A grouping method for optimization of steel skeletal structures by applying a combinatorial search algorithm based on a fully stressed design

van Woudenberg, Tomas R.; van der Meer, Frans P.

DOI
10.1016/j.engstruct.2021.113299

Publication date
2021

Document Version
Final published version

Published in
Engineering Structures

Engineering Structures 249 (2021) 113299

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

A grouping method for optimization of steel skeletal structures by applying a combinatorial search algorithm based on a fully stressed design

Tomas R. van Woudenberg [a,b], Frans P. van der Meer [a,*]

[a] Faculty Civil Engineering & Geosciences, Delft University of Technology, Stevinweg 1, 2628 CN Delft, the Netherlands
[b] Faculty Technology, Amsterdam University of Applied Sciences, Weesperzijde 190, 1097 DZ Amsterdam, the Netherlands[1]

A R T I C L E I N F O

Keywords:
Grouping methods
Structural optimization
Steel structures
Diversity
Principle of commonality

A B S T R A C T

In the design of steel structures, optimization methods can find minimum weight solutions. However, the solutions tend to have a high diversity of profiles, thereby raising costs. Grouping methods provide a solution by limiting the number of unique profiles, while still providing an optimal solution. This study proposes a new grouping method and compares its performance to that of existing methods in eight benchmark problems. In current practice, the grouping is mostly performed manually, relying on an engineer's expertise. This technique requires no additional calculations but fails in finding a light or cheap structure. In general, all other grouping methods perform better than manual grouping. Out of the compared methods, the cardinality constraints method finds the lightest solutions. However, this method requires solving a big optimization problem, thereby increasing computational costs and the variance in the outcome. The new method, 'the fully stressed combinatorial search', groups members by a combinatorial search, which evaluates the estimated weight of a restricted set of groupings based on the weight per unit length of the members of a fully stressed design. Subsequently, optimization of a small search space finds the corresponding optimum profiles. These steps are repeated, in which the fully stressed design uses the result of the previous optimization as its reference design. The loop repeats until the grouping is unchanged, or the result becomes less optimal. This new method finds similar results as the cardinality constraints with less finite element evaluations and higher consistency in the results upon repetition of the analysis.

1. Introduction

In the design of steel structures, optimization methods can find the lightest solution, given a certain profile database. However, optimized solutions tend to have a high diversity of profiles, increasing the cost of the structure. By reducing the number of distinct profiles in the final design by grouping profiles or members, the cost is reduced, according to the principle of commonality [1,2]. However, the exact cost savings from limiting the number of profiles are hard to quantify, which makes it difficult to cover the principle of commonality in the objective function of an optimization problem [3]. Grouping methods provide a solution by forcing the number of distinct profiles to a desired value.

The most popular method is to manually group members, and let an optimization method find the optimal profiles [1,3,4]. Biedermann and Grierson [5] proposed to replace this manual process by a neural network. However, this grouping is strongly dependent on the engineer's expertise. Another possibility provided by Biedermann and Grierson [6] is to group members based on their length. However, the length of a member cannot represent the full structural response. A different approach was taken by Templemann [3] who proposed to solve the grouping problem by reducing the profile database manually to the number of desired profiles. Again, the optimality of this method is strongly dependent on the engineer's expertise.

The structural behavior has been taken into account by several authors [7,8] by evaluating the axial force of a uniform design. For these methods, the full range of forces is divided into the desired number of groups. Toğan and Daloğlu [9] also showed an adaptation of this method by considering the slenderness ratio for compressive beams. Finally,

Abbreviations: ROT, Rules of thumb grouping method; AF, Axial force grouping method; AF+S, Axial force and slenderness grouping method; CC, Cardinality constraints grouping method; UCS, Ungrouped combinatorial search grouping method; FSCS, Fully stressed combinatorial search grouping method.

* Corresponding author.
E-mail addresses: t.r.van.woudenberg@hva.nl (T.R. van Woudenberg), f.p.vandermeer@tudelft.nl (F.P. van der Meer).
[1] Present address.

https://doi.org/10.1016/j.engstruct.2021.113299
Received 11 February 2021; Received in revised form 26 August 2021; Accepted 25 September 2021
Available online 9 October 2021

Mashayekhi, Salajegheh and Dehghani [10] proposed to group members based on their internal axial force, but separate the least-loaded beams in each group. All these methods cannot group tensile and compressive members together in one group, as buckling behavior is present in compressive members. Furthermore, these methods are not applicable to frame structures, as moment and shear forces influence the optimum result.

A thirds class of grouping methods adapt the optimisation method to limit the number of distinct profiles. Barbosa and Lemonge [4] proposed to adapt the encoding of the problem by adding cardinality constraints. In doing so, the optimization methods is altered to a search space which only contains designs which have as maximum the number of desired groups. This concept shows close resemblance with the research performed by Reitman and Hall [11]. They showed that an heuristic approach, in which the grouping or reduced element set is chosen before the optimization of element sizes starts, cannot find the optimum result. Conclusively, the two separate problems should be combined in one optimization problem. The cardinality constraints method does so by changing the encoding of the problem, thereby altering the search space. Barbosa and Lemonge [4] also proposed the option to add an inequality constraint on the number of groups, while Kanno [12] proposed an addition of an equality constraint. These constraints make all undesired designs unfeasible. Finally, adaption of the objective function is also an option, as proposed by Galante and Oñate [13], and Shea, Cagan and Fenves [14]. Although all methods which change the optimization procedure do not exclude the global grouped optimum in their search space, the search space can be large. This leads to an optimization problem which is hard to solve and requires many computations.

The last class of grouping methods performs an ungrouped optimization first, which provides the basis for grouping. Templemann [3] proposed to use a rounding procedure, Provatidis and Ventsanos [2] grouped members based on the mean value and standard deviation of their unit weight per length, Adeli and Sarma [15] proposed a procedure using a multi-criteria cost-optimisation model, and Walls and Elvin [1] proposed to evaluate a reduced set of potential groupings. As forces are redistributed and displacements change with these alternations of the ungrouped optimum, no guarantee is given that the optimum grouped solution can be found. Moreover, the ungrouped optimization is a big optimization problem which is hard to solve.

This research proposes a new grouping method which can find or approach the optimum grouped solution, while keeping the number of computations low.

This paper first illustrates the optimization and grouping problem in Section 2. Section 3 proposes the new grouping method, which is compared with a selection of existing grouping methods in Section 4. Section 5 discusses the new method and the results, followed by conclusions in Section 6.

2. Problem description

In this paper we search for a method to select the steel profiles for a truss or frame structure with given geometry and load. We assume that the cost of the structure can be reduced by limiting the number of distinct profiles in the structure and that the weight of the structure is the main cost driver for a given number of groups.

2.1. Optimization problem

The structural design optimization problem consists of minimizing the weight of the structure:

$$min(W) = \sum_{i=1}^{l} A_i L_i \rho_i \tag{1}$$

with l the number of members, and A_i, L_i, ρ_i the area, length and density of a member i. The profiles of the l members are variable. For a profile database of size m, the total search space of the optimization problem is m^l.

For truss structures, the design is set to normal stress constraints:

$$\frac{\sigma_i}{\sigma_{max}} \leqslant 1 \tag{2}$$

in which σ_i is the stress in a certain member, and σ_{max} the allowable stress. The allowable stress are given by yield stress and buckling stress:

$$\sigma_{\text{buckling}} = \frac{r^2 \pi^2 E}{L^2} \tag{3}$$

with r the radius of gyration, E the Young's modulus and L the length of an element. For problems in which the profiles are specified in area only, the radius of gyration is estimated as $r = \sqrt{4A}$.

For frame structures, the stress limits are specified from AISC-LFRD requirements [16]:

$$\begin{aligned} &\frac{P_u}{\varphi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\varphi_b M_{nx}} + \frac{M_{uy}}{\varphi_b M_{ny}}\right) \leqslant 1 \qquad \text{for } \frac{P_u}{\varphi P_n} \geqslant 0.2 \\ &\frac{P_u}{2\varphi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\varphi_b M_{nx}} + \frac{M_{uy}}{\varphi_b M_{ny}}\right) \leqslant 1 \qquad \text{for } \frac{P_u}{\varphi P_n} < 0.2 \end{aligned} \tag{4}$$

with P_{uk} the required axial tensile strength, P_n the nominal tensile strength, M_{ux} and M_{uy} the required flexural strength in two directions, M_{nx} and M_{ny} the nominal flexural strength in two directions, φ the tensile strength reduction factor (0.9) and φ_b the flexural resistance reduction factor (0.9). For the determination of the nominal compressive strength, the effective length factor as approximated by Dumonteil [17] is used:

$$K = \sqrt{\frac{1.6 G_A G_B + 4(G_A + G_B) + 7.5}{G_A + G_B + 7.5}} \tag{5}$$

in which G_A and G_B are the stiffness ratios of columns and girders at the end joints of an element.

Furthermore, displacement constraints are set:

$$\frac{d_i}{d_{max}} \leqslant 1 \tag{6}$$

with d_i the displacement of a certain part of the structure and d_{max} the allowable displacement. Both stresses and displacements are calculated with the finite element method (FEM).

In a realistic structural design setting, more elaborate design constraints need to be considered. However, the precise selection of constraints does not affect the performance of the grouping methods investigated in this study.

2.2. Grouped optimization problem

Solving the grouped optimization problem consists of (1) grouping l members of a structure, to k groups, and (2) selecting the desired number of k profiles from a profile database of size m. These two subproblems are strongly linked. If the number of profiles is to be exactly equal to k, the first subproblem has N_1 solutions:

$$N_1 = \left\{ \begin{matrix} l \\ k \end{matrix} \right\} = \frac{1}{k(k-1)\cdots 1} \sum_{i=0}^{k} (-1)^i \binom{k}{i} (k-i)^l \tag{7}$$

as defined by the Stirling number of the second kind [18]. The second problem has N_2 solutions:

$$N_2 = \binom{m}{k} = \frac{m(m-1)\cdots(m-k+1)}{k(k-1)\cdots 1} \tag{8}$$

by definition of the binomial coefficient. The optimum grouped solution is one of the N_{total} possibilities:

$$N_{total} = N_1 \cdot N_2 \quad (9)$$

with N_1 and N_2 as defined by Eqs. (7) and (8).

The methods which apply grouping before optimization solve the first problem by an additional analysis. This reduces the search space of optimization to m^k options, which includes solutions which have less than the desired number of profiles and is therefore bigger than N_2. On the other hand, the method of Templemann [3] manually reduces the number of profiles m in the second subproblem to k profiles, resulting in an optimization search space of k^l options. Again, the search space includes solutions which have less than the desired number of profiles and is bigger than N_1. However, in both cases not all N_{total} grouped solutions are included in the search space because of the heuristic solving of one of the subproblems. This leads to a potential exclusion of the optimum grouped solution.

For the methods which change the optimization process to solve the grouping problem, both subproblem are solved indirectly during the optimization process. Therefore, at least all N_{total} grouped solutions are included in the search space, guaranteeing the inclusion of the optimum grouped solution. The cardinality constraint method of Barbosa and Lemonge [4] leads to a search space of $m^k \bullet k^l$ solutions. However, this search space includes duplicate designs as well as designs with a smaller number of groups than the desired number. Therefore, the search space is larger than N_{total}. The search space of the cardinality constraint method can even grow larger than the ungrouped search space of m^l when increasing the number of groups. Specifically, $m^k \bullet k^l > m^l$ if:

$$k > \frac{l \cdot W\left(\frac{m \cdot ln(m)}{l}\right)}{ln(m)} \quad (10)$$

where $W(z)$ is the Lambert W function [19]. For the benchmark problems as presented in Section 4, this inequality holds from $k = 7$ for the 72-bar truss tower and from $k = 8$ for the 112-bar truss dome. The other methods that are applied during the optimization process change the objective or constraint functions, thereby having no influence on the original search space of m^l. All N_{total} grouped solutions are included but only a small part of the search space is part of the desired or feasible N_{total} solutions.

3. New grouped optimization method

A new grouped optimization method is proposed which reduces the search spaces by providing a better estimation for the first subproblem than existing methods, followed by an optimization on the second subproblem in a small search space. This first subproblem is solved by a combinatorial search, in which members are grouped based on their weight per unit length of a fully stressed design. In doing so, only a small but favorable part of all possible N_1 options are considered. In the case of static determinate structures without displacement constraints, the optimum solution is found with the combinatorial search. In other cases, only the grouping of members of the combinatorial search is used as a possible solution to the first subproblem, and the corresponding optimum profiles of the second subproblem are found by solving a reduced optimization problem, with a design space of size m^k, including all N_2 options. As the fully stressed design is dependent on an initial design, which influences the force distribution and global displacements of the structure, the process is repeated to include these effects on the grouping and corresponding profiles. The framework of this method, the fully stressed combinatorial search (FSCS), is shown in Fig. 1.

3.1. Fully stressed design

The fully stressed design is the design with for each member individually the lowest allowable weight. For the first iteration, the heaviest profile of the profile database is initially assigned to all members. For

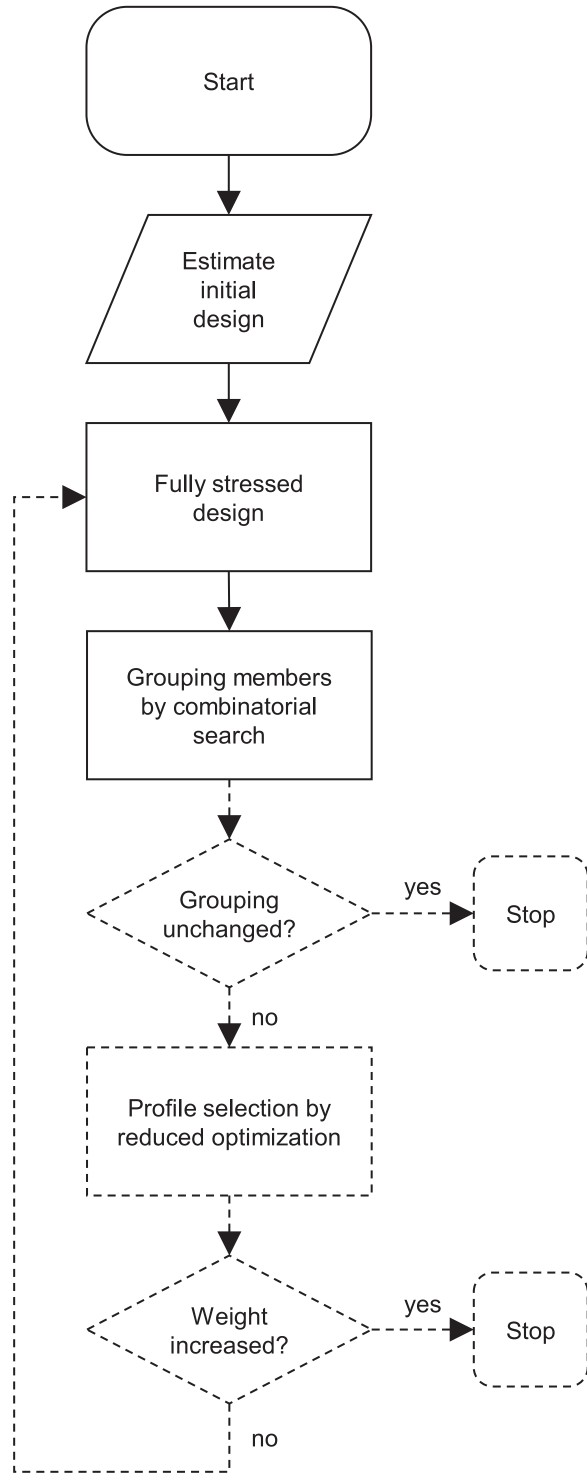

Fig. 1. Framework fully stressed combinatorial search. Dashed steps are not applied for static determinate structures without displacement constraints.

other iterations, the initial design is the result of the grouped optimization from the previous iteration. For each member, the profile list is evaluated from the lightest to the heaviest member, while the other members are equal to the initial design. The first profile for which the design satisfies all stress and displacement constraints is chosen. In effect, the maximum number of FEM calculations for the fully stressed design is $N_{FS,\max}$:

$$N_{FS,max} = m \cdot l \quad (11)$$

The actual number of FEM calculations is less as for each member not the full list of profiles is analyzed, but only up to the first feasible profile. The collection of all individual best profiles together is the output of the fully stressed design.

An example for the fully stressed design in the first iteration in case of a 117-bar braced frame is shown in Fig. 2a. Details of this frame are given in Section 4.3. The figure shows a high diversity in member thicknesses. In this figure, the thickness of members represents their weight per unit length, while members with the same resulting profile have the same color.

3.2. Combinatorial search

The combinatorial search groups members based on their unit weight per length, solving the first grouping subproblem. It takes as input the fully stressed design and evaluates the weight of a reduced set of N_{total}: only combinations are evaluated in which the profiles are increased in weight to k distinct limit profiles. It is identical to the combinatorial search proposed by Walls and Elvin [1], which is one possible solution procedure of the standardization problems proposed by Reitmann [20] and Reitmann and Brent Hall [11].

The combinatorial search evaluates the weight of N_{CS} options:

$$N_{CS} = \binom{n-1}{k-1} = \frac{(n-1)(n-2)\cdots(n-k+1)}{(k-1)(k-2)\cdots 1} \quad (12)$$

by definition of the binomial coefficient in which n is the number of distinct profiles $\{p_1, p_2, \dots p_n\}$ in the fully stressed design, which is equal or less than m. The algorithm for defining the combinations is shown in Table 1, in which the profiles are sorted from heavy to light: weight $p_1 >$ weight $p_2 > \dots >$ weight p_n.

Structural performance is ensured by assigning the heaviest profile in a group to the entire group. For each combination, the weight is evaluated and the grouping with the lowest total weight is the result of the combinatorial search.

An example for optimum grouping following from the combinatorial search in case of the 117-bar braced frame is shown in Fig. 2b. The figure shows that both the columns and bracings are grouped in three groups, while the beams are all grouped in one group. In this figure members with the same member group have the same color, while the thickness has no meaning.

3.3. Reduced optimization problem

For statically determinate structures without displacement constraints, the resulting grouping is the optimum solution for the first subproblem and the corresponding k limit profiles are the optimum profiles of the second subproblem. In that case, the fully stressed combinatorial search method does not require the application of an optimization method.

For other structures, the profile selection for the given grouping is updated in an optimization step, as the k limit profiles might not be optimal. Only the grouping of members from the combinatorial is used in this optimization. This leads to a relatively small search space of m^k options. As the grouping is performed separate of the profile selection, the optimum grouped solution might be excluded.

An example for optimum grouped design in case of the 117-bar braced is shown in Fig. 2c. The figure shows the optimum thicknesses for each of the member groups. In this figure, the thickness of members represents their weight per unit length, while members within a member group have the same color.

3.4. Convergence criteria

The loop of the FSCS incorporates the effect of the initial design of the fully stressed design on static indeterminate structures, which

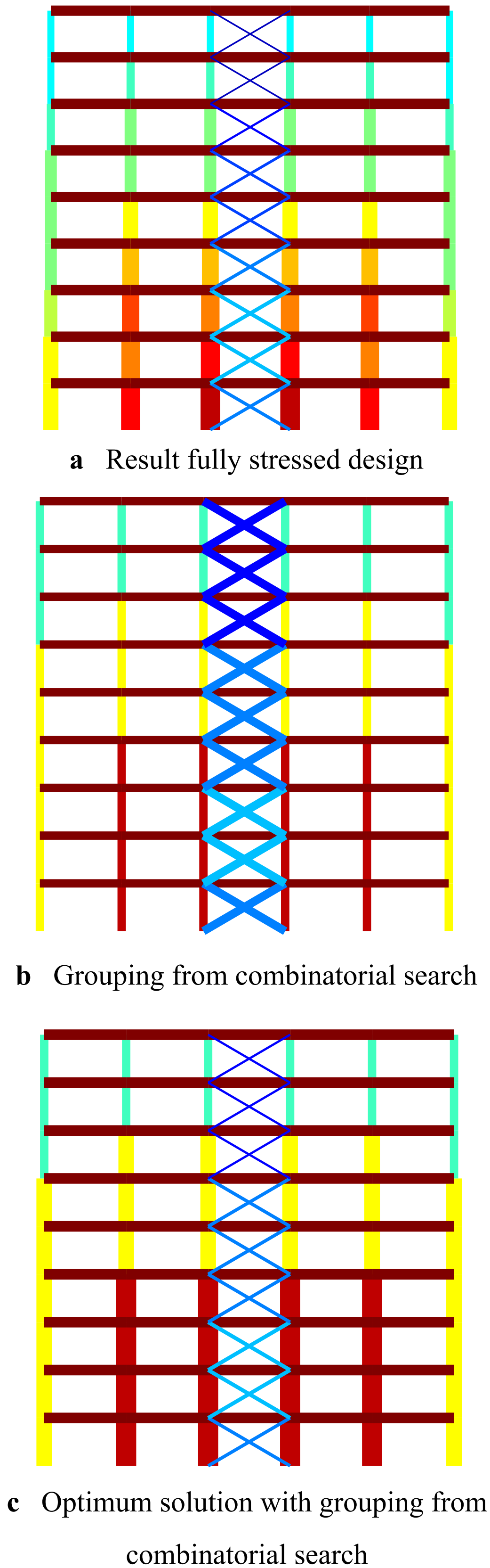

Fig. 2. Example fully stressed combinatorial search for 117-bar braced frame in first iteration.

Table 1
Resulting groups for combinatorial search algorithm of determining N_{CS} possible combinations with grouping of n profiles { $p_1, p_2, \ldots p_n$} to k groups, with weight $p_1 >$ weight $p_2 > \ldots >$ weight p_n. Adaptation of table from Walls and Elvin [1].

Combination	p_1	p_2	•••	p_{k-2}	p_{k-1}	p_k	p_{k+1}	p_{k+2}	•••	p_{n-2}	p_{n-1}	p_n
1	1	2	•••	$k-2$	$k-1$	k	k	k	•••	k	k	k
2	1	2	•••	$k-2$	$k-1$	$k-1$	k	k	•••	k	k	k
⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮
$n-k+1$	1	2	•••	$k-2$	$k-1$	$k-1$	$k-1$	$k-1$	•••	$k-1$	$k-1$	k
$n-k+2$	1	2	•••	$k-2$	$k-2$	$k-1$	k	k	•••	k	k	k
$n-k+3$	1	2	•••	$k-2$	$k-2$	$k-1$	$k-1$	k	•••	k	k	k
⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮
N_{CS}	1	1	•••	1	1	1	1	1	•••	$k-2$	$k-1$	k

changes the grouping of members during the iterations. The loop is stopped when one of two convergence criteria are met:

1. The grouping of members as a result of the combinatorial search is unchanged with respect to the previous iteration.
2. The weight of the optimum solution is higher than the weight of the previous iteration.

For both convergence criteria, the grouped optimum design of the previous iteration is the final grouped design. For the 117 bar braced frame example, the final grouped design with a weight of 24779 kg is shown in Fig. 13.

4. Numerical experiments

In this study, the new FSCS method is compared to a selection of the grouping methods proposed in literature for eight benchmark problems. The optimization in all methods is performed with a genetic algorithm and the structures are evaluated on basic stress and displacement constraints.

4.1. Genetic algorithm

A genetic algorithm [21] is used to solve the optimization problem. A real-valued encoding is used [22], with a tournament selection for constraint handling [23], Laplace crossover [24] and power mutation [25]. As the optimization problem differs per benchmark problem and method, the population size, elite ratio, number of generations to convergence and number of repetitions of the optimization are varied in the experiments, based on the experience of the authors. As these variations influence the comparison criteria of the grouping methods, the options are chosen such that a fair comparison was possible.

Because of the stochastic behavior of the genetic algorithm, each analysis is unique and the resulting optimum weight might differ when the analysis is repeated, especially for optimization problems with many members or available profiles. To check whether the global optimum is found, the analysis is repeated multiple times, as is the custom. However, repeating the analysis may be impractical for application as the computational effort can become high. Therefore, a measure for judging how well a given method is able to find a low-weight solution in a single run is included in the comparison by defining a certainty-criterium. This certainty is the percentage of how many times the optimization converges to the lowest optimum weight found, divided by the total number of analyses. Even when the certainty of finding the optimum is low, the performance of the algorithm is considered good if near-optimum solutions are still found in many cases. Therefore, for solutions in which this certainty is low, the estimated density function of the final weights is plotted using a kernel estimation. This kernel density estimation is similar to a histogram but smoother, which allows an easier identification of typical converging values. The kernel density function is generated by summing up kernel function centered around final weights [26]. The probability density function is defined as an function of the weight w:

$$f_{n,h}(w) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{w-w_i}{h}\right) \tag{13}$$

where n is the number of feasible searches, h is the bandwidth set to a value of the lowest optimum weight divided by 200. w_i are the converged feasible optimum weights and $K(z)$ is the normal kernel function defined by:

$$K(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2} \tag{14}$$

4.2. Selection of existing grouping methods

A selection of existing grouping methods is made in this paper. Furthermore, the ungrouped optimization problem (NG) is solved as well to show the added weight of grouping.

The method which performs a manual grouping based on rules of thumb (ROT) is the first method of the comparison. The grouping is taken from literature, or defined by the authors by the given rules of thumb [3].

Four different variations of the method which perform grouping based on the axial force distribution of a uniform design (AF) are implemented: the first procedure (AF_1) divides the full range of internal forces into equally spaced intervals, the second procedure (AF_2) divides both the compressive and tensile range of internal forces into equally spaced intervals separately, the third procedure (AF_3) is equal to the second procedure, but a separate group is added for members which are in a 10% axial force range around no axial force, and finally the fourth procedure (AF_4) divides the range of absolute internal forces into equally spaced intervals.

The method which is similar to AF, but groups compressive members based on their slenderness (AF + S), is implemented in two similar procedures: the first procedure ($AF + S_1$) groups both full the axial force and slenderness range in equally spaced intervals, while the second procedure ($AF + S_2$) adds a separate group for the 10% range of axial force around zero axial force.

Furthermore, the method which adapts the encoding of the structure by adding cardinality constraints (CC) is treated.

Finally, the method which groups members with a combinatorial search of the ungrouped design (UCS) is treated. The input ungrouped design is the lightest optimum solution of NG. The number of FEM evaluations of this analysis is included in the number of FEM evaluations of UCS.

4.3. Results

For each grouping method the lowest optimum weight, consistency in results and number of FEM evaluations are evaluated. The number of FEM evaluations represents the mean of all runs and includes the FEM evaluations required by both the grouping method and the optimization method.

Besides the weight and performance criteria, this paper graphically shows the resulting design of FSCS. In these figures, the thickness of members represents their weight per unit length, and their colors the group of the member.

The benchmark problems were selected from studies on structural optimization problems. These studies mostly treated none or only a single grouping method while the possible optimum solution to be found is dependent on the grouping method used. Furthermore, varying constraints were used in these studies. Therefore, the only meaningful comparison that can be presented in this paper is a comparison of the performance of grouping methods, not a comparison of the found optimum solutions with respect to other optimum solutions in literature. It is expected that the conclusions of the comparison are identical when the problems are analyzed with other optimization algorithms, as the characteristics of the search space are unchanged.

4.3.1. 18-bar cantilever truss beam

The 18-bar cantilever truss beam is a statically determinate structure without displacement constraints and with a yield stress of 172 MPa. It is loaded on its top nodes. For each member, 25 profiles are available.

As this structure is a statically determinate structure, the optimum design with a given grouping can be found without optimization; only one FEM evaluations is required and the certainty is 100%. For CC, the grouping is variable, therefore it requires application of an optimization methods with multiple FEM evaluations and variance in results.

The problem was solved for four groups. The grouping of ROT was taken from Salajagheh and Vanderplaats [27]: a group for the bottom members, a group for the top members, the verticals, and the diagonals.

The results, which are shown in Table 2, indicated that FSCS, CC and UCS were able to find the optimum result. ROT, AF and AF + S converged to a structure with a higher weight because of the inability of this method to combine both tension and compression member in one group properly.

The optimum grouped solution, which is shown in Fig. 4, groups members in a way that is clearly different from what is obtained with rule of thumb considerations: compressive, tensile and diagonal members are grouped in a way that is not obvious with any degree of engineering experience. The heavier result using ROT is shown in Fig. 3.

4.3.2. 65-bar truss beam

The 65-bar truss beam is a statically determinate structure with a displacement constraint of 60 mm at midspan and a yield stress limit of 350 MPa. It is loaded downwards on its top nodes with a ULS and SLS load case. For each member, 42 equal leg angle profiles are available.

The problem was solved for four groups. The grouping of ROT was taken from Walls and Elvin [1]: a group for the bottom members, a group for the top members, the verticals, and the diagonals.

It was found that the UCS and CC both converged to a similar grouped solution, as shown Table 3. However, UCS required more FEM evaluations because of the need of an ungrouped optimization. Furthermore, while the certainty of UCS was reasonable, the certainty of NG was low, thereby decreasing the practical applicability of UCS. Similarly, CC had a low certainty of finding this solution. The distribution of these weights is plotted in Fig. 6, showing that the NG results are consistently better than those from CC. FSCS found a slightly higher weight solution, but with much less FEM evaluations and a high certainty. This resulting design is shown in Fig. 5. The solution of ROT, AF and AF + S converged to higher weights.

Table 2
Results 18-bar truss cantilever beam.

Method	Weight (kg)	Certainty (%)	FEM evaluations
NG	1855	100	1
ROT	2836	100	1
AF_1	2704	100	1
AF_2	2433	100	1
AF_3	2605	100	1
AF_4	3113	100	1
$AF + S_1$	2492	100	1
$AF + S_2$	2677	100	1
CC	2201	15	26,250
UCS	2201	100	1
FSCS	2201	100	1

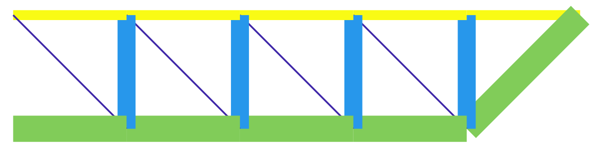

Fig. 3. Result ROT 18-bar cantilever truss.

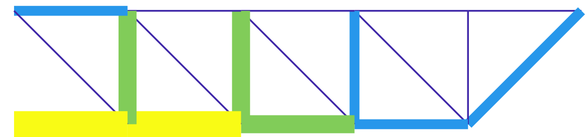

Fig. 4. Result FSCS 18-bar cantilever truss.

Table 3
Results 65-bar truss beam.

Method	Weight (kg)	Certainty (%)	FEM evaluations
NG	1270	0.40	156,024
ROT	1577	80	5021
AF_1	1429	77	6743
AF_2	1445	100	8017
AF_3	1568	40	9025
AF_4	1491	100	9718
$AF + S_1$	1658	100	8995
$AF + S_2$	1621	100	8281
CC	1375	0.52	96,560
UCS	1374	45	159,015
FSCS	1395	80	9803

4.3.3. 72-bar truss tower

The 72-bar truss tower is a statically indeterminate structure with displacement constraints of 6.35 mm at the top nodes in all directions and a yield stress of 172 MPa [28]. It is loaded in two load cases, one vertically downwards, and another one diagonally on the tower. For each member, 25 profiles are available. The final solution is required to be symmetrical.

In this study, the problem was solved for 4 groups. For the ROT grouping the following members are grouped: all verticals, all diagonals in the outer planes, all diagonals in the floor planes and all horizontals.

The experiments, of which the result are shown in Table 4, found that CC and FSCS converged to the same solution, while FSCS did that with the lowest number of FEM evaluations.

The resulting design is shown in Fig. 7. UCS found a slightly higher-weight solution. ROT, AF and AF + S converged to a structure with a high weight, although $AF + S_2$ did fairly well.

4.3.4. 112-bar truss dome

The 112-bar truss dome is a statically determinate structure with displacement constraints and a yield stress of 150 MPa [29]. The top node and four free-spanning nodes should not displace more than 20

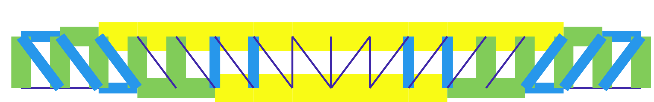

Fig. 5. Result FSCS 65-bar truss beam.

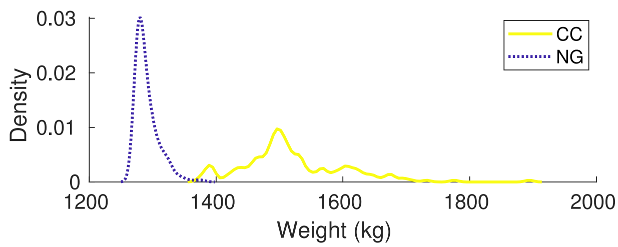

Fig. 6. Distribution optimum weight 65-bar truss beam CC and NG method.

Table 4
Results 72-bar truss tower.

Method	Weight (kg)	Certainty (%)	FEM evaluations
NG	269	38	112,053
ROT	401	100	3414
AF_1	381	100	5330
AF_2	401	100	5938
AF_3	381	100	4912
AF_4	413	100	5824
$AF + S_1$	371	100	5596
$AF + S_2$	315	100	5216
CC	286	100	156,322
UCS	301	100	117,496
FSCS	286	100	3391

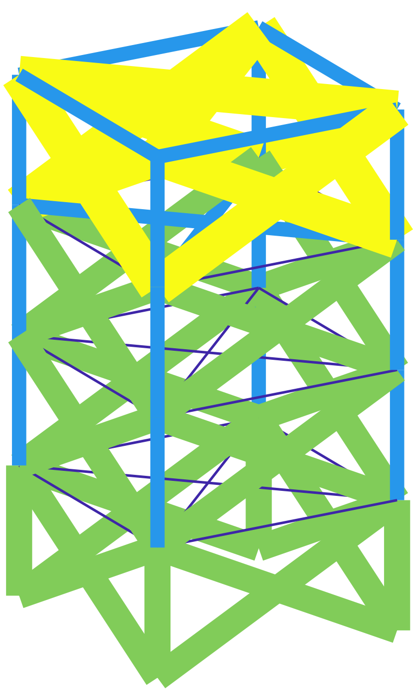

Fig. 7. Result FSCS and CC 72-bar truss tower.

mm. The structure is loaded downwards. For each member, 43 pipe sections are available.

The problem was solved for 3 groups in this study. For the ROT grouping the following members are grouped: all horizontal beams, all diagonal bracings, all members which run directly from a support to the middle node. Furthermore, a point-symmetrical grouping was enforced in all methods, limiting the number of unique members to 16.

CC found the lowest-weight solution as is shown in Table 5. However, it required many FEM evaluations with a low certainty, as shown with the broad distribution of optimum weights in Fig. 9. FSCS found a slightly heavier optimum, but with higher certainty and less FEM evaluations. This solution is shown in Fig. 8. UCS performed third best in terms of weight, but the certainty of the ungrouped solution was low and the number of total FEM evaluations high. ROT, AF and AF + S converged to a structure with a higher weight. This time, $AF + S_2$ is among the worst performing methods for finding a low weight optimum.

Table 5
Results 112-bar truss dome.

Method	Weight (kg)	Certainty (%)	FEM evaluations
NG	2163	0.50	62,266
ROT	3595	50	1692
AF_1	2587	50	2887
AF_2	2713	75	3036
AF_3	3592	100	3064
AF_4	2580	100	2183
$AF + S_1$	2673	80	3030
$AF + S_2$	3536	100	3264
CC	2310	1.0	45,974
UCS	2501	78	63,996
FSCS	2457	60	4206

4.3.5. 160-bar truss tower

The 160-bar truss dome is a statically indeterminate structure with displacement constraints and a yield stress of 147.15 MPa [30]. Both the top node and the nodes which connect to the electricity cables are allowed to displace no more than 80 mm. The structure is loaded in eight load cases, considering self-weight, wind, snapping of different cables and end tower conditions. For each member, 42 equal leg angles are available.

The problem was solved for 6 groups. The grouping of ROT was defined as follows: all vertical beams up to the 6th floor, all verticals and horizontal beams from the 7th to the 9th floor, all vertical and horizontal beams from the 10th to the 12th floor, all bracings up to the 6th floor, all verticals and horizontal bracings from the 7th to 12th floor, and all beams of the outriggers. Furthermore, a symmetric grouping was enforced for each floor level in all methods.

CC and FSCS found similar low-weight solution, close to the NG optimum, as shown in Table 6. However, FSCS required much less FEM evaluations and had a higher certainty. Furthermore, the range of solutions of FSCS was much smaller than the range of solution of CC, as shown in Fig. 11. The FSCS-design is shown in Fig. 10. UCS found a slightly higher weight but required many FEM evaluations, and again ROT, AF and AF + S converged to a structure with a higher weight.

4.3.6. 15-bar unbraced frame

The 15-bar unbraced frame structure has a yield stress of 248 MPa and is loaded both vertically and horizontally [31]. In plane, the effective length factor follows from Eq. (5). Out-of-plane, the effective length

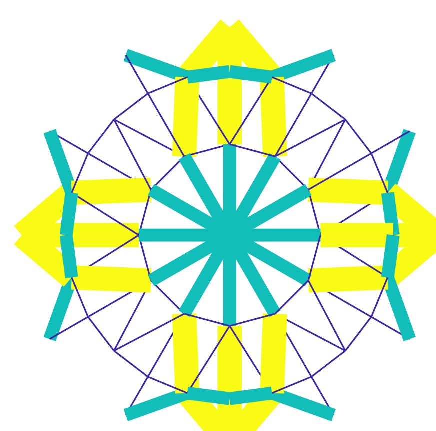

Fig. 8. Top-view result FSCS 112-bar truss dome.

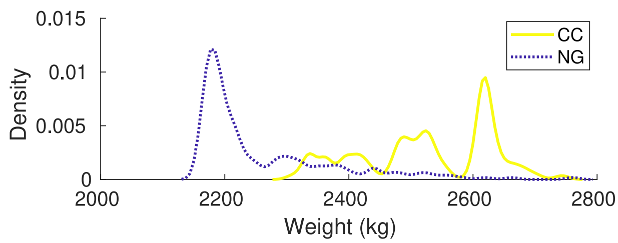

Fig. 9. Distribution optimum weight 112-bar truss dome CC and NG method.

Table 6
Results 160-bar truss tower.

Method	Weight (kg)	Certainty (%)	FEM evaluations
NG	871	0.58	1,767,347
ROT	1015	86	37,411
AF_1	982	43	56,359
AF_2	982	55	49,218
AF_3	943	83	47,469
AF_4	988	100	38,016
AF + S_1	1164	60	26,432
AF + S_2	975	67	33,069
CC	881	0.37	1,253,812
UCS	893	50	1,813,070
FSCS	883	3.4	367,705

factor for the beams and columns is 1/6 and 1, respectively. For each beam, 283 W-sections can be chosen, and for each column the choice is limited to 18 W10 sections.

The problem was solved for 2 groups for columns and 1 for beams. The grouping of ROT had a group for the outer columns and a group for the inner columns.

The results, shown in Table 7, found that the CC performs best with low computational effort. FSCS found a heavier solution, but with more certainty and slightly less FEM evaluations, of which the design is shown in Fig. 12. On the other hand, UCS converged to a suboptimal solution and required more computational effort. ROT found the worst solution, although the difference in weight the other grouped solutions remains small for this case.

4.3.7. 117-bar braced frame

The 117-bar braced frame structure is an adaption of the structure proposed by Walls and Elvin [1]. It has a yield stress of 350 MPa and it is loaded both vertically and horizontal and a drift of 9 mm is allowed between two floors. In-plane, the effective length factor follows from Eq. (5) for the beams and columns, and it equals 1/6 for the bracings. Out-of-plane, the effective length factor for the bracings and beams is 1/6 and for the columns it is 1. For each beam 20 UB sections are available, 15 UC sections for the columns, and 10 equal leg angles for the bracings.

The problem was solved for 1 group for the beams, 3 groups for the columns, and 3 groups for the bracings. The grouping of ROT was based on the original manual grouping by Walls and Elvin [1]: bracings and columns in three consecutive stories.

FSCS found the best solution, with few FEM evaluations and 100% certainty, as shown in Table 8. The corresponding design is shown in Fig. 13. CC converged to a higher weight, with more FEM evaluations and a low certainty, as shown in Fig. 14. UCS found the second-best solution but required many FEM evaluations. Again, ROT resulted in the worst solution.

4.3.8. 147-bar frame

The 147-bar frame structure is a 3D-frame structure [32]. It has a yield stress of 248.2 MPa and it is braced in one direction and unbraced in the other direction. The beams are orientated with their strong axis in the unbraced direction. In this direction, the effective length factor follows from Eq. (5) for the columns. The effective length factor of the

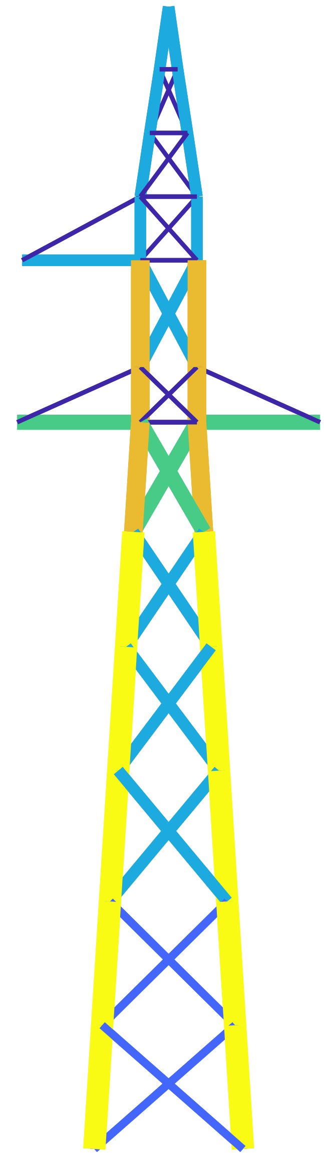

Fig. 10. Side-view result FSCS 160-bar truss tower.

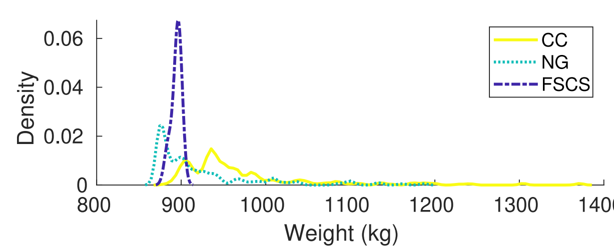

Fig. 11. Distribution optimum weight 160-bar truss tower CC, NG and FSCS method.

Table 7
Results 15-bar unbraced frame.

Method	Weight (kg)	Certainty (%)	FEM evaluations
NG	7699	14	28,948
ROT	8124	100	3122
CC	8014	21	12,843
UCS	8093	57	32,230
FSCS	8081	100	5998

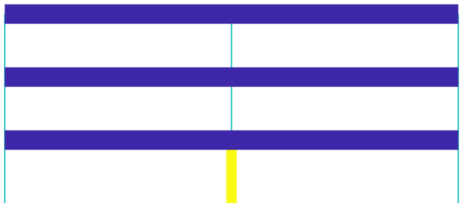

Fig. 12. Result FSCS 15-bar unbraced frame.

Table 8
Results 117-bar frame.

Method	Weight (kg)	Certainty (%)	FEM evaluations
NG	22,657	0.98	182,181
ROT	26,190	100	7251
CC	25,004	0.72	114,166
UCS	24,793	100	189,099
FSCS	24,779	100	21,732

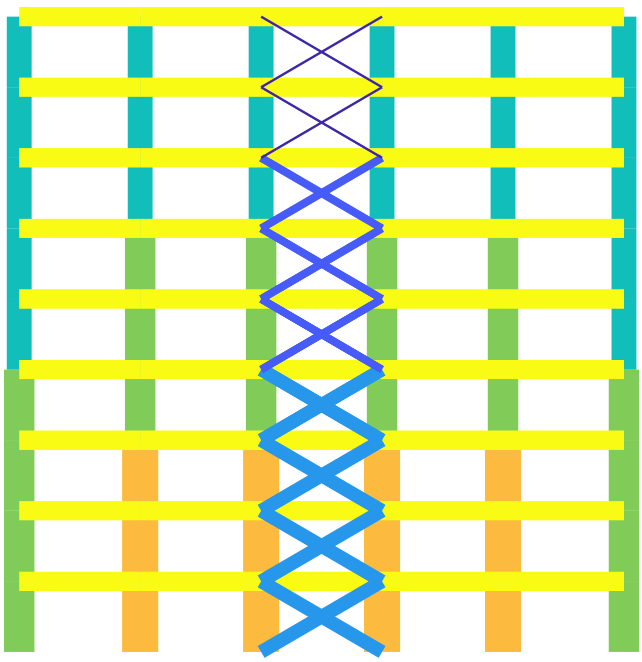

Fig. 13. Result FSCS 117-bar frame structure.

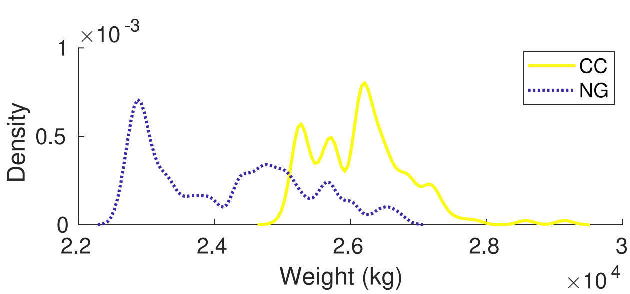

Fig. 14. Distribution optimum weight 117-bar frame CC and NG method.

bracings and beams is 1. In the braced direction, all effective length factors are 1. Furthermore, it is loaded both vertically and horizontally in both directions. The maximum displacement is 30 mm at the top floor and the drift between two floors is limited to 1/400 times the floor height. 25 W-profiles are available for each of the members.

The problem was solved for 6 groups. The grouping of ROT was chosen as follows: the corner and side columns in the first floor, the corner and side columns in the upper two floors, inner columns in the first floor, inner column in the upper two floor, all beams, all bracings.

CC converged to the best solution, as is shown in Table 9. However, the certainty of reaching this solution was low, and many FEM evaluation were required. FSCS required less FEM evaluations and found a slightly heavier solution, of which the resulting design is shown in Fig. 15. Furthermore, although its certainty was low, the bandwidth of its solutions was much smaller than the one of CC, as shown in Fig. 16. UCS found a solution which required many FEM evaluations. As in the other frame problems, ROT resulted in the worst solution.

4.4. Varying number of groups

In the previous analyses, the number of groups was fixed. Solving the problem for multiple number of groups allows an engineer to make a well-argued trade-off between costs and number of groups; for any number of groups the added weight compared to the ungrouped solution can be found. When applied to cost optimization problems, the global cost optimum can be found by evaluating the coast of each of the optimum solutions with different number of groups.

To illustrate how grouped optimization methods can be used to find a cost optimum by sweeping through a range of numbers of groups, the 18-bar cantilever truss problem is revisited with different numbers of groups. Fig. 17 shows the optimum weight found with FSCS for the 18-bar cantilever truss from 1 to 12 groups. 12 groups is the number of profiles in the ungrouped optimization problem, any forced higher number of groups is suboptimal in weight. It is observed that the weight decreases drastically when introducing the first groups, after which further weight reduction from adding more groups is limited. The corresponding designs for 2, 4, 6 and 9 groups are shown as well.

With a suitable cost function, it is conceptually possible to formulate a global cost optimization problem where the number of groups is optimized for the same time as the grouping. However, the search space for the resulting problem would be intractably large. As application of grouping methods reduces the search space drastically, the subdivision of the optimization problem allows to find the global optimum with a lower computation effort and with high certainty.

5. Discussion

The new grouping method uses a fully stressed design and combinatorial search in its grouping process. For big problems with many members or a big profile database, both analyses can result in many computations, reducing the efficiency of this method and leading to a poor scalability: the number of FEM calculations per fully stressed design scales by Eq. (11), the number of weight calculations in the combinatorial search increases given by Eq. (12) and the size of the optimization problem scales by an growing m in m^k. Nonetheless, single-variable optimization methods can be applied for each beam in the fully stressed design. Similarly, optimization and simplification methods are proposed in literature to prevent evaluation of all combinations of the combinatorial search [1,11], while this evaluation is already a cheap computational evaluation only involving a summation and multiplication of the weight per unit length of members with their lengths. The

Table 9
Results 147-bar frame.

Method	Weight (kg)	Certainty (%)	FEM evaluations
NG	24,464	0.96	203,038
ROT	30,380	67	4076
CC	25,656	1	122,616
UCS	26,976	100	207,193
FSCS	26,020	1.3	38,911

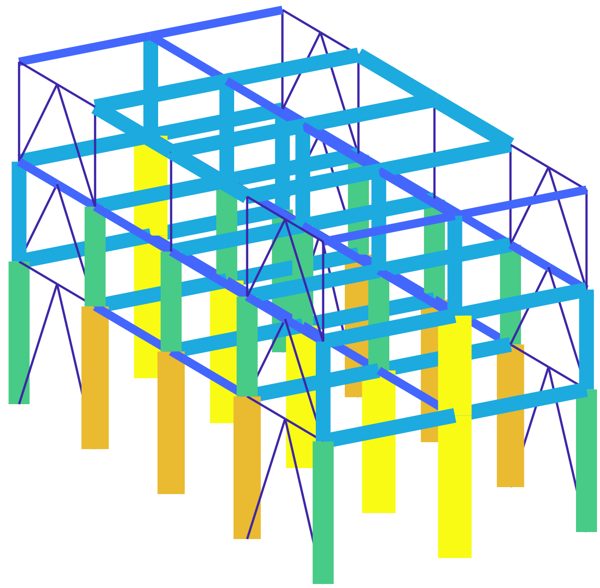

Fig. 15. Result FSCS 147-bar frame.

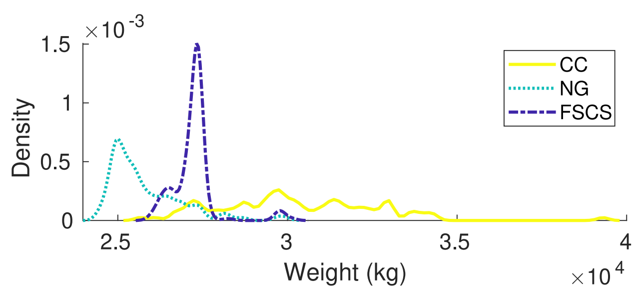

Fig. 16. Distribution optimum weight 147-bar truss dome CC, NG and FSCS method.

increase in size of the optimization problem is seen with other grouping methods as well, reducing efficiency by increasing the number of finite element evaluations. For the cases discussed in this paper the number of finite element evaluations of the optimization algorithm exceeds that of the fully stressed design and combinatorial search. Furthermore, the resulting search space of the fully stressed combinatorial search is always smaller than that of the grouping methods which are applied during optimization, which led to a lower number of finite element calculations and higher certainty with the proposed method for the cases investigated in this study. Finally, for the special case of statically determinate structures without displacement constraints, the fully stressed combinatorial search does not require a computationally heavy optimization algorithm, ensuring the global optimum solution with high efficiency.

Whereas the fully stressed combinatorial search takes the heaviest possible profiles as an initial reference design, adopting a different uniform initial reference design gave a slightly lighter optimum design in a preliminary phase of this study. This difference was found to be marginal, but a non-uniform design might result in an even lighter optimum design. On the other hand, this may require engineering judgement which limits the applicability of the method.

For the manual grouping method, only one grouping possibility per benchmark problem was used. These groupings were taken from literature, or the authors came up with those using their own engineering experience. Another engineer might choose to group members differently. However, given the high number of possible solutions, it is expected that the non-manual grouping methods outperform most engineers by identifying non-intuitive but favorable groupings.

The ability to find the optimum value and the corresponding required number of finite element evaluations are strongly influenced by the choice for the optimization method and its settings; another optimization method or change in options changes the results and the performance. However, the variety of optimization methods and variations is endless, and each problem and grouping method requires varying the options. This makes it impossible to compare the grouping methods independent of the options for the optimization method. Still, it is expected that the relative performance of the different grouping methods does not change much when using a different optimization method, because the size of the search space shows a similar trend as the number of required finite element evaluations.

For the comparison of grouping methods, only problems with a fixed geometry were analyzed in size optimization problems. Nonetheless, the grouping methods are also applicable to shape optimization and topological optimization problems, but these optimization problems introduce a much bigger search space and have an additional requirement of repetitive grouping. The search space is enhanced by addition of the coordinates of the nodes, the number of nodes, and the connectivity of the nodes to the design variables. As this leads to a new optimization problem, the efficiency of the grouping methods on these problems cannot be prospected. Furthermore, the grouping methods should at least alter and evaluate the grouping multiple times, as a changing geometry has significant influence on the force distribution and optimal

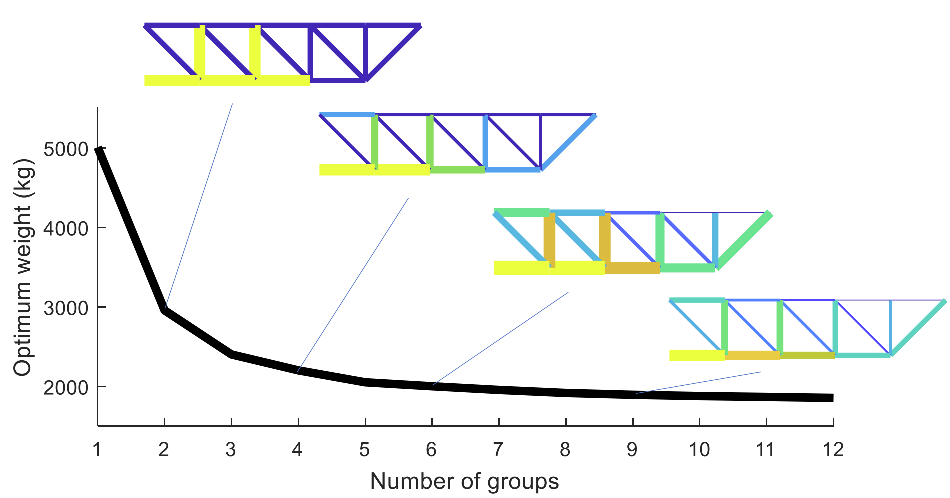

Fig. 17. Results FSCS 18-bar cantilever truss for 1 to 12 distinct profile groups with corresponding designs for 2, 4, 6 and 9 groups.

grouping. This requirement is met by only a few methods. The fully stressed combinatorial search does so by a few iterations of both fully stressed design, combinatorial search and reduced optimization. The grouping methods which change the optimization process also obey the requirement as these methods have a varying grouping during a single optimization run.

The proposed fully stressed combinatorial search method could be applied to a continuous sizing problem by using continuous dimensions instead of discrete dimensions from a profile database. In the proposed method, finding the fully stressed design does not necessarily need continuous dimensions as an input: whether the fully stressed design came from continuous or from discrete dimensions, its results are gathered in groups in the combinatorial search, after which the dimensions are optimized per group to continuous values.

For application to multi-objective optimization it should be noted that the fully stressed design and combinatorial search require a desirable order of profiles. For weight optimization, and to some extent for cost optimization, the order is light to heavy, but the objectives of a multi-objective optimization might require a different or contradicting order of profiles. Other methods which apply grouping during the optimization process are not depending on such an order.

6. Conclusion

In this paper, a new method, the fully stressed combinatorial search, is proposed and tested for grouping in steel skeletal structures. In comparison with other existing methods, it is found that the new method consistently finds low weight solutions with limited computational effort.

Manual grouping is the simplest method to apply, but its performance is strongly dependent on the experience of the engineer. For truss structures, the methods which group members based on axial force perform better but cannot find the global optimum due to the inability of combining compressive and tensile members. Lighter designs are found for several cases with the cardinality constraints method and for two cases also with the ungrouped combinatorial search, although these methods require many finite element evaluations. Finally, the new method converges to designs with similar low weights, while requiring much less finite element evaluations.

Grouped optimization methods can be used for global cost optimization by embedding the procedure in a loop varying the number of groups. In such scenario, where the grouped optimization needs to be performed multiple times to identify a global cost optimum, the proposed method is particularly beneficial in its efficiency and consistency upon repetition.

7. Datasets

The input data and datasets generated during the current study are available in the 4TU.ReserachData repository: https://doi.org/10.4121/12718790.v3.

CRediT authorship contribution statement

Tomas R. van Woudenberg: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Visualization. **Frans P. van der Meer:** Conceptualization, Methodology, Validation, Writing – review & editing, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The presented work is carried out in the context of a Master's thesis of T.R. van Woudenberg at the Delft University of Technology, in cooperation with Royal HaskoningDHV. The graduation committee is thanked for their support: ir. R. Crielaard, dr.ir. J.L. Coenders, ir. S.B. Cox, and ir. J.M. Houben.

References

[1] Walls R, Elvin A. An algorithm for grouping members in a structure. Eng Struct 2010;32(6):1760–8. https://doi.org/10.1016/j.engstruct.2010.02.027.
[2] Provatidis CG, Venetsanos DT. Cost minimization of 2D continuum structures under stress constraints by increasing commonality in their skeletal equivalentsKostenminimierung spannungsbelasteter zweidimensionaler Strukturen. Forsch Im Ingenieurwesen/Engineering Res 2006;70(3):159–69. https://doi.org/10.1007/s10010-006-0026-4.
[3] Templeman AB. Discrete optimum structural design. Comput Struct 1988;30(3): 511–8. https://doi.org/10.1016/0045-7949(88)90284-2.
[4] Barbosa HJC, Lemonge ACC, Borges CCH. A genetic algorithm encoding for cardinality constraints and automatic variable linking in structural optimization. Eng Struct 2008;30(12):3708–23. https://doi.org/10.1016/j.engstruct.2008.06.014.
[5] Biedermann JD, Grierson DE. Training and using neural networks to represent heuristic design knowledge. Adv Eng Softw 1996;27(1-2):117–28. https://doi.org/10.1016/0965-9978(96)00017-8.
[6] Biedermann JD, Grierson DE. A generic model for building design. Eng Comput 1995;11(3):173–84. https://doi.org/10.1007/BF01271283.
[7] Krishnamoorthy CS, Prasanna Venkatesh P, Sudarshan R. Object-oriented framework for genetic algorithms with application to space truss optimization. J Comput Civ Eng 2002;16(1):66–75. https://doi.org/10.1061/(ASCE)0887-3801(2002)16:1(66).
[8] Toğan V, Daloğlu AT. Optimization of 3d trusses with adaptive approach in genetic algorithms. Eng Struct 2006;28(7):1019–27. https://doi.org/10.1016/j.engstruct.2005.11.007.
[9] Toğan V, Daloğlu AT. An improved genetic algorithm with initial population strategy and self-adaptive member grouping. Comput Struct 2008;86(11-12): 1204–18. https://doi.org/10.1016/j.compstruc.2007.11.006.
[10] Mashayekhi M, Salajegheh E, Dehghani M. Topology optimization of double and triple layer grid structures using a modified gravitational harmony search algorithm with efficient member grouping strategy. Comput Struct 2016;172: 40–58. https://doi.org/10.1016/j.compstruc.2016.05.008.
[11] Reitman MI, Brent HW. Optimal structural optimisation. Eng Optim 1990;16: 109–28. https://doi.org/10.1080/03052159008941167.
[12] Kanno Y. Global optimization of trusses with constraints on number of different cross-sections: a mixed-integer second-order cone programming approach. Comput Optim Appl 2016;63(1):203–36. https://doi.org/10.1007/s10589-015-9766-0.
[13] Galante M, Oñate E. Genetic algorithms as an approach to optimize real-world trusses. Int J Numer Methods Eng 1996. https://doi.org/10.1002/(SICI)1097-0207(19960215)39:3<361::AID-NME854>3.0.CO;2-1.
[14] Shea K, Cagan J, Fenves SJ. A shape annealing approach to optimal truss design with dynamic grouping of members. J Mech Des 1997;119:388–94. https://doi.org/10.1115/1.2826360.
[15] Adeli H, Sarma KC. Cost optimization of structures. Chichester, UK: John Wiley & Sons, Ltd; 2006. DOI: 10.1002/0470867353.
[16] American Institute of Steel Construction. Specification for Structural Steel Buildings; 2016.
[17] Dumonteil P. Simple equations for effective length factors. Eng J 1992;29:111–5.
[18] Knuth DE. The art of computer programming: v.4A: Combinatorial algorithms, Pt. 1. Choice Rev Online; 2011. DOI: 10.5860/choice.48-6329.
[19] Corless RM, Gonnet GH, Hare DEG, Jeffrey DJ, Knuth DE. On the LambertW function. Adv Comput Math 1996;5(1):329–59. https://doi.org/10.1007/BF02124750.
[20] Reitman MI. Optimal structural design in the USSR. Appl Mech Rev; 1989. DOI: 10.1115/1.3152419.
[21] Holland JH. Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence; 1975.
[22] Deep K, Singh KP, Kansal ML, Mohan C. A real coded genetic algorithm for solving integer and mixed integer optimization problems. Appl Math Comput 2009;212(2): 505–18. https://doi.org/10.1016/j.amc.2009.02.044.
[23] Deb K. An efficient constraint handling method for genetic algorithms. Comput Methods Appl Mech Eng 2000;186(2-4):311–38. https://doi.org/10.1016/S0045-7825(99)00389-8.
[24] Deep K, Thakur M. A new crossover operator for real coded genetic algorithms. Appl Math Comput 2007;188(1):895–911. https://doi.org/10.1016/j.amc.2006.10.047.
[25] Deep K, Thakur M. A new mutation operator for real coded genetic algorithms. Appl Math Comput 2007;193(1):211–30. https://doi.org/10.1016/j.amc.2007.03.046.
[26] Peter D. H. Kernel estimation of a distribution function. Commun Stat - Theory Methods 1985;14(3):605–20. https://doi.org/10.1080/03610928508828937.

[27] Salajegheh E, Vanderplaats GN. Optimum design of trusses with discrete sizing and shape variables. Struct Optim 1993;6(2):79–85. https://doi.org/10.1007/BF01743339.
[28] Venkaya VB, Khot NS, Reddy VS. Energy distribution in an optimum structural design. Springfield: CFSTI; 1969.
[29] Saka MP. Optimum design of pin-jointed steel structures with practical applications. J Struct Eng (United States) 1990;116(10):2599–620. https://doi.org/10.1061/(ASCE)0733-9445(1990)116:10(2599).
[30] Groenwold AA, Stander N. Optimal discrete sizing of truss structures subject to buckling constraints. Struct Optim 1997;14(2-3):71–80. https://doi.org/10.1007/BF01812508.
[31] Afzali SH, Darabi A, Niazkar M. Steel frame optimal design using MHBMO algorithm. Int J Steel Struct 2016;16(2):455–65. https://doi.org/10.1007/s13296-016-6016-y.
[32] Kazemzadeh Azad S, Hasançebi O. Computationally efficient discrete sizing of steel frames via guided stochastic search heuristic. Comput Struct 2015;156:12–28. https://doi.org/10.1016/j.compstruc.2015.04.009.