Adaptive stabilization of impulsive switched linear time-delay systems
A piecewise dynamic gain approach

Yuan, Shuai; Zhang, Lixian; Baldi, Simone

DOI
10.1016/j.automatica.2019.02.004

Publication date
2019

Document Version
Accepted author manuscript

Published in
Automatica

Citation (APA)
Yuan, S., Zhang, L., & Baldi, S. (2019). Adaptive stabilization of impulsive switched linear time-delay systems: A piecewise dynamic gain approach. *Automatica*, *103*, 322-329. https://doi.org/10.1016/j.automatica.2019.02.004

Adaptive stabilization of impulsive switched linear time-delay systems: a piecewise dynamic gain approach

Shuai Yuan [a,*], Lixian Zhang [a], Simone Baldi [b]

[a] *School of Astronautics, Harbin Institute of Technology, Harbin, 150080, China*

[b] *Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands*

Abstract

In the presence of discontinuous time-varying delays, neither Krasovskii nor Razumikhin techniques can be successfully applied to adaptive stabilization of uncertain switched time-delay systems. This paper develops a new adaptive control scheme for switched time-delay systems that can handle impulsive behavior in both states and time-varying delays. At the core of the proposed scheme is a Lyapunov function with a dynamically time-varying coefficient, which allows the Lyapunov function to be non-increasing at the switching instants. The control scheme, guaranteeing global uniformly ultimate boundedness of the closed-loop system, substantially enlarges the class of uncertain switched systems for which the adaptive stabilization problem can be solved. A two-tank system is used to illustrate the effectiveness of the method.

Key words: Adaptive control; impulsive switched linear systems; time-varying delays; mode-dependent dwell time.

1 Introduction

Thanks to their capability of modeling a wide range of systems with interacting continuous/discrete dynamics [1, 2], hybrid systems with impulsive and switching dynamics, usually called impulsive switched systems, have been attracting fruitful lines of research, encompassing stability, stabilization [3, 4, 5], robust control [6, 7], and others. Dealing effectively with large parametric uncertainty is becoming increasingly crucial when controlling hybrid systems: in several applications it has been recognized that robust controllers may give rise to rather conservative performance in the presence of a large and non-polytopic uncertainty set [8, 9, 10]. Therefore, the design of adaptive control methods to cope with large and non-polytopic parametric uncertainties in hybrid systems is often relevant. To date, adaptive control of a class of hybrid systems, switched systems, has been drawing some attention [11, 12, 13, 14, 15]: the most recent result in this field involves a novel Lyapunov function which is non-increasing at the switching instants [15].

Switched time-delay systems are natural generalizations of switched systems, as time delay is another common problem in hybrid systems. Time delay is typically time-varying, and makes the state of a system evolve based on some delayed information [16, 17, 18, 19]. Stability and stabilization of switched time-delay systems has been intensively studied [16, 20, 4]. However, the two main approaches adopted to deal with time-varying delay, namely the Krasovskii technique and the Razumikhin technique, show some limitations when applied to adaptive control of switched time-delay systems. Since the Krasovskii technique involves the bounded derivatives of the time-varying delays, continuity of time-varying delay at the switching instants should be assumed [16, 21]. If this assumption might be reasonable for non-switched systems, it turns out to be quite restrictive when considering that switching behavior may lead to impulsive delays. On the other hand, even if the Razumikhin technique can handle discontinuous time-varying delays, its application in an adaptive stabilization setting is problematic: as pointed out in [22, 23], the selection of the Razumikhin coefficient is limited in an unknown interval inside which the existence of an adaptive controller is guaranteed. Therefore, addressing discontinuous time-varying delays in adaptive control of uncertain switched systems is not only practically relevant but it also tackles the need to extend the current stabilization tools, which motivates this study.

* Corresponding author

Email addresses: shuaiyuan@hit.edu.cn (Shuai Yuan), lixianzhang@hit.edu.cn (Lixian Zhang), s.baldi@tudelft.nl (Simone Baldi).

Preprint submitted to Automatica *19 January 2019*

In this paper, we develop a new adaptive design for uncertain switched linear systems that can handle impulses in both states and time-varying delays. A stability condition is developed to deal with the impulsive effect of multiple time-varying delays. Based on the stability condition, a new adaptive controller is proposed by solving a family of Riccati equations and LMIs. The adaptive law involves a piecewise dynamic gain which is properly designed to guarantee non-increasing property of the Lyapunov function at the switching instants. With the designed adaptive controller and switching law, global uniform ultimate boundedness of the closed-loop system can be guaranteed while adaptive asymptotic stabilization is still an open problem. The main contribution of this paper is that discontinuities of both the states and the time-varying delays at the switching instants are addressed for the first time in the adaptive stabilization setting. As a matter of fact, the proposed adaptive mechanism substantially enlarges the class of switched linear systems with parametric uncertainties for which the adaptive stabilization can be solved.

This paper is organized as follows: the problem formulation and some useful lemmas are given in Section 2. In Section 3, the adaptive controller is designed. A two-tank system is used to illustrate the proposed method in Section 4. The paper is concluded in Section 5.

Notation: The notations used in this paper are standard: $\mathbb{R}$ and $\mathbb{R}^+$ represent the sets of real numbers and positive real numbers, respectively. The sets of natural numbers and positive integers are denoted by $\mathbb{N}$ and $\mathbb{N}^+$, respectively. The superscript T denotes the transpose of a vector or of a matrix, $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2−norm, and the identity matrix of compatible dimensions is denoted by I. The notation $\mathcal{M} = \{1, 2, \ldots, M\}$ represents the set of subsystem indices and M is the number of subsystems, while $\mathcal{L} = \{1, 2, \ldots, L\}$ represents the set of delay indices and L is the number of delays. We use $*$ as an ellipsis for the terms that are induced by symmetry. For a left-continuous signal $\phi(\cdot)$, the notation $\phi(t^-)$ represents the left limit of $\phi(t)$, i.e., $\phi(t^-) = \lim_{\tau\to t^-} \phi(\tau)$.

2 Problem formulation and preliminaries

Consider the switched linear impulsive system with multiple time-varying delays

$$\begin{aligned}
\dot{x}(t) = &\ \left(A_{\sigma(t)} + \Delta A_{\sigma(t)}(t)\right) x(t) + B_{\sigma(t)}u(t) + w(t) \\
&+ \sum_{\ell=1}^{L} \left(E_{\ell,\sigma(t)} + \Delta E_{\ell,\sigma(t)}(t)\right) x(t - d_{\ell,\sigma(t)}(t)) \\
x_{t_0}(\vartheta) = &\ \psi(\vartheta), \quad \vartheta \in [t_0 - d_{\mathrm{m}}, t_0] \\
x(t_i) = &\ H_{\sigma(t)}x(t_i^-), \quad i \in \mathbb{N}^+
\end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the system input, $w \in \mathbb{R}^n$ is a bounded disturbance with unknown bound $\bar{w}$, i.e., $\|w\| \le \bar{w}$. The matrices $A_p \in \mathbb{R}^{n\times n}$, $E_p \in \mathbb{R}^{n\times n}$, $B_p \in \mathbb{R}^{n\times m}$, and $H_p \in \mathbb{R}^{n\times n}$ are known constant matrices with (A_p, B_p), $p \in \mathcal{M}$, being controllable; $\Delta A_p \in \mathbb{R}^{n\times n}$ and $\Delta E_{\ell,p} \in \mathbb{R}^{n\times n}$ are unknown possibly time-varying matrices. The terms $d_{\ell,p}(\cdot) \in \mathbb{R}$, $\ell \in \mathcal{L}$, $p \in \mathcal{M}$, represent unknown multiple time-varying delays, and $\psi(\vartheta)$ is a continuous initial function for $\vartheta \in [t_0 - d_{\mathrm{m}}, t_0]$ with d_{m} defined in Assumption 1. The switching signal $\sigma(\cdot)$ is a piecewise left-continuous function, taking values from the set $\mathcal{M}$.

Let us represent the sequence of switch-out instants of subsystem p as $\left\{t_{l+1}^{p_{\mathrm{out}}} \mid l \in \mathbb{N}^+\right\}$. Then, the length of the l^{th} active interval of subsystem p is $t_{l+1}^{p_{\mathrm{out}}} - t_l^{p_{\mathrm{in}}}$ for all $l \in \mathbb{N}^+$. The following definitions are given for the class of switching signals and for the type of the stability considered in this work.

Definition 1 [**Mode-dependent dwell time**] [24] Switching signals are said to belong to the *mode-dependent dwell-time admissible* set $\mathcal{D}(\tau_{\mathrm{d}p})$ if for any $p \in \mathcal{M}$ there exists a number $\tau_{\mathrm{d}p} > 0$ such that $t_{l+1}^{p_{\mathrm{out}}} - t_l^{p_{\mathrm{in}}} \ge \tau_{\mathrm{d}p}$ holds for all $l \in \mathbb{N}^+$. Any positive number $\tau_{\mathrm{d}p}$, for which these constraints hold for all $l \in \mathbb{N}^+$, is called *mode-dependent dwell time.*

Definition 2 [**Global uniform ultimate boundedness**] [25] The uncertain switched impulsive system (1) under switching signal $\sigma(\cdot)$ is *globally uniformly ultimately bounded* (GUUB) if there exists a finite positive number b_Γ such that for every initial function x_{t_0}, there exists a finite positive number Γ independent of t_0 such that $\|x(t)\| \le b_\Gamma$ for all $t \ge t_0 + \Gamma$. Any positive number b_Γ for which this condition holds is called *ultimate bound.*

The following assumptions are made.

Assumption 1 There exists a positive constant $d_{\mathrm{m}} \triangleq \sup_{\ell\in\mathcal{L},p\in\mathcal{M},t\ge t_0} d_{\ell,p}(t)$, which is not necessarily known.

Assumption 2 The uncertain matrices $\Delta A_p(\cdot)$ and $\Delta E_p(\cdot)$ satisfy the following matching conditions

$$\Delta A_p(t) = B_p \Xi_p(t), \quad \Delta E_{\ell,p}(t) = B_p \Pi_{\ell,p}(t) \tag{2}$$

with $\|\Xi_p(t)\|^2 \le \xi_p$ and $\|\Pi_{\ell,p}(t)\|^2 \le \zeta_{\ell,p}$ where ξ_p and $\zeta_{\ell,p}$, $p \in \mathcal{M}$, $\ell \in \mathcal{L}$, are unknown positive constants.

Remark 1 Assumption 1 only requires the existence of an upper bound to the multiple time-varying delays. Note that the time-varying delays are allowed be discontinuous at the switching instants due to switching behavior (in other words, the delays are piecewise continuous at the switching instants). Discontinuity excludes the application of the Krasovskii technique, while the Razumikhin technique is intrinsically subject to limitations in the adaptive control setting, as highlighted in [22, 23]. Therefore, a new stability condition needs to be developed for adaptive control of system (1). Assumption 2 is rather standard and widely used in adaptive control or robust control [23, 25] to dominate the parametric uncertainties. Note that Assumption 2 will be relaxed in (24), so as to handle bounded unmatched uncertainties.

The following lemmas are useful for deriving the main results.

Lemma 1 [26] Let $y \in \mathbb{R}^p$, $z \in \mathbb{R}^q$, and M, N be appropriately dimensioned constant matrices. Then, for any positive constant ϵ, it holds that

$$2y^T MNz \leq \epsilon y^T MM^T y + \epsilon^{-1} z^T N^T Nz.$$

Lemma 2 [20] For given positive scalars $\mu \geq 1$, a, and b, which satisfy $0 < b < a\mu/(\mu+1)$, define

$$v \triangleq \frac{1}{c} \operatorname{arctanh} \frac{(\mu-1)c}{\frac{a}{2}(\mu+1)-2b} \tag{3}$$

where $c = \sqrt{a^2/4 - b^2/\mu}$. Let $\varphi(t)$ be the solution of the following initial value problem

$$\begin{aligned} \dot{\varphi}(t) &= -\frac{v}{T}\left(\varphi^2(t) - a\varphi(t) + \frac{b^2}{\mu}\right), \quad t \geq t_s \\ \varphi(t_s) &= \frac{b}{\mu} \end{aligned} \tag{4}$$

with $T > 0$. Then, $\varphi(t)$ exists on $[t_s,\infty)$ and satisfies

$$\varphi(t) = \frac{\frac{a}{2} + c + (\frac{a}{2} - c)\varpi(t)}{1 + \varpi(t)}, \quad t \geq t_s \tag{5}$$

where $\varpi(t) = \frac{a/2+c-b/\mu}{b/\mu-a/2+c} e^{-\frac{2cv}{T}(t-t_s)}$, $\varphi(t_s+T) = b$, and $\dot{\varphi}(t) \geq 0$.

3 Main results

In this section, a new control scheme is proposed based on the solution of a family of LMIs and Riccati equations to guarantee global uniform ultimate boundedness of the closed-loop system. The following lemma extends the results of [22] to switched systems with impulsive behavior, which is crucial to derive the stability results.

Lemma 3 Let $g(\cdot)$ be a left-continuous function with $g(\cdot) \geq 0$ for all $t \geq t_0$ and let $\phi(\cdot) > 0$ be continuous for $t \in [t_0 - d_\mathrm{m}, t_0]$. If there exist positive constants α_1, α_2, α_3 with $\alpha_1 > \alpha_2$ such that

$$\begin{aligned} &\dot{g}(t) \leq -\alpha_1 g(t) + \alpha_2 \sup_{t-d_\mathrm{m} \leq s \leq t} g(s) + \alpha_3, \ t \in [t_i, t_{i+1}) \\ &g(t_{i+1}^-) \geq g(t_{i+1}) \\ &g_{t_0}(\vartheta) = \phi(\vartheta), \quad \vartheta \in [t_0 - d_\mathrm{m}, t_0] \end{aligned} \tag{6}$$

then, we have

$$g(t) \leq \beta_1 + \beta_2 e^{-\rho(t-t_0)}, \quad t \geq t_0$$

where $\beta_1 = \alpha_3/(\alpha_1 - \alpha_2)$, $\beta_2 = \sup_{t_0 - d_\mathrm{m} \leq s \leq t_0} \phi(s) - \beta_1$, and ρ is the unique solution to $\rho = \alpha_1 - \alpha_2 e^{\rho d_\mathrm{m}}$.

PROOF. To facilitate the proof, consider the differential equation

$$\begin{aligned} \dot{f}(t) &= -\alpha_1 f(t) + \alpha_2 \sup_{t-d_\mathrm{m} \leq s \leq t} f(s) + \alpha_3, \quad t \geq t_0 \\ f_{t_0}(\vartheta) &= \sup_{t_0 - d_\mathrm{m} \leq \vartheta \leq t_0} g(\vartheta), \quad \vartheta \in [t_0 - d_\mathrm{m}, t_0]. \end{aligned} \tag{7}$$

Considering that the initial condition $f_{t_0}(\vartheta)$ is positive, we search for a unique positive solution [1] to (7) in the form

$$f(t) = \beta_1 + \beta_2 e^{-\rho(t-t_0)}, \quad t \geq t_0 \tag{8}$$

with $\beta_1 > 0$, β_2, $\rho > 0$ to be determined, which implies that $\sup_{t-d_\mathrm{m} \leq s \leq t} f(s) = f(t - d_\mathrm{m})$. Note that uniqueness of (8) arises from the fact that $f(t)$ is locally Lipschitz guaranteed by continuity of the right-hand side of the differential equation (7). Substituting (8) into (7) leads to

$$\begin{aligned} -\rho\beta_2 e^{-\rho(t-t_0)} = &- \alpha_1\beta_1 + \alpha_2\beta_1 + \alpha_3 \\ &- \alpha_1\beta_2 e^{-\rho(t-t_0)} + \alpha_2\beta_2 e^{-\rho(t-t_0-d_\mathrm{m})} \end{aligned}$$

which gives the solutions to β_1, β_2 and the characteristic equation of ρ

$$\beta_1 = \frac{\alpha_3}{\alpha_1 - \alpha_2}, \ \beta_2 = f_{t_0} - \beta_1, \ \rho = \alpha_1 - \alpha_2 e^{\rho d_\mathrm{m}}$$

where a solution to ρ always exists and is unique due to $\alpha_1 \geq \alpha_2$, and $\beta_2 = \sup_{t_0 - d_\mathrm{m} \leq s \leq t_0} \phi(s) - \beta_1$. Next, we use a proof by contradiction to show that $g(t) \leq f(t)$ for $t \in [t_i, t_{i+1})$. To facilitate the proof, we define a continuous function $h(t) \geq g(t)$ such that $\dot{h}(t) \leq -\alpha_1 h(t) + \alpha_2 \sup_{t-d_\mathrm{m} \leq s \leq t} h(s) + \alpha_3$ for $t \geq t_0$ and $h_{t_0}(\vartheta) = \phi(\vartheta)$

[1] We can use an argument by contradiction to prove positivity. Assume that at time t_l we have $f(t_l) = 0$ and then $\dot{f}(t_l) > 0$. According to (7) and continuity of the derivative of $f(t)$, it follows that there exists an instant $t_r < t_l$ such that $\dot{f}(t_r) = 0$, $f(t_r) > 0$ and $\dot{f}(t) > 0$ for $t \in [t_r, t_l]$, which suggests that $f(t_l) > 0$. This is a contradiction with $f(t_l) = 0$. Therefore, the solution to (7) is always positive.

for $\vartheta \in [t_0 - d_{\mathrm{m}}, t_0]$. Assume that there exists a time instant t_p such that $h(t_p) = f(t_p)$ and $h(t) > f(t)$ for $t > t_p$. It is evident that $h(t) < f(t)$ for $t < t_p$, which results in $\sup_{t_p - d_{\mathrm{m}} < s \le t} h(s) \le \sup_{t_p - d_{\mathrm{m}} < s \le t} f(s)$. According to (7), it follows that $\dot{h}(t_p) < \dot{f}(t_p)$ and hence $h(t) < f(t)$ for $t > t_p$. This leads to a contradiction with the condition that $h(t) > f(t)$ for $t > t_p$. Therefore, we have $h(t) \le f(t)$ and hence $g(t) \le f(t)$ for $t \in [t_i, t_{i+1})$. Considering $g(t_{i+1}^-) \ge g(t_{i+1})$ at the switching instant t_{i+1}, we arrive at $g(t_{i+1}) \le f(t_{i+1})$. This implies, together with (8)

$$g(t) \le \frac{\alpha_3}{\alpha_1 - \alpha_2} + \beta_2 e^{-\rho(t-t_0)}, \quad t \ge t_0$$

where $\beta_2 = \sup_{t_0 - d_{\mathrm{m}} \le s \le t_0} \phi(s) - \frac{\alpha_3}{\alpha_1 - \alpha_2}$. This completes the proof. ■

Remark 2 In [27, Theorem 1], a function V is used, which is continuous for all $t \ge t_0$. In our case, we use the function $g(t)$, which is not continuous because of the switching behavior between different subsystems. In this sense, Lemma 3 can be used to study a larger class of systems as compared to [27, Theorem 1].

Now we are ready to present the stability result using Lemmas 1–3.

Theorem 1 *Suppose that there exist a family of symmetric positive definite matrices $P_p, Q_p, G_p \in \mathbb{R}^{n \times n}$, positive scalars a, b, v, χ τ_p, $\mu \ge 1$, $\varepsilon_{\ell,p}$, $\epsilon_{\ell,p}$, $\varrho_{1,p}$, $\varrho_{2,p}$, $\ell \in \mathcal{L}$, $p \in \mathcal{M}$, such that $b < a\mu/(\mu+1)$, v satisfies (3), and*

$$\begin{bmatrix} \Psi_p & P_p E_{1,p} & \cdots & P_p E_{L,p} \\ * & -\varepsilon_{1,p}^{-1} I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & -\varepsilon_{L,p}^{-1} I \end{bmatrix} < 0 \tag{9a}$$

$$\begin{bmatrix} -\frac{v}{\tau_p} P_p & -G_p \\ * & -\frac{b^2 v}{\mu \tau_p} P_p \end{bmatrix} < 0 \tag{9b}$$

$$\chi P_p - b \sum_{\ell=1}^{L} \left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} \right) I > 0 \tag{9c}$$

$$H_q^T P_q H_q \le \mu P_p \tag{9d}$$

with $\Psi_p = -Q_p + \frac{v}{\tau_p} a P_p + \chi P_p + 2G_p$, where P_p and Q_p, $\varrho_{1,p}$, $\varrho_{2,p}$, $p \in \mathcal{M}$, and κ satisfy the following Riccati equation

$$\begin{aligned} A_p^T P_p + P_p A_p \; &+ \left(\varrho_{1,p}^{-1} + \varrho_{2,p}^{-1} \right) I \\ &- 2\kappa P_p B_p B_p^T P_p = -Q_p. \end{aligned} \tag{10}$$

Then, under Assumptions 1 and 2, the controller

$$u(t) = -\left(\kappa + \frac{1}{2} \hat{\theta}(t) \right) B_{\sigma(t)}^T P_{\sigma(t)} x(t) \tag{11}$$

and the adaptive law

$$\dot{\hat{\theta}}(t) = \gamma \varphi_{\mathrm{m}}(t) x^T(t) P_{\sigma(t)} B_{\sigma(t)} B_{\sigma(t)}^T P_{\sigma(t)} x(t) - \gamma \delta \hat{\theta}(t) \tag{12}$$

with $\gamma > 0$ being a given adaptive gain, $\delta \ge \chi/\gamma$, and

$$\varphi_{\mathrm{m}}(t) = \begin{cases} \varphi(t), & t \in [t_i, t_i + \tau_{\sigma(t)}) \\ b, & t \in [t_i + \tau_{\sigma(t)}, t_{i+1}) \\ \frac{b}{\mu}, & t = t_{i+1} \end{cases} \tag{13}$$

and $\varphi(\cdot)$ as in (5) with $T = \tau_{\sigma(t)}$ and $t_s = t_i$ guarantees that the switched impulsive system (1) is GUUB for any switching signal $\sigma(\cdot) \in \mathcal{D}(\tau_p)$. Moreover, an ultimate bound is given by

$$b_\Gamma = \sqrt{\frac{b\mu \bar{w}^2 \max_{p\in\mathcal{M}} \varrho_{2,p} \overline{\lambda} + \frac{\delta}{2}\theta^2}{b\underline{\lambda}\chi - b\mu \max_{p\in\mathcal{M}} \sum_{\ell=1}^{L} \left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} \right)}} \tag{14}$$

where $\underline{\lambda} \triangleq \min_{p\in\mathcal{M}} \lambda_{\min}(P_p)$, $\overline{\lambda} \triangleq \max_{p\in\mathcal{M}} \lambda_{\max}(P_p)$, and

$$\theta \triangleq \max_{p\in\mathcal{M}} \left\{ \xi_p \varrho_{1,p} + \sum_{\ell=1}^{L} \zeta_{\ell,p} \epsilon_{\ell,p} \right\}. \tag{15}$$

PROOF. In this proof, the time index is sometimes not indicated for compactness, and a delayed signal will be marked with the subscript d, e.g. $x_{\mathrm{d}} = x(t - d_{\ell,p}(t))$. Consider the following Lyapunov function

$$V(t) = \varphi_{\mathrm{m}}(t) x^T(t) P_{\sigma(t)} x(t) + \frac{1}{2\gamma} \tilde{\theta}^2(t) \tag{16}$$

with $\tilde{\theta} = \theta - \hat{\theta}$. It is straightforward that $V(\cdot)$ is continuous during the switching intervals $[t_i, t_{i+1})$, $i \in \mathbb{N}^+$, and discontinuous at the switching instants t_i, $i \in \mathbb{N}^+$. Without loss of generality, we assume that subsystem p is active for $t \in [t_i, t_{i+1})$ and subsystem q is active for $t \in [t_{i+1}, t_{i+2})$. Moreover, to facilitate the analysis of the Lyapunov function, we partition the interval $[t_i, t_{i+1})$ into two parts: $[t_i, t_i + \tau_p)$ and $[t_i + \tau_p, t_{i+1})$, upon which, according to (13), (16) can be recast into

$$V(t) = \begin{cases} \varphi(t) x^T(t) P_p x(t) + \frac{1}{2\gamma} \tilde{\theta}^2(t), & t \in [t_i, t_i + \tau_p) \\ b x^T(t) P_p x(t) + \frac{1}{2\gamma} \tilde{\theta}^2(t), & t \in [t_i + \tau_p, t_{i+1}). \end{cases}$$

The essence of the proof is to show that the Lyapunov function satisfies the conditions in Lemma 3. The proof is organized in three steps:

(a) for $t \in [t_i, t_i + \tau_p)$, the Lyapunov function is shown to satisfy the conditions in Lemma 3 using the LMIs (9a)–(9c), the Riccati equation (10), and the adaptive controller (11)–(13);
(b) for $t \in [t_i+\tau_p, t_{i+1})$, the Lyapunov function is shown to satisfy the conditions in Lemma 3 using the LMIs (9a) and (9c), the Riccati equation (10), and the adaptive controller (11)–(13);
(c) at the switching instant t_{i+1}, the Lyapunov function is shown to be non-increasing due to (9d) and the reset of $\varphi_{\mathrm{m}}(t_{i+1})$.

(*a*) For $t \in [t_i, t_i + \tau_p)$, it can be shown that the time derivative of $V(\cdot)$ is

$$\begin{aligned}\dot{V} \leq \varphi x^T & \left(A_p^T P_p + P_p A_p + \sum_{\ell=1}^{L} \varepsilon_{\ell,p} P_p E_{\ell,p} E_{\ell,p}^T P_p \right) x \\ & + \varphi x^T \left(\sum_{\ell=1}^{L} \epsilon_{\ell,p} P_p \Delta E_{\ell,p} \Delta E_{\ell,p}^T P_p \right) x + \varrho_{1,p}^{-1} \varphi x^T x \\ & + \varrho_{1,p} \varphi x^T P_p \Delta A_p \Delta A_p^T P_p x + 2\varphi x^T P_p B_p u \\ & + \sum_{\ell=1}^{L} \left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} \right) \varphi x_{\mathrm{d}}^T x_{\mathrm{d}} + \varrho_{2,p} \varphi w^T P_p P_p w \\ & + \varrho_{2,p}^{-1} \varphi x^T x - \frac{v}{\tau_p} \left(\varphi^2 - a\varphi + \frac{b^2}{\mu} \right) x^T P_p x - \frac{1}{\gamma} \tilde{\theta} \dot{\hat{\theta}}\end{aligned} \tag{17}$$

where the inequality holds according to Lemma 1 and Lemma 2. Using Assumption 1 and the fact that $\varphi > 0$, (17) is written as

$$\begin{aligned}\dot{V} \leq \varphi x^T & \left(A_p^T P_p + P_p A_p + \sum_{\ell=1}^{L} \varepsilon_{\ell,p} P_p E_{\ell,p} E_{\ell,p}^T P_p \right) x \\ & + \left(\xi_p \varrho_{1,p} + \sum_{\ell=1}^{L} \zeta_{\ell,p} \epsilon_{\ell,p} \right) \varphi x^T P_p B_p B_p^T P_p x \\ & + 2\varphi x^T P_p B_p u + \left(\varrho_{1,p}^{-1} + \varrho_{2,p}^{-1} \right) \varphi x^T x \\ & - \frac{v}{\tau_p} \left(\varphi^2 - a\varphi + \frac{b^2}{\mu} \right) x^T P_p x - \frac{1}{\gamma} \tilde{\theta} \dot{\hat{\theta}} \\ & + \sum_{\ell=1}^{L} \left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} \right) \varphi x_{\mathrm{d}}^T x_{\mathrm{d}} + \varrho_{2,p} \varphi w^T P_p P_p w.\end{aligned} \tag{18}$$

Then, substituting the Riccati equation (10) into (18) yields

$$\begin{aligned}\dot{V} \leq & - \varphi x^T Q_p x + 2\kappa \varphi x^T P_p B_p B_p^T P_p x \\ & + \varphi x^T \left(\sum_{\ell=1}^{L} \varepsilon_{\ell,p} P_p E_{\ell,p} E_{\ell,p}^T P_p \right) x \\ & + \left(\xi_p \varrho_{1,p} + \sum_{\ell=1}^{L} \zeta_{\ell,p} \epsilon_{\ell,p} \right) \varphi x^T P_p B_p B_p^T P_p x \\ & + 2\varphi x^T P_p B_p u + \varrho_{2,p} \varphi w^T P_p P_p w \\ & - \frac{v}{\tau_p} \left(\varphi^2 - a\varphi + \frac{b^2}{\mu} \right) x^T P_p x - \frac{1}{\gamma} \tilde{\theta} \dot{\hat{\theta}} \\ & + \sum_{\ell=1}^{L} \left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} \right) \varphi x_{\mathrm{d}}^T x_{\mathrm{d}}.\end{aligned}$$

With help of the controller (11), the adaptive law (12), and the definition of θ in (15), one has

$$\begin{aligned}\dot{V} \leq & - \varphi x^T Q_p x + \varphi x^T \left(\sum_{\ell=1}^{L} \varepsilon_{\ell,p} P_p E_{\ell,p} E_{\ell,p}^T P_p \right) x \\ & + \sum_{\ell=1}^{L} \left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} \right) \varphi x_{\mathrm{d}}^T x_{\mathrm{d}} + \delta \tilde{\theta} \hat{\theta} \\ & - \frac{v}{\tau_p} \left(\varphi^2 - a\varphi + \frac{b^2}{\mu} \right) x^T P_p x \\ & + \varrho_{2,p} \varphi w^T P_p P_p w.\end{aligned}$$

Furthermore, (9b) directly shows

$$\begin{bmatrix} \varphi x \\ x \end{bmatrix}^T \begin{bmatrix} -\frac{v}{\tau_p} P_p & -G_p \\ * & -\frac{b^2 v}{\mu \tau_p} P_p \end{bmatrix} \begin{bmatrix} \varphi x \\ x \end{bmatrix} < 0$$

which, combined with (9a) by Schur complement, suggests

$$\begin{aligned}\dot{V} \leq & - \chi \varphi x^T P_p x + \varrho_{2,p} \varphi w^T P_p P_p w \\ & + \sum_{\ell=1}^{L} \left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} \right) \varphi x_{\mathrm{d}}^T x_{\mathrm{d}} + \delta \tilde{\theta} \hat{\theta}.\end{aligned}$$

Recalling that $\tilde{\theta} = \theta - \hat{\theta}$ and using $\delta \tilde{\theta} \theta - \delta \tilde{\theta}^2 \leq -\frac{1}{2} \delta \tilde{\theta}^2 + \frac{1}{2} \delta \theta^2$ results in

$$\begin{aligned}\dot{V} \leq & - \chi \varphi x^T P_p x - \frac{\chi}{2\gamma} \tilde{\theta}^2 + \varrho_{2,p} \varphi w^T P_p P_p w \\ & + \sum_{\ell=1}^{L} \left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} \right) \varphi x_{\mathrm{d}}^T x_{\mathrm{d}} + \frac{1}{2} (\frac{\chi}{\gamma} - \delta) \tilde{\theta}^2 + \frac{\delta}{2} \theta^2\end{aligned}$$

where $\chi/\gamma - \delta \le 0$. In addition, the following holds

$$\begin{aligned}\varphi x_{\mathrm{d}}^T x_{\mathrm{d}} &\le \frac{\mu}{\lambda_{\min}(P_p)}\varphi_{\mathrm{d}} x_{\mathrm{d}}^T P_p x_{\mathrm{d}} \\ &\le \frac{\mu}{\lambda_{\min}(P_p)} V_{\mathrm{d}} \\ &\le \frac{\mu}{\lambda_{\min}(P_p)} \sup_{t-d_{\mathrm{m}}\le s\le t} V(s).\end{aligned} \tag{19}$$

Hence, the derivative of V for $t \in [t_i, t_i + \tau_p)$ satisfies

$$\begin{aligned}\dot{V} \le & -\chi V + \frac{\delta}{2}\theta^2 + \varrho_{2,p}\varphi w^T P_p P_p w \\ & + \frac{\mu \sum_{\ell=1}^{L}\left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1}\right)}{\lambda_{\min}(P_p)} \sup_{t-d_{\mathrm{m}}\le s\le t} V(s).\end{aligned} \tag{20}$$

(b) For $t \in [t_i+\tau_p, t_{i+1})$, the Lyapunov function becomes

$$V(t) = b x^T(t) P_{\sigma(t)} x(t) + \frac{1}{2\gamma}\tilde{\theta}^2(t).$$

It follows immediately from (9a) that

$$\begin{bmatrix}\Theta_p & P_p E_{1,p} & \cdots & P_p E_{L,p} \\ * & -\varepsilon_{1,p}^{-1} I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & -\varepsilon_{L,p}^{-1} I\end{bmatrix} < 0$$

with $\Theta_p = -Q_p + \chi P_p$, which, combined with (9c) and following the similar steps from (17) to (20) yields

$$\begin{aligned}\dot{V} \le & -\chi V + \frac{\delta}{2}\theta^2 + \varrho_{2,p} b w^T P_p P_p w \\ & + \frac{\mu \sum_{\ell=1}^{L}\left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1}\right)}{\lambda_{\min}(P_p)} \sup_{t-d_{\mathrm{m}}\le s\le t} V(s).\end{aligned} \tag{21}$$

According to (20) and (21), it holds for $t \in [t_i, t_{i+1})$

$$\begin{aligned}\dot{V} \le & -\chi V + \frac{\delta}{2}\theta^2 + \varrho_{2,p}\varphi_{\mathrm{m}} w^T P_p P_p w \\ & + \frac{\mu \sum_{\ell=1}^{L}\left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1}\right)}{\lambda_{\min}(P_p)} \sup_{t-d_{\mathrm{m}}\le s\le t} V(s).\end{aligned} \tag{22}$$

(c) At the switching instant t_{i+1}, using (9d) and the fact that $\varphi(t_{i+1}^-) = b$ and $\varphi(t_{i+1}) = \frac{b}{\mu}$, one has

$$\begin{aligned}& V(t_{i+1}) - V(t_{i+1}^-) \\ & = \varphi(t_{i+1}) x^T(t_{i+1}) P_q x(t_{i+1}) - b(t_{i+1}^-) x^T(t_{i+1}^-) P_p x(t_{i+1}^-) \\ & = \frac{b}{\mu} x^T(t_{i+1}^-) H_q^T P_q H_q x(t_{i+1}^-) - b x^T(t_{i+1}^-) P_p x(t_{i+1}^-) \\ & = b x^T(t_{i+1}^-)\left(\frac{H_q^T P_q H_q}{\mu} - P_p\right) x(t_{i+1}^-) \\ & \le 0\end{aligned} \tag{23}$$

which implies that (22) holds for all $t \ge t_0$. In light of this, using (9c) and Lemma 3, it readily follows

$$V(t) \le \frac{b\bar{w}^2 \max_{p\in\mathcal{M}} \varrho_{2,p}\lambda_{\max}^2(P_p) + \frac{\delta}{2}\theta^2}{\chi - \frac{\mu\sum_{\ell=1}^{L}\left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1}\right)}{\min_{p\in\mathcal{M}}\lambda_{\min}(P_p)}} + \beta_2 e^{-\rho(t-t_0)}$$

where β_2 is a finite constant dependent on the initial value of the Lyapunov function. This indicates, together with (16), the ultimate bound b_Γ shown in (14). This completes the proof. ■

Remark 3 Some comments are needed to clarify that the family of Riccati equations (10) can be introduced without loss of generality. Since (A_p, B_p) is controllable for all $p \in \mathcal{M}$, one can always find a solution for P_p and Q_p satisfying (10). As a matter of fact, the Riccati equations guarantee a sufficient large stability margin with the only requirement of controllability. In [23], a LMI condition is proposed to design the adaptive controller for time-varying delay without considering switching behavior of the system: however, the absence of a Riccati equation fundamentally requires the system matrix A_p to be Hurwitz, which to a large extent limits the scope of applications of the method in [23].

Remark 4 In contrast with the Razumikhin technique, where an adaptive controller is guaranteed to exist only in an unknown interval, the existence of the adaptive controller (11)–(13) is well defined by the appropriate selection of the constants in Theorem 1. Here are some guidelines for the selection of such constants: after a sufficiently large stability margin has been achieved by the solution of the Riccati equations (10), one can find a feasible μ in (9d); at this point, with a simple grid search over the couple (a, b) (which automatically defines v from (3)), we have that (9a)–(9c) are linear in the decision variables G_p, τ_p^{-1}, $\varepsilon_{\ell,p}^{-1}$, $\epsilon_{\ell,p}^{-1}$. One can either solve a feasibility problem, or preferably, optimize the solution to the LMIs for large τ_p^{-1} (to address a large family of switching laws), or large $\epsilon_{\ell,p}^{-1}$ and $b\underline{\lambda}\chi - b\mu\max_{p\in\mathcal{M}}\sum_{\ell=1}^{L}\left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1}\right)$ (to minimize the ultimate bound b_Γ in (14)).

Remark 5 Different with classic adaptive laws with a constant gain, the proposed design incorporates a piecewise dynamic gain $\varphi_{\rm m}$, entering both the adaptive law (12) and the Lyapunov function (16). Note that (23) suggests that the Lyapunov function (16) is non-increasing at switching instants thanks to the dynamic gain $\varphi_{\rm m}$ in (13). While adaptive asymptotic stabilization of system (1) with time-varying delays is still an open problem, it can be verified that in the absence of disturbances and time-varying delays, asymptotic stability of the adaptive closed-loop system can be derived, which is the first result of its kind since the delays are allowed to be discontinuous at switching instants and constant in between. To verify this claim, use the controller (11) and the adaptive law (12) with $\delta \equiv 0$: then (22) reduces to $\dot{V} \leq -\chi\varphi_{\rm m} x^T P_p x$, and the using of Barbalat's Lemma leads to asymptotic stability. This implies, in the spirit of [15], that the Lyapunov function (16) can lead to less conservative result than standard multiple Lyapunov functions [11], i.e., with $\varphi_{\rm m} \equiv 1$.

Remark 6 Connecting to the previous remark, a question may arise: why cannot the time-interpolation method in [15] (which is also based on a Lyapunov function non-increasing at the switching instants) be adopted to achieve the control objective of this work? Some clarifications are provided as follows: instead of using a constant P_p for each subsystem, [15] relies on a time-varying P_p, $t \in [t_i, t_{i+1})$, obtained by linear interpolation of a set of positive definite matrices (c.f. Lemma 1 in [15]). However, the need for the Riccati equations in (10), which are quadratic in P_p, makes linear interpolation not applicable here.

In many practical cases, the uncertainties may not satisfy the matching conditions shown in (2). For the unmatched case, Assumption 2 can be relaxed into Assumption 3.

Assumption 3 The uncertain matrices $\Delta A_p(\cdot)$ and $\Delta E_{\ell,p}(\cdot)$ satisfy

$$\begin{aligned} \Delta A_p(t) &= B_p \Xi_p(t) + \Delta\Xi_p(t) \\ \Delta E_{\ell,p}(t) &= B_p \Pi_{\ell,p}(t) + \Delta\Pi_{\ell,p}(t) \end{aligned} \tag{24}$$

with $\|\Xi_p(t)\|^2 \leq \xi_p$, $\|\Delta\Xi_p(t)\|^2 \leq \Delta\xi_p$, and $\|\Pi_{\ell,p}(t)\|^2 \leq \zeta_{\ell,p}$, and $\|\Delta\Pi_{\ell,p}(t)\|^2 \leq \Delta\zeta_{\ell,p}$, $p \in \mathcal{M}$, $\ell \in \mathcal{L}$, where ξ_p and $\zeta_{\ell,p}$ are unknown positive constants, and $\Delta\xi_p$ and $\Delta\zeta_{\ell,p}$ are known positive constants.

Remark 7 It is known in adaptive stabilization that unmatched uncertainties as in (24) cannot be addressed by the controller in an adaptive fashion, and the knowledge of the bounds of the unmatched uncertainties is required to guarantee stability of the switched system. In fact, to the best of the authors' knowledge, how to cope with unknown unmatched uncertainties without knowing their bounds is still an open problem both in adaptive control and robust control [25].

Considering the unmatched terms as in (24), we provide the following stability result.

Corollary 1 Suppose that there exist a family of positive definite symmetric matrices $P_p, Q_p, G_p \in \mathbb{R}^{n\times n}$, positive scalars a, b, v, χ τ_p, $\mu \geq 1$, $\varepsilon_{\ell,p}$, $\epsilon_{\ell,p}$, $\iota_{\ell,p}$, $\varrho_{1,p}$, $\varrho_{2,p}$, $\ell \in \mathcal{L}$, $p \in \mathcal{M}$ such that $b < a\mu/(\mu+1)$, v satisfies (3), and

$$\begin{bmatrix} \Psi_p & P_pE_{1,p} & \cdots & P_pE_{L,p} & \sqrt{\Delta\zeta_{1,p}}P_p & \cdots & \sqrt{\Delta\zeta_{L,p}}P_p \\ * & -\varepsilon_{1,p}^{-1}I & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & -\varepsilon_{L,p}^{-1}I & 0 & \cdots & 0 \\ * & * & \cdots & * & -\iota_{1,p}^{-1}I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & \cdots & * & * & \cdots & -\iota_{L,p}^{-1}I \end{bmatrix} < 0$$

$$\begin{bmatrix} -\frac{v}{\tau_p}P_p & -G_p \\ * & -\frac{b^2 v}{\mu\tau_p}P_p \end{bmatrix} < 0$$

$$\chi P_p - b\sum_{\ell=1}^{L}\left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} + \iota_{\ell,p}^{-1}\right) I > 0$$

$$H_q^T P_q H_q \leq \mu P_p$$

with $\Psi_p = -Q_p + \frac{v}{\tau_p}aP_p + \chi P_p + 2G_p$, where P_p and Q_p, $\varrho_{1,p}$, $\varrho_{2,p}$, $p \in \mathcal{M}$, and κ satisfy the Riccati equation

$$\begin{aligned} &\left(A_p + \sqrt{\Delta\xi_p}I\right)^T P_p + P_p\left(A_p + \sqrt{\Delta\xi_p}I\right) \\ &\quad + \left(\varrho_{1,p}^{-1} + \varrho_{2,p}^{-1}\right) I - 2\kappa P_p B_p B_p^T P_p = -Q_p. \end{aligned}$$

Then, under Assumptions 1 and 3, the controller

$$u(t) = -\left(\kappa + \frac{1}{2}\hat{\theta}(t)\right) B_{\sigma(t)}^T P_{\sigma(t)} x(t)$$

and the adaptive law

$$\dot{\hat{\theta}}(t) = \gamma\varphi_{\rm m}(t) x^T(t) P_{\sigma(t)} B_{\sigma(t)} B_{\sigma(t)}^T P_{\sigma(t)} x(t) - \gamma\delta\hat{\theta}(t)$$

with $\gamma > 0$ being a given adaptive gain, $\delta \geq \chi/\gamma$, and $\varphi_{\rm m}$ as defined in (13) guarantees that the switched impulsive system (1) is GUUB for any switching signal $\sigma(\cdot) \in \mathcal{D}(\tau_p)$. Moreover, an ultimate bound is given by

$$b_\Gamma = \sqrt{\frac{b\mu\bar{w}^2 \max_{p\in\mathcal{M}} \varrho_{2,p}\overline{\lambda} + \frac{\delta}{2}\theta^2}{b\underline{\lambda}\chi - b\mu\max_{p\in\mathcal{M}}\sum_{\ell=1}^{L}\left(\varepsilon_{\ell,p}^{-1} + \epsilon_{\ell,p}^{-1} + \iota_{\ell,p}^{-1}\right)}}$$

where $\underline{\lambda} \triangleq \min_{p\in\mathcal{M}} \lambda_{\min}(P_p)$, $\overline{\lambda} \triangleq \max_{p\in\mathcal{M}} \lambda_{\max}(P_p)$, and

$$\theta \triangleq \max_{p\in\mathcal{M}} \left\{ \xi_p \varrho_{1,p} + \sum_{\ell=1}^{L} \zeta_{\ell,p}\epsilon_{\ell,p} \right\}.$$

PROOF. The proof follows similar steps as the proof of Theorem 1, and thus it is omitted. ■

4 Example

Consider the two-tank system taken from [28, 29], and illustrated in Fig. 1. The states of the system are the deviations of reservoir levels with respect to their nominal values, denoted by the dashed lines in Fig.1. The flow between the two reservoirs is proportional to the difference of their levels. We assume that both flow control and level measurement can switch between the first tank (actuator 1-sensor 1) and the second tank (actuator 2-sensor 2). In addition, the pipeline connecting the two tanks gives rise to time delays. The uncertainties in system matrices represent the delayed water flow in the pipeline, which influences the dynamics of the two-tank system. Thus, the two tank system can be modeled as an impulsive switched system

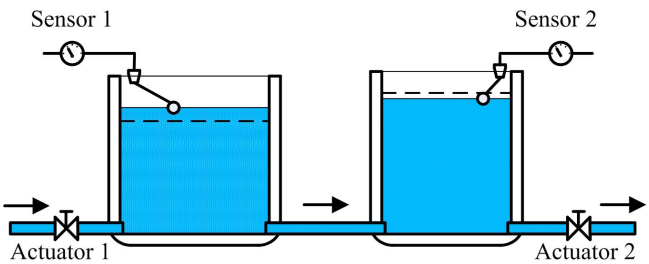

Fig. 1. The two-tank system.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \left(E_{\sigma(t)} + \Delta E_{\sigma(t)}\right) x(t - d_{\sigma(t)}(t)) \\ &\quad + B_{\sigma(t)}u(t) + w(t) \\ x(t_i) &= H_{\sigma(t)}x(t_i^-) \end{aligned}$$

where the following matrices have been taken in line with [28, 29]

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 & -0.2 \\ 0.2 & 0.4 \end{bmatrix}, E_2 = \begin{bmatrix} 0.2 & -0.3 \\ -0.2 & 0.4 \end{bmatrix}$$

$$\Delta E_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0 \end{bmatrix}, \ \Delta E_2 = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, H_1 = \begin{bmatrix} 0.95 & 0 \\ 0 & 1.05 \end{bmatrix}, H_2 = \begin{bmatrix} 1.05 & 0 \\ 0 & 0.95 \end{bmatrix}$$

and $d_1(t) = 0.1(1 - \cos(t))$, $d_2(t) = 0.1(1 + \sin(t))$, and $w(t) = 0.1\cos(5t)$. Let $\varrho_{11} = \varrho_{12} = \varrho_{21} = \varrho_{22} = 0.1$, $\epsilon_1 = \epsilon_2 = 1000$, $\kappa = 10$, $a = 10$, $b = 2$, and $\chi = 0.25$, $\gamma = 1$, $\delta = 0.3$, and

$$Q_1 = \begin{bmatrix} 8 & 1.9 \\ 1.9 & 10 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 2.5 & 0.9 \\ 0.9 & 3 \end{bmatrix}.$$

Solving the Riccati equations (10) and the LMIs (9a)–(9d) results in

$$P_1 = \begin{bmatrix} 0.8661 & 0.6171 \\ 0.6171 & 3.7130 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.9884 & 0.3496 \\ 0.3496 & 0.5164 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 0.0414 & 0.0066 \\ 0.0066 & 0.0227 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.0184 & 0.0062 \\ 0.0062 & 0.0099 \end{bmatrix}$$

the mode-dependent dwell time $\tau_1 = 1.25$, $\tau_2 = 3$, and $\mu = 11.76$. For simulations, the following initial conditions are selected: $x_0 = [1\ 1.5]^T$, $\theta(0) = 1$. To illustrate the effect of the dynamic gain φ_{m} on the Lyapunov function V, we use the function $V_{\mathrm{m}} = \varphi_{\mathrm{m}} x^T P_p x$. Based on the switching signal in Fig. 2, the evolution of V_{m} is given in Fig. 2, which shows that V_{m} and thus V is decreasing at the switching instant [2]. In addition, the system trajectory in Fig. 3 and the state response Fig. 4 are shown to admit an ultimate bound, as expected from the GUUB result of Theorem 1.

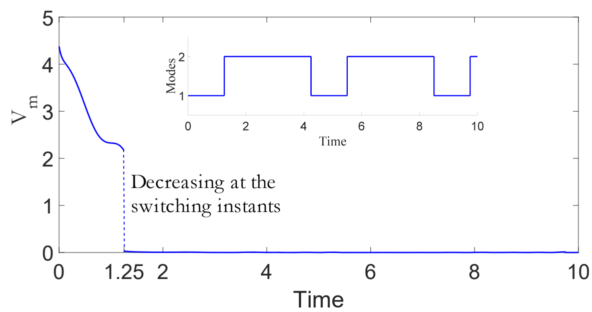

Fig. 2. The evolution of V_{m}.

5 Conclusions

This paper has investigated adaptive stabilization of switched impulsive systems with time-varying and possible discontinuous delays. By solving a family of Riccati equations and LMIs, a novel adaptive controller and a less conservative switching law based on mode-dependent dwell time have been designed. A piecewise dynamic gain has been designed for the adaptive law, which allows the Lyapunov function to be non-increasing at the switching

[2] Since the signal $\tilde{\theta}$ is unknown and continuous for all $t \geq t_0$, the absence of the quadratic term $\tilde{\theta}$ in V_{m} does not impact the non-increasing effect of φ_{m}.

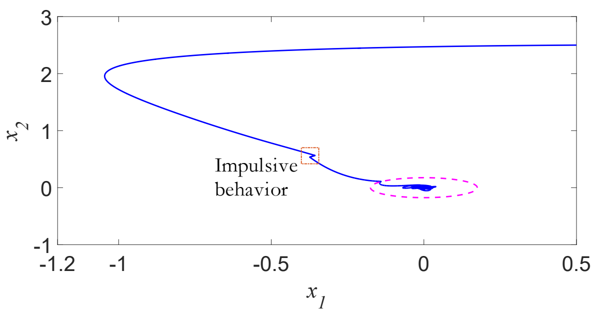

Fig. 3. The state trajectory and the attractive region.

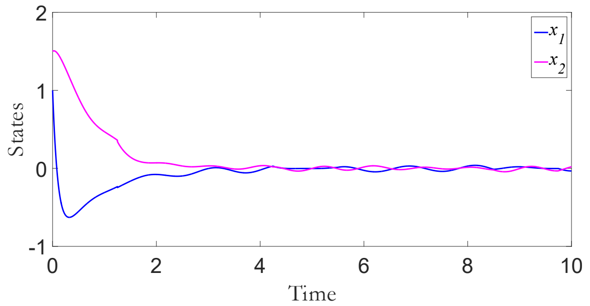

Fig. 4. The state response.

instants. Based on the proposed control scheme, global uniform ultimate boundedness of the closed-loop systems has been guaranteed. A two-tank system is used to illustrate the effectiveness of the control scheme.

References

[1] M. E. Fisher and K. L. Teo. Optimal insulin infusion resulting from a mathematical model of blood glucose dynamics. *IEEE Transactions on Biomedical Engineering*, 36(4):479–486, 1988.

[2] H. Xu, K. L. Teo, and X. Liu. Robust stability analysis of guaranteed cost control for impulsive switched systems. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 38(5):1419–1422, 2008.

[3] W. Zhu. Stability analysis of switched impulsive systems with time delays. *Nonlinear Analysis: Hybrid Systems*, 4(3):608–617, 2010.

[4] J. Liu, X. Liu, and W. Xie. Input-to-state stability of impulsive and switching hybrid systems with time-delay. *Automatica*, 47(5):899–908, 2011.

[5] C. Briat. Stability analysis and stabilization of stochastic linear impulsive, switched and sampled-data systems under dwell-time constraints. *Automatica*, 74:279–287, 2016.

[6] G. Zong, S. Xu, and Y. Wu. Robust H_∞ stabilization for uncertain switched impulsive control systems with state delay: An LMI approach. *Nonlinear Analysis: Hybrid Systems*, 2(4):1287–1300, 2008.

[7] H. Xu and K. L. Teo. Exponential stability with L_2 gain condition of nonlinear impulsive switched systems. *IEEE Transactions on Automatic Control*, 55(10):2429–2433, 2010.

[8] S. Skogestad and I. Postlethwaite. *Multivariable Feedback Control: Analysis and Design*. New York: Wiley, 1996.

[9] C. Olalla, R. Leyva, A. El Aroudi, and I. Queinnec. Robust LQR control for PWM converters: An LMI approach. *IEEE Transactions on Industrial Electronics*, 56(7):2548–2558, 2009.

[10] C. Guiver, C. Edholm, Y. Jin, M. Mueller, J. Powell, R. Rebarber, B. Tenhumberg, and Stuart Townley. Simple adaptive control for positive linear systems with applications to pest management. *SIAM Journal on Applied Mathematics*, 76(1):238–275, 2016.

[11] Q. Sang and G. Tao. Adaptive control of piecewise linear systems: the state tracking case. *IEEE Transactions on Automatic Control*, 57(2):522–528, 2012.

[12] M. di Bernardo, U. Montanaro, and S. Santini. Hybrid model reference adaptive control of piecewise affine systems. *IEEE Transactions on Automatic Control*, 58(2):304–316, 2013.

[13] X. Zhao, X. Zheng, B. Niu, and L. Liu. Adaptive tracking control for a class of uncertain switched nonlinear systems. *Automatica*, 52:185–191, 2015.

[14] Y. Li, S. Tong, L. Liu, and G. Feng. Adaptive output-feedback control design with prescribed performance for switched nonlinear systems. *Automatica*, 80:225–231, 2017.

[15] S. Yuan, B. De Schutter, and S. Baldi. Adaptive asymptotic tracking control of uncertain time-driven switched linear systems. *IEEE Transactions on Automatic Control*, 62(11):5802–5807, 2017.

[16] X. Sun, J. Zhao, and D. J. Hill. Stability and L_2-gain analysis for switched delay systems: A delay-dependent method. *Automatica*, 42(10):1769–1774, 2006.

[17] L. Zhang, E. K. Boukas, and J. Lam. Analysis and synthesis of markov jump linear systems with time-varying delays and partially known transition probabilities. *IEEE Transactions on Automatic Control*, 53(10):2458–2464, 2008.

[18] L. Vu and K. A. Morgansen. Stability of time-delay feedback switched linear systems. *IEEE Transactions on Automatic Control*, 55(10):2385–2390, 2010.

[19] Y. Wang, X. Sun, and F. Mazenc. Stability of switched nonlinear systems with delay and disturbance. *Automatica*, 69:78–86, 2016.

[20] W. Chen and W. Zheng. Delay-independent minimum dwell time for exponential stability of uncertain switched delay systems. *IEEE Transactions on Automatic Control*, 55(10):2406–2413, 2010.

[21] J. Lian, P. Shi, and Z. Feng. Passivity and passification for a class of uncertain switched stochastic time-delay systems. *IEEE Transactions on Cybernetics*, 43(1):3–13, 2013.

[22] S. Oucheriah. Adaptive robust control of a class of dynamic delay systems with unknown uncertainty

bounds. *International Journal of Adaptive Control and Signal Processing*, 15(1):53–63, 2001.

[23] F. Zheng, Q. Wang, and T. Lee. Adaptive robust control of uncertain time delay systems. *Automatica*, 41(8):1375–1383, 2005.

[24] L. Zhang, S. Zhang, and R. D. Braatz. Switched model predictive control of switched linear systems: Feasibility, stability and robustness. *Automatica*, 67:8–21, 2016.

[25] P. Ioannou and J. Sun. *Robust Adaptive Control*. Dover, 2012.

[26] S. Xu and T. Chen. Robust H_∞ control for uncertain stochastic systems with state delay. *IEEE Transactions on Automatic Control*, 47(2):2089–2094, 2002.

[27] A. R. Teel. Connections between Razumikhin-type theorems and the ISS nonlinear small gain theorem. *IEEE Transactions on Automatic Control*, 43(7):960–964, 1998.

[28] F. Blanchini, S. Miani, and F. Mesquine. A separation principle for linear switching systems and parametrization of all stabilizing controllers. *IEEE Transactions on Automatic Control*, 54(2):279–292, 2009.

[29] Y. Chen, S. Fei, and K. Zhang. Stabilization of impulsive switched linear systems with saturated control input. *Nonlinear Dynamics*, 69(3):793–804, 2012.