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A Centralized Model Predictive Control Framework for Logistics Management of Coordinated Supply Chains of Perishable Goods

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Abstract

This paper proposes a centralized model predictive control framework to address logistics management of supply chains of perishable goods. Meeting customer specific requirements is decisive to gain a competitive advantage in supply chain management. This fact motivates stakeholders to address solutions that continuously improve supply chain operations. The solution proposed in this work considers the supply chain as a dynamical system in a state-space representation where different categories of commodities, namely common goods and perishable goods, are included. Additionally, the dynamical model is able to store information of the complete supply chain regarding the quantity of commodities and the due time associated to the perishable goods. A centralized controller then collects the supply chain state information and optimizes the commodity flow based on the model prediction over a fixed time horizon. The model predictive control solution assigns just-in-time commodity flows, schedules production according to customer demand (pull system) and monitors work-in-progress and in-transit commodities. The success of the proposed control approach is demonstrated in a numerical simulation of a three-tier supply chain following three distinct management policies.

KEYWORDS
State-space representation; Centralized Model Predictive Control; Supply Chain Management; Logistics Management; Perishable goods

1. Introduction

The Food and Agriculture Organization of the United Nations (FAO), reported that one third of the food produced for human consumption is wasted (FAO 2011). The majority of food wastage occurs at pre-consumption stages of the supply chain, from production to retailing (see Figure 1). While in developing countries the wastage occurs in the earlier stages of the value chain, namely production and storage, in industrialized countries the wastage comes mostly from the later stages of the value chain, specifically

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distribution and retailing. The lack of coordination between supply chain stakeholders was identified as the most relevant contributing factor to supply chain ineffectiveness.

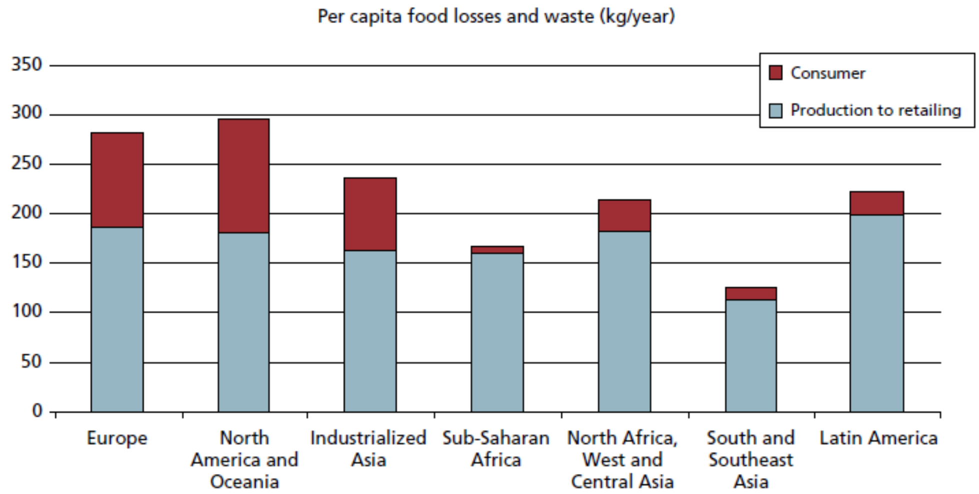


Figure 1. Food wastage, per capita, at pre-consumption and consumption stages of the supply chain, in different regions of the world (FAO 2011).

Supply Chain Management has attracted intensive research attention over the last few years and it has been interpreted by various authors, leading to multiple definitions (Jain, Dangayach, Agarwal & Banerjee 2010). Although there is no unanimous definition, Supply Chain Management focuses on coordinating material, information and financial flows, involving all Supply Chain stakeholders - suppliers, manufacturers, logistics service providers, distributors, retailers and customers - in the decision making process, in order to fulfil customer demand requirements. The goal of Supply Chain Management is to improve the overall performance of the Supply Chain (see Figure 2) (Min & Zhou 2002; Stadler & Kilger 2008). Additionally, Logistics Management (LM) is the component of Supply Chain Management responsible for monitoring and handling the flows of commodities, logistics services and information from the origin to the destination (Kukovic, Topolsek, Rosi & Jereb 2014).

Over the years, Supply Chains have been changing from push to pull perspectives. Manufacturing quality has reached parity across the board, so meeting customer specific requirements has emerged as the next critical opportunity for competitive advantage (Jain et al. 2010). Today, customers demand products and services to be available at any time and to be delivered as fast as possible, while meeting high-quality standards. Consequently, the number of orders are increasing and the quantity per order reducing, simultaneously, leading supply chain stakeholders to incur in high operational costs. The mismatch between customer requirements and operational costs imposes additional logistical effort on coordinating operations efficiently in Supply Chains (Chen, Dong & Xu 2018). The approach proposed in this paper assumes that all customer demand must be satisfied by available stock of commodities at the retailer (Minner & Transchel 2010).

Many Supply Chains deal with perishable commodities. Nahmias (1982) stated that perishable goods are commodities with a fixed lifetime, during which they can be moved across the supply chain or retained in stock and discarded afterwards. For example, perishability is present in:

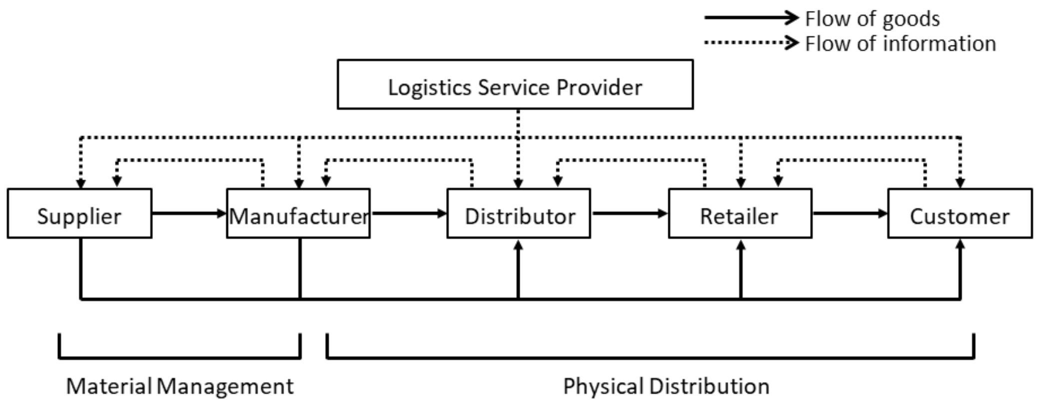


Figure 2. Representation of supply chain dynamics describing the interactions between the stakeholders (adapted from Min & Zhou (2002)).

- dairy industry - from the raw milk that enters the factories and has to be processed under specific conditions, to the intermediate products, whose properties are limited in time, and, finally, the different final products, such as cheese and yoghurt, which are labelled with an expiration date, fixing their shelf-life (Amorim, Meyr, Almeder & Almada-Lobo 2013);
- food industry - fish and seafood should be consumed fresh, requiring specific conservation conditions as their organisms deteriorate quickly (Boziaris & Parlapani 2017), while other type of food, e.g. bananas, go through ripening and long transportation processes being exposed to several different temperature and moisture conditions (Lin, Negenborn, Duinkerken & Lodewijks 2017);
- health industry - blood services are constantly at risk of failing due to shortage of supply and outdating of samples. In 2006, the national estimate of the percentage of outdated blood units, in the United States, was 8%, representing more than 1 million of units (Nagurney, Masoumi & Yu 2012; Whitaker, Green, King, Leibeg, Mathew, Schlumpf & Schreiber 2007).

Hence, perishability is critical to efficiently address Supply Chain operations. Sternbeck & Kuhn (2014) acknowledge that research on Supply Chains of perishable goods lacks integrative approaches which determine delivery plans and target stock levels for retail shops accounting for the impact of complementary supply chain tasks. Additionally, Ivanov, Sokolov & Raguinia (2014) noted that Supply Chain integration approaches need to be strongly supported by technological information systems. The authors define Supply Chains as collaborative cyber-physical systems which combine physical and information subsystems, and processes, which are inter-dependent resulting in coherent management decisions. Some authors developed approaches to improve Supply Chain performance considering integration and multiple tasks simultaneously (Ekşioğlu & Jin 2006; Federgruen, Pastracos & Zipkin 1986; Lee, Chan & Lee 2016; Li 2013; Omar & Zulkipli 2016). However, none of these approaches account for the whole Supply Chain operation from the upstream to the downstream and ignore the transformations done on commodities over the Supply Chain.

Traditionally, Supply Chain Management and Logistics Management rely on ad-hoc heuristics and mathematical programming techniques mostly based on Operational Research (OR) approaches (Li & Marlin 2009; Papageorgiou 2009). The OR approaches, typically, model a Supply Chain as a sequence of individual tasks rather than including the dynamic interactions between supply chain stakeholders (Amorim et al. 2013). Stakeholders should be able to access relevant information in real-time regarding the entire operational process (e.g. capacity limits, task scheduling, transport availability and demand prediction) to ensure effective decision making throughout the Supply Chain (Dreyer, Alfnes, Strandhagen & Kollberg 2009). Thus, in order to replicate closely the behaviour of real Supply Chains, the integration of their multiple processes and the coordination between stakeholders should be explicitly modelled (Rong, Akkerman & Grunow 2011).

Concurrently, control-based techniques are suitable to model the dynamic interactions present in multi-player systems and, therefore, they are useful to coordinate LM of Supply Chains of perishable goods, often subjected to uncertain demand conditions (Mestan, Türkay & Arkun 2006; Pinho, Moreira, Veiga & Boaventura-Cunha 2015). The approach proposed in this paper develops a centralized Model Predictive Control (MPC) framework to coordinate the LM of integrated Supply Chains of perishable goods. MPC has been successfully applied in process industry applications (Wang & Rivera 2009) and in SCM (Alessandri, Gaggero & Tonelli 2011; Fu, Aghezzaf & Keyser 2014; Maestre, de la Peña & Camacho 2009; Subramanian, Rawlings, Maravelias, Flores-Cerrillo & Megan 2013). Recent papers have extended the scope of applicability of this method to Supply Chains of perishable goods (Gaggero & Tonelli 2015; Lin et al. 2017). In MPC, the control algorithm uses current and historical measurements of the system behaviour to predict its behaviour at future time instants. The advantages of using MPC algorithms for optimizing Supply Chain logistics are: i) optimization of the complete Supply Chain operation, ii) design of robust formulations, even in the presence of disturbances such as uncertain demand, iii) integration of functional constraints in the optimization problem regarding limits of operation, namely, production rates, inventory levels and dispatch capacity, and iv) implementation of distinct management policies through the manipulation of the cost function (Pinho et al. 2015). The proposed MPC approach operates under a Global Control Centre. Dreyer et al. (2009) introduced the concept of Global Control Centre as a virtual or physical Supply Chain player which intends to coordinate and integrate the entire operational process in supply networks, by enabling efficient management of material flows, constrained by infrastructural limitations.

The approach proposed in this paper consists of a Model Predictive Control framework to perform logistics management of Supply Chains of perishable goods. The control algorithm optimizes Supply Chain operations considering, simultaneously, production, distribution and inventory management, and the due time of perishable goods. These processes are modelled from a flow perspective, meaning production consists in modifying commodities to produce new ones, distribution concerns the movement of the goods across the different tiers of the Supply Chain and inventory management focuses on holding and dispatching inventory accounting for the time until expiration of perishable goods. The dynamical model of the Supply Chain considers multiple products and distinguishes the flow of commodities according to their nature, type and remaining time until expiration. Perishable goods are monitored either by being perishable or as a result of production. They are assumed to have a known lifetime, leading inventory management to follow an age-based policy. A first-in-first-out (FIFO) issuing policy is assumed at the retailer, meaning perishable products are displayed at shelves by order of arrival. The Model Predictive Control algorithm coordinates the storage and flows between tiers in order to continuously perform the replenishment of the safety stock at the retailer, which depend on the intensity of the customer demand. Consequently, this approach is demand-driven, which means the dynamics of the supply chain operation depends on the intensity of customer demand, a major feature that is usually neglected in the literature (Jain et al. 2010).

This paper builds on the work of Nabais, Negenborn, Carmona-Benítez, Mendonça, Lourenço & Botto (2013), which proposes a multi-agent model predictive control approach to manage demand driven supply chains. This work addresses the logistics management of Supply Chains in general, not considering the specificities of the commodities. Besides, it only addresses the movement of commodities between multiple tiers, ignoring important Supply Chain processes such as production and inventory management. The approach proposed in this paper adopts a similar Model Predictive Control strategy as Nabais et al. (2013) but applies it to Supply Chains of perishable goods, tracking their due time over the entire Supply Chain. Besides, it categorizes commodities according their nature and models Supply Chain processes, namely production, distribution and inventory management based on that categorization. The goal of this paper is to present a systemic framework to perform logistics management of supply chains of perishable goods. Hence, the contributions of the current paper can be summarized as follows: i) the ability to track the due time associated to perishable goods over the entire Supply Chain, using a state-space representation of the dynamical model of the Supply Chain; ii) it addresses logistics management processes from a flow perspective: production consists in transforming commodities either to produce new commodities or to modify the ones that already exist; distribution focuses on the movement of commodities across the multiple tiers; and inventory management stores and assigns commodity flow accounting for the due time of perishable goods; and iii) managerial insights are drawn, concerning the length of the prediction horizon of the Model Predictive Control algorithm and distinct management goals. This is made using a demand-driven approach to logistics management, continuously replenishing the variations in the safety stock of the most downstream tier caused by customer demand.

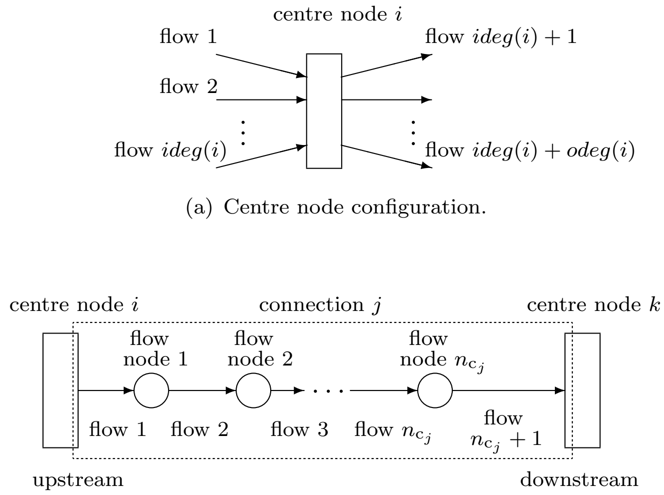
This paper is organized as follows. In Section 2, a state-space representation describes the dynamics of Supply Chains of perishable goods. Logistics Management is formulated using a centralized Model Predictive Control framework in Section 3. The performance of the proposed approach is tested through numerical simulations in Section 4. The Logistics Management of a three-tier supply chain is evaluated by analysing its response to three distinct management policies. In Section 5, conclusions are drawn and future research topics are indicated. Section 1 of the Appendix contains a description of the notation used in this paper.

2. Modeling

From a system dynamics point of view, Supply Chains consist of large and complex networks of nodes, representing physical locations, and links, associated to the movement of commodities between the nodes. Modelling Supply Chains requires a trade-off between capturing the main dynamics of the system and considering hypothesis and assumptions to reduce complexity and avoid redundancy.

2.1. Conceptual Approach

The Supply Chain model is conceived taking into consideration two aspects: i) Supply Chain design, representing the possible paths and processes commodities can follow from upstream to downstream; and ii) commodity categorization.



(a) Centre node configuration.

(b) Connection between two centre nodes.

Figure 3. Basic components of the proposed Supply Chain model (Nabais et al. 2013).

2.1.1. Supply Chain Design

At a macroscopic level, a Supply Chain is characterized by two major properties (Nabais et al. 2013):

- potential: related to the storage capability in specified areas, namely factories, distribution centres, warehouses and retail shops, where commodities can be produced, modified or simply stored. These locations are modelled as centre nodes. Centre node i has an in-degree, $ideg(i)$, and an out-degree, $odeg(i)$, corresponding to the flows of commodities entering and exiting the centre node, respectively (see Figure 3(a));
- flow: related to production and transport of commodities between physical locations. The movement of commodities is modelled using connections, which are composed of a succession of flow nodes each having an in-degree and out-degree equal to one. Connection j is composed of n_{c_j} flow nodes and $n_{c_j}+1$ flows (see Figure 3(b)), where n_{c_j} is the number of flow nodes belonging to connection j. Connections follow a pull-push flow perspective: pulling commodities from the upstream node of the connection and pushing them to the downstream node.

2.1.2. Commodity Categorization

This paper assumes that commodities in Supply Chains are categorized into two categories, according to their nature:

- common goods: common goods do not deteriorate over time. The number of different common goods (e.g. packages, ore, sand) in the Supply Chain is defined as n_{g}. They are consumed directly at the downstream of the Supply Chain or used to produce new goods at production connections, either common goods or perishable goods;
- perishable goods: the economic value of perishable goods is limited in time. The number of different perishable goods (e.g. bananas, pears, blood) in the Supply Chain is defined as n_{p}. This way, $n_{\mathrm{dt_p}}$ is the number of time instants until due

time for perishable good p, right after being available in the Supply Chain. For each perishable good, the remaining time until due time is considered in $\mathrm{dt_p}$, which is a time-varying property representing the number of time instants until perishable good p loses its economic value. For the sake of readability, from now on this property will be mentioned as due time. Perishable goods need to be available at the retailer before due time.

The total number of commodities, n_{comm}, in the Supply Chain is given by $n_{\text{comm}} = n_{\text{g}} + n_{\text{p}}$. The model stores information regarding the commodity quantity and the due time, in case of perishable goods. The commodity quantity can be measured discretely, by units, or continuously, by volume or weight, depending on the type of commodity. The number of flow classes, n_{class}, tracked throughout the Supply Chain, is defined as the sum of the different common goods plus the maximum due time of each perishable good, i.e.,

$$n_{\text{class}} = n_{\text{g}} + \sum_{i=1}^{n_{\text{p}}} n_{\text{dt}_i}. \tag{1}$$

2.2. Supply Chain Model

The total number of nodes n_{t} in the model is related to the Supply Chain design and is given by

$$n_{\text{t}} = n_{\text{cn}} + \sum_{j=1}^{n_{\text{c}}} n_{\text{c}_j}, \tag{2}$$

where n_{cn} is the number of centre nodes and n_{c} is the number of connections.

Apart from the most downstream node, for each node in the Supply Chain a state-space vector $\bar{\mathbf{x}}_i(k)$, $i = 1, \ldots, n_{\text{t}} - 1$, is defined as follows

$$\bar{\mathbf{x}}_i(k) = \begin{bmatrix} x_i^1(k) \\ x_i^2(k) \\ \vdots \\ x_i^{n_{\text{g}}}(k) \\ x_i^{1,1}(k) \\ \vdots \\ x_i^{1,n_{\text{dt}_1}}(k) \\ \vdots \\ x_i^{n_{\text{p}},1}(k) \\ \vdots \\ x_i^{n_{\text{p}},n_{\text{dt}n_{\text{p}}}}(k) \end{bmatrix}, \tag{3}$$

where $x_i^{\text{g}}(k)$ is the quantity of common good g at node i at time instant k, $x_i^{\text{p},\text{dt}_{\text{p}}}(k)$ is the amount of perishable good p with remaining due time dt_p at node i at time instant k, and $n_{\text{dt}_{\text{p}}}$ is the maximum due time for perishable good p. The quantities $x_i^{\text{g}}(k)$ and

$x_i^{\mathrm{p,dt_p}}(k)$ represent the flow classes, thus the length of each state-space vector, $\bar{\mathbf{x}}_i(k)$, is $n_{\bar{\mathrm{x}}_\mathrm{i}} = n_{\mathrm{class}}$. The demand is seen as a disturbance $\mathbf{d}(k)$ at the most downstream node of the supply chain. Mathematically, it is represented as a vector of dimension $n_{\mathrm{d}} = n_{\mathrm{comm}}$. This means that the due time of perishable goods is not tracked at the most downstream node, usually corresponding to the retail shop, where a FIFO issuing policy is assumed at the selling shelves. For this reason, the state-space vector of the most downstream node $\bar{\mathbf{x}}_{n_\mathrm{t}}(k)$ resumes to the quantity of each commodity type at the node and its dimension is n_{comm}. In order to quantify the wastage and loss of perishables in the Supply Chain, the additional state-space vector $\bar{\mathbf{x}}_{\mathrm{OD}}(k)$ accounts for the quantity of perishable goods that become overdue across the Supply Chain, before reaching the shelves of the retail shop, i.e.,

$$\bar{\mathbf{x}}_{n_\mathrm{t}}(k) = \begin{bmatrix} x_{n_\mathrm{t}}^{1}(k) \\ \vdots \\ x_{n_\mathrm{t}}^{n_{\mathrm{comm}}}(k) \end{bmatrix}, \bar{\mathbf{x}}_{\mathrm{OD}}(k) = \begin{bmatrix} x_{\mathrm{OD}}^{1}(k) \\ \vdots \\ x_{\mathrm{OD}}^{n_\mathrm{p}}(k) \end{bmatrix}, \tag{4}$$

where $x_{n_\mathrm{t}}^{\mathrm{comm}}(k)$ represents the quantity of commodity of type *comm* at the most downstream node n_t and $x_{\mathrm{OD}}^{\mathrm{p}}(k)$ describes the quantity of perishable good p that became overdue. It is assumed that perishable goods delivered at the retailer before due time are sold on time to customers and perishable goods that become overdue lose their economical value. By merging all these individual state vectors, the overall state vector $\mathbf{x}(k)$ of the entire supply chain is obtained,

$$\mathbf{x}(k) = \begin{bmatrix} \bar{\mathbf{x}}_1(k) \\ \bar{\mathbf{x}}_2(k) \\ \vdots \\ \bar{\mathbf{x}}_{n_\mathrm{t}}(k) \\ \bar{\mathbf{x}}_{\mathrm{OD}}(k) \end{bmatrix}. \tag{5}$$

The state vector dimension is $n_\mathrm{x} = n_{\mathrm{t}-1} n_{\mathrm{class}} + n_{\mathrm{comm}} + n_\mathrm{p}$, corresponding to the product between the number of flow classes handled at each node and the number of nodes existing in the Supply Chain, plus the number of commodities at the most downstream node and the number of perishable goods. For the case there are no perishable goods in the Supply Chain, the number of flow classes to track reduces to the number of common goods, n_g, and the state-space dimension is simply $n_\mathrm{x} = n_\mathrm{t} n_\mathrm{g}$. The commodity is always accessible through the state vector, whether in the production, transport or storage stages of the Supply Chain.

Consider $u_j^{\mathrm{p,dt_p}}(k)$ as the quantity of perishable good p with remaining due time $\mathrm{dt_p}$ to be pulled from node j at time instant k. For all admissible flows in the Supply Chain, a control action vector $\bar{\mathbf{u}}_j(k)$ is defined, with dimension $n_{\bar{\mathrm{u}}_\mathrm{j}} = n_{\mathrm{class}}$. All $\bar{\mathbf{u}}_j(k)$ $(j = 1, \ldots, n_\mathrm{f})$, where n_f is the number of flows in the Supply Chain, are merged to form the overall control action vector $\mathbf{u}(k)$ with dimension $n_{\mathrm{u}} = n_{\mathrm{f}} n_{\mathrm{class}}$,

$$\bar{\mathbf{u}}_j(k) = \begin{bmatrix} u_j^1(k) \\ u_j^2(k) \\ \vdots \\ u_j^{n_{\mathrm{g}}}(k) \\ u_j^{1,1}(k) \\ \vdots \\ u_j^{1,n_{\mathrm{dt},1}}(k) \\ \vdots \\ u_j^{n_{\mathrm{p}},1}(k) \\ \vdots \\ u_j^{n_{\mathrm{p}},n_{\mathrm{dt}n_{\mathrm{p}}}}(k) \end{bmatrix}, \mathbf{u}(k) = \begin{bmatrix} \bar{\mathbf{u}}_1(k) \\ \bar{\mathbf{u}}_2(k) \\ \vdots \\ \bar{\mathbf{u}}_{n_{\mathrm{f}}}(k) \end{bmatrix}. \tag{6}$$

The Supply Chain is fully observable since the output of the model is actually the state of the system, $\mathbf{y}(k) = \mathbf{x}(k)$. The model of the Supply Chain can be represented in a state-space form as follows:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}_{\mathrm{u}}\mathbf{u}(k) + \mathbf{B}_{\mathrm{d}}\mathbf{d}(k), \tag{7}$$

$$\mathbf{y}(k) = \mathbf{x}(k) \tag{8}$$

where $\mathbf{A}$ (dimension $n_{\mathrm{x}} \times n_{\mathrm{x}}$), $\mathbf{B}_{\mathrm{u}}$ (dimension $n_{\mathrm{x}} \times n_{\mathrm{u}}$) , $\mathbf{B}_{\mathrm{d}}$ (dimension $n_{\mathrm{x}} \times n_{\mathrm{d}}$) are the state-space matrices. The Supply Chain state at the next time instant, $\mathbf{x}(k+1)$, is determined using (7) as a function of the current Supply Chain state $\mathbf{x}(k)$ plus the contribution from the control action $\mathbf{u}(k)$ and the demand disturbance $\mathbf{d}(k)$. The control action $\mathbf{u}(k)$ corresponds to the flow of commodities between nodes and is constrained by the available transport resources.

2.3. Model Insight

In this paper Supply Chain design takes into account the following features:

- Supply Chain processes - Supply Chain design is able to split production from distribution and considers production activities first, regardless the position in the Supply Chain;
- flow and potential properties - Supply Chain design addresses separately and sequentially the flow component (connections and flow nodes) and the potential component (centre nodes);
- flow perspective - Supply Chain design follows the path of the commodities through the Supply Chain, from the upstream to the downstream.

Based on the aforementioned design criteria, supply chain nodes should be ordered using the following policy: i) connections related to production are addressed first (flow nodes are numbered from upstream to downstream); ii) connections related to distribution are addressed second (flow nodes are numbered from upstream to downstream); and, finally, iii) centre nodes are addressed (centre nodes are numbered from upstream to downstream). Flows are also numbered sequentially from upstream to downstream.

3. Logistics Management

The proposed approach assumes the existence of a Global Control Centre, which consists of an additional node, external to the Supply Chain, which continuously collects information regarding the operation of the Supply Chain. The information collected by the Global Control Centre consists of the commodity quantity and due time of perishable goods at each node, the current storage capacity of the centre nodes, the current transport capacity of the connections, and predictions on transport capacity, storage capacity and customer demand profile in future time instants. Global Control Centre compiles this data and runs a centralized model predictive control algorithm to determine the flow assignment to implement in the system, in order to optimize the Supply Chain overall performance. Then, the flow assignment decisions are communicated to the agents responsible for the assignment of commodity flow, located at the centre nodes. The main advantage of using model predictive control is the ability to include constraints and predictions about the system behaviour and handle disturbances within an unified modelling and optimization framework (Maciejowski 2002). At each time sample, the controller gathers information related to the updated state of the system, predictions on the storage capacity, transport capacity and demand profile over a defined prediction horizon, N_{p}, and formulates an optimization problem considering a control-relevant cost function, which evaluates the performance of the Supply Chain. The output of the optimization problem is the sequence of future control actions, which optimizes the supply chain behaviour over the prediction horizon N_{p}. The predicted control action regarding the first time step is implemented in the system and the overall system state is updated. At the next time step, the process is repeated considering the updated state system and the new system predictions.

3.1. Problem Formulation

This paper presents a systemic framework to perform logistics management of supply chains of perishable goods. The cost function adopted is a linear function which associates weights $\mathbf{q}_i(k)$, $i = 1, \ldots, n_{\mathrm{t}}$, to the nodes and weights $\mathbf{q}_{\mathrm{OD}}(k)$ to the overdue perishable goods, over the prediction horizon, N_{p} . The cost function depends on the current state of the system, control actions and predicted demand, over the prediction horizon N_{p}. The weights of the cost function may vary according to the management policies as different Supply Chain goals require distinct operational behaviour. This type of cost function is usually used in linear programming and mixed integer linear programming formulations of Supply Chain optimization problems (Papageorgiou 2009). Thus, the cost function of the MPC algorithm is defined as:

$$J\left(\tilde{\mathbf{x}}_k\right) = \sum_{l=0}^{N_{\mathrm{p}}-1} \mathbf{q}\left(k+l\right)\mathbf{x}\left(k+1+l\right), \tag{9}$$

where $\tilde{\mathbf{x}}_k$ is a vector composed with the state vectors of each time instant k, over the prediction horizon N_{p},

$$\tilde{\mathbf{x}}_k = \left[\begin{array}{ccc} \mathbf{x}^{\mathrm{T}}(k+1) & ,\ldots, & \mathbf{x}^{\mathrm{T}}(k+N_{\mathrm{p}}) \end{array}\right]^{\mathrm{T}}. \tag{10}$$

The MPC optimization problem for the logistics management of the Supply Chain can be formulated as follows:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k\right) \tag{11}$$

$$\text{s.t.} \quad \mathbf{x}(k+1+l) = \mathbf{A}\mathbf{x}(k+l) + \mathbf{B}_{\mathrm{u}}\mathbf{u}(k+l) + \mathbf{B}_{\mathrm{d}}\mathbf{d}(k+l), \tag{12}$$

$$\mathbf{x}(k+1+l) \geq \mathbf{0}, \tag{13}$$

$$\mathbf{u}(k+l) \geq \mathbf{0}, \tag{14}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}(k+1+l) \leq \mathbf{x}_{\max}, \tag{15}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}(k+l) \leq \mathbf{u}_{\max}, \tag{16}$$

$$\mathbf{x}(k+l) \geq \mathbf{P}_{\mathrm{xu}}\mathbf{u}(k+l), \quad l = 0, \ldots, N_{\mathrm{p}} - 1. \tag{17}$$

where $\tilde{\mathbf{u}}_k$ corresponds to the control action vectors computed for each time instant k, over the prediction horizon N_{p},

$$\tilde{\mathbf{u}}_k = \left[\begin{array}{ccc} \mathbf{u}^{\mathrm{T}}(k) & , \ldots, & \mathbf{u}^{\mathrm{T}}(k+N_{\mathrm{p}}-1) \end{array}\right]^{\mathrm{T}}, \tag{18}$$

where $\mathbf{x}_{\max}$ $(n_{\mathrm{t}}n_{\mathrm{comm}} \times 1)$ is the maximum storage capacity per node, $\mathbf{u}_{\max}$ $(n_{\mathrm{f}}n_{\mathrm{comm}} \times 1)$ corresponds to the available transport capacity, according to the supply chain layout, $\mathbf{P}_{\mathrm{xu}}$ $(n_{\mathrm{x}} \times n_{\mathrm{u}})$ is the projection matrix from the control action set $\mathcal{U}$ into the state-space set $\mathcal{X}$, $\mathbf{P}_{\mathrm{xx}}$ $(n_{\mathrm{t}}n_{\mathrm{comm}} \times n_{\mathrm{x}})$ is the projection matrix from the state-space set $\mathcal{X}$ into the maximum storage capacity set $\mathcal{X}_{\max}$, $\mathbf{P}_{\mathrm{uu}}$ $(n_{\mathrm{f}}n_{\mathrm{comm}} \times n_u)$ is the projection matrix from the control action set $\mathcal{U}$ into the available moving capacity set $\mathcal{U}_{\max}$. Constraints (13)–(17) are necessary to obtain feasible and meaningful control actions, which means:

- non-negativity of states and control actions: negative storage at the nodes and negative flows of commodities are not physically possible. The non-negativity of states and control actions is imposed by constraints (13)–(14);
- maximum storage capacity: each Supply Chain node has to respect its storage capacity limits. This feature is captured in constraint (15);
- maximum control actions: the maximum handling capacity to move commodities between nodes, limited by resource availability, is represented by constraint (16);
- flow conservation: not all control actions that satisfy constraints (14) and (16) are feasible. The flow of commodities to be moved from a node must never exceed the amount of commodities stored in that node. Constraint (17) illustrates this restriction.

4. Numerical Experiments

In this section, the centralized MPC framework is applied to perform Logistics Management of a Supply Chain composed a manufacturer, a distributor, a retailer and a global control centre. The Supply Chain handles both common goods and perishable goods, considering three distinct management policies of increasing complexity. The three management policies can be described as:

- safety stock policy - it focuses on filling the inventory of the retailer in order to satisfy customer demand;
- waste minimization policy - it intends, simultaneously, to reduce the quantity

of overdue perishable goods in the Supply Chain and fill the inventory of the retailer;

- just-in-time policy - it intends to make commodities available at the retailer shelves as fast as possible, while reducing the amount of overdue perishable goods and filling the inventory of the retailer.

4.1. Case Study Description

4.1.1. Static Configuration

The sampling time considered in the simulation is one day. The Supply Chain is composed of one manufacturer, one distributor and one retailer, and its activity is monitored by a global control centre (see Figure 4). The logistics operations of the

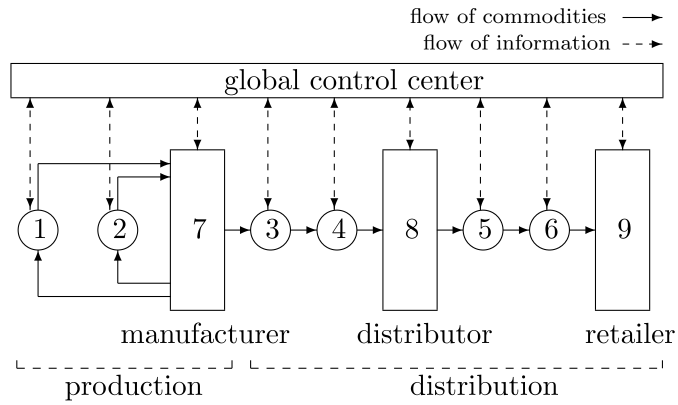


Figure 4. Supply chain composed of one manufacturer, one distributor, one retailer and a global control center, handling two common goods and two perishable goods (vertical boxes - centre nodes; circles - flow nodes; horizontal box - external control node).

Supply Chain consist of:

- production - occurs at the upstream of the Supply Chain and the manufacturer is responsible for the production flow assignment. Perishable goods are produced from common goods. There are two production lines with a lead time of 2 days. They are composed of two flows and one flow node;
- distribution - consists in transporting the commodities from the manufacturer to the retailer and it is assured by a logistics service provider. Transportation connections have a lead time of 3 days. Therefore, they are composed of three flows and two flow nodes;
- inventory management - consists in monitoring the inventory level at the retailer and replenish it when it goes under the desired safety stock level.

The Supply Chain storage and transport capacity limits are discriminated per commodity at each node and connection (Table 1 and Table 2). They are assumed constant over the entire simulation. Besides, four different commodities are supplied to the market: two common goods, G_1 and G_2, and two perishable goods, P_1 and P_2, manufactured from the common goods, with a lifetime of 14 days. The commodity quantity is measured in units. The common goods are, initially, available at the manufacturer. They are consumed to produce the perishable goods in separate production lines at the manufacturer, according to a specific ratio (see Table 3). Once produced and made

Table 1. Maximum storage capacity, per commodity, at centre nodes (manufacturer, distributor and retailer)

		total	commodities			
			G_1	G_2	P_1	P_2
Capacity	Manufacturer	4200	2000	2000	100	100
	Distributor	200	50	50	50	50
	Retailer	100	25	25	25	25

Table 2. Maximum handling capacity, per commodity, at connections, discriminated by flow nodes and flows

			total	commodities			
				G_1	G_2	P_1	P_2
flow nodes		node 1	2	0	0	2	0
		node 2	3	0	0	0	3
		node 3	14	4	5	2	3
		node 4	14	4	5	2	3
		node 5	14	4	5	2	3
		node 6	14	4	5	2	3
connections	connection 1	1^{st} flow	2	0	0	2	0
		2^{nd} flow	2	0	0	2	0
	connection 2	1^{st} flow	3	0	0	0	3
		2^{nd} flow	3	0	0	0	3
	connection 3	1^{st} flow	14	4	5	2	3
		2^{nd} flow	14	4	5	2	3
		3^{rd} flow	14	4	5	2	3
	connection 4	1^{st} flow	14	4	5	2	3
		2^{nd} flow	14	4	5	2	3
		3^{rd} flow	14	4	5	2	3

Table 3. Ratio of common goods, G_1 and G_2, consumed to produce perishable goods, P_1 and P_2, and their maximum due time, $n_{\mathrm{dt_p}}$.

		common goods		$n_{\mathrm{dt_p}}$
		G_1	G_2	
perishable goods	P_1	1	1	14
	P_2	2	1	14

available at the manufacturer, the perishable goods have 14 days to be delivered at the retail shop. Once delivered at the retailer, the due time of the perishable goods is no longer tracked and it is assumed that the retail shop follows a FIFO policy at the shelves. Common goods are also delivered at the retail shop and sold directly to the customers. The lead time for common goods to be available at the retail shop is 6 days, while for perishable goods is 8 days, due to the production operation.

The Supply Chain graph, $\mathcal{G}$, relates center nodes, flow nodes and flows (see Figure 4).

The Supply Chain graph can be represented by its incidence matrix, $\mathbf{D}(\mathcal{G})$:

$$\mathbf{D}(\mathcal{G}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

The rows of $\mathbf{D}(\mathcal{G})$ represent the nodes of the Supply Chain, while the columns consist of the flows. For example, row 1 describes flow 1 entering in node 1 and flow 2 exiting node 1. Matrices $\mathbf{A}$, $\mathbf{B}_{\mathrm{u}}$ and $\mathbf{B}_{\mathrm{d}}$ for this case study are presented in the Appendix.

4.1.2. Dynamic Configuration

The Supply Chain behaviour is simulated over 100 days and customer selling starts 20 days after the beginning of the simulation. At the start of the simulation, there is only inventory of common goods at the manufacturer, in a sufficiently large quantity to fulfil the expected demand at the retailer, over the entire simulation period. This means that all perishable goods must be produced. The demand profiles of the four distinct commodities were generated using gamma distributions (considering the describing parameters values: $k = 2$ and $\theta = 1$) (Burgin 1975). The customer demand profiles are assumed to be deterministic. Furthermore, it is assumed predictions of the demand are available and match the demand.

The weights of the cost function of the optimization problem describe the management policy, therefore three distinct sets of weights are assigned to the three different management policies:

- Management Policy 1 (MP 1) – safety stock policy - the weights associated to the inventory at the retailer depend on the commodity quantity. The inventory is bounded by a lower limit and an upper limit. If the commodity quantity is below the lower limit, the weights are highly negative to promote the increase of commodity quantity; if the commodity quantity is above the lower limit and below the upper limit, the weights are slightly negative to impel the commodity quantity to reach the upper limit. In turn, if the commodity quantity is above the upper limit, the weights are highly positive to penalize the excess of commodity quantity and decrease it below the upper limit;
- Management Policy (MP 2) – waste minimization policy - the cost function has two components: one associated to the penalty of overdue perishable goods and the other related to the safety stock at the retailer described in management policy 1. The weights associated to overdue perishable goods are higher than the weights associated to the safety stock since reducing wastage is prioritized relatively to keeping the safety stock desired level at the retailer;
- Management Policy 3 (MP 3) – just-in-time policy - the cost function has three components: one associated to the penalty of inventory at the distributor, intending to reduce it to the minimum level possible and the other two accounting for the safety stock and waste minimization described previously.

Table 4. Cost function weights associated to the supply chain state considering three distinct management policies.

			Commodities			
			G_1	G_2	P_1	P_2
Management Policy 1	Flow nodes		1	1	1	1
	Manufacturer		0	0	0	0
	Distributor		0	0	0	0
	Retailer	$x_9 <=$ LL	−10	−10	−200	−200
		LL $< x_9 <=$ UL	−1	−1	−100	−100
		$x_9 >$ UL	10	10	100	100
	Overdue		–	–	0	0
Management Policy 2	Flow nodes		1	1	1	1
	Manufacturer		0	0	0	0
	Distributor		0	0	0	0
	Retailer	$x_9 <=$ LL	−10	−10	−200	−200
		LL $< x_9 <=$ UL	−1	−1	−100	−100
		$x_9 >$ UL	10	10	100	100
	Overdue		–	–	75	125
Management Policy 3	Flow nodes		1	1	1	1
	Manufacturer		0	0	0	0
	Distributor		5	5	5	5
	Retailer	$x_9 <=$ LL	−10	−10	−200	−200
		LL $< x_9 <=$ UL	−1	−1	−100	−100
		$x_9 >$ UL	10	10	100	100
	Overdue		–	–	75	125

The weights of the cost function are assumed to be constant over the entire simulation. Besides, the inventory lower limit (LL) at the retailer is 3 units and the upper limit (UL) is 6 units, for all commodities. The cost function parameters (inventory limits and weights) are presented in Table 4.

4.2. Numerical Results

The performance of the Supply Chain is evaluated for the three different Management Policies using the following performance measures:

- overdue perishable goods - the total quantity of perishable goods that became overdue before being delivered at the retail shop;
- overproduction of perishable goods - the difference between the demand of perishable goods at the retailer and the quantity of perishable goods produced;
- total commodity movements - total amount of commodity movements between the manufacturer and the retailer;

- distributor inventory usage - total commodity quantity stored at the distributor, over the entire simulation;
- retailer inventory error - difference between the desired safety stock level at the retailer and the retailer inventory level;
- computation time - the amount of time, in seconds, of the entire simulation;
- commodity movements per due time - amount of commodity movements for each due time.

Supply Chain logistics management is analysed considering two dimensions:

- prediction horizon, N_p - for each Management Policy, the logistical behaviour of the Supply Chain is studied by varying the prediction horizon, from 8 time steps, which correspond to the lead time of the perishable goods, to 15 time steps, which covers the entire lifetime of perishable goods. The customer demand profile is the same for all different simulations;
- customer demand profile - for each Management Policy, the logistics management of the Supply Chain is analysed imposing three different demand profiles. The prediction horizon, N_p, is fixed for all simulations.

4.2.1. Insights in management policies regarding the prediction horizon, N_p

Figure 5 illustrates the customer demand scenario (Scen 1), discriminated by commodity.

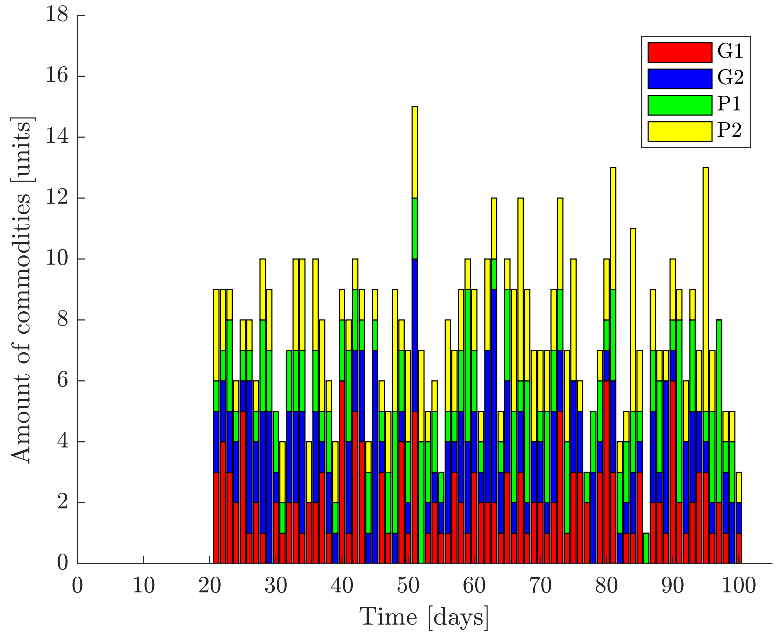


Figure 5. Profile of the customer demand, discriminated by commodity (Scen 1).

Table 5 and table 6 present the values of the performance measures for each prediction horizon sampled and for each Management Policy. Regarding overdue perishable goods, MP 2 and MP 3 can minimize and, sometimes, eliminate wastage, for higher

N_p. In turn, MP 1 produces high quantities of overdue perishables, which increase with N_p. While MP 2 and MP 3 define the minimization of wastage as a management goal, resulting in less overdue commodities for higher N_p, MP 1 does not account for it, so a higher N_p produces more overdue commodities. In terms of overproduction, a higher N_p leads to an excess of perishable goods for all management policies. Production is not considered in the cost function, directly, because the MPC algorithm acts based on the customer demand intensity. However, by looking further ahead, MPC algorithms with higher N_p, anticipate the customer demand and assign more production flows. Total commodity movements and distributor inventory usage also increase with the growth of N_p. Nevertheless, comparing the commodity movements and the distributor inventory usage for the three management policies, MP 3 performs Supply Chain operations more efficiently, which was expected due to its just-in-time characteristic. In turn, higher N_p results in smaller retailer inventory error, meaning the framework is able to replenish the safety stock at the retailer and keep it close to desired level. MP 2 is the management policy that manages the safety stock more efficiently, as its inventory error values are the lowest (negative values mean the real inventory level was above the desired inventory level). Although, MP 3 performs operations more efficiently, MP 2 manages better the safety stock at the retailer, which is decisive to satisfy the customer demand. Concerning dimensionality, a higher N_p lead to larger computation time because it adds additional constraints to the optimization problem. The computation time of the simulations grow exponentially with N_p, as can be seen in Table 6.

Table 7 details the flows per due time, showing that MP 1 moves many goods with low due times and allow them to get overdue. In turn, MP 2 and MP 3 handle perishable goods in a similar way, moving high commodity quantities of perishable goods with higher due times and low commodity quantities with shorter due times flows. The difference between MP 2 and MP 3 regarding the number of movements of common goods, which is lower for MP 3, is associated to the fact that, in MP 3, common goods are not supposed to be stored at the distributor. Thus, they only move downstream of the chain to replenish retailer safety stock, otherwise, they are kept at the manufacturer and do not create inventory at the distributor.

Figure 6 presents the time evolution of inventory at the distributor and the retailer, simultaneously, considering $N_\mathrm{p} = 12$. Regarding inventory usage, at the beginning of simulation, while there is no customer demand, MP 2 and MP 3 fill the inventory of the retailer with perishable goods to avoid wastage. When customer demand starts, the main driver of Supply Chain operation is replenishing the safety stock at the retailer. MP 1 and MP 2 perform similarly, after customer selling starts because their main concern is retailer inventory. In turn, besides managing the safety stock, MP 3 intends to minimize the inventory at the distributor, which is demonstrated to be lower, compared to the other two management policies.

To conclude, the prediction horizon, N_p, is a tuning parameter of the MPC algorithm and the following conclusions can be taken: i) N_p should be equal or higher than the lead time of the whole supply chain, ii) prediction horizons higher than the lifespan of perishable goods do not increase algorithm performance, so prediction horizon should be set between supply chain lead time and the highest lifespan of perishable goods iii) decision makers need to find the right trade-off between increasing supply chain performance and increasing dimensionality, as higher prediction horizons increase performance at cost of a higher computational effort.

Table 5. Performance analysis of the distinct management policies for different prediction horizons, N_{p}, considering quantity of overdue goods, overproduction and the total number of commodity movements

Prediction Horizon	Overdue Goods			Overproduction			Total Commodity Movements		
	MP 1	MP 2	MP 3	MP 1	MP 2	MP 3	MP 1	MP 2	MP 3
$N_{\mathrm{p}} = 8$	33	–	–	39	–	–	4006	–	–
$N_{\mathrm{p}} = 10$	33	–	–	41	–	–	4160	–	–
$N_{\mathrm{p}} = 12$	36	2	2	42	19	19	4175	4027	3844
$N_{\mathrm{p}} = 14$	35	0	0	41	21	21	4152	4032	3915
$N_{\mathrm{p}} = 15$	41	0	0	44	24	24	4170	4055	4046

Table 6. Performance analysis of the distinct management policies for different prediction horizons, N_{p}, considering the usage of the distributor inventory, retailer inventory error and the computation time (in seconds)

Prediction Horizon	Distributor Inventory			Retailer Inventory Error			Computation time [s]		
	MP 1	MP 2	MP 3	MP 1	MP 2	MP 3	MP 1	MP 2	MP 3
$N_{\mathrm{p}} = 8$	1370	–	–	440	–	–	68	–	–
$N_{\mathrm{p}} = 10$	2274	–	–	128	–	–	111	–	–
$N_{\mathrm{p}} = 12$	2466	1934	825	131	52	196	197	203	136
$N_{\mathrm{p}} = 14$	2424	2001	871	68	-10	90	286	281	224
$N_{\mathrm{p}} = 15$	2656	1993	1032	108	-14	-9	334	301	271

Table 7. Number of commodity movements per due time in the Supply Chain to measure the risk of perishable goods to become overdue, considering different prediction horizons, N_{p}.

Prediction Horizon		Commodities Due Times														
		Common Goods	dt_1	dt_2	dt_3	dt_4	dt_5	dt_6	dt_7	dt_8	dt_9	dt_{10}	dt_{11}	dt_{12}	dt_{13}	dt_{14}
MP 1	$N_{\mathrm{p}} = 8$	1988	6	20	12	19	36	63	78	110	226	247	247	339	322	1029
	$N_{\mathrm{p}} = 10$	2145	9	14	18	25	43	76	82	116	229	238	246	327	313	1021
	$N_{\mathrm{p}} = 12$	2142	7	24	31	59	82	91	113	155	218	242	253	301	265	938
	$N_{\mathrm{p}} = 14$	2139	15	34	40	62	76	110	147	157	219	238	239	274	228	918
	$N_{\mathrm{p}} = 15$	2139	21	40	71	64	72	102	129	175	215	229	237	273	233	926
MP 2	$N_{\mathrm{p}} = 8$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	$N_{\mathrm{p}} = 10$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	$N_{\mathrm{p}} = 12$	2136	10	10	8	2	4	4	2	28	287	291	294	317	319	970
	$N_{\mathrm{p}} = 14$	2142	12	10	8	0	4	6	6	24	285	289	292	314	319	979
	$N_{\mathrm{p}} = 15$	2132	12	10	8	0	2	6	4	28	286	295	298	321	326	999
MP 3	$N_{\mathrm{p}} = 8$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	$N_{\mathrm{p}} = 10$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	$N_{\mathrm{p}} = 12$	1953	10	10	8	6	4	2	6	28	289	293	302	307	311	970
	$N_{\mathrm{p}} = 14$	2025	12	10	8	6	4	2	6	28	289	293	299	304	311	976
	$N_{\mathrm{p}} = 15$	2123	12	10	8	6	4	2	6	28	290	297	306	311	318	997

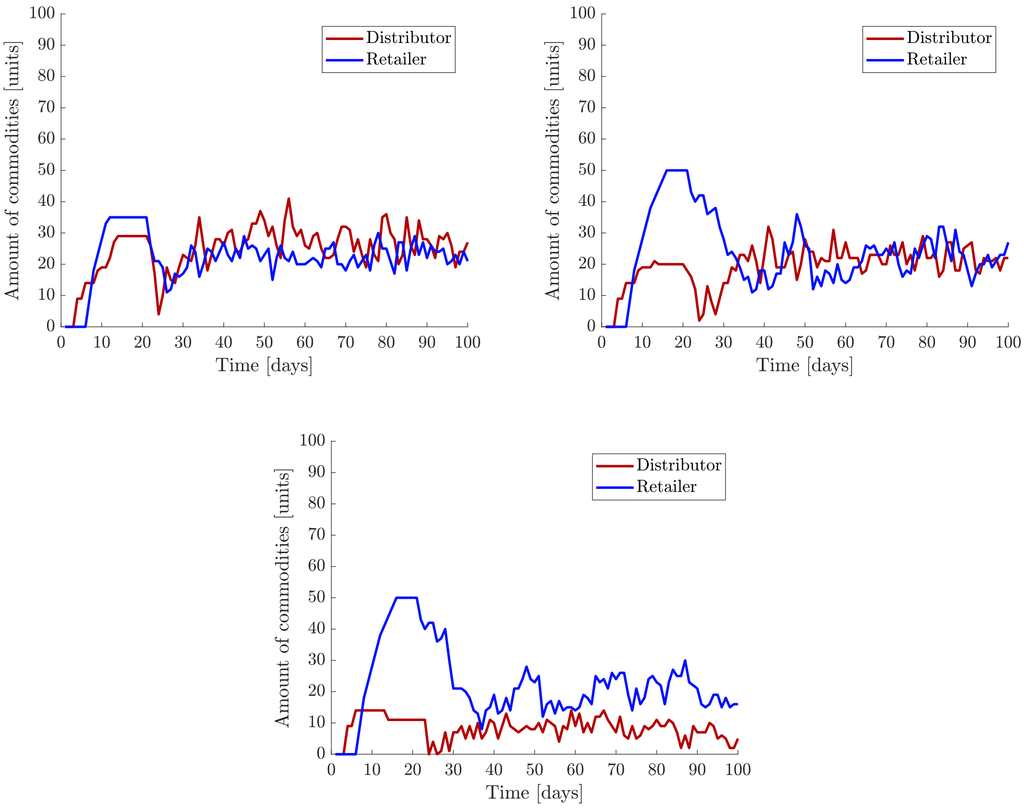


Figure 6. Storage evolution at distributor and retailer for the three distinct management policies considering $N_\mathrm{p} = 12$: Management Policy 1 (up-left), Management Policy 2 (up-right), Management Policy 3 (down-center).

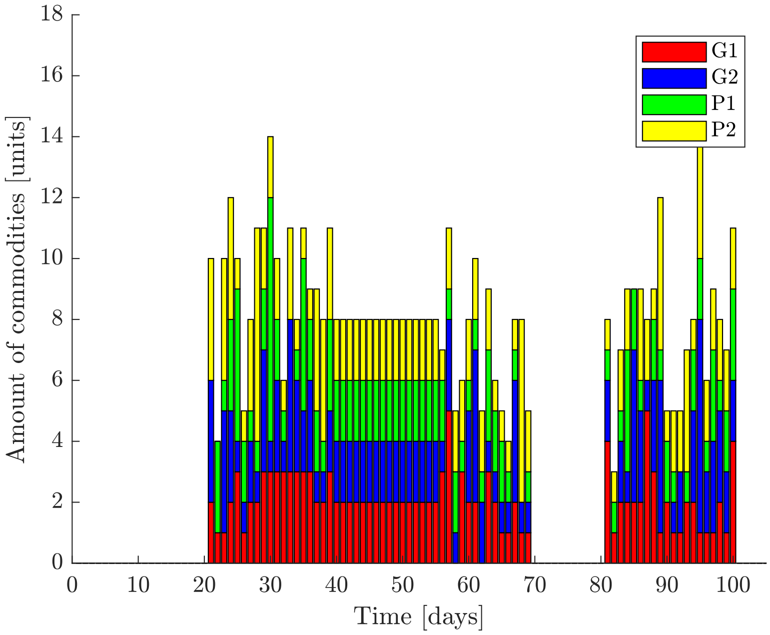
4.2.2. Insights in management policies regarding customer demand profile

In order to study the resilience of the proposed framework, two more demand scenarios are considered (see Figure 7).

Hereupon, logistics management of the case study Supply Chain is performed for the three management policies, considering a fixed prediction horizon, $N_\mathrm{p} = 12$, dealing with three distinct demand scenarios:

- Scenario 1 (Scen 1) - the demand pattern is irregular, presenting many oscillations. The maximum demand is 15 units and the minimum 1 unit;
- Scenario 2 (Scen 2) - the demand pattern presents some oscillations but it assumes a constant demand of 8 units between day 40 and day 60. Also, between day 70 and day 80, there is no customer selling. The maximum demand is 15 and the minimum demand is 0.
- Scenario 3 (Scen 3) - the demand pattern is almost constant. There is a constant demand of 4 units, 1 of each commodity, with few fluctuations. The maximum demand is 5 units and the minimum is 3 units.

Table 8 and table 9 present the values of the performance measures for each demand scenario and for each management policy. Concerning wastage, MP 3 is the most efficient management policy, presenting the lowest values of overdue perishable goods and overproduction. MP 2 also presented low wastage values. Total commodity movements



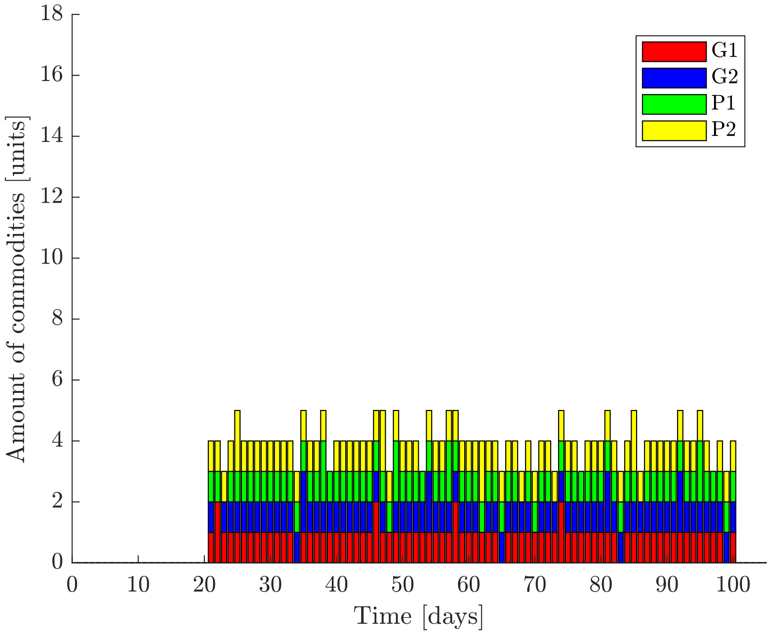


Figure 7. Profile of the customer demand scenario 2 (top) and customer demand demand scenario 3 (bottom), discriminated by commodity.

and distributor inventory usage measures confirm MP 1 as the least efficient management policy and MP 3 as the best policy. However, analysing the retailer inventory error, regarding the ability to replenish safety stock desired levels, and, consequently customer satisfaction, MP 2 is the management policy that presents the lowest retailer error values for demand scenarios 1 and 2. In demand scenario 3, MP 2 produces excessive inventory. The main objective of the logistics management is to satisfy the customer and the MP 2 is the management policy that better fulfils customer demand and, in case of Supply Chain disruption, it responds faster than the other 2 policies due to the desired safety stock levels. Nevertheless, MP 3 is the management policy that optimizes more efficiently Supply Chain operations. It is also a riskier management policy than MP 2, because it is more likely to fail to satisfy the demand if there are uncertainties in Supply Chain, such as disruptions and delays. Table 10 illustrates that MP 3 involves more risk than MP 2, because it moves higher commodity quantities with lower due times . In terms of dimensionality, high intensity demand profiles result in longer simulations, according to the values of the computation time, where demand scenario 1, that is the most intense profile, generate the longer simulations.

Table 8. Performance analysis of the distinct management policies for different customer demand scenarios, considering quantity of overdue goods, overproduction and the total number of commodity movements

Management Policies	Overdue Goods			Overproduction			Total Commodity Movements		
	Scen 1	Scen 2	Scen 3	Scen 1	Scen 2	Scen 3	Scen 1	Scen 2	Scen 3
MP 1	36	54	55	42	51	77	4175	3892	2505
MP 2	2	6	13	19	26	43	4027	3743	2352
MP 3	2	4	8	19	25	41	3844	3571	2241

Table 9. Performance analysis of the distinct management policies for different customer demand scenarios, considering the usage of the distributor inventory, retailer inventory error and the computation time (in seconds)

Management Policies	Distributor Inventory			Retailer Inventory Error			Computation time [s]		
	Scen 1	Scen 2	Scen 3	Scen 1	Scen 2	Scen 3	Scen 1	Scen 2	Scen 3
MP 1	2466	2102	1815	131	93	−313	197	167	124
MP 2	1934	1578	1429	52	22	−485	203	148	113
MP 3	825	823	603	196	165	−201	136	161	126

Table 10. Number of commodity movements per due time in the Supply Chain to measure the risk of perishable goods to become overdue, considering different demand scenarios.

Customer Demand Scenario		Commodities Due Times														
		Common goods	dt_1	dt_2	dt_3	dt_4	dt_5	dt_6	dt_7	dt_8	dt_9	dt_{10}	dt_{11}	dt_{12}	dt_{13}	dt_{14}
MP 1	Scen 1	2142	7	24	31	59	82	91	113	155	218	242	253	301	265	938
	Scen 2	1920	14	26	23	21	16	65	117	157	240	240	254	312	280	953
	Scen 3	1181	14	34	39	49	51	59	77	74	110	131	153	206	178	675
MP 2	Scen 1	2136	10	10	8	2	4	4	2	28	287	291	294	317	319	970
	Scen 2	1944	18	18	14	2	2	4	2	2	265	274	283	300	305	947
	Scen 3	1181	9	12	16	26	20	42	22	36	133	143	177	199	193	558
MP 3	Scen 1	1953	10	10	8	6	4	2	6	28	289	293	302	307	311	970
	Scen 2	1784	20	20	16	11	8	4	1	0	262	271	280	289	298	938
	Scen 3	1085	12	26	18	46	18	48	22	42	145	153	159	147	179	548

5. Conclusions and future research

In this paper, a centralized Model Predictive Control framework is proposed to address Logistics Management of coordinated Supply Chains of perishable goods. The proposed framework models the Supply Chain as a dynamical system, capable of tracking commodity quantity and the due time of perishable goods over the entire Supply Chain. Furthermore, it addresses simultaneously, production, distribution and inventory management processes of the Supply Chain. A centralized MPC algorithm, triggered by a Global Control Centre, optimizes Supply Chain operations in order to satisfy customer demand, assuming all demand is satisfied by available stock at the retailer.

The performance of the proposed framework is analysed through the behaviour of a Supply Chain following three distinct management policies, for different prediction horizons and customer demand profiles. The numerical experiments revealed that different management goals lead to different logistical decisions. The results obtained show that the algorithm performed accordingly to the goals set in the cost function for the distinct management policies. This framework is meant to be a decision support tool for Supply Chain decision makers and stakeholders.

The proposed approach is innovative in terms of tracking the due time of perishable goods over the entire supply chain. Furthermore, it is modular, flexible and scalable. This opens the gate to apply this framework as a decision support tool to help solve coordination problems of supply chain of perishable goods. The following future developments are foreseen: i) include uncertainty in the framework by considering uncertain storage and transport capacities, uncertain and unknown customer demand and forecast modules; and ii) split the Supply Chain system into smaller subsystems, managed by control agents, and apply a distributed model predictive approach, where the multiple subsystems and agents communicate and collaborate to achieve Supply Chain overall goals.

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Appendix

.1. Notation

The following table describes the notation adopted in this paper. The symbols are sorted according to the order they appear in the text:

Symbol	Meaning
$ideg(i)$	in-degree of centre node i
$odeg(i)$	out-degree of centre node i
n_{c_j}	number of flow nodes belonging to connection j
n_{g}	number of different common goods
n_{p}	number of different perishable goods
$n_{\mathrm{dt_p}}$	number of time instants until due time for perishable good p
$\mathrm{dt_p}$	time-varying property representing the number of time instants until perishable good p gets overdue
n_{comm}	number of different commodities
n_{class}	number of flow classes accounting for the properties of all commodities
n_{t}	total number of nodes
n_{cn}	number of center nodes
n_{c}	number of connections
$\bar{\mathbf{x}}_i(k)$	state-space vector for node i at time instant k
$x_i^{\mathrm{g}}(k)$	quantity of common good g at node i at time instant k
$x_i^{\mathrm{p,dt_p}}(k)$	quantity of perishable good p with remaining due time dt_p at node i at time instant k
$\mathbf{d}(k)$	disturbance vector at time instant k
n_{d}	length of disturbance vector $\mathbf{d}(k)$

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Symbol	Meaning
$\bar{\mathbf{x}}_{n_\mathrm{t}}(k)$	state-space vector for the downstream node n_t
$\bar{\mathbf{x}}_{\mathrm{OD}}(k)$	state-space vector for overdue perishable goods
$x_{n_\mathrm{t}}^{\mathrm{comm}}(k)$	commodity quantity of each commodity type *comm* at the downstream node n_t
$x_{\mathrm{OD}}^{\mathrm{p}}(k)$	quantity of perishable good p that became overdue
$\mathbf{x}(k)$	overall state-space vector
n_x	length of overall state-space vector $\mathbf{x}(k)$
$u_j^{\mathrm{p},\mathrm{dt}_\mathrm{p}}(k)$	quantity of perishable good p with remaining due time dt_p to be pulled from node j at time instant k
$\bar{\mathbf{u}}_j(k)$	control action for flow j
n_f	number of links
$\mathbf{u}(k)$	overall control action vector
n_u	length of overall control action vector $\mathbf{u}(k)$
$\mathbf{y}(k)$	output vector
$\mathbf{A}$	state-space matrix associated to the state vector $\mathbf{x}(k)$
$\mathbf{B}_\mathrm{u}$	state-space matrix associated to the control action vector $\mathbf{u}(k)$
$\mathbf{B}_\mathrm{d}$	state-space matrix associated to the disturbance $\mathbf{d}(k)$
N_p	prediction horizon of model predictive control algorithm
$J\left(\tilde{\mathbf{x}}_k\right)$	cost function of the optimization problem
$\mathbf{q}_i(k)$	cost function weight associated to node i
$\tilde{\mathbf{x}}_k$	vector composed of the state-space vectors, for each time instant k, over the prediction horizon N_p

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Symbol	Meaning
$\tilde{\mathbf{u}}_k$	vector composed of the control actions vectors, for each time instant k, over the prediction horizon N_{p}
$\mathbf{x}_{\max}$	maximum storage capacity for all nodes
$\mathbf{u}_{\max}$	available transport capacity for all flows
$\mathbf{P}_{\mathrm{xu}}$	projection matrix from the control action set $\mathcal{U}$ into the state-space set $\mathcal{X}$
$\mathcal{U}$	control action set
$\mathcal{X}$	state-space set
$\mathbf{P}_{\mathrm{xx}}$	projection matrix from the state-space set $\mathcal{X}$ into the maximum storage capacity set $\mathcal{X}_{\max}$
$\mathcal{X}_{\max}$	maximum storage capacity set
$\mathbf{P}_{\mathrm{uu}}$	projection matrix from the control action set $\mathcal{U}$ into the available moving capacity set $\mathcal{U}_{\max}$
$\mathcal{U}_{\max}$	available moving capacity set

.2. Model Matrices

This section presents the matrices $\mathbf{A}$, $\mathbf{B}_{\mathrm{u}}$ and $\mathbf{B}_{\mathrm{d}}$ for the Supply Chain case study analysed in Section 4 “Numerical Experiments”.

Matrix $\mathbf{A}_{\mathrm{g}}$ accounts for the two common goods, while $\mathbf{A}_{\mathrm{p}}$ is the matrix of each perishable good with $\mathrm{dt}_p = 14$,

$$
\mathbf{A}_{\mathrm{g}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad
\mathbf{A}_{\mathrm{p}} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Matrix $\mathbf{A}_{\mathrm{node}}$ is the commodity matrix for all nodes, except the most downstream, where $\mathbf{A}_{\mathrm{comm}}$ is the commodity matrix, not considering the due time of perishable goods. In turn, $\mathbf{A}_{\mathrm{OD}}$ accounts for the overdue perishable goods,

$$
\mathbf{A}_{\mathrm{node}} = \begin{bmatrix} \mathbf{A}_{\mathrm{g}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{\mathrm{p}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{p}} \end{bmatrix} \quad
\mathbf{A}_{\mathrm{comm}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
\mathbf{A}_{\mathrm{OD}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

The model matrix $\mathbf{A}$ is the result of $\mathbf{A}_{\mathrm{node}} \otimes \mathbf{I}_{\mathrm{n_t}-1}$ plus the influence of the most downstream node and the overdue perishable goods,

$$
\mathbf{A} = \begin{bmatrix}
\mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{comm}} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{OD}}
\end{bmatrix}
$$

The model matrices $\mathbf{B}_{\mathrm{node1}}$ and $\mathbf{B}_{\mathrm{node2}}$ regarding production are built as follows:

$$
\mathbf{B}_{\mathrm{g}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad
\mathbf{B}_{\mathrm{ratio1}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
\mathbf{B}_{\mathrm{ratio2}} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$\mathbf{B}_{\mathrm{p}} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

$$\mathbf{B}_{\mathrm{node1}} = \begin{bmatrix} \mathbf{B}_{\mathrm{g}} & \mathbf{B}_{\mathrm{ratio1}} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\mathrm{p}} & \mathbf{0} \end{bmatrix} \quad \mathbf{B}_{\mathrm{node2}} = \begin{bmatrix} \mathbf{B}_{\mathrm{g}} & \mathbf{0} & \mathbf{B}_{\mathrm{ratio2}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathrm{p}} \end{bmatrix}$$

The matrix accounting for the overdue perishable goods along the Supply Chain $\mathbf{B}_{\mathrm{OD}}$ is built from:

$$\mathbf{B}_{\mathrm{OD1}} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$\mathbf{B}_{\mathrm{OD2}} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$\mathbf{B}_{\mathrm{OD}} = \begin{bmatrix} \mathbf{0} & \mathbf{B}_{\mathrm{OD1}} & \mathbf{B}_{\mathrm{OD2}} \end{bmatrix}$$

$$\mathbf{B}_{\mathrm{tp}} = \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix}$$

At the most downstream node, due time of perishable goods is no longer tracked, only the total amount of perishable goods through matrix $\mathbf{B}_{\mathrm{comm}}$:

$$\mathbf{B}_{\mathrm{comm}} = \begin{bmatrix} \mathbf{A}_{\mathrm{g}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\mathrm{tp}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathrm{tp}} \end{bmatrix}$$

The model matrix $\mathbf{B}_{\mathrm{u}}$ results from the combination of the incidence matrix, $\mathbf{D}(\mathcal{G})$ of the Supply Chain and the previous matrices $\mathbf{B}$ described:

$$\mathbf{D}(\mathcal{G}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{B}_{\mathrm{u}} = \begin{bmatrix} \mathbf{B}_{\mathrm{node1}} & -\mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathrm{node2}} & -\mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & -\mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & -\mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & -\mathbf{A}_{\mathrm{node}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & -\mathbf{A}_{\mathrm{node}} \\ -\mathbf{B}_{\mathrm{node1}} & \mathbf{A}_{\mathrm{node}} & -\mathbf{B}_{\mathrm{node2}} & \mathbf{A}_{\mathrm{node}} & -\mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{\mathrm{node}} & -\mathbf{A}_{\mathrm{node}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{\mathrm{comm}} \\ \mathbf{B}_{\mathrm{OD}} & \mathbf{B}_{\mathrm{OD}} & \mathbf{B}_{\mathrm{OD}} & \mathbf{B}_{\mathrm{OD}} & \mathbf{B}_{\mathrm{OD}} & \mathbf{B}_{\mathrm{OD}} & \mathbf{B}_{\mathrm{OD}} & \mathbf{B}_{\mathrm{OD}} & \mathbf{B}_{\mathrm{OD}} & \mathbf{B}_{\mathrm{OD}} \end{bmatrix}$$

Finally, the model matrix $\mathbf{B}_{\mathrm{d}}$ is described as follows:

$$\mathbf{B}_{\mathrm{d}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{A}_{\mathrm{comm}} \\ \mathbf{0} \end{bmatrix}$$