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How proper similitude can improve our understanding of crack closure and plasticity in fatigue

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Abstract

The appropriateness of some common similitude principles with respect to describing and predicting fatigue damage propagation is discussed. Linear elastic fracture mechanics have provided a basis to describe damage growth using stress intensity factors or strain energy release rates, both related to the work of Griffith and Irwin. The fatigue crack growth equations presented in the literature are discussed, and it is demonstrated that the principles of similarity in current methodologies have not yet been well established. As a consequence, corrections for the stress ratio effect are misunderstood. An alternative principle of similitude using cyclic work and strain energy release is proposed.

Keywords: Fatigue, Crack growth, Similitude, Crack closure, Linear Elastic Fracture Mechanics

Nomenclature

A	Area, or Crack surface	[mm^2]
a	Crack length	[mm]
C	Correction factor or Constant	
ε	Strain	[-]
G	Strain Energy Release Rate	[N/mm]
γ_e	Surface energy per unit area	[mJ/mm^2]
K	Stress Intensity Factor	[MPa√mm]
K_t	Stress Concentration Factor	[-]
L	Length	[mm]
m	Exponent	
N	Number of load cycles	[cycles]
n	Exponent	
P	Applied load	[N]
P	Pressure	[Pa]
Q	Heat	[mJ]
R	Stress ratio S_{min}/S_{max}	[-]
r_p	Plastic zone size	[mm]
S	Global stress	[MPa]
S_y	Yield strength	[MPa]
U	Strain energy	[mJ]
V	Volume	[mm^3]

Y	Crack closure correction	[-]

1. Introduction

Although many researchers these days may not realise this explicitly, anywhere in the field of engineering and science, a principle of similarity is adopted. For example, to predict when quasi-static failure occurs in a complex 3-dimensional Finite Element simulation, often stresses are correlated under the assumption that similarity between stresses in the model and in a tensile specimen will yield similar consequences.

Scrutinizing the vast number of papers in the literature on fatigue damage growth, illustrates that most discussions on the validity of proposed methodologies narrow down to the question: what is the most appropriate principle of similitude? Once a principle of similitude has been agreed upon, most studies tend to follow this approach without further questioning the fundamentals underlying this principle [1]. From an engineering perspective this is preferred, because continuous questioning of basic principles will hinder progress in research and technology. However, from an academic or scientific perspective, one may expect continuous criticism with respect to fundamentals of selected principles of similarity.

In this paper the appropriateness of the principles of similitude currently adopted for fatigue damage growth within the context of linear elastic fracture mechanics is questioned. Various observations seem to indicate that currently trends are being misinterpreted simply because similitude has not been well established.

2 Reviewing current fatigue approaches

2.1 Stress and strain based fatigue approaches

Traditionally, mechanics of materials has been dealt with using stresses and strains. For most quasi-static loading conditions, this principle seems appropriate and has proven its usefulness in the field of science and engineering.

Once fatigue as a degradation or wear-out phenomenon was acknowledged [1], engineers and scientists initially approached the problem using these similarity principles at hand, i.e. engineering stresses and strains. The early days of fatigue research are characterised by studies and papers that propose approaches based on stress and strain. For example, August Wöhler proposed to plot the observed failure life against the stress amplitude [1], because he regarded this as being most decisive for the destruction of material cohesion. According to him, the maximum stress is of influence only in so far as the higher it is, the lower is the stress amplitude which leads to failure. This principle of similitude has never really been questioned and most engineering handbooks [2,3,4] presently utilise these S-N curves for design.

2.2 Crack propagation approaches

At some point, a distinction was made between the phases of fatigue. The first phase covers the nucleation and propagation of microscopically small cracks, while the second phase covers the propagation of macroscopically sized cracks [5].

Crack propagation required a different approach compared to the evaluation of the fatigue initiation life. People attempted to relate the rate of propagation to a variety of parameters representing similarity. Hence, various crack propagation equations were proposed, which erroneously often are referred to as crack propagation laws, like for instance 'the Paris law' [6,7,8,9]. The field of crack growth description is characterised by an engineering approach rather than a scientific approach, as illustrated by the many corrections to parameters describing the conditions. Thus it is the author's opinion that the word 'law' should be considered highly inappropriate in this field in particular.

The first well known crack growth relation was proposed by Head [10,11], which was based upon a mechanical model using rigid-plastic work hardening assuming a constant plastic zone size. After correction for the increase in plastic zone size proportional to the crack length [12], this equation was modified to [13]

$$\frac{da}{dN} = \frac{CS^2 a}{S_y - S} \tag{1}$$

Frost and Dugdale [12] observed that the propagation of cracks in metallic materials seem to correlate to the cube of the stress rather than its square, i.e.

$$\frac{da}{dN} = \frac{S^3 a}{C} \tag{2}$$

McEvily and Illg [14] proposed another formulation based upon a fictitious crack tip radius and using the stress concentration factor K_t, resulting in

$$\frac{da}{dN} = K_t S_{nom} \tag{3}$$

with K_t obviously a function of the given crack tip radius. Meanwhile, Paris [15] proposed to adopt Irwin's [16] Stress Intensity Factor (SIF) K, arguing that this parameter reflects the influence of external load and geometry. His relation can be formulated as

$$\frac{da}{dN} = C\Delta K^n = C\left(\Delta S\sqrt{\pi a}\right)^n \tag{4}$$

where n is commonly between 2-4 for metallic materials. Frost et al. [17] reanalysed existing data with equation (4) and concluded that it was less satisfactory than equation (2). Another formulation was proposed by Liu [18,19] who hypothesized that the saturation of hysteresis energy absorbed by the material during every cycle could be used as a criterion. The resulting formulation may be written as

$$\frac{da}{dN} = CS^2 a \tag{5}$$

Paris and Erdogan discussed these crack growth formulations in detail in [13], but in their discussion they seem to suggest that correlation between empirical relation and data validates the empirical relation. Obviously, all above empirical crack growth relations correlate to data, but that is simply because of their empirical nature. However, both authors correctly conclude in [13] that more data should be employed to verify whether any of these formulations is appropriate.

Reviewing the above equations, one observes that the general observation of all of the above mentioned authors is that the crack propagation rate correlates to the applied stress and the crack length according to

$$\frac{da}{dN} \sim S^n a^m \tag{6}$$

Where n may range between 2 and 4, and m between 1 and 2. But the fundamental question to be asked is: What does this mean?

2.3 *Capturing the cyclic nature*

What may be observed reviewing the above referenced literature is the ambiguous use of either S or ΔS. Where, for example, originally the relation was proposed in terms of S, other papers refer to the original relation while using ΔS. In the end, it seems that generally it is deemed appropriate to use ΔS to represent fatigue crack growth, which seems in agreement with the original stress based approaches for fatigue life that describe the fatigue life using the stress amplitude S_a.

2.4 *On the Stress Intensity Factor range ΔK for similitude*

At a given point most people applied the SIF concept to describe similitude in fatigue crack propagation [20]. The SIF is generally referred to as 'the controlling variable for analysing crack-extension rates' [21]. To describe the cyclic nature of fatigue loading, in general, the Paris relation given by equation (4) is adopted. However, the SIF range does not provide a comprehensive description of similitude, which is illustrated by the vast amount of data reported that show an apparent stress ratio effect [22,23, 24,25].

Although state-of-the art, it seems incorrect to consider the SIF concept beyond any dispute. Various authors [26-32] have discussed the inappropriateness of using a single parameter ΔK to describe crack growth. Indeed, where the load (or stress) cycle requires 2 parameters to be described, it does not seem reasonable to assume that a single parameter equivalent to ΔS suffices to describe growth as result of that load cycle.

Consequently, two major lines of reasoning may be identified while reviewing the literature. Either an effective SIF range ΔK_{eff} was derived based on a proposed stress ratio correction, for which the observations of Elber [33,34] on crack closure were introduced to substantiate the correction

$$\Delta K_{eff} = K_{max} - K_{op} = U\Delta K = (0.5 + 0.4R)\Delta K \tag{7}$$

Or, in disagreement with concepts of crack closure, authors proposed to describe the problem with two parameters similar to how the load cycle itself is described by two parameters [35,36]

$$\Delta K_{eq} = \Delta K(1-R)^{\gamma} = \Delta K^{(1-\gamma)} K_{max}^{\gamma} \tag{8}$$

Either way, both perspectives propose a correction to the cyclic component of the SIF, implicitly or explicitly correcting for mean stress or stress ratio. The primary difference between the two views concerns the opinion about what phenomenon or mechanism necessitates the correction.

To conclude this brief review, the observation that one can make while reviewing the vast number of crack propagation relations, is that they are completely phenomenological and not derived from physics. In educating engineers, often emphasis is put on dimensional analysis when formulating equations and relations. This obviously does not apply to the phenomenological equations reviewed in this paper. A quick dimensional analysis can illustrate the physical inappropriateness of most equations, unless the fit parameters are assumed to also have units for which no physical reasoning is given.

What seems to be missing in the literature is the scientific dissection of the fatigue problem from a physics perspective in order to identify the appropriateness of using these stresses to describe the cyclic nature. In general, people tend to acknowledge (implicitly) that a stress cycle relates to cyclic work, but equations are rarely scrutinised for their appropriateness for describing the cyclic work applied.

3 Key observations justifying the current analysis

There is a variety of observations reported in the literature, that at first glance seem to be non-related, or attributed to completely different aspects or mechanisms. However, when investigating these observations more closely, one can observe some peculiarities, that seem to require further explanation.

3.1 Same stress ratio effect – different reason

Let us begin with the phenomenon termed stress ratio effect, observed when plotting data in the form of a Paris relation, i.e. da/dN versus ΔK. For metallic materials it seems the general opinion that this effect is caused by plasticity induced crack closure, as first observed by Elber [33,34]. Occasionally other closure mechanisms are considered, but plasticity induced closure is considered the most important or dominant mechanism [5].

This explanation for the stress ratio effect requires, however, a second thought. The problem here is that identical stress ratio effects have been reported for ply-delamination propagation [36-41], in-plane fatigue testing of composites [42,43,44], and fatigue testing of polymers [45]. The problem with these observations is that most of these material systems do not exhibit significant plasticity. Fatigue in composites and ply-delamination growth in composite systems generally occur in a rather brittle manner. Nonetheless, a similar stress ratio effect is observed [46]. The fundamental question to be asked is: Why?

It seems that people do realise the discrepancy, but since plasticity induced closure seems an accepted explanation for the stress ratio effect in metals, other mechanisms are studied to explain the stress ratio effect for these other material systems, like roughness closure, or visco-elastic phenomena. The problem is that in many cases, a good explanation cannot be given. For example, it was demonstrated in [47] that closure or shielding mechanisms do not explain the stress ratio effect in ply-delamination growth. So why should plasticity induced crack closure explain the stress ratio effect in metals, if similar explanations cannot be given for similar stress ratio effects observed in other material systems?

3.2 Stress ratio correction – crack closure or….?

For crack growth in metals, it is the general perception that this effect is predominantly, or even solely, induced by the phenomenon of plasticity induced crack closure. The first correction for crack closure was proposed by Elber [33,34], described by

$$Y = 0.4R + 0.5 \tag{9}$$

with $Y = \Delta S_{eff}/\Delta S$. This correction was later modified by a number of researchers, amongst which Schijve [48,49], Newman [50, 51] and De Koning [52]. These crack closure corrections can be graphically presented in the form of S_{op}/S_{max} versus R, as illustrated in Figure 1. For this representation equation (9) is rewritten to [48, 49]

$$\frac{S_{op}}{S_{\max}} = 0.4R^2 + 0.1R + 0.5 \tag{10}$$

Originally, this expression was only validated for $0.1<R<0.7$. Extending the curve over the entire range of $-1<R<1$, as illustrated in Figure 1, yields an increase for larger negative R values. As this was deemed physically unrealistic, Schijve [48,49] proposed

$$\frac{S_{op}}{S_{\max}} = 0.12R^3 + 0.21R^2 + 0.22R + 0.45 \tag{11}$$

Newman proposed an even more complex expression [50, 51] which for the constraint factor $\alpha = 1$ yields a curve that is very similar to the others, see Figure 1.

Now the peculiar aspect with these corrections is that they in fact represent a mean stress correction. Mean stress effects have been observed before when plotting for example fatigue S-N curves, as reported by Wöhler. Then what is the difference between these mean stress effects and the aforementioned crack closure corrections?

To answer that question, the crack closure corrections could be recalculated from S_{op}/S_{max} versus R to S_a versus S_m. This is easily done with equation (11) using the failure and fatigue strength of 2024-T3, considering that Schijve developed his equation specifically for this alloy.

If we consider that

$$S_m = \frac{1+R}{2} S_{\max}$$
$$S_a = \frac{\Delta S_{eff}}{2} = \frac{S_{\max} - S_{op}}{2} = \frac{1 - S_{op}/S_{\max}}{2} S_{\max} \quad (12)$$

then equation (12) can be rewritten to S_a versus S_m assuming for failure S_{ult} = 483 MPa [53]. The result is illustrated in Figure 2, where the curve is compared to the Gerber parabola for the fatigue limit for aluminium 2024-T3 (S_f = 138 MPa [53]).

The similarity and correlation between the two curves is striking. In particular, if one considers that the one curve claims to correct for plasticity induced crack closure in the wake of macroscopic cracks, while the other one corrects for a case where macroscopic cracks are deemed to be absent or at least insignificant.

Then the fundamental question to be asked here is: If these corrections are so similar, aren't they correcting for the same physical principle, instead of correcting for completely different mechanisms?

3.3 Observation by Paris on influence of Young's modulus

With the above two observations, the question arises whether the stress ratio effect is really a physical effect, or whether it is an artefact of the similarity parameters used. Here, another observations seems of particular interest, especially because it has been reported by Paris. In [54], Paris reviews earlier observations reported by Anderson, and Harris and himself [55], where it was identified that the measured fatigue crack growth rates (in inert environments) were identical for various base materials and their alloys, if the K value was normalized with the elastic modulus E. Interestingly, Paris notes that these observations were done with normalizing by E, but not with normalizing by the density. Obviously, there is no physical reason to expect that normalising K by the density would help, but it illustrates the engineering approach of people used to work with quasi-static properties like strength, and specific strength.

Noteworthy in this context is Paris' statement

> "With every physical model proposed to date, nothing has shown better results for the comprehensive data than this simple normalization taking ΔK over E. Pondering on this fact over the years has led to stating that perhaps this should be explained with some reasoned physical model before anyone claims a correct model or more detailed effects!"[54]

The fundamental question to be asked is: Why would a similarity parameter like K need to be divided by the Young's modulus E?

The answer to that question is given later in this paper.

3.4 Threshold dependence on stress ratio

Mackay [56] reported a study on the crack growth rates in the low ΔK regime for two common aerospace aluminium grades, i.e. 2024-T3 and 7075-T6 (clad). The study is of interest, because the threshold SIF range ΔK_{th} is reported for various stress ratios between 0.05 and 0.6. The conclusion of the study is that the ΔK_{th} were dependent on R in a predictable manner. The dependency is illustrated in Figure 3. For 2024-T3 the data from Mackay could be extended with the data provided by Taylor [57], as illustrated in Figure 4.

The threshold SIF range ΔK_{th} obtained by Mackay and Taylor can be recalculated to threshold SERR range ΔG_{th} using the relation between SIF and SERR derived by Irwin [16]

$$\Delta G_{th} = G_{\max,th} - G_{\min,th} = \frac{1}{E}\left(K^2_{\max,th} - K^2_{\min,th}\right) = \frac{1}{E}\left[\left(\frac{\Delta K_{th}}{1-R}\right)^2 - \left(\frac{R\Delta K_{th}}{1-R}\right)^2\right] \qquad (13)$$

These threshold SERR range values calculated with equation (13) are also plotted against the stress ratio R in Figure 3 and Figure 4. What becomes immediately obvious is that when ΔG_{th} is plotted against R, there is no stress ratio dependency. Note the 1/E in equation (13) following from the relation between K and G, with reference to the observation by Paris discussed in the previous section.

The fundamental question to be asked here is: Is the stress ratio dependency something physical, or just an artefact of the selected similitude parameter ΔK?

4 Current hypothesis

Thus, in this paper it is hypothesized that the strain energy release is the correct approach to describe the similitude in fatigue damage growth. The argument supporting this hypothesis is that rather than applying cyclic load or cyclic stress to a structure or specimen, one applies cyclic work.

5 Compliance with the 1st and 2nd law of thermodynamics

Since all processes in nature are governed by energy principles, obeying the principal laws of thermodynamics, it is believed that the similitude should be sought in parameters reflecting the strain energy release or applied work.

In a theoretically elastic problem the elastic strain energy stored under the application of a quasi-static stress S is described by

$$U = \frac{1}{2}P\delta = \frac{1}{2}SA\varepsilon L = \frac{1}{2}S\varepsilon V = \frac{S^2}{2E}V \qquad (14)$$

with, P the applied load, δ the elongation, $\frac{1}{2}S\varepsilon$ the strain energy density and V the volume of the specimen.

Griffith and Irwin used this concept to develop a fracture criterion for a panel containing a crack under quasi-static loading. They deduced that once fixed-grip conditions are considered, i.e. the displacement remains constant, the additional work induced by crack extension remains zero. This implies that conservation of internal energy yields that the release in strain energy due to crack extension should be equated to the energy needed to create the additional crack surfaces.

The fact that the strain energy release here is a consequence of crack extension and not primarily the driver is not of interest; if the released strain energy exceeds the energy required to further extend the crack growth, then unstable crack growth is achieved. Hence, a stability criterion could be developed.

This seems to have been misinterpreted by others when addressing fatigue problems. In the case of fatigue loading, fixed grip conditions certainly do not apply. During the fatigue cycle, the strain energy and applied work continuously change.

Another aspect that seems to be ignored in current approaches is that Griffith and Irwin considered instantaneous crack extension for a static problem with an already applied stress S, thus assuming the theoretical elastic problem of work equalling internal energy up to that stress S. In fatigue, however, the crack extension and plastic deformation occur while loading the sample from S_{min} to S_{max}. Hence, equation (14) is not completely correct here. This can be illustrated with Figure 5, where for a CCT specimen the corresponding force displacement curve is plotted. At any crack length a, one could assume perfect linear elastic behaviour (indicated by dotted lines) if brittle behaviour is assumed. However, while loading from P_{min} to P_{max} the crack extends with Δa. This extension does not occur instantaneously at P_{max}, but propagates with the increase of loading. This increase is not linear with the load level (indicated by the arrow), but occurs at higher load levels. Thus during the load cycle, increments of crack extension can be identified that are at another effective crack length, corresponding to another linear elastic force-displacement curve, effectively yielding a non-linear force-displacement curve.

The discrepancy with common linear elastic methods, based on for example G_{max}, ΔG, K_{max}, or ΔK, can be illustrated with Figure 6, where for the problem illustrated in Figure 5, the strain energy can be equated to the area under the load-displacement curves. In displacement controlled problems, the amount of applied work (and internal strain energy) is less than calculated with equation (14), while for force-controlled problems the amount exceeds the amount calculated with equation (14).

To summarize, the process of fatigue could be written with an equation in terms of energy similar to Griffith's proposal but such that it represents the irreversible fatigue process. Thus for a single fatigue cycle this could be for example

$$U_0 + U_{\uparrow} \rightarrow U_0^* + U_{\downarrow} + U_a + U_{pl} \tag{15}$$

where U_0 represents the monotonic strain energy available at minimum load described by $\frac{1}{2}P_{min}\delta_{min}$, $U_{\uparrow}$ represents the work applied by the test machine to the specimen during the loading part of the cycle, $U_{\downarrow}$ the work applied by the specimen to the test machine during unloading, U_a the energy dissipated to create new fracture surfaces and U_{pl} the energy dissipated in plasticity. Note that $U_{\uparrow}$ and $U_{\downarrow}$ relate to the cyclic work or energy, i.e. the energy associated to $\Delta P = P_{max} - P_{min}$, but that they are not the same in magnitude in case of hysteresis or energy dissipation. With the reduction in strain energy due to energy dissipation mechanisms like plasticity and crack growth, also a portion of the monotonic energy is dissipated, which at the end of the load cycle results in $U_0^* < U_0$.

6 Equating damage growth to applied work

The difference between $U_\uparrow$ and $U_\downarrow$ is somewhat difficult to measure for a single fatigue cycle. The solution to that would be to measure at any load cycle during the fatigue test the applied work to the specimen, i.e. $U_{N=i}$. In case of displacement controlled tests, this will yield a reduction of strain energy measured against the number of load cycles, of which the derivative could be written as dU/dN.

It does not require a lot of reasoning to say that the derivative of strain energy with respect to the fatigue cycle can be decomposed as

$$\frac{dU}{dN} = \frac{dU}{dA}\frac{dA}{dN} \qquad (16)$$

with which the correlation between crack extension over a single load cycle and strain energy release rate is illustrated. However, one should keep in mind that this dU/dA represents an average strain energy release rate G_{av} which is not equal to $G_{max} - G_{min}$ nor to $(\sqrt{G_{max}} - \sqrt{G_{min}})^2$. Nor is it equal to the critical Strain Energy Release Rate (SERR) G_c [58].

This can be illustrated by elaborating further on the illustrations in Figure 6, as presented in Figure 7 for two different stress ratios. All load displacement curves in Figure 7 refer to the problem illustrated in Figure 5, assuming brittle crack growth, i.e. all energy dissipation relates to crack extension and not to plasticity.

Regardless of whether the comparison is made based on equal minimum or maximum load (both cases illustrated in Figure 7), the stress ratio evidently has an effect on the actual cyclic work applied $U_\uparrow$, the strain energy dissipated dU, and on the crack extension da. This implies that the stress ratio, or mean stress, has a similar effect on both dU and da for a given cycle. Hence, the characteristic equation (16) in which both dU/dN and da/dN are plotted against each other, may not exhibit a stress ratio effect as generally observed in Paris relationships.

Then how should damage or crack growth be related correctly to the applied loading through the applied cyclic work?

To begin with, it is generally acknowledged with the strain energy release rate concept that during a single load cycle hysteresis occurs, which equals the strain energy dissipated during the load cycle. In equation form, one could thus state that

$$U_0 + U_\uparrow = U_0^* + U_\downarrow + \frac{dU}{dN} \qquad (17)$$

With this equation, it is important to realise that here the roads towards development of a fundamental theory and a prediction model may depart. In a prediction model, parameters are equated that relate consequence (i.e. crack growth) to the cause (i.e. applied load or work).

However, in a fundamental theory, only these parameters of similitude are considered that directly equate to the mechanisms, but may not necessarily link the original cause with the consequence. Thus where a prediction model would try to quantify $U_\uparrow$ in terms of loading as a parameter to describe da/dN, the fundamental theory could focus on relating dU/dN to da/dN. It is the author's opinion that first the fundamental theory should be disclosed, before an appropriate prediction model can be proposed.

Now let us start hypothesizing on the fundamental theory for illustration. The energy is dissipated primarily by crack extension and formation of plasticity [59]. Thus it could be argued that

$$\frac{dU}{dN} = \frac{dU_a}{dN} + \frac{dU_{pl}}{dN} = \frac{dU_a}{dA}\frac{dA}{dN} + \frac{dU_{pl}}{dV_{pl}}\frac{dV_{pl}}{dN} \tag{18}$$

With U_a the strain energy release due to crack extension da and U_{pl} the strain energy released due to formation of additional plasticity volume V_{pl}. Here, the term dU_a/dA is obviously the effective strain energy release rate G_{eff}

$$G_{eff} = \frac{dU_a}{dA} \tag{19}$$

related to the extension of the crack with length da in one cycle. But one should note, that despite the cyclic nature of the load cycle, this term is written as G_{eff} and not ΔG_{eff}. Hence, it is not calculated as the difference between an artificial strain energy release rate at S_{max} and one at S_{min} (both related to instantaneous crack extension), but represents an actual strain energy release rate during a single load cycle.

Reviewing equation (18) reveals that it is obviously dimensionally correct. It also illustrates how the crack growth rate da/dN relates to the strain energy release rate G_{eff} together in relation to the actual strain energy dissipated. It represents not merely an empirical correlation between two parameters, it constitutes the physics of the problem! With reference to [58,60], it may be argued that the G_{eff} in equation (19) is to be considered a material characteristic independent of the applied load cycle.

Additionally, as described by Irwin, plasticity has a significant contribution to the strain energy dissipation and Griffith's theory could be modified for plasticity [62]. But rather than putting it all together like the resistance proposed by Irwin and later by Orowan [61]

$$R = 2\left(\gamma_e + \gamma_p\right) \tag{20}$$

it seems more appropriate to dissect the energy dissipation over two dissipative mechanisms as in equation (18). Hence, the strain energy release rate due to the formation of additional plasticity is defined as dU_{pl}/dV_{pl}. Now, Irwin has illustrated how a circular plastic zone size for plane stress relates to the crack length (unit width assumed)

$$r_p = \frac{1}{2\pi}\left(\frac{K_I}{S_y}\right)^2 = \frac{1}{2\pi}\left(\frac{S\sqrt{\pi a}}{S_y}\right)^2 = \frac{a}{2}\left(\frac{S}{S_y}\right)^2 \tag{21}$$

with S_y the yield strength of the material. This means that the plasticity growth rate could be approximated to relate to the crack growth rate by taking the derivative of the square of equation (21)

$$\frac{dV_{pl}}{dN} \sim \frac{d\left(r_p\right)^2}{dN} = \frac{a}{2}\left(\frac{S}{S_y}\right)^4 \frac{dA}{dN} \tag{22}$$

with the maximum size determined with the maximum stress in the load cycle, i.e. $S = S_{max}$. Combining equation (18) and (22) yields for arbitrary thicknesses

$$\frac{dU}{dN} = \left[\frac{dU_a}{dA} + \frac{a}{2}\left(\frac{S}{S_y}\right)^4 \frac{dU_{pl}}{dV_{pl}}\right]\frac{dA}{dN} \tag{23}$$

Note that the term between the straight brackets is equal to dU/dA in equation (16). The energy dissipation dU/N due to plasticity in case of force controlled fatigue tests is thus linearly related to the crack length a. Relating the SERR G=dU/dA to the resistance R in equation (20), implies that

$$\begin{aligned} \gamma_e &\sim \frac{dU_a}{dA} \\ \gamma_p &\sim \frac{a}{2}\left(\frac{S}{S_y}\right)^4 \frac{dU_{pl}}{dV_{pl}} \end{aligned} \tag{24}$$

7 Further discussion on principles of similitude

7.1 Example: Crack propagation in aluminium sheet

The above discussion illustrates that the fundamental theory for describing fatigue crack growth could equate the strain energy dissipation, revealed by strain energy release, to the crack extension and formation of plasticity. That having said, one must be careful in selecting the boundaries of the system for which energy preservation is considered. This could be important when stress energy density is taken as parameter of similitude [63-66], which implicitly homogenises the strain energy over the entire volume. Here, the system was taken equal to the entire specimen illustrated in Figure 5 without specifying any detail about the direct vicinity of the crack tip.

This can be illustrated with an actual test. For the current study, a 6 mm thick Aluminium 2024-T3 Alclad sheet specimen has been tested with L = 300 mm and W = 160 mm (see Figure 5). The crack growth was monitored optically at the side using a digital camera and recorded with the number of applied load cycles. The test was executed in load controlled conditions on a closed-loop 250 kN MTS fatigue testing machine with a maximum stress of S_{max} = 75 MPa, and R = 0.05.

Together with the crack extension, the applied force P and the displacement δ were recorded, which allows calculation of the strain energy U applied to the specimen at each load cycle N. The resultant a-N and U-N curves are provided in Figure 8. The crack growth rate da/dN and rate with which the strain energy changes dU/dN is obtained by taking the derivative of both curves.

Instead of applying data reduction techniques as proposed by for example ASTM standards [67], equations were fitted straight through the curves in Figure 8 with a coefficient of determination close to 1. Derivatives were taken analytically using these equations providing a clear trend in da/dN versus dU/dN. The result is presented in Figure 9 both using double logarithmic and linear scales.

One should note that in this approach any energy dissipation mechanisms present are assumed to be included in the dU/dN. Looking at the correlation in both curves in Figure 8 immediately illustrates the purpose of so-called geometry correction factors: despite applying constant amplitude loading, the applied cyclic work is substantially increasing in these conditions, leading to an increase in da/dN.

The curve presented in Figure 9 seems close to linear, but if approximated with a power law, it would yield an exponent less than 1. The linear trend indicated in this figure assumes that once twice the amount of energy is dissipated in a single load cycle, also twice the amount of crack area has been created.

The non-linearity of this curve indicates that with increasing crack growth rates more energy is dissipated in other mechanisms compared to the dissipation by crack formation. One can safely assume that plasticity will be the most dominant dissipation mechanisms to cause this deviation from the linear trend.

7.2 Example: Delamination growth in carbon fibre reinforced polymer composites

Considering cyclic work and strain energy release rather than cyclic stress does not only apply to fatigue cracking in metals, it applies to any damage extension under cyclic loading in any engineering material or system. This can be illustrated with two examples. The first is provided by Pascoe et al. [68,69] who determined the reduction in strain energy in displacement controlled fatigue disbond tests in adhesively bonded aluminium double cantilever beam (DCB) specimens. The decrease in measured strain energy over the load cycles is subsequently plotted against da/dN in line with equation (16). The first observation by Pascoe is that the power law describing the relationship has a power of about 0.8 (similar to the case in the previous section), and that the stress ratio has negligible or no effect on the relationship.

A similar example is provided by Yao et al. [70] where displacement controlled fatigue interlaminar ply-delamination tests were performed on carbon fibre DCB tests, see Figure 10. The obtained delamination growth curve and the reduction in applied strain energy U_{tot} are given in Figure 11. Similarly, equations were fitted through the data with R^2 close to 1, of which subsequently the derivative was taken analytically to enable plotting da/dN against dU/dN.

Figure 12 presents the reduction in strain energy per cycle against the crack growth rate da/dN, clearly indicating that this trend is fairly linear over the entire regime. Small deviations were reported by Yao that are attributed to failure or pull-out of bridging fibres.

These examples by Pascoe and Yao teach us several lessons:

- The stress ratio effect is not a physical effect, i.e. it is only there, where incorrect parameters of similitude have been selected.
- The SERR dU/da in equation (16) is neither defined by $G_{max} – G_{min}$ or $(\sqrt{G_{max}} - \sqrt{G_{min}})^2$, nor is it equal to the critical SERR G_c. The relationship between dU/dN and dA/dN suggests that a single relationship between dU/dA and dA/dN can be derived.
- Describing fatigue damage growth based on plastically dissipated energy alone [71,72] may not always be appropriate, as it assumes that the ratio between strain energy release due to crack extension and plasticity formation is constant. The power 0.8 in Pascoe's data and similar values in the data provided in section 7.1 suggests that this ratio may not be constant, as was suggested before by Broberg [73,59].
- There may be dissipating mechanisms related to shielding mechanisms, like for example fibre bridging in composites, but in terms of dU/dN bridging fibres only contribute once they fail, not when they are intact, as demonstrated by Yao et al. [70].

It is therefore recommended to dissect the SERR into terms that describe the dissipating mechanisms individually. Thus $G_a = dU_a/dA$ is related to crack extension only, and for example $G_{pl} = dU_{pl}/dV_{pl}$ is related to the formation of plasticity. In doing so, the SERR due to crack extension G_a could be related to a single material characteristic. In addition, a relationship characterized by a power law with power different from 1, could be quantitatively interpreted by evaluating the ratio between the two terms.

7.3 Prediction models

Comparing the equations (17) and (23) illustrates that what describes crack growth is the offered strain energy and work at the left-hand side of equation (17), i.e. U_0 and $U_\uparrow$. Substitution of equation (23) into equation (17) illustrates that a model predicting crack extension da should be based on

$$\frac{da}{dN} = f\left(U_0 + U_\uparrow\right) \tag{25}$$

This is illustrated by the example study [74], where the applied load cycle is governed by

$$U_\uparrow = \frac{1}{2} S_{\max} \varepsilon_{\max} - \frac{1}{2} S_{\min} \varepsilon_{\min} \tag{26}$$

instead of $S_{max} – S_{min}$. As demonstrated in [74], most of the so-called crack closure corrections are explained by this difference in selection of similitude parameters, explaining the observations presented in section 3. Only a small difference is observed between the relationship based on cyclic energy and these crack closure corrections, exactly in that range of stress ratios where one could expect an influence of closure, i.e. $R < 0.3$.

These closure or shielding mechanisms imply a non-linearity in the load-displacement curve at the minimum load, which affects the applied cyclic work to the system. Closure mechanisms like plasticity induced crack closure, or roughness closure, should therefore aim at quantifying the difference between calculated applied linear elastic work and the actual work [74].

In addition, studies on for example the threshold in fatigue should aim at understanding and describing the minimum applied work before strain energy is dissipated, i.e. before the crack starts to extend. Only that portion of the work beyond the energy threshold is expected to contribute to growth, and should therefore be considered in prediction models. Here one should consider that this threshold energy includes the monotonic energy below the minimum load in case of positive stress ratios. This means that here a little effect of stress ratio may be present, which could be accounted for by taking

$$U_{\uparrow,eff} = U_0 + U_{\uparrow} - U_{threshold} \qquad (27)$$

Obviously, this formulation implies that the threshold is related to a minimum (i.e. threshold) load below which no energy dissipation due to crack extension occurs. This could be equivalent to a minimum crack tip opening.

8. Conclusions and recommendations

The similitude parameters commonly applied in fatigue evaluations were reviewed and discussed. The general conclusion is that the SIF range ΔK is not the appropriate parameter for similitude, because it does not reflect well the cyclic nature of the work applied.

The required additional corrections for the stress ratio, seem to compensate this shortcoming, but also seem to be misunderstood, illustrated by the plasticity induced crack closure explanation. In addition, the crack growth relationships like the Paris equation are purely phenomenological, and they lack any fundamental substantiation using the physics of the crack growth problem. Dimensional analysis reveals that constants in the equations have dimensions, which cannot be explained from the perspective of physics.

When based on the SERR, the relationships tend to equate parameters to crack growth that are the consequence of that growth rather than the driver. Nonetheless, the parameters are often inconsistently considered crack driving parameters.

It is therefore recommended to first develop fundamentally the theory that describes how the offered strain energy (work) is dissipated in the individual mechanisms, without the intention to develop prediction models. Once the theory is sound and verified, one should attempt to further develop it into a prediction model.

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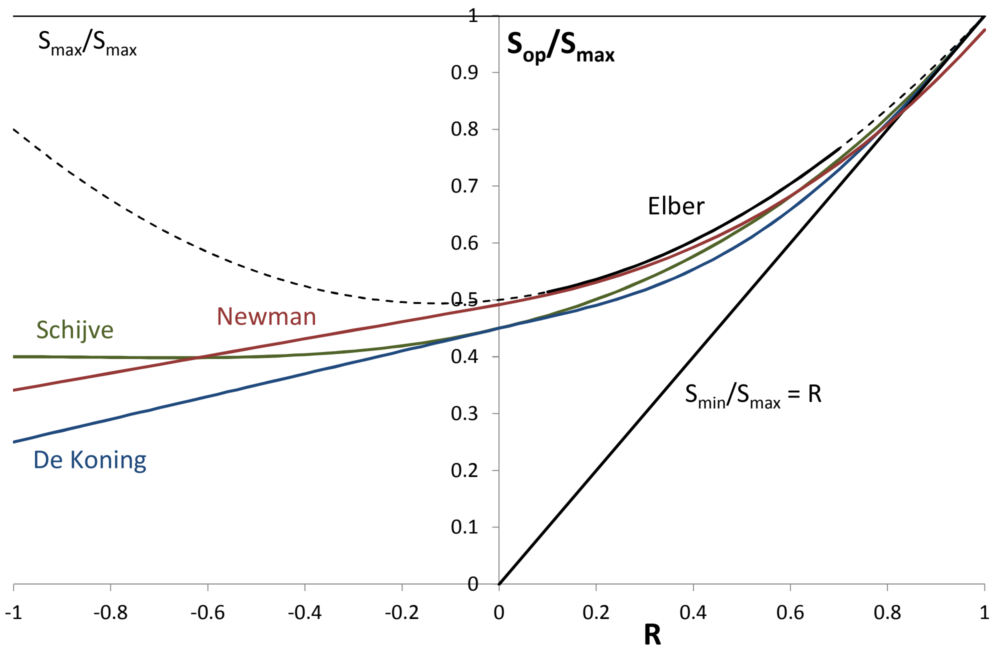

Figure 1 Illustration of various crack closure corrections proposed for the range of -1<R<1 [33,34,48,49,50,51,52]

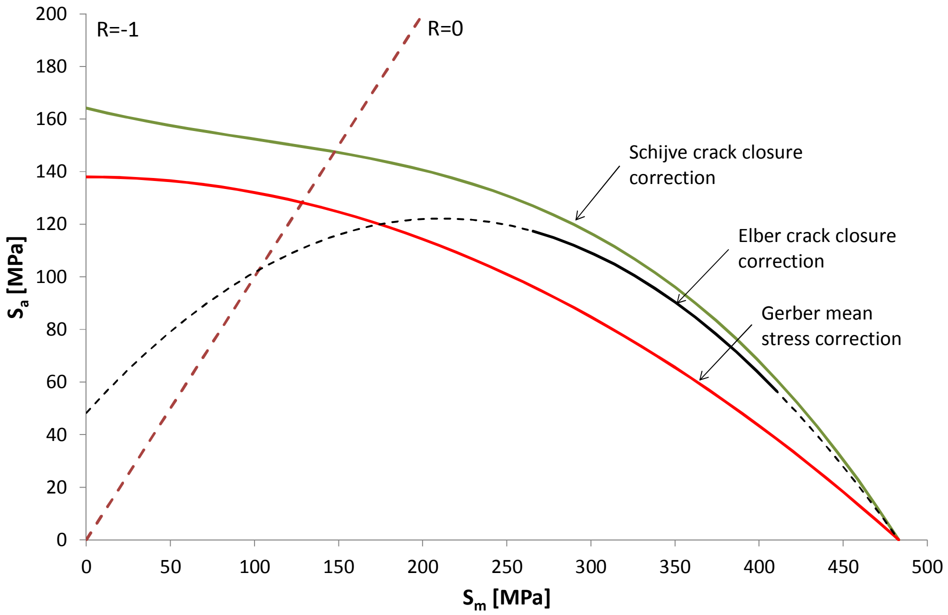

Figure 2 Correlation between the Gerber parabola describing the fatigue limit of 2024-T3 and the crack closure corrections of Schijve and Elber for the same alloy.

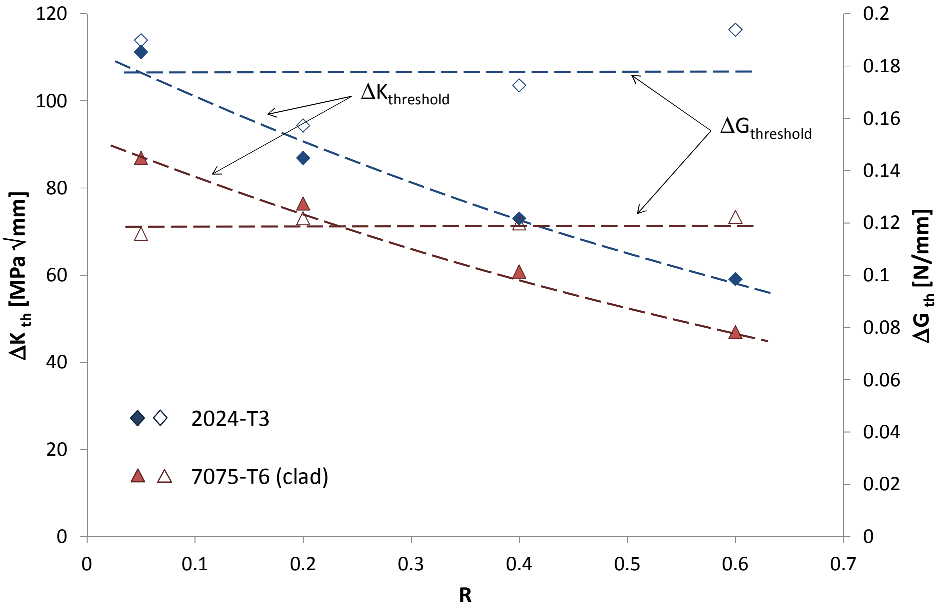

Figure 3 Correlation between the threshold SIF ΔK_{th} and stress ratio R, and the threshold SERR ΔG_{th} and R (data for 2024-T3 and 7075-T6 from [56]).

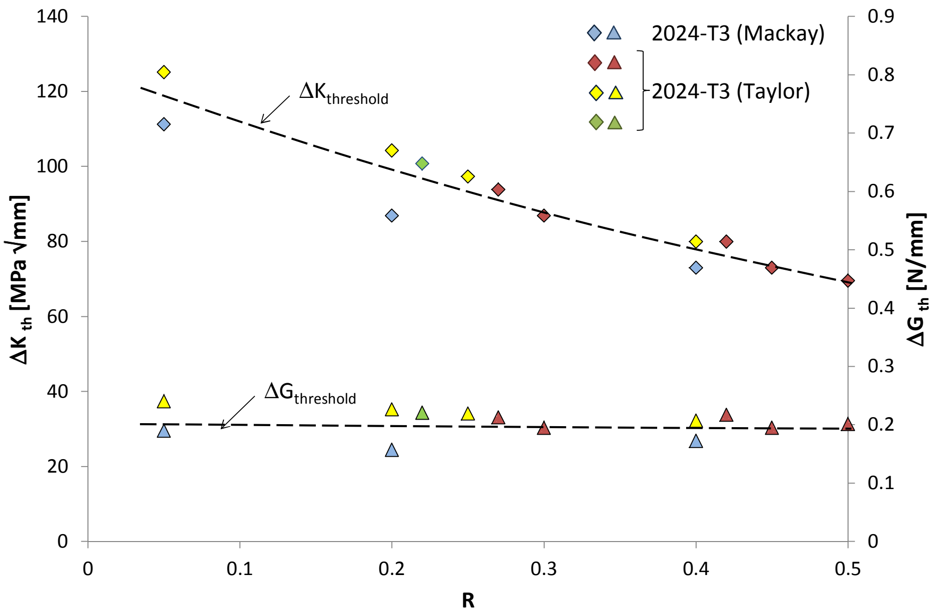

Figure 4 Correlation between the threshold SIF ΔK_{th} and stress ratio R, and the threshold SERR ΔG_{th} and R (data for 2024-T3 from [56,57]).

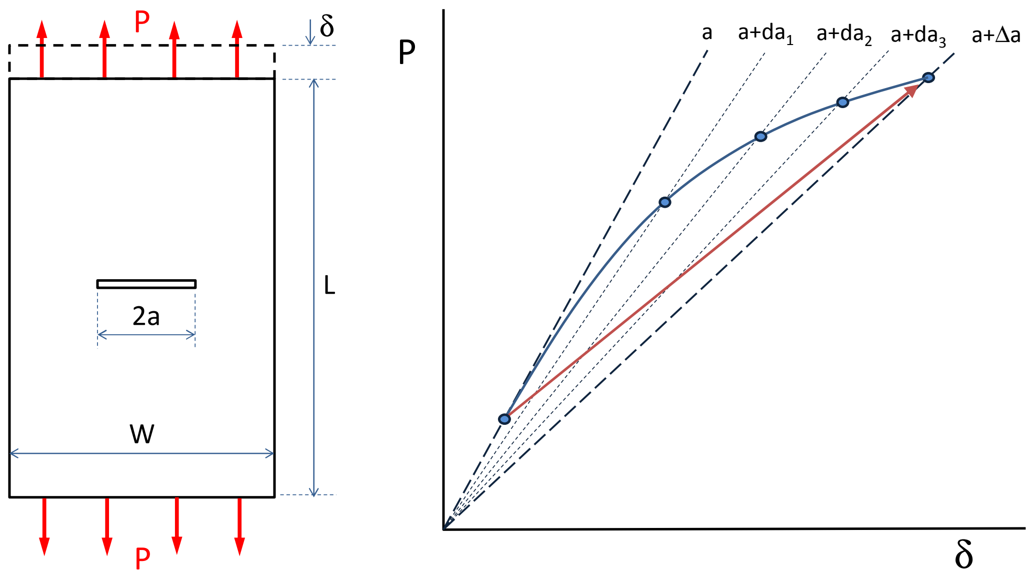

Figure 5 Illustration of a Centre-Cracked Tension (CCT) panel loaded by applied load P inducing an elongation δ = f(a) (left), and the corresponding force displacement diagram for crack increments da_i during the extension Δa created in the part of the load cycle from P_{min} to P_{max}.

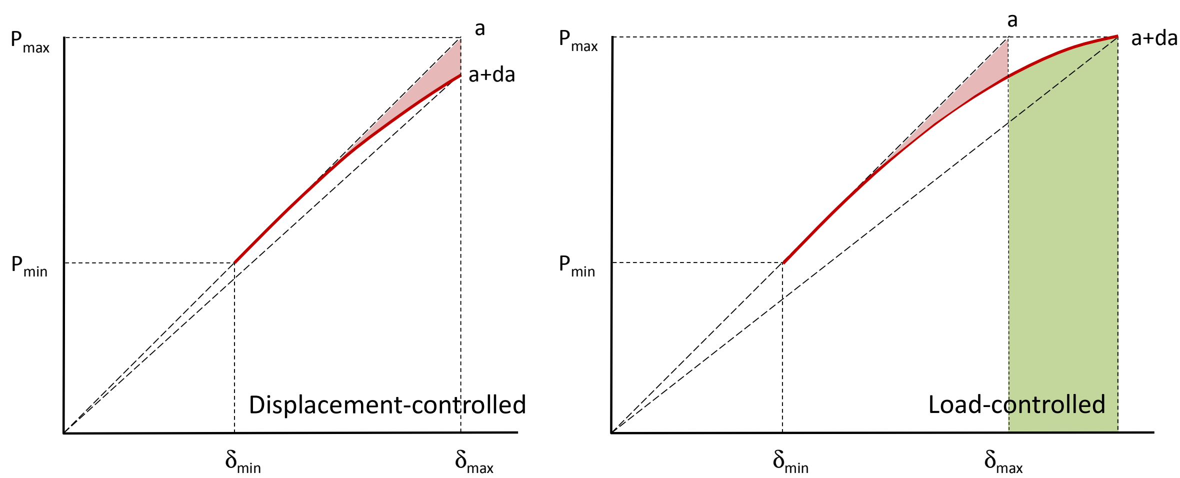

Figure 6 Illustration of the non-linear response during the uploading part of the fatigue cycle induced by the energy dissipating mechanism of crack extension if brittle behaviour is assumed (shaded areas represent difference with common linear elastic approaches: pink = work not applied, green = additional work applied)

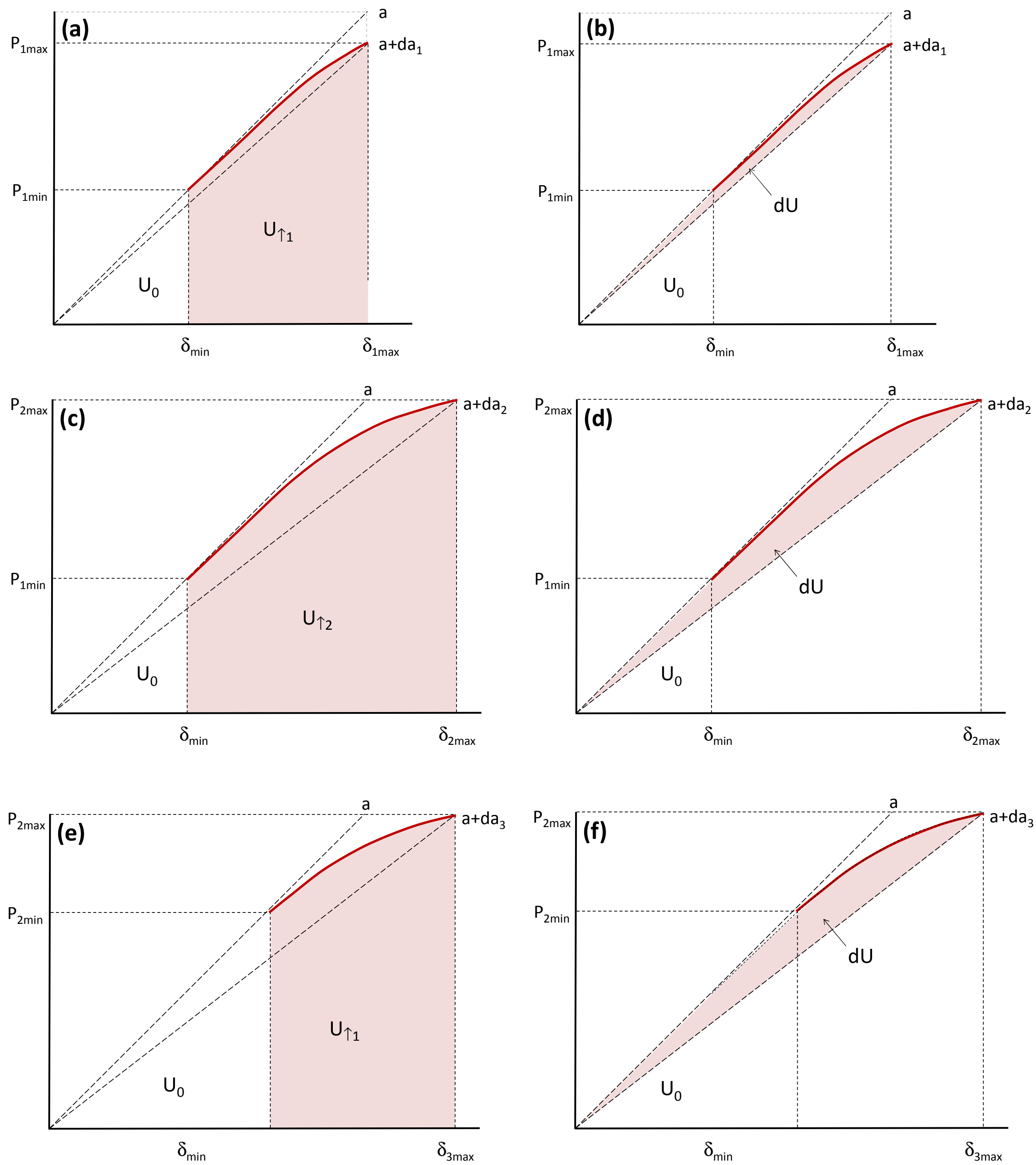

Figure 7 Illustration of the relation between applied work $U_{\uparrow}$ (a,c,e) and the strain energy release dU (b,d,f) for three different stress ratios; $R_{(e,f)} = P_{2min}/P_{2max} > R_{(a,b)} = P_{1min}/P_{1max} > R_{(c,d)} = P_{1min}/P_{2max}$ (brittle crack growth assumed – no plasticity)

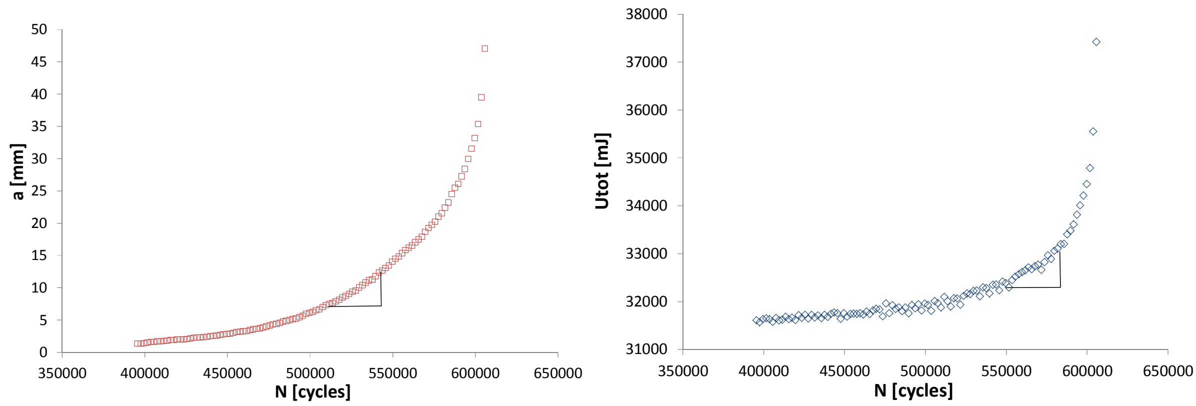

Figure 8 **Measured crack growth (left) against the applied number of load cycles, and (right) the measured strain energy $U_{tot} = ½P_{max}\delta_{max}$ plotted against the number of load cycles.**

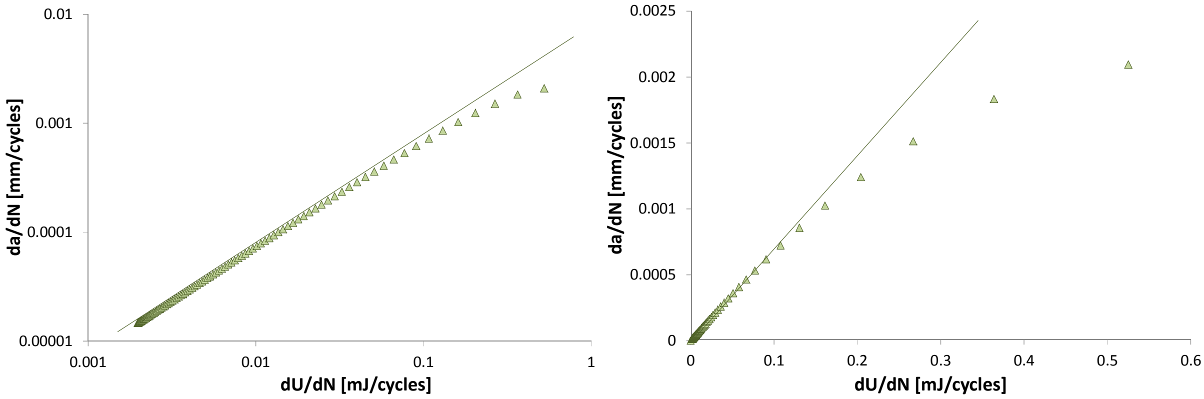

Figure 9 **Correlation between da/dN and dU/dN taken from Figure 8, presented on double logarithmic scale (left) and linear scale (right).**

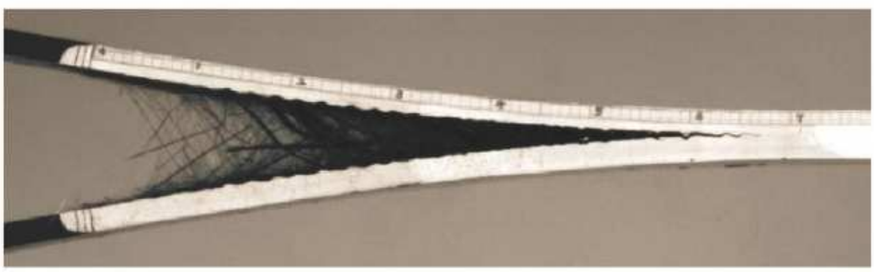

Figure 10 **Illustration of delamination growth in a DCB specimen made of carbon fibre polymer composite with fibre bridging as a crack shielding mechanism [70].**

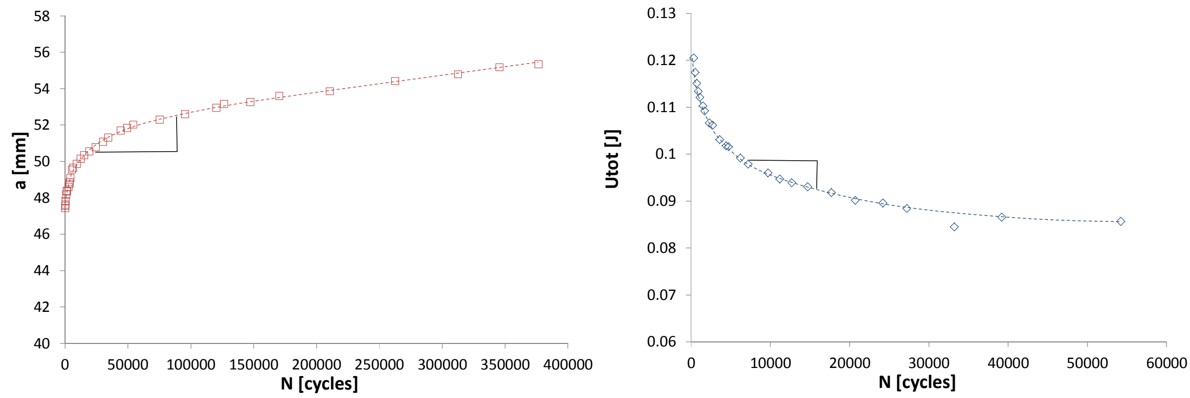

Figure 11 Measured delamination growth (left) against the applied number of load cycles, and (right) the measured strain energy $U_{tot} = ½P_{max}\delta_{max}$ plotted against the number of load cycles, data from [70].

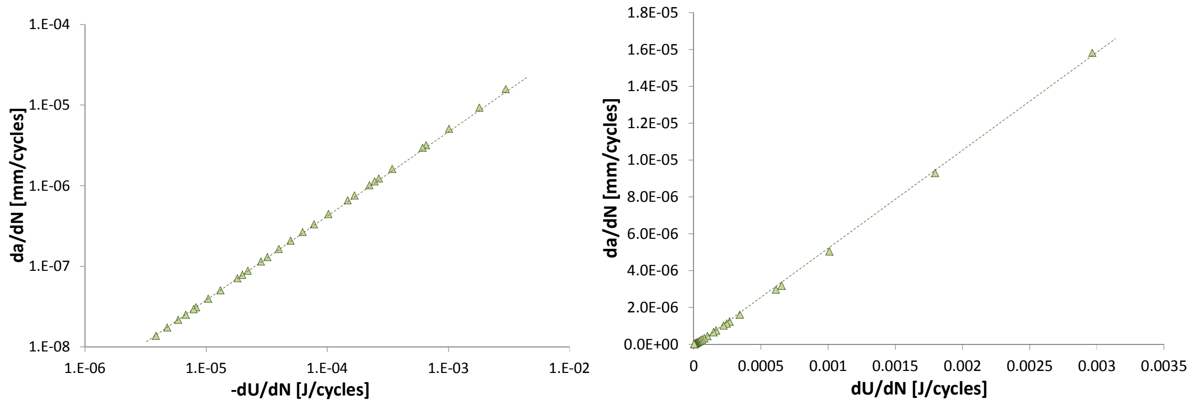

Figure 12 Correlation between da/dN and dU/dN taken from Figure 11, presented on double logarithmic scale (left) and linear scale (right).