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Rostampour, Vahab; Keviczky, Tamás

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Distributed Stochastic Model Predictive Control Synthesis for Large-Scale Uncertain Linear Systems

Vahab Rostampour, and Tamás Keviczky

Abstract— **This paper presents an approach to distributed stochastic model predictive control (SMPC) of large-scale uncertain linear systems with additive disturbances. Typical SMPC approaches for such problems involve formulating a large-scale finite-horizon chance-constrained optimization problem at each sampling time, which is in general non-convex and difficult to solve. Using an approximation, the so-called scenario approach, we formulate a large-scale scenario program and provide a theoretical guarantee to quantify the robustness of the obtained solution. However, such a reformulation leads to a computational tractability issue, due to the large number of required scenarios. To this end, we present two novel ideas in this paper to address this issue. We first provide a technique to decompose the large-scale scenario program into distributed scenario programs that exchange a certain number of scenarios with each other in order to compute local decisions. We show the exactness of the decomposition with a-priori probabilistic guarantees for the desired level of constraint fulfillment. As our second contribution, we develop an inter-agent soft communication scheme based on a set parametrization technique together with the notion of probabilistically reliable sets to reduce the required communication between each subproblem. We show how to incorporate the probabilistic reliability notion into existing results and provide new guarantees for the desired level of constraint violations. A simulation study is presented to illustrate the advantages of our proposed framework.**

I. INTRODUCTION

Distributed model predictive control (MPC) has been an active research area in the past decades, due to its applicability in different domains such as power networks [1], chemical process control [2], and building automation [3]. For such large-scale dynamic systems with state and input constraints, distributed MPC is an attractive control scheme. In distributed MPC one replaces large-scale optimization problems stemming from centralized MPC with several smaller-scale problems that can be solved in parallel. These problems make use of pieces of information from other subsystems to design a distributed MPC. In the presence of uncertainties, however, the main challenge in the formulation of distributed MPC is how the controllers should exchange pieces of information through a communication scheme among subsystems (see, e.g. [4], and references therein). This highlights the necessity of developing distributed control strategies to cope with the uncertainties in subsystems while at the same time minimizing information exchange through a communication framework.

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The authors are with Delft Center for Systems and Control, Delft University of Technology, Mekelweg 2, 2628 CD, Delft, The Netherlands. {v.rostampour, t.keviczky}@tudelft.nl

To handle uncertainties in distributed MPC, some approaches are based on robust MPC [5]. Assuming that the uncertainty is bounded, a robust optimization problem is solved at each sampling time, leading to a control law that satisfies the constraints for all admissible values of the uncertainty. The resulting solution using such an approach tends to be very conservative in many cases. Tube-based MPC, see for example [6] and the references therein, was considered in a plug-and-play decentralized setup in [7], and it has been recently extended to distributed control systems [8] for a collection of linear stochastic subsystems with independent dynamics. While in [8] coupled chance constraints were considered such that the uncertainties are independent at each time step, in this paper we consider a chance constraint on the feasibility of trajectories of dynamically coupled subsystems with uncertainties that might be correlated through time steps. Our approach is motivated by [7] to reduce the conservativeness of the control design. Other representative approaches for stochastic MPC (SMPC) of a single stochastic system include affine parametrization of the control policy [9], the randomized (scenario) approach [10], and the combined randomized and robust approach [11] based on [12]. None of these approaches, to the best of our knowledge, have been considered in distributed control strategies.

This paper aims to propose a solution for distributed SMPC by promoting the scenario MPC technique to the distributed case in a more systematical approach. Scenario MPC approximates SMPC via the so-called scenario (sample) approach [13], [14], and if the underlying optimization problem is convex with respect to the decision variables, finite sample guarantees can be provided. Following such an approach, the computation time for a realistic large-scale system of interest becomes prohibitive, due to the fact that the number of samples to be extracted tends to be very high, and consequently leads to a large number of constraints in the resulting optimization problem. To overcome the computational burden caused by the large number of constraints, in [15], [16] a heuristic sample-based approach was used in an iterative distributed fashion via dual decomposition such that all subsystems collaboratively optimize a global performance index. In another interesting work [17], a multi-agent consensus algorithm was presented to achieve consensus on a common value of the decision vector subject to random constraints such that a probabilistic bound on the tails of the consensus violation was also established. However, in most of the aforementioned references the aim to reduce communication among subsystems, which we refer to as agents, has not been addressed.

Our work in this paper differs from the aforesaid references in two important aspects which have not been, to the best of our knowledge, considered in literature. A decomposition technique based on the large-scale system dynamics is employed to distribute the resulting centralized scenario optimization problem at each sampling time and a novel communication scheme is introduced to reduce communication between the small-scale problems. The main contributions of this paper are twofold: 1) We provide a technique to decompose the large-scale scenario program into distributed scenario programs that exchange a certain number of scenarios with each other in order to compute local decisions. We show the exactness of the decomposition with a-priori probabilistic guarantees for the desired level of constraint fulfillment under some mild conditions. 2) We develop an inter-agent soft communication scheme based on a set parametrization technique together with the notion of probabilistically reliable set to reduce the required communication between each subproblem. We show how to incorporate the probabilistic reliability notion into existing results and provide new guarantees for the desired level of constraint violations.

Notations

$\mathbb{R}, \mathbb{R}_+$ denote the real and positive real numbers, and $\mathbb{N}, \mathbb{N}_+$ the natural and positive natural numbers, respectively. We operate within the n-dimensional space $\mathbb{R}^n$ composed of column vectors $u, v \in \mathbb{R}^n$. The Cartesian product over n sets $\mathcal{X}_1, \cdots, \mathcal{X}_n$ is given by: $\prod_{i=1}^n \mathcal{X}_i = \mathcal{X}_1 \times \cdots \times \mathcal{X}_n = \{(x_1, \cdots, x_n) : x_i \in \mathcal{X}_i\}$. The cardinality of a set $\mathcal{A}$ is denoted by $|\mathcal{A}| = A$. We denote a block-diagonal matrix with blocks X_i, $i \in \{1, \cdots, n\}$, by $\text{diag}_{i\in\{1,\cdots,n\}}(X_i)$, and a vector consisting of stacked sub vectors x_i, $i \in \{1, \cdots, n\}$, by $\text{col}_{i\in\{1,\cdots,n\}}(x_i)$. Given a metric space Δ, its Borel σ-algebra is denoted by $\mathfrak{B}(\Delta)$. Throughout the paper, measurability always refers to Borel measurability. In a probability space $(\Delta, \mathfrak{B}(\Delta), \mathbb{P})$, we denote the N-Cartesian product set of Δ by Δ^N and the respective product measure by $\mathbb{P}^N$.

II. Problem Statement

Consider a discrete-time uncertain linear system with additive disturbance in a compact form as follows:

$$x_{k+1} = A(\delta_k)x_k + B(\delta_k)u_k + C(\delta_k)w_k \ , \tag{1}$$

with a fixed initial condition $x_0 \in \mathbb{R}^m$. Here $k \in \mathcal{T} := \{0, 1, \cdots, T-1\}$ denotes the time instance, $x_k \in \mathcal{X} \subset \mathbb{R}^m$ and $u_k \in \mathcal{U} \subset \mathbb{R}^p$ correspond to the state and control input, respectively, and $w_k \in \mathbb{R}^n$ represents an additive disturbance. The system matrices $A(\delta_k) \in \mathbb{R}^{m\times m}$ and $B(\delta_k) \in \mathbb{R}^{m\times p}$ as well as $C(\delta_k) \in \mathbb{R}^{m\times n}$ are random, since they are known functions of an uncertain variable δ_k that influences the system parameters at each time step k. $\boldsymbol{w} := \{w_k\}_{k\in\mathcal{T}}$ and $\boldsymbol{\delta} := \{\delta_k\}_{k\in\mathcal{T}}$ are defined on some probability spaces $(\mathcal{W}, \mathfrak{B}(\mathcal{W}), \mathbb{P}_{\boldsymbol{w}})$ and $(\Delta, \mathfrak{B}(\Delta), \mathbb{P}_{\boldsymbol{\delta}})$, respectively. $\boldsymbol{w}$ and $\boldsymbol{\delta}$ are two independent random processes. The support sets $\mathcal{W}$ and Δ of $\boldsymbol{w}$ and $\boldsymbol{\delta}$, respectively, together with their probability measures $\mathbb{P}_{\boldsymbol{w}}$ and $\mathbb{P}_{\boldsymbol{\delta}}$ are entirely generic. In fact, $\mathcal{W}$, Δ and $\mathbb{P}_{\boldsymbol{w}}$, $\mathbb{P}_{\boldsymbol{\delta}}$ do not need to be known explicitly. Instead, the only requirement is availability of a "sufficient number" of samples, which will become concrete in later parts of the paper. Such samples can be for instance obtained by a learned model from available historical data [18].

Consider the state and control input constraint sets to be compact convex in the following form

$$\mathcal{X} := \{\, x \in \mathbb{R}^m : \ G\,x \le g \,\}\,,\ \mathcal{U} := \{\, u \in \mathbb{R}^p : \ H\,u \le h \,\}\,,$$

where $G \in \mathbb{R}^{q\times m}$, $g \in \mathbb{R}^q$, and $H \in \mathbb{R}^{r\times p}$, $h \in \mathbb{R}^r$. In order to find a stabilizing full-information controller that leads to admissible control inputs $\boldsymbol{u} := \{u_k\}_{k\in\mathcal{T}}$ and satisfies the state constraints, we follow the traditional MPC approach. The design relies on the standard assumption of the existence of a suitable pre-stabilizing control law, e.g., [7, Proposition 1]. To cope with the state prediction under uncertainty and disturbance, we employ a parametrized feedback policy [9] for (1) and split the control input, $u_k = Kx_k + v_k$, with $v_k \in \mathbb{R}^p$ as a free correction input variable to compensate for disturbances.

The control objective is to minimize a cumulative quadratic stage cost of a finite horizon cost $J(\cdot) : \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}$ that is defined as follows:

$$J(\boldsymbol{x}, \boldsymbol{u}) = \mathbb{E}\left[\sum_{k=0}^{T-1} \left(x_k^\top Q x_k + u_k^\top R u_k\right) + x_T^\top P x_T\right] , \tag{2}$$

$Q \in \mathbb{R}^{m\times m}_{\succeq 0}$, and $R \in \mathbb{R}^{p\times p}_{\succ 0}$. Consider $\boldsymbol{x} := \{x_k\}_{k\in\mathcal{T}}$, $(A, Q^{\frac{1}{2}})$ to be detectable and P to be the solution of the discrete-time Lyapunov equation:

$$\mathbb{E}[\, A_{cl}(\delta_k)^\top P A_{cl}(\delta_k) \,] + Q + K^\top R K - P \preceq 0 \,, \tag{3}$$

for the closed-loop system, $A_{cl}(\delta_k) = A(\delta_k) + B(\delta_k)K$. Each stage cost term is taken in expectation $\mathbb{E}[\cdot]$, since the argument x_k is a random variable. Note that two major difficulties arising in stochastic and distributed MPC, namely recursive feasibility [19] and stability, are not in the scope of this paper, and they are subject of our ongoing research work. Using $\boldsymbol{v} = \{v_k\}_{k\in\mathcal{T}}$, consider now the following stochastic control problem:

$$\min_{\boldsymbol{v}\in\mathbb{R}^{Tp}} \quad J(\boldsymbol{x}, \boldsymbol{u}) \tag{4a}$$

$$\text{s.t.} \quad x_{k+1} = A(\delta_k)x_k + B(\delta_k)u_k + C(\delta_k)w_k \,, \tag{4b}$$

$$\mathbb{P}[\, x_{k+\ell} \in \mathcal{X} \,,\ \ell \in \mathbb{N}_+ \,] \ge 1 - \varepsilon \,, \tag{4c}$$

$$u_k = Kx_k + v_k \in \mathcal{U} \ , \quad \forall k \in \mathcal{T} \,, \tag{4d}$$

where x_0 is initialized based on the measured current state, and $\varepsilon \in (0, 1)$ is the admissible state constraint violation parameter of the large-scale system (1). Even though $\mathcal{U}$ and $\mathcal{X}$ are compact convex sets, due to the chance constraint on the state trajectory, the feasible set of the optimization problem in (4) is a non-convex set, in general.

To handle the chance constraint (4c), we recall a scenario-based approximation [20]. w_k and δ_k at each sampling time $k \in \mathcal{T}$ are not necessarily independent and identically distributed (i.i.d.). In particular, they may have time-varying distributions and/or be correlated in time. We assume that a "sufficient number" of i.i.d. samples of the disturbance

$\boldsymbol{w} \in \mathcal{W}$ and $\boldsymbol{\delta} \in \Delta$ can be obtained either empirically or by a random number generator. We denote $\mathcal{S}_{\boldsymbol{w}} := \{\boldsymbol{w}^{(1)}, \cdots, \boldsymbol{w}^{(N_s)}\} \in \mathcal{W}^{N_s}$ and $\mathcal{S}_{\boldsymbol{\delta}} := \{\boldsymbol{\delta}^{(1)}, \cdots, \boldsymbol{\delta}^{(N_s)}\} \in \Delta^{N_s}$ as sets of given finite samples (scenarios). Following the approach in [2], we approximate the expected value of the objective function empirically by averaging the value of its argument for some number of different scenarios $N_{\bar{s}}$, which plays a tuning parameter role.

We are now in a position to formulate an approximated version of the proposed stochastic control problem in (4) using the following finite horizon scenario program:

$$\min_{\boldsymbol{v}\in\mathbb{R}^{Tp}} \; J(\boldsymbol{x},\boldsymbol{u}) \tag{5a}$$

$$\text{s.t.} \quad x_{k+1}^{(i)} = A(\delta_k^{(i)})x_k^{(i)} + B(\delta_k^{(i)})u_k^{(i)} + C(\delta_k^{(i)})w_k^{(i)}\,, \tag{5b}$$

$$x_{k+\ell}^{(i)} \in \mathcal{X}\,,\; \ell \in \mathbb{N}_+\,,\; \forall \boldsymbol{w}^{(i)} \in \mathcal{S}_{\boldsymbol{w}}\,,\; \forall \boldsymbol{\delta}^{(i)} \in \mathcal{S}_{\boldsymbol{\delta}}\,, \tag{5c}$$

$$u_k^{(i)} = Kx_k^{(i)} + v_k \in \mathcal{U}\,, \quad \forall k \in \mathcal{T}\,, \tag{5d}$$

where superscript (i) indicates a particular sample realization. The solution of (5) is the optimal control input sequence $\boldsymbol{v}^* = \{v_k^*, \cdots, v_{k+T-1}^*\}$. Based on the MPC paradigm, the current input is implemented as $u_k := Kx_k + v_k^*$ and we proceed in a receding horizon fashion. Note that new scenarios are needed at each sampling time $k \in \mathcal{T}$.

In the following theorem, we restate the explicit theoretical bound of [14, Theorem 1] which measures the finite scenarios behavior of (5).

Theorem 1: Let $\varepsilon, \beta \in (0,1)$ and $N_s \geq \mathsf{N}(\varepsilon, \beta, Tp)$, where

$$\mathsf{N}(\varepsilon,\beta,Tp) := \min\left\{ N \in \mathbb{N} \,\middle|\, \sum_{i=0}^{Tp-1} \binom{N}{i} \varepsilon^i (1-\varepsilon)^{N-i} \leq \beta \right\}.$$

If the optimizer of problem (5), $\boldsymbol{v}^* \in \Re^{Tp}$, is applied to the discrete time dynamical system (1) for a finite horizon of length T, then, with at least confidence $1-\beta$, the original constraint (4c) is satisfied for all $k \in \mathcal{T}$ with probability more than $1-\varepsilon$.

It was shown in [14] that the above bound is tight. The interpretation of Theorem 1 is as follows: when applying $\boldsymbol{v}^*$ in a finite horizon control problem, the violation of the feasibility of the state trajectory remains below ε with confidence $1-\beta$:

$$\mathbb{P}^{N_s}\left[\, \mathcal{S}_{\boldsymbol{w}} \in \mathcal{W}^{N_s}, \mathcal{S}_{\boldsymbol{\delta}} \in \Delta^{N_s} \,:\, \text{Vio}(\boldsymbol{v}^*) \leq \varepsilon \,\right] \geq 1-\beta\,,$$

with $\text{Vio}(\boldsymbol{v}^*) := \mathbb{P}[\, \boldsymbol{w} \in \mathcal{W}, \boldsymbol{\delta} \in \Delta \,:\, x_{k+\ell} = A_{cl}(\delta_k)x_k + B(\delta_k)v_k^* + C(\delta_k)w_k \notin \mathcal{X}\,, \ell \in \mathbb{N}_+ \,|\, x_k = x_0\,]$, where $A_{cl}(\delta_k) = A(\delta_k) + B(\delta_k)K$. It is worth mentioning that the proposed constraint on the control input in (5d) is also met in a probabilistic sense, due to the nature of the scenario approach that appears in (5).

Remark 1: One can obtain an explicit expression for the desired number of scenarios N_s as in [21], where it is shown that given $\varepsilon, \beta \in (0,1)$ and e the Euler constant, then $N_s \geq \frac{e}{e-1}\frac{1}{\varepsilon}\left(Tp + \ln\frac{1}{\beta}\right)$. It is important to note that N_s is used to construct the sets of scenarios, $\mathcal{S}_{\boldsymbol{w}}$, $\mathcal{S}_{\boldsymbol{\delta}}$ to obtain a probabilistic guarantee for the desired level of feasibility, while the number of scenarios $N_{\bar{s}}$ is just a tuning variable to approximate the objective function empirically.

III. Distributed Scenario MPC

We consider a partitioning of the system dynamics (1) through a decomposition into M subsystems and let $\mathcal{M} = \{1, 2, \cdots, M\}$ be the set of subsystem indices. The state variables x_k, control input signals u_k and the additive disturbance w_k, can be considered as $x_k = \text{col}_{i\in\mathcal{M}}(x_{i,k})$, $u_k = \text{col}_{i\in\mathcal{M}}(u_{i,k})$, and $w_k = \text{col}_{i\in\mathcal{M}}(w_{i,k})$, respectively, where $x_{i,k} \in \mathbb{R}^{m_i}$, $u_{i,k} \in \mathbb{R}^{p_i}$, $w_{i,k} \in \mathbb{R}^{n_i}$, and $\sum_{i\in\mathcal{M}} m_i = m$, $\sum_{i\in\mathcal{M}} p_i = p$, $\sum_{i\in\mathcal{M}} n_i = n$. The following assumption is important in order to be able to partition the system parameters.

Assumption 1: It is assumed that the control input and the disturbance variables of the subsystems are decoupled,. e.g. $u_{i,k}$ and $w_{i,k}$ only affect subsystem $i \in \mathcal{M}$ for all $k \in \mathcal{T}$. The state and control input constraint sets of each subsystem $i \in \mathcal{M}$ have the following form: $\mathcal{X}_i := \{\, x \in \mathbb{R}^{m_i} : \; G_i\, x \leq g_i \,\}, \mathcal{U}_i := \{\, u \in \mathbb{R}^{p_i} \;:\; H_i\, u \leq h_i \,\}$, such that $\mathcal{X} = \prod_{i\in\mathcal{M}} \mathcal{X}_i$, $\mathcal{U} = \prod_{i\in\mathcal{M}} \mathcal{U}_i$, and $G = \text{diag}_{i\in\mathcal{N}}(G_i)$, $H = \text{diag}_{i\in\mathcal{N}}(H_i)$, $g = \text{col}_{i\in\mathcal{N}}(g_i)$, $h = \text{col}_{i\in\mathcal{N}}(h_i)$.

We are now able to decompose the large-scale system matrices $B(\delta_k) = \text{diag}_{i\in\mathcal{M}}(B_i(\delta_k)) \in \mathbb{R}^{m\times p}$, $C(\delta_k) = \text{diag}_{i\in\mathcal{M}}(C_i(\delta_k)) \in \mathbb{R}^{m\times n}$, as well as $A(\delta_k) \in \mathbb{R}^{m\times m}$ as follows:

$$A(\delta_k) = \begin{bmatrix} A_{11}(\delta_k) & \cdots & A_{1M}(\delta_k) \\ \vdots & \ddots & \vdots \\ A_{M1}(\delta_k) & \cdots & A_{MM}(\delta_k) \end{bmatrix},$$

where $A_{ij}(\delta_k) \in \mathbb{R}^{m_i\times m_j}$, $B_i(\delta_k) \in \mathbb{R}^{m_i\times p_i}$, and $C_i(\delta_k) \in \mathbb{R}^{m_i\times n_i}$. Define the set of neighboring subsystems of subsystem i as follows:

$$\mathcal{N}_i = \left\{ j \in \mathcal{M}\backslash i \;\middle|\; A_{ij}(\delta_k) \neq \mathbf{0} \right\}\,, \tag{6}$$

where $\mathbf{0}$ denotes a matrix of all zeros with proper dimension. Note that if subsystems are decoupled, they remain decoupled regardless of the uncertainties $\delta_{i,k}$ for all $i \in \mathcal{N}$. Consider now a large-scale network that consists of M interconnected subsystems, and each subsystem can be described by an uncertain discrete-time linear time-invariant system with additive disturbance of the form

$$\begin{cases} x_{i,k+1} &= A_{ii}(\delta_k)x_{i,k} + B_i(\delta_k)u_{i,k} + q_{i,k} \\ q_{i,k} &= \sum_{j\in\mathcal{N}_i} A_{ij}(\delta_k)x_{j,k} + C_i(\delta_k)w_{i,k} \end{cases}, \tag{7}$$

where for each subsystem i, $x_{i,k} \in \mathcal{X}_i \subseteq \mathbb{R}^{m_i}$, $u_{i,k} \in \mathcal{U}_i \subseteq \mathbb{R}^{p_i}$, and $w_{i,k} \subseteq \mathbb{R}^{n_i}$. Following Assumption 1, one can consider a linear feedback gain matrix K_i for each subsystem $i \in \mathcal{M}$ such that $K = \text{diag}_{i\in\mathcal{M}}(K_i)$. Using K_i in each subsystem, we assume that there exists P_i for each subsystem $i \in \mathcal{M}$ such that $P = \text{diag}_{i\in\mathcal{M}}(P_i)$ to preserve the condition in (3). Consider now the objective function of each subsystem $i \in \mathcal{M}$ in the following form:

$$J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) := \mathbb{E}\left[\sum_{k=0}^{T-1} \left(x_{i,k}^\top Q_i x_{i,k} + u_k^\top R_i u_{i,k} \right) + x_{i,T}^\top P_i x_{i,T} \right],$$

where $Q_i \in \mathbb{R}_{\succeq 0}^{m_i\times m_i}$, $R_i \in \mathbb{R}_{\succ 0}^{p_i\times p_i}$ such that $Q = \text{diag}_{i\in\mathcal{M}}(Q_i)$, and $R = \text{diag}_{i\in\mathcal{M}}(R_i)$. Note that $\boldsymbol{x}_i = \text{col}_{k\in\mathcal{T}}(x_{i,k})$ and $\boldsymbol{u}_i = \text{col}_{k\in\mathcal{T}}(u_{i,k})$ such that $\boldsymbol{x} =$

$\text{col}_{i\in\mathcal{M}}(\boldsymbol{x}_i)$ and $\boldsymbol{u} = \text{col}_{i\in\mathcal{M}}(\boldsymbol{u}_i)$. For sake of simplicity of the mathematical notations, a decomposition of multiplicative uncertainty δ_k is not considered. We however note that such a decomposition is straightforward by considering $\delta_{i,k}$ for each subsystem $i \in \mathcal{M}$. This leads to $A_{ii}(\delta_{i,k})$, $B_i(\delta_{i,k})$, $C_i(\delta_{i,k})$, and an effect on the state coupling matrices between subsystems $A_{ij}(\delta_{i,k})$ for all $j \in \mathcal{N}_i$ of each $i \in \mathcal{M}$.

Consider $\boldsymbol{v}_i = \text{col}_{k\in\mathcal{T}}(v_{i,k})$ such that $\boldsymbol{v} = \text{col}_{i\in\mathcal{M}}(\boldsymbol{v}_i)$, we decompose the proposed formulation in (5) using the following finite horizon scenario program for each subsystem $i \in \mathcal{M}$:

$$\min_{\boldsymbol{v}_i\in\mathbb{R}^{Tp_i}} J_i(\boldsymbol{x}_i, \boldsymbol{u}_i) \tag{8a}$$

$$\text{s.t.} \quad x^{(i)}_{i,k+1} = A_{ii}(\delta^{(i)}_k)x^{(i)}_{i,k} + B_i(\delta^{(i)}_k)u^{(i)}_{i,k} + q^{(i)}_{i,k}\,, \tag{8b}$$

$$x^{(i)}_{i,k+\ell} \in \mathcal{X}_i\,,\ \ell \in \mathbb{N}_+\,,\ \forall \boldsymbol{w}^{(i)}_i \in \mathcal{S}_{\boldsymbol{w}_i}\,,\ \forall \boldsymbol{\delta}^{(i)} \in \mathcal{S}_{\boldsymbol{\delta}} \tag{8c}$$

$$u^{(i)}_{i,k} = K_i x^{(i)}_{i,k} + v_{i,k} \in \mathcal{U}_i\,, \quad \forall k \in \mathcal{T}\,, \tag{8d}$$

where $\boldsymbol{w}_i = \text{col}_{k\in\mathcal{T}}(w_{i,k}) \in \mathcal{W}_i$ such that $\mathcal{W} = \prod_{i\in\mathcal{M}} \mathcal{W}_i$. $\mathcal{S}_{\boldsymbol{w}_i} := \{\boldsymbol{w}^{(1)}_i, \cdots, \boldsymbol{w}^{(N_{s_i})}_i\} \in \mathcal{W}^{N_{s_i}}_i$ denotes a set of given finite samples (scenarios) of disturbance in each subsystem $i \in \mathcal{M}$, such that $\mathcal{S}_{\boldsymbol{w}} = \prod_{i\in\mathcal{M}} \mathcal{S}_{\boldsymbol{w}_i}$. Note that we use indices in parenthesis to refer to each scenario of the random variables, e.g.(i), whereas indices without parenthesis refer to each subsystem $i \in \mathcal{M}$.

Remark 2: The proposed constraint (8c) represents an approximation of the following chance constraint on the state of each subsystem $i \in \mathcal{M}$:

$$\mathbb{P}[\, x_{i,k+\ell} \in \mathcal{X}_i\,,\ \ell \in \mathbb{N}_+\,] \geq 1 - \varepsilon_i\ , \tag{9}$$

where $\varepsilon_i \in (0, 1)$ is the admissible state constraint violation parameter of each subsystem (7). One can also consider $\alpha_i = 1 - \varepsilon_i$ as the desired level of state feasibility parameter of each subsystem (7).

In the following proposition, we provide a connection between the proposed optimization problem in (8) and the optimization problem in (5).

Proposition 1: Given Assumption 1, the proposed optimization problem in (8) is an exact decomposition of the optimization problem in (5).

Proof: The reader is referred to the technical report of this paper in [22] with more details and complete proofs. ■

The following theorem can be considered as the main result of this section to quantify the robustness of the solutions obtained by (8).

Theorem 2: Let $\varepsilon_i, \beta_i \in (0, 1)$ be chosen such that $\varepsilon = \sum_{i\in\mathcal{M}} \varepsilon_i \in (0, 1)$, $\beta = \sum_{i\in\mathcal{M}} \beta_i \in (0, 1)$, and $N_{s_i} \geq \mathsf{N}(\varepsilon_i, \beta_i, Tp_i)$ for all $i \in \mathcal{M}$. If $\boldsymbol{v}^* = \text{col}_{i\in\mathcal{M}}(\boldsymbol{v}^*_i)$, the collection of the optimizers of problem (8) for all subsystem $i \in \mathcal{M}$, is applied to the discrete-time dynamical system (1) for a finite horizon of length T, then, with at least confidence $1 - \beta$, the original constraint (4c) is satisfied for all $k \in \mathcal{T}$ with probability more than $1 - \varepsilon$.

Proof: The reader is referred to the technical report of this paper in [22] with more details and complete proofs. ■

The interpretation of Theorem 2 is as follows. In the proposed distributed scenario program (8), each subsystem $i \in \mathcal{M}$ can have a desired level of constraint violation ε_i and a desired level of confidence level β_i. To keep the robustness level of the collection of solutions in a probabilistic sense (4c) for the discrete-time dynamical system (1), these choices have to follow a certain design rule, e.g. $\varepsilon = \sum_{i\in\mathcal{M}} \varepsilon_i \in (0, 1)$ and $\beta = \sum_{i\in\mathcal{M}} \beta_i \in (0, 1)$. This yields a fixed ε , β for the large-scale system (1) and the individual ε_i , β_i for each subsystem $i \in \mathcal{M}$. It is important to mention that in order to maintain the violation level for the large-scale system with many partitions, i.e. $|\mathcal{N}| = N \uparrow$, the violation level of individual agents needs to decrease, i.e. $\varepsilon_i \downarrow$, which leads to very conservative results for each subsystem, since the number required samples needs to increase, i.e. $N_{s_i} \uparrow$. Addressing such a limitation is subject of our ongoing research work.

An important key feature of the proposed distributed scenario program in (8) compared to the optimization problem in (5) is as follows. Using the proposed distributed framework, we decompose a large-scale scenario program (5) with N_s number of scenarios into $M = |\mathcal{M}|$ small-scale scenario programs (8) with N_{s_i} number of scenarios. This yields a significant reduction in the computation time complexity of scenario programs compared to (5) by using the proposed distributed scenario program (8). Using the subsystem dynamics in (7), agent $i \in \mathcal{N}$ substitutes $q^{(i)}_{i,k}$ in the proposed scenario optimization problem (8) with $q^{(i)}_{i,k} = \sum_{j\in\mathcal{N}_i} A_{ij}(\delta^{(i)}_k)x^{(i)}_{j,k} + C_i(\delta^{(i)}_k)w^{(i)}_{i,k}$, where $\delta^{(i)}_k$ and $w^{(i)}_{i,k}$ are the local scenarios of random variables that are available in each subsystem by definition $\boldsymbol{w}^{(i)}_i \in \mathcal{S}_{\boldsymbol{w}_i}$ and $\boldsymbol{\delta}^{(i)} \in \mathcal{S}_{\boldsymbol{\delta}}$, and taking into consideration that the interaction dynamics model $A_{ij}(\delta^{(i)}_k)$ by each neighboring agent $j \in \mathcal{N}_i$ is also available for agent $i \in \mathcal{N}$. Hence, the only information that subsystem $i \in \mathcal{M}$ needs is an N_{s_i} number of samples of the state variable $\boldsymbol{x}^{(i)}_j = \text{col}_{k\in\mathcal{T}}(x^{(i)}_{j,k}) \in X_j := \mathcal{X}^T_j$ from all its neighboring subsystems $j \in \mathcal{N}_i$ at each $k \in \mathcal{T}$. It is important to note that even though the proposed distributed scenario program in (8) yields a reduction of computation time complexity, it however requires more communication between each subsystem, since at each $k \in \mathcal{T}$ all neighboring agents $j \in \mathcal{N}_i$ of agent i should send a set of scenarios of the state variable $\mathcal{S}_{\boldsymbol{x}_j} \in X^{N_{s_i}}_j$ to the agent $i \in \mathcal{M}$.

IV. Information Exchange Scheme

When (8) is applied to the large-scale scenario program (5), all neighboring agents $j \in \mathcal{N}_i$ of agent $i \in \mathcal{M}$ should send a set of scenarios of the state variable $\mathcal{S}_{\boldsymbol{x}_j} := \{\boldsymbol{x}^{(1)}_j, \cdots, \boldsymbol{x}^{(N_{s_i})}_j\}$ to agent i at each sampling time $k \in \mathcal{T}$. It is of interest to address the issue of how an agent $j \in \mathcal{N}_i$ can send the contents of $\mathcal{S}_{\boldsymbol{x}_j}$ to agent $i \in \mathcal{M}$.

We propose the following two schemes: 1) following our proposed setup in (8) to achieve a probabilistic guarantee for the obtained solution, agent $i \in \mathcal{M}$ requests from its neighboring agents to send the complete set of data $\mathcal{S}_{\boldsymbol{x}_j}$, element by element such that the number of required samples N_{s_i}, is chosen according to Theorem 2 in order to have a given probabilistic guarantee for the optimizer $\boldsymbol{v}^*_i$. We refer to this scheme as a *hard communication protocol* between

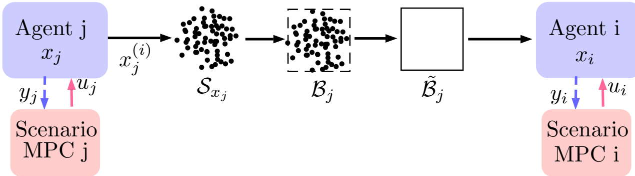

Fig. 1. Pictorial representation of the proposed inter-agent soft communication scheme. $\mathcal{S}_{\boldsymbol{x}_j}$ is the set of $\tilde{N}_{s_i}$ scenarios, $\mathcal{B}_j$ is the parametrized set used in the optimization problem (10), and $\tilde{\mathcal{B}}_j$ is the solution of the optimization problem (10). y_i and y_j are just to present the state measurements which are sent to controllers.

agents. Its advantage is that it is simple and transmits exactly the contents of $\mathcal{S}_{\boldsymbol{x}_j}$, but due to possibly high values of N_{s_i}, it may turn out to be too costly in terms of required communication bandwidth. 2) to address this shortcoming, we propose another scheme, where agent $j \in \mathcal{N}_i$ sends instead a suitable parametrization of a set that contains all the possible values of data with a desired level of probability (*the level of reliability*) $\tilde{\alpha}_j$. By considering a simple family of sets, for instance boxes in $\mathbb{R}^{m_j}$, communication cost can be kept down at reasonable levels. We refer to this scheme as a *soft communication protocol* between agents (see Figure 1). Such a scheme may be understood as a cascading scenario scheme similar to the one in [23], where a sufficient number of scenarios was determined in order to establish a probabilistic feasibility for two cascading scenario programs subject to a similar source of uncertainty. Our soft communication setting however differs from [23], since each agent is subject to its own uncertainty source. We aim to incorporate the reliability notion of such a soft communication scheme into the feasibility bound of each agent, in addition to determining the number of required scenarios that can be obtained as a corollary of our results presented so far.

We now describe the soft communication protocol in more detail. The neighboring agent $j \in \mathcal{N}_i$ has to first generate $\tilde{N}_{s_i}$ samples of $\boldsymbol{x}_j$ in order to build the set $\mathcal{S}_{\boldsymbol{x}_j}$. It is important to notice that in the soft communication protocol the number $\tilde{N}_{s_i}$ of samples generated by agent j may be different than the one needed by agent i, which is N_{s_i}, as will be remarked later. Let us then introduce $\mathcal{B}_j \subset \mathbb{R}^{m_j}$ as a bounded set containing all the elements of $\mathcal{S}_{\boldsymbol{x}_j}$. We assume for simplicity that $\mathcal{B}_j$ is an axis-aligned hyper-rectangular set [12]. This is not a restrictive assumption and any convex set, e.g. ellipsoids and polytopes, could have been chosen instead as described in [24]. We can define $\mathcal{B}_j := [-\boldsymbol{b}_j, \boldsymbol{b}_j]$ as an interval, where the vector $\boldsymbol{b}_j \in \mathbb{R}^{m_j}$ defines the hyper-rectangle bounds.

Consider now the following optimization problem that aims to determine the set $\mathcal{B}_j$ with minimal volume:

$$\left\{ \begin{array}{ll} \min\limits_{\boldsymbol{b}_j \in \mathbb{R}^{m_j}} & \|\boldsymbol{b}_j\|_1 \\ \text{s.t.} & \boldsymbol{x}_j^{(l)} \in [-\boldsymbol{b}_j, \boldsymbol{b}_j]\ , \ \forall \boldsymbol{x}_j^{(l)} \in \mathcal{S}_{\boldsymbol{x}_j} \\ & l = 1, \cdots, \tilde{N}_{s_i} \end{array} \right. , \tag{10}$$

where $\tilde{N}_{s_i}$ is the number of samples $\boldsymbol{x}_j \in \mathcal{S}_{\boldsymbol{x}_j}$ that neighboring agent j has to take into account in order to determine $\mathcal{B}_j$. If we denote by $\tilde{\mathcal{B}}_j = [-\tilde{\boldsymbol{b}}_j, \tilde{\boldsymbol{b}}_j]$, the optimal solution of (10) computed by the neighbor agent j, then for implementing the soft communication protocol, agent j needs to communicate only the vector $\tilde{\boldsymbol{b}}_j$ along with the level of reliability $\tilde{\alpha}_j$ to the agent i.

Definition 1: A set $\tilde{\mathcal{B}}_j$ is called $\tilde{\alpha}_j-$reliable if

$$\mathbb{P}\left[\boldsymbol{x}_j \in X_j \,:\, \boldsymbol{x}_j \notin [-\tilde{\boldsymbol{b}}_j, \tilde{\boldsymbol{b}}_j] \right] \leq 1 - \tilde{\alpha}_j\,, \tag{11}$$

and we refer to $\tilde{\alpha}_j$ as the level of reliability of the set $\tilde{\mathcal{B}}_j$.

We now provide the following theorem to determine $\tilde{\alpha}_j$ as the level of reliability of the set $\tilde{\mathcal{B}}_j$.

Theorem 3: Fix $\tilde{\beta}_j \in (0,1)$ and let

$$\tilde{\alpha}_j = \sqrt[\tilde{N}_{s_i} - m_j]{\frac{\tilde{\beta}_j}{\binom{\tilde{N}_{s_i}}{m_j}}}\ . \tag{12}$$

We then have $\mathbb{P}^{\tilde{N}_{s_i}}\Big[\{\boldsymbol{x}_j^1, \cdots, \boldsymbol{x}_j^{\tilde{N}_{s_i}}\} \in X_j^{\tilde{N}_{s_i}} : \mathbb{P}\big[\boldsymbol{x}_j \in X_j : \boldsymbol{x}_j \notin [-\tilde{\boldsymbol{b}}_j, \tilde{\boldsymbol{b}}_j]\big] \leq 1 - \tilde{\alpha}_j \Big] \geq 1 - \tilde{\beta}_j$.

Proof: The proof is a direct result of [13, Theorem 1] with some algebraic manipulations. We refer the reader to [22] for a complete proof. ■

Theorem 3 implies that given an hypothetical new sample $\boldsymbol{x}_j \in X_j$, agent $j \in \mathcal{N}_i$ has a confidence of at least $1 - \tilde{\beta}_j$ that the probability of $\boldsymbol{x}_j \in \tilde{\mathcal{B}}_j = [-\tilde{\boldsymbol{b}}_j, \tilde{\boldsymbol{b}}_j]$ is at least $\tilde{\alpha}_j$. Therefore, one can *rely on $\tilde{\mathcal{B}}_j$ up to $\tilde{\alpha}_j$ probability*. The number of samples $\tilde{N}_{s_j}$ in the proposed formulation (10) is a design parameter chosen by the neighboring agent $j \in \mathcal{N}_i$. We however remark that one can also set a given $\tilde{\alpha}_j$ as the desired level of reliability and obtain from (12) the required number of samples $\tilde{N}_{s_i}$.

When an agent $i \in \mathcal{M}$ and its neighbor $j \in \mathcal{N}_i$ adopt the soft communication scheme, there is an important effect on the probabilistic feasibility of agent i, following Remark 2. Such a scheme introduces some level of stochasticity on the probabilistic feasibility of agent i, due to the fact that the neighboring information is only *probabilistically reliable*. This will affect the local probabilistic robustness guarantee of feasibility as it was discussed in Theorem 2 and consequently in Theorem 1. To accommodate the level of reliability of neighboring information, we need to marginalize a joint cumulative distribution function (cdf) probability of $\boldsymbol{x}_i$ and the generic sample $\boldsymbol{x}_j \in X_j$ appearing in Theorem 3. We thus have the following theorem, which can be regarded as the main theoretical result of this section.

Theorem 4: Given $\tilde{\alpha}_j \in (0,1)$ and a fixed $\alpha_i \in (0,1)$, the state trajectory of a generic agent $i \in \mathcal{M}$ is probabilistically $\bar{\alpha}_i$–feasible for all $\boldsymbol{w}_i \in \mathcal{W}_i$, $\boldsymbol{\delta} \in \Delta$, i.e.,

$$\mathbb{P}\left[x_{i,k+\ell} \in \mathcal{X}_i\,,\, \ell \in \mathbb{N}_+\right] \geq \bar{\alpha}_i\,, \tag{13}$$

where $\bar{\alpha}_i = 1 - \frac{1-\alpha_i}{\tilde{\alpha}_i}$ such that $\tilde{\alpha}_i = \prod_{j \in \mathcal{N}_i}(\tilde{\alpha}_j)$.

Sketch of Proof: The proof consists of two important steps. Given α_i as the lower bound on the joint cdf$(\boldsymbol{x}_i, \boldsymbol{x}_j)$, one needs to marginalize it to obtain the cdf$(\boldsymbol{x}_i)$ using the results in Theorem 3, and then, proceeding with the fact that the information of neighboring agents are conditionally independent, one can obtain the above assertion. We refer the reader to [22] for a complete proof and details. ■

Following the statement of Theorem 4, it is straightforward to observe that if for all $j \in \mathcal{N}_i$, $\tilde{\alpha}_j \to 1$ then $\bar{\alpha}_i \to \alpha_i$. This means that if *the level of reliability* of the neighboring information is one, $\mathbb{P}\big[\, \boldsymbol{x}_j \in \tilde{\mathcal{B}}_j \;:\; \forall j \in \mathcal{N}_i \big] = 1$, then, the state feasibility of agent i will have the same probabilistic level of robustness as the hard communication scheme, $\mathbb{P}\big[\, \boldsymbol{x}_i \in X_i \,\big] \geq \alpha_i$. Combining this result with the statement of Theorem 2, the proposed soft communication scheme introduces some level of stochasticity on the feasibility of the large-scale system as in (4c). In particular, $\varepsilon_i \in (0,1)$ the level of constraint violation in each agent $i \in \mathcal{M}$ will increase, since it is proportional with the inverse of $\prod_{j \in \mathcal{N}_i} (\tilde{\alpha}_j) \in (0,1)$, and therefore, $\varepsilon = \sum_{i \in \mathcal{M}} \varepsilon_i \in (0,1)$ will also increase. After receiving the parametrization of $\tilde{\mathcal{B}}_j$ and the level of reliability $\tilde{\alpha}_j$, agent $i \in \mathcal{M}$ should immunize itself against all possible variation of $\boldsymbol{x}_j \in \tilde{\mathcal{B}}_j$ by taking the worst-case of $\tilde{\mathcal{B}}_j$, similar to the worst-case reformulation proposed in [11, Propostion 1]. It is important to notice that in this way, we decoupled the sample generation of agent $j \in \mathcal{N}_i$ from agent $i \in \mathcal{M}$.

V. Proposed Distributed Scenario MPC

We summarize our proposed distributed scenario MPC in Algorithm 1 such that agents communicate with each other by using our proposed soft inter-agent communication scheme. Note that in case of the hard communication scheme, each agent needs to generate N_{s_i} scenarios and send exactly all of them to all its neighboring agents $j \in \mathcal{N}_i$. In other words, the following changes have to be made in Algorithm 1. $\tilde{N}_{s_i}$ will be substituted by N_{s_i} in Step 9 and Step 10 will be removed. Steps 11 and 12 will send and receive exactly N_{s_i} samples, respectively.

In Algorithm 1 it is assumed that the feedback control gain matrices K_i for all agent $i \in \mathcal{M}$ are given (3), and the coupling terms $A_{ij}(\delta_k)$ are known between each agent $i \in \mathcal{M}$ and its neighboring agents $j \in \mathcal{N}_i$. It is important to note that Step 5 of Algorithm 1, initializes $\tilde{\mathcal{B}}_j$ for all neighboring agents $j \in \mathcal{N}_i$ to be used for the initial iteration in Step 8, and then, at each iteration all agent $i \in \mathcal{M}$ will send and receive $\tilde{\mathcal{B}}_j$ from all its neighboring agents $j \in \mathcal{N}_i$ as in Steps 11 and 12, respectively.

Algorithm 1 Distributed Scenario MPC

1: **Decompose** the large-scale dynamical system (1) into M agents as the proposed form in (7)
2: **Determine** the index set of neighboring agents $\mathcal{N}_i$ for each agent $i \in \mathcal{M}$
3: **For** each agent $i \in \mathcal{M}$ **do**
4: **fix** initial state $x_{i,0} \in \mathcal{X}_i$, $\varepsilon_i \in (0,1)$, and $\beta_i \in (0,1)$ such that $\varepsilon = \sum_{i \in \mathcal{M}} \varepsilon_i \in (0,1)$, $\beta = \sum_{i \in \mathcal{M}} \beta_i \in (0,1)$
5: **initialize** $\tilde{\mathcal{B}}_j$ for all neighboring agents $j \in \mathcal{N}_i$
6: **determine** $N_{\bar{s}_i} \in (0,+\infty)$ to approximate the objective function, and N_{s_i} following Theorem 2 to approximate the chance constraint (9) in an equivalent sense
7: **generate** $N_{\bar{s}_i}$, N_{s_i} scenarios of $\boldsymbol{w}_i$, $\boldsymbol{\delta}$ to determine the sets of $\bar{\mathcal{S}}_{\boldsymbol{w}_i}$, $\bar{\mathcal{S}}_{\boldsymbol{\delta}}$ and $\mathcal{S}_{\boldsymbol{w}_i}$, $\mathcal{S}_{\boldsymbol{\delta}}$
8: **solve** the proposed optimization problem in (8) by taking into account the worst-case of $\tilde{\mathcal{B}}_j$ and determine $\boldsymbol{v}_i^*$
9: **generate** $\tilde{N}_{s_i}$ scenarios of $\boldsymbol{x}_i$ using the dynamical system of agent i in form of (7) and $\boldsymbol{v}_i^*$ together with $\mathcal{S}_{\boldsymbol{w}_i}$, $\mathcal{S}_{\boldsymbol{\delta}}$
10: **determine** set $\tilde{\mathcal{B}}_i$ by solving the optimization problem (10)
11: **send** the set $\tilde{\mathcal{B}}_i$ to all neighboring agents $j \in \mathcal{N}_i$
12: **receive** the sets $\tilde{\mathcal{B}}_j$ from all neighboring agents $j \in \mathcal{N}_i$
13: **apply** the first input of solution $u_{i,k}^* = K_i x_{i,k} + v_{i,k}^*$ into the uncertain subsystem (7)
14: **measure** the state and substitute it as the initial state of the next step $x_{i,0}$
15: **set** $k \leftarrow k+1$ and **goto** Step (7)
16: **End for**

VI. Numerical Study

To numerically illustrate the functionality of our proposed approach, we simulate a building climate comfort system with three rooms. Following the system dynamics in (1), we define the system parameters as follows:

$$A = \begin{bmatrix} 0.2 & 0.3 & 0 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0 & 0.4 \end{bmatrix}, B = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, C = \begin{bmatrix} 0.02 \\ 0.02 \\ 0.02 \end{bmatrix},$$

such that $A(\delta_k) = A + \delta_k$ and $B(\delta_k) = B + \delta_k$ as well as $C(\delta_k) = C + \delta_k$, where $\delta_k \in \mathbb{R}$ is generated from a normal distribution with a mean value 0, variance 1 and a maximal magnitude of 0.01 at each sampling time. The system matrices are a simplified model of three-room

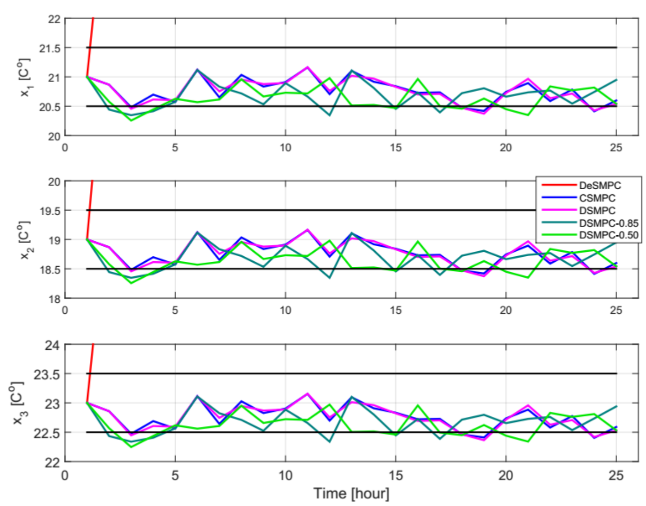

Fig. 2. A-posteriori feasibility validation of the obtained results.

building such that the states $x_{i,k}$ for $i = 1,2,3$, denote the temperature of rooms. δ_k represents the modeling errors, losses through windows, and $w_k \in \mathbb{R}$ can be realized as the outside weather temperatures such that it can vary within 10% of its nominal scenario at each sampling time. The initial states are $[21 \;\; 19 \;\; 23]^\top$ and the objective is to keep the temperature of rooms with our desired lower $[20.5 \;\; 18.5 \;\; 22.5]^\top$ and $[21.5 \;\; 19.5 \;\; 23.5]^\top$ upper bounds at the minimum control unit production u_k. The control input u_k are also constrained to be within -1.5 [kWh] and 1.5 [kWh] for all three rooms, due to actuator saturation. The initialization of the $\tilde{\mathcal{B}}_j$ for all neighboring agents $j \in \mathcal{N}_i$ as in Step 5 in Algorithm 1 can be done for instance by assuming the initial temperature of the neighboring rooms are known.

We simulate four different problem formulations in Fig. 2. The "blue" line shows the results obtained via centralized SMPC (CSMPC) using (5), the "magenta" presents the results obtained by using distributed SMPC (DSMPC) via (8), the "dark green" and "light green" lines show the results obtained via DSMPC with the proposed soft communication scheme with $0.85-$reliability (DSMPCS-0.85) as described in Definition 1 and DSMPCS-0.50 both following Algorithm 1, respectively, in a closed-loop control system framework. For comparison purposes, we also present the results obtained via decoupled SMPC (DeSMPC) using the "red" line, where the impact of coupling neighboring dynamics in (7) are relaxed. The "black" lines indicate the bounds of the three dynamically coupled systems.

Figure 2 illustrates our other two main contributions more precisely: 1) the obtained results via CSMPC (blue line) and DSMPC (magenta line) are practically equivalent throughout the simulation; this is due to Proposition 1 and Theorem 2. Actually, the solutions via DSMPC are slightly more conservative compared to the results via CSMPC, and this is a direct consequence of Theorem 2. In fact the level of violation in CSMPC is considered to be $\varepsilon = 0.05$ and leading to $\varepsilon_i = 0.0167$ for all agents due to Theorem 2, which is more restrictive. 2) the proposed soft communication scheme yields less conservative solutions as explicitly derived in Theorem 4, and can be clearly seen in Figure 2 with the obtained results via DSMPCS-0.85 (dark green) and DSMPCS-0.50 (light green). Following Theorem 4 the new violation level using DSMPCS-0.85 is $\bar{\varepsilon}_i = 0.0231$, and using DSMPCS$-0.50$ is $\bar{\varepsilon}_i = 0.0668$. It is important to notice that the violation level of global chance constraint will increase to $\bar{\varepsilon} = 0.0702$ and $\bar{\varepsilon} = 0.2004$ using DSMPCS-0.85 and DSMPCS-0.50, respectively.

VII. Conclusions

In this paper we presented a rigorous approach to distributed stochastic model predictive control (SMPC) using the scenario-based approximation. We then provided a novel inter-agent soft communication scheme to minimize the amount of information exchange between each subsystem. Using a set-based parametrization technique, we introduced a reliability notion and quantified the level of feasibility of obtained solutions via the distributed SMPC integrated with the so-called soft communication scheme in a probabilistic sense. The theoretical guarantees of the proposed distributed SMPC framework coincide with its centralized counterpart. Our current research direction concentrates on extending the proposed framework to cope with the case of a common uncertainty source and to formally address the recursive feasibility and stability of the closed-loop.

References

[1] A. N. Venkat, I. A. Hiskens, J. B. Rawlings, and S. J. Wright, "Distributed MPC strategies with application to power system automatic generation control," *IEEE transactions on control systems technology*, vol. 16, no. 6, pp. 1192–1206, 2008.

[2] V. Rostampour, P. Mohajerin Esfahani, and T. Keviczky, "Stochastic nonlinear model predictive control of an uncertain batch polymerization reactor," in *IFAC Conference on Nonlinear Model Predictive Control (NMPC)*, vol. 48, no. 23. Elsevier, 2015, pp. 540–545.

[3] Y. Ma, S. Richter, and F. Borrelli, "DMPC for building temperature regulation," *Control and Optimization with Differential-Algebraic Constraints*, vol. 23, p. 293, 2012.

[4] J. Lavaei and A. G. Aghdam, "Control of continuous-time LTI systems by means of structurally constrained controllers," *Automatica*, vol. 44, no. 1, pp. 141–148, 2008.

[5] C. Conte, M. N. Zeilinger, M. Morari, and C. N. Jones, "Robust distributed model predictive control of linear systems," in *European Control Conference (ECC)*. IEEE, 2013, pp. 2764–2769.

[6] M. Cannon, B. Kouvaritakis, S. V. Rakovic, and Q. Cheng, "Stochastic tubes in model predictive control with probabilistic constraints," *IEEE Transactions on Automatic Control*, vol. 56, no. 1, pp. 194–200, 2011.

[7] S. Riverso, M. Farina, and G. Ferrari-Trecate, "Plug-and-play decentralized model predictive control for linear systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 10, pp. 2608–2614, 2013.

[8] L. Dai, Y. Xia, Y. Gao, and M. Cannon, "Distributed stochastic MPC of linear systems with additive uncertainty and coupled probabilistic constraints," *IEEE Transactions on Automatic Control*, vol. PP, no. 99, pp. 1–8, 2016.

[9] P. Hokayem, E. Cinquemani, D. Chatterjee, F. Ramponi, and J. Lygeros, "Stochastic receding horizon control with output feedback and bounded controls," *Automatica*, vol. 48, no. 1, pp. 77–88, 2012.

[10] M. Lorenzen, F. Allgöwer, F. Dabbene, and R. Tempo, "Scenario-based stochastic MPC with guaranteed recursive feasibility," pp. 4958–4963, 2015.

[11] V. Rostampour and T. Keviczky, "Robust randomized model predictive control for energy balance in smart thermal grids," in *European Control Conference (ECC)*. IEEE, 2016, pp. 1201–1208.

[12] K. Margellos, P. Goulart, and J. Lygeros, "On the road between robust optimization and the scenario approach for chance constrained optimization problems," *Trans. on Automatic Control*, vol. 59, no. 8, pp. 2258–2263, 2014.

[13] G. C. Calafiore and M. C. Campi, "The scenario approach to robust control design," *Transactions on Automatic Control*, vol. 51, no. 5, pp. 742–753, 2006.

[14] M. C. Campi and S. Garatti, "The exact feasibility of randomized solutions of uncertain convex programs," *SIAM Journal on Optimization*, vol. 19, no. 3, pp. 1211–1230, 2008.

[15] Y. Long, S. Liu, L. Xie, and K. H. Johansson, "A scenario-based distributed stochastic MPC for building temperature regulation," in *Automation Science and Engineering (CASE), 2014 IEEE International Conference on*. IEEE, 2014, pp. 1091–1096.

[16] S. Liu, A. Sadowska, H. Hellendoorn, and B. De Schutter, "Scenario-based distributed model predictive control for freeway networks," in *International Conference on Intelligent Transportation Systems (ITSC)*. IEEE, 2016, pp. 1779–1784.

[17] G. C. Calafiore and D. Lyons, "Random convex programs for distributed multi-agent consensus," in *European Control Conference (ECC)*. IEEE, 2013, pp. 250–255.

[18] G. Papaefthymiou and B. Klockl, "Mcmc for wind power simulation," *IEEE transactions on energy conversion*, vol. 23, no. 1, pp. 234–240, 2008.

[19] J. Löfberg, "Oops! I cannot do it again: Testing for recursive feasibility in MPC," *Automatica*, vol. 48, no. 3, pp. 550–555, 2012.

[20] G. Schildbach, L. Fagiano, C. Frei, and M. Morari, "The scenario approach for stochastic model predictive control with bounds on closed-loop constraint violations," *Automatica*, vol. 50, no. 12, pp. 3009–3018, 2014.

[21] T. Alamo, R. Tempo, A. Luque, and D. R. Ramirez, "Randomized methods for design of uncertain systems: Sample complexity and sequential algorithms," *Automatica*, vol. 52, pp. 160–172, Feb. 2015.

[22] V. Rostampour and T. Keviczky, "Distributed stochastic model predictive control synthesis for large-scale uncertain linear systems," *arXiv*, 2017. [Online]. Available: https://arxiv.org/pdf/1703.06273.pdf

[23] K. Margellos, M. Prandini, and J. Lygeros, "On the connection between compression learning and scenario based single-stage and cascading optimization problems," *IEEE Transactions on Automatic Control*, vol. 60, no. 10, pp. 2716–2721, 2015.

[24] V. Rostampour and T. Keviczky, "Probabilistic energy management for building climate comfort in smart thermal grids with seasonal storage systems," *arXiv*, 2017. [Online]. Available: https://arxiv.org/pdf/1611.03206.pdf