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Cooperative r-Passivity Based Control for Mechanical Systems ⋆

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Abstract: In this work we consider the problem of cooperative end-effector control between heterogeneous fully actuated agents when varying-time delays and/or packet loss are present. We couple agents via outputs encoded with task-space coordinates and velocities that are transformed into wave-variables to overcome the destabilising effects of the communication network. The scheme poses dynamic requirements on the agents which are locally satisfied with feedback control that integrates subtasks, such as joint-limit avoidance or local tracking, when there are redundant degrees-of-freedom. The proposed approach extends existing methods to task-space control. The approach is robust to network effects, applies to nonlinear systems and is scalable by design. The tuning task is simplified considerably by separation of the cooperative and non-cooperative control terms. We demonstrate the efficacy of the proposed approach experimentally.

Keywords: Passivity-Based Control, Nonlinear Control, Co-operative control, Time-delays, r-Passivity, Scattering, Navigation Functions

1. INTRODUCTION

This work concerns cooperative distributed control of heterogeneous nonlinear systems in *realistic* network conditions. Cooperation between heterogeneous systems opens up new avenues for automation in domains such as the medical, logistic or aerospace industries (Wurman et al. (2008), Leitner (2009)). The considered type of interactions are especially useful for aligning systems of different types, e.g., alignment of space-vessels or automated industrial picking and packing. Although cooperative control methods for systems of different types exist, their use is often facilitated by heuristic, systems specific inter-agent couplings. Our method decouples cooperative and local behaviour of agents, which greatly increases the application scope and reduces the tuning requirements compared to other methods. The effects of time-delays and communication packet loss adversely affect stability properties of typical cooperative controllers and in some cases may result in drift. Mitigating these effects is particularly challenging due to the nonlinear dynamics of the agents and the various couplings that can be introduced between them. In this work we apply passivity theory to overcome the aforementioned issues. Passive systems are characterised by their inability to instantaneously supply energy; externally supplied energy is either stored or dissipated. The Scattering Transformation (ST), introduced for bilateral teleoperation applications in Niemeyer and Slotine (1991), transforms the networked input and output of a system into signals with a defined energy, such that the network becomes passive. In Chopra and Spong (2006) this approach was used to derive a convergent Multi-Agent Scheme (MAS) for passive systems. The scheme achieves synchronisation of passive outputs (i.e., velocities). It was shown in Chopra and Spong (2005) that the coordinate states may be encoded with the velocities in the outputs $\mathbf{r}$ which, by local feedback, can be rendered passive. This r-passivity approach was used in Chopra and Spong (2006) to show state convergence of the MAS. In this work we consider a modification of the outputs $\mathbf{r}$ for task-space control. We incorporate formation and leader-follower control, of which synchronisation is a special case. The resulting scheme poses cooperative dynamical requirements on the individual agents for which we develop a novel local controller. Our control law designs the cooperative and redundant dynamical behaviour of the agent separately, making it simple to tune and easily deployable in different scenarios.

Networks where the standard ST architecture is applied achieve passivity in the presence of arbitrary *constant* communication delays in continuous time. In Lozano et al. (2002) the result is extended to preserve passivity in the presence of *time-varying* delays in continuous-time. In Berestesky et al. (2004) it is shown that time-varying delays and packet loss in discrete-time both lead to the absence of data on the receiver side of an edge when the network is sampled. The algorithm developed in the same work reconstructs wave-variables in a passive manner, thereby remaining passive under improved performance. The work in Liu and Puah (2014) introduces a passive reconstruction method, called Wave-Variable Modulation (WVM), that may be applied locally while extracting the maximum energy. This reconstruction is incorporated into a Communication Management Module (CMM) that connects an agent to other agents in a passive manner in the presence of network effects. This concept is the key to robust and scalable communication.

In this work, we propose a control method and demon-

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strate its ability experimentally. The main contributions are the following:

(i) Extension of r-passivity to task-space coordinates, validation of coordinate convergence under a large set of time-varying delays in the MAS from Chopra and Spong (2006) and a modified scheme that achieves leader-follower- or leaderless formation control. (Section 3).
(ii) Development of a control law for fully actuated mechanical systems that achieves r-passivity while allowing control over the excess degrees-of-freedom (Section 4).
(iii) Experimental validation of the proposed method using a robotic manipulator and differential drive robots (Section 5).

Relevant background is reviewed first in Section 2.

2. PRELIMINARIES

2.1 System Modelling

In this work, agents are modelled with the Hamiltonian dynamics

$$\begin{bmatrix}\dot{\mathbf{q}}_i\\ \dot{\mathbf{p}}_i\end{bmatrix} = \begin{bmatrix}\mathbf{0}_{n_i} & \mathbf{I}_{n_i}\\ -\mathbf{I}_{n_i} & \mathbf{0}_{n_i}\end{bmatrix}\begin{bmatrix}\frac{\partial H_i}{\partial \mathbf{q}_i}\\ \frac{\partial H_i}{\partial \mathbf{p}_i}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_{n_i\times m_i}\\ \mathbf{F}_i\end{bmatrix}\boldsymbol{\tau}_i, \quad (1a)$$

$$H_i = \tfrac{1}{2}\mathbf{p}_i^T\mathbf{M}_i^{-1}\mathbf{p}_i + V_i(\mathbf{q}_i), \quad (1b)$$

where $\mathbf{q}_i, \mathbf{p}_i \in \mathbb{R}^{n_i}$ are the coordinates and momenta respectively, the Hamiltonian H_i is the sum of kinetic and potential energy in the system and the inputs $\boldsymbol{\tau}_i \in \mathbb{R}^{m_i}$ enter the system via input matrix $\mathbf{F}_i \in \mathbb{R}^{n_i\times m_i}$. We consider fully actuated systems, for which $n_i = m_i$. For these systems we can assume $\mathbf{F}_i = \mathbf{I}_{n_i}$ without loss of generality, due to invertibility of $\mathbf{F}_i$. We define the end-effector coordinates $\mathbf{z}_i \in \mathbb{R}^l$, which in general relate to the coordinates via a nonlinear transformation $\mathbf{z}_i = \mathbf{a}_i(\mathbf{q}_i)$. Additionally, we define the related Jacobian $\mathbf{J}_i \in \mathbb{R}^{l\times n_i}$ such that $\dot{\mathbf{z}}_i$ may be expressed as $\dot{\mathbf{z}}_i = \mathbf{J}_i\dot{\mathbf{q}}_i$. We are now ready to formally define a problem description.

2.2 Problem Definition

Consider N heterogeneous mechanical agents, each described by (1a), (1b). The systems communicate over a directed, bidirectional and strongly connected graph $\mathcal{G}(\Sigma, \mathcal{E})$. The cooperative objectives are specified by the desired final inter-agent distances $\mathbf{z}^*_{ij} = -\mathbf{z}^*_{ji}$. Agents assigned as leaders are given a reference $\mathbf{z}^*_i$. The bidirectional edge pair $\mathcal{E}_{ij}, \mathcal{E}_{ji} \in \mathcal{E}$ is subject to distinct and possibly time-varying delays $T_{ij}(t), T_{ji}(t) \geq 0$. The problem is solved iff

$$\lim_{t\to\infty} ||\dot{\mathbf{q}}_i|| = 0,\ \forall\ i \in \mathcal{I}_N, \quad (2a)$$

$$\lim_{t\to\infty} ||\mathbf{z}_i - \mathbf{z}^*_i|| = 0,\ \forall\ i \in \mathcal{I}_L, \quad (2b)$$

$$\lim_{t\to\infty} ||\mathbf{z}_i(t - T_{ij}(t)) - \mathbf{z}_j(t) + \mathbf{z}^*_{ij}|| = 0,\ \forall (i,j) \in \mathcal{E}, \quad (2c)$$

where $\mathcal{I}_N = \{1, \ldots, N\}$ and $\mathcal{I}_L$ the set of leader indices.

2.3 Passivity

A passive system is a system in which energy can only increase by the energy delivered to it externally. The received energy is either stored or dissipated. Formally we may state passivity as follows.

Definition A system is said to be passive if there exists a C^1 storage function $V > 0$, $V(\mathbf{0}) = 0$ and a dissipation function $S \geq 0$ for which the passivity relation

$$V(\mathbf{x}(t)) - V(\mathbf{x}(0)) = \int_0^t \mathbf{y}(s)^T\boldsymbol{\tau}(s)ds - \int_0^t S(s)ds, \quad (3)$$

holds. The equivalent differential passivity relation obtained by differentiation is given by

$$\dot{V} + S = \boldsymbol{\tau}^T\mathbf{y}. \quad (4)$$

Systems for which $S(\mathbf{x}) > 0\ \forall\ \mathbf{x} \neq \mathbf{0}$ are said to be strictly passive. Systems for which $S(\mathbf{x}) = 0$ are said to be lossless.

2.4 Multi-Agent Passivity-Based Control with Communication Delays

Typical cooperative control schemes relay agent outputs over the network, which cannot be guaranteed to be passive in the presence of delays. The Scattering Transformation (ST) (Niemeyer and Slotine (1991)) replaces these outputs $(\mathbf{r}_j, \mathbf{r}_i)$ with wave references $(\mathbf{r}_{js}, \mathbf{r}_{is})$ for cooperative control. The wave references are computed from the wave variables $(\mathbf{s}^-_{ij}, \mathbf{s}^+_{ji})$, that are communicated over the network. Figure 1 depicts this cooperative control scheme as introduced by Chopra and Spong (2006).

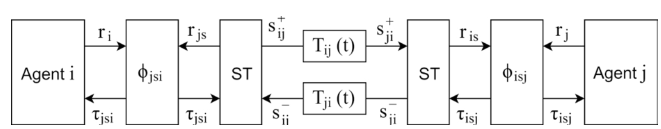


Fig. 1. The cooperative control scheme in Chopra and Spong (2006), with notation adapted to this work.

The wave-variables and wave references are computed from the ST as

$$\mathbf{s}^+_{ij} = \frac{1}{\sqrt{2b}}(-\boldsymbol{\tau}_{jsi} + b\mathbf{r}_{js})\ ;\ \mathbf{s}^-_{ji} = \frac{1}{\sqrt{2b}}(\boldsymbol{\tau}_{isj} - b\mathbf{r}_{is}), \quad (5)$$

$$\mathbf{r}_{js} = \frac{1}{b}(-\sqrt{2b}\mathbf{s}^-_{ij} - \tau_{jsi})\ ;\ \mathbf{r}_{is} = \frac{1}{b}(\sqrt{2b}\mathbf{s}^+_{ji} - \tau_{isj}), \quad (6)$$

where $b > 0$ is the virtual impedance of the network. The variables $\mathbf{s}^-_{ij}, \mathbf{s}^+_{ji}$ result from propagation through the delayed network, described by the transport equations

$$\mathbf{s}^-_{ij}(t) = \mathbf{s}^-_{ji}(t - T_{ji}(t))\ ;\ \mathbf{s}^+_{ji}(t) = \mathbf{s}^+_{ij}(t - T_{ij}(t)). \quad (7)$$

The ST ensures losslessness of the network in the presence of *arbitrary constant* delays. In Lozano et al. (2002), this result is strengthened to time-varying delays in continuous-time by introducing a delay-dependent gain $f(t)$ on the receiving side of the network that satisfies

$$f(t) \leq 1 - \frac{\partial T}{\partial t}. \quad (8)$$

This extension holds under the following assumption:

Assumption 1: The delays do not increase faster than time, i.e.,

$$\frac{\partial T_{ij}(t)}{\partial t} \leq 1,\ \forall\ t,\ (i,j) \in \mathcal{E}_{ij}. \quad (9)$$

The cooperative controls are described by a passive function $\boldsymbol{\phi}_{jsi}(\mathbf{r}_{jsi})$, with $\mathbf{r}_{jsi} = \mathbf{r}_{js} - \mathbf{r}_i$, the difference between the wave references and the agent outputs. The cooperative control input for each agent is computed as the sum of inputs on each of its edges,

$$\boldsymbol{\tau}_i = \sum_{j\in\mathcal{N}_i} \boldsymbol{\phi}_{jsi}(\mathbf{r}_{jsi}),\ \forall\ i \in \mathcal{I}_N. \quad (10)$$

The scheme depicted in Figure 1 effectively consists of three distinct passive system components:

- The agents (V_i, S_i),
- The network with ST (V^{ij}_{channel}, 0),
- The cooperative controls (V_{jsi}, S_{jsi}).

The cooperative control scheme with agents (1a), (1b), ST (5) - (7) and controls (10) is shown to be globally stable in Chopra and Spong (2006) using the Lyapunov-Krasovskii candidate

$$V = \sum_{i\in\mathcal{I}_L} V_i + \sum_{(i,j)\in\mathcal{E}} V^{ij}_{\text{channel}} + \sum_{i\in\mathcal{I}_L}\sum_{(i,j)\in\mathcal{E}} V_{jsi}, \quad (11)$$

with its derivative given by

$$\dot{V} = -\sum_{i\in\mathcal{I}_L} S_i - \sum_{i\in\mathcal{I}_L}\sum_{(i,j)\in\mathcal{E}} S_{jsi} \le 0.$$

This implies that asymptotically

$$S_{jsi} \to 0\ \forall\ (i,j) \in \mathcal{E}, \quad S_i \to 0\ \forall\ i. \quad (12)$$

Synchronisation properties of the system depend on the choice of cooperative controls and outputs.

3. COOPERATIVE R-PASSIVITY BASED CONTROL

In this work we build on Chopra and Spong (2006) by considering cooperative control of task-space coordinates $\mathbf{z}$. The following section introduces cooperative controls and outputs that achieve cooperative consensus.

3.1 Modified outputs and passive cooperative controls

The coordinates $\mathbf{q}_i$ are unobservable from the typical passive outputs $\dot{\mathbf{q}}_i$, which leads to coordinate drift in the presence of time-varying delays and packet loss, as shown in Chopra and Spong (2005). We adapt the solution in Chopra and Spong (2005) designed for synchronisation of the states $\mathbf{q}_i$, to synchronisation of end-effector coordinates $\mathbf{z}_i$, resulting in the outputs

$$\mathbf{r}_i = \dot{\mathbf{z}}_i + \lambda\mathbf{z}_i, \quad \lambda > 0. \quad (13)$$

Notice that the objectives (2c) are observable from these outputs. To achieve these objectives we define a potential function $V_c \ge 0$, for which

$$V_c(\mathbf{r}_{jsi}) = 0 \iff \mathbf{r}_{jsi} = \mathbf{0}. \quad (14)$$

The gradient descend based control inputs are computed as

$$\boldsymbol{\phi}_{jsi} = -\mathbf{K}_{ij}\frac{\partial V_c(\mathbf{r}_{jsi})}{\partial \mathbf{r}_{jsi}}, \quad (15)$$

where $\mathbf{K}_{ij} = \mathbf{K}_{ji}$ is the control gain on the edge $\mathcal{E}_{ij}$. A storage function of these controls is $V_{jsi} = 0$. Substitution in (4) yields

$$S_{jsi} = \frac{\partial^T V_c(\mathbf{r}_{jsi})}{\partial \mathbf{r}_{jsi}}\mathbf{K}_{ij}\mathbf{r}_{jsi} \ge 0.$$

Since the gradient of V_c is odd, these cooperative controls are strictly passive.

3.2 Synchronisation of the network

In the following, we denote systems that are passive with respect to the input-output pair $(\mathbf{r}, \boldsymbol{\tau}_c)$ as *r-passive*. We will show that the proposed modifications to the scheme Chopra and Spong (2006) result in synchronisation of the cooperative coordinates $\mathbf{z}$. For this result the following assumption is necessary.

Assumption 2: Agents are strictly r-passive with damping on their cooperative velocities, i.e., there exists a V_i, $S_i(\dot{\mathbf{z}}_i)$ for which

$$S_i = \boldsymbol{\tau}^T_{c,i}\mathbf{r}_i - \dot{V}_i \ge 0,\ \forall\ i, \quad (16)$$

holds.

We will develop a controller that satisfies Assumption 2 by design in Section 4.

Theorem 1. In the setting of Problem I the scheme depicted in Figure 1 with agents satisfying Assumption 2, outputs (13), the ST (5), (6) with delay dependent gains (8) and controls (10), (15) achieve (2c) with $\mathbf{z}^*_{ij} = -\mathbf{z}^*_{ij} = \mathbf{0}\ \forall\ (i,j) \in \mathcal{E}$, in the presence of time-varying delays satisfying Assumption 1.

Proof. Since all components in the system are r-passive, Lyapunov candidate (11) leads to asymptotic properties outlined in (12). By design of V_c, we have

$$S_{jsi} \to 0 \iff \mathbf{r}_{jsi} \to \mathbf{0},\ \boldsymbol{\tau}_{jsi} \to \mathbf{0},\ \forall\ (i,j) \in \mathcal{E}, \quad (17)$$

while Assumption 2 and (12) imply $\mathbf{r}_i \to \lambda\mathbf{z}_i$. Hence we may write

$$\dot{\mathbf{z}}_{js} = -\lambda(\mathbf{z}_{js} - \mathbf{z}_i),\ \forall\ (i,j) \in \mathcal{E},$$

which is a linear system where the input $\lambda\mathbf{z}_i$ is constant. The coordinate transformation $\hat{\mathbf{z}}_{js} = \mathbf{z}_{js} - \mathbf{z}_i$ yields

$$\dot{\hat{\mathbf{z}}}_{js} = -\lambda\hat{\mathbf{z}}_{js},\ \forall\ j,$$

which is Hurwitz and hence converges exponentially to $\hat{\mathbf{z}}_{js} = \mathbf{0}$. Reverting the transformation shows that

$$\lim_{t\to\infty}\mathbf{z}_{js} = \lim_{t\to\infty}\mathbf{z}_i, \quad \lim_{t\to\infty}\dot{\mathbf{z}}_{js} = \mathbf{0},\ \forall\ (i,j) \in \mathcal{E}. \quad (18)$$

Hence locally the outputs converge to the wave references. To show cooperative convergence we substitute (5) and (7) in (6), which yields

$$\lim_{t\to\infty}\mathbf{r}_{js} = \lim_{t\to\infty}\frac{1}{b}(-\boldsymbol{\tau}_{isj}(t-T_{ji}(t)) + b\mathbf{r}_{is}(t-T_{ji}(t)) - \boldsymbol{\tau}_{jsi})$$

and a similar description for $\mathbf{r}_{is}$ for all edge pairs. Since $\boldsymbol{\tau}_{jsi} \to \mathbf{0}$ and $\boldsymbol{\tau}_{isj} \to \mathbf{0}$, we obtain

$$\lim_{t\to\infty}\mathbf{r}_{js}(t) = \lim_{t\to\infty}\mathbf{r}_{is}(t - T_{ji}(t)),\ \forall\ (i,j) \in \mathcal{E}. \quad (19)$$

Using Assumption 2 and (18) we finally obtain

$$\lim_{t\to\infty}\mathbf{z}_j(t - T_{ji}(t)) - \mathbf{z}_i = \mathbf{0},\ \forall\ (i,j) \in \mathcal{E}. \quad (20)$$

Which together with a strongly connected network implies (2c) is achieved. □

3.3 Leaders and formation control

The above result can be extended to formations by modifying the input of the cooperative controls as

$$\hat{\mathbf{r}}_{jsi} = \mathbf{r}_{js} - \mathbf{r}_i - \frac{\lambda}{2}\mathbf{z}^*_{ij},\ \forall\ (i,j) \in \mathcal{E}. \quad (21)$$

Asymptotic properties (12) then imply $\hat{\mathbf{r}}_{jsi} \to \mathbf{0}$. Following the proof of Theorem 1 results in

$$\lim_{t\to\infty}\mathbf{z}_{js} - \mathbf{z}_i = \tfrac{1}{2}\mathbf{z}^*_{ij},\ \forall\ (i,j) \in \mathcal{E}. \quad (22)$$

Substituting the unmodified relation (19) in (22) yields

$$\mathbf{z}_j(t-T_{ji}) - \mathbf{z}_i = \mathbf{z}^*_{ji}.\ \forall\ (i,j) \in \mathcal{E}. \tag{23}$$

Hence objective (2c) is satisfied.

To include leaders we define an additional cooperative potential function V_i^* and its associated control function

$$\phi_i^* = \mathbf{K}_i^* \frac{\partial V_i^*(\mathbf{z}_i^* - \mathbf{z}_i)}{\partial(\mathbf{z}_i^* - \mathbf{z}_i)},\ \forall\ i \in \mathcal{I}_L. \tag{24}$$

The cooperative control input with leaders is given by

$$\tau_i = \phi_i^*(\mathbf{z}_i^* - \mathbf{z}_i) + \sum_{j\in\mathcal{N}_i} \phi_{jsi}(\mathbf{r}_{jsi}). \tag{25}$$

4. LOCALLY R-PASSIVE AGENT CONTROLS

The results in the previous section rely on r-passivity of individual agents (Assumption 2). In this section we present our main results: a control law that renders fully actuated mechanical systems r-passive.

Theorem 2. Agent i with dynamics (1a) - (1b), with $n_i = m_i$, controlled by

$$\boldsymbol{\tau}_i = \mathbf{M}_i\mathbf{J}_i^\dagger\left(\boldsymbol{\tau}_{c,i} - \mathbf{K}_{z,i}\dot{\mathbf{q}}_i\right) + \frac{\partial H_i}{\partial \mathbf{q}_i}, \tag{26a}$$

$$\mathbf{K}_{z,i} = \mathbf{J}_i\left((\lambda+\gamma_i)\mathbf{I}_{n,i} - \mathbf{M}_i^{-1}\dot{\mathbf{M}}_i\right) + \dot{\mathbf{J}}_i, \tag{26b}$$

with $\gamma_i > 0$ a tuning parameter, render the agent passive w.r.t. the storage and dissipation function

$$V_i = \tfrac{1}{2}\mathbf{r}_i^T\mathbf{r}_i + \tfrac{1}{2}\gamma_i\lambda\mathbf{z}_i^T\mathbf{z}_i, \qquad S_i = \gamma_i\dot{\mathbf{z}}_i^T\dot{\mathbf{z}}_i.$$

Proof. The following proof is constructive. The derivative of the storage function is

$$\begin{aligned}\dot{V}_i &= \dot{\mathbf{r}}_i^T\mathbf{r}_i + \gamma_i\lambda\dot{\mathbf{z}}_i^T\mathbf{z}_i \\ &= (\dot{\mathbf{r}}_i + \gamma_i\dot{\mathbf{z}}_i)^T\mathbf{r}_i - \gamma\dot{\mathbf{z}}_i^T\dot{\mathbf{z}}_i.\end{aligned}$$

Substituting in differential passivity relation (4) yields

$$(\dot{\mathbf{r}}_i + \gamma_i\dot{\mathbf{z}}_i)^T\mathbf{r}_i = \boldsymbol{\tau}_{c,i}^T\mathbf{r}_i.$$

Hence the l-dimensional dynamics that achieve r-passivity are given by

$$\boldsymbol{\tau}_{c,i} = \dot{\mathbf{r}}_i + \gamma_i\dot{\mathbf{z}}_i = \ddot{\mathbf{z}}_i + (\lambda+\gamma_i)\dot{\mathbf{z}}_i.$$

Which may be expanded as

$$\begin{aligned}\boldsymbol{\tau}_{c,i} &= \mathbf{J}(\dot{\mathbf{M}}_i^{-1}\mathbf{p}_i + \mathbf{M}_i^{-1}\dot{\mathbf{p}}_i) + \dot{\mathbf{J}}_i\dot{\mathbf{q}}_i + (\lambda+\gamma_i)\mathbf{J}_i\dot{\mathbf{q}}_i, \\ &= \mathbf{J}_i\mathbf{M}_i^{-1}\dot{\mathbf{p}}_i + \mathbf{K}_{z,i}\dot{\mathbf{q}}_i,\end{aligned}$$

Define the feedback $\boldsymbol{\tau}_{c,i} = \hat{\boldsymbol{\tau}}_{c,i} + \mathbf{K}_{z,i}\dot{\mathbf{q}}_i$ such that

$$\hat{\boldsymbol{\tau}}_{c,i} = \mathbf{J}_i\mathbf{M}_i^{-1}\dot{\mathbf{p}}_i. \tag{27}$$

To find the corresponding set of n-dimensional system dynamics, note that $\mathbf{J}_i\mathbf{M}_i^{-1}\mathbf{M}_i\mathbf{J}_i^\dagger = \mathbf{I}_{l\times l}$, hence

$$\mathbf{J}_i\mathbf{M}_i^{-1}\left(\mathbf{M}_i\mathbf{J}_i^\dagger\hat{\boldsymbol{\tau}}_{c,i} - \dot{\mathbf{p}}_i\right) = \mathbf{0}. \tag{28}$$

To obtain passivity, we match these dynamics with the plant momenta equations, i.e.

$$\dot{\mathbf{p}}_i = \mathbf{M}_i\mathbf{J}_i^\dagger\hat{\boldsymbol{\tau}}_{c,i} = -\frac{\partial H_i}{\partial \mathbf{q}_i} + \boldsymbol{\tau}_i. \tag{29}$$

The proposed control laws satisfy this equation. □

This result matches the desired cooperative dynamics with the cooperative behaviour of the plant. In the common case that $n_i > l$, the system possesses uncontrolled degrees-of-freedom. These coordinates may be controlled using subtask optimisation (see Hsu et al. (1988)). Consider the subtask potential $V_{s,i}(\mathbf{q}_i) \geq 0$ which is minimised if the local objective $\mathbf{q}_i^*$ is achieved. The null space definition $\mathbf{J}_i^\perp = (\mathbf{I}_{n_i} - \mathbf{J}_i^\dagger\mathbf{J}_i)$ from Hsu et al. (1988) can be used to influence only the redundant degrees-of-freedom. This definition satisfies

$$\mathbf{J}_i\mathbf{J}_i^\perp = \mathbf{0}_{l\times n_i}, \quad \mathbf{J}_i^\perp\mathbf{J}_i^\dagger = \mathbf{0}_{n_i\times l}, \quad \mathbf{J}_i^\perp\mathbf{J}_i^\perp = \mathbf{J}_i^\perp. \tag{30}$$

The subspace tracking error may then be defined by

$$\mathbf{e}_{s,i} = \mathbf{J}_i^\perp\left(\dot{\mathbf{q}}_i + \frac{\partial V_{s,i}}{\partial \mathbf{q}_i}\right) \triangleq \mathbf{J}_i^\perp\mathbf{w}_i. \tag{31}$$

The second property of (30) prevents the subtask optimisation from modifying the cooperative behaviour.

Theorem 3. Agent i with dynamics (1a) - (1b), where $n_i = m_i$ and

$$\begin{aligned}\boldsymbol{\tau}_i = \frac{\partial H_i}{\partial \mathbf{q}_i} &+ \mathbf{M}_i\mathbf{J}_i^\dagger\left(\boldsymbol{\tau}_{c,i} - \mathbf{K}_{z,i}\dot{\mathbf{q}}_i\right) \\ &- \mathbf{M}_i\mathbf{J}_i^\perp\left(\frac{\partial V_{s,i}}{\partial \mathbf{q}_i} + \mathbf{K}_{v,i}\dot{\mathbf{q}}_i + \dot{\mathbf{J}}_i^\perp\mathbf{w}_i\right),\end{aligned} \tag{32}$$

with $K_{z,i}$ defined by (26b) and

$$\mathbf{K}_{v,i} = \frac{\partial^T}{\partial \mathbf{q}_i}\left(\frac{\partial V_{s,i}}{\partial \mathbf{q}_i}\right) + \mathbf{I}_{n_i} - \mathbf{M}_i^{-1}\dot{\mathbf{M}}_i, \tag{33}$$

achieve r-passivity w.r.t. the storage and dissipation functions (2), while $\mathbf{e}_{s,i} \to \mathbf{0}$.

Proof. Consider the following Lyapunov candidate

$$U_i = \frac{1}{2}\mathbf{e}_{s,i}^T\mathbf{e}_{s,i}. \tag{34}$$

The derivative is

$$\begin{aligned}\dot{U}_i &= \mathbf{w}_i^T\mathbf{J}_i^\perp\dot{\mathbf{w}}_i + \mathbf{w}_i^T\mathbf{J}_i^\perp\dot{\mathbf{J}}^\perp\mathbf{w}_i. \\ &= \mathbf{w}_i^T\mathbf{J}_i^\perp\left(\ddot{\mathbf{q}}_i + \frac{\partial\dot{V}_{s,i}}{\partial \mathbf{q}_i}\right) + \mathbf{w}_i^T\mathbf{J}_i^\perp\dot{\mathbf{J}}^\perp\mathbf{w}_i. \\ &= \mathbf{w}_i^T\mathbf{J}_i^\perp\Bigg[\dot{\mathbf{M}}_i^{-1}\mathbf{M}_i\dot{\mathbf{q}}_i - \mathbf{J}_i^\perp\left(\frac{\partial V_{s,i}}{\partial \mathbf{q}_i} + \mathbf{K}_{v,i}\mathbf{q}_i + \dot{\mathbf{J}}_i^\perp\mathbf{w}_i\right) \\ &\qquad + \frac{\partial\dot{V}_{s,i}}{\partial \mathbf{q}_i}\Bigg] + \mathbf{w}_i^T\mathbf{J}_i^\perp\dot{\mathbf{J}}_i^\perp\mathbf{w}_i.\end{aligned}$$

Where we substituted the dynamics described by the momenta $\dot{\mathbf{p}}_i$. Substituting the proposed $\mathbf{K}_{v,i}$ yields

$$\dot{U}_i = -\mathbf{w}_i\mathbf{J}_i^\perp\mathbf{w}_i = -\mathbf{e}_{s,i}^T\mathbf{e}_{s,i} \leq 0. \tag{35}$$

Hence $\mathbf{e}_{s,i} \to \mathbf{0}$.

A vanishing subspace tracking error, $\mathbf{e}_{s,i} \to \mathbf{0}$, was shown in Hsu et al. (1988) to result in tracking of the subtasks as long as its vector lies in the null space. This result implies that all velocities in the null space of $\mathbf{J}_i$ go to zero, since convergence of V_s implies $\mathbf{w}_i = \dot{\mathbf{q}}_i$. As a consequence of the cooperative scheme and cooperative damping, velocities in the range space of $\mathbf{J}_i$ also converge to zero. Hence we obtain $\mathbf{q}_i \to \mathbf{0},\ \forall\ i \in \mathcal{I}_N$, which implies satisfaction of objective (2a). Since Theorem 1 satisfied objectives (2b), (2c), we conclude that the continuous-time problem is solved.

4.1 Cooperative mass matrices

The approach outlined in Theorem 2 is a top down approach: the desired storage function leads directly to a control law. In the case of Theorem 2 the cooperative mass matrix of each system is the identity matrix, hence systems become homogeneous in their cooperative dynamics. In many cases it may be beneficial to tune the cooperative behaviour per system, e.g. for increased efficiency, overcoming unmodelled friction or faster responses in a particular cooperative subspace. The following result generalises Theorem 2 to general mass matrices.

Theorem 4. Theorem 2 with the modified cooperative control law

$$\hat{\boldsymbol{\tau}}_{c,i} = \mathbf{M}_{z,i}\boldsymbol{\tau}_{c,i} - \mathbf{K}_{z,i}\dot{\mathbf{q}}_i + \tfrac{1}{2}\dot{\mathbf{M}}_{z,i}\mathbf{M}_{z,i}^{-1}\mathbf{r}_i, \tag{36}$$

$$\hat{\mathbf{K}}_{z,i} = \mathbf{J}_i(\lambda\mathbf{I}_{n,i} - \mathbf{M}_i^{-1}\dot{\mathbf{M}}_i) + \dot{\mathbf{J}}_i + \gamma_i\mathbf{M}_{z,i}\mathbf{J}_i. \tag{37}$$

results in r-passivity with respect to the storage and dissipation function

$$V = \tfrac{1}{2}\mathbf{r}_i^T\mathbf{M}_{z,i}^{-1}\mathbf{r}_i + \tfrac{1}{2}\lambda\gamma_i\mathbf{z}_i^T\mathbf{z}_i, \qquad S_i = \gamma_i\dot{\mathbf{z}}_i^T\dot{\mathbf{z}}_i, \tag{38}$$

while asymptotic properties remain unchanged.

Proof. The storage function derivative is

$$\dot{V}_i = \mathbf{r}_i^T\mathbf{M}_{z,i}^{-1}\dot{\mathbf{r}}_i + \tfrac{1}{2}\mathbf{r}_i^T\dot{\mathbf{M}}_{z,i}^{-1}\mathbf{r}_i + \gamma_i\dot{\mathbf{z}}_i^T(\mathbf{r}_i - \dot{\mathbf{z}}_i), \tag{39}$$

Which, using differential passivity (4), leads to the input description

$$\boldsymbol{\tau}_{c,i} = \mathbf{M}_{z,i}^{-1}\dot{\mathbf{r}}_i + \tfrac{1}{2}\dot{\mathbf{M}}_{z,i}^{-1}\mathbf{r}_i + \gamma_i\dot{\mathbf{z}}_i$$

$$\mathbf{M}_{z,i}\boldsymbol{\tau}_{c,i} = \dot{\mathbf{r}}_i - \tfrac{1}{2}\dot{\mathbf{M}}_{z,i}\mathbf{M}_{z,i}^{-1}\mathbf{r}_i + \gamma_i\mathbf{M}_{z,i}\dot{\mathbf{z}}_i.$$

The proposed cooperative control law satisfies this equation and results in the system dynamics (27). □

A simple application of Theorem 4 is shaping for a scaled identity mass $\mathbf{M}_{z,i}^{-1} = \eta_i\mathbf{I}_l$. Since $\dot{\mathbf{M}}_{z,i} = \mathbf{0}_{l\times l}$, the modification to the control law is a simple gain of $\frac{1}{\eta_i}$ on the received cooperative input $\boldsymbol{\tau}_{c,i}$. Hence the system is virtually made heavier by reducing its input.

This concludes the approach in continuous-time. We have solved the continuous-time problem for fully actuated mechanical systems using local controllers (32) to achieve r-passivity of agents and Theorem 1 to achieve cooperative objectives. The proposed scheme can be applied in a discrete-time context with time-varying delays and packet loss by applying a Communication Management Module (CMM). In the following experiments we apply Wave-Variable Modulation (WVM) as developed in Liu and Puah (2014). This CMM buffers incoming messages, which are timestamped, and is read by the controller at the network frequency. With decreasing delays, two packets may arrive at the same instance. In this case we keep only the most recent packet. Increasing delays and packet loss may lead to an empty buffer when sampling (Berestesky et al. (2004)). In these cases WVM locally computes a passive reconstruction of the wave-variable, increasing performance while remaining stable.

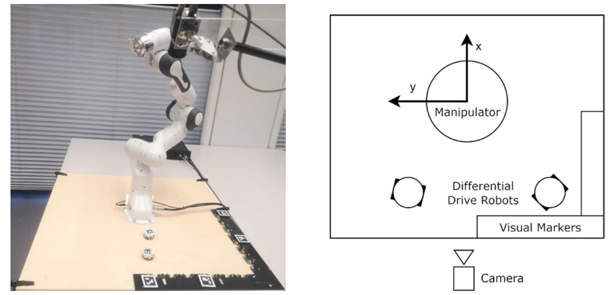
5. RESULTS

Experimental results of the proposed control scheme are presented in the following. The setup, depicted in Figures 2(a) and 2(b), consists of a 7 Degree-Of-Freedom (DOF) robotic manipulator [1] and two Differential Drive robots [2].

[1] Franka Emika Panda, see https://www.franka.de/technology/

[2] GCtronic Elisa3, see https://www.gctronic.com/doc/index.php/Elisa-3

Our experiments demonstrate the proposed control scheme in an abstract picking and packing problem, e.g., the driving robots could be carrying items that need to be grasped and moved by the manipulator to another cart.



(a) Photo. (b) Schematic

Fig. 2. The experimental setup

The manipulator is fully actuated with $n = 7$. We define the position of the end-effector in 3D space as cooperative coordinates such that $\mathbf{z} = (x, y, z) \in \mathbb{R}^3$. The matrices $\mathbf{M}(\mathbf{q}), \mathbf{J}(\mathbf{q})$ required for control are obtained from the robot controller over a wired Ethernet connection and gravitational forces are internally compensated. The sampled data control loop is implemented at a rate of 1 kHz. Matrix derivatives $\dot{\mathbf{M}}, \dot{\mathbf{J}}$ are obtained by numeric differentiation and low-pass filtering.

The differential drive robots are actuated by two speed-control motors. We use the model and feedback linearisation around the hand position derived in Lawton et al. (2003). Virtual accelerations are integrated internally to generate speed outputs from force inputs. The resulting cooperative model is a point-mass at the hand position. We have $n = l = 2$ and $\mathbf{z} = (x, y) \in \mathbb{R}^2$. The world reference frame is located at the manipulator base. The positions of the Elisa3 agents in this frame are measured via a camera detection system. The controllers of both systems and the communication network run on a single computer. Thus, to illustrate performance with respect to time-varying delays and packet loss, a network is emulated artificially. To limit the slope of the random delays we generate delays via a random walk process with $0.2\ s \leq T \leq 0.4\ s, \dot{T} \leq 0.5$. We use a Bernoulli dropout model with $p = 0.05$ to emulate packet loss. The methods introduced in this work are implemented in ROS and C++ (source code available at https://github.com/oscardegroot/ROS_rPBC).

The objective in the following experiment is to form a circle in the ground plane with a radius of 0.1 m between the three agents. The height of the manipulator is constrained by defining it as a leader coordinate with 0.3 m as reference and 1.0 as gain. The main parameter values are summarised in Table 1. Joint-limits of the manipulator

Table 1. Experimental parameter values.

λ	γ	κ	f_N	$\mathbf{K}_{ij}$	R_w	ϵ	α
1.0	0.05	0.05	100 Hz	$1.5\mathbf{I}_n$	30.0	0.0004	35.0

are evaded using a quadratic local potential around the central angle of the joints. The transient response is sped up by setting its mass to $\eta = 0.015$. Similarly for the differential drive robots we set $\eta = 0.05$. Figures 3 and 4 show the trajectories and cooperative coordinates of the agents respectively. The circular formation is achieved by the agents. Notice that the height of the leader converges to the reference of 0.3 m. Although the effects of the delays are visible in slight oscillations of the trajectories, their deteriorating effect on the performance is small and the system converges without drift. The cooperative velocities depicted in Figure 5 converge to zero.

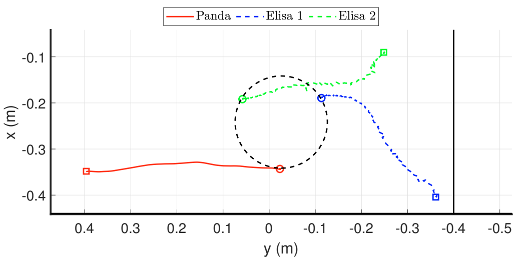


Fig. 3. Trajectories of the agents in the ground plane. The initial and final positions of the agents are denoted with a square and circle respectively. The bounds in the x and y axes are drawn as a black solid line.

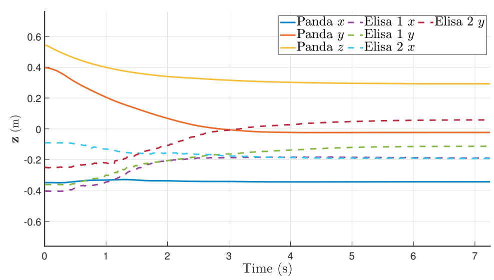


Fig. 4. Cooperative coordinate evolution over time.

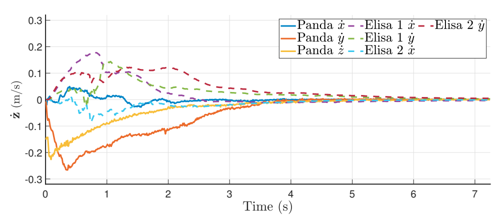


Fig. 5. Cooperative velocities over time.

To show the robustness of the proposed method we perform multiple consensus experiments with the manipulator and a single differential drive robot. In the first set of experiments, the minimum- and maximum delay are increased in 200 ms increments, while packet loss is disabled. In the second set of experiments delays are varied between 200 ms and 400 ms, while packet loss is increased in 5% increments. We plot the last time where the Euclidean distance between agents exceeds 0.02 m ("settling time"). The results are visible in Figure 6. The systems settle for

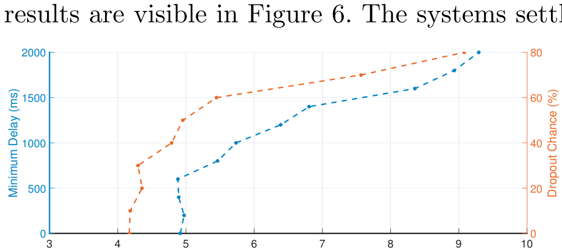


Fig. 6. Robustness results of the presented method towards time delays and packet loss.

all the presented delays, at the expense of performance. We claim that this trend proceeds for higher delays. For packet loss higher than 80% we start to observe drift. The presented results motivate the applicability of the proposed approach in a wide range of network conditions.

6. CONCLUSIONS AND FUTURE RESEARCH

In this work we developed a unified cooperative control scheme for nonlinear fully actuated heterogeneous agents using passivity theory. Where existing methods selectively achieve robustness to time-delay or cooperative nonlinear control, we have derived a method that accomplishes both. We have additionally shown that the coordinates of each agent may be split into local and cooperative coordinates which may be influenced separately via control. Accordingly, the designer is provided with an intuitive set of weights to tune for appropriate local and cooperative behavior. The set of introduced cooperative controls based on Navigation Functions incorporates constraints such that obstacles, other agents or singularities may be avoided. We have successfully demonstrated the efficacy of the approach experimentally.

Future work will be focussed on the extension of the proposed methodology to underactuated systems. Matching of the underactuated degree, which is the key ingredient of Interconnection- and Damping Assignment Passivity-Based Control (IDA-PBC), may be applicable to the proposed framework in the case of underactuated systems.

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