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DOI
10.1016/j.oceaneng.2022.110579

Publication date
2022

Document Version
Final published version

Published in
Ocean Engineering

Citation (APA)

Important note
To cite this publication, please use the final published version (if applicable). Please check the document version above.
Impact of cyclic strain accumulation on the tilting behaviour of monopiles in sand: An assessment of the Miner’s rule based on SANISAND-MS 3D FE modelling

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A B S T R A C T

Offshore monopiles accumulate permanent tilt under long-lasting cyclic environmental loads. Accurate prediction of monopile tilt is key to assessing their serviceability, and requires a fundamental understanding of loading history effects. While both experimental and numerical studies are shedding light on this matter, this work uses step-by-step implicit 3D FE modelling to investigate loading history effects in the response to cyclic lateral loading of monopiles in sand and to identify links between local soil behaviour and relevant features of global pile behaviour. For this purpose, the recently developed SANISAND-MS model is adopted to achieve a reliable simulation of sand’s cyclic ratcheting. In particular, the validity of an up-scaled Miner’s rule for monopile tilting under multi-amplitude cyclic loading is assessed based on the results of 3D FE parametric analyses, with emphasis on the role played by the engineering idealisation of random environmental loading. The validity of such a rule has been numerically investigated both in terms of local soil element response and global foundation behaviour — for the particular case of a large-diameter monopile. In respect, the effect of the loading history idealisation is presented, and it is concluded that Miner’s rule does not always rigorously apply to all the cases considered herein. The translation of irregular loading histories into a regular version with loading packages sorted in ascending amplitude order is shown to be a reasonable approach, at least when the possibility of cyclic pore pressure build-up is disregarded.

1. Introduction

Monopile foundations for offshore wind turbines (OWTs) must be designed to guarantee that the superstructure remains fully functional through the whole lifetime environmental cyclic loading. In this regard, avoiding the excessive cyclic accumulation of lateral monopile tilt is extremely relevant to design (Arany et al., 2015), as is also testified by the growing body of offshore geotechnical research dedicated to the subject (Cuéllar et al., 2009; LeBlanc et al., 2010b; Klinkvort and Hededal, 2014; Byrne et al., 2015; Staubach and Wichmann, 2020). The tilting response of monopiles is affected by a number of governing factors, including monopile geometry, soil type, and loading conditions. The influence of these factors has been investigated experimentally by several authors, mostly with regard to the drained cyclic response of monopiles in sand (LeBlanc et al., 2010a; Klinkvort and Hededal, 2013; Richards et al., 2019). Relevant numerical modelling research has also been carried out, most often to develop simplified engineering methods for the serviceability and fatigue analysis of monopiles — see, e.g., the 0D macroelement models proposed by Houlsby et al. (2017) and Page et al. (2018), as well as Abadie et al. (2019, 2020).

Recently, three-dimensional finite element (3D FE) simulations have been increasingly adopted to explore the fundamental mechanisms that govern monopile tilting (Jostad et al., 2020; Liu et al., 2021), also with respect to the effects of the monopile installation method (Staubach et al., 2020; Fan et al., 2021; Bienen et al., 2021). The 3D FE analysis of cyclic soil–monopile interaction can be built on either ‘explicit’ or ‘implicit’ constitutive modelling of cyclic soil behaviour. ‘Explicit’ modelling takes the number of loading cycles as a model input to compute monopile deformation over a given loading period (Achmus et al., 2009; Jostad et al., 2014; Wichmann et al., 2017). In such a way, the accumulated soil strains over each loading cycle are exclusively evaluated at one selected time. Generally, extensive experimental programmes are required to support explicit modelling calculations. In contrast, ‘implicit’ methods compute the cyclic soil response incrementally, i.e., in a step-by-step manner, so that the overall behaviour of...
the considered system is obtained as a result of usual time integration. Nowadays, ‘implicit’ 3D FE modelling approaches are being increasingly adopted to study the cyclic performance of monopile lateral behaviour under cyclic loading (Cuéllar et al., 2014; Tasiopoulou et al., 2021; Liu et al., 2021; Liu and Kaynia, 2021).

Most of the abovementioned experimental and computational studies exclusively consider monopiles subjected to regular, single-amplitude cyclic loading histories. However, real environmental loads occur in irregular repetitions with varying amplitude, average level, and frequency. In offshore design practice, irregular loading histories are often idealised as regular using methods such as the rainflow counting method (Kaggwa et al., 1991), which leads to convert the original loading history into a sequence of multiple loading packages with cyclic amplitude and number of cycles varying from one package to the next. While more research on the response of monopiles to multi-amplitude loading cycles is certainly needed (Page et al., 2021), the well-known Miner’s rule has been often adopted to conceptualise loading history effects in the tilting response of monopiles. Miner’s rule is an empirical postulate that was originally proposed to describe cyclic fatigue mechanisms in metals (Miner et al., 1945; Wilkins, 1956), and later applied to cyclically loaded soils as well (Stewart, 1986; Wichmann et al., 2010). However, the upsampling of Miner’s rule from the soil element level to the global foundation scale level can only be accomplished through further studies on the response of offshore foundations to multi-amplitude cyclic loading, with focus on the implications of adopting idealised/regular loading histories for engineering design purposes. Currently, the studies on this aspect have been mainly performed mainly on 1 g experimental (LeBlanc et al., 2010a; Abadie et al., 2018; Luo et al., 2020; Ma et al., 2021) and ‘explicit’ 3D FE modelling (Wichtmann et al., 2017) perspectives.

The present study builds on ‘implicit’ 3D FE modelling of cyclic monopile–sand interaction, in which the quality of the soil constitutive model plays a crucial role. In this regard, the SANISAND-MS model with ratcheting control proposed by Liu et al. (2019) is adopted herein to perform parametric studies on the response of monopiles to multi-amplitude cyclic loading. Using advanced constitutive models in 3D FE simulations allows to study in detail geotechnical mechanisms that force-resultant macro-models can only incorporate as a priori constitutive assumptions (Abadie et al., 2018). Similar considerations apply to the ‘meso-scale’ SANISAND-MS model, whose formulation and calibration could be supported by parallel micro-mechanical studies (Alonso-Marroquin and Herrmann, 2004; Kawamoto et al., 2018). In what follows, the ability of SANISAND-MS to simulate at the soil element level a cyclic behaviour that is compliant with Miner’s rule is first explored in Section 2. The upsampling of Miner’s rule to the global monopile scale is then discussed based on 3D FE results presented in Section 3 in terms of cyclic pile head displacement, shear force and moment distributions, and lateral soil reactions (i.e., \( p – y \) response). The implications of idealising the external loading history are discussed in Section 4 with respect to two different cases, in which realistic loading histories of 1 h and 20 min duration are considered in their original and idealised versions. The ultimate goal of this work is to exploit state-of-the-art 3D FE modelling to investigate relevant loading history effects in the cyclic lateral response of monopiles. The results and observations presented in the following will provide further insight into lesser-known aspects of cyclic soil–pile interaction, and will support the enhancement of existing simplified methods for engineering analysis/design.

2. SANISAND-MS modelling of cyclic sand behaviour

2.1. Model highlights

SANISAND-MS model is an elasto-plastic implicit constitutive model, originally proposed by Liu et al. (2019) to improve the simulation of cyclic sand ratcheting with respect to the previous SANISAND parent model by Dafalias and Manzari (2004). The model is formulated in the framework of bounding surface plasticity, and complies with well-established critical state principles through the use of the state parameter introduced by Been and Jeffries (1985), which allows to capture the dependence of sand’s behaviour both on current void ratio and mean effective stress. Following the work of Corti et al. (2016), the ‘memory surface’ concept has been introduced into the SANISAND constitutive formulation to enhance the simulation of cyclic strain accumulation, i.e., to control the development of cyclic ratcheting. As is shown in Fig. 1, SANISAND-MS features four main model surfaces – namely, bounding, yield, dilatancy and memory surfaces – and has been quantitatively validated against experimental data from the literature — more details about model formulation and validation may be found in Liu et al. (2019), Liu and Pisanò (2019) and Liu et al. (2020). The applicability of the resulting 3D FE SANISAND-MS framework to the analysis of cyclically loaded monopiles has been recently demonstrated by Liu et al. (2021) and Liu and Kaynia (2021), though with exclusive reference to single-amplitude loading cycles.

2.2. Model performance under multi-amplitude loading: compliance with Miner’s rule at soil element level

It is customary in geotechnical research to validate cyclic constitutive models against laboratory test results obtained under single-amplitude repeated loading. However, soils under offshore foundations are normally subjected to highly irregular, non-stationary cyclic loads, whose impact on the interpretation and modelling of cyclic soil behaviour is still an open research question (Andersen, 2015; Zografou et al., 2018). This section provides new evidence about the performance of SANISAND-MS under multi-amplitude cyclic loading. In particular, compliance with the above mentioned Miner’s rule is verified to expand the validation basis of the model, and boosts confidence in its use for the solution of geotechnical boundary value problems. Miner’s rule has been proposed to estimate the cumulative fatigue damage of metals using an linear accumulation approach. The rule can be expressed using the equation below:

\[
f_D = \sum_{i=1}^{N} \frac{n_i(a_{i,amp})}{N_i(a_{i,amp})}
\]

where \( f_D \) is the total damage that can be set as one (i.e., failure) for design purpose. \( N_i \) is the total number of cycles to failure when subjected to a constant amplitude \( a_{i,amp} \), and \( n_i \) is the number of the applied load cycles under the same load amplitude. Miner’s rule is a linear damage rule that assumes the effects of the sequence of applied load amplitudes can be ignored. Focusing on the design aspects of accumulation of foundation displacement (and thus accumulation of the strain in local soil response), the essence of Miner’s rule in relation to sand’s cyclic behaviour can be summarised by the following two statements:
For each sequence, the accumulated total strain $\varepsilon_{acc}$ was recorded at the end of each cycle when $q = q^{max}$, without accounting for the very first cycle:

$$\varepsilon_{acc} = \sqrt{(\varepsilon_{ax}^{acc})^2 + 2(\varepsilon_{rad}^{acc})^2}$$  

(2)

where $\varepsilon_{ax}$ and $\varepsilon_{rad}$ denote the axial and radial strain components – see also Wichtmann et al. (2010) and Liu et al. (2019).

Fig. 2 shows a generally good agreement between experimental and SANISAND-MS results over the 10^5 cycles considered in the testing programme (4 packages comprising 25 000 cycles each). In particular, Fig. 2a displays gradual monotonic increase in $\varepsilon_{acc}$ with the number of cycles $N$, at a rate depending on the current cyclic load amplitude. In Fig. 2b obvious strain accumulation occurs during the second package ($q^{max} = 60$ kPa), at a rate faster than observed during the first segment of lower amplitude ($q^{max} = 20$ kPa); the lower cyclic load in the third stage leads to practically no ratcheting, finally re-activated by the highest load amplitude in the fourth package. Strain accumulation patterns fully consistent with Miner’s rule/statement 1 can also be observed for the third (Fig. 2c) and fourth (Fig. 2d) load sequences. The last case in Fig. 2d corroborates the idea of cyclic strain accumulation being dominated by the highest load amplitudes, as they are experienced by the soil for the first time.

The results in Fig. 2, however, do not seem to fully confirm the validity of Miner’s rule/statement 2. Both the experimental and simulation results indicate that applying load packages with large amplitude results in larger accumulated strain. The experimental results in Figs. 2c and 2d indicate the achievement of an accumulated strain equal to 2% after 10^5 cycles, which is larger than the final value of ~1.7% that is visible in Figs. 2a and 2b. Although $\varepsilon_{acc} - N$ trends are specific to the considered load sequence (which challenges the general validity of Miner’s rule), the residual strain $\varepsilon_{acc}$ ($N = 10^5$) lies in all cases within

### Table 1

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Critical state</th>
<th>Yield surface</th>
<th>Plastic modulus</th>
<th>Dilatancy</th>
<th>Memory surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
<td>$\gamma$</td>
<td>$M_c$</td>
<td>$\delta_c$</td>
<td>$\zeta$</td>
<td>$m$</td>
</tr>
<tr>
<td>110</td>
<td>0.05</td>
<td>1.27</td>
<td>0.712</td>
<td>0.049</td>
<td>0.845</td>
</tr>
<tr>
<td>$b_0$</td>
<td>$c_0$</td>
<td>$n^d$</td>
<td>$\alpha_o$</td>
<td>$n^p$</td>
<td>$\mu_0$</td>
</tr>
<tr>
<td>3.95</td>
<td>1.01</td>
<td>2.0</td>
<td>1.06</td>
<td>1.17</td>
<td>200</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>$\beta$</td>
<td></td>
<td></td>
<td></td>
<td>0.0005</td>
</tr>
</tbody>
</table>

For the considered load sequences, the following load packages were performed on dry, medium-coarse Karlsruhe sand ($D_0 = 0.55$ mm, $D_{10} = 0.29$ mm, $e_{max} = 0.874$, $e_{min} = 0.577$, maximum (dry) unit weight $\gamma_{max} = 16.5$ kN/m^3, minimum (dry) unit weight $\gamma_{min} = 13.9$ kN/m^3, coefficient of uniformity $C_u = D_{05}/D_{10} = 1.8$).

The corresponding SANISAND-MS model parameters were previously identified by Liu et al. (2019) with respect to single-amplitude cyclic loading conditions (see Table 1).

All the results considered in the following, both experimental and numerical, are associated with the following triaxial test settings: $p_{ts} = 200$ kPa (mean stress at the beginning of, and during, cyclic loading), $q^{acc} = 150$ kPa (average deviatoric stress during cyclic loading), and initial, pre-cyclic relative density set to $D_r = 59\%$ ($e_{ts} = 0.698$, initial void ratio) in numerical simulations to represent the experimental range of 58%-63%. The soil response to cyclic load packages of four different deviatoric stress amplitude ($q^{max} = 20, 40, 60, 80$ kPa) was considered with respect to the following load sequences (Fig. 2):

- (a) 20 kPa → 40 kPa → 60 kPa → 80 kPa;
- (b) 20 kPa → 60 kPa → 40 kPa → 80 kPa;
- (c) 60 kPa → 20 kPa → 60 kPa → 40 kPa;
- (d) 80 kPa → 60 kPa → 40 kPa → 20 kPa.
a relatively narrow range, i.e., 1.7–2.0%. It may thus be argued that the specific arrangement of the cyclic packages may only have a weak impact on \( \eta^{\text{acc}} (N = 10^7) \). In other words, only a few, high-amplitude packages seem to affect soil ratcheting in the longest term. Fig. 2 support the application of SANISAND-MS to cyclic soil–structure interaction problems involving multi-amplitude cycles, with soil parameters that can still be calibrated against more common single-amplitude test data.

### 3. Upscaling of Miner’s rule

After evaluating the compliance with Miner’s rule of SANISAND-MS simulations performed at the soil element level (Fig. 2), this section aims to assess whether the rule may be upscaled to monopile–sand systems subjected to multi-amplitude lateral cyclic loading. While the validity of such upscaling has been recently supported by several experimental studies (Li et al., 2015; Truong et al., 2019; Abadie et al., 2018), the rest of this work will seek further insight using SANISAND-MS 3D FE modelling.

#### 3.1. 3D FE model

All the results presented in the following have been obtained using the OpenSees FE simulation platform (McKenna, 2011), within which an original implementation of SANISAND-MS was previously developed (Liu et al., 2019). Fig. 3 shows the 3D FE model adopted in this study, which features:

1. an elastic tubular monopile, with diameter \( D = 5 \) m, embedded length \( L = 20 \) m, and wall thickness \( t \approx 10 \) cm;
2. a soil box domain with dimensions \( W_1 = 30 \) m, \( W_2 = 35 \) m, \( L + B = 30 \) m, as indicated in Fig. 3. The symmetry of the problem at hand (one-directional lateral loading) was exploited to reduce the computational costs, so that only half of the soil domain was modelled;
3. the soil is assumed to be dry (fully drained conditions) and characterised by the SANISAND-MS parameters given in Table 1. Initial values of relative density equal to \( D_r = 30\% \) and \( D_r = 70\% \) are exclusively considered in the following;
4. lateral load \( H \) applied with an eccentricity \( e_{ecc} = 4D = 20 \) m above the soil surface. The load application point was connected at the mudline with the rest of the 3D pile by means of an elastic Timoshenko beam. Boundary conditions were imposed on the soil domain to obtain a fully fixed bottom surface, a free upper surface, and no horizontal displacement along the direction perpendicular to the lateral surface. Detailed description of FE solution algorithms and model verification are provided by Liu et al. (2021).

#### 3.2. Simulation programme with multi-amplitude regular cyclic loading

A first set of SANISAND-MS 3D FE simulations was performed to investigate the response of the reference monopile to multi-amplitude regular cyclic loading, i.e., under a sequence of different cyclic load packages of different, though constant, amplitude. As is shown in Fig. 4, each cyclic load package is completely defined by its average amplitude \( H_{\text{ave}} \) and regular cyclic variation \( H_{\text{cyc}} \). Alternatively, the dimensionless amplitude and asymmetry load factors introduced by LeBlanc et al. (2010a) – \( \zeta_h \) and \( \zeta_e \), respectively – may be also used after defining a reference horizontal load \( H_{\text{ref}} \) (here taken as the horizontal load that monotonically induces a lateral pile displacement at the soil surface equal to 0.1\( D \)):

\[
\zeta_h = H_{\text{max}}/H_{\text{ref}} = (H_{\text{ave}} + H_{\text{cy}})/H_{\text{ref}} \tag{3}
\]

\[
\zeta_e = H_{\text{max}}/H_{\text{min}} = (H_{\text{ave}} - H_{\text{cy}})/(H_{\text{ave}} + H_{\text{cy}}) \tag{4}
\]

where \( H_{\text{max}} \) and \( H_{\text{min}} \) denote the maximum and minimum lateral load values during a given cyclic package. Based on Liu et al. (2021), the reference horizontal load for the monopile in Fig. 3 is \( H_{\text{ref}} = 15450 \) kN and \( H_{\text{ref}} = 26800 \) kN for \( D_r = 30\% \) and \( D_r = 70\% \), respectively. In this work, the monopile is subjected to packages of pure one-way cyclic loading (i.e., \( \zeta_e \geq 0 \)), with maximum \( \zeta_e \) never larger than 0.3. A summary of the considered loading cases with regular cyclic packages is provided in Table 2. Load history effects are investigated by simulating different application patterns of the selected load packages, namely with increasing, mixed, and decreasing amplitude. Due to the computational costs that are typical of implicit 3D FE calculations, individual packages of 100 cycles each are exclusively considered: while this limitation should be carefully borne in mind prior to any extension to real environmental loading histories, it will still be possible to infer useful lessons about cyclic soil–monopile interaction under relatively complex lateral loading sequences.

#### 3.3. Load history effects on global monopile cyclic response

Several numerical studies on OWTS foundation behaviour under cyclic loading of constant amplitude have been recently published for both cases of drained (Staubach and Wichmann, 2020; Tasiopoulos et al., 2021; Staubach et al., 2021; Liu et al., 2021) and undrained (Tasiopoulos et al., 2021; Liu and Kaynia, 2021) conditions. This section adds to the understanding of cyclic monopile behaviour by investigating the load history effects associated with the application of regular multi-amplitude cyclic load sequences.

3D FE results indicate the monopile lateral displacement (unless otherwise specified, always referred to the value at the soil surface level) evolves at a significant rate when the current load amplitude is the highest experienced until that moment — for each cycle, the respective pile lateral displacement is the value associated with the average load level \( H = H_{\text{ave}} \). This is clearly visible for the simulation cases in loose sand (\( D_r = 30\% \)) – see Figs. 5a and 6a – where the increasing cyclic amplitude of subsequent load packages induces progressively
Table 2
Regular multi-amplitude cyclic loading cases for the reference monopile in loose and dense sand.

<table>
<thead>
<tr>
<th>Order</th>
<th>( \Delta r ) [%]</th>
<th>( H_{ave} ) [kN]</th>
<th>( H_{cy} ) [kN]</th>
<th>( \zeta_{b} )</th>
<th>( \zeta_{c} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increasing</td>
<td>30</td>
<td>15 450 2310</td>
<td>770 → 1540 → 2310</td>
<td>0.2 → 0.25 → 0.3</td>
<td>0.5 → 0.21 → 0</td>
</tr>
<tr>
<td>Mixed</td>
<td>70</td>
<td>26 800 4020</td>
<td>1540 → 770 → 2310</td>
<td>0.2 → 0.25 → 0.3</td>
<td>0.21 → 0.5 → 0</td>
</tr>
<tr>
<td>Decreasing</td>
<td>70</td>
<td>26 800 4020</td>
<td>2310 → 1540 → 770</td>
<td>0.3 → 0.25 → 0.2</td>
<td>0 → 0.21 → 0.5</td>
</tr>
<tr>
<td>Increasing</td>
<td>70</td>
<td>26 800 4020</td>
<td>4020 → 1540 → 2310</td>
<td>0.3 → 0.25 → 0.2</td>
<td>0 → 0.21 → 0.5</td>
</tr>
</tbody>
</table>

Fig. 5. Influence of the cyclic loading sequence on the evolution of monopile tilt — force–displacement response at the pile head. (a)–(c), \( H_{ave} = 2310 \) kN, \( \Delta r = 30\% \); (d)–(f), \( H_{ave} = 4020 \) kN, \( \Delta r = 70\% \).

higher lateral displacement accumulation rates. Conversely, negligible lateral displacement is observed in Figs. 5c and 6c for the second and third packages, both featuring an amplitude smaller than the first. The intermediate case in Figs. 5b–6b also shows displacement accumulation trends in full agreement with an up-scaled version of Miner’s rule.

Qualitatively similar results have also been obtained for the case of dense foundation sand (\( \Delta r = 70\% \)): (1) when increasing the cyclic amplitude through the subsequent load packages, the lateral displacement of the pile head accumulates significantly from package to package (see Figs. 5d and 6d); (2) Figs. 5e and 6e show again displacement accumulation trends in full agreement with an upscaled Miner’s rule; (3) if the loading package of largest cyclic amplitude is applied first, a lower pile head displacement accumulation rate results for the following packages — see Figs. 5f and 6f.

It is worth noting in Figs. 5f and 6f that, although with an accumulation rate that is much lower than during the second and third packages, non-negligible pile tilting still occurs during the last weakest package. Such phenomenon has also been experimentally observed by Barari et al. (2017). One possible reason is that for dense sand, notwithstanding the assumption of drained conditions in this work, many element are sheared under (nearly) constant volume conditions — thus mobilising a soil response that may be referred to as of an ‘undrained-type’, as indicated by Fig. 7c. During this process, soil shear straining develops until its strain-saturation state (Fig. 7d). The continually developing pile deflection may be a direct consequence of
Fig. 6. Influence of the cyclic loading sequence on the evolution of monopile tilt — pile head lateral displacement against the number of loading cycles. (a)-(c), $H_{ave} = 2310$ kN, $D_r = 30\%$; (e)-(f), $H_{ave} = 4020$ kN, $D_r = 70\%$.

the shear/deviatoric strain evolution of surrounding soil. However, in loose sand, the stress evolution in most elements reaches a steady state at a moderate $p$ level (Fig. 7c), while both the volumetric and shear strains evolve only slightly (Fig. 7b).

With regard to the considered loading sequences, the values of accumulated lateral displacement after 300 cycles are not exactly identical but lie within a very narrow range (i.e., within 0.105–0.11 m for loose sand and 0.29–0.32 m for dense sand) without an obvious dependence on the specific package application sequence. Such conclusion agrees with the experimental (1 g, pile) observations from LeBlanc et al. (2010a) and Ma et al. (2021). However, one may notice that for other foundation type (e.g., suction caisson as studied by Luo et al. (2020)), loading history with parcels in the order of decreasing cyclic amplitude results in larger accumulated displacement/rotation.

Comparing the accumulated pile head displacement in all three loading histories (parcels with increasing, mix and decreasing amplitude), simulation results of loose sand case show less obvious variation than that of dense sand case. It indicates that dense sand is more sensitive to the load sequence (as also observed by Barari et al. (2017)).

The different pile head displacement accumulation patterns in Figs. 5 and 6 indicate, for monopiles both in loose and dense sand, that an upscaling Miner’s idealisation may only be partially valid.

Distributions of internal shear force along the monopile are plotted in Figs. 8a and 9a for the cases of $D_r = 30\%$ and $D_r = 70\%$, respectively — for all cyclic sequences in Table 2, shear forces values are associated with the last cycle in each loading package when $H = H_{ave} = H_{ave} + H_{cy}$. In all instances, the absolute value of the shear force tends to decrease down to about three-quarters of the pile embedded length, which approximately coincides with the location of the monopile rotation point. For a given sand relative density, (nearly) the same shear force distribution is simulated in relation to a certain maximum load amplitude, which confirms the expected dominance of general equilibrium requirements over loading history effects. The corresponding bending moment distributions are presented in Figs. 8b and 9b. In the case of monopile in loose sand (Fig. 10), almost identical distributions of shear force and bending moment result for three considered loading sequences (Table 2), which confirms the validity of the upscaling Miner’s rule for the reference
monopile in loose sand — cf. to Figs. 5a–5c and Figs. 6a–6c: it may thus be suggested that, for the considered loading condition ($\zeta_b \leq 0.3$ and $\zeta_c \geq 0$, drained) and duration ($N = 300$), load history effects are practically negligible in terms of geotechnical and structural response of the monopile. In contrast, the case of dense sand in Fig. 11 reveals shear force and bending moment profiles that are package-dependent when $H = H_{ave}$, with a clear impact of the loading history even though all force/moment profiles are associated with the same external load level on the monopile.

Additional insight into the interaction between monopile and soil may be gained by studying the evolution of the lateral soil reaction at selected depths (integral over the pile circumference) against the pile deflection. Cyclic $p - y$ relationships are shown in Fig. 12 for two representative depths, namely equal to $0.25D$ and $3.5D$ below the soil surface. It may generally be observed that the lateral soil reaction also seems to follow a ‘Miner-type’ evolution against the pile deflection at the considered depth, as is possible to appreciate in Fig. 5. Substantial variations in lateral soil reaction and pile deflection accumulation are visible at both depths when the load parcel of largest amplitude applied. Slightly larger resistance force is assembled at soil depth $= 3.5D$. Overall, very similar soil reaction ranges and pile deflections are obtained for a given $D_r$, with only minimal loading history effects.

Fig. 7. Stress path evolution (deviatoric stress $q$ against mean effective stress $p$ response) and strain evolution (plastic volumetric strain $\varepsilon_{vol}$ against $\varepsilon_q$ response). Simulation results on element A in Fig. 3.

Fig. 8. Influence of the cyclic loading sequence on the distributions of (a) shear force and (b) bending moment along the monopile $= D_r = 30\%$. For the three loading sequences in Table 2, results extracted at the end of each load package ($N = 100$) at the correspond maximum load level ($H = H_{max}$).
Fig. 9. Influence of the cyclic loading sequence on the distributions of (a) shear force and (b) bending moment along the monopile – $D_r = 70\%$. For the three loading sequences in Table 2, results extracted at the end of each load package ($N = 100$) at the maximum load level ($H = H_{\text{max}}$).

Fig. 10. Influence of the cyclic loading sequence on the distributions of (a) shear force and (b) bending moment along the monopile – $D_r = 30\%$. For the three loading sequences in Table 2, results extracted at the end of each load package ($N = 100$) at the average load level ($H = H_{\text{ave}}$).

3.4. Links to the ‘terminal density’ concept from 3D FE results

Sandy soils are known to achieve a constant density (‘terminal density’) when subjected to prolonged cyclic loading (Lackenby et al., 2007). The attainment of a terminal density state depends on the soil initial condition (void ratio, inherit fabric, etc.) and loading/boundary conditions (confining pressure, stress path, etc.). This has been demonstrated through the results of triaxial (Lackenby et al., 2007) and oedometer tests (Park and Santamarina, 2019), and confirmed on a constitutive modelling basis (Liu and Pisano, 2019).

This section aims to investigate the role of the (mean) effective stress state on sand’s ‘terminal density’ behaviour in the reference boundary value problem – a monopile under lateral cyclic loading. Based on the discussion about Fig. 7d, an obvious evolution of the plastic volumetric strain can still be observed in dense sand under a loading history with load packages ordered in decreasing amplitude order, which means that a trend to approach a terminal density has not been fully triggered in this case. For this reason, the discussion on terminal density in this section is limited to the simulations associated with $D_r = 30\%$.

In Fig. 13, the mechanism behind the ‘meso-scale’ Miner-type $D_r$ evolution is revealed. The relevance of the soil stress state is highlighted by tracking the evolution of the confining pressure and the Lode angle. In this work, the Lode angle is defined in the range $\theta = [0^\circ \ldots 60^\circ]$ (from triaxial compression to extension). However, the colour-bar in Fig. 13 is restrained to its real variation range (i.e., $10^\circ \ldots 40^\circ$) for better visualisation and support to quantitative observations. Plots are associated with a state of the system associated with a load level of $H = H_{\text{ave}}$ for each cycle.

Under the three considered loading cases, the relative density $D_r$ of an element keeps evolve on the condition that the mean effective stress $\sigma$ or the Lode angle vary as well. Only when both the mean effective stress $\sigma$ and the Lode angle achieve a relatively stable state (for instance, $N = 50\ldots 100$ in Fig. 13a, $N = 100\ldots 200$ in Fig. 13b and...
Fig. 11. Influence of the cyclic loading sequence on the distributions of (a) shear force and (b) bending moment along the monopile – $D_r = 70\%$. For the three loading sequences in Table 2, results extracted at the end of each load package ($N = 100$) at the average load level ($H = H_{ave}$).

Fig. 12. Cyclic soil resistance force against pile deflection under different loading sequences. (a)-(c): $D_r = 30\%$ and (d)-(f) $D_r = 70\%$. 
with the following load histories:

To seek confirmation of this assumption, two sets (set \( A \)) represent the value under the real irregular loading sequence. To transform irregular loading into idealised loading (i.e., from \( A - 1 \) to \( A - 2 \)), the idealised ascending-amplitude load history ceases to evolve significantly. These observations lead to regard macro-Miner’s rule as a direct consequence of local sand’s ratcheting behaviour.

\[ N = 150\sim300 \text{ in Fig. 13c), } D_r \text{ ceases to evolve significantly. These simulation results are in further agreement with the notion of ‘terminal density’, though under stress paths and loading conditions that are more complex than in the aforementioned experimental studies. Apart from confirming the validity of the ‘terminal density’ concept, a local response consistent with Miner’s rule is also exemplified in Fig. 13 in terms of } D_r - N \text{ trends. These observations lead to regard macro-Miner’s rule as a direct consequence of local sand’s ratcheting behaviour.} \]

4. Impact of the loading history idealisation

Monopiles are in reality subjected to irregular cyclic loading. In engineering design practice, however, irregular loading is traditionally transformed into idealised regular load parcels characterised by average load levels that are the same for simplification, but with a different arrangement of the load amplitudes — typically with load parcels of uniform amplitude being arranged in ascending amplitude order. After idealising the loading process, it is assumed that the accumulated pile head displacement at the end of the idealised load history is representative of its value under the real irregular loading sequence. To seek confirmation of this assumption, two sets (set \( A \) and \( B \), each set contains two load cases) of simulations are performed for the same reference monopile considered so far in dense sand, in combination with the following load histories:

A-1 20 min recorded time domain load history of an offshore wind turbine (extracted from a 1-h storm). The load level is re-scaled so that the maximum load \( H_{\text{max}} = 15.7 \text{ MN} \) (it corresponds to \( \zeta_b = 0.58 \)) as a high estimation of a load level.

A-2 the idealised load history transformed from the irregular load history \( A - 1 \) using Rain Flow Counting method (Kaggwa et al., 1991). The idealised load history includes 5 load parcels (324 loading cycles). The average load \( H_{\text{ave}} = 5.4 \text{ MN} \). The amplitude of the load parcels ranges from 1.026 to 10.266 MN. In this way, the maximum load level in \( A - 2 \) is the same as in \( A - 1 \).

B-1 The 1-h load storm (the same as the recorded storm load for \( A - 1 \)) with the load level be re-scaled so that \( H_{\text{max}} = 7.6 \text{ MN} \) (it corresponds to \( \zeta_b = 0.27 \)).

B-2 the idealised load history transformed from the irregular load history \( B - 1 \) using Rain Flow Counting method. The idealised load history includes 12 load parcels (900 loading cycles). The average load \( H_{\text{ave}} = 3.75 \text{ MN} \). The amplitude of the load parcels ranges from 0.2 to 3.85 MN. In this way, the maximum load level in \( B - 2 \) is the same as in \( B - 1 \).

When translating irregular loading into idealised loading (i.e., from \( A - 1 \) and \( B - 1 \) to \( B - 1 \) and \( B - 2 \)), each regular loading cycle has a period of around 4 s.

4.1. Pile head displacement

The simulation cases in this section are limited to monopile behaviour in dense sand only in order to better consider the real life design situation. Pile head lateral displacements obtained from the two sets of loading conditions are plotted in Fig. 14a (load histories \( A - 1 \) and \( A - 2 \), the magnitude of the displacements is associated with the left \( y \)-axis of the plot) and Fig. 14b (load histories \( B - 1 \) and \( B - 2 \)). For convenient relating of displacement evolution pattern to the changing of loading amplitude, the load histories are also included in the figure (the magnitude is associated with the right \( y \)-axis).

Loading history effect is first investigated through the comparison of accumulated displacement at the end of loading histories \( A - 1 \) and \( A - 2 \) – the corresponding load level is about \( H = 8.8 \text{ MN} \), as indicated in Fig. 14. Monopile subjected to irregular load history \( A - 1 \) accumulates lateral displacement of 0.445 m, which is about 17% higher than that accumulated after \( A - 2 \) – the idealised ascending-amplitude load history (0.37 m, which is extracted at the same load level as the ending point of load history \( A - 1 \)) if take the displacement after irregular loading as the base (i.e., \((0.445-0.37)/0.445 = 17\%\)). One may assume that representing the irregular load history with idealised regular loading cycles featured as constant average load and load parcels of increasing amplitude being arranged in ascending amplitude order.
amplitude leads to the conservative estimation of the final accumulated pile displacement. Almost the same maximum pile head displacement for loading cases $A - 1$ and $A - 2$ (around 0.454 m as indicated in Fig. 14a) occurs near the end of the loading histories — which seems to indicate that the idealisation of the loading history has minor effects on the pile maximum head displacement. To further assess the generality of the two assumptions, the accumulated pile head displacement caused by loading histories $B - 1$ and $B - 2$ are also investigated.

The loading histories $B - 1$ and $B - 2$ differ from $A - 1$ and $A - 2$ on the following aspects: (1) smaller peak loading amplitude (7.86 MN instead of 15.7 MN); (2) longer duration ($A - 1$ re-scales the first 20 min of the 1-h storm, $B - 1$ contains the complete 1-hour history); and (3) different average load level. However, for $A - 1$ and $B - 1$, the peak loading amplitude occurs at the same time — $t = 2.5 - 2.8$ min. The following observations may be drawn: (1) the accumulated pile head displacements at the end of the two loading histories are almost the same ($0.082$ m for $B - 1$ and $0.08$ m for $B - 2$) — simulation against idealised loading history no longer show conservatism in this case; (2) the maximum pile head displacement equals $0.097$ m for $B - 1$ and $0.103$ m for $B - 2$, which indicates that, similarly to the $A - 1 - A - 2$ simulation, the idealisation of the loading history idealisation has minor effects on the pile maximum head displacement.

Miner’s rule and the simulation results (both on monopile in dense and loose sand) in the previous section indicate that pile lateral displacement accumulates rapidly when the load parcel with maximum ever load amplitude is applied. This is demonstrated again by the simulations on the idealised regular loading cases as illustrated in Fig. 14. However, the statement 1 of Miner’s rule (which suggests that no displacement is accumulated if the current stress amplitude is smaller than the ever experienced maximum load amplitude) is not fulfilled for monopile behaviour in dense sand domain. In simulation cases of irregular loading, pile head displacement keeps accumulating after the peak load amplitude being applied.

The maximum load amplitude during the irregular loading $A - 1$ occurs at time $t = 2.5 - 2.8$ min. However, lateral pile head displacement accumulation keep evolving obviously until $t = 5$ min — after which, the displacement accumulation still continued but with much lower speed. For irregular loading $B - 1$, the maximum pile head displacement occurs at $t = 37.6$ min — to compare, the maximum load presents at $t = 2.5 - 2.8$ min.

Two possible reasons can lead to this: (1) the ever-changing (which is not significant though) average load level in an irregular load history; (2) duration of the peak load in the irregular load history is too short so that the soil state has not reached its cyclically stable state under such load amplitude levels. The verification (or falsification) of these two assumptions needs further investigations both from experimental and numerical perspectives.

4.2. Soil state around monopile

The study of the distribution of soil relative density and stress state helps to predict long term monopile behaviour from local soil states point of view. Fig. 15a shows the change of relative density $D_r$ in the entire soil domain at the end of the irregular loading storm $B - 1$.

Loading $B - 1$ is nearly one-way loading with only several minutes’ exception. Densification mainly occurs near pile head in front of the loading direction and below pile toe. Fig. 15b indicates that after the idealised loading history $B - 2$ being applied, densification occurs at the same location, but also around the middle part of the pile in addition. However, the slightly different soil state in the middle part of the soil seems have minor effects on the pile global tilting: pile head displacement at the end of the loading are quite similar ($0.082$ m for $B - 1$ against $0.08$ m for $B - 2$) in the two simulation cases.

The stress redistribution (indicated by $p/p_r$) ratio shows greater similarity at the end of the loading $B - 1$ (Fig. 16a) and loading $B - 2$ (Fig. 16b). Significant increase of $p/p_r$ is observed in both cases in front of the loading direction at around 1~$2D$ depth. Mean effective stress reduction occurs in the reverse direction of the loading at shallow layers and in the front direction near pile toe. It should be noticed that such a stress distribution corresponds to nearly one-way cyclic loading event.
5. Conclusion

In this study, validation of Miner’s rule is confirmed at element test level against high-cyclic triaxial test results, under loading conditions of multi-amplitude parcels with different orders. Then, implicit 3D FE modelling was combined with the memory-enhanced, bounding surface SANISAND-MS model to numerically analyse monopile tilt in dry sand under lateral cyclic loading. Loading cases with regular parcels of multi-amplitude and different parcel sequence are evaluated. Miner’s rule is partially validated under drained condition when focusing on the pile head displacement — ordering of loading parcels has minor effects on the accumulated pile head displacement. Monopile response, saying shear force distribution, bending moment distribution and cyclic \( p - y \) evolution, is also presented. The similarities observed from the ‘Miner’ type numerical results between mesoscale (soil stress points) and macroscale (whole soil–pile system) confirm the close link between cyclic soil behaviour and monopile tilting response. Loading idealisation effects on monopile lateral behaviour are studied under conditions of varying load levels and duration. Pile head displacement triggered by irregular and idealised regular loading histories are close to each other, which confirms the validity of using idealised loading history of ascending-amplitude loading parcels to represent for the irregular loading history, in the premise of fully drained condition. Future work will continue to explore the role of relevant geometrical/geotechnical/loading factors, including load multi-directionality and hydromechanical effects related to pore pressure buildup.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Acknowledgement**

The authors gratefully acknowledge the support from the Wave Loads and Soil Support for Extra-Large Monopiles (WAS-XL) project (NFR grant 268182).

**References**


