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Comments on "A proportional-integral extremum-seeking controller design technique" [Automatica 77 (2017) 61–67]☆

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ABSTRACT

In the proof of [Guay and Dochain (2017), Th.1], Equation 6 is incorrect.

In this note, we show by a counterexample that Equation 6 in (Guay & Dochain, 2017),

$$\frac{\partial h}{\partial x}\left(\pi(\hat{u})\right) g(\pi(\hat{u})) = \frac{\partial h}{\partial x}(\pi(\hat{u}))\frac{\partial \pi(\hat{u})}{\partial \hat{u}}. \tag{1}$$

which is used in the proof of Theorem 1 (page 65, left column, line 16) to obtain a negative Lyapunov derivative, is incorrect. This error also appears in Guay (2016, Equ. 6), Guay and Atta (2018, Equ. 16) and Guay, Vandermeulen, Dougherty, and McLellan (2018, p. 505, left column, line 27).

As a class of counterexamples, we consider linear systems with strongly convex quadratic cost, specifically, in the notation of (Guay & Dochain, 2017),

$$f(x) = Ax, \quad g(x) = B, \quad h(x) = \tfrac{1}{2}x^\top Qx + x^\top p, \tag{2}$$

where $A \in \mathbb{R}^{n\times n}$ is Hurwitz, $B \in \mathbb{R}^{n\times m}$, $Q \in \mathbb{R}^{n\times n}$ is positive definite, $p \in \mathbb{R}^n$. In Guay and Dochain (2017, Assum. 4), we take $k_g = 0$. Therefore, the steady-state mapping is given by

$$\pi(\hat{u}) = -A^{-1}B\hat{u}. \tag{3}$$

Then, we have that $\frac{\partial h}{\partial x}(x) = Qx + p$, hence $\frac{\partial h}{\partial x}(\pi(\hat{u})) = -QA^{-1}B\hat{u} + p$, and the left-hand side of (1) is

$$\tfrac{\partial h}{\partial x}(\pi(\hat{u}))g(\pi(\hat{u})) = (p^\top - \hat{u}^\top B^\top A^{-\top}Q)B. \tag{4}$$

Since $\frac{\partial \pi}{\partial \hat{u}} = -A^{-1}B$, the right-hand side of (1) is given by

$$\tfrac{\partial h}{\partial x}(\pi(\hat{u}))\tfrac{\partial \pi}{\partial \hat{u}} = -(p^\top - \hat{u}^\top B^\top A^{-\top}Q)A^\top B. \tag{5}$$

Considering that the vectors in (4) and (5) are not equal in general (unless $A = -I$ as in Krilašević & Grammatico, 2020), we conclude that Guay and Dochain (2017, Equ. 6) is incorrect.

Equation 6 in Guay and Dochain (2017) is used in the proof of Theorem 1 on page 65, while applying $\frac{\partial h}{\partial x}(\pi(\hat{u}))g(\pi(u))\tilde{u} = \frac{\partial \ell}{\partial \hat{u}}(\hat{u})\tilde{u} \leq -\alpha_u\|\tilde{u}\|^2$, for all $\hat{u}$. Next, we show that this incorrectness of Guay and Dochain (2017, Equ. 6) compromises the Lyapunov-based proof of Guay and Dochain (2017, Thm. 1) for our counterexample in (2). Let u^* be the minimizer of $l(\hat{u})$ and let $\tilde{u} := u^* - \hat{u}$. From (4), it holds

$$\begin{aligned}
&\tfrac{\partial h}{\partial x}(\pi(\hat{u}))g(\pi(\hat{u}))\tilde{u} = (p - \hat{u}^\top B^\top A^{-\top}Q)B\tilde{u} \\
&+ (p - u^{*\top}B^\top A^{-\top}Q)B\tilde{u} - (p - u^{*\top}B^\top A^{-\top}Q)B\tilde{u} \\
&= \tilde{u}^\top B^\top A^{-\top}QB\tilde{u} + (p - u^{*\top}B^\top A^{-\top}Q)B\tilde{u} \\
&= \tfrac{1}{2}\tilde{u}^\top B^\top(A^{-\top}Q + QA^{-1})B\tilde{u} \\
&\quad + (p - u^{*\top}B^\top A^{-\top}Q)B\tilde{u}.
\end{aligned} \tag{6}$$

The Lyapunov analysis in Guay and Dochain (2017) relies on the strict negative-definiteness of the quadratic term in $\tilde{u}$, as it is used to majorize other positive terms (see for example, the last inequality of the left column on page 65). However, for our case in (2), the matrix $M := B^\top(A^{-\top}Q + QA^{-1})B$ in (6) is not always negative definite.

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