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On the physical interpretation of the R-ratio effect and the LEFM parameters used for fatigue crack growth in adhesive bonds

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Abstract

The available models for the prediction of fatigue crack growth in adhesive bonds rely on the similitude principle. In most cases, one of three similitude parameters based on the strain energy release rate (SERR) is used; i.e. G_{\max} , $(\Delta\sqrt{G})^2$, or ΔG . In all cases it is usually observed that keeping the similitude parameter constant, and changing the *R*-ratio, results in a different crack growth rate. In this paper it is shown that this apparent '*R*-ratio' effect is caused because the selected similitude parameter does not define a unique load cycle; a single value of the similitude parameter could correspond to infinitely many load cycles. The strain energy dissipation approach is used to show that the resistance to fatigue crack growth is related to the maximum applied load. The amount of energy available for crack growth is shown to be related to the applied cyclic work. With these relationships the *R*-ratio effects reported in literature can be qualitatively explained, purely in terms of the actual applied load cycle. Although it is possible that the material behaviour also depends on the *R*-ratio, the magnitude of these effects can only properly be determined if the applied load cycle is correctly described first.

Keywords: Adhesive Bonds, Fatigue Crack Growth, LEFM, R-ratio

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10 1. Introduction

Since the pioneering work of Roderick et al. [1, 2] and Mostovoy and Ripling [3], linear elastic fracture mechanics (LEFM) has remained the main framework for thinking about fatigue crack growth (FCG) in adhesive bonds. Although the use of LEFM has led to good prediction models for FCG, an understanding of the underlying physics remains lacking. As a result, calibration of FCG prediction models requires large and expensive test campaigns. Furthermore, the limits of the models' validity are unclear.

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Nomenclature					
A	Curve fit parameter (N/mm)	η	Coefficient (mm^2)		
a	Crack length (mm)	γ	Mean stress sensitivity parameter		
C	Curve fit parameter	ν	Poisson's ratio		
d	Displacement (mm)	σ_s	Interlaminar tensile strength		
E	Young's modulus (MPa)	Subs	cripts		
G	Strain energy release rate $(mJ/mm^2,$	0	At zero force		
	m N/mm)	с	Critical		
ΔG	Strain energy release rate range $(mJ/mm^2 N/mm)$	cc	Closure		
G^*	Energy dissipation per unit crack growth (mJ/mm^2)	cyc	Cyclic		
		d	With respect to displacement		
K	Stress intensity factor $(MPa\sqrt{mm})$	eff	Effective		
k	Number of divisions	Ι	Mode I		
ΔK	Stress intensity factor range (MPa $\sqrt{\text{mm}}$)	II	Mode II		
N	Cycle number	\min	Minimum		
n	Curve fit parameter	\max	Maximum		
P	Force (N)	mon	Monotonic		
R	Load ratio	pl	Plastic		
t	Time (s)	р	With respect to force		
U	Strain energy (mJ)	R	With respect to <i>R</i> -ratio		
w	Width (mm)	$^{\mathrm{th}}$	Threshold		
w	Weight factor	tot	Total		

The reason for the lack of physical understanding is that the models for FCG prediction rely on the similitude principle. While this is a very powerful principle for producing predictions, it does not necessarily produce physical insight. For true insight it is not enough to identify that something is a similitude parameter, it is also necessary to be able to explain why it is a similitude parameter. This paper will discuss how misunderstanding the limitations of the similitude principle has led to misinterpretation of the *R*-ratio effect.

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For the characterisation and prediction of FCG usually a parameter based on the strain energy release rate (SERR) is selected as a similitude parameter, e.g: G_{max} , $\Delta\sqrt{G}$, or ΔG (as defined below). This paper will discuss the physical interpretation of these parameters. Furthermore a new method of characterising FCG will be introduced by measuring the change of strain energy in the system. It will be shown how this

new view provides clues to the physical relevance of the LEFM similitude parameters.

2. The similitude principle and its limitations

Above it was mentioned that the LEFM methods for prediction of FCG rely on the similitude principle. It therefore seems appropriate to start with a brief summary of what the similitude principle is, and what its limitations are. Fundamentally, the similitude principle is a tool for making valid comparisons. When using the similitude principle, a certain parameter is selected, e.g. stress. The assumption is then that if the similitude parameter has the same value in two different cases, the material behaviour will be the same. For example, say a material undergoes tensile failure at a certain value of stress during a tensile test on a laboratory specimen. Then

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if in a full-scale structure the same value of the stress is reached somewhere in the structure, tensile failure will also occur there. Even if the external loads on the structure, and the geometry of the structure itself, are completely different than during the laboratory test.

The important point here, is that the similitude principle is based on *consistency*, rather than on physical understanding. As long as the same value of the chosen similitude parameter always results in the same

- ⁴⁰ behaviour, it is a good similitude parameter. Whether the parameter actually has physical relevance is a secondary matter. Thus finding a good similitude parameter should not be confused with gaining an understanding of the underlying physics. Returning to the example of tensile stress, that a material always fails at a certain stress value makes stress a good similitude parameter. However it does not offer any explanation as to *why* the material fails at that value of stress.
- ⁴⁵ Due to a lack of a direct connection to the underlying physics, there may be limitations to the use of a similitude parameter that are not directly apparent. For example, the ultimate tensile stress will not correctly predict failure for an object that contains cracks. This is true, even if one corrects for the reduction in cross-sectional area and concentration of stress due to the presence of the cracks [4]. These limitations are not necessarily a problem, as long as one is aware of them. However in order to understand when and why a certain similitude parameter is no longer applicable, an understanding of the physics is necessary.

Over the past 50 years, the similitude principle has been successfully employed to predict FCG rates. However it is now time to move beyond those predictions into understanding why the selected similitude parameters are appropriate, and to understand when and why they stop working.

3. Application of the similitude principle to fatigue crack growth

At their heart, most of the current approaches to fatigue crack growth prediction in adhesive bonds (and fibre reinforced composites) are based on a correlation between a SERR parameter and the crack growth rate [5], i.e:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = Cf\left(G\right)^n\tag{1}$$

where C and n are parameters determined by curve fitting, and f(G) is some function of the SERR. Usually one of the following is selected:

$$f(G) = G_{\max} \tag{2}$$

$$f(G) = \Delta \sqrt{G} = \sqrt{G_{\max}} - \sqrt{G_{\min}}$$
(3)

$$f(G) = \Delta G = G_{\max} - G_{\min} \tag{4}$$

where G_{max} is the SERR at maximum load and G_{min} is the SERR at minimum load.

Equation 1 was selected based on the famous relationship found by Paris et al. [6–8] for crack growth in metals:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K^n \tag{5}$$

where $\Delta K = K_{\text{max}} - K_{\text{min}}$ is the range of the stress intensity factor (SIF). Paris et al. did not have a direct physical justification for selecting ΔK as a similitude parameter. ΔK

was selected because Paris et al. assumed that the material behaviour at the crack tip was governed by the crack-tip stress field, which can be characterised by K. While it indeed seems logical that the crack-tip stress field governs the crack growth behaviour, it is a bit of a jump to go from this premise straight to selecting ΔK as the similitude parameter. Why should it be the range of K, and only the range of K, that is used as a similitude parameter? After all, ΔK by itself does not provide a unique description of a load

 $_{65}$ cycle. In fact there are an infinite number of different load cycles that all have the same ΔK , but will result

in different crack growth rates. Furthermore, how does using a time-independent parameter such as ΔK explain why there is only a finite amount of crack growth within a single cycle?

These questions have gone largely unanswered (and perhaps even unasked) due to the simple fact that ΔK does work as a similitude parameter. That the same ΔK will give the same crack growth rate (as long as the material and *R*-ratio are kept constant), for two different geometries and far-field applied load cycles, is not in question. What should be questioned is *why* this is so.

The lack of a clear physical justification for selecting ΔK is reflected in the uncertainty over which similitude parameter to use for FCG in adhesive bonds and composites. Originally the form of equation 1 was selected based on the success of equation 5 at predicting FCG in metals. Because K is difficult to compute for layered materials, K was replaced in equation 5 by a function of the SERR, which is allowed given the equivalence between the SIF and the SERR established by Irwin [9]:

$$G = \frac{K^2}{E'} \tag{6}$$

where E' is equal to the Young's modulus (E) in the case of plane stress, and in case of plane strain is equal to:

$$E' = \frac{E}{1 - \nu^2} \tag{7}$$

with ν the material's Poisson's ratio.

As Rans et al. have pointed out [10], if one wishes to maintain the same similitude basis as Paris et al., (i.e. ΔK) then $\Delta\sqrt{G}$ should be selected as similitude parameter. Nevertheless, the most used similitude parameters for crack growth in adhesives and composites are G_{max} and ΔG . Although these parameters apparently are suitable similitude parameters, it is important to realise that they no longer rely on the same basis for similitude as equation 5. Whereas ΔK , and thus $\Delta\sqrt{G}$, takes the crack-tip stress amplitude as the basis of similitude, G_{max} relates to the maximum crack-tip stress, and ΔG relates to the externally applied cyclic work.

- Regardless of which similitude parameter is selected, in many cases an *R*-ratio effect is seen. That is to say, if the ratio between the minimum and maximum applied loading changes, then the resulting crack growth rate will also change. This *R*-ratio effect was already considered in metals by Paris et al. [6, 7] and in the work of Roderick et al. [2] on fibre reinforced polymers bonded to an aluminium sheet. Since then an *R*-ratio effect has been reported by many researchers, e.g. [11–27].
- It is a fact of mathematics that a single parameter cannot uniquely define a cycle. Therefore, whether one selects G_{\max} , $\Delta\sqrt{G}$, or ΔG as similitude parameter, in every case there are an infinite number of load cycles that each produce the same value for the chosen parameter. Thus keeping one of these parameters constant, and then changing the *R*-ratio means applying a different load cycle to the object. That a different load cycle results in a different crack growth rate does not seem very surprising. Nevertheless most approaches to
- •• dealing with the *R*-ratio do not seem to approach the issue with this view. As will be discussed further below, many approaches to dealing with the *R*-ratio effect seem to assume that keeping the similitude parameter constant means that the driving force for crack growth is constant. The *R*-ratio effect is then treated as a change in response of the material to the applied load, rather than a change in the applied load itself.

In metals, the *R*-ratio effect is usually explained using the crack closure concept introduced by Elber [25, 28], although some researchers question this concept [29–31]. Even though the mechanical behaviour of adhesives and fibre reinforced polymer composites is different to that of metals, crack closure has been reported in adhesives and composites by several researchers [32, 33]. In analogy to the approach used in metals, Pirondi and Nicoletto accounted for a change in *R*-ratio by introducing an effective SERR range as similitude parameter, defined as:

$$\Delta G_{eff} = G_{\max} - G_{cc} \tag{8}$$

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researchers. This method also requires further investigation to verify whether indeed no R-ratio exists when $G_{\min} > G_{cc}$, which is what the closure concept implies.

An older approach to taking the R-ratio into account can be found in the works of Hojo et al. [12, 14] and Atodaria et al. [15–17]. Hojo et al. proposed the relationship [12, 14]:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K^{(1-\gamma)n} K_{\mathrm{max}}^{\gamma n} \tag{9}$$

where γ is an empirical mean stress sensitivity parameter. Atodaria et al. proposed the similar formulation [15–17]:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C \left[\left(\sqrt{G} \right)_{average}^{\gamma} \left(\Delta \sqrt{G} \right)^{1-\gamma} \right]^n \tag{10}$$

where $\sqrt{G_{average}}$ is a weighted average, determined as:

$$\sqrt{G}_{average} = \left[\frac{1}{k} \sum_{\sqrt{G_{th}}}^{\sqrt{G_{max}}} \left(\sqrt{G}\right)^w\right]^{\frac{1}{w}}$$
(11)

with k the number of increments into which the range from $\sqrt{G_{th}}$ to $\sqrt{G_{max}}$ is divided, and w an experimental weight factor.

Hojo et al. [12] based the formulation of equation 9 on the observation that for a constant da/dN value, if ΔK was plotted against (1-R), a good fit through the data points was given by an equation of the form:

$$\Delta K = \Delta K_{R=0} \left(1 - R \right)^{\gamma} \tag{12}$$

where $\Delta K_{R=0}$ is the value of ΔK extrapolated to R = 0. This is clearly a phenomenological approach, that works to predict the crack growth rate, but does not offer an explanation of the *R*-ratio effect in terms of the underlying physics.

Atodaria et al. [15–17] recognised that fatigue crack growth may occur at more points in the cycle than just at maximum load. Hence the use of the average SERR $G_{average}$. Furthermore they noticed opposite *R*-ratio effects depending on whether $G_{average}$ or ΔG was kept constant when varying the *R*-ratio. This was also seen in the present data, as will be discussed further below. As a result of these opposite *R*-ratio effects Atodaria et al. decided on an equation that multiplied $\sqrt{G_{average}}$ and $\sqrt{\Delta G}$. Again, this is a phenomenological relationship, rather than one based on an underlying physical theory.

Allegri et al [34] propose the model:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C \left(\frac{G_{\mathrm{IImax}}}{G_{\mathrm{IIc}}}\right)^{\frac{n}{(1-R)^2}} \tag{13}$$

Allegri et al. do not explain why this equation was chosen, but its similarity to the model proposed by Andersons et al [18] suggests it may be based on a similar line of reasoning. Andersons et al. propose the equation:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C_I \left(\frac{K_I}{K_{Ic}}\right)^{n_I} \tag{14}$$

where

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$$C_I = \frac{n_I}{2\pi \left(n_I - 2\right)} \left(\frac{K_{Ic}}{\sigma_s}\right)^2 \tag{15}$$

where σ_s is the static interlaminar tensile strength. The *R*-ratio was taken into account by modifying the coefficient C_I and exponent n_I according to:

$$n_R = \frac{n_I}{1-R} \tag{16}$$

$$C_R = \frac{n_I - 2}{n_I - 2(1 - R)} n_I \tag{17}$$

These expressions were derived by assuming the validity of the Goodman relationship, describing the fatigue strength at a given R-ratio as a function of the fatigue strength for R = 0.

Based on fractographic observations Khan [26] proposed a model superimposing a monotonic contribution, related to G_{max} , and a cyclic contribution related to ΔG , which resulted in the equation:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C_1 G_{\mathrm{max}}^{n_1} + C_2 \Delta G^{n_2} \tag{18}$$

A final approach to including the R-ratio effect is formulating a Hartman-Schijve [35] / Priddle [36] type equation, such as done by Andersons et al. [19], and Jones, Kinloch, et al. [37–41]. Jones et al. propose the equation:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C \left[\frac{\Delta\sqrt{G} - \Delta\sqrt{G_{th}}}{\sqrt{1 - \frac{\sqrt{G_{max}}}{\sqrt{A}}}} \right]^n \tag{19}$$

where C, n, and A are parameters that are determined by curve fitting. $\Delta\sqrt{G_{th}}$ is the threshold value of $\Delta\sqrt{G}$, defined as the value of $\Delta\sqrt{G}$ for which $da/dN = 10^{-7}$ mm/cycle, and determined separately for each experiment. Thus, apart from including both $\Delta\sqrt{G}$ and G_{max} in the equation, Jones et al. also account for the R-ratio by assuming that $\Delta\sqrt{G_{th}}$ is a function of R.

Khan [26] attempted to base his model on features of the fracture surface, and Pirondi and Nicoletto [33] based their model on crack closure measurements. In the other cases the form of the model was chosen ¹²⁰ purely based on the form of the da/dN curve, rather than on an underlying conceptual model of the physics involved in the problem. All these models are based on at least two parameters that describe the load cycle (e.g. G_{max} and R, or G_{max} and $\Delta\sqrt{G}$). Nevertheless, none of the authors involved explicitly state that two parameters are required because it is impossible to uniquely define the applied load cycle with only one parameter.

This can be most clearly seen in the latest work of Jones et al. [41]. Jones et al. claim that in equation $19 \ \Delta\sqrt{G}$ represents the crack driving force. The basis for this claim is that the *R*-ratio effect that is seen when using $\Delta\sqrt{G}$ as a similitude parameter, is the same as that which is seen when using ΔK as a similitude parameter. I.e. keeping $\Delta\sqrt{G}$ or ΔK constant and increasing *R*, increases the crack growth rate. Of course given equation 6, $\Delta\sqrt{G}$ and ΔK are equivalent parameters (see also [10]). The observation that the *R*-ratio effect is the same for both parameters is then somewhat redundant.

Apart from that, Jones et al.'s interpretation of $\Delta\sqrt{G}$ as crack driving force rests on the assumption that ΔK can be considered to be the crack driving force. However, as was pointed out above, the similitude principle relies solely on consistency. Thus the mere fact that $\Delta\sqrt{G}$ or ΔK perform well as similitude parameters for predicting crack growth rate, does not prove that interpreting these parameters as the crack driving force is valid.

It is also clear from examining equation 19 that in order for the crack growth rate to be the same in two different cases, it is not $\Delta\sqrt{G}$ that must be the same, but the entire term between the square brackets. In other words, in equation 19, $\Delta\sqrt{G}$ is *not* a similitude parameter. Rather the similitude parameter is:

Similitude parameter =
$$\frac{\Delta\sqrt{G} - \Delta\sqrt{G_{th}}}{\sqrt{1 - \frac{\sqrt{G_{max}}}{\sqrt{A}}}}$$
(20)

since any combination of $\Delta\sqrt{G}$ and G_{max} that results in the same value of equation 20, will result in the same crack growth rate.

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In [41] Jones et al. not only identify $\Delta\sqrt{G}$ as the crack driving force, they also give an overview of the different *R*-ratio effects that are seen in literature when using the different possible SERR-based similitude parameters. They show that when using G_{max} or ΔG as the similitude parameter, increasing the *R*-ratio results in a decrease of the crack growth rate. They then label this effect as 'incorrect', because they claim that increasing *R* should result in an increase of the crack growth rate, based on the behaviour seen when ΔK is used as a similitude parameter. More specifically, Jones et al. argue that an increase in the mean load, should result in an increase of the crack growth rate.

In fact, whether using G_{max} , $\Delta\sqrt{G}$, or ΔG as similitude parameters, the *R*-ratio effect can be explained in each case if one simply considers the crack growth rate to be a function of the entire load cycle, which cannot be uniquely described by only one parameter. This will be further elaborated in the following sections of the paper.

Griffith already identified the concept of an energy balance in crack growth [4]. Extending these ideas to fatigue, the amount of crack growth in a cycle must be related to 1) how much energy is required per unit of crack growth and 2) how much energy is available for crack growth during the entire fatigue cycle. For fatigue crack growth in metals, this idea was formulated by Bodner [42] as:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{\mathrm{d}a}{\mathrm{d}U_{pl}} \frac{\mathrm{d}U_{pl}}{\mathrm{d}N} \tag{21}$$

where dU_{pl}/dN is the total amount of plastic dissipation in the cycle (which is a measure of the available energy), and da/dU_{pl} is the amount of crack growth created per unit of plastic energy dissipation. Inverting da/dU_{pl} , gives dU_{pl}/da , the amount of energy dissipation per unit of crack growth. This is the amount of energy required for crack growth, which provides a measure of the resistance to crack growth.

In adhesive bonds, the amount of plasticity is likely limited when compared to crack growth in metals. Therefore rather than just using the plastic energy dissipation dU_{pl}/dN it is more appropriate to use the total energy dissipation dU/dN. This will be further discussed in the following sections. Furthermore, a set of data from fatigue crack growth experiments conducted by the authors [43, 44] will be used to illustrate the relationship between the SERR similitude parameters on the one hand, and the required and available energy on the other.

4. Test set-up and data analysis

order to enhance visibility of the crack.

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The fatigue crack growth experiments were performed on double cantilever beam (DCB) specimens, consisting of two aluminium 2024-T3 arms, bonded with Cytec FM94 epoxy adhesive. The nominal specimen width was 25 mm. The specimens were manufactured by bonding two aluminium plates according to the cure cycle specified by the adhesive manufacturer (1 hour at 120 °Cand 6 bar (0.6 MPa)). As this is an industry standard cure cycle it was assumed that following this cure cycle would produce a 100% curing of the adhesive, but this was not checked. After curing the plates were cut into strips, which were then milled to the desired dimensions. One side of the specimens was coated with diluted typewriter correction fluid in

4.1. Test procedure

Testing was conducted on an MTS 10 kN fatigue testing machine, under displacement control and at a frequency of 5 Hz. Force and displacement were measured by the fatigue testing machine and crack length was measured using a camera aimed at the side of the specimen. At the last calibration, the error in the force measurement was found to be 0.64% of the true value at 100 N force, and the error in the displacement was found to be 0.02% of the true value at 5 mm displacement. The resolution of the images taken by the camera varied slightly from test to test, depending on the exact distance between camera and specimen. Nevertheless, for all tests it was on the order of 20 pixels/mm, i.e. 0.05 mm/pixel. The scale of the photographs was determined by attaching a piece of graphing paper to the specimens, which was included in the photograph. The paper was graduated in 1 mm squares.

Prior to each test the specimen was loaded quasi-statically with a displacement rate of 1 mm/min until a maximum in the force and visual onset of crack growth were observed. Then the load was removed. From this quasi-static test the critical displacement d_c was determined, at which the force was maximum. The maximum displacement for the fatigue test was set to be equal to or slightly smaller than d_c . The minimum displacement was then set in order to obtain the desired *R*-ratio. The initial set of experiments had been aimed at obtaining certain values of $\Delta G/G_{\text{max}}$ and four different R_p ratios were obtained, i.e. $R_p = 0.036; 0.29; 0.61$ and $R_p = 0.86$, where:

$$R_p = \frac{P_{\max}}{P_{\min}} \tag{22}$$

for subsequent tests R_d was set to one of these four values, where:

$$R_d = \frac{d_{\max}}{d_{\min}} \tag{23}$$

Table 1: Mean values and standard deviation of the measured R_p and R_d for the tests performed as part of this research. The grouping used in presentation of the data is also shown.

	R_d		R_p		
Experiment	Mean	Standard deviation	Mean	Standard deviation	Group
B-001-II	0.10	$4.0 \cdot 10^{-4}$	0.036	0.0060	R = 0.036
B-002-I	0.88	$4.6 \cdot 10^{-4}$	0.86	0.0015	R = 0.86
B-002-II	0.74	$3.5 \cdot 10^{-4}$	0.61	0.015	R = 0.61
C-001-I	0.33	0.0010	0.29	0.0047	R = 0.29
C-002-D	0.67	0.0087	0.61	0.010	R = 0.61
D-002-I	0.29	$2.8597 \cdot 10^{-4}$	0.29	0.0017	R = 0.29
E-001-I	0.29	0.012	0.24	0.012	R = 0.29
E-001-II	0.29	$3.6\cdot10^{-4}$	0.27	0.0021	R = 0.29
E-002-I	$2.3\cdot 10^{-4}$	$6.3 \cdot 10^{-4}$	-0.022	0.0056	R = 0.036
E-002-II	$-9.3\cdot10^{-5}$	$4.5 \cdot 10^{-4}$	0.014	0.0047	R = 0.036
E-003-I	0.61	$7.6 \cdot 10^{-4}$	0.60	0.0029	R = 0.61
E-003-II	0.61	$3.94 \cdot 10^{-4}$	0.62	0.0027	R = 0.61

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Since the force-displacement curve did not go exactly through the origin, for the experiments, $R_d \neq R_p$. Table 1 shows the measured mean value and standard deviation of R_p and R_d for the experiments in this paper. For clarity of presentation the data was divided into 4 groups, corresponding to the obtained Rratios. This is also indicated in table 1. Due to the low crack growth rates obtained at R = 0.86 only one specimen was tested at this R-ratio, after which it was decided to focus on testing at lower R-ratios. More details on the experimental procedure can be found in [45]. The measured specimen dimensions, as well as the raw data, are available online: [43, 44].

185 4.2. Crack length measurement

Photographs were taken at regular intervals of once every 100 cycles at the start of the test, which was increased to once every 1000 cycles once the crack growth was determined to have slowed sufficiently. The photographs were taken while the specimen was held at maximum displacement. As mentioned above, the resolution of the images was roughly 0.05 mm/pixel.

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After the test the photographs were analysed to find the crack length as a function of the number of cycles. A power-law curve was fit through these data points and then the derivative was taken in order to find the crack growth rate da/dN. As a measure of the goodness of fit, table 2 shows the root mean square error (RMSE) values. An error propagation analysis was performed in order to estimate the error in the calculated da/dN values, which is also shown in table 2. This was based on the 95% confidence interval for the fitting parameters of the *a* vs *N* fit, assuming a normal distribution.

4.3. Strain energy release rate calculation

Strain energy release rates were calculated according to the compliance calibration (CC) method described in ASTM standard D5528-01 [46], i.e.

$$G = \frac{nPd}{2wa} \tag{24}$$

Table 2: Root mean square error (RMSE) for the fit of crack length a as a function of number of cycles N , and the maximum
propagated error in da/dN based on the 95% confidence interval of the fitting parameters.

Experiment	Absolute RMSE (mm)	RMSE relative to lowest a value	Maximum error in $\mathrm{d}a/\mathrm{d}N$
B-001-II	0.41	0.34%	0.59%
B-002-I	0.77	1.25%	2.81%
B-002-II	0.85	0.95%	1.65%
C-001-I	0.61	0.99%	0.77%
C-002-D	1.1	1.24%	3.29%
D-002-I	0.38	0.40%	20.2%
E-001-I	0.16	0.28%	2.11%
E-001-II	0.87	0.83%	70.7%
E-002-I	0.44	0.67%	24.6%
E-002-II	0.57	0.46%	24.9%
E-003-I	0.50	0.88%	23.3%
E-003-II	0.30	0.37%	21.4%



Figure 1: Test set-up showing the position of the crack growth camera with insert showing the loading block attachment (a), and definition of load application and crack length a (b).

Table	3: CC Correct	ion parame	eters.
	Specimen	\overline{n}	
	B-001-II	3.267	
	B-002-I	1.863	
	B-002-II	3.936	
	C-001-I	3.198	
	C-002-D	2.741	
	D-002-I	3.122	
	E-001-I	2.832	
	E-001-II	3.769	
	E-002-I	3.060	
	E-002-II	3.868	
	E-003-I	3.160	
	E-003-II	3.635	

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where P is the force, d is the displacement, w is the specimen width, a is the crack length, and n is a correction coefficient. This coefficient is equal to the slope of a line through the log(compliance) versus log (crack length) data [46]. A correction coefficient was determined for each specimen individually, as shown in table 3.

ASTM standard D5528-01 provides three methods for the calculation of the SERR. As the resulting Gvalues were found to not differ much [43], the CC method was selected due to its ease of use.

Based on the accuracy of the force and displacement measurements, the 95% confidence bound of the curve fit used to calculate n, and estimating the error in the crack length measurements to be on the order of 5 pixels, the error in the calculated G values is estimated to be less than 2%.

4.4. Energy dissipation

Every 100 cycles the minimum and maximum displacement and force was recorded by the fatigue machine. By assuming linear elastic behaviour the entire force-displacement curve could then be reconstructed. The assumption of linear elasticity was verified with one specimen by applying 10 load cycles at a displacement rate of 1 mm/min at intervals of 10,000 cycles and recording the full force-displacement curve [47]. With the force-displacement curve, the amount of strain energy in the system, U, can be determined as the area under this curve, i.e. assuming linear elasticity:

$$U = \frac{1}{2}P(d - d_0)$$
(25)

where d_0 is the displacement for which the force is zero. Based on the accuracy of the force and displacement measurements, the error in the calculated U value is estimated to be 0.64%.

As shown in figure 2 the amount of strain energy in the system will decrease as the test progresses and the crack grows, as the specimen is loaded in displacement control. Thus by fitting a function through 210 the data for strain energy vs number of cycles and then taking the derivative, one can obtain dU/dN, the change in strain energy per cycle. As a measure of the goodness of these fitting functions, table 4 shows the resulting RMSE values. Table 4 also shows the estimated maximum error in dU/dN, based on on an error propagation analysis of the 95% confidence interval for the fitting parameters of the U vs N fit.



Experiment	Absolute RMSE (mJ)	RMSE relative to lowest U_{tot} value	Maximum error in dU/dN
B-001-II	3.0	0.92%	0.09%
B-002-I	9.0	1.4%	0.88%
B-002-II	2.1	1.2%	0.14%
C-001-I	2.1	0.87	0.11%
C-002-D	2.5	0.71%	0.10%
D-002-I	0.81	0.21%	0.03%
E-001-I	1.0	0.52%	0.15%
E-001-II	1.2	0.36%	0.22%
E-002-I	2.1	0.76%	0.41%
E-002-II	1.1	0.43%	0.12%
E-003-I	1.4	0.54%	0.77%
E-003-II	1.5	0.37%	15.%

Table 4: Root mean square error (RMSE) for the fit of strain energy U_{tot} as a function of number of cycles N, and the maximum propagated error in dU/dN based on the 95% confidence interval of the fitting parameters.



Figure 2: Calculation of the energy dissipation dU/dN, as well as the definitions of the monotonic energy, U_{mono} , the cyclic energy U_{cyc} , and the total energy U_{tot} .



Figure 3: Crack growth rate plotted versus G_{max} , as well as a schematic representation of the effect of keeping G_{max} constant and changing the *R*-ratio.

Figure 2 also shows that the total strain energy in the system U_{tot} can be subdivided into two parts. The first part is the monotonic energy U_{mono} , which is added to the specimen during the first load cycle. This is equal to the area under the curve from $(d_0,0)$ to (d_{\min},P_{\min}) . If there was no crack growth this energy would remain in the specimen for the entire duration of the test. However if there is crack growth, then some of this energy may be released.

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The second part is the cyclic energy or applied work U_{cyc} , which is equal to the area under the curve from (d_{\min}, P_{\min}) to (d_{\max}, P_{\max}) . This energy is added to the specimen during the loading portion of the fatigue cycle, or in other words, it is the work performed on the specimen by the fatigue machine. If no dissipation occurs it would of course all be returned to the fatigue machine during the unloading portion of the load cycle. However, if crack growth or other dissipative mechanisms do occur some of this energy will be dissipated, rather than returned to the fatigue machine. The specimen is loaded in displacement control, which means that any energy dissipated due to crack growth will not be replaced. Therefore U_{cyc} decreases as the test progresses and the crack grows. Since it is U_{cyc} that is added to the specimen during each fatigue cycle (and returned again each cycle during the unloading, minus any dissipated energy), it shall be referred to by the term applied work in this paper.

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5. Results - *R*-ratio effect

Figures 3 through 6, show the crack growth rates measured in the experiments plotted against the typical SERR similitude parameters of G_{max} , $(\Delta\sqrt{G})^2$, and ΔG . In order to maintain consistency of the units, the example of Rans et al. [10] has been followed, and $(\Delta\sqrt{G})^2$ has been used as a similitude parameter, rather than $\Delta\sqrt{G}$. $(\Delta\sqrt{G})^2$ is defined as:

$$(\Delta\sqrt{G})^2 = \left(\sqrt{G_{\max}} - \sqrt{G_{\min}}\right)^2 \tag{26}$$

Apart from the test data, the figures also illustrate schematically the effect of keeping the similitude parameter constant, while changing the R-ratio, on the applied G cycle and on the applied work (the area under the force-displacement line).

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Let us first examine the case of using G_{max} as a similitude parameter, as shown in figure 3. It is clear that if G_{max} is kept constant, increasing the *R*-ratio results in a reduction of the crack growth rate. Jones



Figure 4: Crack growth rate plotted versus G_{max} . The error bars show the estimated 95% confidence intervals for the da/dN values. Note that at this scale the error bars for some experiments are too small to be clearly visible.



Figure 5: Crack growth rate plotted versus $(\Delta\sqrt{G})^2$, as well as a schematic representation of the effect of keeping $(\Delta\sqrt{G})^2$ constant and changing the *R*-ratio.



Figure 6: Crack growth rate plotted versus ΔG , as well as a schematic representation of the effect of keeping ΔG constant and changing the *R*-ratio.

et al [41] have labelled this effect as 'incorrect', because they expect an increased R-ratio to lead to an increased mean load, and therfore to an increased crack growth rate.

To examine whether the observed R-ratio effect is merely the cause of measurement uncertainty, figure 4 shows again the results displayed in figure 3, but now including error bars indicating the estimated 95% confidence interval for da/dN. It is clear that even if the measurement uncertainty is taken into consideration, an R-ratio effect remains.

As illustrated schematically in figure 3, keeping G_{max} constant while increasing R corresponds to keeping G_{max} constant and increasing G_{min} . This will indeed lead to an increase of the mean load. However, it will also lead to a reduction of both the load range (i.e. ΔG or $\Delta \sqrt{G}$) and the applied work.

The effect of increasing R while keeping $(\Delta\sqrt{G})^2$ constant is shown in figure 5. From the experimental data it is clear that increasing R, with $(\Delta\sqrt{G})^2$ constant, results in an increase of the crack growth rate. Now let us examine the effect on the load cycle. Increasing R for constant $(\Delta\sqrt{G})^2$ results in an increased mean and maximum load. Furthermore, if $(\Delta\sqrt{G})^2$ is kept constant and R is increased, the applied work U_{cyc} performed on the specimen is also increased. This can be shown with some algebra as follows [45]: From the definitions of R and G we have:

$$(\Delta\sqrt{G})^2 = (1-R)^2 G_{\text{max}}$$
⁽²⁷⁾

Then keeping $(\Delta \sqrt{G})^2$ constant and changing the *R*-ratio gives:

$$\Delta \sqrt{G})_2^2 = (\Delta \sqrt{G})_1^2 \tag{28}$$

$$(1 - R_2)^2 G_{\max,2} = (1 - R_1)^2 G_{\max,1}$$
⁽²⁹⁾

$$G_{\max,2} = \frac{(1-R_1)^2}{(1-R_2)^2} G_{\max,1}$$
(30)

setting $R_2 = R, R_1 = 0$, one can then obtain which value of G_{max} will produce the desired $(\Delta \sqrt{G})^2$ value, for any *R*-ratio:

$$G_{\max} = \frac{1}{(1-R)^2} G_{\max,R=0}$$
(31)

Assuming linear elasticity, the applied work can be written as:

$$U_{cyc} = \frac{1}{2} P_{\max} d_{\max} - \frac{1}{2} P_{\min} d_{\min}$$
(32)

$$= (1 - R^2) P_{\max} d_{\max} \tag{33}$$

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Introducing for convenience the coefficient η , defined as:

$$\eta = \frac{2wa}{n} \tag{34}$$

one can rewrite equation 33 to:

$$U_{cyc} = \left(1 - R^2\right) \frac{G_{\max}}{\eta} \tag{35}$$

Thus if $(\Delta \sqrt{G})^2$ is kept constant, and the *R*-ratio is increased from 0, U_{cyc} can be obtained by inserting equation 31 into equation 35 to give:

$$U_{cyc} = \frac{(1-R^2)}{(1-R)^2} \frac{1}{\eta} G_{\max,R=0}$$
(36)

Since $(1-R)^2 < (1-R^2)$, this means U_{cyc} will increase if $(\Delta\sqrt{G})^2$ is kept constant while R is increased.

The last case is using ΔG as a similitude parameter, as shown in figure 6. For the current experimental results there is no clear systematic *R*-ratio effect. In literature usually a reduction of the crack growth rate is reported [10, 41] if ΔG is kept constant and *R* is increased. From the definition of the *R*-ratio it follows that in order to keep ΔG constant and increase *R*, the mean load and also the maximum load must increase. The effect on U_{cyc} can again be determined using the approach that was introduced above. The definitions of ΔG and *R* give [45]:

$$\Delta G = \left(1 - R^2\right) G_{\max} \tag{37}$$

therefore, keeping ΔG constant and increasing R from zero implies that G_{max} must equal:

$$G_{\max} = \frac{1}{1 - R^2} G_{\max, R=0}$$
(38)

inserting this into equation 35 gives:

$$U_{cyc} = \left(1 - R^2\right) \frac{G_{\max}}{\eta} \tag{39}$$

$$= \frac{(1-R^2)}{(1-R^2)} \frac{1}{\eta} G_{\max,R=0}$$
(40)

$$=\frac{1}{\eta}G_{\max,R=0}\tag{41}$$

in other words: if R is increased, but ΔG is kept constant, then U_{cyc} will remain constant as well.

This section has presented the experimentally determined effect on the crack growth rate of changing the R ratio while keeping one of the SERR similitude parameters $(G_{\max}, (\Delta\sqrt{G})^2, \text{ or } \Delta G)$ constant. It has also presented the effect of changing the R-ratio on the mean load and the applied work U_{cyc} based on a theoretical linear elastic force-displacement curve. Table 5 summarises these results. It is clear that the effect of the R-ratio depends on what is considered to be the fundamental similitude parameter that is being held constant. Furthermore it is clear that there is no direct relationship between a change in either the mean load or maximum load and the crack growth rate. Increasing the mean load could lead to either an increase or a decrease of the crack growth rate, depending on which parameter is being kept constant, and the same goes for the maximum load and U_{cuc} .

Table 5 therefore indicates that the *R*-ratio effect should not be thought of as a modification of the fundamental relationship between a single SERR parameter and the crack growth rate. Rather, the crack growth rate is a function of the entire load cycle, which can only be described by using multiple parameters, e.g. G_{max} and ΔG , or $(\Delta \sqrt{G})^2$ and *R*. The question then becomes whether it is possible to explain the qualitative relationships shown in 5 based on such a multiple parameter viewpoint, and whether a physical interpretation of the SERR parameters can be hypothesised that explains these relationships.

The next section will use the energy approach to further explore the physical meaning of the SERR similitude parameters. In section 7 it will be shown how this physical interpretation can explain the relationships shown in table 5.

Table 5: Effect of changing the *R*-ratio while keeping one SERR parameter constant. In the present research for constant ΔG no clear effect of *R*-ratio on da/dN could be identified, but in literature a decrease of da/dN for an increase of *R* is usually reported [41].

	Effect of increasing R on:			
Constant parameter	$\mathrm{d}a/\mathrm{d}N$	$\mathrm{mean}\ G$	$\max G$	U_{cyc}
G_{\max}	decrease	increase	$\operatorname{constant}$	decrease
$(\Delta \sqrt{G})^2$	increase	increase	increase	increase
ΔG	decrease	increase	increase	$\operatorname{constant}$



Figure 7: Crack growth rate versus energy dissipation per cycle. A power-law curve fit through the combined data for all experiments is also shown. [48].

6. Results - Energy dissipation

As discussed in section 4, both the energy dissipation per cycle dU/dN and the crack growth rate da/dNwere determined from the measured data. Figure 7 shows a comparison of energy dissipation and the crack growth rate in each cycle. From figure 7 it is clear that there is a very strong correlation between the crack growth rate and the energy dissipation, as would be expected. With the exception of one outlier (experiment B-002-II; R = 0.61), the data for all the experiments falls within a narrow band, irrespective of *R*-ratio. The reason for the offset between the data for experiment B-002-II and the other experiments could not be determined. Although the curves for the different experiments collapse to a narrow band, upon closer inspection a small *R*-ratio effect can still be seen.

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Before discussing the *R*-ratio effect further, it should be noted that the exponent of a power-law curve fit through the data shown in figure 7 is not equal to 1. This means that the amount of energy dissipated per unit of crack growth was *not constant* over the course of the test. At high crack growth rates, more energy was dissipated per unit of crack growth, than at low crack growth rates. In other words, if the energy dissipation in a cycle dU/dN is increased by a factor of 2, the crack growth rate will only increase by a factor of $2^{0.86} \approx 1.8$. This suggests that at high crack growth rates additional dissipative mechanisms are activated that do not contribute to crack growth.

The energy dissipation per unit of crack growth can be represented with the parameter G^* , introduced



Figure 8: Energy dissipation versus G_{max} and $(\Delta \sqrt{G})^2$ for a crack growth rate of 10^{-4} mm/cycle, including a linear fit through the data. The data for experiment B-002-II was excluded as an outlier in the determination of the linear fit. Each point corresponds to a different experiment [45].

in [45] as:

$$G^* = -\frac{1}{w} \frac{\mathrm{d}U/\mathrm{d}N}{\mathrm{d}a/\mathrm{d}N} \tag{42}$$

- G^* can be interpreted as the average strain energy release rate during a cycle. However, it should be noted that G^* is not necessarily equal to the mean value of the applied G cycle. Since G^* represents the amount of energy that must be dissipated to create a unit of crack growth, it can be interpreted as a measure of the material's resistance to crack growth; if G^* is higher, more energy must be dissipated to create the same amount of crack growth. As mentioned above, the fact that the exponent of the power-law fit through the dU/dN vs da/dN data is not equal to one, implies that G^* is not constant during a crack growth test.
- To further investigate this matter, it is useful to examine the energy dissipation for a fixed amount of crack growth. Figures 8 and 9 show the energy dissipation measured during each test at the point where the crack growth rate was equal to 10^{-4} mm/cycle. Since the crack growth rate was the same for each point, G^* can readily be estimated by dividing dU/dN by $25 \cdot 10^{-4}$ (the crack growth rate times the nominal
- width). This is shown on the right hand axis of each panel. It is clear that the amount of energy dissipation was different in the different tests, even though the crack growth rate was the same. This implies that the resistance to crack growth, i.e. the amount of energy required per unit of crack growth, was different in each test.

To find out which parameter(s) control(s) the crack growth resistance, in the figures the energy dissipation is plotted against the different LEFM parameters. It is clear there is a strong linear correlation between the energy dissipation and G_{max} , and a weaker correlation between the energy dissipation and $(\Delta\sqrt{G})^2$. As before, experiment B-002-II is an outlier. The reason for this could not be ascertained. However, given that the behaviour of B-002-II was anomalous, even when compared to the other experiments performed at the same *R*-ratio, it was considered to be appropriate to exclude the data point for B-002-II in the determination of the linear fit.



Figure 9: Energy dissipation versus ΔG for a crack growth rate of 10^{-4} mm/cycle [45].

Although the energy dissipation is correlated to G_{max} and $(\Delta \sqrt{G})^2$, there does not appear to be a correlation between the energy dissipation and ΔG . It should be noted that in figure 8, G_{max} and $(\Delta \sqrt{G})^2$ are not independent, since the crack growth rate of 10^{-4} mm/cycle is the result of a load cycle that is defined by a combination of G_{max} and $(\Delta \sqrt{G})^2$. Thus a higher G_{max} implies a lower matching $(\Delta \sqrt{G})^2$ and vice versa. This means that figure 8 cannot be used to tell whether the energy dissipation is correlated to $G_{\rm max}$, $(\Delta\sqrt{G})^2$, or the combination of both. However, this can be decided by plotting G^* as a function of these parameters over the entire test range, as is done in figure 10

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This figure shows that the correlation between G^* and $(\Delta \sqrt{G})^2$ depends on the *R*-ratio, whereas there is a linear correlation between G^* and G_{\max} that is not affected by R. This implies that G^* , and therefore the resistance to crack growth, is correlated to $G_{\rm max}$, and not to $(\Delta \sqrt{G})^2$. A possible explanation is that 320 the resistance to crack growth, i.e. how much energy must be dissipated per unit of crack growth, mainly depends on the maximum stress at the crack tip. At higher crack tip stresses, it may be expected that more mechanisms are activated that dissipate energy, without contributing to crack growth. For example the higher the crack tip stresses, the more plastic strain – and thus energy dissipation – there will be around the crack tip, which might have a shielding effect. As G_{max} provides a measure for the crack-tip stress, it 325 makes sense that G_{\max} is correlated to the resistance to crack growth.

Of course crack growth resistance is only half the story. If the energy dissipation per unit of crack growth is known, then the total amount of crack growth achieved in the cycle will depend on the total amount of energy dissipated in that cycle [42, 45]. As discussed in section 4.4, dU/dN provides a measure for the energy dissipation in a cycle. To investigate what determines the magnitude of dU/dN, the same strategy 330 as before was followed, only instead of keeping the crack growth rate fixed, now G^* was kept fixed. This is shown in figures 11 through 13. These figures show the energy dissipation for the point in each test at which G^* was equal to 0.7 mJ/mm². Since G^* was equal for all these data points, there is a one-to-one

- correspondence between $\mathrm{d}U/\,\mathrm{d}N$ and $\mathrm{d}a/\,\mathrm{d}N$ for these figures.
- In figures 12 and 13 power law fits through the data are shown. In the left panel of figure 12) (dU/dN)335 vs $(\Delta \sqrt{G})^2$) the data for experiment B-002-II (R = 0.61) is anomalous. The reason for this could not be ascertained. However, given that the behaviour of B-002-II was so different, compared to the other tests at the same R-ratio, it was considered appropriate to exclude the data point for B-002-II when producing the curve fit. Similarly in the left panel of 13 (dU/ dN vs U_{tot}) the data point for B-002-I (R = 0.86) does not match the rest of the data. Again the cause for this behaviour could not be determined. Since B-002-I was 340
- the only experiment conducted at R = 0.86 the possibility that this behaviour is caused by the very high *R*-ratio cannot be ruled out. However, this would have to be cause by some sudden change in behaviour, occurring for 0.61 < R < 0.86. Therefore, the data point for B-002-I was neglected as an outlier when



Figure 10: G^* as a function of both G_{max} and $(\Delta \sqrt{G})^2$, including a linear fit of G^* versus G_{max} . The data for experiment B-002-II was excluded from this fit as an outlier [45].

producing the curve fit shown in the graph.

- Per the first law of thermodynamics, the total amount of energy dissipated in a cycle must be equal to the amount of energy available for crack growth. Thus dU/dN is a measure for the total amount of energy available in each cycle. When G^* is fixed, there is no correlation between G_{max} and dU/dN, see figure 11. However, as can be seen in figures 12 and 13, there is a clear correlation between dU/dN and ΔG , $(\Delta \sqrt{G})^2$, U_{cyc} , and U_{tot} .
- The exact nature of these correlations is at present still unclear, as is the question whether they are all meaningful. After all, the parameters ΔG , $(\Delta \sqrt{G})^2$, U_{cyc} and U_{tot} are not independent in this representation: increasing one will result in an increase of all the others. On the basis that fatigue crack growth must ultimately be driven by the energy input into the system during the load cycle, the correlation between dU/dN and U_{cyc} would seem to be the most fundamental relationship. However, the present graphs don't provide enough evidence to draw this as a firm conclusion.
- Another question that needs to be investigated further is why the relationship between the amount of energy available for crack growth (dU/dN) and the applied work (U_{cyc}) is not linear. For the shown G^* value, increasing U_{cyc} by a factor of 2 will increase the amount of available energy by a factor of $2^{3.78} \approx 13.7$. This implies that at higher levels of applied work, also a greater portion of that work is available for crack growth [45]. The reason for this is not yet clear.

What is clear in any case is that the amount of energy available for crack growth is correlated to U_{cyc} , U_{tot} , $(\Delta\sqrt{G})^2$, or ΔG . These parameters depend mainly on the load range. On the other hand, the resistance to crack growth is correlated to G_{max} , i.e. the maximum load. Thus the results presented in this section point to a physical interpretation of the meaning of the SERR parameters, and their influence on crack growth.

 G_{max} relates to the maximum applied load and therefore correlates to the resistance to crack growth. On the other hand, ΔG and $(\Delta \sqrt{G})^2$ relate to the load range, and to the applied work, and thereby form a measure for the amount of energy available for crack growth (dU/dN). The total amount of crack growth that will occur in a cycle then depends on the combination of the amount of energy available for crack growth, for which dU/dN is a measure, and the amount of energy required for crack growth, for which G^* is a measure.



Figure 11: Energy dissipation as a function of G_{max} for a fixed value of $G^* = 0.7 \text{mJ/mm}^2$. Since G^* is fixed, each value of dU/dN corresponds to a different crack growth rate. Each point corresponds to a different experiment.



Figure 12: Energy dissipation as a function of $(\Delta\sqrt{G})^2$ (left panel) and ΔG (right panel) for a fixed value of $G^* = 0.7 \text{ mJ/mm}^2$, including power-law fits through the curves. For the dU/dN versus $(\Delta\sqrt{G})^2$ fit, the data for experiment B-002-II was excluded as an outlier. Since G^* is fixed, each value of dU/dN corresponds to a different crack growth rate [45].



Figure 13: Energy dissipation as a function of U_{tot} (left panel) and U_{cyc} (right panel) for a fixed value of $G^* = 0.7 \text{ mJ/mm}^2$, including power-law fits through the curves. For the U_{tot} versus $(\Delta \sqrt{G})^2$ fit, the data for experiment B-002-I was excluded as an outlier. Since G^* is fixed, each value of dU/dN corresponds to a different crack growth rate [45].

370 7. Discussion

With the physical interpretation for G_{max} , $(\Delta \sqrt{G})^2$, and ΔG presented in the previous section, the *R*-ratio effect when using these parameters as similitude parameters (table 5) can now be qualitatively explained.

7.1. Explanation of the R-ratio effect

- If G_{max} is used as similitude parameter, then keeping G_{max} constant and increasing R results in a decrease of the load range, and thus $(\Delta\sqrt{G})^2$ and ΔG , and a decrease in the applied work (U_{cyc}) . As G_{max} is constant, so is G^* , implying the amount of energy required per unit of crack growth remains constant. The net effect is that the crack growth rate decreases.
- Keeping $(\Delta\sqrt{G})^2$ constant and increasing R results in an increase of G_{max} and an increase of U_{cyc} . The increase of G_{max} corresponds to a greater resistance to crack growth (as measured by G^*). However the increase in U_{cyc} corresponds to an increase in the amount of available energy. The resistance correlates linearly with G_{max} , whereas the correlation of the amount of available energy (as measured by dU/dN) with U_{cyc} is non-linear, with an exponent greater than 1. Thus the increase in available energy will outweigh the increase in resistance to crack growth, and the net effect will be an increase of the crack growth rate. Note
- that it is the increase in the cyclic work that drives the increased crack growth rate, whereas the increased mean load (and therefore also increased maximum load) in fact causes an increase in the resistance to crack growth, and therefore a decrease in the crack growth rate. In other words, keeping $(\Delta\sqrt{G})^2$ constant and increasing the mean load produces two opposite effects on the crack growth rate. That the crack growth rate in the end increases is due to the net result of these two opposite effects.
- Finally, keeping ΔG constant and increasing R also results in an increase of G_{\max} , but in this case U_{cyc} remains constant. Thus again the resistance to crack growth is increased, while the amount of available energy now remains constant. Thus the net result is a decrease of the crack growth rate, which is what is usually reported in literature [10, 41], although the effect was not very visible for the present results (figure 6). Note that this shows that the role of the mean load is usually misinterpreted in the literature. The crack growth rate increases only when U_{cyc} increases. Any increase of the mean load that also results in an increase of the maximum load, will in fact *lower* the crack growth rate, rather than cause it to increase.

The arguments presented above should make it clear that the *R*-ratio effect that is reported in literature is largely caused by an incorrect description of the load cycle. The amount of crack growth in a cycle is determined by the entire load cycle, which it is impossible to describe with a single parameter. It is only by using a two-parameter based description of the load cycle (e.g. G_{max} and ΔG , or $(\Delta \sqrt{G})^2$ and *R*, or some other combination) that the load cycle is uniquely defined, and similitude is maintained.

Furthermore, calling the *R*-ratio effect a mean load effect is incorrect. The effect of changing the mean load will depend on whether only the load range changes, or also the maximum load. In other words, the effect of changing the mean load will depend on what other load parameters are kept constant.

⁴⁰⁵ Previous research has identified physical mechanisms that would also produce an *R*-ratio effect (e.g. crack closure). It is not the contention of the authors that these mechanisms do not exist. However, in order to properly measure their effect on crack growth, it is important to first properly describe the applied load cycle. Only once a correct and unique description of the applied load cycle is used, can the effect of physical mechanisms such as crack closure be determined.

410 7.2. Generality of the results

This paper presents results only for mode I crack growth, for only one material type, i.e. an epoxy adhesive, and for only one specimen geometry. Thus a discussion on the generality of the results presented here is appropriate.

Firstly, it should be acknowledged that it is not yet clear how to apply the energy dissipation methodology employed here to the case of mode II cracks, especially for *R*-ratios smaller than zero (i.e. reversed shear). The key argument of this paper is that the *R*-ratio effect can largely be explained by the observation that the resistance to crack growth correlates to G_{max} (and thus the maximum load), whereas the amount of energy available for crack growth correlates to U_{cyc} (and thus to the load range). Whether these correlations also hold for mode II crack growth requires further investigation.

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The experiments described here were all performed at the same loading frequency (5 Hz). It is known that loading frequency can affect the crack growth rate. Therefore the specific numerical correlations found here may be expected to be frequency dependent. However, as long as visco-elastic effects remain negligible, it is expected that the general trends that were identified will remain applicable.

- In the present study, the crack lengths were all measured from the side of the specimen. Post-mortem investigation showed that the crack front was curved. The crack length at the centre of the specimen was several millimetres longer than near the edges. This curvature may introduce errors into the calculation of G [49, 50] and G^* , as for both calculations a straight front was assumed. Similar curvature was seen for cracks of different lengths, which suggests that the curvature remains more or less constant during the growth of the crack. Since this paper looked mainly at the crack growth rate, it is thought that a constant
- $_{430}$ curvature would not have much effect on the results discussed here. Similarly, although the curvature might introduce an error into the calculation of G, it is thought that this error will be constant. Therefore, although the numerical values of the presented correlations may be affected, the existence and general form of the correlations is still valid.

After the fatigue tests were performed the specimens, with the exception of B-001-II, were opened and the fracture surfaces were examined. Adhesive residue was observed on both fracture surfaces, indicating that cohesive failure took place inside the adhesive layer. Thus the relationships that were identified in the previous chapter may only apply to cohesive failure. Indeed there is already some evidence that a change of failure mechanisms may change these relationships [45]. A more detailed fractographic analysis may be found in [45].

- Another point that needs investigation is whether the correlations mentioned above also hold for other materials. A strong correlation between crack growth rate and energy dissipation has been found in other types of materials, including glass-fibre / epoxy composites [51, 52], carbon-fibre / epoxy [53], and metals [54–60]. Therefore it seems very likely that the energy method used here could also be used to characterise fatigue crack growth in other materials. Whether the correlations between crack growth resistance and
- available energy on the one hand, and the load cycle on the other, that were found in this case are also applicable to other materials needs to be investigated further. However, the literature already provides some clues. For example Ranaganathan et al. [59, 60] have shown that the amount of energy needed per unit of crack growth is not necessarily a constant, but can depend on the applied load. More recently Hogeveen investigated fatigue crack growth in aluminium 2024 [54]. He found that if the maximum load was increased,
- the roughness of the fracture surfaces also increased. This would likely result in an increase in the amount of energy required per unit of projected crack area, giving effectively an increase in the resistance to crack growth. Alderliesten [31] examined *R*-ratio corrections in metals. He concluded that if one formulates an *R*-ratio correction based on keeping U_{cyc} constant, one obtains numerical values that are very close to the *R*-ratio corrections based on the plasticity induced crack closure concept. This suggests that also in metals U_{cyc} correlates to the driving force for crack growth.

Clearly, more research is needed before the results of this paper can be generalised to cover all materials. However the literature does provide indications that also in other materials crack growth resistance is correlated to the maximum load. At the same time, based on basic mechanics the amount of cyclic energy is always related to the load range. It also seems logical that putting more energy into the system would also result in more energy available for crack growth. This is further backed by the analysis of Alderliesten [31]. Thus the results presented in this paper are expected to hold for other materials as well.

In particular it is expected that two different trends may be active, i.e. that the resistance to crack growth, and the amount of energy available for crack growth are affected by different properties of the load cycle, and may in fact counteract each other (e.g. both resistance to crack growth, and amount of

- energy available might increase). Any net '*R*-ratio effect' that is identified may therefore be the result of two different trends, which may even have opposite effects on the crack growth rate. *R*-ratio effects in all materials should be investigated from this point of view.

8. Conclusion

By measuring energy dissipation, and using this approach to characterise the fatigue crack growth behaviour of an adhesive bond, a physical interpretation was found for the commonly used LEFM similitude parameters: G_{max} , $(\Delta\sqrt{G})^2$, and ΔG . It was shown that the resistance to crack growth (in terms of required energy dissipation per unit of crack growth) correlates to the maximum applied load, and therefore G_{max} provides a measure of the resistance.

The energy dissipation per cycle dU/dN provides a measure of the amount of energy available for crack growth. It was found that dU/dN depends on the load range and/or the applied work U_{cyc} . Thus $(\Delta\sqrt{G})^2$ and ΔG provide a measure for the amount of available energy.

The total amount of crack growth depends on both the available energy and the resistance to crack growth, i.e. it depends on both the maximum load, and the load range. These physical interpretations were used to show that the *R*-ratio effects that are usually reported in literature, are caused by relying on an incomplete description of the load cycle. Keeping one of the LEFM similitude parameters constant and changing the *R*-ratio amounts to applying a different load cycle the specimen. Therefore the *R*-ratio effect should primarily be understood as the effect of applying a different load cycle, rather than a change in material behaviour as such. Various physical mechanisms (e.g. crack closure) have been suggested as the cause of the *R*-ratio effect. However, in order to correctly measure their effect, it is important to use a complete and unique (in the mathematical sense) description of the applied load cycle.

Furthermore it was shown that the mean stress (or mean load) effect is misinterpreted in the literature. Any increase of the mean load that also results in an increase of the maximum load will in fact result in an increase in the resistance to crack growth. The crack growth rate will only increase if U_{cyc} is increased. When using ΔK (or, equivalently, $(\Delta \sqrt{G})^2$) as a similitude parameter, it is often reported that the crack

growth rate increases when the mean load is increased. However, in this case both the maximum load and U_{cyc} are increased. The increase of U_{cyc} , and thus the amount of available energy, outweighs the increase of the resistance related to the increased maximum load. Thus the resulting net effect is an increased crack growth rate. It should be kept in mind that this increase is due to the net result of two competing effects, and is not a simple function of the increase in mean load.

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