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Review

A review on predicting critical collapse pressure of flexible risers for ultra-deep oil and gas production

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A R T I C L E I N F O

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A B S T R A C T

Flexible riser is a key enabler for the oil and gas production in ultra-deep water which transports production fluids between floating production systems and subsea wells. As oil and production heads to water depths in excess of 3000 m, high hydrostatic pressure has been one primary challenge facing the riser operators. Excessive hydrostatic pressure may cause collapse failure of flexible risers and thus predicting the critical collapse pressure is of significant importance to their anti-collapse design. Collapse is a complex phenomenon related to the material properties, the geometry of the pipe and its overall surface topography and, therefore, makes the prediction of critical pressure challenging. Related prediction approaches of flexible risers have been developed for decades, yet a comprehensive review of their predictive capabilities, efficiency and drawbacks is lacking. This paper reviews the recent advances on collapse studies of flexible risers and highlights the gaps in existing prediction methods, aiming to facilitate the current anti-collapse design and be a baseline for future utilization of flexible risers in deeper water expansion.

1. Introduction

As the offshore oil industry continuously moves into ever deeper water, there is an increasing demand for the development and qualification of production riser systems to enable this expansion. Flexible riser, a primary riser device for floating production, is being required to meet such a demand.

Flexible riser is one kind of flexible pipes which transports fluid between subsea facilities and topside structures [1], as shown in Fig. 1. It consists of multiple layers of wound metal bands and extruded polymers. The polymeric layers work as sealing, anti-wear and/ or heat-insulated components while the metallic layers withstand the imposed loads, e.g. radial inward forces, internal pressure and axial tension [2,3]. The function and the most commonly used materials of each layer are listed in Table 1 [4,5]. This pipe-like structure has been applied to shallow water production over four decades with an established technology due to the advantages of flexibility and corrosion resistance. However, cost and technical challenges increase significantly with water depth, requiring the development of the flexible riser technology.

High valued external pressure in deep water fields (increases about ten atmospheres for every 100 m of water depth) makes flexible riser vulnerable to be collapsed, especially for the curved portion within the touchdown zone, as shown in Fig. 2 [6]. Anti-collapse capability is usually regarded as an essential qualification factor for those ultra-deep water flexible risers. With flexible risers being contemplated for water depths of nearly 3000 m, their anti-collapse capability may govern riser design and the final production cost [7,8].

At present, a comprehensive overview of collapse studies of flexible risers under deep sea environment is lacking. Such an overview is urgently needed to highlight current research gaps and pave the way for the future development. This paper is intended to introduce the development of collapse studies of flexible risers and elucidate the limitations of existing available prediction methods, which is organized as: following the introduction, Section 2 clarifies the common collapse types of flexible risers in deep water and the problems lie in standards with regard to the prediction of critical collapse pressure. Section 3 is focused on the existing prediction approaches of critical pressure of flexible risers while Section 4 elaborates the studies related to the factors that affect the collapse resistance of the flexible risers. Section 5 concludes the work.

2. Collapse failure of flexible risers

Harsh operating environments in deep/ ultra-deep water fields impose a variety of potential failure modes on flexible risers, such as collapse, burst, lateral buckling/ bird-caging buckling and fatigue, etc.

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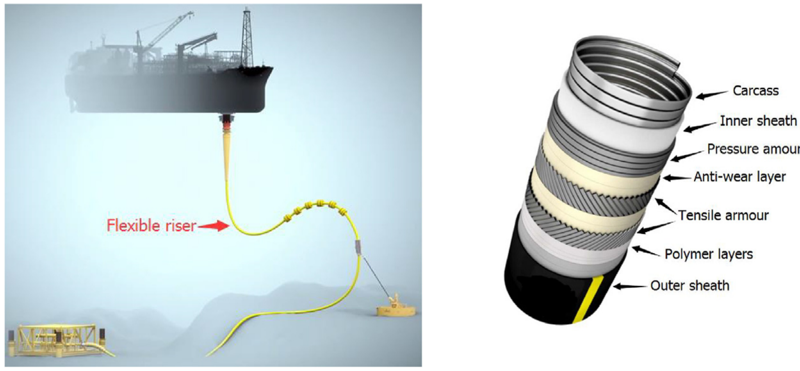

Fig. 1. Flexible riser and its layer configuration.

Table 1
Name, material and function of each layer within a typical flexible riser.

Layer	Material	Function
Carcass	Duplex steel	External pressure resistance
Pressure armor	Carbon steel	Hoop and radial load resistance
Tensile armor	Carbon steel	Axial and torsional load resistance
Inner sheath	HDPE, XLPE, PA, PVDF	Internal fluid containment
Outer sheath	HDPE, PA, TPE	External fluid barrier
Anti-wear layer	PA, PVDF, HDPE	Abrasion resistance
Insulation layer	PP, PVC, PU	Thermal insulation

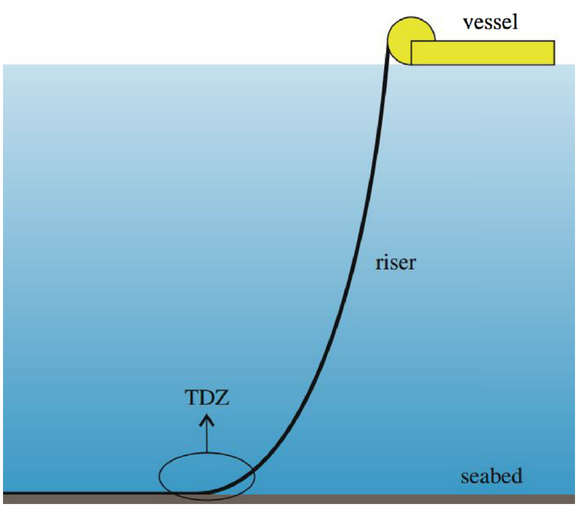

Fig. 2. Touch down zone during installation [6].

Among those different failure modes, collapse failure is always a primary challenge for riser operators to cope with. The anti-collapse capability of a flexible riser usually decides the pipe wall thickness and governs the manufacturing cost. Additionally, the replacement of a collapsed flexible riser is also very costly. Considering that, the understanding of collapse failure and related riser performance characteristics is important for designing reliable flexible riser systems [9].

Collapse of flexible risers refers to radial buckling of the internal carcass structures under external hydrostatic water pressure, as shown in Fig. 3. This failure is commonly divided into two types, dry and wet collapse, depending on the annulus conditions of risers [10]. Dry collapse may occur when the outer sheath is intact and all layers within the riser play a role together to resist the collapse. In this scenario, the interlocked carcass and the pressure armor are the main layers for collapse resistance, as they contribute the most to radial stiffness. If the outer sheath is breached, the seawater floods the annulus and then the external pressure acts directly on the inner sheath. This situation, named wet collapse, represents the most extreme loading conditions

Fig. 3. Collapse failure of the flexible riser [11].

since the whole external loading is resisted by the carcass alone. Other layers, mainly the pressure armor, just act as constraints to the carcass.

According to the most recent survey of flexible pipe failure/ damage mechanisms carried out by O'Brien et al. [12], the outer sheath damage remains the most common failure, as shown in Fig. 4. For the sections of flexible risers lying on the seabed, their external sheath may be worn out due to many small movements (see Fig. 5) [13]. That increases the risk of wet collapse and therefore requires the carcass layer, the main component for collapse resistance, should be strong enough when facing a wet annulus environment.

Various standards have been developed with regard to the design of flexible risers. Among them, API 17B and 17J are two widely acceptable specifications that issued by American Petroleum Institute [10,14]. For a flexible riser that applied for ultra-deep water production, however, those specifications are not able to provide an available approach to

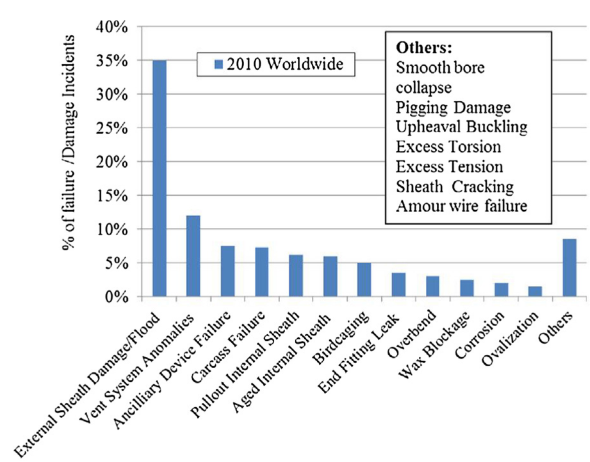

Fig. 4. Flexible pipe failure/damage mechanisms [12].

Fig. 5. Damaged outer sheath.

calculate the critical collapse pressure of the carcass. In their latest versions (2014), no prescriptive methodology except only a safety factor was given to guide the anti-collapse design of the interlocked carcass. It reveals that standardized methods have not been established yet for the anti-collapse design of flexible risers and therefore, how to predict the critical pressure is still a gray area for riser designers [15].

Over-conservative design of the carcass is adopted by riser manufacturers to reduce the latent collapse risks of flexible risers in deep sea environment. However, that leads to a heavier carcass, which needs more pairs of tensile armor and payloads of floating vessels to withstand the additional weight. As a result, it increases the costs in production, installation and operation [16].

To improve the ultra-deep water performance of flexible risers, various factors should be considered to address their current design limits. As one of the key factors that generally governs final riser wall thickness, overall weight as well as the costs, the critical pressure of the carcass is thus required to be well-determined with reliable and sophisticated methodologies [17].

3. Prediction approaches of critical pressure

Collapse studies have been conducted extensively by many researchers since the inception of the flexible risers around the 1970s [18]. Experiments are the most reliable way to predict the critical pressure of riser products. Although such kind of experiments are costly, they are the foundation to develop related analytical and numerical models. Buckling theories of rings are adopted by researchers to develop the analytical models. However, most of them are limited to highly simplified models due to the complexity of the carcass profile. By contrast, numerical simulation, such as Finite Element Analysis (FEA), has less limitations in modeling the carcass with its actual profile and therefore, becomes a suitable alternative of expensive experimental studies. Mostly, the prediction of critical pressure of flexible risers is performed on numerical models, aided by the calibration of experimental tests.

3.1. Hydrostatic tests

Over the past decades, numerous experimental programs have been performed to assess the critical pressure of flexible pipes that prepared for deep water environment. Such kind of tests are conducted with specialized hyperbaric chambers, which are very few in the world [19]. Although the experimental tests can be a reliable way to measure the critical pressure of the flexible pipes, they require a substantial cost. Besides, the high capacity chambers are always limited for an

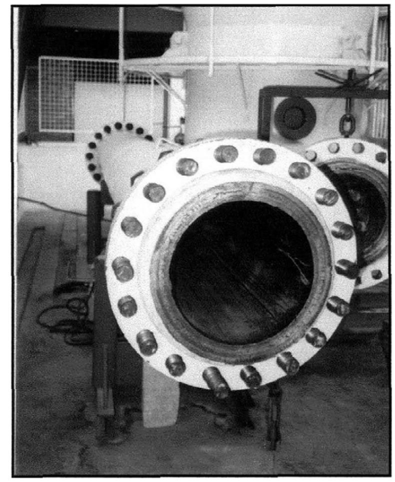

Fig. 6. Hyperbaric chamber used in Souzas experimental tests [20].

increasing demand. For the most part, the experimental tests have been an approach that helping develop and calibrate the corresponding numerical models. Souza [20] performed the collapse tests of flexible pipes at the COPPE/UFRJ Submarine Technology Laboratory. The tests were conducted in a horizontal hyperbaric chamber with a capacity of 10,000 psi, as shown in Fig. 6. The samples with two different internal diameters, 4 and 8 in., were placed in that hyperbaric chamber and pressurized to collapse. The curves of loading pressure versus time were recorded to validate the effectiveness of her numerical models. Due to the internal diameter limitation of that chamber, all the samples were test with no curvature. The test results showed that the collapse might cause the opening of the interlocked carcass layer but that opening would be negligible when there was a pressure armor inside the pipe structures.

Since the flexible risers are curved in the touchdown zone, this curvature effect weakens the anti-collapse capability of the riser structures. Clevelario et al. [21] conducted curved collapse tests of flexible pipes to investigate the curvature effect, as shown in Fig. 7. The samples of two different internal diameter, 4 and 6, remained layers consist of the carcass, inner liner, the pressure armor for both straight and curved tests. The curved samples contained additional tensile armors to withstand the axial compression loads generated by the end cap effect [22]. They were bent to 1.5 times the storage bending radius (SBR) [23] to investigate the curved collapse behavior under external pressure. These test curvature radii were determined by the global analyses of the test samples (see in Fig. 8), which could not be reached in all possible environmental and operational conditions. Each samples curved collapse pressure was recorded and compared with its straight counterpart. The test results shown that all the curved samples had collapse strength reduction beyond 10%, indicating the importance of

Fig. 7. Curved collapse test samples [21].

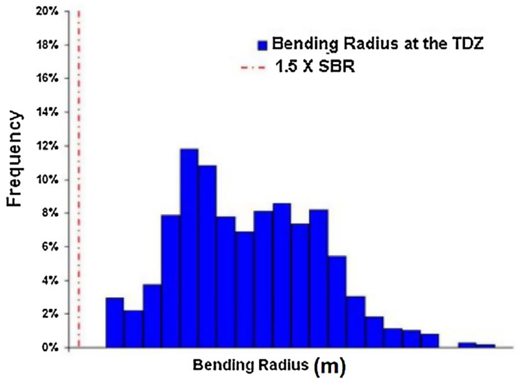

Fig. 8. Global analysis result – TDZ bending radius histogram [21].

pipe curvature in the collapse analyses of flexible risers.

3.2. Numerical simulation

Although the experimental tests could offer the engineers a physically intuitive observation on the anti-collapse performance of flexible pipes, relatively high cost associated with experiments hinders its wide application in the reality. In this regard, numerical simulation is developed to predict the critical pressure of flexible risers as an alternative.

Since the carcass and pressure armor are the main components for preventing from wet collapse, the rest layers, e.g. tensile armors, antiwear and insulation layers, are generally omitted in the numerical models [24]. Numerical models are often divided into two types: 3D full FE model and equivalent FE models (2D or 3D). The 3D full model refers to modeling the interlocked layers with their actual rolled shapes, as shown in Fig. 9. Such kind of FE models preserve the layer geometric details and therefore can be used to investigate the issues related to stress concentrations [24]. However, due to the onerous modeling procedure and time-consuming computation, the full FE models are impractical for design purposes of flexible pipes. To simplify the collapse analyses, the treatment of the interlocked metallic layer as an equivalent layer is adopted by researchers. Therefore, various equivalent layer methods are developed to construct a homogeneous pipe that displays the same collapse behavior as the carcass layer.

Considering the helicoidal geometry of carcass imposes a directional dependency on the structural mechanical properties, a fictitious orthotropic shell was built based on the analogy between grids and plates [25], as shown in Fig. 10. This idea was first proposed by Cruz and Dias [26], who took the strip spiral carcass layer as a grid with distinct stiffness in two orthogonal directions. By assuming that both the shell and the carcass have the same stiffness (membrane, bending and torsion), they determined the equivalent properties of that orthotropic shell.

This method is often used to study the responses of carcass layer subjected to axial loads [27–29] or crush [3,30–32] due to the orthotropic mechanical properties of the equivalent shell. Since the carcass layer, however, takes responsibility for radial resistance only, the treatment of helical carcass wire as a homogenous ring by discarding its lay angle in collapse studies is more acceptable to academics. Therefore, the lay angle effect on collapse problems was neglected in the equivalent ring methods, allowing to solve the collapse pressure with analytical ring models [33]. This effect was later investigated by Neto and Martins [34], which evidenced the fact that the lay angle has negligible effect in collapse prediction.

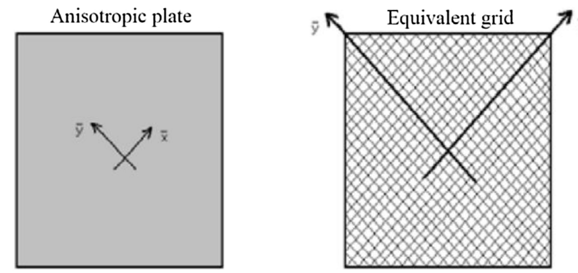

Fig. 10. Analogy between plates and grids.

Many equivalent methods were proposed in terms of the cross-section area, the thickness or other possibilities [35–39]. Area equivalent method [35] is carried out based on the equivalence of cross-sectional areas. As the cross-sectional area is the only parameter considered in this method, the actual material distribution in the carcass profile is not accounted and hence the accuracy of the prediction results is always doubtful [40].

Considering that collapse of ring-like structures is a bending-dominated problem [33], some equivalent ring methods came forward to build this equity based on the structural bending stiffness. One bending stiffness equivalence method employed by Loureiro and Pasqualino [37] originates from the above-mentioned equivalent orthotropic shell method, which obtains the equivalent thickness by equating the sectional bending stiffness between the carcass and the ring. Another similar bending stiffness equivalence method proposed by Martins et al. [38] requires the ring model has the same bending stiffness per unit axial length of the carcass. However, those two methods neglect the self-contact issue of the carcass, leading to an overestimation of the actual structural bending stiffness.

Since most equivalent methods were unable to consider the material elastic-plasticity, Tang et al. [39] proposed a method bases on the circumferential strain energy equivalence. That strain energy of the carcass was obtained through numerical model which could account for self-contact issues. In this model, only the hoop strain was generated in the carcass layer due to the applied Dirichlet-type boundary conditions [41]. However, that boundary conditions enhance the structural stiffness of the carcass, lowering its absorbed strain energy. As a result, their equivalent model gave an underestimated prediction on the critical pressure of the carcass.

Fig. 9. Details of the carcass profile and the 3D full FE model.

Table 2
Summary of existing equivalent ring methods.

Equivalent method	Authors	Geometric factors		Material factors		FEM required
		Section geometry	Initial imperfections	Linear elasticity	Elastic-plasticity	
Bending stiffness per unit area	Cruz et al. (1997)	Y	N	Y	N	N
Bending stiffness per unit length	Martins et al. (2003)	Y	N	Y	N	N
Area equivalent	Zhang et al. (2003)	Y	N	N	N	N
Thickness equivalent (with a faction fill coefficient)	Chen et al. (2015)	Y	N	N	N	N
Strain energy equivalent	Tang et al. (2015)	Y	Y	Y	Y	Y

Table 2 summarizes the common equivalent layer methods and gives an overview toward their own characteristics. To gain an insight of the reliability of the existing equivalent layer methods, Lloyds Register Energy [42] conducted an investigation on the prediction accuracy of the mathematic models with different equivalent layer methods. The results shown a considerable variation existed in the predictions between different equivalent methods, indicating that further development of these methods is needed.

3.3. Analytical methods

Owing to the geometric complexity of the interlocked layer profile, analytical approach has been limited to highly simplified analytical models, aided by experimental calibration. The analytical models for flexible pipe collapse are developed based on a general ring buckling model [18]. With the help of the equivalent layer methods, an equivalent ring model could be constructed and the differential equation for that bending ring takes the form

$$\frac{d^2w}{d\theta^2} + w = -\frac{MR^2}{EI} \tag{1}$$

where θ is the angle along the circumference, M is the bending moment due to the loading, R is the mean radius of the ring, EI is the bending stiffness of the ring.

By using this equation, the critical pressure p_{cr} of the ring within the elastic limit can be obtained as

$$p_{cr} = \frac{3EI}{R^3} \tag{2}$$

Since the flexible pipe is a concentric structure, Glock [43] presented a closed-form analytical solution for the critical pressure of an elastic cylinder that confined in a rigid cavity

$$p_G = \frac{E}{1-v^2}\left(\frac{t}{D}\right)^{11/5} \tag{3}$$

Chen et al. [36] considered the outside layers as a spring support to the inner carcass and thus proposed a formulation to that spring-supported ring model

$$p_c = \frac{3EI}{R^3} + \frac{1}{4}k \tag{4}$$

where k is a constant that related to the bending stiffness of both inner liner and pressure armor.

However, the above-mention equations are built based on elastic collapse merely. For a flexible riser that applied to deep water environment, it is more likely to be collapsed in the plastic range [36]. Clinedinst [44] suggested that replacing Young's modulus with a “reduced modulus” to consider the plastic collapse of pipes. This reduced modulus is a function of the stress-strain curve of the pipe material and its cross-section. For a rectangular cross-section, the reduced modulus can be expressed as

$$E_r = \frac{4E\frac{d\sigma}{d\varepsilon}}{\left(\sqrt{E} + \sqrt{\frac{d\sigma}{d\varepsilon}}\right)^2} \tag{5}$$

where $d\sigma/d\varepsilon$ is the slope of the stress-strain curve of the material at the stress σ and strain ε caused by the critical load.

Bai et al. [45] regarded the initial yielding pressure $p_{e,y}$ as the plastic collapse pressure of pipes and therefore an equation for critical pressure calculation is given as

$$p_{e,y}^2 - \left[\frac{\sigma_y t}{R} + \left(1 + 6\frac{w_0}{t}\right)p_{e,cr}\right]p_{e,y} + \frac{\sigma_y t}{Rp_{e,cr}} = 0 \tag{6}$$

where σ_y is the material yield stress, w_0 is the maximum initial radial deviation from a circle, $p_{e,cr}$ is the elastic critical pressure that calculated through Eq.(2).

This equation is proposed based on the assumption given by Jacobsen [46], who defined the buckling pressure as the pressure $p_{e,y}$ at the onset of material yielding in the extreme outer fiber. However, this typically underestimated the critical pressure pc, because fail does not occur until the elastic-plastic boundary has penetrated some way through the wall thickness, as shown in Fig. 11 [47,48].

4. Pipe imperfections and curvature

For a flexible riser that operates in ultra-deep water, its critical pressure is affected by many factors, such as ovalization, layer gap and pipe curvature. To ensure the structural safety of flexible risers during installation and operation, a lower bound collapse concept is always adopted to consider the possible worst geometric configuration and material properties [35]. Since the collapse resistance of flexible risers is sensitive to the structural imperfections and pipe curvature, it is necessary to take those factors into account when conducting such a lower bound collapse prediction [21,35]. Related studies have been carried out by scientists to quantify those factors and introduce them into prediction models.

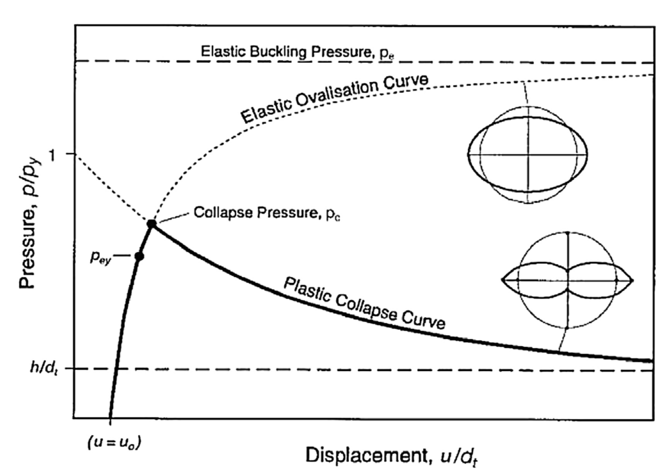

Fig. 11. Elastic ovalization and plastic collapse curves defining collapse pressure [48].

4.1. Pipe imperfections

4.1.1. Geometric imperfections

Practical pipe structures are manufactured to specified tolerances, and as a result always deviate to some degree from an ideally perfect geometric shape [49]. This poses geometric imperfections to practical riser structures and brings safety hazards to their ultra-deep water application. Many radial buckling studies of pipe structures have been conducted to investigate the effect of various geometric imperfections on collapse capacity of pipes [33,50–52]. Those works indicate that the collapse capacity is strongly influenced by initial ovalization and gap as both of them are global imperfections affecting the whole cross-section of pipe structures [45,53]. Once the cross section of the metallic layer is fully yielded, increasing pressure will first compress the pipe uniformly inward and then ovalize it owing to those small nonuniformities [54]. As stated by Kyriakides [49],"when the structure stays elastic it is not imperfection sensitive. By contrast, when inelasticity sets in it becomes imperfection sensitive".

API 17J defines the ovalization (out of roundness) as:

$$\Delta_{ad} = \frac{D_{max} - D_{min}}{D_{max} + D_{min}} \tag{7}$$

It requires the operators to take initial ovalization into account in their collapse analyses. A minimum ovality of 0.2% should be used if no other data exists [14]. Numerical techniques were employed by researchers to gain an insight into the effect of initial ovalization on riser collapse. Neto and Martins [34] modeled the carcass with a set of initial ovalization and computed the critical pressure numerically. The finite element models exhibited clear reduction of critical pressure due to the increasing initial ovalization. Considering that API 17B [10] allowed the collapse analyses to take the pressure armor into account, Malta et al. [55] studied the effects of ovalization on confined collapse modes of carcass with 2D FE models. In their work, the carcass layer was confined within the pressure armor and two types of symmetry initial ovalization condition (singly or doubly) were imposed to the carcass. According to the analysis results, they found the symmetry initial ovalization condition did have an impact on the post-buckling behavior (eight/heart shape collapse mode [56], see Fig. 12) of carcass. The doubly initial ovalization always caused an eight collapse mode while the singly ovalization interfered the final modes together with the pressure armor thickness. Although the relationship between critical pressure and collapse modes is still under investigation, some studies [57–59] indicated that the heart shape buckling pattern might yielded a lower critical pressure.

Layer gap between the inner sheath and the pressure armor is being paid attention to after more and more researchers considered the pressure amour in their collapse studies [4,60,61]. This imperfection reduces the supporting capacity of pressure armor when facing wet collapse. Two factors may trigger the occurrence of the layer gap: volume change of the aging polymer and extrusion into the adjacent interlocked layers [62]. Although no gap is created during the riser manufacture, the factory acceptance test (FAT) causes a volume loss of the polymer layers during first pressurization of the flexible riser. This leads to some unclosed gap between layers, which will act as the initial gap for subsequent pressure loadings of flexible risers and practical operations [63].

Numerical modeling is a main approach to address this issue. An airbag technique was adopted by Axelsson and Skjerve [62] to investigate the sensitivity of collapse pressure on radial gaps between riser layers. In their 3D FE models, the gap was simulated as an airbag layer which would not affect the behavior of the surrounding layers. With the increase of the thickness of this airbag, the critical pressure of the carcass dropped significantly. A similar phenomenon was observed by Neto and Martins [64]. In their work, the loading pressure peaked twice during the radial deflection of the carcass. A lower one was reached prior to the gap closure, followed by another larger collapse pressure after the carcass came in contact with the pressure armor, indicating that a premature radial stiffness reduction to the riser structures caused by the nonzero initial gap.

4.1.2. Material imperfection

Due to the cold work during the carcass manufacturing process, strain hardening may occur, causing a high degree of stress-induced material an-isotropy [65]. This cold work makes the material properties varied throughout the formed profile of the carcass strip, as shown in Fig. 13, complicating the collapse behavior of carcass layer along with the geometric anisotropy.

Adopting an average stress-strain curve to represent the non-homogeneous material behavior of the overall wire's cross-section has been one available way for researchers to incorporate this material imperfection. This approach was first employed by Zhang et al. [35], who provided the equivalent layer a typical stress-strain curve according to the cold work level of the original carcass. However, the authors admitted that the level of cold work was difficult to measure directly and depended on "a number of factors related to the design and manufacturing process". To address this problem, Nogueria and Netto [66] proposed a methodology to estimate the average stress-strain curve of the carcass. They first applied loads to the crown point of a half sectional carcass wire specimen and recorded its load-displacement curve. A corresponding FE model of the specimen was then constructed, attempting to reproduce experimental load-displacement curve by repeatedly adjusting the model's stress-strain curve. This onerous method was improved by Lacerda et al. [67,68], who simplified the average stress-strain curve as a bilinear curve. This bilinear curve was decided by three material parameters, Young's modulus, yield stress and tangent modulus. The Young's modulus was determined by the linear portion of the load-displacement curve of the test specimen, and the yield stress and the tangent modulus were calibrated by the rest elastic-plastic

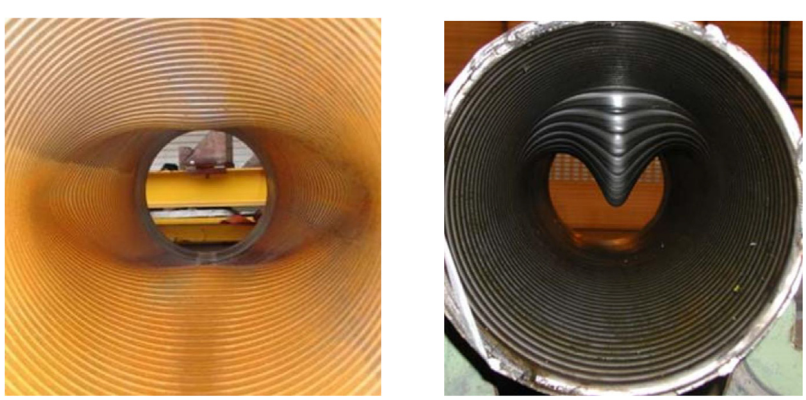

Fig. 12. Eight shape (left) and heart shape (right) collapse modes [56].

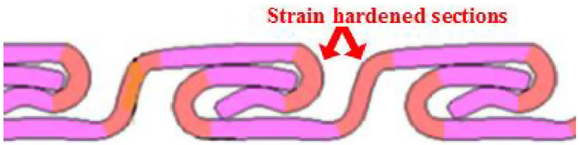

Fig. 13. Material imperfection induced by cold work.

portion. If a simple bilinear curve was not able to reproduce the experimental results accurately, a tri-linear curve could also be employed.

Using average stress-strain curves is a compromise to the limitation of current techniques on the measurement of cold work level, which means discarding the geometric details of the carcass profile. To avoid an incorrect predictions of stress concentrations, Axelsson [62] constructed a FE model with the actual carcass profile and applied different stress-strain curves to the corresponding cold formed sections. Considering that there was a relationship between the steel hardness H_v and yield stress σ_y which approximately followed the form as below [69–72]:

$$H_v \approx 3\sigma_y \tag{8}$$

a hardness measurement technique was used to estimate the representative yield strength of the carcass curved sections, helping define those stress-strain curves.

4.1.3. Residual stress

Residual stress is a factor that may quicken the occurrence of plasticity and cause the early collapse of the flexible riser [73]. It is generated from two stages during the pipe manufacture (see in Fig. 14): one is the roll bending stage where the metallic wire experiences a sequence of bending and twisting events; another is the interlocking stage, where the profiled wire is wound onto a bobbin [74]. Owing to the practical difficulties, there is no available post-deformation stress relief operation could be conducted to the flexible riser products [75]. Those products, therefore, contain unknown magnitude of residual stress in the cross-section of their armor wires.

Estimating the residual stress accurately is of great importance since the plastic yielding is caused by the summation of applied stress and residual stress [77], but how to achieve that is a tough task. Numerical approaches are adopted by some researchers to make the first attempt. Tang et al. [78] simulated the cold-forming process of the carcass wire with FE software MARC and obtained the distribution of residual stress along the carcass profile. Those residual stresses were then input into an identical model which would be ran in ANSYS for the followed collapse analyses. Although the numerical results shown the residual stresses cause a significant decrease (nearly 8%) on collapse pressure, the lack of test data made it less persuasive.

To facilitate the stress analyses of flexible risers, an establishment of preliminary studies for the measurement techniques of residual stress is required. Conventional destructive methods such as hole-drilling [79] are no longer applicable as they are unable to measure the stress distribution along the interlocked layer profile. By contrast, non-destructive approaches are gaining popularity among researchers for their advantages of determining the stress state in-situ on the manufactured risers. Fernando et al. [75] first used the X-ray diffraction method to measure the distribution of residual stress in pressure armor. This

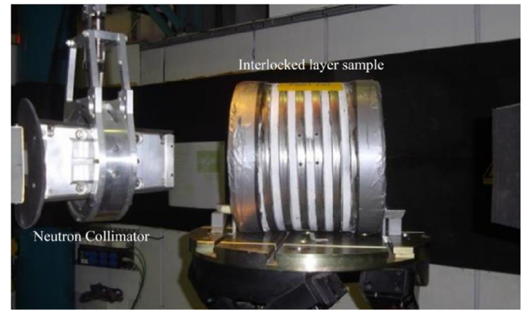

Fig. 15. Neutron diffraction technique [81].

method evaluated the magnitude of the residual stress by measuring the changes in the spacing of the lattice planes between pre- and post-manufactured pressure armor wires [80]. However, this method could only measure the stress state near the armor surface and failed to give a correct evaluation on the magnitudes of stresses along the wires cross-section. In this regard, another measurement technique, the Neutron Diffraction method, was adopted in their later research [81] for its large penetration depth [82]. This technique, as shown in Fig. 15, was similar to X-ray diffraction but due to its different scattering properties (neutrons interact primarily with the nuclei of atoms), additional information could be obtained [83]. With the aid of the neutron diffraction method, three orthogonal strains (hoop, axial and radial based on riser coordinate) at gauge points in the wires cross section were measured, and thus the residual stress distribution on the whole wire section was depicted. Despite the limited gauge points was unable to cover all localized hot spots on the wire cross section, this technique performed a potential way to measure the residual stress in the interlocked layers within a manufactured flexible risers.

4.2. Curvature effect

During deep-water installation and operation, the flexible risers experience bending within the touchdown zone. This bending condition may affect the structural stability of flexible risers for service in such extreme water depth and lead to curved collapse with the combination of external hydraulic pressure. Since the curvature affects many factors, such as the changes in void fraction of carcass layers and gap between layers [21], which makes curved collapse one particular issue still not fully addressed in current technical resources of flexible risers.

The complexity of curvature effect poses barriers to analytical approach and limits most curved collapse studies to finite element methods. Efforts were made by Loureiro [84] to develop the analytical model of riser curved wet collapse. A treatment of curvature as an additional ovalization was adopted in his analytical model, stemming from the bending study of Brazier [85] and Guarracino [86] on thin cylindrical tubes. This additional ovalization was induced by the ovalized inner sheath since pipes undergone a progressive flattening during bending due to the Von Karman effect [87]. Although bending a carcass alone without reaching its MBR (minimum bending radius) did not introduce high stresses [62], it had to withstand the ovalizing pressure imposed by the inner sheath and thus gains some additional ovalization. Based on the ovalization pressure equation given by Brazier, this

Fig. 14. Manufacture process of carcass layer [76].

additional ovalization was worked out by the author. Finally, a new initial ovalization was obtained by adding it to the straight-pipe initial ovalization, transforming the curvature effect into an initial geometric imperfection issue. This analytical model was examined by the author himself and other researchers [42] with different FE models. However, those work indicated that the wet collapse pressure predicted by this analytical model was overestimated, deviating considerably from the numerical results.

Compared with analytical methods, numerical modeling seems to be a better option to many researchers for its ability in visualizing the curvature effect during riser collapse. A 3D full finite element model which considered all the cross-sectional details of interlocked carcass and pressure armor was constructed by Neto et al. [88]. This time-consuming model was built with a length adequate to isolate the influence of the boundary conditions. Two case studies were performed on that 3D model to investigate the curvature effect on wet and dry collapse of flexible pipes. Based on the numerical results, a preliminary conclusion was drawn by Neto that the curvature could cause a significant reduction of the critical pressure in wet collapse while that reduction was negligible in dry collapse. This full model was later simplified by the authors themselves due to its impracticality in flexible pipe anti-collapse design [24]. With the aid of displacement coupling and appropriate kinematic constraints at the cutting edges, the length of the original model was reduced to two pitches. This simplified model was employed to study the curvature effect on dry collapse further. The investigation results of curvature effect on dry collapse of flexible pipes presented in this study was in accordance with the previous conclusion drawn by the authors.

Another simplified 3D solid model comprised carcass, inner sheath and pressure armor was carried out by Lu et al. [89] to investigate the curved wet collapse. Interlocked layers in this model were modeled as helical strips (like a spring ring) while the inner sheath was simulated by a continuous cylindrical wall. To simulate the bent collapse of a 6-in. flexible pipe, this model was first bent to a given bend radius (3.6 m) and then subjected to external pressure. According to the numerical results, a clear onset of collapse was captured from the pressure vs. displacement curve and the critical pressure was 5% lower than that of a straight pipe approximately. However, as the authors stated, this simplified model required sufficient test data to calibrate and its prediction accuracy needed further inspection.

5. Conclusion and discussion

As oil and gas exploitation moves toward ultra-deep water fields, many challenges related to anti-collapse capabilities of flexible riser occur. Ultra-deep water collapse study of flexible risers is a complicated task but of great importance to the oil and gas industry. The main purpose of this review is to provide researchers in this field a set of relevant references required for their research and highlight the barriers in the prediction of critical pressure. These barriers can be concluded as:

i) The lack of an effective equivalent layer method; Due to the neglect or incorrect consideration of contact issues of the carcass, all those equivalent layer methods fail to capture the actual structural stiffness of the carcass. As a result, the equivalent models usually lead to inaccurate prediction of critical pressure. Moreover, almost all the existing methods are developed based on only one certain structural property equivalence, which is inadequate for an equivalent model to perform a similar collapse behavior of the carcass.
ii) Imperfections generated from manufacture process; Up to date, imperfection studies of flexible risers mostly concentrate on the geometric imperfections. The studies on material stress hardening and residual stress are relatively barren. For the most part, imperfection investigations are limited to finite element analysis, lacking the verification of experimental data. Additionally, there is always a demand of developing analytical models for sensitivity analyses instead of revaluating through repeated numerical model adjustments.
iii) Complex collapse behavior under combined external and bending loads; Curved collapse of flexible risers is a difficult issue as bending state brings changes to many parameters (e.g. pitch, layer gap and void fraction of interlocked layers). The numerical models used for curvature study often require adequate axial lengths to eliminate boundary effects, making the curved collapse analyses an onerous task. Although some simplified curved collapse models have been proposed, they are not well-examined due to the lack of experimental data and thus cannot be an reliable tool in reality.

For the increasing water depth facing by the riser operators, an accurate and reliable collapse prediction technique would provide a well-determined operation limit for their products, helping save costs and increase their confidence. For the future trends of flexible riser, such a technique will also be a powerful tool to be compatible with the composite material and new riser system technologies.

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