulation: Importance Splitting MC simulation is a technique to simulate R runs from one stopping time to the next stopping time over a strictly increasing sequence of stopping times. A possible way to define such stopping times is by means of hitting times of a sequence of strictly decreasing subsets, in the state space of the process [38], [40]. Importance splitting is also called sequential MC simulation because the R runs are first completed until the next stopping, before conducting a simulation of R runs to the next stopping time. At each stopping time τ there are R realizations of $\{X_\tau^i\}, i \in \{1,\ldots,N\}$. These R realizations at τ are typically referred to as R particles. Prior to starting a MC simulation until the next stopping time, copies are made of particles according to their probabilistic weights; this is the importance splitting step (e.g., [41]).

Applying importance splitting MC simulation for aircraft collision probability in time period $[0,T]$ (Definition 11), is estimated by evaluating the probability of the process reaching larger, closed, nested sets $E^{ij} = E_m^{ij} \subset E_{m-1}^{ij} \subset \ldots \subset E_1^{ij}$, where E_1^{ij} should be reached first, before E_2^{ij} is reached, and so on [19]. The probability of collision is now factorized using a decreasing sequence of conflict volumes $E_m^{ij},\ldots,E_1^{ij}$, where for $1 \le k \le m$,

$$E_k^{ij}{=}\{(x,y){\in}\mathbb{R}^{2n} : s_t^i{=}M_s(X_t^i), s_t^j{=}M_s(X_t^j), v_t^i{=}M_v(X_t^i),$$
$$v_t^j = M_v(X_t^j), s_t^{ij} = s_t^i - s_t^j, v_t^{ij} = v_t^i - v_t^j, |s^{ij}{+}\Delta v^{ij}| \le d_k,$$

for some $\Delta{\in}[0,\Delta_k]$, with d_k and Δ_k the parameters of conflict definition at level k, $d_{k+1} < d_k$, $\Delta_{k+1} < \Delta_k$. In [19], Table 10.5 shows an example of conflict level parameter values.

Let $\tau_{1,k}^{ij}$ denote the first hitting time of E_k^{ij}, $k \in \{1,2,\ldots,m\}$, i.e., $\tau_{1,k}^{ij} = \inf_t\{t > 0 : (X_t^i, X_t^j) \in E_k^{ij}\}$. Then the probability that a collision occurs in time period $[0,T]$ is,

$$P(\tau_{1,m}^{ij} < T) = \prod_{k=1}^{m} P(\tau_{1,k}^{ij} < T | \tau_{1,k-1}^{ij} < T). \tag{16}$$

Using importance splitting MC simulation, the probability that a collision occurs in time period $[0,T]$ is estimated as,

$$\hat{P}(\tau_{1,m}^{ij} < T) = \prod_{k=1}^{m} \hat{P}(\tau_{1,k}^{ij} < T | \tau_{1,k-1}^{ij} < T), \tag{17}$$

where each term is estimated through MC simulation from the stopping time at level $k-1$ to the stopping time at level k with R copies from the particles that had arrived at the stopping time of level $k-1$.

After a sequence of m MC simulations with R particles each, the probability that a collision occurs in time period $[0,T]$ is estimated as,

$$\hat{P}(\tau_{1,m}^{ij} < T) = \prod_{k=1}^{m} \left[\frac{\sum_{r=1}^{R} \mathbf{1}_{\tau_k^{ij,r}\in[0,T] | \tau_{k-1}^{ij,r}\in[0,T]}}{R} \right]. \tag{18}$$

[38] proved that eq. (18) converges to $P(\tau_{1,m}^{ij} < T)$ for $R \to \infty$, provided that the process $\{X_t^i\}, i \in \{1,\ldots,N\}$ is a strong Markov process, i.e., the Markov property is satisfied at any stopping time.

Remark 3: [41] (pages 183-188) have compared straightforward MC simulation versus importance sampling MC simulation and importance splitting MC simulation on the probability of conflict estimation for a simple example encounter scenario between two aircraft. Both acceleration methods showed to work better than straightforward MC simulation.

Remark 4: In rare event simulation literature the consensus is that in contrast with importance sampling, importance splitting can be scaled to high-dimensional simulation models [42], [43], [41] (page 196).

Remark 5: [44] combine importance sampling with importance splitting MC simulation for complex GSHS.

IV. Power hierarchy of safety risk models

To analyze the risk of air traffic conflict and collision, the quantitative models above make use of information about the deviations in aircraft position and velocity. The multi-layer processes that are involved with such deviations are various, from errors or failures in the technical systems, human errors, inaccurate estimation of weather conditions, etc. Such multi-layer processes are captured in safety risk models that go beyond the top layer models in Section III. The aim of this section is to present the most frequently encountered safety risk analysis models employed for risk assessment. Following [45] and [12], these models are presented in a modeling power hierarchy (see Fig. 1).

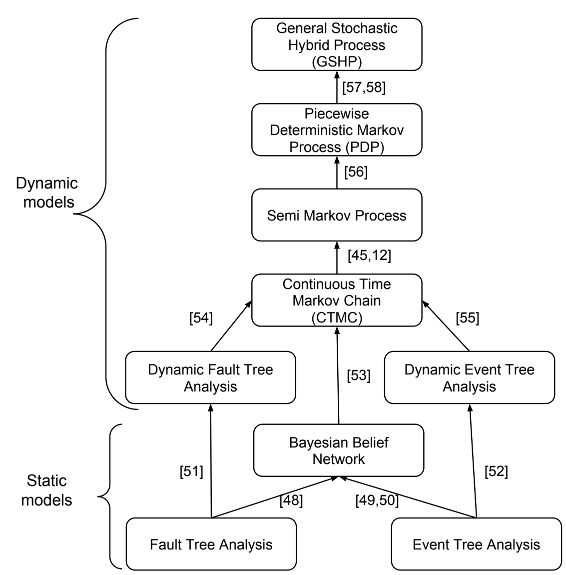

Fig. 1. Power hierarchy models for safety risk analysis, where arrows show formal transformation possibilities, including references.

Fig. 1 illustrates, in a bottom-up fashion, the increasing capability of the models to formalize and analyze risk. Below we describe the models and relations in Fig. 1 in more detail.

A. Static models

Firstly, we discuss the static models that are time-invariant and the mapping relationships between them.

Fault Tree Analysis (FTA): A fault tree [46] is an acyclic graph with $basic$ nodes, corresponding to initiating faults, or gates, which are subject to logical operations (AND/OR). The basic nodes are bi-modal (failed/not failed). Failure probabilities are specified for each mode. The root of the fault tree (top event) corresponds to the event under assessment. FTA assesses the probability of occurrence of the top event of the fault tree. This probability is computed by means of minimum cut sets of the fault tree. The minimum cut set is a list of minimal, necessary and sufficient events which, if all occur, then the top event occurs. A fault tree may have several minimum cut sets. The occurrence of at least one of these minimum cut sets leads to the occurrence of the top event. The occurrence probability of a given minimum cut set is the product of the occurrence probabilities of all events belonging to this set. The occurrence probability of the top event is now the sum of occurrence probabilities of all min cut sets of the fault tree.

Event Tree Analysis (ETA): An event tree [47] is constructed starting with an initiating event that triggers a reaction and leads to other events. This procedure is iterated for each intermediate event until all possible states of the system with undesired consequences have been added. Events are associate with probabilities of success/failure (in general, an event can be described by one, two or more modes, rather than the failed/not failed modes of fault trees). The tree consists of several paths in the form of chains of events. Each path results in an outcome. ETA assesses the probability of occurrence of each outcome corresponding to a path in the tree. The severity of each outcome is determined based on the detrimental/beneficial events that occurred along the corresponding path.

Bayesian Belief Networks (BBNs): A BBN is a directed, acyclic graph with nodes $A_i, 1 \leq i \leq n$, representing random variables, each with a finite set of mutually exclusive states, M the set of arcs of the graph and conditional probabilities P over the nodes, i.e., $P_{A_i}(A_i|f(A_i))$, where $f(A_i)$ returns the set of parents of node A_i in the graph. BBNs possess the causal Markov property, i.e., a random variable depends only on its direct causes (parent variables) and is independent of the rest of the variables. Thus, the joint distribution of $A_1, \ldots, A_n$ is given by $P_{A_1,\ldots,A_n}(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i|f(A_i))$. The probability distribution of a particular variable is found by taking the marginals of the joint probability distribution with respect to this variable. Arrow [B] in Fig. 1, established by [48], shows that a fault tree can be mapped into a BBN. Arrow [Bf] in Fig. 1, established by [49], [50], shows that an event tree can be mapped into a BBN.

B. Dynamic extension of static models

Dynamic extension of static models aims to take account of dynamic evolution of model components over time. Below we introduce the main dynamic extensions used for safety risk assessment and we discuss the power hierarchy relations between models.

Dynamic Fault Tree (DFT): DFT extends the fault tree by specifying the probabilities of component failures as a function of time (see arrow [D] in Fig. 1, established by [51]). A DFT is a bipartite directed acyclic graph, where the nodes represent either failures or are gates. Gates are connected to failure nodes by means of arcs. Gates are either boolean, as in the case of standard FTs, or dynamic gates, which model temporal and functional dependencies between failure events. Similar to standard FT, the top event of a DFT represents the hazard under assessment.

Dynamic Event Tree (DET): DET extends the event tree (see arrow [s] in Fig. 1, established by [52]) by defining the time it takes to go from one state to another. This is specified by means of pairs of start-state and end-state, the time of transition from the start-state to the end-state, and the transition probability of reaching this end-state, given that the system is currently in the start-state. The DET evaluation output is in the form of a set of states that system can be in (referred to as terminating states) and the associated times this states are expected to be reached. Initial conditions are specified in terms of ”root” state-time pairs with associated probability 1.

C. Stochastic dynamic models

For the use of MC simulation at the top layer, there is a large spectrum of stochastic dynamic models allowed. However, if the top layer also uses an analytical technique, such as Generalized Reich model, Markov chain approximation or Importance splitting, then a mathematically unambiguous integration requires a stochastic dynamic model that falls in the class of Markov process models. Markov process models form a large class of stochastic dynamic system models, where a future state of a system is conditionally independent of its past states, given the current state of the system.

Continuous-time Markov chain (CTMC): A CTMC is specified by a set of discrete states, transition rates that specify the jumps from one state to another, and the initial state probability. The time between any two consecutive jumps is assumed to be exponential. CTMC possesses the Markov property according to which its future state, given that the present state of the CTMC is known, is conditionally independent of the past states. CTMC are also used as approximate models in the top layer (see Markov chain approximations in Section III-F). Static models can be extended to CTMCs. [53] shows that a BBN can be converted to a CTMC. [54] shows that a DFT can be converted to a CTMC. [55] shows that a DET can be converted to a CTMC.

Semi-Markov process: CTMC are further extended to Semi-Markov process by [45], [12], where the dynamic evolution of the system is governed by ordinary differential equations instead of exponential inter-jump times and stationary transition rates.

Piecewise Deterministic Markov Process (PDP): Semi Markov processes are further extended to piecewise deterministic Markov processes (PDP) by [56]. PDP are non-diffusion Markov processes whose evolution between consecutive jumps are governed by ordinary differential equations. Jumps occur according to state dependent transition rates or boundary hittings, and a transition probability measure for the new state after a jump. A PDP satisfies the strong Markov property [56].

General Stochastic Hybrid Markov Process (GSHP): At the top of the hierarchy of stochastic processes, GSHPs are extended from PDPs by [57], [58]. GSHPs are non-linear continuous-time hybrid-state stochastic processes. Compared with PDPs, GSHPs include diffusion by means of Brownian motion. Consequently, the evolution of the continuous state component is governed by stochastic differential equations. A GSHP satisfies the strong Markov property [57].

V. Model specification and validation

The multiple layers and entities of an ATM operation typically lead to a large stochastic dynamic model. To manage the development of such large model, there is need for a systematic approach in model specification. In specifying a complex Markov process model, Petri nets have proven their effectiveness. A complementary formal model specification is agent-based modeling of a complex system.

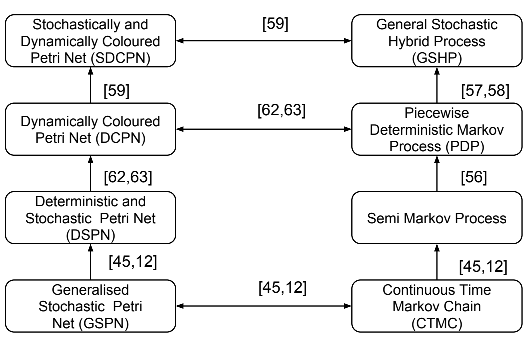

Fig. 2. Petri nets as modeling formalism for Markov process models, where arrows show formal transformation possibilities, including references.

A. Petri nets

A Petri net [59], [60] is a bipartite, directed graph (F, K, Q, H), with F and K disjoint sets of places and transitions, Q the set of directed arcs that connect places to transitions and transitions to places, H the number of tokens residing at each place. The dynamics of the events is given by the dynamics of the tokens from one place to another due to firing of transitions. Formal transformations have been established between Petri nets and stochastic processes (see Fig. 2). CTMCs are shown to be isomorphic to GSPNs in [45], [12]. Deterministic and Stochastic Petri nets (DSPNs) further extend GSPNs as they accommodate both exponentially firing transitions and constant-duration transitions [45], [12]. In turn, DSPNs are extended to Dynamically Colored Petri nets (DCPNs) by [61], [62], where the tokens have colors that change over time according to ordinary differential equations, while the tokens reside in a place. Colors are applied to tokens to differentiate between tokens residing in the same or different places. DCPNs are shown to be isomorphic to PDPs in [61], [62]. Finally, Stochastically and Dynamically Colored Petri nets (SDCPN), which extend the class of DCPNs in [63], assume that tokens have colors that change dynamically and stochastically over time according to SDEs. SDCPNs are shown to be isomorphic to GSHPs in [63].

Remark 6: Other hybrid Petri net formalisms that have similarities with DCPN have been developed in [64], [65], [66]. For the Fluid Stochastic Petri Net (FSPN) [65] the relation between FSPN and PDP type of Markov processes has been characterized in [67], [68].

B. Agent-based modeling (ABM)

Simulation of an ABM of a complex socio-technical system is increasingly recognized as a powerful approach to identifying and understanding exhibiting emergent behavior [69]. For a safety-critical ATM design this is of value both for nominal as well as rare emergent behaviors. Multiple definitions for an agent are in use in different domains such as "an agent is an autonomous system situated within a part of environment, which senses that environment and acts on it, over time, in pursuit of its own agenda and so as to effect what it senses in the future" [70]; "an agent is anything that can be viewed as perceiving its environment through sensors and acting upon that environment through actuators" [71].

For a large complex socio-technical system such as ATM, agent-based modelling [72] provides the tools for analyzing, modeling, and designing the whole system in terms of its agents, each with its own set of local tasks and capability. The integration of the agents can then be achieved by modeling the interactions among the agents. So agent-based modeling provides abstraction levels that make it simpler and more natural to deal with the scale and complexity of problems in an ATM design. Agent components can be described at a high level of abstraction, yet they support a systematic compositional modeling approach [73]. Moreover, ABM allows to model non-functional hazards that are typical for complex socio-technical systems like ATM [74], [75], [76].

In an ATM design, different actors, hardware, software are interacting in the complex socio-technical system. In developing an ABM, agents may be humans (e.g., air traffic controllers, pilots), systems, organizations, and other entities that pursue a certain goal. In addition to these active agents, there is a need to model reactive agents (such as air- ground communication system and aircraft), as well as non- agents (such as airspace structure and weather), and all interactions between agents. In [77] it has been explained how ABM can be combined with other advanced modeling methods in safety risk modeling and analysis of changing ATM operations.

C. Model validation

Once a model of a given ATM operation (design) has been developed, this model will be used to conduct quantitative assessments. In doing so, one should be aware that the assessment results obtained apply to the model. So the question is: to what extend do the results obtained for the model apply to the given ATM operation (design)? The answering of the latter question is commonly referred to as model validation. Following [78] model validation is defined to mean "substantiation that a computerized model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model". A model should be developed for a specific purpose (or application) and its validity determined with respect to that purpose [79]. Hence, model validation is determining whether the similarity between behavior and output of the simulation model and the behavior and output of the given ATM operation (design) is such that the model's intended purpose is realized. Model validation plays a different role in the early design phase than it does in the pre-operational phase. In the latter phase, the magnitude of uncertainty in the safety case must be sufficiently small in order to pass applicable safety criteria. However, in the early design phase there are many uncertainties for which design requirements and solutions remain to be developed.

Both regarding model behavior and model output, data-based model validation is preferred, i.e. comparing model generated data with realistic data [79], [80], [81]. Typically, air traffic conflict and collision model applications apply data-based validation to their sub-models. This, however, is not the same as a data-based validation of the behavior and output of the entire model, under various conditions. The low probability of serious conflicts in ATM makes that data-based validation of the behaviour and output of the entire model is not feasible. Even for an existing ATM operation it is not feasible to collect such data for all relevant conditions. Fortunately there are other model validation techniques that can be used [79]. One is animation and tracing, i.e. showing and evaluating the dynamic and stochastic behavior of the entire model in a graphical form, including tracing (backtracking) of the sequence of events that have happened in the model. Another is comparing the outputs and behavior of the developed model against results from another model for (part of) the operation. A third one is testing if under degenerate and extreme conditions, the impact on the entire model behavior and output is plausible. A fourth one is face value validation, which means that subject matter experts are asked whether the model behavior and output is reasonable. This may trigger valuable requests in collecting and showing additional animation and tracing results. A fifth one is predictive validation, in which the model is used to predict (forecast) behavior, and then a comparison is made between results obtained from conducting dedicated experiments on the ATM operation (design), e.g. through field tests or human-in-the-loop simulation.

Inherent to the very nature of an ATM operation (design), an ATM model will include various kinds of aleatory and epistemic uncertainties. In order to assess if the ATM model possesses a satisfactory range of accuracy consistent with the intended application of the model [78], two important complementary model validation techniques are Sensitivity Analysis (SA) and Uncertainty Quantification (UQ). Sensitivity analysis (SA) aims to measure how sensitive the output of the entire model is to single and joint changes in model parameter values [82]. Uncertainty quantification (UQ) aims to estimate the levels of uncertainty in the output of the model as a result of aleatory and epistemic uncertainties in the parameters of the model and of potential differences between model and ATM operation (design) considered [83], [80], [81]. Because the level of uncertainty at the output of the model is the product of the level of uncertainty at the input multiplied by the sensitivity of the model, SA and UQ form two sides of one coin. It is also of interest to notice that SA and UQ can be applied to a model of a safety-critical operation, though not directly to the true operation. This explains why model-based SA and UQ has gained significant interest both in safety science [84] and in aerospace science [85].

VI. Applications of air traffic conflict and collision models

To estimate the probability of an aircraft conflict or collision for a specific operation, typically a model from Section III is employed together with models from Section IV and Section V. Table II shows a series of such applications from literature for risk assessment in air transportation.

A. *Collection and evaluation of applications*

The collection of papers in Table II consists of applications published during the last three decades that address complex air traffic scenarios, and that delivered outcomes in the form of quantified estimates of conflict and/or collision probabilities. The applications are organized along their year of publication, with the most recent applications at the bottom of Table II. Over these three decades in time, the applications in Table II (36 in total) evolve from conventional ATC (17x), through TCAS (2x), to early ATM design (15x) and Unmanned Aerial Vehicle (UAV) (2x).

Table II shows 3 applications on retrospective safety risk estimation, i.e. evaluation of past incidents and accidents. Most papers consider scenarios involving two aircraft; only five applications consider scenarios involving more than two aircraft. Almost all papers (33x) aim for prospective applications, i.e., to assess the potential risk of a novel or changed ATM operation.

About half of the papers in Table II address an early design (17x). About half of the papers estimate conflict probabilities (15x), some estimate serious conflict probabilities (4x), several estimate near collision probabilities (12x) and many estimate collision probabilities (25x).

Regarding the use of specific models from Section III, we see a frequent use of the (generalized) Reich model (9x). During the last two decades we see a steady increase in the use of MC simulation (21x). This has been started by [86] and [18], respectively for conflict and collision probability estimation. All applications involving more than two aircraft (5x) make use of MC simulation. Using importance sampling/splitting MC simulation for estimation of collision probabilities has gained ground during the last decade (6x) for the evaluation of an early design. There are three applications that do not make use of any model from Section III; two are retrospective applications [87] and [88], and one prospective application [52]. Of the 33 applications that make use of models from section III, 9 assess more than one event type; each of these 9 make use of MC simulation.

Ref.	Application			Section II					Section III								Section IV						Section V				
		Prospective	Early design	Number aircraft	Conflict	Serious conflict	Near collision	Collision	Gas law model	Paielli &Erzberger model	Reich model	Generalized Reich model	Markov chain approximation	Monte Carlo simulation	Importance Sampling/ Importance Splitting	PBC	FTA	ETA	BBN	DET	PDP	GSHP	DCPN / FSPN	SDCPN	ABM	Sensitivity analysis	Uncertainty quantification
[20]	Aircraft collision	✓	-	N	-		-	✓	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[89]	Procedural ATC horizontal separation	✓	-	2	-	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[90]	Procedural ATC horizontal separation	✓	-	2	-	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[91]	Conventional ATC vertical separation	✓	-	2	-	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[92]	Conventional ATC en-route	✓	-	2	-	-	-	✓	-	-	✓	-	-	-	-	-	✓	✓	-	-	-	-	-	-	-	✓	✓
[30]	Free flight en-route	✓	✓	2	✓	-	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[86]	Free Flight en-route	✓	✓	2	✓	-	-	-	-	-	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-
[18]	Four ATM concepts parallel routes	✓	✓	2	-	-	-	✓	-	-	-	✓	-	✓	-	-	✓	-	-	-	-	✓	✓	-	-	-	-
[93]	Conventional ATC en-route	✓	-	2	✓	-	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[94]	Conventional ATC horizontal separation	✓	-	2	-	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[95]	Conventional ATC en-route	✓	-	2	✓	-	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[96]	Converging runways	✓	-	2	-	-	-	✓	-	-	-	✓	-	✓	-	-	✓	-	-	-	✓	-	✓	-	-	✓	✓
[97]	Conventional ATC	-	-	2	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	-	-	-	-	-	-
[98]	Advanced ATM	✓	✓	2	✓	-	-	-	-	-	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-
[87]	TCAS	✓	-	2	-	-	✓	-	-	-	-	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-
[99]	Conventional ATC en-route	✓	-	2	✓	-	-	-	-	-	-	–	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[100]	Free flight parallel routes	✓	-	2	-	-	-	✓	-	-	-	✓	-	✓	-	-	✓	-	-	-	✓	-	✓	-	-	✓	✓
[101]	ASAS in Terminal Movement Area	✓	✓	2	-	-	✓	✓	-	-	-	-	-	✓	✓	-	-	-	-	-	-	✓	-	✓	✓	-	-
[102]	Worldwide probabilistic air transport safety	-	-	1	-	-	-	✓	-	-	-	-	-	-	-	-	✓	✓	✓	-	-	-	-	-	-	-	-
[35]	Mediterranean free flight en-route	✓	✓	8	✓	✓	✓	✓	-	-	-	-	-	✓	✓	✓	-	-	-	-	-	✓	-	✓	✓	-	-
[103]	Active runway crossing	✓	✓	2	-		-	✓	-	-	-	-	-	✓	-		✓	-	-	-	✓	-	✓	-	✓	✓	✓
[88]	Advanced ATM en-route	✓	✓	2	✓	-	✓	✓		-	-	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-
[15]	TCAS	✓		2	-	-	✓	✓	-	-	-	-	-	✓	-	-	-	-	$\checkmark^2$	-	-	-	-	-	-	-	-
[104]	Advanced ATM en-route	✓	✓	2	-	-	✓	✓	-	-	-	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	✓	-
[105]	Conventional ATM with improved CNS	✓	-	2	✓	✓	-	-	-	-	-	-	-	✓	-	-	-	-	-	-	-	-	✓	-		✓	-
[106]	Advanced ATM en-route	✓	✓	2	✓	-	-	✓	✓	-	-	-	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-
[52]	Advanced ATM en-route	✓	✓	2	✓	-	✓	✓	-	-	-	-	-	-	-	-	✓	-	-	✓	-	-	-	-	-	✓	-
[107]	Visual flight rule airspace	✓	-	250	-	-	-	✓	-	-	-	-	-	✓	✓	-	-	-	-	-	-	-	-	-	-	-	-
[108]	Advanced ATM en-route	✓	✓	2	✓	-	-	-	-	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-
[109]	Advanced free flight en-route	✓	✓	8	✓	✓	✓	✓	-	-	-	-	-	✓	✓	✓	-	-	-	-	-	✓	-	✓	✓	✓	-
[110]	Active runway crossing	-	-	2	-	-	-	✓	-	-	-	-	-	✓	-	-	-	-	-	-	-	-	$\checkmark^1$	-	-	✓	-
[111]	Advanced free flight en-route	✓	✓	2	✓	-	✓	✓	-	-	-	-	-	$\checkmark^3$	-	-	$\checkmark^4$	-	-	✓	-	-	-	-	-	✓	-
[112]	UAV	✓	✓	100	-	-	✓	-	-	-	-	-	-	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-
[113]	UAV	✓	✓	2	-	-	✓	-	-	-	-	-	-	✓	✓	-	-	-	$\checkmark^2$	-	-	-	-	-	-	-	-
[114]	Advanced ATM en-route	✓	✓	8	✓	✓	✓	✓	-	-	-	-	-	✓	✓	✓	-	-	-	-	-	✓	-	✓	✓	✓	-
[115]	Conventional ATC	✓		2	-	-	-	✓	-	-	✓	-	-	-	-	-	-	✓	-	-	-	-	-	-	-	-	-

TABLE II

INTEGRATED USE OF THE MODELS FROM SECTIONS II-V FOR APPLICATIONS TO AIR TRAFFIC OPERATIONS. $\checkmark$ = USED, $-$ = NOT USED, $\checkmark^1$ COLORED STOCHASTIC PN (CSPN), $\checkmark^2$ DYNAMIC BBN, $\checkmark^3$ SIMULATION FOR CONFLICT PROBABILITY, $\checkmark^4$ RELIABILITY DIAGRAM.

The use of specific models from Section IV in combination with models from Section III has started two decades ago by [92] and [18], and has become common practice during the last decade. FTA is used 12x in combination with the (generalized) Reich model or with (rare event) MC simulation. Table II also shows that the use of strong Markov process models (PDP or GSHS) from Section IV is 6x combined with importance sampling/splitting MC simulation from Section III. BBN is used 4x, of which 2 times a dynamic BBN in combination with MC simulation. Other models from Section IV are used less frequently: ETA in one retrospective and two prospective applications; DET in two prospective applications. The use of models from Section V is clearly on the rise during the last decade: Petri net (CSPN, DCPN, FSPN or SDCPN) modeling formalism is used 10x; ABM is used 5x; Sensitivity analysis is used 11x; Uncertainty quantification is used 4x.

B. Discussion of model restrictions

The trend in the use of conflict and collision models in the applications in Table II form a reflection of various model restrictions in handling the collection of applications. Over the three decades considered, the type of applications have clearly evolved from modeling of existing ATM operations, and studying the effect of changes in separation minima, to the modeling and analysis of future ATM designs. During the second decade novel modeling approaches have been developed and often applied to conflict probability estimation. However, many of the applications in Table II demonstrate that from a safety perspective the estimation should also address less frequent events such as serious conflicts, near collisions and collisions, and involving more than 2 aircraft. In spite of the great insight provided both by the Paielli and Erzberger model and the Markov chain approximation model, these models fall short in assessing rare events beyond conflicts between two aircraft. At the same time, the ICAO adopted Reich model is running out of steam due to its restriction to stationary processes and encounters between 2 aircraft. Although the Generalized Reich model overcomes the stationarity restriction of the Reich model, it inherits the Reich model restriction to work only for aircraft that fly in an organized route structure.

From the static models, FTA and BBN have shown their effective use when the underlying statistical information is available from an existing ATM operation. However the effective use of ETA is questionable, specifically when multiple agents work concurrently on a dynamically evolving encounter. A reasonable expectation is that DET, i.e. a dynamic extension of ETA, may resolve these ETA shortcomings to a certain extent.

For two advanced ATM applications [52] and [111] propose to go further; they provide an initial estimation of (near) collision probabilities through using DET in combination with reliability diagrams for technical system failures and MC simulation for the estimation of conflict probabilities between 2 aircraft. However, their DET has a similar limitation like FTA, ETA and BBN, in the sense that it does not model non-functional hazards, and neither is able to identify unknown (positive or negative) emergent behavior.

For one of the applications in Table II, [116] has made a comparison between an assessment by MC simulation of an Agent-based model [103] versus an ETA based assessment [117], both for the same active runway crossing operation. The two studies have been compared on the following aspects [116]: i) Modeling capabilities, i.e. in handling complexity, dynamics, performance variability, interaction/concurrency, and emergent behavior; ii) Aspects in using the models, i.e. required expertise, capturing contextual conditions, transparency, and incorporating uncertainties; iii) Risk results obtained, i.e. differences in findings, comparison to real data, and feedback to design. On each of these aspects the identified differences are significant, including an order of magnitude difference in estimated conditional collision risks. The overall conclusion is that the MC simulation of ABM reveals rare behaviors that are not well captured by an ETA approach.

As demonstrated in many applications, MC simulation has become an indispensable approach in prospective estimation of serious conflict and (near) collision probabilities. The reason is that it forms a universal means in getting around various restrictions of the models described above. However, making MC simulation work for the assessment of a complex socio-technical ATM operation (design) poses novel challenges. One of the key challenges is to remedy the extremely high computer time required to run rare event MC simulations, even on a very powerful computer.

In several applications, the initial condition of a MC simulation is defined through a complementary modelling approach, such as FTA, ETA, dynamic BBN or DET. Illustrative applications using FTA-based conditional MC simulation are [103], [88], [104], [106]. Illustrative applications using dynamic BBN-based conditional MC simulation are [15], [113]. An illustrative application using FTA and DET-based conditional MC simulation is [111]. Potential limitations of such conditional MC simulation are threefold. First of all, the conditions that are not modeled in the FTA, the dynamic BBN or the DET are not considered by the MC simulation. Secondly, the simulated aircraft trajectory path cannot influence the conditions specified by the FTA, the dynamic BBN or the DET. Thirdly, the number of aircraft in the MC simulation scenario is limited to the number of aircraft captured in the adopted FTA, BBN or DET.

The above limitations do not apply when conditional MC simulation is used within the context of formal mathematical conditions. For example, in applying importance splitting, the mathematical conditions assure that no safety critical paths are missed. Illustrative applications of importance splitting are [107] and several applications that use an Agent-based Model.

Because the ATM system is a complex socio-technical system, a natural way to enable MC simulation is to develop an Agent-based Model (ABM) of the given ATM design. An important advantage of ABM is that it forms a natural framework to capture non-functional hazards [74], [75], [76]. Illustrative applications using ABM are [35], [101], [103], [109], [114]. To assure compliance with the mathematical conditions of importance sampling/splitting, powerful hybrid Petri nets are used for the specification of the ABM.

As has been explained in section V, the sensitivity analysis and uncertainty quantification plays an increasingly important role in providing effective feedback to the design of an advanced ATM operation. However, uncertainty quantification easily asks for an increase of two orders in magnitude of the number of MC simulation runs to be conducted. This explains why in few ATM applications uncertainty quantification has been used.

VII. Conclusions

Estimating the probability of air traffic conflict and collision is of interest for air traffic as it can provide valuable insight into the safety level of current air traffic operations and of future air traffic design. This paper has presented the main air traffic conflict and collision probability estimation models proposed in the literature. Since each model proposes its own definitions and notations, a unified mathematical framework for air traffic conflict and collision notations and definitions has been proposed. Using this framework, the mathematical core of existing air traffic conflict and collision directed models has been presented and the advances in estimating the probability of air traffic conflict and collision have been outlined.

Section II has provided a unifying reach probability setting of conflicts and collisions between multiple aircraft in a volume of airspace. Conflicts and collisions are directly related to the top-layer of air traffic, which consists of the four-dimensional (time and space) trajectories flown by aircraft. The reach probability framework has been well developed in control literature, though with focus on conflicts. The generalization of the reach probability framework to serious conflicts, near collisions and collisions extends this reach probability framework to safety research.

Section III has provided a review of models from literature that are able to quantify conflict and collision probabilities in the top layer. This reveals a valuable spectrum of models. From the extreme event domain there are the Rice and Reich models, the latter of which is ICAOs standard for collision risk assessment between aircraft flying in an organized route system. From the control under uncertainty domain there are the Paielli and Erzberger and the Markov chain approximation models, both aimed for capturing conflict probabilities. From the simulation domain there is MC simulation, importance sampling/splitting, and Periodic Boundary Condition (PBC).

Section IV has provided a review of models that can capture the realization of aircraft trajectories as a result of interactions between a series of layers in ATM, including pilots, air traffic controllers and a multitude of technical systems on-board as well as on the ground. These models have been presented in a model power hierarchy, including the literature source for each identified relation. This power hierarchy shows, in a bottom-up fashion, the increasing capabilities of the models to formalize and analyze risk, from static, time-invariant models such as fault trees, event trees, Bayesian networks, where the structure of the models is static, to dynamic, time-variant models such as dynamic fault trees, continuous-time Bayesian networks and Markov processes, where the evolution of a system is characterized over time. Through the power hierarchy of the generic risk models reviewed, it is shown that there exists means to enhance the specification and analysis of lower-level models.

Section V has provided an overview of models that support the development of advanced models in level IV. One set of such models consists of hybrid Petri nets (DCPN, FSPN and SDCPN) that provide the syntax to systematically develop a stochastic hybrid mathematical model of the various layers and interactions in the ATM operation. The second set of supporting models is Agent-based Modelling (ABM), which provide the socio-technical semantics to systematically develop a model for the ATM operation considered. The third set of supporting models concerns model validation. Here it is well explained that model validation in the early design phase differs significantly from model validation in the pre-operational phase. In the early phase there are many uncertainties and options that remain to be tamed by the design. Receiving feedback from sensitivity analysis and uncertainty quantification forms very valuable information to a design team. During later phases it is increasingly more expensive for making significant changes in the design, as a result of which combinations of unexpected sensitivities and uncertainties may lead to a negative safety case.

Section VI has provided an overview of use of various models from sections III, IV and V in papers that have produced quantified conflict and collision probability estimates for various applications to ATM operations, both conventional and advanced designs, during the last three decades. During the first of these three decades the Reich model was the main model used. The second decade has shown a period of experimentation in applying novel models from sections III, IV and V to various applications. During the last decade we notice a general trend to make use of (rare event) MC simulation in combination with various supporting models from Sections IV and V in order to evaluate early designs. This coincides with the control-directed explanation provided by [7]. Section VI subsequently explains how specific restrictions of various conflict and collision probability estimation models explain the trend identified in Table II.

As has been described in the introduction, the key motivation for providing this review is to strengthen the early phase of ATM design. In this early phase the design space is very large and fully open with many options. From a control perspective this is the ideal situation, though the many uncertainties typically bring shivers to developers of a safety case. As it has been well described by [10] in this early design phase the switching between the many options is relative easy in comparison to a similar switching in a later design phase. However in order to make a rational choice in the early design phase there is need for modeling and analysis of various options in the large ATM design space against various key performance indicators, including safety. The survey provided in this paper shows that there are now several relatively novel models available for application in the early phase of future ATM design, which make it possible to provide safety analysis feedback to an early ATM design.

There also are multiple directions for further developments, such as: i) Further development of agent-based modeling of complex safety-critical socio-technical systems, including ATM; ii) Further development of rare event MC simulation techniques; iii) Further development of SA and UQ techniques for models of complex socio-technical systems; and iv) Development of dedicated software tools in support of the development of MC simulation models of given ATM operation design.

As has been explained by [118], [119] there also is a great opportunity for a new generation of advanced ATM design experts. The current approach in future ATM design works open loop, i.e. ATM designers are accustomed to work without receiving the kind of feedback addressed in this paper. The latter is clearly illustrated by the many applications in Table II that have addressed ATM designs that had been conceived at least five years earlier. Through making use of the successful modeling and analysis approaches reviewed in this paper, a new design generation of future complex socio-technical ATM can work in a closed loop, i.e. they can receive timely quantitative feedback already during the early design phase. This also will promote a new design generation that will make develop and make use of control design methods that handle socio-technical systems.

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