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ARTICLE

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Reliability-based calibration of design code formulas: Application to shear resistance formulas for reinforced concrete members without shear reinforcement

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Abstract

This paper presents a reliability-based calibration method for design code formulas. The method is demonstrated on the shear design formulas in Eurocode 2 and *fib* Model Code 2010 (MC2010). We found that the partial factor γ_c in the current Eurocode 2 is about 20% lower than the optimal value and, thus, provides an insufficient safety margin. The obtained optimal partial factor γ_R in the (modified) Eurocode 2 and MC2010 formulas is 1.53 and 1.36, respectively. The difference stems from higher accuracy and, hence, lower uncertainty of the MC2010 model in predicting experimental results. Hence, on average, the MC2010 formula leads to about 13% larger design resistances compared to Eurocode 2 given that the target reliability for both design formulas is the same. To stimulate and facilitate future structural code development and derivation of partial factors, we make the used computer code freely available.

KEYWORDS

design, model uncertainty, partial factor, reinforced concrete, reliability-based calibration, shear formula

1 | INTRODUCTION

The development of structural design formulas is an on-going process fuelled by the development of "new" structural materials (e.g., fiber-reinforced and recycled aggregate concrete), innovative structural designs, and improved insights into structural behavior (see, e.g., the recent revision process of the shear resistance formula in Eurocode 2, in which an empirical formula is replaced by a formula based on a clear physical background[1,2]). To accommodate these recurring updates and extensions of the design code, a sound and transparent approach for the calibration of structural design formulas is needed. Moreover, clarity is required on how the involved uncertainties are defined and partitioned, and how these uncertainties contribute to each partial factor.

Most partial factors in the current Eurocodes relevant for concrete structures are derived based on expert judgment and calibrations to previous design procedures, but sound background documents are missing (the original CEB and *fib* background reports are difficult to access). Hence, the origin of standardized partial factors is often unclear and it is not exactly known which uncertainties were intended to be covered by them. An example of this is the partial factor for concrete γ_c in Eurocode 2.[3] The review of background documents and consultation of experts revealed contradictory explanations on the uncertainties associated to this partial factor.[4] The definition of γ_c is further complicated because it is used with the same

Discussion on this paper must be submitted within two months of the print publication. The discussion will then be published in print, along with the authors' closure, if any, approximately nine months after the print publication.

value of 1.5 in most concrete-related design formulas. These formulas cover failure modes with well-established physics-based models (e.g., flexural failure) and also failure modes with less understood physics and empirical models (e.g., shear resistance without shear reinforcement). To align the reliability level of different resistance formulas, standardization committees decided to adjust regression coefficients in the latter category of models as, for example, occurred with the factor $C_{Rd,c}$ in the shear resistance formula.[5] Although it may still lead to safe designs, these decisions further increase the confusion regarding the coverage of (model) uncertainties in design formulas and they hinder the understanding of the design formula for users who are not aware of the background. This, in turn, can lead to incorrect usage of the formulas and unsafe decisions.

Reliability-based calibration of structural design formulas is a more coherent and principled method to derive partial factors. It aims to ensure that reliability levels of design scenarios are as close as possible to target levels, respecting all uncertainties in the basic random variables on the action and resistance sides. Reliability-based code calibration has been performed since the 1970s[6] and individual efforts were published over the last decade in various context.[7–18] Despite these contributions, reliability-based calibration is still not generally practiced by researchers in the Eurocode standardization committees of construction material specific design codes, probably because the method is perceived as too complex.[19] On the other hand, the second-generation Eurocodes have made clear improvements in this respect. In the reliability background document for Eurocode,[20] a general guide on code calibration is given. Furthermore, the prEN 1992 Annex A provides a detailed guidance of adjustment of partial factors for materials using the so-called design value method.

This paper presents a reliability-based calibration method for design code formulas, combining the best practices from structural reliability and code calibration among others reported in the references above, and being consistent with ISO 2394[21] and Thoft-Cristensen and Baker.[22] In addition, we adopt an inverse codified design approach that allows us to consider thousands of design scenarios. Furthermore, we consider multiple actions and load combination rules, and use probabilistic models of the basic random variables that are broadly accepted in the standardization community. To lower the barrier for researchers who are not familiar with but willing to perform reliability-based calibration of design formulas, and in the spirit of open and reproducible research, the developed code is open-sourced.[23] We hope that this will stimulate and facilitate sound and transparent code development in the future.

A reliability-based calibration method neglects the direct consideration of failure consequences and costs in the partial factor calibration. These aspects can be included when using a risk-based approach, see, for example, Refs 24,25. Partial factor calibration using a risk-based approach would require a number of additional assumptions, uncertainties and unknows, for example, estimating failure consequences and interest rates for 50–100 years into the future.[25,26] In this paper, we intentionally accept the target reliabilities from Eurocode, implicitly assuming that risk optimal designs are obtained when these target reliabilities are met.

After a description of the reliability-based calibration method (Section 2), we demonstrate the method on the shear design formulas for reinforced concrete members without shear reinforcement in Eurocode 2[3] and *fib* Model Code 2010 (MC2010)[27] (Sections 3 and 4). Furthermore, the impact of several modeling assumptions on the calibrated partial factor is investigated. The outcomes are discussed in Section 5 and the main conclusions are drawn in Section 6. With some adjustments, the method can be adopted for the calibration of other design code formulas as well.

2 | RELIABILITY-BASED CALIBRATION OF DESIGN FORMULAS

2.1 | Main steps

Prior to performing a reliability-based calibration of a (set of) structural design formula(s), several important decisions have to be made[21,22]:

- First, target reliability should be determined that expresses the level of safety for which structures (or structural members) should be designed. Appropriate target reliabilities are balanced among various aspects such as failure consequences, the costs and efforts to reduce the probability of failure and the design service life. Recommendations for them are provided in Eurocode 0,[28] ISO 2394,[21] and the JCSS probabilistic model code.[29]
- Second, decisions should be made on the number of partial factors, on their association with the parameters in the design formula(s) and on which uncertainties they need to cover. The choice for the number of partial factors is a trade-off between a practical convenience on the one hand (which calls for a lower number), and economic designs for a wide range of structures on the other (which calls for a larger number).
- Third, the range of design scenarios to be covered by the design formula(s) needs to be defined. This includes

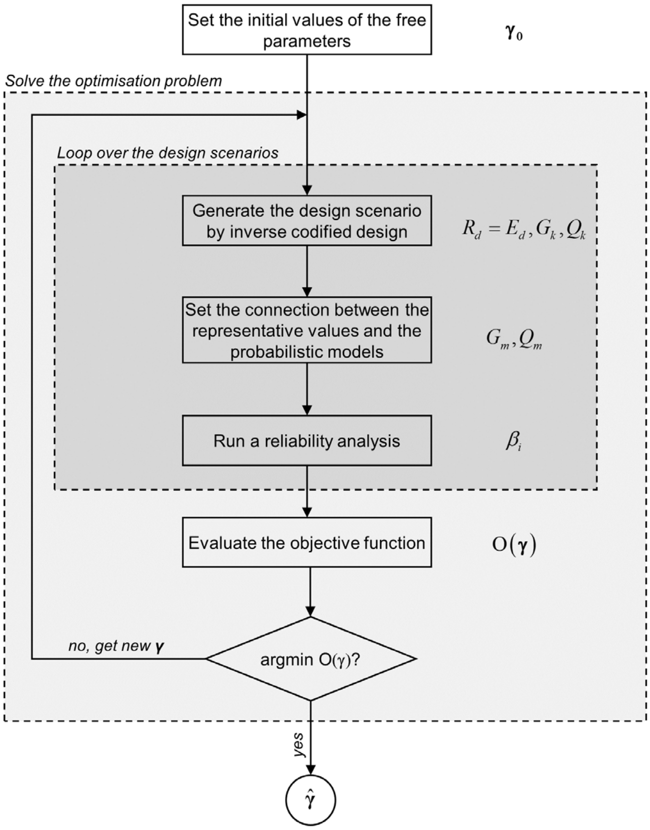

FIGURE 1 Schematic overview of the main steps in calculating partial factor(s) of a design formula

the specification of load type, load combination rules, and load ratios on the action side and the type of structures and material and geometry parameters on the resistance side. Also, the prevalence weights of these design scenarios should be determined, which are based on the frequency at which each design scenario is expected to occur in practice.

- Fourth, the limit state function(s) should be formulated and the probabilistic models and representative values of the limit state function parameters should be selected. A limit state function defines the boundary between safe and unsafe structures (or structural members), for example, the load effect (E) should not exceed the resistance (R).
- Fifth, a reliability analysis method(s) needs to be selected to compute the failure probability and reliability index of each design scenario.
- Finally, to calibrate the partial factor(s), an objective function is needed that minimizes the differences between the reliability indices of all design scenarios and the target reliability.

The above items form the ingredients of the reliability-based calibration method. The goal of the method is to find an optimum set of partial safety factors (described by a set of γ) that is optimum in various design scenarios according to the requirements identified by the code makers. From a mathematical point of view, the calibration of the design formula(s) is treated as an optimization problem:

$$\hat{\boldsymbol{\gamma}} = \text{argmin} O(\boldsymbol{\gamma}) \text{ where } O : \mathbb{R}^n \mapsto \mathbb{R}, \tag{1}$$

where O is the objective function; $\boldsymbol{\gamma}$ is the vector of free parameters (i.e., partial factors) in the design formula(s) to be calibrated; and $\hat{\boldsymbol{\gamma}}$ is the optimal (calibrated) vector of free parameters.

$\hat{\boldsymbol{\gamma}}$ is obtained by numerically solving the (nonlinear) optimization problem, starting from a first proposal solution $\boldsymbol{\gamma}_0$. Finding $\hat{\boldsymbol{\gamma}}$ can be a computationally expensive task because for each iteration one needs to solve a series of reliability analyses over all considered design scenarios. Figure 1 provides a schematic overview of the calculation steps. In subsequent sections, some of the presented steps are discussed in more detail, for example, how design scenarios are generated (Section 2.2), how the connection is made between the representative values and probabilistic models (Section 2.3) and what the optimization functions are that can be used for the calibration of partial factors (Section 2.4).

2.2 | Inverse codified design

In a conventional (semi-probabilistic) design, the design load effect E_d can be calculated given the present actions and resistance-related parameters, such as dimensions of a member, reinforcement layout or material strength, are determined such that $R_d \leq E_d$. Implementing this conventional design for a wide range of design scenarios can be difficult because it may lead to designs with unrealistic material and geometry parameters in some cases and/or one needs to implement a large number of constraints, for example, product availability and constructability rules.

To overcome this difficulty, we turn the conventional design approach around and compute the load effect (E_d) for a predefined resistance while still fulfilling the design requirement $R_d \leq E_d$. We refer to this as the *inverse codified design* approach. This approach starts with calculating the design resistance R_d given $\boldsymbol{\gamma}$ and representative values associated with a considered design scenario. The load effect is determined assuming full utilization of the design resistance:

$$R_d = E_d. \tag{2}$$

The goal of inverse codified design is to find the characteristic values of the actions contributing to E_d.

Without loss of generality, we demonstrate the approach for one design scenario that is governed by a single variable action (Q) and one permanent action (G). The design scenario is characterized by a design resistance R_d and a load effect ratio χ:

$$\chi = \frac{Q_\mathrm{k}}{Q_\mathrm{k} + G_\mathrm{k}}, \tag{3}$$

where Q_k and G_k are the characteristic values of the variable action load effect and the permanent action load effect, respectively. These are the two unknowns that we seek to find while ensuring full utilization: $R_\mathrm{d} = E_\mathrm{d}$ (G_k, Q_k) (Equation 2). Equation (3) can be rewritten as:

$$Q_k = \frac{G_k \cdot \chi}{1 - \chi}, \tag{4}$$

and this equation can be used to express E_d as the function of a single unknown: G_k, for example, using the simple (Equation 6.10 in Eurocode 0[28]) and advanced (Equation 6.10a,b in Eurocode 0[28]) load combination rules for persistent or transient design situations. Hence, Equation (2) becomes:

$$R_\mathrm{d} = E_\mathrm{d}(G_\mathrm{k}). \tag{5}$$

From this, G_k can be obtained by using a root-finding algorithm after which Q_k can be calculated with Equation (4).

In the reliability-based calibration method, the inverse codified design is applied for all design scenarios, including the ones with multiple variable actions. If multiple variable actions are present, then the procedure is essentially the same as above because each variable action ($Q_{\mathrm{k},i}$) can be expressed as the function of a fixed load ratio (χ_i) and the permanent load (Equation 4).

2.3 | Connect the semi-probabilistic format to the reliability-based format

To calculate the reliability level of a design obtained by a semi-probabilistic format, we need to connect the semi-probabilistic formulation (see Section 2.2) to the reliability analysis formulation (also called *full-probabilistic analysis*). A semi-probabilistic approach checks the compliance of a structure on the level of the design load and resistance, where a full-probabilistic approach compares the calculated reliability index of a structure with a standardized target reliability value. The components of the connection between the semi-probabilistic formulation and the reliability analysis formulation are listed below (details are given for the considered cases in Section 4):

a. Performance function (g): the limit state expressed in the semi-probabilistic limit state function (Equation 2) is used to formulate the performance function $g = R - E$, where random variables are involved in contrast to design and representative values.
b. Load combination: the same load combination rules of Eurocode 0[28] used in the semi-probabilistic format are also used in the reliability analysis. The only difference is that in the latter, random variables are used instead of design and representative values.
c. Probabilistic models: the uncertainty in each selected parameter is represented using a univariate distribution function. Each representative value of the semi-probabilistic format needs to be connected to a corresponding distribution. The distributions are taken from the literature up to the mean value of the distributions. The mean (x_mean) and representative values (x_repr) are then connected through a function:

$$x_\mathrm{mean} = h_X(x_\mathrm{repr}). \tag{6}$$

h_X is taken from the literature or in the absence of a reference and based on our judgment. In this paper, we use two classes of h_X function:

1. Assumes x_repr to be a fractile of the distribution of X corresponding to a fixed probability of non-exceedance (P_repr):

$$F_X(x_\mathrm{repr}, x_\mathrm{mean}) = P_\mathrm{repr}, \tag{7}$$

where F_X is the cumulative distribution function of X. This is often used for material properties and actions.[13]

2. Assumes x_mean to be shifted with respect of x_repr by a fixed offset (x_shift):

$$x_\mathrm{mean} = x_\mathrm{repr} + x_\mathrm{shift}. \tag{8}$$

This format is often recommended to connect the representative (nominal) value of geometrical parameters with its corresponding mean.[30]

Having (a)–(c) defined uniquely connects the semi-probabilistic formulation to the reliability analysis formulation and, in turn, allows computation of the reliability level using standard reliability methods.

2.4 | Objective function

The objective function (Equation 1) measures the discrepancy between the design scenario reliability levels (β_i) and the target reliability (β_target). We use the following

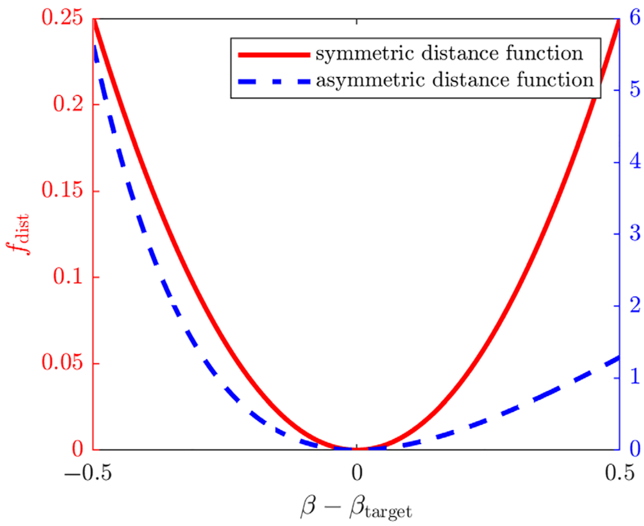

FIGURE 2 Comparison of the symmetric squared (Equation 10) and asymmetric Hansen and Sørensen[31] (Equation 11) distance functions

weighted integral to quantify the total discrepancy across design scenarios:

$$O(\gamma) = \int_{D_{p_{\text{DS}}}} w(p_{\text{DS}}) \cdot f_{\text{dist}}\left(\gamma,\ \beta_{\text{target}},\ p_{\text{DS}}\right) \cdot \mathrm{d}p_{\text{DS}}, \tag{9}$$

where w is the design scenario prevalence weight function; p_{DS} is the vector of design scenario characterizing parameters (Section 4.1.1); $D_{p_{\text{DS}}}$ is the domain of p_{DS} (Section 4.1.1); f_{dist} is the distance function.

For the distance function, a simple symmetric function is suggested by ISO 2394[21]:

$$f_{\text{dist}} = \left(\beta(\gamma,\ p_{\text{DS}}) - \beta_{\text{target}}\right)^2. \tag{10}$$

An asymmetric objective function (in terms of reliability index) might be more justified because the consequences of structural failure are often much larger than the cost of overdesign (the total cost functions are not symmetric either). One popular, asymmetric distance function is proposed by Hansen and Sørensen.[31] The formula penalizes more heavily negative deviations from the target reliability (Figure 2):

$$f_{\text{dist}} = 4.35 \cdot \left(\beta(\gamma,\ p_{\text{DS}}) - \beta_{\text{target}}\right) + \exp\left[-4.35 \cdot \left(\beta(\gamma,\ p_{\text{DS}}) - \beta_{\text{target}}\right)\right] - 1. \tag{11}$$

We consider both objective functions in this paper.

3 | DESCRIPTION OF THE SHEAR RESISTANCE MODEL

3.1 | EN1992-1-1 shear design formula for RC members without shear reinforcement

We demonstrate the presented reliability-based calibration method on the Eurocode 2[3] shear design formula for RC members without shear reinforcement (Equations 6.2a and 6.2b). For convenience, we present the design equations with a notation that is based on Eurocode 2 but adjusted to the topic and goal of this paper:

$$V_{\text{Rd,c}} = \frac{\theta_{\text{R,repr}}}{\gamma_{\text{R}}} \cdot \max\left(V_{\text{Rk,c,base}},\ V_{\text{Rk,c,min}}\right), \tag{12}$$

where $\theta_{\text{R,repr}}$ is the representative value of resistance model uncertainty (to get the codified Eurocode 2 formula it should take value 1.0); and γ_{R} is the resistance related partial factor (to get the codified Eurocode 2 formula it should take the value of the concrete material partial factor (γ_{C})).

The base shear resistance is defined as:

$$V_{\text{Rk,c,base}} = \left[C_{\text{R,c,base}} \cdot k \cdot \left(100 \cdot \rho_{\text{l}} \cdot f_{\text{ck}}\right)^{1/3} + k_1 \cdot \sigma_{\text{cp}}\right] \cdot b_{\text{w}} \cdot d, \tag{13}$$

where $V_{\text{Rk,c,base}}$ is the base shear resistance in N; f_{ck} is the concrete characteristic compressive strength in MPa; k is the size effect factor; $C_{\text{R,c,base}}$ is a coefficient with recommended value of 0.18 (= 0.12 · 1.5); ρ_{l} is the longitudinal reinforcement ratio; b_{w} is the smallest width of the cross section in the tensile area in mm; d is the effective depth in mm; and σ_{cp} is the mean compressive stress in the cross section in MPa.

The lower bound on the shear resistance (in N) is defined as:

$$V_{\text{Rk,c,min}} = \left(\nu_{\min} + k_1 \cdot \sigma_{\text{cp}}\right) \cdot b_{\text{w}} \cdot d, \tag{14}$$

where $\nu_{\min}$ is the minimum shear stress at which the shear failure may occur.

$\nu_{\min}$ can be computed as:

$$\nu_{\min} = C_{\text{R,c,min}} \cdot k^{3/2} \cdot f_{\text{ck}}^{1/2}, \tag{15}$$

where $C_{\text{R,c,min}}$ is a coefficient with recommended value of 0.0525 (= 0.035 · 1.5).

Although less relevant in this context, it is noteworthy that the design formula is dimensionally inconsistent.

In this paper, we focus on calibrating the formula for flexural shear failure of reinforced concrete members without prestressing; hence, we make the following simplifications:

- We neglect the effect of prestressing, that is, $\sigma_{cp} = 0$ MPa.
- We also neglect the Eurocode load reduction factor for a load that is within *2d* from the support.

3.2 | *fib* MC2010 level II shear design formula for RC members without shear reinforcement

The reliability-based calibration method is also demonstrated on the *fib* MC2010[27] shear design formula for RC members without shear reinforcement (Equation 7.3–17 in the Model Code; abbreviated as MC2010 in this paper). After similar adjustments as discussed in Section 3.1, the design shear resistance can be formulated as:

$$V_{\mathrm{Rd,c}} = \frac{\theta_{\mathrm{R,repr}}}{\gamma_{\mathrm{R}}} \cdot k_v \cdot \sqrt{f_{\mathrm{ck}}} \cdot z \cdot b_{\mathrm{w}}, \quad (16)$$

where $\theta_{\mathrm{R,repr}}$ is the representative value of resistance model uncertainty (to get the codified MC2010 formula it should take value 1.0); γ_{R} is the resistance-related partial factor (to get the codified MC2010 formula it should take the value of the concrete material partial factor (γ_{C})); k_{v} is a factor that accounts for longitudinal strain and size effect; and z is the internal lever arm in mm.

For the level II approximation, k_{v} can be computed as:

$$k_v = \frac{0.4}{1 + 1500 \cdot \varepsilon_x} \cdot \frac{1300}{1000 + k_{dg} \cdot z}, \quad (17)$$

where ε_{x} is the longitudinal strain at the mid-depth of the effective shear depth; and k_{dg} is a factor that accounts for the maximum aggregate size d_{g}.

The parameters ε_{x} and k_{dg} are calculated using Equations (7.3–16) and (7.3–20) in MC2010,[27] respectively. In anticipation on the explanation of the design scenarios in Section 4.1.1, it is important to mention that ε_{x} among others depends on the maximum bending moment of the shear span and, therefore, relates to the shear span-to-depth ratio (a/d) when the member is loaded by a point load and k_{dg} needs the maximum aggregate size d_{g} as input.

4 | CALIBRATION OF THE RESISTANCE PARTIAL FACTOR

4.1 | Problem formulation

In the calibration, we focus on the resistance side and determine the partial factor γ_{R} for the shear resistance models presented in the previous chapter. This section presents the input for the considered cases, which involve a reference case (R1) and its various alternations.

For the reference case, we consider a target reliability (β_{target}) of 4.7 and a 1-year reference period, which corresponds to reliability class RC2 according to Eurocode 0.[28] According to Eurocode 0,[28] this target reliability can be deemed to be equivalent to a target reliability of 3.8 over a 50-year reference period.

4.1.1 | Design scenarios

The discussed shear resistance models are usually applied in the context of the design of new concrete slabs in buildings and infrastructure (such as concrete floors, slab bridges, and tunnel roofs). For the calibration, we define design scenarios to cover the application domain. A design scenario is a structural member design and its corresponding loading along with sufficient information to carry out a semi-probabilistic assessment (or design). To define a design scenario, we use a number of characterizing parameters (p_{DS}) essential in defining the member design and loading conditions. The parameters are discretized over a selected domain, that is, each discretized parameter forms a vector.

Table 1 gives an overview of the design scenarios and their corresponding parameter ranges. The second and the third columns in Table 1 show typical ranges of the characterizing parameters for building and bridge applications. The last column shows the discretized parameters based on these ranges, which we use to define the design scenarios.

The following actions are considered in the design scenarios: permanent load (including the self-weight of the structure and other permanent load), traffic load, imposed load, snow load, and wind load. We use the simple (Equation 6.10 in Eurocode 0[28]) and advanced (Equation 6.10a,b in Eurocode 0[28]) load combination rules for persistent or transient design situations. We combine the permanent load with one variable action (traffic) or two variable actions (snow-wind, snow-imposed, or wind-imposed). The ratio between the permanent and variable loads is varied through the load ratios χ_1 and χ_2 (see Section 2.2). By definition, χ_1 and χ_2 can take values between 0 and 1. We discretize them in a range from 0.1 to 0.9 with a step size of 0.1.

The design scenarios ($D_{p_{\mathrm{DS}}}$) are formed as the Cartesian product of the parameter vectors in Table 1: $D_{p_{\mathrm{DS}}} = p_{\mathrm{DS},1} \times p_{\mathrm{DS},2} \ldots \times p_{\mathrm{DS},n}$. The design scenarios differ in prevalence, which is taken into account by a weight function (w) in the objective function of Equation (9). We compute the weights based on the following assumptions:

- The load ratio (χ) dependent part of the weight function is based on (χ_w, w_χ) pairs as reported in Ellingwood et al.[6] (for convenience, they are reproduced in Table 2). We use the same (χ_w, w_χ) pairs for all variable actions. Our weight function is the linear interpolation between the pairs. For cases with two variable actions, the interpolation grid is formed as $\chi_w \otimes \chi_w$ and $w_\chi \otimes w_\chi$ for load ratios and weights, respectively. The load ratio-dependent part of the weight function is assumed to be independent of the rest (not χ) of the p_{DS} parameters.
- All design scenarios formed by the rest of the p_{DS} parameters ($D_{p_{\mathrm{DS}}/\chi}$) are assumed to have the same weight: 1.0. This is assumed to be valid even if the number of components in one or more $p_{\mathrm{DS},i}$ vectors is changed.
- All four considered load combinations are assumed to have the same weight: 1.0.

Combined with Equation (9), the above assumptions can be mathematically expressed as:

$$O(\gamma) = \sum_{\text{load combinations}} \sum\nolimits_{D_{p_{\mathrm{DS}}/\chi}} \sum\nolimits_{D_\chi} w(\chi) \cdot f_{\mathrm{dist}}\left(\gamma, \beta_{\mathrm{target}}, p_{\mathrm{DS}}\right), \tag{18}$$

where χ is χ_1 for load combinations with a single variable action and (χ_1, χ_2) for two variable actions; $D\chi$ is the domain of χ; and $D_{p_{\mathrm{DS}}/\chi}$ are design scenarios formed by the Cartesian product of p_{DS} without χ.

TABLE 1 Considered application domains and the discrete set of characterizing parameters for the calculated design scenarios

	Concrete floor slabs	Concrete slab bridges	Characterizing design parameters
d_{nom} [mm]	120–300	300–800	[150, 300, 450, 600, 750]
f_{ck} [MPa]	20–40	30–70	[20, 40, 60, 80]
$\rho_{\mathrm{l,nom}}$ [%]	0.4–0.8	0.8–1.2	[0.5, 1.0, 1.5]
d_{g} [mm][a]	4–16	8–24	[8, 16, 24]
a/d [−][a]	2.0–4.0		[2.0, 3.0, 4.0]
χ_1 [−]	0.1–0.9		[0.1:0.1:0.9]
χ_2 [−]	0.1–0.9		[0.1:0.1:0.9]

[a]Only relevant for the *fib* Model Code 2010 shear design formula.

If one has more information about design scenario prevalence, then it is recommended to be accounted for in the weight function. We deem the above assumptions to be acceptable for the purpose of this paper.

4.1.2 | Performance function and probabilistic models

As mentioned in Section 2.3, the formulation of the performance function emerges from the semi-probabilistic limit state function (Equation 2) albeit the parameters are now treated as random variables:

$$g = \theta_{\mathrm{R}} \cdot V_{\mathrm{R}} - \theta_{\mathrm{E}} \cdot V_{\mathrm{E}}, \tag{19}$$

where V_{R} is the shear resistance; θ_{R} is the resistance model uncertainty; V_{E} is the cross-sectional shear force (load effect); and θ_{E} is the load effect model uncertainty.

V_{R} is obtained from one of the shear resistance models described in Section 3. The shear resistance model parameters d, f_{c}, A_{sl}, and b_{w} are considered as random variables. Table 3 summarizes their representative values and probabilistic models. The probabilistic model of θ_{R} is inferred from the shear database reported by the ACI-DAfStb joint committee.[34] The details of this inference are provided in Annex A and the main results are summarized in Tables A1 and A2. In case of the Eurocode 2 formula, the maximum likelihood estimates of the log-normal distribution parameters are *mean* = 1.137, *cv* = 0.2378. The coefficient of determination (R^2 value) of the mean prediction of the calibrated model is 0.79. For the MC2010 formula, the estimated log-normal distribution parameters are *mean* = 1.34, *cv* = 0.19, and the R^2 value is 0.92. The computed coefficients of variations are in reasonable agreement with Marí et al.,[35] where values of 0.21 and 0.19 were obtained for the Eurocode 2 and MC2010 formulas, respectively. Sigrist et al.[36] and Sykora[37] obtained smaller coefficients of variations for the MC2010 formula, showing values of 0.11 and 0.14, respectively. Marí et al.[38] computed a coefficient of variation of 0.28 for the Eurocode 2 formula, which is a bit larger than we obtained. The observed differences likely stem from the different choices regarding the used shear database and the assumed underlying distribution type.

TABLE 2 Assumed prevalence weights with respect to the load ratios based on Ellingwood et al.[6]

Load ratio, χ_w	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Weight, w_χ	0.00	0.26	0.93	1.0	0.77	0.26	0.08	0.00	0.00

Note: To ease the interpretation, the weights are scaled to reach 1.0 with the largest weight.

TABLE 3 Representative values and probabilistic models of resistance parameters

Symbol	Description	Distribution	Mean	CV	P_r	γ	x_r	Reference
d [mm]	Effective depth	Normal	$x_r + 10$	$10/x_{mean}$			d_{nom}	[32]
$f_{c[}$	Concrete compressive strength	Log-normal	$f_{c,mean}$	0.15	0.05	1.5	$f_{ck} = F_X^{-1}(P_r)$	[6,33]
A_{sl}	Longitudinal reinforcement area	Normal	x_r	0.02			$\rho_{l,nom} \cdot b_{w,nom} \cdot d_{nom}$	[32]
b_w	Width of cross section	Normal	x_r	$5/x_{mean}$			1000	[32]
θ_R	Resistance model uncertainty[a]	Log-normal	Table A1	Table A1			Table A2	See Annex A

Note: Subscript *r*: representative value. Subscript *nom*: nominal value (from design specifications and/or technical drawings).
[a]Depends on the selected resistance model, see Annex A.

V_E is obtained from the same load combination rules used in the semi-probabilistic format (see Section 4.1.1). This means that the reliability problem is formulated as a time-independent one, that is, the simultaneous occurrence of time-variant action values is approximated by using the standardized semi-probabilistic combination factors. This approximation (semi-probabilistic load combination factors) is sometimes used in the structural reliability literature (e.g., Refs.[16,33]) and expected to lead to design values within 5% of those obtained with probabilistic (random process-based) load combinations.[39,40] An important limitation of our study is that in the absence of annual load combination factors in the Eurocode, we use the "general" load combination factors, which are presumably for a 50-year reference period. Through the standardized load combination factors, we indirectly use the fixed load sensitivity factor of the Eurocode. This limitation could be resolved by using a probabilistic load combination approach, such as Turkstra's rule or the Ferry Borges Castanheta method.[32] Baravalle[41] and Beck and Souza Jr.[11] demonstrate the probabilistic load combination approach in the context of calibrating the Eurocode and Brazilian design code formulas, respectively.

Each contribution of the permanent and variable loads to V_E in the load combination rule is obtained from multiplication of the load effect with its corresponding load model uncertainty, for example, $\theta_G \cdot V_G$, $\theta_W \cdot V_W$, etc. The representative values and probabilistic models of all considered actions and load (effect) model uncertainties are reported in Table 4. The same table also presents the adopted load combination factors.

All random variables in the performance function are assumed to be mutually independent.

4.2 | Analyses and results

This section presents the main results of the reliability-based calibrations. An overview of the calibrations and the obtained optimal values for the partial factor γ_R is provided in Table 5. The reliability-based calibration method is implemented in MATLAB and the code is made freely available.[23] The reliability calculations are performed using the First-Order Reliability Method (FORM) implemented in the structural reliability software FERUM.[50] In case, the design scenarios are designed with Eurocode 2 using Equation (12), the reliability of each design is computed as a parallel system, where the two components of the parallel system are formed by $V_{R,c,base}$ and $V_{R,c,min}$, respectively. The system problem is treated by solving two (component level) reliability problems: one with $V_{R,c,base}$ in the performance function of Equation (19) and another with $V_{R,c,min}$. The reliability of the parallel system problem is calculated according to Ditlevsen,[51] for which the correlation between the components is estimated based on the component level sensitivity factors. In line with Equation (12), the same resistance model uncertainty random variable is used for the two components (failure modes), or equivalently, resistance model uncertainties are assumed to be fully correlated. The load effect model uncertainty is modeled the same way.

4.2.1 | Reference cases (R1)

In this section, we first present our reference cases in the calibration, in which the reference calibration R1-1 considers the full set of design scenarios for the Eurocode 2 shear design formula and results in an optimal γ_R of 1.53. This value is close to $\gamma_C = 1.5$[3]; however, it should be noted that we use $\theta_{R,repr} \approx 0.85$ (Table A2) while the equivalent of the codified Eurocode 2 formula has $\theta_{R,repr} = 1.0$.

Figure 3 shows a subset of the reliability indices (β) of the design scenarios designed with the calibrated γ_R. The subset corresponds to fixed design scenario defining resistance parameters f_{ck}, d_{nom}, $\rho_{l,nom}$, d_g, and a/d, and varying load ratios χ_1 and χ_2. Henceforth, we refer to this subset as the *illustrative design scenario subset*. Each

TABLE 4 Representative values and probabilistic models of action and load combination parameters

Symbol	Description	Distribution	Mean	CV	P_r	γ	x_r	Reference
Permanent load								
V_G	Internal force from load	Normal	x_r	0.10		1.35	$V_{G,nom}$	[16]
θ_G	Model uncertainty	Log-normal	1.0	0.05			x_{mean}	Expert judgment
ξ	Reduction factor	Deterministic					0.85	[28]
Snow								
V_{S1}	$= s_1 \cdot c_1$, internal force from load	Non-param.[a]	1.0[a]	0.625[a]	$F_X(x_r) = 0.976$[a]	1.5	$s_{1,r} \cdot c_{1,r}$	
s_1	Ground snow load	Gumbel	1.0	0.60	0.98		$F_X^{-1}(P_r)$	[42]
c_1	Ground-to-roof conversion factor	Normal	1.0	0.15			x_{mean}	[43]
θ_S	Model uncertainty	Log-normal	1.0	0.10			x_{mean}	[33]
$\psi_{0,S}$	Load combination factor	Deterministic					0.5	<1000 m[44]
Wind								
V_{W1}	$= q_{ref,1} \cdot c_{e,1} \cdot c_{p,1} \cdot c_{d,1}$, internal force from load	Non-param.[a]	1.0[a]	0.408[a]	$F_X(x_r) = 0.990$[a]	1.5	$q_{ref,1,r} \cdot c_{e,1,r} \cdot c_{p,1,r} \cdot c_{d,1,r}$	
$q_{ref,1}$	Wind pressure	Gumbel	1.0	0.27	0.98		$F_X^{-1}(P_r)$	[33]
$c_{e,1}$	Exposure factor	Log-normal	1.0	0.15	0.94		$F_X^{-1}(P_r)$	[32,33]
$c_{p,1}$	Pressure coefficient	Gumbel	1.0	0.20	0.76		$F_X^{-1}(P_r)$	[33]
$c_{d,1}$	Dynamic amplification factor	Log-normal	1.0	0.15	0.50		$F_X^{-1}(P_r)$	[32]
θ_W	Model uncertainty	Log-normal	1.0	0.10			x_{mean}	[33]
$\psi_{0,W}$	Load combination factor	Deterministic					0.6	[45]
Traffic								
V_{T1}	Internal force from load	Gumbel	1.0	0.075	$1–2.13 \times 10^{-5}$	1.35	$F_X^{-1}(P_r)$	Expert judgment[46,47]
θ_T	Model uncertainty	Normal	1.0	0.142			x_{mean}	Expert judgment[47]
$\psi_{0,T}$	Load combination factor	Deterministic					0.8	[48]
Imposed								
V_{I1}	Internal force from load	Gumbel	1.0	0.53	0.98	1.5	$F_X^{-1}(P_r)$	[41]
θ_I	Model uncertainty	Log-normal	1.0	0.10			x_{mean}	[33,49]
$\psi_{0,I}$	Load combination factor	Deterministic					0.7	[28]
θ_E	Load effect model uncertainty	Log-normal	1.0	0.10			x_{mean}	[32], [b]

Note: The parameters of all time-dependent random variables are provided for a 1-year reference period. Subscript *r*: representative value. Subscript *nom*: nominal value (from design specifications and/or technical drawings).
[a]Derived from the probabilistic models and representative values of its components (see the Description and x_r columns).
[b]Using the recommended model for shear forces in frames.

subplot of Figure 3 belongs to a different combination of variable actions. Most reliability indices of the prevalent design scenarios are centered on β_{target} while design scenarios with low or zero prevalence weights are farther away from it.

The left subplots of Figures 4 and 5 show the squared FORM sensitivity factors (α^2) belonging to the random variables in the reliability analyses for the traffic load and wind-imposed load combinations of the illustrative design scenario subset. From these plots, we can identify the dominant random variables in a particular reliability analysis and in the illustrative design scenario subset. It is salient that the resistance model uncertainty θ_R is one

TABLE 5 Summary of the calibrated partial factors for the considered cases

ID	$\widehat{\gamma}_R$	Resistance model	Load combina-tion rule[a]	Load combi-nations	β_{target}	K_{FI}	Objective[b]	Design scenarios	Remarks[c]
R1-1	1.526	EN1992-1-1 full	Simple	All*	4.7	1.0	Symm.	All[d]	Reference calibration.
R1-2	1.526	EN1992-1-1 full	Simple	All*	4.7	1.0	Symm.	Reduced[e]	
R2-1	1.473	EN1992-1-1 base	Simple	All*	4.7	1.0	Symm.	Reduced[e]	
R2-2	1.804	EN1992-1-1 full	Simple	All*	4.7	1.0	Symm.	Reduced[e]	$\theta_{R,repr} = 1.0$, hence corresponds to current EN1992-1-1. $\widehat{\gamma}_R = 1.526\cdot(1/0.846) = 1.804$.
R2-3	1.503	MC2010 level II	Simple	All*	4.7	1.0	Symm.	All[d]	
R2-4	1.364	MC2010 level II	Simple	All*	4.7	1.0	Symm.	All with reduced f_{ck} range	Only f_{ck} values of 20 and 40 MPa
R2-5	1.363	MC2010 level II	Simple	All*	4.7	1.0	Symm.	Reduced[e]	
R2-6	1.367	EN1992-1-1 full	Simple	All*	4.7	1.0	Symm.	Reduced[e]	[e] With f_{ck}/γ_c in Equation (13) instead of f_{ck}. $\widehat{\gamma}_R = 1.526\cdot(1/1.39)^{(1/3)} = 1.367$.
R3-1	1.512	EN1992-1-1 full	Advanced	All*	4.7	1.0	Symm.	Reduced[e]	
R3-2	1.594	EN1992-1-1 full	Simple	Traffic	4.7	1.0	Symm.	Reduced[e]	
R3-3	1.424	EN1992-1-1 full	Simple	All* but traffic	4.7	1.0	Symm.	Reduced[e]	
R3-4	1.417	EN1992-1-1 full	Simple	All*	4.7	1.0	Symm.	Reduced[e]	Same as R1-1 but without random variable θ_E.
R4-1	1.585	EN1992-1-1 full	Simple	All*	4.7	1.0	Asymm.	Reduced[e]	
R4-2	1.616	EN1992-1-1 full	Simple	All*	5.2	1.1	Symm.	Reduced[e]	RC3 reliability class.
R4-3	1.457	EN1992-1-1 full	Simple	All*	4.2	0.9	Symm.	Reduced[e]	RC1 reliability class.
R4-4	1.450	EN1992-1-1 full	Simple	All*	4.7	1/0.95	Symm.	Reduced[e]	Same as R1-1 but with 95% utilization ratio in the semi-probabilistic design. $\widehat{\gamma}_R = 1.526\cdot0.95 = 1.450$.

*Traffic, snow-wind, snow-imposed, and wind-imposed.

[a]Simple: Equations (6–10) of Eurocode 0[28]; advanced: Equations (6–10a-b) of Eurocode 0.[28]

[b]Symm.: Equation (9) and Equation (10); asymm.: Equation (9) and Equation (11).

[c]Unless otherwise stated in the Remarks column, the value of $\theta_{R,repr}$ is taken from Table A1 with matching resistance model.

[d]Table 1.

[e]Reduced design scenario space: $d_{nom} = 300$ mm; $f_{ck} = 40$ MPa; $\rho_{lnom} = 1\%$, $dg = 16$ mm, $a/d = 3$ instead of the corresponding ranges in Table 1.

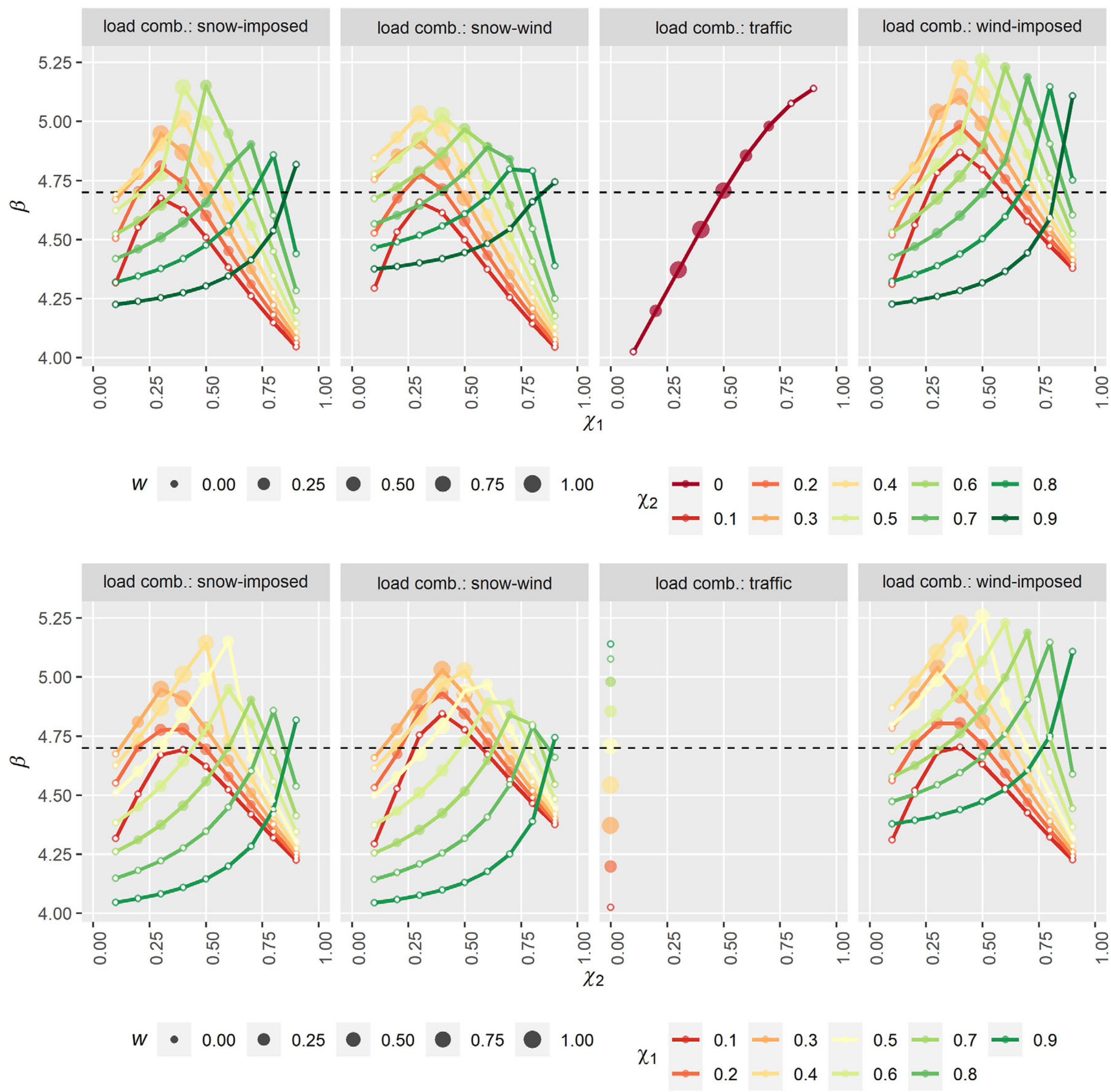

FIGURE 3 Reliability indices (β) obtained using the calibrated partial factor γ_R for case R1-1 for the design scenarios with $f_{ck} = 40$ MPa, $d_{nom} = 300$ mm, $\rho_{l,nom} = 1\%$ and varying load ratios χ_1 (top) and χ_2 (bottom). The horizontal dashed line indicates β_{target}. The size of the circles corresponds to the prevalence weight (w) of the design scenario. An empty circle highlights a design scenario with zero prevalence weight.

of the most dominant random variables. For traffic load design scenarios, its α^2 value is about 0.70 irrespective of the χ_1 value. For the two variable loads design scenarios, θ_R is typically dominant when χ_1 and χ_2 are relatively low, that is, when the permanent load is governing the cross-sectional shear force; however, this dominance diminishes when χ_1 and χ_2 increase and ${\alpha_{VQ1}}^2$ and/or ${\alpha_{VQ2}}^2$ become(s) larger. The above dominance of θ_R and its dependence on χ also persist across all design scenarios (left subplot of Figure 6). The α^2 values of the other resistance-related random variables are relatively small in all the design scenarios, indicating that their impact on the calibrated partial factor is limited.

The right subplots of Figures 4 and 5 present the α^2 plots for the load effect (E) and resistance (R), which combine the α^2 of all load effect- and resistance-related random variables, respectively. These plots show that the assumptions of Eurocode 0[48] on $\alpha_R = 0.80$ and $\alpha_E = 0.70$ are greatly inaccurate for most design scenarios, particularly for the two variable loads design scenarios with χ values larger than 0.5. In general, this observation holds true across the majority of the design scenarios as illustrated by the median α^2 values of R (right subplot of Figure 6).

The β and α^2 plots for design scenarios with other values for f_{ck}, d_{nom}, and $\rho_{l,nom}$ are very similar to

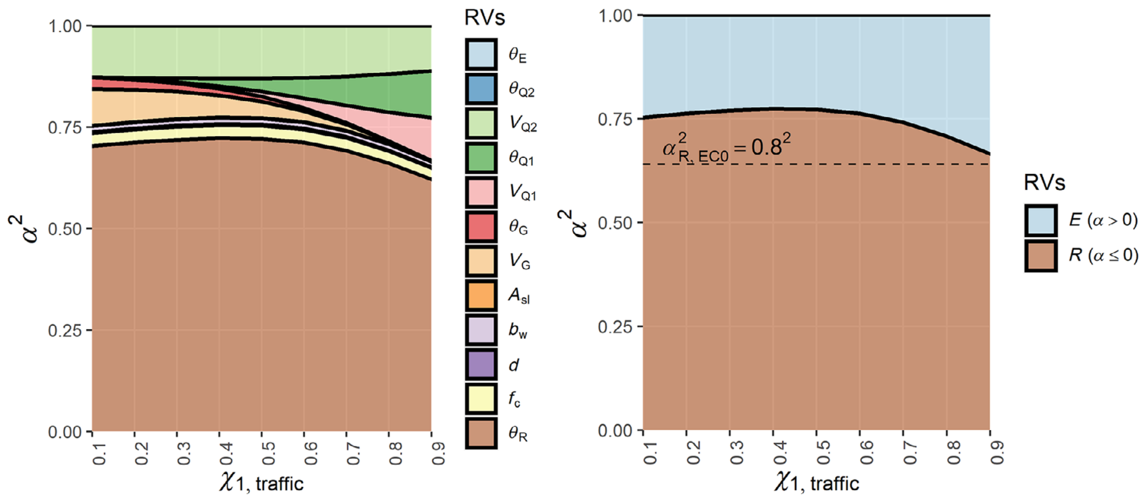

FIGURE 4 Squared sensitivity factor (α^2) plots belonging to the random variables in the reliability analyses presented in Figure 3 for the design scenarios with traffic load, $f_{ck} = 40$ MPa, $d_{nom} = 300$ mm, $\rho_{l,nom} = 1\%$ and a varying load ratio χ_1. Left: α^2 of all random variables. Right: α^2 of E and R that combines the load effect- and resistance-related random variables, respectively. The horizontal dashed line in the right plot indicates $\alpha_R^2 = 0.8^2$, which is commonly assumed for R, for example, in Eurocode.

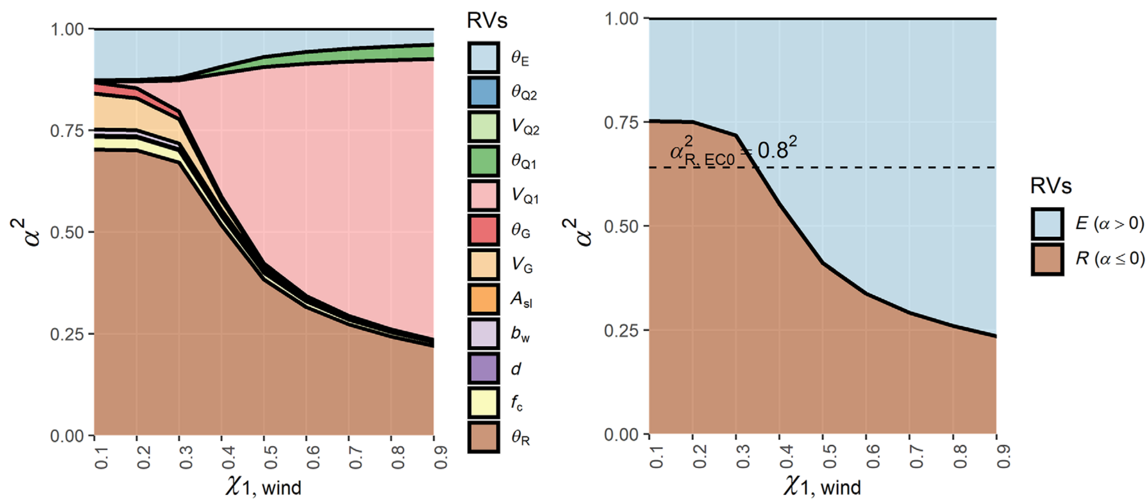

FIGURE 5 Squared sensitivity factor (α^2) plots belonging to the random variables in the reliability analyses presented in Figure 3 for the design scenarios with wind-imposed loads $f_{ck} = 40$ MPa, $d_{nom} = 300$ mm, $\rho_{l,nom} = 1\%$ and a varying load ratio χ_1. Left: α^2 of all random variables. Right: α^2 of E and R that combines the load effect- and resistance-related random variables, respectively. The horizontal dashed line in the right plot indicates $\alpha_R^2 = 0.8^2$, which is commonly assumed for R, for example, in Eurocode

those in Figures 3–5. For these plots, the reader is referred to Rózsás et al.[52]

In calibration case R1-2, we investigate the impact of the discretization of the design scenario defining resistance parameters by taking fixed values for f_{ck} (40 MPa), d_{nom} (300 mm), and $\rho_{l,nom}$ (1%) in the semi-probabilistic designs. The resulting optimal γ_R of 1.53 is practically the same as for case R1-1 (see Table 5). Compared to R1-1, in R1-2 the number of design scenarios decreases from 15,120 to 252 and the wall clock time for the calibration decreases from about an hour to less than a minute. For convenience, the calibration cases presented in the following subsections use the reduced set of design scenarios taken from R1-2.

4.2.2 | Resistance-side variations (R2)

In this section, we present and discuss results related to resistance-side variations, such as different shear resistance models, the number of partial factors, and their association with resistance model parameters. The related cases are identified by IDs of format R2-x in Table 5.

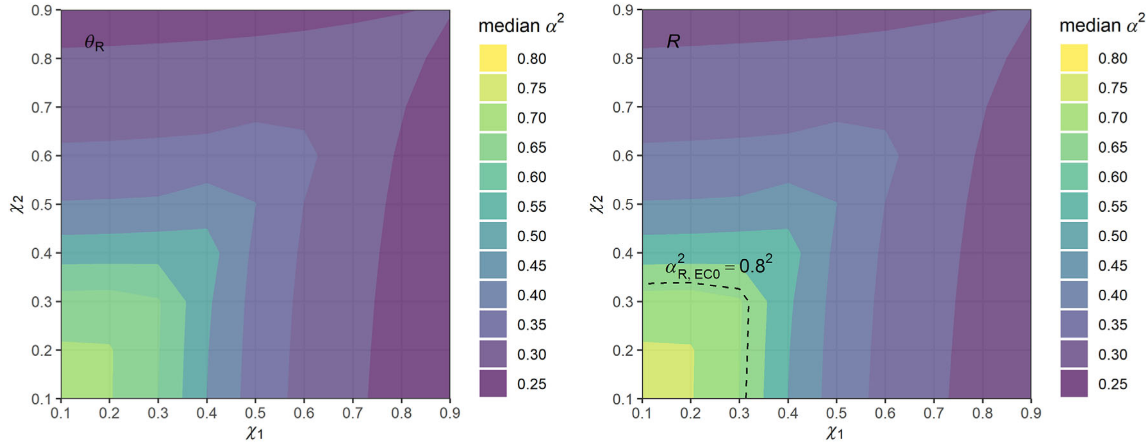

FIGURE 6 Median α^2 values over all R-1-1 design scenarios with two variable actions. Left: For random variable θ_R. Right: For R that sums the α^2 values of all resistance-related random variables ($\alpha < 0$). The dashed line is the $\alpha^2_R = 0.8^2$ isoline commonly assumed for R, for example, in Eurocode.

Case R2-1 calibrates only the base formula (Equation 13) of the Eurocode 2 shear resistance model. The impact of this variation on the optimal γ_R is small, leading to the value of 1.47, which is less than 5% smaller than that of the reference case (R1-2). This is not surprising because the estimated distribution parameters of θ_R for the base formula are similar to those of the full formula (see Table A1).

In case R2-2, we set $\theta_{R,repr}$ in Equation (12) to 1.0, instead of using the calibrated value of 0.85 from Table A2. This is equivalent to the currently codified Eurocode 2 shear formula with the difference that we use the symbol γ_R instead of γ_C. The calibration leads to an optimal value for γ_R of 1.8, which is, surprisingly, much larger than the γ_C of 1.5 in the Eurocode. We cannot find a satisfactory explanation for this because we have insufficient information about how the Eurocode 2 shear formula was conceived and particularly how it covers the resistance model uncertainty. If the experiments in the ACI-DAfStb shear database are representative of the application domain of the Eurocode 2 shear formula, then our results indicate that on average structures designed using this shear formula are considerably below the standardized target reliability. Note that the calibrated shear formulas of R2-2 and the reference case R1-2 are the same: the $\theta_{R,repr}/\gamma_R$ ratio is the same ($\approx$0.55) in both formulas. This happens because the two cases only differ in a single multiplication coefficient in the semi-probabilistic format.

Cases R2-3, R2-4, and R2-5 are used to investigate the impact of using a different resistance model, namely, the MC2010 model. R2-3 works with a full set of design scenarios (now including d_{dg} and a/d as well, see Table 1). Compared to R2-3, R2-4 has a smaller number of design scenarios because it considers only 20 and 40 MPa for f_{ck}. R2-5 uses the same design scenarios as R1-2. There is a considerable ($\sim$10%) difference between the optimal partial factor of R2-3 and R2-4 although they only differ in design scenarios. This is due to how the shear resistance formula depends on f_{ck}: at $f_{ck} = 70$ MPa there is an abrupt drop in the predicted resistance (through k_{dg}, Equation 7.3–20 in fib[27]). While the semi-probabilistic format predicts considerably different resistances when f_{ck} is 69.9 MPa and f_{ck} is 70.1 MPa, the reliability levels of the corresponding designs are practically the same. In the inverse design this materializes in substantially different reliability levels, which in turn leads to an increased optimal partial factor to accommodate this situation. This is a flaw in the resistance formula and highlights a possible adverse consequence of using physically unrealistic formulas. The problem occurs only in the vicinity of the 70 MPa transition and is illustrated for some selected design scenarios in Figure 7. Cases R2-4 and R2-5 are defined to avoid this problem, that is, the f_{ck} values of 60 and 80 MPa are not included. Like in the reference cases, going from R2-4 to R2-5 by eliminating the discretization of the design scenario defining resistance parameters hardly affects γ_R. Compared to R2-4, in R2-5 the number of design scenarios decreases from 68,040 to 252 and the wall clock time for the calibration decreases from a few hours to less than a minute.

The coefficients of determination (R^2) presented in Section 4.1.2 and Table A1 show that the MC2010 model has a better agreement with the experimental results than the Eurocode 2 model. As a more recently developed shear design formula, it is considered to be more accurate than Eurocode 2 and, therefore, its estimated coefficient of variation for θ_R is smaller. This has a considerable impact on the optimal γ_R, a reduction of 0.16 (i.e., about 12%) when comparing R2-4 and R2-5 to the

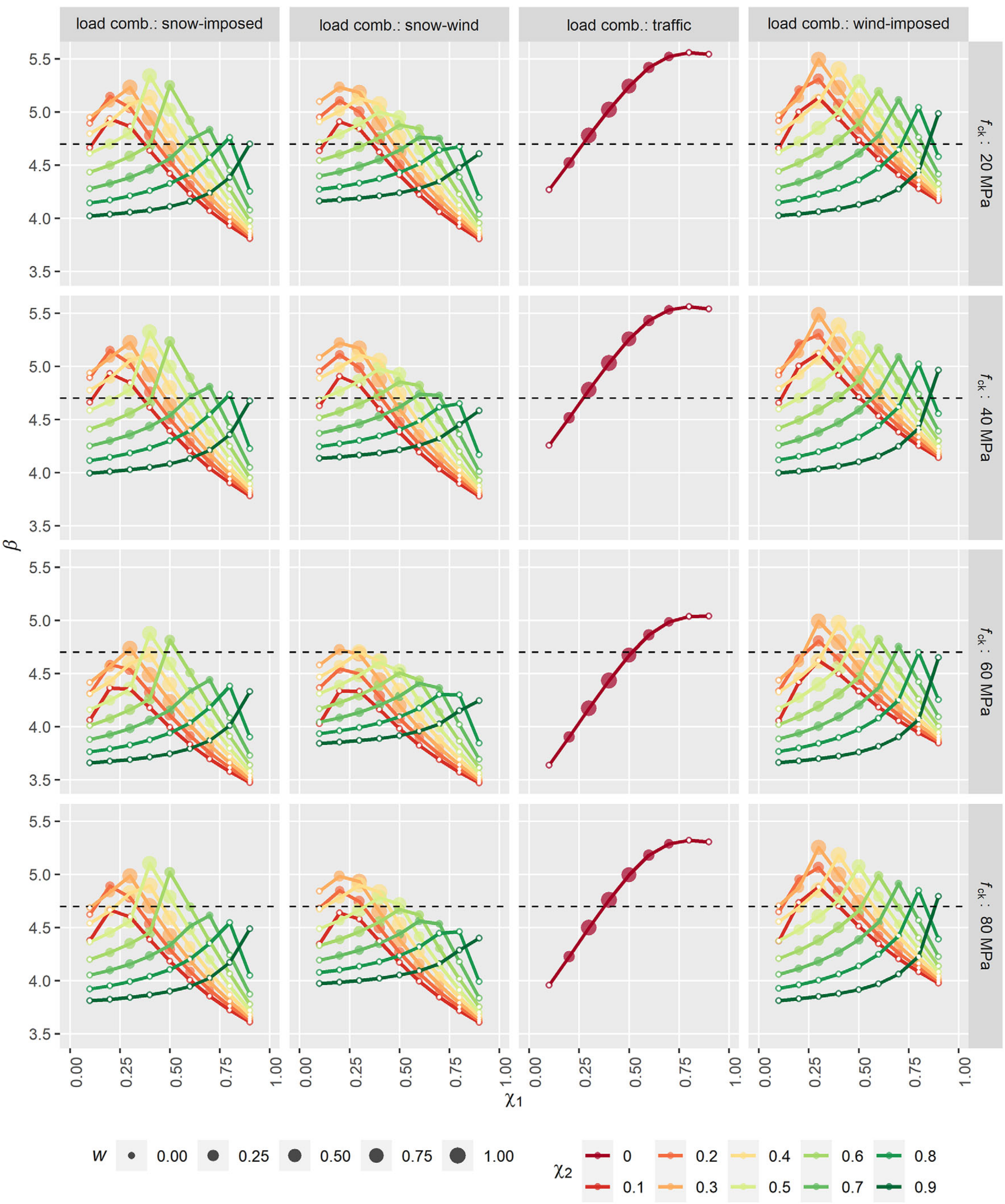

FIGURE 7 Reliability indices (β) obtained using the calibrated partial factor γ_R for case R2-3 and displayed for the design scenarios with $d_{\mathrm{nom}} = 300$ mm, $\rho_{\mathrm{l,nom}} = 1\%$. The horizontal dashed line indicates β_{target}. The size of the circles corresponds to the prevalence weight (w) of the design scenario. An empty circle highlights a design scenario with zero prevalence weight.

reference case R1-2. The practical relevance of more accurate shear resistance models becomes clearer when the comparison is made on the level of design resistances. Considering cases R1-1 and R2-4 (that have the full set of design scenarios) and using their optimal partial factors, the ratio between the average design resistance from MC2010 and the average design resistance from Eurocode 2 is 1.13. This means that on average MC2010 leads to about 13% more economical structural designs compared to Eurocode 2 given that the target reliability for both design formulas is the same.

Instead of using a single partial factor in the shear design formula, it fits better the semi-probabilistic safety format to assign distinct partial factors to distinct uncertainty sources. For example, it makes sense to differentiate between the partial factor related to concrete material

(γ_c) and the partial factor related to resistance model uncertainty (γ_R). Case R2-6 performs a calibration with these two separate partial factors. We assume that γ_c solely covers the concrete material uncertainty (this encompasses all variability and uncertainty between concrete factory, between recipe, between mix, within mix variability and conversion from test specimen to within structural member strength, as in line with the CEN technical report N228[53]) and that γ_R solely covers the resistance model uncertainty (this encompasses all variability and uncertainty that affect the resistance of the structural member—including the geometrical uncertainties—but not covered by material uncertainty or load model uncertainty). We think that γ_c should be associated directly to the parameter of the concrete material property in the design formula, hence, we replace f_{ck} in Equation (13) with f_{ck}/γ_c. We further assume that γ_c is equal to 1.39.[27] The optimal γ_R after calibration in this case is 1.37 and, as shown in the last column of Table 5, this value could also be obtained after some simple arithmetic operations. The advantage of a design formula like this is that the coverage of uncertainty (or the safety margin) is clearly attributed and is not hidden in model regression coefficients.

4.2.3 | Load-side variations (R3)

Complementary to the previous section, here we present load-side variation results. The related cases are identified by IDs of format R3-x in Table 5.

Switching from the simple load combination rule to the advanced one (R3-1) has a negligible (<1%) impact on the optimal partial factor. This supports the decision in the Eurocode to use the same partial factor value irrespective of the load combination rule.

Next, we investigate the impact of calibrating only with the traffic load combination (R3-2) and calibrating with all other load combinations except the traffic load (R3-3). This is motivated by the observation in R1-1 that the reliability indices of the design scenarios associated to traffic load combinations with the largest prevalence weights are below the target reliability index, whereas it is the opposite for the other load combinations (Figure 3). The optimal partial factors are 1.59 and 1.42 for R3-2 and R3-3, respectively. Although the differences with respect to the reference case R1-1 are noticeable, they may still be considered as sufficiently small (≈5%–7%) to opt for a single partial factor for all load combinations for the ease of use.

In case R3-4, we perform a calibration without the θ_E random variable while keeping the rest of the model, including the semi-probabilistic format and representative values, the same. The motivation for this case comes from the lack of agreement in the structural reliability literature whether to explicitly and separately consider this uncertainty from the load model uncertainty as a random variable or not. For example, Sørensen[16] neither considers load model uncertainty nor load effect model uncertainty. Pacheco[15] considers the load effect model uncertainty but not the load model uncertainty, Gulvanessian[54] and Nadolski et al.[13] consider a single load model uncertainty and no explicit load effect model uncertainty. Meinen and Steenbergen[33] consider both components explicitly. Even if one assumes that some of the differences are only on the level of the used terms, in most cases it cannot be deciphered from the used models what they cover. The optimization without θ_E leads to a partial factor of 1.42, which is markedly smaller than the reference value. This difference could be explained by the relatively large importance of θ_E among the action-side random variables for load combinations without traffic (see its large sensitivity factor α in Figure 5). Case R3-4 shows that the decision to disregard θ_E in the calibration can be non-conservative and should be taken with care. It should be noted that if the mean of θ_E was smaller than one (e.g., due to conservative modeling assumptions regarding boundary conditions) then an opposite effect might be found.

4.2.4 | Other variations (R4)

This section deals with variations that do not fall into any of the previous categories but are relevant from a reliability-based calibration and standardization point of view. The related cases are identified by IDs of format R4-x in Table 5.

The use of the asymmetric objective function of Equation (11) in case R4-1, instead of the symmetric one of Equation (10), has a small impact (<4%) on the optimal partial factor. Because the design scenarios with negative deviations from the target reliability are more heavily penalized, the optimal γ_R value in R4-1 is slightly larger compared to the reference case (R1-1).

With the cases R4-2 and R4-3, we calibrate the Eurocode 2 shear formula considering the reliability classes RC3 ($\beta_{target} = 5.2$) and RC1 ($\beta_{target} = 4.2$), respectively. Aiming for a different target reliability in these classes, Eurocode assumes that the same partial factors can still be used by applying a multiplication factor K_{FI} to the loads (see Annex B in Eu[28]). The calibrations show optimal partial factors of 1.62 and 1.46 for the R4-2 and R4-3 cases, respectively. These noticeable differences compared to the reference case (<7%) indicate that for the highest reliability class on average the safety margin slightly reduces while for the lowest reliability class it slightly increases. However, the differences are small enough to favor a single partial factor for all reliability classes.

In case R4-4, we investigate the impact of considering a 95% utilization ratio in the semi-probabilistic design instead of full utilization as in Equation (2). The optimal partial factor proportionally reduces with 5%, resulting in a value of 1.45.

5 | DISCUSSION

The presented results illustrate the impact of various modeling decisions on the optimal partial factor when calibrating two shear design formulas. Considering all the analyzed cases, the most impactful modeling decisions are the selected probabilistic models of the random variables and the connection of the representative values to these probabilistic models. Due to the lack of data and the tiny probabilities involved in structural reliability, these models cannot be unambiguously established just relying on data, hence, they are recommended to be based on agreements within the community.[55] Although some steps are made toward these agreements (see, e.g., the recommendations of the Joint Committee of Structural Safety [JCSS]), they are incomplete, especially regarding the connection of representative values to probabilistic models. Moreover, relevant Eurocode background documents are lacking, for example, to what fractile does the representative or characteristic value of the concrete compressive strength or a resistance model belong. These circumstances make all calibration studies, including this one, somewhat arbitrary. Although we paid a lot of attention to selecting the probabilistic models, other decisions could have been made that would have probably led to different results (see, e.g., case R3-4). This arbitrariness can only be overcome by the mentioned community level agreements that would create a fixed reference framework for calibration and, in turn, allow the assessment of current standards and the comparison of different calibration approaches and safety formats. This agreement should be established at least on the level of individual standards such as the Eurocode, but ideally on an even more general level.

The representative or characteristic values of random variables are anchor points that establish the relation between semi-probabilistic and probabilistic methods. Definitions of representative values for design formula parameters are essential but often not available in standards. Particularly for the resistance model uncertainty, a clear definition of its representative value ($\theta_{\mathrm{R,repr}}$) is missing. In this paper, we defined $\theta_{\mathrm{R,repr}}$ as the value that belongs to 5% non-exceedance probability for the shear resistance[56] under the considered design scenarios (Table 1) and corresponding representative values and probabilistic models (Tables 3 and 4), see Annex A.1.4.2. $\theta_{\mathrm{R,repr}}$ can be computed through solving an optimization problem. Using this definition of $\theta_{\mathrm{R,repr}}$ (at the level of the shear resistance instead of θ_{R}) can be seen as a step toward a statistically consistent design formula for which inputs with a certain representative value type lead to outputs with the same representative value type (i.e., mean value inputs should yield a mean value output, characteristic value inputs should yield a characteristic value output, etc.). How $\theta_{\mathrm{R,repr}}$ is defined can have a significant impact on the calibrated partial factor, as illustrated by case R2-2: with $\theta_{\mathrm{R,repr}} = 1.0$, $\hat{\gamma}_{\mathrm{R}}$ increases by 18% and reaches 1.80. Therefore, clarity and community level agreement on $\theta_{\mathrm{R,repr}}$ is needed.

The presented analyses consider a target reliability (β_{target}) of 4.7 related to a 1-year reference period, based on the reliability class RC2 in Eurocode 0.[28] It is derived from an annual target reliability of 3.8 by assuming mutually independent annual failure events. However, often some dependence is present, which would lead to annual targets smaller than 4.7. For example, the JCSS probabilistic model code[29] recommends an annual β_{target} of 4.2 for the matching consequence class.

For pragmatic reasons (e.g., more manageable optimization problem) we focused on the calibration of a single partial factor, although it would be better to simultaneously calibrate multiple (ideally all) codified factors. For example, this was already done in the 80s in the USA[6] and more recently work has been done in Europe as well.[9,20]

6 | CONCLUSIONS

This paper presents a reliability-based calibration of selected shear design formulas in Eurocode 2 and *fib* MC2010. The used method is general and can be applied to other design formulas and multiple partial factors as well. The method could be improved by using a probabilistic load combination approach rather than using the standardized combination factors in the reliability analysis. Besides calibrated partial factors, the method provides insights into the variation of the safety level of considered design scenarios and the importance of each random variable in the reliability analyses. These insights can be valuable in setting future research agendas on the development of design formulas, for example, collecting more (experimental) data to better quantify uncertainties of certain dominant random variables.

Based on the performed calibrations of the shear design formulas, we draw the following main conclusions:

- The optimal resistance model uncertainty partial factor ($\widehat{\gamma}_R$) of the current Eurocode 2 shear formula is 1.8 (case R2-2). For this case, variable $\widehat{\gamma}_R$ is interchangeable with γ_C in the standardized formula, which indicates that on average structures designed with a γ_C of 1.5 are considerably below the standardized target reliability provided that the experiments in the used ACI-DAfStb shear database are representative of the application domain of the Eurocode 2 shear formula.
- For the modified Eurocode 2 shear formula with $\theta_{R,repr}$ of 0.85 (case R1-1) and the modified MC2010 shear formula with $\theta_{R,repr}$ of 1.08 (case R2-4), $\widehat{\gamma}_R$ is 1.53 and 1.36, respectively. This reduction of about 12% indicates that the use of a shear formula that agrees better with experimental results (the MC2010 has a 16% higher R^2 score to the results in the ACI-DAfStb shear database) can significantly impact $\widehat{\gamma}_R$. On average, this leads to about 13% larger design resistances for the MC2010 compared to Eurocode 2 given that the target reliability for both design formulas is the same.
- The resistance model uncertainty θ_R is one of the most dominant random variables with $\alpha_{\theta R}{}^2$ values ranging from 0.25 (when χ_1 and χ_2 are large) to 0.75 (when χ_1 and χ_2 are small). The $\alpha_{\theta R}{}^2$ value is generally more than 90% of the squared sensitivity factor of the combined resistance-related random variables ($\alpha_R{}^2$), showing that the other resistance parameters hardly impact $\widehat{\gamma}_R$.
- The run-time to perform a calibration reduces from an order of hours to less than a minute when taking fixed values for the resistance parameters in the design scenarios (hence reducing the number of design scenarios) without affecting the resulting $\widehat{\gamma}_R$ (compare R1-1 with R1-2 and R2-3 with R2-4).
- Fixing the α_R and α_E values conceals a large variation that can be demonstrated through reliability analyses. Fixed values might work well on average, but they should be applied with care to particular situations, for example, high variable to total load ratio cases.
- In line with Eurocode's assumptions, $\widehat{\gamma}_R$ hardly changes (<7%) for the different load combination rules and reliability classes.

We make the code used for the presented analyses publicly and freely available.[23] We hope that others will collaborate on the calibration of other design formulas, aiming for statistically sound and transparent next generation Eurocodes.

AUTHOR CONTRIBUTIONS

Árpád Rózsás quantified the model uncertainty. Árpád Rózsás and Arthur Slobbe implemented the computer code for the calibration of the shear design formula and the visualization of the results. Árpád Rózsás set up and performed the calculations with the assistance of Arthur Slobbe. Arthur Slobbe, Árpád Rózsás, and Yuguang Yang drafted the main body of the text and prepared the figures. Yuguang Yang defined the design scenarios. Yuguang Yang discussed and presented the outcomes in Eurocode 2 Task Group on shear (CEN-TC250-SC2-WG1-TG4).

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DATA AVAILABILITY STATEMENT

All supporting code is publicly available in a GitHub repository: https://github.com/TNO/shear_calibration. The reliability-based calibration results can be viewed and interacted with through an online dashboard: https://rozsasarpi.shinyapps.io/visualize_calibration_results/.

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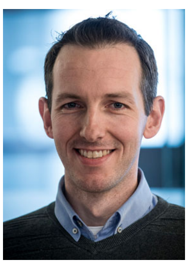

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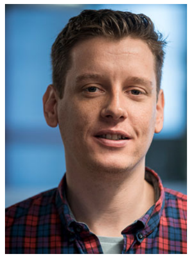

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ANNEX A

A.1 | INFERENCE OF SHEAR RESISTANCE MODEL UNCERTAINTY BASED ON LABORATORY EXPERIMENTS

A.1.1. | Overview

This annex summarizes the estimation of the shear resistance model uncertainty, which is defined—following the common practice in structural reliability, for example, Refs.[58–60]—as the ratio of experimental resistance and model-predicted resistance:

$$\theta_{\mathrm{R}} = \frac{V_{\mathrm{exp}}}{V_{\mathrm{R}}}, \tag{A1}$$

where V_{exp} is the resistance from experiments; and V_{R} is the resistance from model prediction.

Section A.1.2 describes the experimental data, Section A.1.3 covers the considered resistance models, and Section A.1.4 contains the details and results of parameter estimation.

A.1.2. | Experimental data

We use the shear database reported by the ACI-DAfStb joint committee.[34] The most important requirement toward the experimental data is that it is representative of the intended application domain of the formula. Being limited by the technical capacities of laboratories and the cost of executing large-scale experiments, the number of experiments on specimens with large dimensions and low reinforcement ratios is believed to be underrepresented in the database.[61] We are unaware of a more representative dataset, hence, we use the ACI-DafStb database in this paper; however, if more and/or more representative data becomes available the model uncertainty estimation should be updated.

The used database contains 744 experiments. The measured parameters present in the Eurocode 2 resistance formula are visually summarized in Figure A1. The figure shows that most of the tested specimens are relatively small, that is, $b_w < 500$ and $d < 500$ mm. For interpretation of the parameters, see Section 3. The database listed the mean values of the relevant variables, including concrete strength. These values are used directly in the calibration.

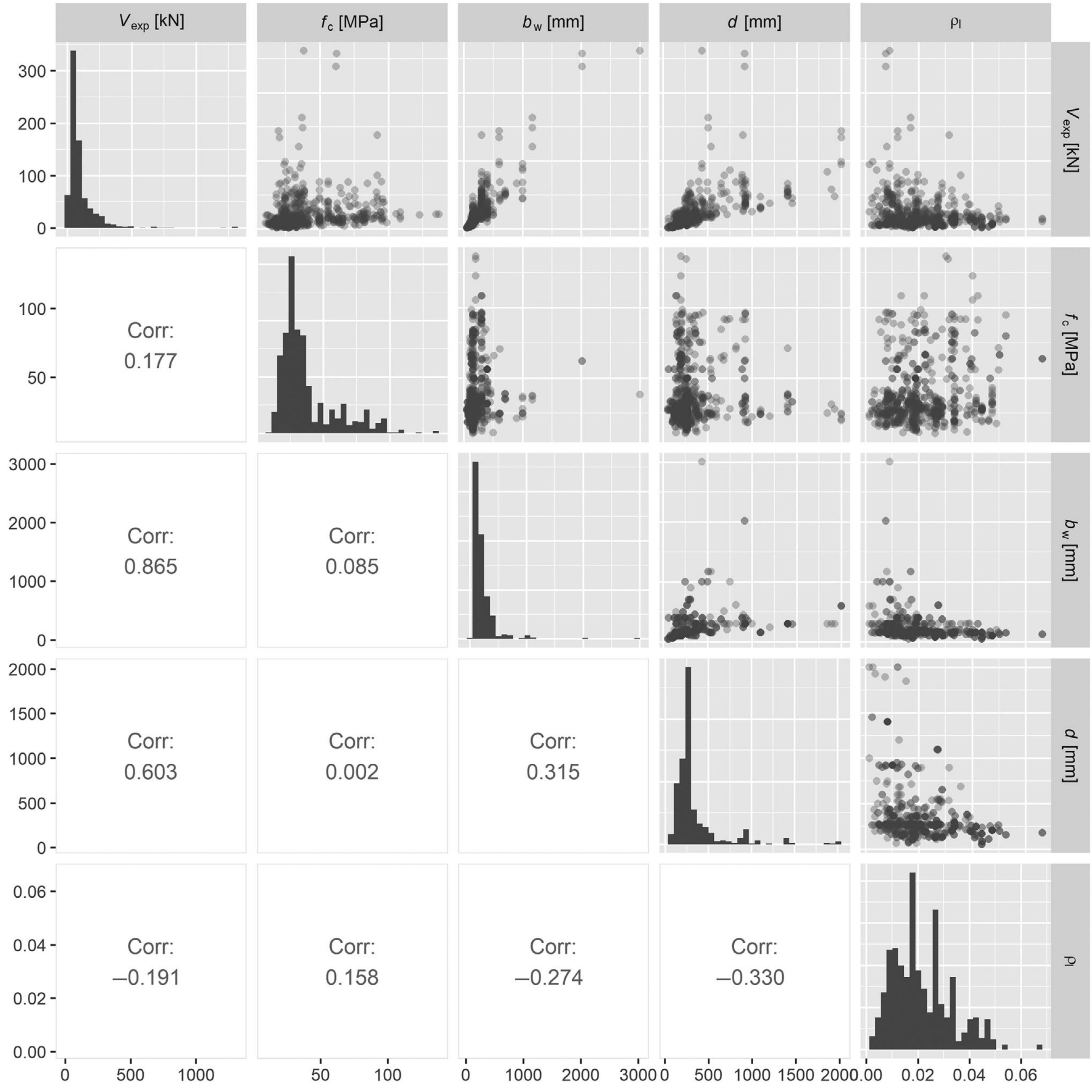

FIGURE A1 Overview of the used experimental database with respect to the parameters present in the Eurocode 2 resistance formula. Upper triangle: Pairwise scatter plots. Diagonal: Histogram of the parameter values. Lower triangle: Pairwise Pearson correlation coefficients

A.1.3. | Model predictions.

A.1.3.1. | EN1992-1-1

We consider two variants of the current EN1992-1-1 formula:

- The *full* model as expressed in Equation (12). Considering Equation (A1), we formulate the model uncertainty as:

$$\theta_R = \frac{V_{exp}}{V_{\mathrm{Rd,c}}\left(\theta_{\mathrm{R,repr}} = 1.0,\ \gamma_R = 1.0,\ C_{\mathrm{R,c,base}} = 0.18,\ C_{\mathrm{R,c,min}} = 0.0525\right)}, \tag{A2}$$

where the parameters of $V_{\mathrm{Rd,c}}$ that are not explicitly mentioned in Equation (A2) are used with their experimentally measured values or if not available with the values reported in Section 3.1.

- The *base* model as expressed in Equation (13). The motivation behind this variant is that $\nu_{\min}$ in Equation (14) describes the boundary between bending failure and shear failure. It is used to indicate the minimum shear stresses under which shear failure can still occur. Considering Equation (A1), we formulate the model uncertainty as:

$$\theta_R = \frac{V_{exp}}{V_{\mathrm{Rd,c}}\left(\theta_{\mathrm{R,repr}} = 1.0,\ \gamma_R = 1.0,\ C_{\mathrm{R,c,base}} = 0.18,\ V_{\mathrm{Rk,c,min}} = 0\right)}, \tag{A3}$$

where the parameters of $V_{\mathrm{Rd,c,min}}$ that are not explicitly mentioned in Equation (A3) are used with their experimentally measured values or if not available with the values reported in Section 3.1.

A.1.3.2. | *fib* MC2010 level II

Considering Equations (16) and (A1), we formulate the model uncertainty as:

$$\theta_R = \frac{V_{exp}}{V_{\mathrm{Rd,c}}\left(\theta_{\mathrm{R,repr}} = 1.0,\ \ \gamma_R = 1.0\right)}, \tag{A4}$$

where the parameters of $V_{\mathrm{Rd,c}}$ that are not explicitly mentioned in Equation (A2) are used with their experimentally measured values or if not available with the values reported in Section 3.2.

A.1.4. | Parameter estimation

A.1.4.1. | EN1992-1-1 full

In line with the common practice in structural reliability, the model uncertainty is assumed to be log-normally distributed[30]:

$$\theta_{\mathrm{R}} \sim \ln \mathcal{N}(mean, cv). \tag{A5}$$

After the distribution type is fixed, the task is to estimate the distribution parameters. For convenience, we parametrize the two-parameter log-normal distribution with its mean (*mean*) and coefficient of variation (*cv*). We use the maximum likelihood method to estimate the parameters, where the likelihood function is formulated by assuming independent and identically distributed observations. Note that any bias (or general unrepresentativeness) present in the data is preserved by the fitted distribution function. The maximum likelihood method is deemed to be sufficient to estimate the distribution parameters due to the large number of observations in comparison with the number of estimated parameters, and because our prior knowledge about the model uncertainty is vague.

The maximum likelihood estimates of the log-normal distribution parameters are $mean = 1.137$, $cv = 0.2378$ (also summarized in Table A1). The fitted log-normal distribution is shown in Figure A2 and the observed and predicted resistances are compared in Figure A3. The plots indicate that the log-normal model is a reasonable choice for the data. The R^2 value of the mean prediction of the calibrated model is 0.79, which—in the authors'

TABLE A1 Summary of the calibrated resistance model uncertainties

Resistance model	Model uncertainty	*Mean*	*cv*	n_{lb}[a]	R^2[b]
EN1992-1-1 full	Equation (A2)	1.137	0.2378	7	0.787
EN1992-1-1 base	Equation (A3)	1.138	0.2376	NA	0.786
MC2010	Equation (A4)	1.344	0.1924	NA	0.918

Abbreviation: NA, not applicable.

[a]Number of specimens for which the lower bound of the model is active.

[b]Coefficient of determination. Computed by using the mean prediction of the calibrated resistance model.

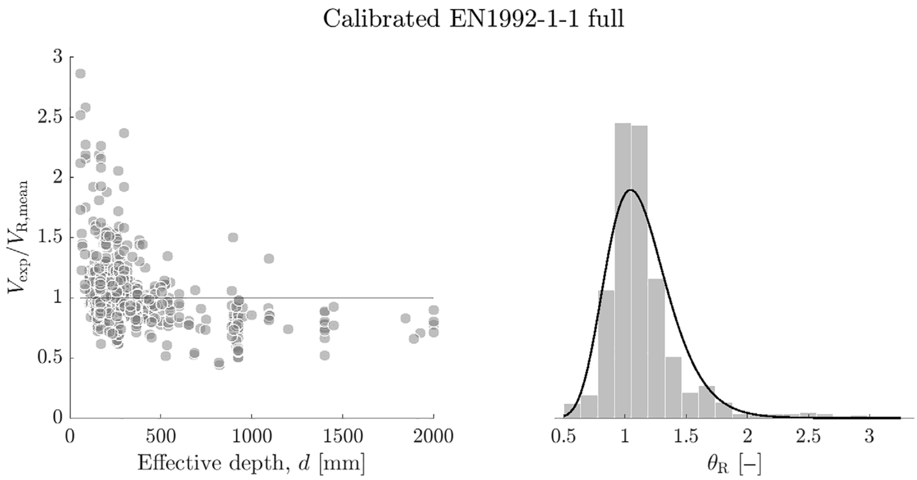

FIGURE A2 Left: Ratio of experimental resistance and mean model prediction of the calibrated current EC2 formula. Right: Histogram and fitted distribution of θ_R

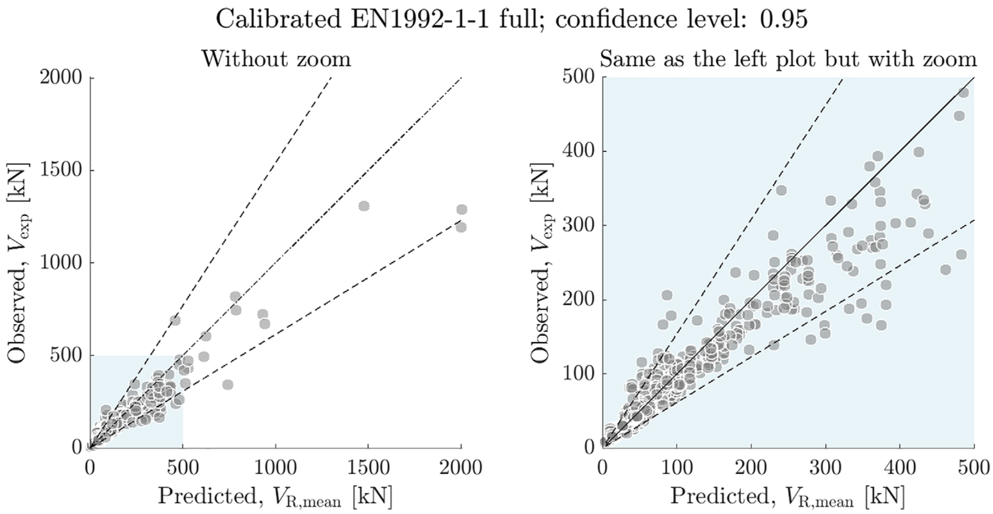

FIGURE A3 Experimental resistance and mean model prediction of the calibrated current EC2 formula. Solid line: Perfect prediction; dashed lines: 95% prediction uncertainty interval of the calibrated model ("added" to the perfect prediction)

TABLE A2 Summary of the calibrated representative values (θ_{repr}) of the resistance model uncertainties for various cases

Resistance model	Load combination rule	Load combinations	Design scenarios	Objective	θ_{repr} [a]	Related case ID(s)
EN1992-1-1 full	Simple	All[b]	All[c]	Symm.	0.8468	R1-1
EN1992-1-1 full	Simple/advanced	Any	Reduced[d]	Symm./asymm.	0.8460	R1-2, R2-6, R3-1, R3-2, R3-3, R3-4, R4-1, R4-2, R4-3, R4-4
EN1992-1-1 base	Simple/advanced	Any	Reduced[d]	Symm./asymm.	0.8178	R2-1
MC2010	Simple	All[b]	All	Symm.	0.9633	R2-3
MC2010	Simple	All[b]	All with reduced f_{ck}[e]	Symm.	1.075	R2-4
MC2010	Simple/advanced	All[b]	Reduced[d]	Symm./asymm.	1.079	R2-5

[a]Computed to lead to 5% non-exceedance probability for the shear resistance (V_R) under the considered design scenarios (Table 1) and corresponding representative values and probabilistic models (Tables 3 and 4, respectively). For details see Section A.1.4.2.

[b]Traffic, snow-wind, snow-imposed, and wind-imposed.

[c]Table 1.

[d]Reduced design scenario space: $d_{nom} = 300$ mm; $f_{ck} = 40$ MPa; $\rho_{lnom} = 1\%$, $dg = 16$ mm, $a/d = 3$ instead of the corresponding ranges in Table 1.

[e]As in Table 1 but the discretization of f_{ck} is reduced to [20, 40] MPa.

opinion—indicates an acceptable accuracy considering the empirical and dimensionally inconsistent nature of the formula.

A.1.4.2. | Representative value

In the semi-probabilistic format, we need a representative value of θ_{R}, for example, see Equation (12). In the absence of a widely accepted recommendation on defining the representative value (θ_{Rrepr}), we use the following definition: the value of θ_{Rrepr} should lead to a characteristic resistance (V_{Rk}) that has a 5% non-exceedance probability.[56] In general, this cannot be met for all design scenarios so we aim to minimize the total deviation from the target non-exceedance probability across all design scenarios. This formulation leads to an optimization problem very similar to the reliability-based calibration of the partial factor (Section 4) with the following differences and similarities:

- θ_{Rrepr} plays the role of γ_{R}.
- The target reliability changes to $-\Phi^{-1}(0.05) = 1.64$.
- The same design scenarios and prevalence weights are considered.
- The symmetric objective function is used.
- The performance function in the reliability analysis:

$$g = \theta_{\mathrm{R}} \cdot V_{\mathrm{R}} - V_{\mathrm{Rk}} \tag{A6}$$

All the aspects and assumptions not explicitly mentioned are the same as for the partial factor calibration. Although this formulation requires solving an optimization problem to obtain the representative value, it has the advantage of making the V_{Rk} formula, on average, correspond to a known non-exceedance probability and, thus, making its interpretation easier. The calibrated model uncertainty representative values for the considered cases are summarized in Table A2. For the cases with reduced design scenarios, the target reliability can be exactly met for all design scenarios at the same time due to the mathematical form of the resistance formulas.

A.1.4.3. | Summary of results

The fitting procedure described in Section A.1.4.1 is repeated for the other considered shear resistance models. The results are summarized in Table A1.