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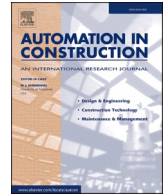
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# Dynamic control for construction project scheduling on-the-run

O. Kammouh<sup>a,\*</sup>, M. Nogal<sup>b</sup>, R. Binnekamp<sup>b</sup>, A.R.M. Wolfert<sup>b</sup>

<sup>a</sup> Faculty of Technology, Policy, and Management, Delft University of Technology, Delft, the Netherlands

<sup>b</sup> Faculty of Civil Engineering and Geosciences, Delft University of Technology, the Netherlands

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## ABSTRACT

Construction project management requires dynamic mitigation control to ensure a project's timely completion. Current mitigation approaches are usually performed by an iterative Monte Carlo (MC) analysis which does not reflect (1) the project manager's goal-oriented behavior, (2) contractual project completion performance schemes, and (3) stochastic dependence between construction activities. Therefore, the development statement within this paper is to design a method and implementation tool that properly dissolves all of the aforementioned shortcomings ensuring the project's completion date by finding the most effective and efficient mitigation strategy. For this purpose, the Mitigation Controller (MitC) has been developed using an integrative approach of nonlinear stochastic optimization techniques and probabilistic Monte Carlo analysis. MitC's applicability is demonstrated using a recent Dutch large infrastructure construction project showing its added value for dynamic control on-the-run. It is shown that the MitC is a state-of-the-art decision support tool that a-priori automates and optimizes the search for the best set of mitigation strategies on-the-run rather than a-posteriori evaluating the potentially sub-optimal and over-designed mitigation strategies (as commonly done with modern software such as Primavera P6).

## 1. Introduction

Probabilistic Monte Carlo (MC) simulations are frequently used to estimate the project's completion date and cost at a given required probability level. Preliminary cost and time estimates in construction projects are rarely accurate estimates of the actual figures obtained at the end of the project. These initial estimates are vital as they have the potential to determine the probability of success of a project (i.e., timely and on-budget completion). [11] proposed a methodology to accurately estimate construction material quantities during an early project phase. Their approach was compared to the state of the practice, and the results showed an improvement in the accuracy of preliminary project cost estimates.

Estimating costs and time at an early stage of a construction project involves aleatory uncertainty, which is irreducible. Therefore, providing better initial estimates cannot guarantee on-budget and on-time delivery of the project. For this reason, project control and risk mitigation are inevitable. Project control allows for instantaneous response to unexpected events or variabilities that can potentially impact the cost/time of the project. Controlling projects as they are executed can help reduce the

overrun cost due to delays or costs variability. [22] presented a methodology for monitoring construction progress and quality using real-time data from a commercial building during the execution phase. They demonstrated that financial benefits could be achieved by employing novel automated control methods over classical ones. Other studies have focused on utilizing delay mitigation measures and finding their effect on effective and efficient project success [10,28,37,45]. Furthermore, [3] proposed a method for choosing optimal actions for crashing activities durations. The proposed method allows reducing the cost of schedule crashing actions and the cost of delays. It also enables increasing the robustness of the schedule by reducing differences between the actual and the as-planned schedules.

Two different approaches to scheduling and control can be identified in the literature: the "offline" and the "online" scheduling [13] [14]. The former, also referred to as proactive [36], consists of constructing a schedule that considers all potential future disruptions [16]. The latter is considered a continuous activity, with decisions on the 'processes' timing or scope, made and verified as the works progress and for a short planning horizon. Nevertheless, all previous attempts to control projects do not reflect the project manager goal-oriented control behavior. This

\* Corresponding author.

E-mail addresses: [O.Kammouh@tudelft.nl](mailto:O.Kammouh@tudelft.nl) (O. Kammouh), [m.nogal@tudelft.nl](mailto:m.nogal@tudelft.nl) (M. Nogal), [r.binnekamp@tudelft.nl](mailto:r.binnekamp@tudelft.nl) (R. Binnekamp), [r.wolfert@tudelft.nl](mailto:r.wolfert@tudelft.nl) (A.R.M. Wolfert).

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will be further elaborated on in the following sub-sections.

### 1.1. Modeling limitations of the classical probabilistic scheduling approach

The classical MC simulation approach for selecting and incorporating mitigation measures to control project delays does not reflect the goal-oriented control behavior of a project manager. To reflect this behavior, each MC iteration should, in case the completion date is not met, incorporate an optimization process to find the closest completion date to the contractual target completion date at the lowest construction costs. Moreover, this approach is usually performed by repeated Monte Carlo (MC) simulations with different combinations of mitigation measures until a subset of mitigation measures that meet the required probability level at the target completion date is identified [4,20]. All of this means that within every MC simulation, the *same* set of mitigation measures is selected regardless of the reason or amount of delay occurring in each simulation iteration. In real life, however, the project manager would proactively respond to a given scenario by choosing only a specific and optimal set of mitigation measures to control the given project completion date effectively and efficiently. Finally, another drawback of the classical MC approach is that it is too conservative since it incorporates a complete set of mitigation measures permanently rather than keeping these measures tentative for allocation. It is evident that in a MC iteration where the delay is relatively small, it will not be necessary to use all available mitigation measures. So, the resulting MC cumulative probability curve for project completion time (S-curve) does not reflect reality resulting in ineffective and/or inefficient construction project control.

### 1.2. The mitigation controller concept

To resolve the MC limitations and/or drawbacks mentioned above, the Mitigation Controller concept (MitC) was recently introduced [19]. On this basis, a new software tool has also been released [17,18]. Using an integrative approach of both Monte Carlo (MC) techniques and dynamic control optimization theory, the MitC removes these MC limitations and/or drawbacks by reflecting the goal-oriented mitigation behavior of the project manager in each simulation. This is achieved by solving an optimization problem within every MC iteration, where one should understand that each MC iteration should represent a possible real-life scenario. Therefore, in each scenario, one should integrate the most effective goal-oriented mitigation strategy. Hereby, the MitC tool automates the selection process of the most cost-efficient set of mitigation measures (i.e. a mitigation strategy) at the most time-effective construction project's completion. The MitC in its current form has two limitations that are addressed in this paper:

1. The MitC in its current form does not allow for contractual completion performance schemes (penalties and rewards on major contract milestones);
2. The MitC in its current form does not account for stochastic correlations between the duration of the construction activities.

### 1.3. Contractual completion performance schemes (penalty/reward)

It is prevalent to penalize late completion of construction projects and/or reward early completion. Accounting for the penalty could result in a mitigation strategy that does not necessarily use the maximum capacity of the mitigation measures because it could be the case that paying the penalty is more affordable than paying for mitigating the time overrun. On the other hand, considering rewards in the analysis could produce a mitigation strategy that expedites the project's completion date to an earlier date than the planned one. This is the case when the reward is larger than the additional mitigation cost. There have been several attempts to account for penalty and, to less extent,

reward in project mitigation problems [8,12,15,23,32]; however, these models are not designed specifically for construction projects. Construction projects are mostly one-off projects that have a pre-established price. Cost overruns can be disastrous for a construction project compared to, for instance, software or manufacturing projects where a cost overrun can be compensated for in the product's price.

[29] introduced a model to mitigate project delays in EPC projects (engineering, procurement, and construction) that includes penalty schemes. However, this model does not account for potential rewards in case of early completion of the project. This means that the project duration cannot be less than the target duration, even in cases where it would be more financially beneficial to reduce the project's duration further.

All of the available models described above do not consider and/or integrate MC simulation and control optimization, as in the original MitC. Therefore, all of these previously developed models lack a certain connection with actual construction practice. Finally, none of these models can reflect the correlation between construction activities, which is introduced in the next section.

### 1.4. Stochastic correlation between construction activities

The scheduled construction activities are modeled independently within current state-of-the-art scheduling software such as Primavera P6,<sup>1</sup> as is also the case within the original MitC concept. However, in practice, the construction relations<sup>2</sup> can be stochastically correlated. These correlations result from construction execution under similar conditions such as weather, site condition, supervision, labor skills, etc. [31]. The significance of modeling such correlations has been highlighted and investigated in previous studies [9,38,39]. In [38], it has already been shown that results coming from scheduling models that do not consider correlations could have substantial errors. In the aforementioned paper, it has even been claimed that accounting for correlations is more important than the choice of distribution of the activities.

There is a limited number of project management approaches in the literature that allow modeling the correlations between random variables. In [44], a method was introduced to account for the correlation between project activities in repetitive projects. However, his approach is only applicable in projects that constitute repetitive tasks or in repeated projects; hence, it does not extend to other types of correlations that exist in non-repetitive projects/activities. [43] proposed to explicitly impose correlations between the random variables as inputs. To do that, a prior decision on the correlations between the variables must be provided. This approach is systematic but rather impractical since assuming those correlation coefficients is tedious. Simulation-based correlation approaches are another way to account for correlations between the random variables [21,26,41]. For example, [26] introduced a new Monte Carlo-based model—the correlated schedule risk analysis model (CSRAM)—to evaluate construction networks under uncertainty with correlated activities and risk factors. This study provides a great capability to model uncertainty correlations in construction projects. However, due to its current mechanism, this method suffers from scalability limitations when modeling complex cause-and-effect relationships. Hence, it is only suitable for small to medium-size projects.

Moreover, Touran and Wiser [35], Touran [34], and [27] focused on the correlation of cost items in construction. Although they only consider the correlation between the cost items, they provide important insights that can be used to tackle the correlations between other random variables (durations, risks, etc.).

<sup>1</sup> the durations of activities are sampled from predefined duration stochastic distributions which are uncorrelated

<sup>2</sup> Typical other type of relations such as soft relation between directly linked start-to-finish relations are currently being develop for the MitC in close corporation with PrimaNed B.V.

In addition to their limitations, none of the papers mentioned above did consider correlations of construction activities' durations in an integrative MC and dynamic project control context. Therefore the development statement for this paper is to include stochastic correlation of construction activities within the original MitC concept and its related software.

### 1.5. Development goals and novelties

The primary goal of this paper is to introduce the extended MitC concept focusing on two recent integrated developments that contribute to better reflecting real-life construction scheduling and cost scenarios:

- i) Enabling contractual project completion performance schemes (i.e., penalties and rewards on the major contract milestone(s))
- ii) Introducing a rigorous approach to account for stochastic correlations between the durations of the construction activities.

These developments will also add value to the body of products as they are usable in different (software) scheduling applications. To incorporate the new contributions in the original MitC concept, a new mathematical model of the simulation and optimization problem has been formalized. The extended MitC has also already been converted into an Open Software tool with a friendly Graphical User Interface (GUI) and can be accessed at <https://github.com/mitigation-controller/mitc>.

The remaining document is organized as follows. Section 2 describes the fundamental description of the original MitC concept as a basis for this paper. Section 3 introduces the modeling approach for incorporating contractual performance schemes. Section 4 presents the correlation modeling approach of activities' durations. Section 5 describes the optimization problem of the extended MitC. Section 6 presents a demonstrative example of a real construction project. Finally, Section 7 includes conclusions and further developments.

## 2. The mitigation controller concept description and considerations

The Mitigation Controller was previously introduced to automate and reflect the goal-oriented behavior of the project manager when selecting and incorporating mitigation measures in construction projects. In its current form, the MitC allows modeling different scheduling risks and uncertainties: e.g., the uncertainty bandwidth in the activities' durations, the uncertainty bandwidth in mitigated durations of the mitigation measures, the uncertainty in the costs of the measures; and the inclusion of predefined project risk events. This paper inherits and builds upon the original MitC concept; hence, the basic starting points will be briefly summarized in the following subsections 2.1–2.4 to make this paper independently readable. For a complete description of the multi-objective optimization in the original MitC, the reader is referred to [19].

### 2.1. Uncertainty modeling of construction activities durations and mitigation measures

In construction project scheduling, over or under-estimation of the activities' duration causes cost overrun or underrun. Therefore, considering the uncertainty in construction activities' duration is necessary to capture a reliable project duration estimate. In probabilistic project planning, Beta-PERT is the most used probabilistic distribution that enables the uncertainties to be considered [25]. The Beta-PERT is a continuous probability distribution that requires three parameters: the minimum outcome  $a$ , most-likely outcome  $b$ , and maximum outcome  $c$  that a random variable can take. The Beta-PERT distribution takes into account that the 'most likely' case is more likely to occur which is reflected in a multiplier for that estimate. The expected value of the

distribution can be calculated as follows:

$$E[X] = \frac{a + 4b + c}{6} = \mu \quad (1)$$

To mitigate construction delays, mitigation measures are usually applied.<sup>3</sup> Every mitigation measure  $j$  is associated with a mitigation capacity  $m_j \in \mathbb{R}^+$ , which represents the reduction in the duration of the activity if mitigation  $j$  is applied. These mitigation actions, however, also have uncertainty and must be accounted for.

In this paper, the Beta-PERT distribution is adopted to model the variations in duration. Each activity duration,  $d_i \in \mathbb{R}^+$ , and mitigation measure capacity,  $m_j$ , are given three estimates: minimum, most-likely, and maximum, with probability density functions  $f(d_i; a, b, c)$  and  $f(m_j; a, b, c)$  respectively. The three estimates are used to construct a Beta-PERT distribution for each activity duration and mitigation measure capacity. The distributions are then used for random sampling in the MC simulation.

### 2.2. Uncertainty modeling of the mitigation cost

Mitigating construction delay involves a mitigation cost. The cost of mitigation is determined by the type of mitigation measure used. Some mitigation strategies include assigning extra workers to accelerate the implementation of critical construction activities, while other strategies consist of adding resources to speed up the construction process, such as additional equipment. The relationship between the mitigated duration and cost of the mitigation measures is nonlinear [2]. There are cases where the mitigated duration and cost are strictly related. For example, if a mitigation measure includes allocating extra personnel to accomplish a task, or renting more trucks to speed up the building process, then the cost is determined by the number of days the extra personnel worked or the vehicles were rented. There are, however, cases where the cost is not proportional to the mitigation capacity, such as one-off spending measures (e.g., buying additional tools, paying for acquiring a license to be able to work outside the regular time, etc.). The MitC accounts for these different cases in the determination of the mitigation cost by relating the mitigation cost to the mitigation capacity variation and mitigation type, as follows:

$$c_{j,min/max} = c_{j,l} \left( 1 - \frac{m_{j,l} - m_{j,min/max}}{m_{j,l}} \eta_j \right) \quad (2)$$

where  $c_{j,min/max} \in \mathbb{R}^+$  are the minimum and maximum mitigation costs that relate to the minimum and maximum mitigating capacities  $m_{j,min/max} \in \mathbb{R}^+$ ,  $c_{j,l} \in \mathbb{R}^+$  is the most-likely mitigation cost that corresponds to the most-likely mitigation capacity,  $m_{j,l} \in \mathbb{R}^+$ .  $\eta_j \in [0, 1]$  is a measure that indicates the degree of positive correlation between the cost variation and the mitigated capacity variation. If the variation in cost is fully proportional to the variation in the mitigation capacity,  $\eta_j$  is set equal to 1. If, on the contrary, the variation in cost is independent of the variation in the mitigation capacity,  $\eta_j$  is set equal to zero. In that case, the minimum and maximum cost estimates will be equal to the most-likely value. The value of  $\eta_j$  can be simply obtained by performing a correlation analysis for the mitigated duration and mitigation cost of mitigation measure  $j$  using data from previous construction projects. If past data is not available, the value of  $\eta_j$  shall be estimated with logical reasoning or expert knowledge.

Using the min/max values of mitigation costs calculated in Eq. (2) and the min/max values of mitigation capacities, the actual mitigation cost in a specific iteration is calculated by means of interpolation.

<sup>3</sup> It is important to note that the mathematical impact modeling is described here: i.e., crashing or compacting activity duration by acceleration measures. However the types of mitigation measures in practice can both increase production speeds and/or introducing an alternative (execution) work method or others.

### 2.3. Risk events modeling

Various risks can impact a project throughout its execution phase leading to partial or whole construction operation being interrupted for an extended period of time. The discovery of polluted soil and the failure of concrete casting are two examples of risks during construction. Ignoring these risks results in project delays and/or cost overruns. The MitC accounts for risk events as follows; every risk item is associated with a random variable  $D_e \in \mathbb{R}^+$  that denotes the disruption duration caused by a risk event  $e$ . The probability distribution of this random variable is a mixed discrete-continuous distribution, expressed as follows:

$$f(d_e; p_e, a_e, b_e, c_e) = \begin{cases} 1 - p_e & \text{if } X_e = 0 \\ f(d_e^*; a_e, b_e, c_e)p_e & \text{if } X_e = 1 \end{cases} \quad (3)$$

where  $X_e = \{0, 1\}$  is a discrete random variable that represents the occurrence (or non-occurrence) of risk event  $e$ . If risk event  $e$  occurs,  $X_e$  takes the value 1 with probability  $p_e$ , while it takes 0 with probability  $q_e = 1 - p_e$  if a risk event does not occur; that is,

$$f(X_e; p_e) = \begin{cases} q_e = 1 - p_e & \text{if } X_e = 0 \\ p_e & \text{if } X_e = 1 \end{cases} \quad (4)$$

while  $d_e^*$  is the outcome of the random variable  $D_e^* \in \mathbb{R}^+$ , which denotes the disruption duration caused by a risk event  $e$  given that the risk event occurs (i.e.,  $X_e = 1$ ).  $D_e^*$  is assumed to follow the Beta-PERT distribution with a probability density function  $f(d_e^*; a_e, b_e, c_e)$ , where  $a_e$ ,  $b_e$ , and  $c_e$  are the minimum, most-likely, and maximum outcomes, respectively.

### 2.4. Relations between construction activities, mitigation measures, and risk events

Mitigation measures are used to mitigate project delays to complete the project on time. The activity crashing technique is adopted as the mitigation method. Hence, in this paper, a mitigation measure is equivalent to activity crashing: i.e., shortening or compacting the activity duration by a measure. The MitC couples mitigation measures with project activities in such a way that one mitigation measure can affect more than one activity. The relations between the mitigation measures and the construction activities are given by the relation matrix in Eq. (5). The relation parameter  $r_{ij}$  takes the value of 1 when mitigation measure  $j$  intervenes upon activity  $i$ , zero otherwise.

$$[r_{ij}] = \begin{bmatrix} r_{11} & \dots & r_{1J} \\ \vdots & \ddots & \vdots \\ r_{I1} & \dots & r_{IJ} \end{bmatrix} \quad (5)$$

where  $I$  is the number of activities and  $J$  is the number of mitigation measures.

Similarly, each risk event can impact several activities at the same time. The relation parameter  $s_{ie}$  takes the value of 1 when risk event  $e$  affects activity  $i$  and zero otherwise. The corresponding relation matrix is expressed as follows:

$$[s_{ie}] = \begin{bmatrix} s_{11} & \dots & s_{1E} \\ \vdots & \ddots & \vdots \\ s_{I1} & \dots & s_{IE} \end{bmatrix} \quad (6)$$

where  $E$  is the total number of potential risk events. Last but not least, it is worthwhile to mention that the MitC currently does not allow for mitigation by introducing soft relation between directly linked start-to-finish relations (currently under development).

Note that the mitigation measures and risk events can affect different activities in different ways. In this case, risk events or mitigation measures targeting different activities can be introduced more than once in the risk/mitigation register, where every insertion targets a different activity.

### 3. Inclusion of contractual completion performance scheme (penalties/rewards)

In the original Mitigation Controller concept, contractual performance schemes (penalty/reward) cannot be considered in the optimization simulations. This section removes this limitation so that any contractual performance scheme can be considered in the simulation. The objective is no longer to achieve the target completion date of the project, as it was in the original MitC, but to reduce the total net cost (i.e., cost of mitigation measures  $\pm$  penalties/reward).

Fig. 1 shows the possible scenarios that could occur after the mitigation process, where “scenario” means “a possible project state after mitigation”. A particular scenario will only occur if it corresponds to the minimum net cost. In Scenario 1, the mitigated duration is lower than the target duration. The net cost in this case is the mitigation cost minus the reward of early completion. In Scenario 2, the mitigated duration is larger than the target duration. The net cost is then the mitigation cost plus the penalty of late completion. In Scenario 3, the mitigated duration is equal to the target duration. The net cost, in this case, is the cost of the selected mitigation measures.

In real-life construction projects, the reward is composed of two parts. The first is a lump-sum amount that is acquired if the project is finished before the deadline and the second is the daily operating costs that are saved because of the project's early completion. Penalties, on the other hand, are normally imposed per range of time delay, and there is usually a cap for the penalties that cannot be exceeded. For the sake of simplicity, penalties and rewards are considered as daily dependent in this paper (i.e., reward/day of early finish and penalty/day of delay).

### 4. Inclusion of stochastic construction activity correlation modeling

The project's completion time is impacted by two things: the uncertainty in the durations of activities and the occurrence of risk events. Here, we focus on the first part, the durations uncertainty. In the context of this study, activities correlation signifies the relationships between the variation in the durations of the activities. The aim is to obtain realistic results by indirectly capturing the correlations between the activities' durations.

The uncertainties in the durations of the activities are caused by several factors, such as site conditions, weather, and labor skills, which can impact the timely execution of construction activities. These factors may simultaneously impact several activities in a particular project and may result in activity durations to be correlated [1,5,24,30,42]. For example, if the weather condition turns out to be bad at a certain time, it will increase the duration of all weather-sensitive activities executed at that time. Similarly, the duration of weather-sensitive activities can decrease to some extent when the weather condition is good. When this correlation effect happens to several activities along a network path, the uncertainty band of the path's duration may considerably change. If the path is critical or near critical, it will cause a change (i.e., increase or decrease) in the uncertainty band of the project's duration. Increased uncertainty band in the project's duration may increase the project's timely completion uncertainty.

Normally, the durations of activities in a project are treated independently [40], i.e., within every MC iteration, the durations of activities are sampled from predefined distributions independently from one another. This is backed by the assumption that the durations of activities are statistically independent. This assumption is reasonable only if they do not share factors causing delays. In the following section, a new approach for modeling the correlations among the durations of project activities is presented.

The difference between these uncertainty factors and risks is that the risk always incurs a delay to the project and does not necessarily target an activity. The uncertainty factor, on the other hand, can only target activities and can cause an increase or decrease in the duration.



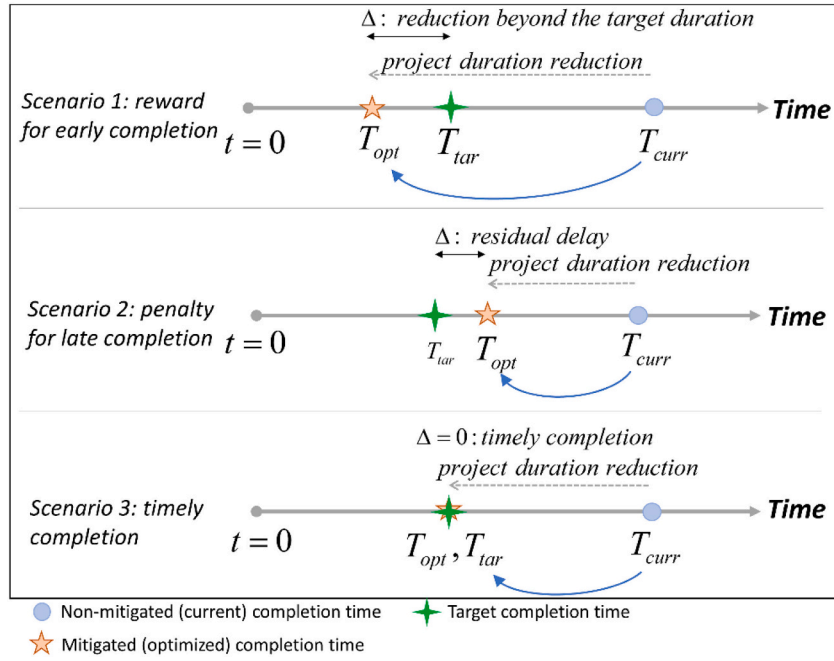


Fig. 1. Possible optimization scenarios.

#### 4.1. Formalizing the correlation model of the activities' duration distributions

The uncertainty band within the durations of construction activities is the result of several factors (e.g., weather conditions, labor skills, resources availability, etc.). The proposed correlation model assumes that correlation between the activities' durations occurs only when activities share the same uncertainty-triggering factors. For example, assume Activity 1 is sensitive to labor and weather, and Activity 2 is sensitive to weather and resources. The two activities are partially correlated as they only share the weather as a common factor.

Without loss of generality, the uncertainty of the duration of activity  $i$  can be split into (a) the duration uncertainty caused by independent factors affecting only activity  $i$  (e.g., effectiveness in the geotextile installation), and (b) the uncertainty introduced by factors  $f = \{1, 2, \dots, F\}$  that are common to several activities (e.g., weather conditions). Fig. 2 depicts the two sources of uncertainty in the durations of Activities 1 and 2. Three correlated factors have been considered, each of them with different influence upon Activities 1 and 2; e.g., Factor 3 does not influence Activity 1.

The effect of these common factors is relative; each factor can either cause an increase or decrease in the durations of the affected activities. To represent the uncertainty introduced, every factor is assigned a probability distribution with an expected value equal to zero. Hence, the outcome drawn from the distribution,  $U_f = u_f$ , can be positive or negative.

To mathematically express that, the random variable *activity duration*,  $D_i \in \mathbb{R}^+$ , is split into the random variable *uncorrelated duration*,  $D_{i, \text{uncorr}} \in \mathbb{R}^+$ , and the random variables *shared uncertainty*,  $U_{i, f} \in \mathbb{R}$  (see Fig. 2). Eq. (7) expresses the relationship between the random variables.

$$D_i = D_{i, \text{uncorr}} + \sum_{f=1}^{f=F} U_{i, f}, \quad \forall i \in \mathcal{I} \quad (7)$$

where  $U_{i, f}$  represents the contribution of the shared factor  $f$  to the uncertainty of the random variable  $D_i$ ,  $\mathcal{I} = \{1, 2, \dots, I\}$  is the set of planned activities.  $U_{i, f}$  can be obtained from the uncertainty distributions of the correlated factors  $U_f$  through the relation parameter  $v_{fi}$  using Eqs. (8) and (9).

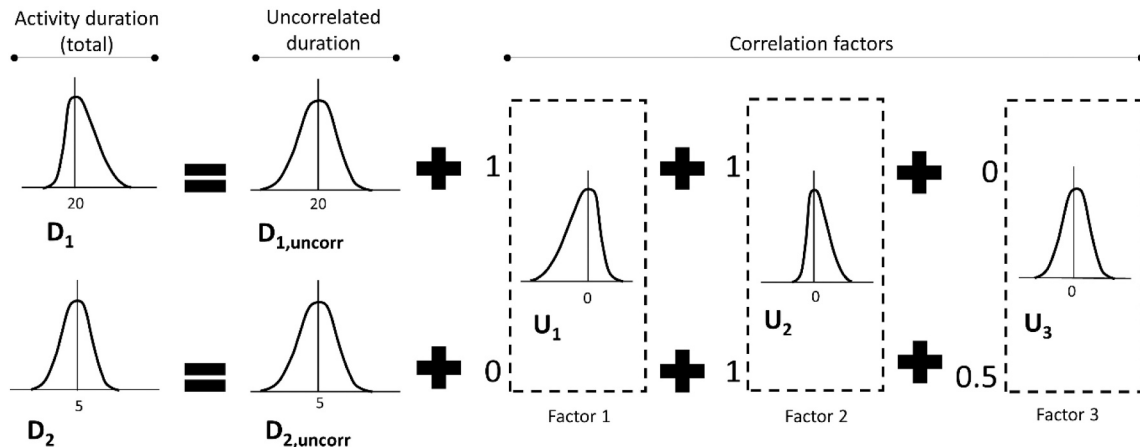


Fig. 2. Sources of uncertainty in the durations of two activities.

$$[v_{fi}] = \begin{bmatrix} v_{11} & \dots & v_{1f} \\ \vdots & \ddots & \vdots \\ v_{f1} & \dots & v_{ff} \end{bmatrix} \quad (8)$$

The relation parameter  $v_{fi}$  defines the extent to which an uncertainty factor affects the activities. It can take values between [0,1], where 0 means no impact and 1 means full impact. Note that uncertainty factors can impact only activities executed at the same time (e.g., when those activities have the same predecessor). If an uncertainty factor impacts activities executed at different times, this factor should be introduced twice in the database.

The term  $\sum_{f=1}^{f=F} U_{if}$  in Eq. (6) can then be rewritten using the matrix form as follows:

$$\sum_{f=1}^{f=F} U_{if} = U_f^T v_{fi} \quad \forall i \in \mathcal{I} \quad (9)$$

where  $U_f^T$  refers to the transposed  $U_f$ , that is,  $U_f^T = [U_1 \dots U_F]$ .

The random variables in Eq. (7) are subject to the following assumptions:

- Assumption 1:  $D_{i, \text{uncorr}}$  and  $U_{i, f}$  ( $\forall f \in F$ ) are random variables that follow the Beta-PERT distribution with probability density functions  $f(d_{i, \text{uncorr}}; a_{i, \text{uncorr}}, b_{i, \text{uncorr}}, c_{i, \text{uncorr}})$  and  $f(u_{i, f}; a_{i, f}, b_{i, f}, c_{i, f})$ , respectively.
- Assumption 2:  $D_i$ , the summation of  $D_{i, \text{uncorr}}$  and  $U_{i, f}$ , also follows a Beta-PERT distribution with a probability density function  $f(d_i; a_i, b_i, c_i)$ .
- Assumption 3:  $D_{i, \text{uncorr}}$  and  $U_{i, f}$  are statistically independent.

Assumptions 1 and 2 are only compatible if the Beta-PERT distribution has the reproductive property. Reproducibility of distribution means that when adding two random variables with the same probability distribution, the resulting random variable follows the same probability distribution with defined distribution parameters (see [6,7]). This assumption is discussed in Section 4.2.

Since  $D_{i, \text{uncorr}}$  and  $U_{i, f}$  are statistically independent, the mean can be, regardless of the distribution type, written as follows,

$$\mu_{D_i} = \mu_{D_{i, \text{uncorr}}} + \sum_{f=1}^{f=F} \mu_{U_{i, f}} \quad \forall i \in \mathcal{I} \quad (10)$$

Given that  $U_{i, f}$  has an expected value equal to zero,  $\mu_{D_i} = \mu_{D_{i, \text{uncorr}}}$ .

The relationships between the pessimistic and optimistic values for the random variables are given as follows,

$$\begin{cases} a_{D_i} = a_{D_{i, \text{uncorr}}} + \sum_{f=1}^{f=F} a_{U_{i, f}} \\ c_{D_i} = c_{D_{i, \text{uncorr}}} + \sum_{f=1}^{f=F} c_{U_{i, f}} \end{cases} \quad \forall i \in \mathcal{I} \quad (11)$$

Regarding the parameter  $b$ , it is obtained by introducing Eqs. (10) and (11) to Eq. (1), resulting in

$$b_{D_i} = \frac{6\mu_{D_i} - a_{D_i} - c_{D_i}}{4} = b_{D_{i, \text{uncorr}}} + \sum_{f=1}^{f=F} b_{U_{i, f}} \quad \forall i \in \mathcal{I} \quad (12)$$

In classical stochastic scheduling,  $D_i$  is the only random variable whose probability distribution definition is required as input. In the proposed approach, the probability distributions of  $U_f$  are also required as input along with the relation parameter  $v_{fi}$ , which can be estimated by expert opinion based on the experience of construction managers and historical data. Given the probability distributions of  $D_i$  and  $U_{i, f}$ , the probability distribution of  $D_{i, \text{uncorr}}$  can be derived using Eqs. (7)–(12). Note that estimating the probability distribution of  $D_{i, \text{uncorr}}$  can be very difficult otherwise. Once the probability distribution of  $D_{i, \text{uncorr}}$  is defined, random samples from  $D_{i, \text{uncorr}}$  and  $U_{i, f}$  can be obtained and

introduced in the MC simulation accounting for the existing correlation among factors.

#### 4.2. Validation of the assumption of reproducibility of the Beta-PERT distribution

This section discusses the validity of the assumption of reproducibility of the Beta-PERT distribution, which implies that the summation of two independent random variables,  $X_1$  and  $X_2$ , following Beta-PERT distributions,  $X_1 \sim \text{PERT}(x_1; a_1, b_1, c_1)$  and  $X_2 \sim \text{PERT}(x_2; a_2, b_2, c_2)$ , results in a random variable,  $X_3$ , that also follows a Beta-PERT probability distribution,  $X_3 \sim \text{PERT}(x_3; a_3, b_3, c_3)$ , whose parameters are defined as follows;

$$\begin{cases} a_3 = a_1 + a_2 & (13.1) \\ b_3 = b_1 + b_2 & (13.2) \\ c_3 = c_1 + c_2 & (13.3) \end{cases} \quad (13)$$

The rationale behind Eqs. (13.1) and (13.3) is that the minimum (or maximum) value of the resulting distribution cannot be other than the summation of the smallest (or largest) values that the random summands can take. Regarding parameter  $b$ , Eq. (13.2) is obtained as follows; Since  $X_1$  and  $X_2$  are statistically independent, the following relationship holds regardless of the distribution type:

$$\mu_{X_3} = \mu_{X_1} + \mu_{X_2} \quad (14)$$

By introducing Eqs. (1) into (14), the following relation is obtained:

$$\mu_{X_3} = \frac{a_3 + 4b_3 + c_3}{6} = \frac{a_1 + 4b_1 + c_1}{6} + \frac{a_2 + 4b_2 + c_2}{6} \quad (15)$$

Then, accounting for the relations in Eqs. (13.1) and (13.3), Eq. (13.2) is obtained. Note that, whereas the relations in Eqs. (13.1) and (13.3) are valid for any probability distribution, Eq. (15) assumes the Beta-PERT distribution for  $X_3$ .

To verify to which extent  $X_3$  follows a Beta-PERT distribution, 1000 different samples, with a 50-sample size each, have been created by summing two samples that follow the Beta-PERT distribution. The independent samples are created by considering random parameters in the range of  $[-100, 100]$  guaranteeing  $a < b < c$ . Then, the resulting samples are compared against the hypothesized distribution obtained with the parameters defined in Eqs. (13.1)–(13.3). The Kolmogorov-Smirnov Goodness-of-Fit test is used to determine whether each sample conforms to the corresponding hypothesized probability distribution. It can be concluded that in 95% of the 1000 simulated cases, it cannot be rejected the null hypothesis that states that “the sample is consistent with the hypothesized

distribution” at a 0.01 significance level. In other words, it is accepted that the summation of two independent random variables following Beta-PERT distributions results in a random variable that also follows a Beta-PERT probability distribution. The percentage decreases when more variables are added, as shown in Table 1.

We conclude that the Beta-PERT distribution has a *pseudo-reproducibility* property when the number of summands is small. Therefore, the approach is valid for activities affected up to 4–5 factors. The MitC user must consider that when more than 4 factors affect a given activity, they should use other probabilistic distributions with the reproducibility property.

#### 5. The mathematical formulation of the extended MitC concept

This section introduces the mathematical formulation of the

**Table 1**  
Probability of acceptance of the null hypothesis (alpha = 0.01).

Number of summands	2	3	4	5
Probability of acceptance	95%	84%	66%	49%

extended MitC concept enabling (1) contractual project completion performance schemes and (2) stochastic dependence between construction activities.

### 5.1. The MC simulation approach within MitC

The core of the MitC is the ability to integrate the inherent uncertainties that govern all variables by utilizing a Monte Carlo technique that captures the stochastic behavior of the variables.

Fig. 3 provides the main steps of the MitC. In **Step 1**, data about the project network are organized in a machine-readable structure. The required data includes:

- The construction project activities, correlation factors, mitigation measures, and potential risk events;
- Three estimates for the duration of every activity.
- Three estimates for the shared uncertainty of every shared factor;
- Three estimates for the capacity of every mitigation measure: the time reduction gained by implementing a mitigation measure;

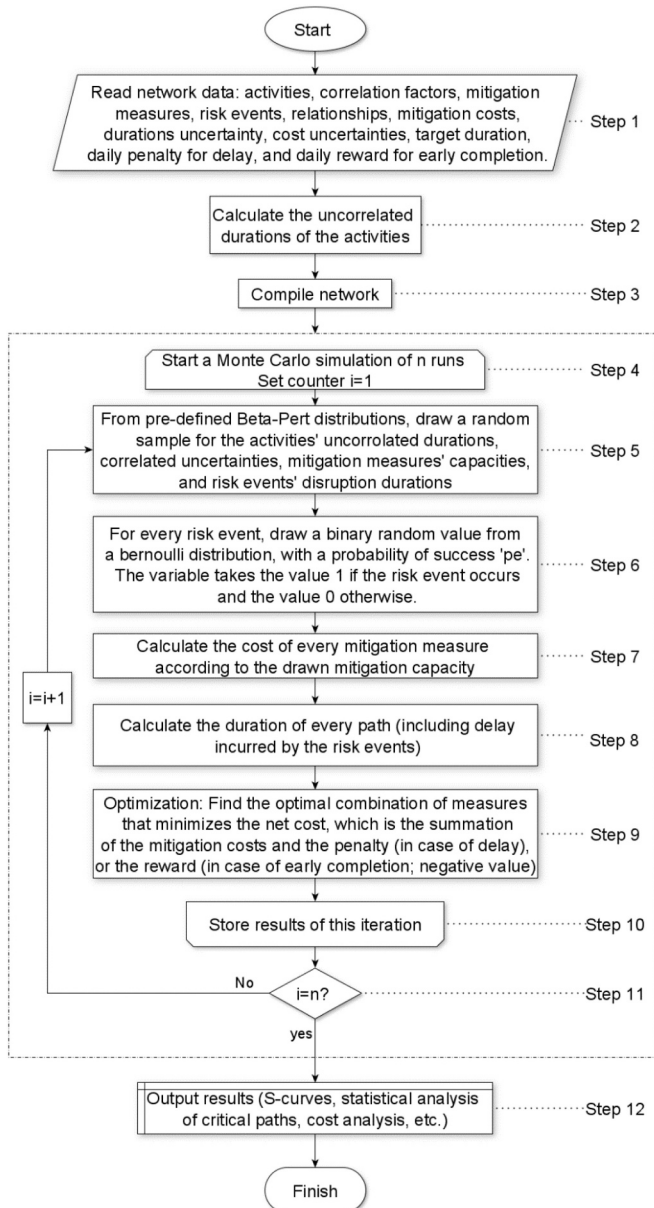


Fig. 3. Main steps of the improved Mitigation Controller.

- The most-likely cost of every mitigation measure with the corresponding relation factor  $\eta_j$ , which defines how much the cost changes when the mitigation capacity changes;
- Three estimates for the disruption duration of each risk event: the delay incurred if a risk event occurs;
- A probability of occurrence  $p_e$  for every risk event;
- The relationships between the construction activities and the correlation factors: which factors are shared by which activities;
- The relationships between the mitigation measures and the activities: the activities whose durations are reduced as a result of implementing a mitigation measure;
- The relationships between the risk events and the activities: the activities that are impacted by the occurrence of a risk event;
- The planned completion time of the construction project, or target duration,  $T_{tar}$ ;
- The daily penalty for late completion; and
- The daily reward for early completion.

$T_{tar}$  is the target (i.e., desired/planned) project completion duration. Therefore,  $T_{tar}$  is always equal or lower than the current completion time of the project  $T_{curr}$ , which is related to the completion time without mitigation.

In **Step 2**, the uncorrelated durations of construction activities are derived following Section 4.1.

The network is validated and compiled in **Step 3**. The network is first validated using a set of rules to ensure that no errors exist in the network structure and data input. This can prevent, for instance, open ends networks from further processing. Network compilation refers to the process of determining all potential critical paths and their durations. There are several ways to determine the critical path (see for instance [33]). In this paper, every path is represented by a set of activities arranged chronologically. The duration of the path is calculated as the sum of the durations of all activities that constitute this path, given that the activities are of a *finish-to-start* type.

In **Step 4**, the MC simulation is initiated by creating a loop of  $n$  iterations and setting the counter to 1. The parameter  $n$  is defined by the user based on a predefined convergence tolerance or confidence level.

In **Step 5**, the uncorrelated durations of the construction activities, the shared uncertainties, the capacities of the mitigation measures, and the disruption durations of the risk events are sampled from predefined Beta-PERT distributions, which are constructed using the estimates of the activities' durations, correlation factors, mitigation measures, and risk events, defined in Step 1.

In **Step 6**, a random binary variable for every risk event is sampled from a Bernoulli distribution, with a probability of success  $p_e$ . The variable takes the value of 1 if the risk occurs (i.e., *success*) and 0 otherwise.

In **Step 7**, the cost of every mitigation measure is computed according to the drawn value of the mitigation capacity, using Eq. (2).

After drawing a random sample for the durations of the activities and risk events, the durations of all paths are evaluated (**Step 8**).

The optimization is carried out in **Step 9**, where the optimal mitigation strategy that minimizes the net cost is identified. The net cost is the summation of the costs of mitigation measures and the penalty (in case of delay) or the reward (in case of early completion; negative value). The mathematical formulation of the optimization problem is described in Section 5.

In **Step 10**, the results of the current iteration are stored in the memory; such as the optimal mitigation strategy, the mitigation cost, and the project completion time.

**Step 11** is the final step of the current MC iteration, after which the procedure in Steps 4–9 is repeated until the total number of iterations is reached.

Once the MC simulation is concluded, a statistical analysis is performed on the results of the  $n$  iterations to obtain, for instance, the mitigation cost distribution, the completion time distribution, and the frequency of each selected mitigation strategy. More details on the



analysis are provided in the illustrative example.

## 5.2. The optimization approach within MitC

The following is the mathematical formalization of the optimization problem. Assume a set of construction activities  $\mathcal{J} = \{1, 2, \dots, I\}$  and a set of all project schedule paths  $\mathcal{K} = \{1, 2, \dots, K\}$ . Every path  $k \in \mathcal{K}$  constitutes a sequence of project activities. The relationship between the activities and the paths is represented by a parameter  $p_{k,i}$  taking the value of 1 when activity  $i$  is included in the path  $k$  and zero otherwise. Delays in the scheduled activities will cause the paths durations to be longer than the target completion time of the project,  $T_{tar} \in \mathbb{N}$ . To mitigate the delays in the activities' durations, mitigation measures are usually identified and implemented. The set of the identified mitigation measures is denoted by  $\mathcal{J} = \{1, 2, \dots, J\}$ . The implementation of a mitigation activity  $j$  is represented by the variable  $x_j \in \{0, 1\}$ .

The objective of the optimization problem executed within every run of MC is to select the most effective set of mitigation measures that minimize the net cost. Fig. 4 depicts the logic behind the mathematical formulation of the optimization problem. The project network is represented by a set of all possible paths. The duration of the project is the duration of the path with the longest duration (i.e., critical path). In most cases, there is more than one path whose durations are larger than the project target duration. When mitigating only the critical path, another path will become the critical path. This new critical path must then be mitigated. Therefore, to avoid such an iterative approach, all paths whose durations are larger than the target duration must be mitigated in a single iteration.

Hence, the objective of the optimization problem is to reduce the duration of all paths whose durations are larger than the target duration stated in the contract so that the net cost is minimum. In Fig. 4, path  $k = 1$  is the critical path, before and after mitigation. The duration of this path after mitigation is lower than the target duration. The net cost, in this case, is the cost of the applied mitigation measures minus the reward obtained of early completion.

Fig. 5 provides a graphical representation of the optimization problem, which corresponds to the first scenario in Fig. 1 where the mitigated duration is lower than the target duration.

The objective function of the optimization problem can be mathematically expressed as follows:

$$\min_x \sum_{j \in \mathcal{J}} c_j x_j + \Delta_1 \times P - \Delta_2 \times R \quad (16)$$

where  $x_j \in \{0, 1\}$  is a binary variable that represents the implementation of a mitigation activity  $j$ ,  $\Delta_1$  is the project delay after implementing the

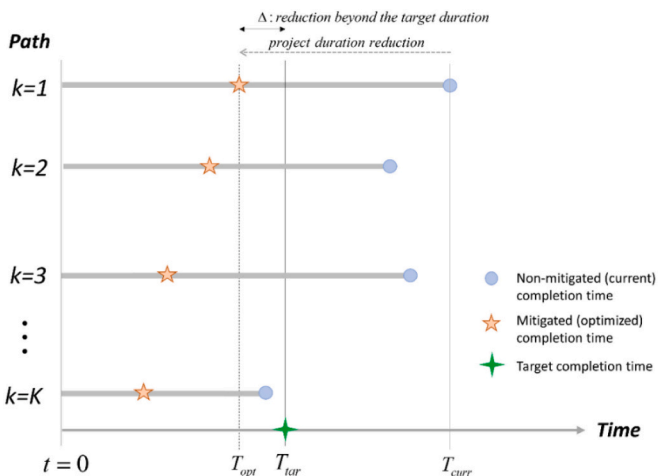


Fig. 4. Rationale behind the optimization problem.

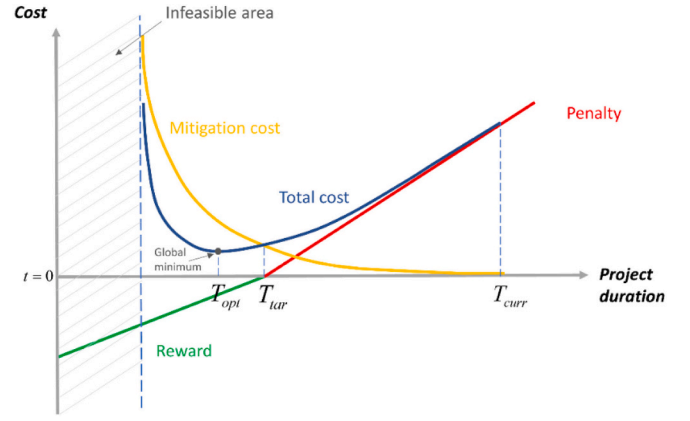


Fig. 5. Graphical representation of the optimization problem.

mitigation measures,  $\Delta_2$  is the duration reduction beyond the target duration,  $P$  is the daily penalty,  $R$  is the daily reward.

The project can either be delayed or not. Therefore,  $\Delta_1$  and  $\Delta_2$  cannot occur simultaneously. Hence, the objective function is subject to the following constraint:

$$\Delta_1 \times \Delta_2 = 0 \quad (17)$$

The objective function is also subject to the following compatibility constraints:

$$d_k^0 - MitDur_k \leq T_{tar} + \Delta_1 - \Delta_2 \quad \forall k \in \mathcal{K} \quad (18)$$

where  $d_k^0$  is the current duration (before optimization) of path  $k$ ,  $MitDur_k$  is the mitigated duration from path  $k$ , given by:

$$MitDur_k = \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{J}} p_{k,i} r_{i,j} m_j x_j \quad \forall k \in \mathcal{K} \quad (19)$$

where  $p_{k,i}$  is a relation parameter which takes the value of 1 when activity  $i$  is included in the path  $k$  and zero otherwise.

The optimization problem above is clearly nonlinear (see Eq. (17)). The optimization problem can be linearized by adding a translation of value  $e$  to both variables,  $\Delta_1$  and  $\Delta_2$ , obtaining the following optimization system:

$$\min_x \sum_{j \in \mathcal{J}} c_j x_j + (\Delta_1 - e) \times P - (\Delta_2 - e) \times R \quad (20.1)$$

s.t.

$$(\Delta_1 - e) \times (\Delta_2 - e) = 0 \quad (20.2) \quad (20)$$

$$d_k^0 - MitDur_k \leq T_{tar} + (\Delta_1 - e) - (\Delta_2 - e) \quad \forall k \in \mathcal{K} \quad (20.3)$$

$$MitDur_k = \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{J}} p_{k,i} r_{i,j} m_j x_j \quad \forall k \in \mathcal{K} \quad (20.4)$$

By assuming  $e = \{\Delta_1, \Delta_2\}$ , the translation guarantees that Eq. (20.2) always fulfills, hence, this equation is not required anymore. As a result, the optimization model expressed in terms of  $\Delta_1^*$  and  $\Delta_2^*$  (with  $\Delta_i^* = \Delta_i - e$ ,  $i = \{1, 2\}$ ) becomes linear as follows;

$$\min_x \sum_{j \in \mathcal{J}} c_j x_j + \Delta_1^* \times P - \Delta_2^* \times R \quad (21.1)$$

s.t.

$$d_k^0 - MitDur_k \leq T_{tar} + \Delta_1^* - \Delta_2^* \quad \forall k \in \mathcal{K} \quad (21.2) \quad (21)$$

$$MitDur_k = \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{J}} p_{k,i} r_{i,j} m_j x_j \quad \forall k \in \mathcal{K} \quad (21.3)$$

## 6. Demonstrative case of a Dutch construction project

### 6.1. Case description

This section illustrates the extended MitC with an application to a real construction project. The project is executed by the Dutch highway agency Rijkswaterstaat, and it is one of several sub-projects for expanding the road network connecting Schiphol Airport, Amsterdam, and Almere (SAA). For illustrative purposes, only the major construction activities have been considered in this example. The project's completion date can be translated into a maximum duration of 1466 days (i.e., target duration  $T_{tar} = 1466$  days). This duration can be achieved if all construction activities are executed as planned. However, activities durations have uncertainty and can take longer or shorter than planned. This uncertainty can lead to a delay in the project delivery date. Besides, risk events can occur and hence incur additional delays. In this example, a list of 19 risk events is identified and included. To mitigate the delay incurred by the risk events and the duration uncertainty of the activities, mitigation measures are introduced. A total of 19 mitigation measures are identified to mitigate the project delay. These measures are used by the MitC to optimize the project duration so that the net cost (cost of mitigation measures and penalty/reward) is minimum. The penalty for every day of delay and the reward for every day of early finish have been assumed in this example to equal 3000 euros/day.

It is noted that the figures used in this example are a congruent representation of the actual figures. The real figures have been scaled and reduced into a simplified case to preserve the confidentiality requirements.

The rest of this section includes data and details needed to run the simulation as well as results and interpretations. The example presented hereafter can be reproduced using the MitC software tool (<https://github.com/mitigation-controller/mitc>).

### 6.2. Level-1 project schedule

The construction project analyzed here comprises 37 activities. Table 2 displays the project activities with their three-point duration estimates. The three-point estimates of the activities' durations are used to construct the Beta-PERT distributions  $f(d_i; a_i, b_i, c_i)$ . The last column in the table includes the predecessors of the activities. The original/planned project's duration (1466 days) is computed using the most-likely duration of each activity. The Gantt chart of the project, as well as more information about the project activities, can be found in [19].

### 6.3. Construction activities' correlation

To account for the correlation among the activities' durations, the shared uncertainties are needed (see Section 4.1). Table 3 lists the uncertainty factors with their three-point estimates (Min, Most-Likely, Max), which are used to build the Beta-PERT probability distributions of the shared uncertainties. The last column in the table contains the relationships between the uncertainty factors and the project activities. For example, Factor 1 is shared by both activities 10 and 11.

Applying Eqs. (7)–(12) and using the information in Tables 2–3, one can obtain the probability distributions of the uncorrelated durations,  $D_{i, uncorr}$ . Note that the shared factors in Table 3 are introduced by the authors for demonstrative purposes. In real construction projects, this list of factors could be longer.

### 6.4. Mitigation measures on-the-run

The mitigation measures used in this example are listed in Table 4 together with their three-point capacity estimates (Min, Most-Likely, Max). These measures are the actual measures used in the construction project SAA. Each mitigation measure has a mitigation cost. The Most-Likely costs of the mitigation measures, which are the authors'

**Table 2**

Activities' duration and relationships.

ID	Activity description	Duration (days)			Predecessor activities
		Optim.	Most-Likely	Pess.	
0	Start (dummy activity)	0	0	0	
1	Contract date	0	0	0	0
2	Financial Close	0	0	0	0
3	Design	819	920	1435	1
4	Acquiring the certificate of commencement	105	130	194	1
5	Commencement certificate is acquired	0	0	0	2, 4
6	Date of commencement	0	0	0	5
7	Maintain existing road assets A9 / A1 / A6	976	1284	1836	6
8	Conditioning, Cables and Conducts, permits	168	200	268	6
9	Preload	324	395	525	6
10	Constructing a new Aqueduct	200	260	341	6
11	Constructing a Canal bridge	285	335	492	6
12	Construction works in the southern A1 lane	113	128	189	9, 10, 11
13	Commissioning of the southern A1 new lane	0	0	0	12
14	Producing parts of Railbridge part 1	223	251	366	6
15	Producing parts of Railbridge part 2	194	220	350	14
16	Assembling a railway bridge on location	559	674	971	14
17	Moving Railway Bridge in place during Train Free Period	0	0	0	16
18	Road works northern A1 lane	109	130	191	17
19	Commissioning of the Northern A1 new lane	0	0	0	18
20	Road and construction works new part junction Diemen	477	530	848	13
21	Build new viaducts A6	304	400	532	6
22	Build second Hollandse bridge	286	340	459	6
23	Road and construction works junction Muiderberg	716	930	1237	6
24	Road works eastern part A6	90	100	130	21, 22
25	A6 East ready	0	0	0	24
26	Reconstruction western part A6	324	400	532	25
27	Commissioning A6	0	0	0	18, 23, 26
28	Road works existing part Diemen junction	71	90	130	19, 20
29	Request Availability Certificate	0	0	0	28
30	Assess and obtain Availability Certificate	16	20	27	29
31	Demolition old A1 (part 1)	54	61	91	30
32	Demolition old A1 (part 2)	23	30	48	31
33	Greenery for old A1	77	90	129	31
34	Applications and obtaining partial completion certificates	104	120	161	30
35	Request Completion Certificate	0	0	0	33
36	Obtaining the Certificate of Completion	17	20	27	35
37	Scheduled Completion Date	0	0	0	36
38	End (dummy activity)	0	0	0	3, 7, 8, 15, 27, 32, 34, 37

**Table 3**  
Shared uncertainty factors.

ID Shared uncertainty factor	Shared uncertainty (days)			Relations with activities
	Min.	Most-Likely	Max.	
1 Weather <a href="#">condition 1</a>	-45	0	72	10, 11
2 Soil composition	-50	0	100	21, 22, 23, 7
3 Crew performance	-10	0	50	12, 23
4 Soil composition	-45	0	110	20, 26
5 Equipment availability 1	-20	0	100	15, 16
6 Site availability	-5	0	100	16, 20
7 Procurement, fabrication or assembly	-1	0	55	7, 20
8 Project control and management	-20	0	50	8, 9
9 Design or documentation accuracy	-5	0	15	32, 33
10 Owner-driven changes	0	0	45	18, 20
11 Issues with contractor	-20	0	50	3, 4
12 Issues with supplier	-20	0	100	7, 14
13 Equipment availability 2	-80	0	90	7, 16
14 Weather <a href="#">condition 2</a>	-140	0	100	7, 23

suggestions, are listed in Column 7 of the table, while the minimum (*Min*) and maximum (*Max*) estimates, listed in Columns 6 and 8, respectively, are calculated using Eq. (2). The relationship factor  $\eta_j$  is set to 0.5 for all mitigation measures. This value implies a partial relation between the variation in the mitigation costs and the variation in the mitigation capacities. The relations between the mitigation measures and activities are given in the last column of the table where, for instance, mitigation measure with 'ID = 9' mitigates the duration of activity with 'ID = 14'. Eq. (5) is used to introduce these relations into the model. Note that each mitigation measure can affect more than one activity, although it is not the case in this example.

### 6.5. Project risk events

Risk events are an additional source of uncertainty that can negatively affect the project completion time. The amount of delay induced by every risk event is here referred to as *risk duration*.

The identified risk events and their three-point duration estimates are listed in Table 5. These are obtained from one of the co-authors who was involved as a director both during the tender and the execution phases of this project. Every risk event is associated with an occurrence probability  $p_e$ . The relations between the risk events and activities are

**Table 4**  
Mitigation measures' durations, relationships, and costs.

ID	Mitigation description	Mit. Capacity (days)			Cost (euros) ( $\eta = 0.5$ )			Relations with activities
		Min.	Most-likely	Max.	Min.	Most-Likely	Max.	
1	Extra engineering design office personnel	99	101	101	118,846	120,000	120,000	3
2	Extra software design capacity	14	14	14	30,000	30,000	30,000	4
3	Extra maintenance engineers	103	127	127	136,153	150,000	150,000	7
4	Extra administrators for permitting	43	51	57	44,307	48,000	50,769	8
5	Applying extra preloading material	41	51	51	677,884	750,000	750,000	9
6	Adding extra on-site construction flow	92	101	107	190,384	200,000	205,769	10
7	Extra prefab construction capacity	117	127	129	144,230	150,000	151,153	11
8	Additional M&E engineers	51	51	51	60,000	60,000	60,000	12
9	Additional welding equipment and personnel	53	64	64	90,909	100,000	100,000	14
10	Extra temporary soil measures	45	51	53	235,576	250,000	254,807	15
11	Ancillary on standby	201	203	222	199,038	200,000	209,615	16
12	Additional M&E engineers	14	14	14	30,000	30,000	30,000	18
13	Additional excavation capacity	96	101	101	121,394	125,000	125,000	20
14	Additional concrete workers / carpenters	82	101	107	67,788	75,000	77,163	21
15	Temporary ancillary construction and re-work	70	76	84	1,442,307	1,500,000	1,576,923	22
16	Additional excavation capacity	60	76	82	134,615	150,000	155,769	23
17	Additional asphalt equipment and personnel	101	101	107	200,000	200,000	205,769	26
18	Additional removal works	43	51	53	69,230	75,000	76,442	28
19	Additional equipment and personnel	10	14	16	214,285	250,000	267,857	30

given in Column 6 of the table where, for instance, risk event with 'ID = 4' affects the duration of activity with 'ID = 8'.

### 6.6. Results and benefits of MitC

The extended MitC concept is applied to control the SAAone's project completion date in an effective and efficient manner. The MitC has been programmed using Matlab® R2019b. The optimization problem was solved using the optimization toolbox in Matlab. The linprog function was specifically used to solve the linear optimization problem.

#### 6.6.1. Comparing different contractual completion performance schemes

Four cases are analyzed in this paper: 1) No penalty for delay or reward early completion, 2) High penalty for every day of delay (i.e.,  $10^9$  euros/day) and No reward for early completion, 3) High penalty for every day of delay (i.e.,  $10^9$  euros/day) and High reward for every day of early completion (i.e.,  $10^9$  euros/day), and 4) penalty (3000 euros/day) and reward (3000 euros/day) exist simultaneously with the same order of magnitude as the average cost of the mitigation measures (i.e., typical case).

Fig. 6 shows the results of the four simulations that correspond to the four cases. Every simulation produces the cumulative probability curves of the project's completion time (S-curves) for three scenarios. The first scenario (*No Mit*) accounts for the stochastic uncertainties in the durations but without considering any mitigation measure. The second scenario (*ALL Mit*), also called *Permanent*, considers all available mitigation measures simultaneously in the analysis without performing any optimization. The third scenario (*MitC*), also called *Tentative*, considers the optimal mitigation strategy in every MC iteration, which is obtained by applying the optimization problem of MitC. In all three scenarios, the risk events are included. The first and second scenarios are implemented for comparative reasons.

**Case 1.** No penalty, No reward: since there is neither a penalty nor a reward, the MitC does not apply any mitigation measure because applying mitigation measures would incur additional costs. Therefore, the curve of the *MitC* coincides with the curve of *No Mit*. In such a case, there is a probability of 0.8 of finishing the work in 1620 days (delay of 154 days).

**Case 2.** High penalty, No reward: regularly, the contractual penalty is much higher than the average cost of mitigation measures. In this case, the MitC optimizes the construction project's duration intending to avoid any delay, if possible. It finds the most effective mitigation measures that achieve the originally planned duration ( $T_{tar} = 1466$  days).

**Table 5**

Durations, relationships, and probabilities of the identified risk events.

ID	Risk event description	Risk duration (days)			Affected activities	$p_e$
		Min.	Most-Likely	Max.		
1	Rejection of preliminary design	96	105	119	3	0.2
2	failure of EDP audit	13	14	15	4	0.05
3	Existing condition Hollandse Brug deviates from maintenance plan made during the tender phase	63	70	78	7	0.15
4	Unexpected gas conducts	35	35	41	8	0.2
5	The consolidation rate is lower than estimated	34	35	40	9	0.1
6	Piling machines failure	14	14	15	10	0.1
7	Delay in the delivery of prefabricated elements	19	21	25	11	0.2
8	Dynamic Traffic Management equipment / software not functioning	20	21	22	12	0.25
9	Production equipment failure	20	21	23	14	0.05
10	Construction site subsides	13	14	15	15	0.05
11	Failure of ancillary equipment	33	35	41	16	0.1
12	Dynamic Traffic Management equipment / software not functioning	20	21	21	18	0.25
13	Discovery of polluted soil	13	14	14	20	0.05
14	Failure of concrete casting	13	14	14	21	0.05
15	Main pillar subsides	65	70	71	22	0.02
16	Discovery of polluted soil	25	28	32	23	0.05
17	Insufficient quality of base layer	39	42	47	26	0.02
18	Discovery of asphalt with too high PAK percentage	13	14	17	28	0.05
19	Additional scope of work (misc.)	130	140	160	30	0.01

Achieving early completion of the project requires additional mitigation measures. Therefore, the MitC avoids early completion of the project since no rewards are provided and because the cost would be higher if additional measures are applied.

In this case, where the penalty is high and no reward is provided, the probability of project completion at the target duration is maximized (probability of 0.97). This is the reason why the MitC curve intersects with the Permanent curve at this point. Since the MitC avoids early completion (no reward), the probability of finishing the project before the target duration is very low (see the steep slope of the curve before the target duration). Case 2 can be seen as date-oriented optimization since the optimization targets a specific date (i.e., the target duration).

**Case 3.** High penalty, High reward. In this case, the MitC aims at achieving the lowest duration possible of the construction project so that delay is avoided and reward is gained. As can be seen in the figure, the MitC curve coincides with the Permanent curve. It is important to note that although the two curves coincide, the MitC does not use all available mitigation measures as in the case of Permanent since some of the measures might be not effective, for instance when a mitigation measure improves the duration of an activity that is not on the critical path in a given MC iteration. This means that the MitC will result in fewer costs even though it yields the same results as Permanent. Now, the probability of finishing in e.g., 1400 days is 0.8, whereas in the previous case it was negligible.

**Case 4.** penalty (3000 euros/day) and reward (3000 euros/day) exist simultaneously with the same order of magnitude as the average cost of mitigation measures: this is the case in typical projects where there is a penalty for delay and reward for early completion. In every MC

iteration, the MitC searches for the most effective mitigation measures that produce the lowest possible net cost considering penalties and rewards. Depending on the durations of activities and the risk events in a specific iteration, the MitC might choose to go for either an early completion or delay as long as the net cost is minimum. The target duration, therefore, does not govern the optimization problem. In this case, there is a probability of 0.80 of finishing the work in 1510 days (delay of 44 days), which translates into the best scenario from an economic perspective.

#### 6.6.2. Effect of the stochastic construction activity correlations

Fig. 7 compares the cumulative probability of project completion time for the two cases “with” and “without” considering correlations between activities. It is shown that accounting for correlation among the activity durations changes the probability distribution of the project completion time. This is justified and expected because forcing a correlation between activity durations changes the number of possible scenarios. For instance, if raining, all weather-sensitive activities will tend to take longer, and unrealistic scenarios where some weather-sensitive activities are delayed and others are taking less time will not be possible in the absence of any mitigation measure. Hence, such unrealistic scenarios should be eliminated from the set of potential scenarios. If the correlation is not forced, unrealistic scenarios can be considered among the potential scenarios. Ignoring the existing correlations produces a modeling error that may lead to over or underestimating the project duration.

#### 6.6.3. Cost analysis

This section compares the estimated net cost obtained from the MC simulation for the two mitigation strategies, MitC and Permanent, where penalty and reward are both equal to 3000 euros/day (Case 4). The net cost is the sum of the mitigation cost, rewards in the event of early completion, and penalties in the event of late completion. A substantial cost reduction could be achieved by employing the MitC compared to the scenario where all mitigation measures are applied simultaneously. This is shown in Fig. 8 where the cost probability is obtained for the Tentative and Permanent strategies. The highest costs reported were 0.88 M Euros when considering measures as being Tentative while 4.56 M Euros when considering measures as permanent. This implies that the classical probabilistic approach (Permanent) overestimates costs by about 3.68 M Euros (81%). The overestimation of the costs results from the ineffective use of mitigation measures.

Due to the stochastic uncertainties, the most effective mitigation strategy can change throughout the different iterations. Hence, a criticality index (CI) can be derived for every mitigation measure representing the number of times the mitigation measure was included in the set forming the optimal mitigation strategy. Fig. 9 classifies the mitigation measures according to their CI. The y-axis represents the probability that a mitigation measure is included in the optimal mitigation strategy. Several measures have not been selected in any iteration (e.g., IDs: 1, 4, 5, 6, 7, 10, 14, 15, 16, 17, 19), and some were only selected in an insignificant number of iterations (e.g., 3 and 11). Only one mitigation measure (i.e., measure 2) has been used in all simulation iterations. This suggests that the mitigation measures that are not used or with a low probability of being used can be excluded from the list of possible mitigation measures, while some mitigation measures should be prioritized as they are more frequently selected throughout the simulation. More discussions on the criticality of mitigation measures can be found in [19].

## 7. Conclusions and further developments

Classical probabilistic construction planning theory and its applications are not reflecting the project manager goal-oriented control behavior. Both scientific researchers and state-of-the-art scheduling software (e.g. Primavera P6) do not consider a project manager's real-



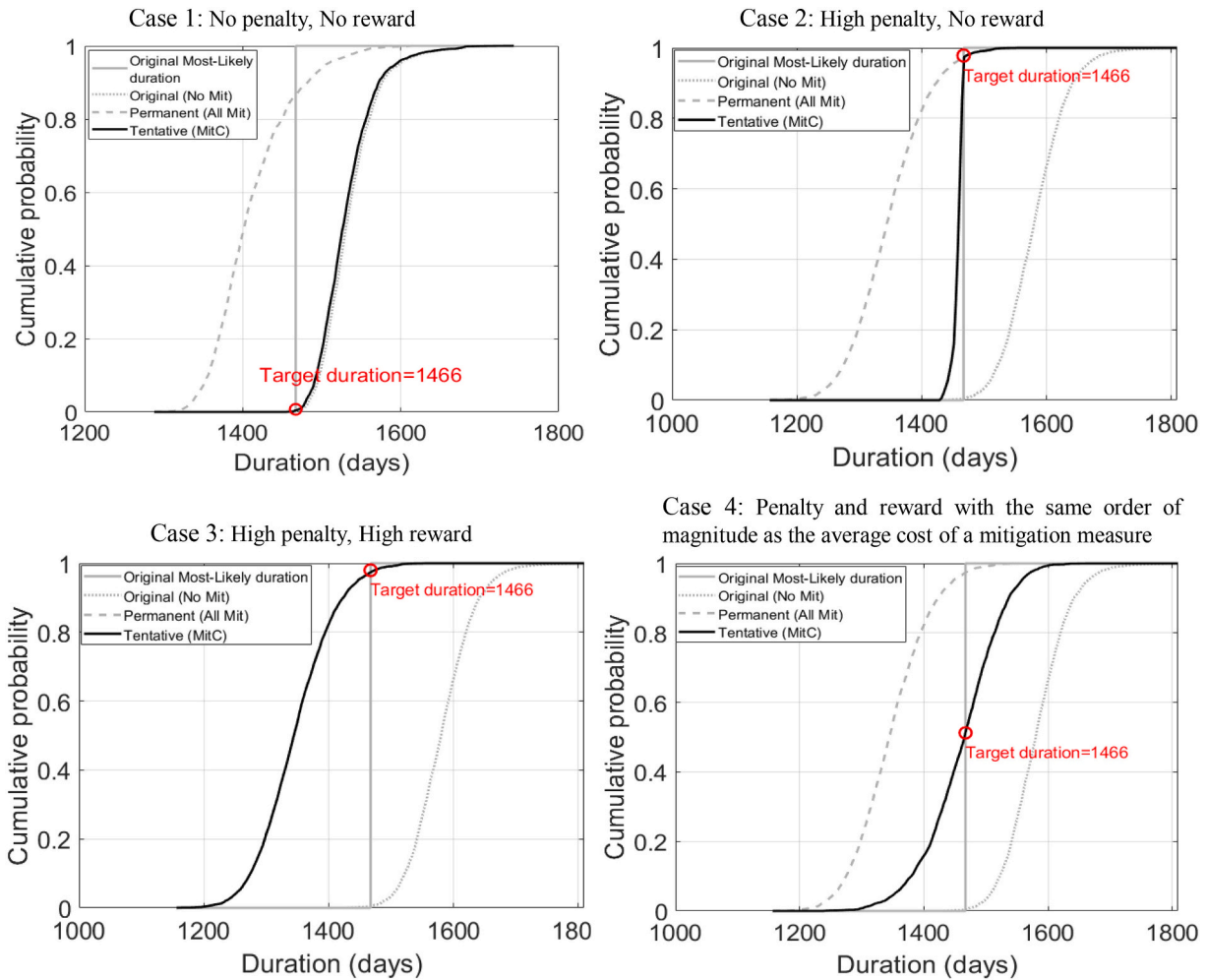


Fig. 6. Cumulative probability of project completion time for four cases.

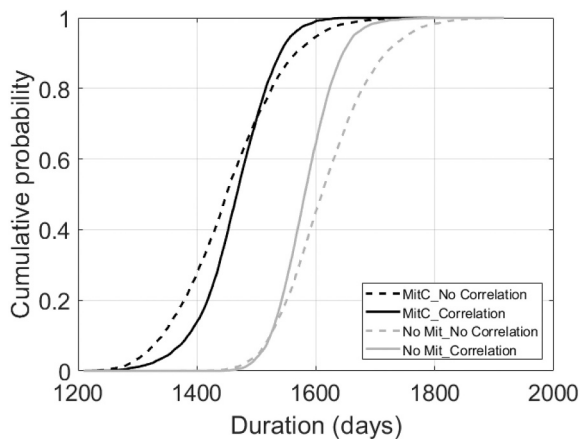


Fig. 7. Cumulative probability of project completion time with and without considering correlations among activities.

life goal-oriented and control behavior during project execution. The Mitigation Controller (MitC) concept was recently developed to integrate the goal-oriented behavior of the project manager as part of the classical Monte Carlo analysis. The MitC is considered a support tool for automating the process of searching for the most effective set mitigation strategy that guarantees the required probability level of meeting the target availability date of construction projects. This paper builds upon

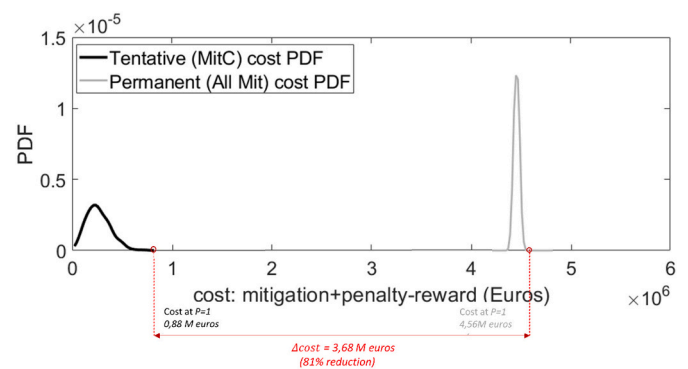


Fig. 8. Cost distribution obtained from the MC simulation for the *Tentative (MitC)* and *Permanent (ALL Mit)* mitigation strategies.

the original MitC by incorporating contractual performance schemes (penalty/reward) in the optimization problem, providing new insights on the mitigation strategies of construction projects. It also introduces a novel approach that captures the correlations of construction activities' durations and models them as such.

The final product is an extended MitC (i.e., MitC) that combines the PERT scheduling approach with Monte Carlo simulation to generate the project completion probability curve (i.e., the S-curve). It considers several factors while computing the optimal mitigation strategy, such as the stochastic variations in the activities' durations, mitigation

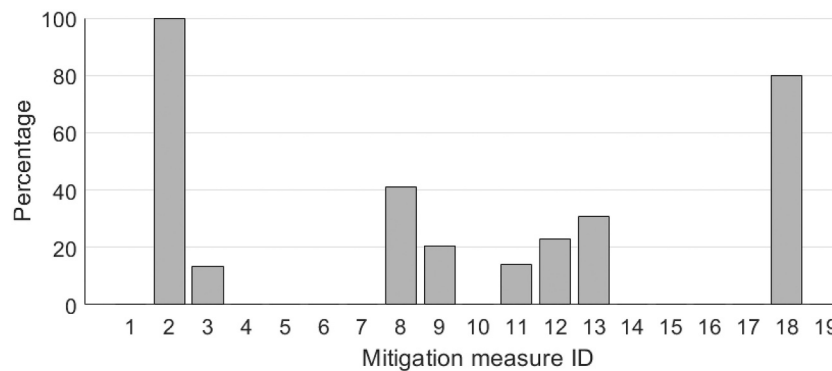


Fig. 9. Criticality index (CI) of the mitigation measures.

capacities (i.e., crashing duration), and mitigation costs. It also considers the occurrence probability of the potential risk events that could impact the normal execution of the construction activities. The extended MitC provides a realistic estimate for the project's timely completion probability by modeling the project manager's goal-oriented control behavior and by considering correlations among project activities.

The results of the MitC show that the classical probabilistic construction planning is overly conservative both in time and in the number of required mitigation measures, resulting in over-spending a significant amount of money. Furthermore, it is shown that considering the penalty and reward in the delay mitigation process changes the optimization objective. The MitC allows for different optimization objectives depending on the values of penalty, reward, and the costs of the mitigation measures. In all cases, the MitC selects only the most effective mitigation measures that achieve the lowest net cost. Most savings are not necessarily achieved by completing the project exactly on time. That is, allowing for delay or expediting the project to finish in advance can be even more advantageous, depending on the respective penalty/reward scheme. Finally, it is demonstrated that modeling correlations among activities have a large effect on the estimated completion time of the project, and neglecting it would lead to a significant modeling error. As a result, more accurate S-curves are obtained because a number of non-feasible scenarios are removed by considering the statistical correlation between the construction activities' durations.

Future work will be oriented towards extending the MitC to address the existing limitations and accommodate more features of interest to the construction industry. The following are potential future improvements of the MitC which will also contribute to the body of knowledge in a wide spectrum of applications.

- The MitC should allow for optimization on criteria other than costs alone. An application that will be investigated is the optimization where Lost Vehicle Hours (LVHs) is the predominant optimization criterion.
- Risks related to mitigation measures are currently defined using a PERT distribution. Further iterations of the MitC will allow for choosing different distributions.
- The proposed tool is only able to model a *finish-to-start* relation type. Other relation types of activities will also be considered in future research.
- Another aspect that will be included in future research is the consideration of resources limitation. This paper assumes unlimited personnel and material resources, allowing unlimited simultaneous mitigation measures.
- In the methodology described in this paper, mitigation measures work by crashing the construction activities; i.e., the scheduling network structure stays intact. In reality, changes to the network structure can be made to avoid delays at a minimum cost level. Further research will be carried out to not only incorporate

mitigation measures but also incorporate automated changes in the scheduling network structure to account for schedule interruptions.

#### Data availability statement

Some or all data, models, or code generated or used during the study are available in a repository online in accordance with funder data retention policies. The repository is located here: <https://github.com/mitigation-controller/mitc>

#### Declaration of Competing Interest

The authors have no conflicts of interest to declare. All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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