



Delft University of Technology

## On-line survivable routing in WDM networks

Beshir, AA; Kuipers, FA; Orda, Ariel; Van Mieghem, PFA

### Publication date

2009

### Document Version

Accepted author manuscript

### Published in

21st International Teletraffic Congress

### Citation (APA)

Beshir, AA., Kuipers, FA., Orda, A., & Van Mieghem, PFA. (2009). On-line survivable routing in WDM networks. In s.l. (Ed.), *21st International Teletraffic Congress* (pp. 1-8). ITC.

### Important note

To cite this publication, please use the final published version (if applicable). Please check the document version above.

### Copyright

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

### Takedown policy

Please contact us and provide details if you believe this document breaches copyrights. We will remove access to the work immediately and investigate your claim.

# On-Line Survivable Routing in WDM Networks

Anteneh Beshir, Fernando Kuipers, Piet Van Mieghem

Network Architectures and Services  
Delft University of Technology  
Mekelweg 4, 2628 CD Delft  
The Netherlands

Emails: {A.A.Beshir, F.A.Kuipers, P.F.A.VanMieghem}@tudelft.nl

Ariel Orda

Department of Electrical  
Engineering, Technion,  
Israel Institute of Technology  
Haifa, Israel 32000

Email: ariel@ee.technion.ac.il

**Abstract**—In WDM networks, survivable routing and wavelength assignment (SRWA) involves assigning link-disjoint primary and backup lightpaths. In the on-line SRWA problem, a sequence of requests arrive and each request is either accepted or rejected based only on the input sequence seen so far. For special networks, we establish on-line algorithms with constant and logarithmic competitive ratios. It is not possible to obtain good competitive ratios in general topologies. Hence, we propose heuristic schemes and evaluate their performance by way of simulations. The building blocks in these schemes are 2-approximation algorithms (*MSA* and *ESA*) that we establish for the minimum disruption link-disjoint paths (MDLDP) problem. These approximations require far less memory and computation time than the best-known exact solution of the MDLDP problem. We use these three algorithms as heuristics for the on-line SRWA problem for infinite and finite duration requests and we show that, in terms of on-line performance, our algorithms do as well as (even at times better than) the exact algorithm of the MDLDP problem. We also provide an exact ILP formulation to solve the infinite duration off-line SRWA problem.

## I. INTRODUCTION

In optical networks employing wavelength-division multiplexing (WDM), the enormous capacity of a fiber is divided into several non-overlapping wavelength channels that can transport data independently. These wavelength channels make up *lightpaths*, which are used to establish point-to-point optical connections that may span several fiber links without using routers. In wavelength-selective WDM networks, a lightpath connection between a source and a destination must have the same wavelength in all links along its route. In wavelength-interchanging WDM networks, the nodes have the capability to convert a wavelength at an incoming link to a different one at an outgoing link. Unfortunately, the high price of wavelength converters makes them less desirable. Therefore, in this paper we only focus on wavelength-selective networks.

In WDM networks, provisioning lightpaths involves not only routing, but also wavelength assignment and this problem is referred to as the *routing and wavelength assignment (RWA)* problem. Due to the tremendous amount of data transported, survivability, which is the ability to reconfigure and re-establish communication upon failure, is indispensable in WDM networks. Since in reality not all the links fail at the same time, we consider the *single-link failure model*, where at most a single link fails at any given time. The *survivable routing and wavelength assignment (SRWA)* problem is to assign, given a set of lightpath requests, link-disjoint primary

and backup lightpaths to each request so that the total number of accepted requests is maximized.

For a single request, the SRWA problem can be solved with Suurballe's algorithm [9], if the primary and backup lightpaths use the same wavelength (for different wavelengths, it is NP-complete). But, in practice, lightpath requests arrive over time and the decision to accept or reject a request is made without any knowledge of future requests, yet maintaining the goal to maximize the total number of accepted requests. This version of the SRWA problem is called *on-line SRWA*. An algorithm is said to be an on-line algorithm if, for any arbitrary input sequence  $\sigma$ , at any point in the sequence a decision is made based on the input seen so far and without any knowledge of the future. On the other hand, an off-line algorithm is assumed to know the whole input sequence. Thus, the performance of an on-line algorithm  $A$  can at best be as good as an optimal, but usually non-implementable, off-line algorithm  $OPT$ .

*Definition 1:* An on-line algorithm  $A$  is said to be  $\rho$ -competitive if for any input sequence  $\sigma$ ,

$$\mathcal{B}(A, \sigma) \geq \frac{1}{\rho} \mathcal{B}(OPT, \sigma)$$

where  $\mathcal{B}(X, \sigma)$  is the number of accepted requests by algorithm  $X$  for the input sequence  $\sigma$ . The smallest such  $\rho$  is called the *competitive ratio* of the algorithm.

In Section II, we provide algorithms for the on-line SRWA problem with constant and logarithmic competitive ratios for specific networks. In Section III, we introduce rerouting of lightpaths to improve the practical performance of on-line routing. To this end, we discuss a related problem called the *minimum disruption link-disjoint paths (MDLDP)* problem and provide two 2-approximation algorithms for solving it. An algorithm is a 2-approximation algorithm for MDLDP if for any request, the number of lightpaths rerouted by its solution is at most twice that of the optimal algorithm. In Sections IV and V, we employ these algorithms as heuristics to solve the on-line SRWA with rerouting problem for requests of infinite and finite duration, respectively.

## II. ON-LINE SRWA

The on-line survivable routing and wavelength assignment (SRWA) problem is defined as follows.

*Problem 1: On-line SRWA:* The physical optical network is modeled as an undirected graph  $G(\mathcal{N}, \mathcal{L})$ , where  $\mathcal{N}$  is

a set of  $N$  nodes and  $\mathcal{L}$  is a set of  $L$  links. Each fiber link has a set of  $W$  wavelengths,  $\Lambda = \{w_1, w_2, \dots, w_W\}$ . A sequence of lightpath requests  $\sigma$  arrive over time. Each request  $i \in \sigma$  is represented by  $(s_i, t_i)$ , where  $s_i, t_i \in \mathcal{N}$  are its source and destination nodes, respectively. The on-line SRWA problem is to allocate for each request link-disjoint primary and backup lightpaths such that (1) the same wavelength is used on all links of the primary and backup lightpaths, (2) no two lightpaths having the same wavelength can share a link, and (3) the decision to accept or reject a request is based only on the input sequence seen so far. The objective is to maximize the number of accepted requests.

Before addressing the on-line SRWA problem, we consider the on-line SRWA problem without survivability (on-line RWA) and other related problems that have been studied in the literature.

**Problem 2: On-line Maximum Disjoint Paths (MDP)**

**Problem:** Given are a graph  $G(\mathcal{N}, \mathcal{L})$  and a sequence of requests. For each request  $(s_i, t_i)$ , find a path  $P_i$  that connects  $s_i$  and  $t_i$  such that no two paths share the same link. The objective is to maximize the total number of accepted requests.

The MDP problem is NP-complete [5]. Since lightpaths on the same wavelength are not allowed to share a link, the on-line MDP problem is equivalent to the on-line RWA problem with  $W = 1$ . Awerbuch *et al.* [3] have shown that if there is a  $\rho$ -competitive algorithm for the on-line MDP problem, then a  $(\rho + 1)$ -competitive algorithm can be obtained for the on-line RWA problem by employing the on-line MDP algorithm on each wavelength.

The on-line MDP problem has been widely studied in the literature. The  $\Omega(N^a)$ , where  $a = \frac{2}{3}(1 - \log_4 3)$  lower bound given by Bartal *et al.* [4] for randomized algorithms shows that it is not possible to find a good competitive ratio for general networks. In fact, most of the work in the literature has been restricted to special networks such as lines, trees, lattices, tree of rings, etc.

**Problem 3: On-line  $k$  Maximum Disjoint Paths ( $k$ -MDP)**

**Problem:** Given are a graph  $G(\mathcal{N}, \mathcal{L})$  and a sequence of requests. For each request  $(s_i, t_i)$ , find  $k$  link-disjoint paths  $P_{i1}, \dots, P_{ik}$  that connect  $s_i$  and  $t_i$  such that no two paths of different requests share the same link. The objective is to maximize the total number of accepted requests.

A simple upper-bound of any non-preemptive on-line algorithm for  $k$ -MDP is  $O(\frac{L}{k})$ . Suurballe's [9] algorithm ( $k = 2$ ) has a competitive ratio equal to this upper-bound. For example in Figure 1, if the input sequence is  $(s, t)$  followed by  $(s, a_1)$ ,  $(a_1, a_2), \dots, (a_y, t)$ ,  $(s, b_1)$ ,  $(b_1, b_2), \dots, (b_y, t)$  and all links have equal cost, the off-line algorithm accepts  $O(N)$  requests (i.e., all except the first), but the on-line algorithm accepts only the first two requests. Since in this example  $L = O(N)$ , the competitive ratio is of the same order as the upper-bound.

Using the same argument provided by Awerbuch *et al.* [3], a  $(\rho + 1)$ -competitive algorithm for the on-line SRWA problem ( $W > 1$ ) can be derived from a  $\rho$ -competitive algorithm of the on-line 2-MDP problem. Hence, in the remainder of this section, we provide algorithms and corresponding competitive

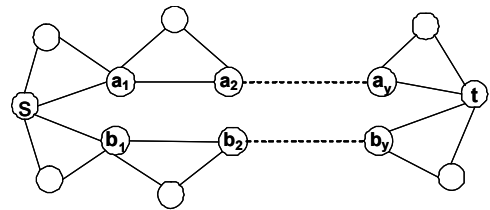


Fig. 1. An example where Suurballe's algorithm attains the upper bound.

ratios for the on-line 2-MDP problem, which forms the basis for the on-line SRWA problem, in star-of-rings, tree-of-rings, and lattice networks. Even though these are simple networks, not only do they help us gain insight into the problem, but they are also used in real networks (e.g., the SURFnet network in the Netherlands resembles a star-of-rings<sup>1</sup>).

**A. Star-of-rings network**

**Algorithm 1**  $Star\_Alg(G, s, t)$

- Accept a request if it is the first request so far that uses the ring(s) to which the source and destination nodes belong.
- Reject, otherwise.

$Star\_Alg(G, s, t)$  is 2-competitive if the number of rings is greater than 1. For a single ring, it is optimal. Figure 2 shows an example where  $Star\_Alg(G, s, t)$  is 2-competitive for the input sequence  $(a, b)$ ,  $(b, c)$ ,  $(a, d)$ . In this example, the on-line algorithm accepts only the first request, while the off-line algorithm accepts the last two requests.

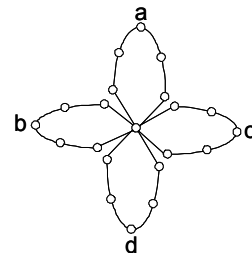


Fig. 2. A star of rings containing four rings.

**B. Tree-of-rings network**

**Algorithm 2**  $Tree\_Alg(G, s, t)$

- Replace each ring by a single link so that the whole tree of rings is substituted by the underlying tree topology.
- Each 2-MDP request in the tree of rings is equivalent to a corresponding MDP request in the underlying tree.
- Use the algorithm of Awerbuch *et al.* [2], which has  $O(\log N)$  competitive ratio for a tree of  $N$  nodes, to solve the on-line MDP problem.

From [2], it follows that  $Tree\_Alg(G, s, t)$  is  $O(\log \Upsilon)$ -competitive, where  $\Upsilon$  is the number of rings.

<sup>1</sup><http://www.surfnet.nl/en>

### C. Lattice network

The  $O(\log N)$ -competitive algorithm given by Kleinberg and Tardos [6] for the on-line MDP problem can, with a slight modification, be used for solving the on-line 2-MDP problem with an  $O(\log N)$ -competitive ratio. We have described the modified algorithm in detail in a technical report [12].

### III. ON-LINE SRWA WITH REROUTING

In Section II, we provided algorithms for the on-line 2-MDP problem in specific networks, which can be used to derive corresponding algorithms for the on-line SRWA problem. Unfortunately, it is not possible to attain a good competitive ratio for general networks [4]. In this section, we explore the idea of rerouting lightpaths to improve performance. Although rerouting does not improve the competitive ratio, we show through simulations that it can increase the acceptance rate considerably. In wavelength-selective WDM networks, a rerouting procedure may be path rerouting (i.e., changing the route of a lightpath while keeping the wavelength), wavelength rerouting (i.e., changing the wavelength while keeping the path) or a combination of both. Compared to path rerouting, wavelength rerouting does not need extra path computation (as it retains the same path), facilitates control and, if the rerouted lightpath is moved to a vacant route on another wavelength, it incurs less traffic disruption [7]. We therefore focus on wavelength rerouting.

Generally, the wavelength rerouting problem is NP-complete [7]. It consists of solving the three possible scenarios presented below. The second and the third scenarios make the problem hard to solve. Figure 3 shows the different scenarios. The labels on the links represent already existing lightpaths.

- 1) When the lightpaths to be rerouted are on the same wavelength, they can be moved to vacant wavelengths in parallel without any conflict (since they do not share links). For example, in Figure 3(a), a new lightpath from node 1 to 5 can be accepted on wavelength  $w_2$  by rerouting lightpath  $p_3$  to  $w_1$  and  $p_4$  to  $w_3$  in parallel.
- 2) When the lightpaths are on different wavelengths, moving to vacant wavelengths can be done sequentially while checking for conflicts. For example, in Figure 3(b), a new lightpath from node 1 to 5 can be accepted on  $w_1$  by first rerouting  $p_4$  to  $w_3$  and then  $p_1$  to  $w_2$ .
- 3) Moving to a vacant wavelength may not be sufficient, and it may be necessary to swap the wavelengths of lightpaths. For example, in Figure 3(c), a new lightpath from node 1 to 4 can be accepted on  $w_2$  by swapping the wavelengths of  $p_2$  and  $p_3$ .

In the literature and the remainder of this paper, the term *wavelength rerouting* is used to refer to the reduced problem, i.e., assigning a lightpath by moving existing lightpaths on the same wavelength to vacant wavelengths in parallel. Xue [11] has shown that this problem can be solved in  $O(WN \log N + WL)$  time.

On-line SRWA with wavelength rerouting involves assigning link-disjoint primary and backup lightpaths for new requests by rerouting, if necessary, already existing lightpaths.

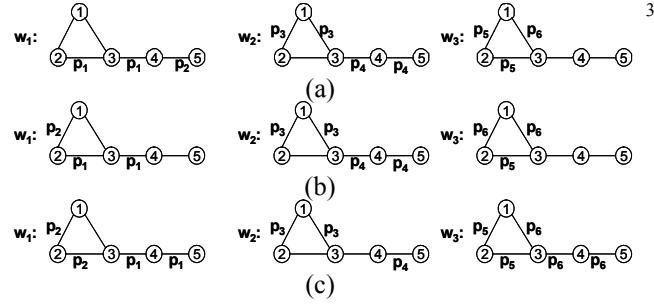


Fig. 3. Different scenarios of wavelength rerouting: (a) moving to vacant, (b) sequential rerouting, and (c) swapping.

When rerouting lightpaths, the number of lightpaths rerouted should be kept to a minimum. This leads us to consider the minimum-disruption link-disjoint paths (MDLDP) problem. The MDLDP problem is NP-complete when the primary and backup lightpaths use different wavelengths. However, it is polynomially solvable for the same wavelength [10]. We consider the polynomially-solvable version.

**Problem 4: Minimum Disruption Link-Disjoint Paths (MDLDP):** The physical optical network is modeled as an undirected graph  $G(\mathcal{N}, \mathcal{L})$ , where  $N = |\mathcal{N}|$  and  $L = |\mathcal{L}|$ . Each fiber link has a set  $\Lambda = \{w_1, w_2, \dots, w_W\}$  of  $W$  wavelengths. Given a request  $i$ , the MDLDP problem is to allocate on the same wavelength link-disjoint primary and backup lightpaths for request  $i$ , while minimizing the number of lightpaths to be rerouted.

Wan and Liang [10] provided an  $O(WL^5 \log N)$  exact algorithm for solving the MDLDP problem. We refer to this algorithm as *WLA*. *WLA* has a very high running time and requires a large amount of memory. This makes it less suitable, especially in an on-line setting where the algorithm has to be invoked whenever a new request arrives. We propose two 2-approximation algorithms with considerably less running times and memory requirements.

#### A. 2-Approximation Algorithms for MDLDP

We provide two 2-approximation algorithms for MDLDP: *MSA* and *ESA*. *MSA* is a modified version of Suurballe's algorithm [9] with a running time of  $O(WN \log N + WL)$  and *ESA* is an extended algorithm with a running time of  $O(WN^2 \log N + WNL)$ . This is a significant reduction from the  $O(WL^5 \log N)$  running time of the exact *WLA* algorithm with at most twice as much lightpaths being rerouted.

In our notation, we use  $p$  to represent a lightpath and  $P$  to represent any path. A lightpath on wavelength  $w_i$  is said to be *reroutable*, if and only if all of its links are free on at least one other wavelength  $w_j$ , i.e., no lightpath is using these links on  $w_j$ . A path  $P$  from  $s$  to  $t$  is said to *traverse* a lightpath  $p$  if it shares at least one link with  $p$ . Let  $\mathcal{P}_k$  be the set of lightpaths on wavelength  $w_k$ ;  $\mathcal{P}'_k \subseteq \mathcal{P}_k$  be the set of reroutable lightpaths on wavelength  $w_k$ ;  $\mathcal{P}''_k = \mathcal{P}_k \setminus \mathcal{P}'_k$  be the set of non-reroutable lightpaths on wavelength  $w_k$ ; and  $\Lambda_{(i,j)}$  be the set of free wavelengths on fiber link  $(i, j)$ .

We identify  $W$  subgraphs,  $G_k = G(\mathcal{N}, \mathcal{L}_k)$ ,  $\mathcal{L}_k = \{(i, j) \in$

$\mathcal{L} \mid w_k \in \Lambda_{(i,j)}$  or  $\exists p \in \mathcal{P}'_k$  such that link  $(i,j)$  belongs to lightpath  $p$ . The cost of a link  $(i,j)$  in subgraph  $G_k$  is  $cost_k(i,j) = \epsilon$ , if  $(i,j)$  is a free link, where<sup>2</sup>  $2N\epsilon < 1$ ;  $cost_k(i,j) = 1$  otherwise. However, the cost  $cost_k(P)$  of a path  $P$  in subgraph  $G_k$  is the sum of the cost of its free links and the number of *distinct* reroutable lightpaths traversed by  $P$ , i.e., multiple links belonging to a lightpath are counted only once. Thus, the shortest path between two nodes traverses the minimum number of reroutable lightpaths. Note that any lightpath that is traversed by the shortest path is encountered only once, i.e., in a single segment (of possibly multiple links).

---

**Algorithm 3**  $MSA(G, s, t)$ 


---

- 1) For each  $G_k, k = 1, \dots, W$ 
    - a) In graph  $G_k$ , find the shortest path from  $s$  to  $t$ .
    - b) Graph  $G'_k$  is obtained by directing each link  $(i,j)$  of the shortest path from  $t$  to  $s$ , setting the cost of the free links on the shortest path as  $cost_k(j,i) = -cost_k(i,j)$  and the cost of *all links* of lightpaths that are traversed by the shortest path to zero.
    - c) Find the shortest path from  $s$  to  $t$  in  $G'_k$ .
    - d) If the shortest path exists in  $G'_k$ , remove all the overlapping links between the two paths in  $G_k$  to obtain the solution.
  - 2) Choose the best solution among all wavelengths.
- 

In Step 1a of the  $MSA$  algorithm, we find the shortest path from  $s$  to  $t$  (using an algorithm such as the one given in [11]). In Step 1b, the cost of all links belonging to lightpaths traversed by the shortest path is set to zero so that these links are preferred in the second path and the lightpaths are not counted twice. Similarly, the cost of free links on the shortest path is set to  $-\epsilon$ .

*Theorem 1:*  $MSA$  is a 2-approximation algorithm for the MDLDP problem.

*Proof:* Since the best solution is chosen after independently considering each wavelength, it suffices to consider only the wavelength that provides the best solution. Assume that for this wavelength, given that a solution of  $MSA$  that traverses a total of  $K$  lightpaths, there is an optimal solution that traverses less than  $\frac{K}{2}$  lightpaths, which would violate the claim of 2-approximation. Our intention is to prove that the assumption is wrong.

Let  $\ell(P)$  represent the number of lightpaths traversed by a path  $P$  and  $\ell(\{P_1, P_2\})$  represent the number of distinct lightpaths traversed by paths  $P_1$  and  $P_2$ , where  $\ell(\{P_1, P_2\}) \leq \ell(P_1) + \ell(P_2)$ .

Let  $\{P_1^*, P_2^*\}$  be the optimal solution. In  $MSA$ , let  $P_1$  be the first shortest path that is obtained in Step 1a and  $P_2$  be the second shortest path that is obtained in Step 1c.

Let  $\mathcal{Q}$  be the set of alternating lightpaths of the optimal solution  $\{P_1^*, P_2^*\}$ , i.e., lightpaths with segments in both  $P_1^*$

and  $P_2^*$ . Let  $\mathcal{S}$  be the set of links of lightpaths  $p \in \mathcal{Q}$ .

$\ell(\{P_1^*, P_2^*\}) < \frac{K}{2}$  implies that  $\ell(P_1^*) < \frac{K}{2}$  and  $\ell(P_2^*) < \frac{K}{2}$ . Hence, the first shortest path returned by  $MSA$  must have  $\ell(P_1) < \frac{K}{2}$ . Since  $\ell(\{P_1, P_2\}) = K$ , the second shortest path returned by  $MSA$  should have  $\ell(P_2) > \frac{K}{2}$ . However,  $MSA$  can find a path  $P_2$  from the set of links of  $P_1^*, P_2^*$  and  $\mathcal{S}$ . If  $P_1$  also contains any of these links, they are redirected in Step 1b of  $MSA$  and are assigned a cost of zero. Since no new lightpaths are added  $\ell(P_2) < \frac{K}{2}$ , which is a contradiction. ■

The 2-approximation is attained in the worst case when  $\ell(P_1) = \ell(P_2) = \ell(\{P_1^*, P_2^*\})$  and  $P_1$  and  $P_2$  do not have common lightpaths as shown in Figure 4(a).  $P_1 = \{s, 3, t\}$ ,  $P_2 = \{s, 4, t\}$ ,  $P_1^* = \{s, 1, 2, t\}$ , and  $P_2^* = \{s, 5, 6, t\}$ ;  $\ell(\{P_1, P_2\}) = 2$  and  $\ell(\{P_1^*, P_2^*\}) = 1$ .

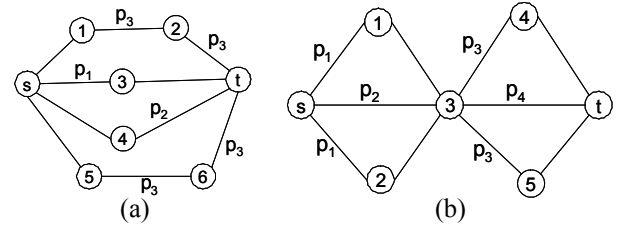


Fig. 4. (a) A worst case for  $MSA$  that leads to a 2-approximation and (b) an example where  $ESA$  fails.

The example in Figure 4(a) can exactly be solved if  $P_1$  leaves the source node through node 1 or node 5. We can achieve this by extending the  $MSA$  algorithm so that it checks the shortest path through any given node  $u \in \mathcal{N} \setminus \{s, t\}$ . This is exactly what our extended algorithm  $ESA$  does. As it can be seen later in Section III-C,  $ESA$  has a significantly improved performance in solving the MDLDP problem. But, it fails for cases like the one in Figure 4(b), where  $P_1 = \{s, 1, 3, t\}$ ,  $P_2 = \{s, 2, 3, 5, t\}$ ,  $P_1^* = \{s, 1, 3, 4, t\}$ , and  $P_2^* = \{s, 2, 3, 5, t\}$ ;  $\ell(\{P_1, P_2\}) = 3$  and  $\ell(\{P_1^*, P_2^*\}) = 2$ .

In  $ESA$ , for each node  $u \in \mathcal{N} \setminus \{s, t\}$ , we find link-disjoint paths from  $s$  to  $t$ , where the first path is forced to go through  $u$ . In Step 1a-ii, the cost of all links on  $P_{s-u}$  and all links belonging to lightpaths on  $P_{s-u}$  (except those of the lightpath on the last link, if there is any) is set to infinity. This is to prevent the same links from being used again in  $P_{u-t}$  and to make sure that any lightpath in  $P_{s-t}$  is traversed in at most one segment. For the lightpath on the last link, since our interest is to find the shortest path from  $s$  to  $t$  through  $u$ , it can still be encountered *just* after  $u$ . Therefore, its links, except those in  $P_{s-u}$ , will have a cost of zero. In Step 1a-iii, the shortest path from  $u$  to  $t$  is found. If  $P_{s-u}$  and  $P_{u-t}$  share nodes, then the algorithm does not proceed to finding the second shortest path. Instead, it skips to searching for the solutions of the remaining nodes. Once the path through  $u$  is found by concatenating  $P_{s-u}$  and  $P_{u-t}$ , the links on this path are directed from  $t$  to  $s$  in Step 1a-iv. In Step 1b, all the solutions are compared and the one that traverses the minimum number of lightpaths is chosen. If there are multiple solutions that traverse the same number of lightpaths, the one with the

<sup>2</sup>Using such a cost, the longest possible link-disjoint paths made up of only free links have a total cost that is less than any link-disjoint pair of paths that cross a lightpath.

**Algorithm 4**  $ESA(G, s, t)$ 

- 
- 1) For each  $G_k, k = 1, \dots, W$ 
    - a) For each node  $u \in \mathcal{N} \setminus \{s, t\}$ :
      - i) In graph  $G_k$ , find the shortest path  $P_{s-u}$  from  $s$  to  $u$ .
      - ii) Graph  $G'_k$  is obtained from  $G_k$  by setting the cost of all links on  $P_{s-u}$  and each link belonging to lightpaths on  $P_{s-u}$  to infinity except for links of the lightpath (if any) in the last link of  $P_{s-u}$ . For the lightpath in the last link, all its links except the ones in  $P_{s-u}$  will have a cost of zero.
      - iii) In graph  $G'_k$ , find the shortest path  $P_{u-t}$  from  $u$  to  $t$ . If  $P_{s-u}$  and  $P_{u-t}$  share nodes, go to Step 1a-i if there are remaining nodes whose shortest paths have not been found, otherwise go to Step 1b. If  $P_{s-u}$  and  $P_{u-t}$  do not share nodes, the shortest path through  $u$  is found by concatenating the two.
      - iv) Graph  $G''_k$  is obtained from  $G_k$  by directing each link  $(i, j)$  along the shortest path from  $t$  to  $s$ . The cost of free links on the shortest path is set to  $cost_k(j, i) = -cost_k(i, j)$  and the cost of all links belonging to lightpaths on the shortest path is set to zero.
      - v) In graph  $G''_k$ , find the shortest path from  $s$  to  $t$ .
      - vi) If the shortest path exists, remove all the overlapping links.
    - b) Choose the best solution among all nodes.
  - 2) Choose the best solution among all wavelengths.
- 

*smallest hopcount* is chosen. Since  $ESA$  includes  $MSA$ , it is at worst a 2-approximation algorithm.

### B. Reroutability Status Update Procedure

Once a lightpath request is accepted and its link-disjoint lightpaths are determined, it affects the reroutability of other lightpaths. These lightpaths include the rerouted lightpaths, and lightpaths that are using the same link, but on different wavelengths. In addition, the reroutability of the new lightpaths has to be identified. Once a request is accepted, its primary and backup lightpaths are treated independently, i.e., each can be rerouted to a different wavelength independently of the other. Hence, as in [8], for each lightpath, we dynamically keep track of such information as its hopcount, its wavelength, how many of its links are free on other wavelengths and to which other wavelengths it can be rerouted to. This is done as follows.

- 1) When a new lightpath  $p$  is assigned without rerouting other lightpaths on wavelength  $w_k$ :
  - We create new reroutability status information for  $p$ , e.g., how many of its links are free on other wavelengths and the wavelengths it can be rerouted to. This takes  $O(NW)$  time.

- After checking whether  $p$  is reroutable or not, we assign the costs of its links on wavelength  $w_k$ . This takes  $O(N)$  time.
  - In addition, the reroutability status information of lightpaths using the same fiber link, but on other wavelengths, should be updated. If  $q$  is such a lightpath, the number of its links that are free on wavelength  $w_k$  is decremented by one for each link that  $p$  and  $q$  have in common. Thus, if  $q$  was reroutable to wavelength  $w_k$ , it is not any more. Since, in the worst case, there are  $O(NW)$  such lightpaths, this takes  $O(NW)$  time.
- 2) When a new lightpath  $p$  is assigned by rerouting some lightpaths on wavelength  $w_k$ :
    - All the aforementioned operations are performed.
    - If  $q$  is a rerouted lightpath, the costs of its links on the new wavelength, and its reroutability status on  $w_k$  should be updated. This takes  $O(N)$  time and in the worst case  $O(N)$  lightpaths are rerouted. Therefore, the total running time is  $O(N^2)$ .
  - 3) When the holding time of lightpath  $p$  expires:
    - All its links on wavelength  $w_k$  will be free links and their cost is updated accordingly. This takes  $O(N)$ .
    - For any lightpath  $q$  that uses the same fiber link, but a different wavelength, the reroutability status information is updated. The number of its free links on wavelength  $w_k$  is increased by one and if this equals to the hopcount of  $q$ , then  $q$  is reroutable to  $w_k$ . This will take  $O(NW)$  time.

The total running time of the reroutability update procedure is  $O(N^2 + NW)$ . We employ this procedure when solving the on-line SRWA problem using the MDLDP algorithms.

### C. Simulation Study

We proceed to compare our 2-approximation algorithms ( $MSA$  and  $ESA$ ) with the exact algorithm ( $WLA$ ) in solving the MDLDP problem. In order to simulate a wide range of possibilities, we generate dynamic traffic, where requests arrive according to a Poissonian distribution (arrival rate  $r$ ) with exponential holding times of mean 1. For each request, we record the results of our algorithms in comparison to  $WLA$ . The *approximation ratio* represents the ratio of the number of lightpaths traversed by an approximation algorithm to the number of lightpaths traversed by  $WLA$ . It is averaged for all accepted requests over 10 iterations, each 5000 requests. The source and destination nodes are randomly selected with all nodes having equal probability of being selected.

We consider three networks: an ARPANET network (Figure 5), an Erdős-Rényi random network ( $N = 50$ , link density  $\alpha = 0.2$ , i.e., the average total number of links is  $\alpha \cdot O(N^2)$ ), and a  $7 \times 7$  lattice network, each with  $W = 10$  wavelengths. In all our simulations, the approximation ratio attained by  $ESA$  never exceeded 1.00004. The approximation ratios of both  $ESA$  and  $MSA$  in comparison to the exact algorithm were much smaller than 2. Table I shows the average simulated

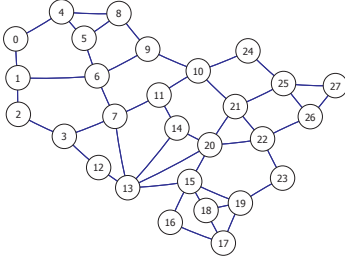


Fig. 5. ARPANET network

approximation ratios of *MSA*, in terms of the number of lightpaths rerouted, when compared to *WLA*, which returns the exact number for a given request.

TABLE I

APPROXIMATION RATIOS OF THE *MSA* ALGORITHM IN THE THREE NETWORKS FOR DIFFERENT ARRIVAL RATES ( $r$ ) FOR  $W = 10$ .

ARPANET		RANDOM		LATTICE	
$r$	Approx. ratio	$r$	Approx. ratio	$r$	Approx. ratio
10	1.0105	20	1.0185	20	1.0208
15	1.0095	40	1.0209	30	1.0172
20	1.0055	60	1.0309	40	1.0147
25	1.0170	80	1.0326	50	1.0119
30	1.0128	110	1.0275	60	1.0093
35	1.0100	120	1.0169	70	1.0073

In Sections IV and V, we use the aforementioned MDLDP algorithms to heuristically solve infinite and finite duration on-line SRWA, respectively. For each case, we compare the performances of *MSA*, *ESA* and *WLA*.

#### IV. INFINITE DURATION ON-LINE SRWA

In the infinite duration on-line SRWA problem, lightpaths stay indefinitely once they arrive. The off-line SRWA problem, where all the requests are known beforehand, can be described as a network flow problem. For this, we provide ILP formulations under two cases: case 1, when both the primary and backup lightpaths have to use the same wavelength and case 2, when they can use different wavelengths.

##### Indices:

- $i = 1, \dots, F$  ID of requests ( $F$  in total)
- $w = 1, \dots, W$  ID of wavelengths
- $\mathcal{N}(u)$  Set of nodes adjacent to node  $u$

##### Variables (integers):

- $\gamma_{i,w,u,v}$  is 1 (or  $-1$  depending on the flow direction) if the primary or backup lightpaths of request  $i$  use wavelength  $w$  on link  $(u, v) \in \mathcal{L}$ ; 0 otherwise.
- $x_{i,w}$  *Case 1 (same wavelength)*: is 1 if request  $i$  is accepted and uses wavelength  $w$ ; 0 otherwise. *Case 2 (different wavelengths)*: is 0 if neither the primary nor the backup lightpaths of request  $i$  are on wavelength  $w$ ; 1 if either the primary or the backup lightpath of request  $i$  is on wavelength  $w$ ; 2 if both the primary and the backup lightpaths of request  $i$  are on wavelength  $w$ .
- $y_i$  is 1 if request  $i$  is accepted; 0 otherwise.

##### Objective:

Maximize the number of accepted requests.

$$\text{Maximize: } \sum_{i=1}^F y_i$$

##### Constraints

*Antisymmetry constraints*: Since the graph is undirected, the flow is in both directions.

$$\gamma_{i,w,u,v} = -\gamma_{i,w,v,u} \quad \forall (u, v) \in \mathcal{L}; 1 \leq i \leq F; 1 \leq w \leq W.$$

*Conservation constraints*: If a given node is not the source or destination of a given request, then any flow related to the request that enters the node has to leave the node.

$$\sum_{v \in \mathcal{N}(u)} \gamma_{i,w,u,v} = 0 \quad \forall u \in \mathcal{N} \setminus \{s_i, t_i\}; 1 \leq i \leq F; 1 \leq w \leq W.$$

*Capacity constraints*: Only a single lightpath can use a given wavelength on a certain link.

$$\sum_{i=1}^F \gamma_{i,w,u,v} \leq 1 \quad \forall (u, v) \in \mathcal{L}; 1 \leq w \leq W.$$

*Disjointness constraints*: The primary and the backup lightpaths of a request should be link-disjoint.

$$\sum_{w=1}^W \gamma_{i,w,u,v} \leq 1 \quad \forall (u, v) \in \mathcal{L}; 1 \leq i \leq F.$$

##### Equations

For *Case 1*: Lightpaths of a request on a given wavelength.

$$\sum_{v \in \mathcal{N}(s_i)} \gamma_{i,w,s_i,v} = 2 \cdot x_{i,w} \quad 1 \leq i \leq F; 1 \leq w \leq W.$$

$$\sum_{v \in \mathcal{N}(t_i)} \gamma_{i,w,v,t_i} = 2 \cdot x_{i,w} \quad 1 \leq i \leq F; 1 \leq w \leq W.$$

For *Case 2*: Lightpaths of a request on a given wavelength.

$$\sum_{v \in \mathcal{N}(s_i)} \gamma_{i,w,s_i,v} = x_{i,w} \quad 1 \leq i \leq F; 1 \leq w \leq W.$$

$$\sum_{v \in \mathcal{N}(t_i)} \gamma_{i,w,v,t_i} = x_{i,w} \quad 1 \leq i \leq F; 1 \leq w \leq W.$$

An accepted request has link-disjoint primary and backup lightpaths.

$$\sum_{w=1}^W x_{i,w} = 2y_i \quad 1 \leq i \leq F.$$

Solving the given ILP formulation for large networks and high number of requests is not feasible. Therefore, we use the algorithms of the MDLDP problem to solve the on-line SRWA problem sequentially. Clearly, this approach will not guarantee an optimal solution. However for small networks, we show that the results obtained by these algorithms are close to the optimal off-line solution (given by the ILP for case 2), which does not need rerouting. Tables II and III show comparisons, in terms of the number of rejected requests, of our algorithms (*MSA* and *ESA*), *WLA* and without

rerouting ( $W/R$ ) against the optimal ILP formulation for small random networks with link density  $\alpha$  ( $N = 10$  with 20 requests and  $N = 12$  with 30 requests) and  $W = 4$ . We observe that rerouting performs better (though marginally, since the network is small and the number of requests are few) than without rerouting and our algorithms perform as good as (and at times better than)  $WLA$ .

TABLE II  
NUMBER OF REJECTED REQUESTS FOR  $N = 10$  AND 20 REQUESTS.

$\alpha$	$W/R$	$WLA$	$MSA$	$ESA$	$Optimal$
0.2	15	14	14	14	12
0.3	13	12	12	11	7
0.4	8	6	6	6	3
0.5	7	6	6	7	3
0.6	6	5	5	4	1

TABLE III  
NUMBER OF REJECTED REQUESTS FOR  $N = 12$  AND 30 REQUESTS.

$\alpha$	$W/R$	$WLA$	$MSA$	$ESA$	$Optimal$
0.2	23	22	22	22	21
0.3	25	25	25	25	24
0.4	12	11	11	10	6
0.5	5	3	3	3	2
0.6	4	3	3	3	0

## V. FINITE DURATION ON-LINE SRWA

Finite duration SRWA requests arrive to and depart from the network over time. Thus, any two lightpaths can share requests as long as they do not overlap in time. We use the algorithms of MDLDP as heuristics to solve the finite duration on-line SRWA problem.

We use the same scenarios as in Section III-C for our simulations. Figures 6-11 show comparisons of the performance of our algorithms ( $MSA$  and  $ESA$ ) with  $WLA$  in terms of the percentage of rejections. The given results are (a) for different number of requests with a constant arrival rate, and (b) for different arrival rates with a constant number of requests. A comparison of these algorithms with the case of no rerouting ( $W/R$ ) shows that rerouting of lightpaths decreases the percentage of rejections significantly. *In addition, we observe that both  $MSA$  and  $ESA$  perform similarly to  $WLA$ , which has much higher running time and memory requirements.* The need to have a small running time becomes more pronounced in an on-line setting, where the algorithm has to be invoked repeatedly whenever a request arrives.

## VI. CONCLUSIONS

In WDM optical networks, where lightpaths carry a tremendous amount of data, survivability is of paramount importance. In practice, lightpath requests arrive over time and a decision on whether to accept or deny a request should be made without any knowledge of the future requests. Therefore, it is necessary to have an on-line solution scheme with good performance to deal with survivable routing and wavelength assignment (SRWA). In this paper, we have studied on-line

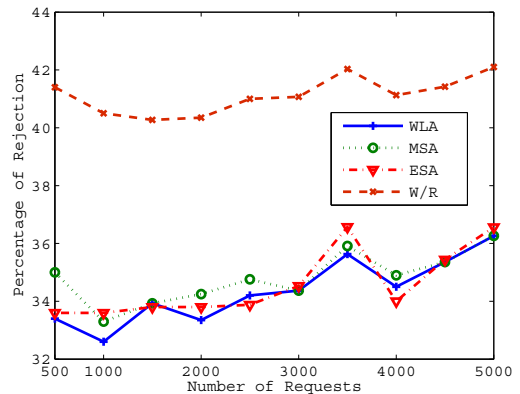


Fig. 6. Rejection rates of  $MSA$ ,  $ESA$ ,  $WLA$  and without rerouting for different number of requests for the ARPANET network. ( $W = 10$ ,  $r = 40$ )

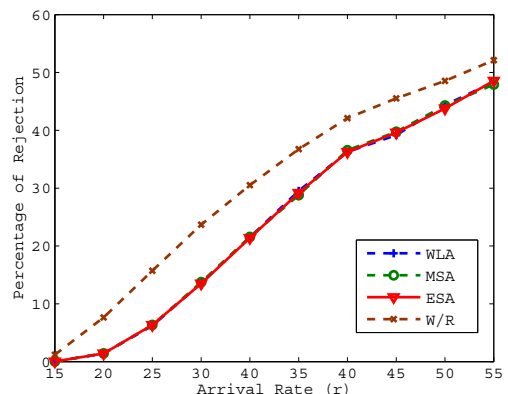


Fig. 7. Rejection rates of  $MSA$ ,  $ESA$ ,  $WLA$  and without rerouting for different arrival rates of the ARPANET network. ( $W = 10$  and 5000 requests)

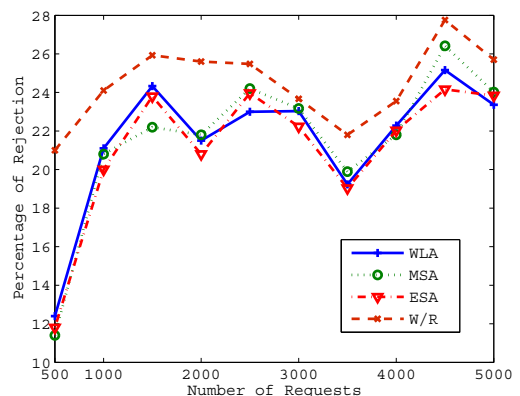


Fig. 8. Rejection rate of  $MSA$ ,  $ESA$ ,  $WLA$  and without rerouting for different number of requests for the random network. ( $N = 50$ ,  $\alpha = 0.2$ ,  $W = 10$ ,  $r = 120$ )



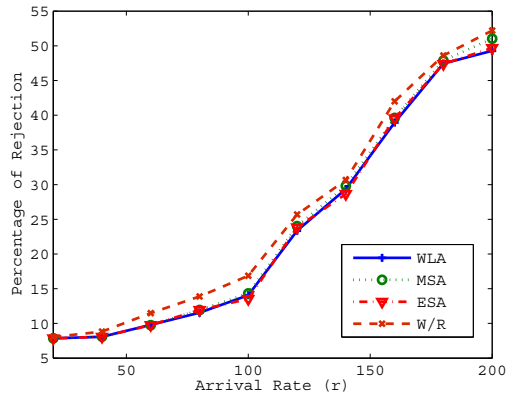


Fig. 9. Rejection rates of *MSA*, *ESA*, *WLA* and without rerouting for different arrival rates for the random network. ( $N = 50$ ,  $\alpha = 0.2$ ,  $W = 10$  and 5000 requests)

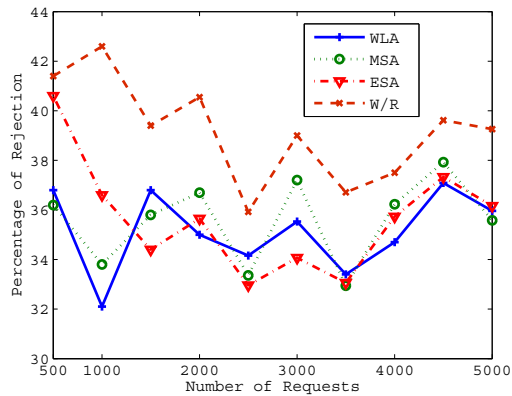


Fig. 10. Rejection rates of *MSA*, *ESA*, *WLA* and without rerouting for different number of requests for the lattice network. ( $N = 49$ ,  $W = 10$ ,  $r = 60$ )

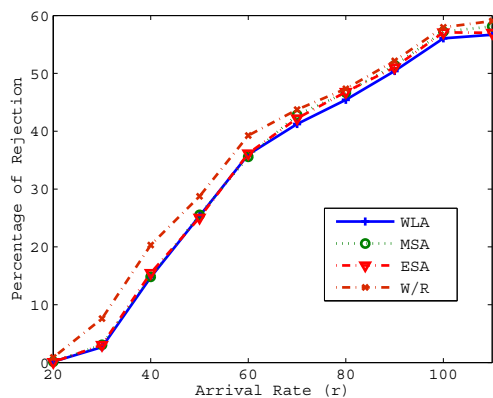


Fig. 11. Rejection rates of *MSA*, *ESA*, *WLA* and without rerouting for different arrival rates in the lattice network. ( $N = 49$ ,  $W = 10$  and 5000 requests)

SRWA and have provided constant and logarithmic competitive ratios for special networks. For general networks, it is not possible to have algorithms with good competitive ratios. Since the competitive ratio reflects a worst-case performance, we considered lightpath rerouting, which generally improves the acceptance rate, but not the competitive ratio. To serve this purpose, we studied the Minimum Disruption Link-Disjoint Paths (MDLDP) problem, for which we provided two 2-approximation algorithms: *MSA* and *ESA*. We have shown through simulations that these algorithms perform close to the best-known exact algorithm for MDLDP, which incurs a very high time-complexity. We subsequently applied all considered MDLDP algorithms as heuristics for infinite and finite duration on-line SRWA. For infinite duration SRWA, these algorithms performed close to the optimal off-line solution (for which we provided an ILP formulation). For finite duration SRWA, we considered Poissonian distributed input sequences with exponential holding times. In these scenarios, our algorithms performed as good as (and at times even better than) the exact algorithm of the MDLDP problem, but in a much faster time. These findings suggest that our algorithms are more suitable for dealing with the (on-line) SRWA problem.

#### ACKNOWLEDGEMENTS

This work has partly been supported by the GigaPort project, which is led by SURFnet and funded by the Dutch Ministry of Economic Affairs under grant number BSIK03020 and partly by the Next Generation Infrastructures foundation (<http://www.nginfra.nl>).

#### REFERENCES

- [1] B. Awerbuch, Y. Azar, and S. Plotkin, "Throughput-competitive on-line routing," *Proc. of the 34th IEEE FOCS*, pp. 32-40, 1993.
- [2] B. Awerbuch, Y. Azar, A. Fiat, and A. Rosen, "Competitive non-preemptive call control," *Proc. of the 5th ACM-SIAM Symp. on Discrete Algorithms*, pp. 312-320, 1994.
- [3] B. Awerbuch, Y. Azar, A. Fiat, S. Leonardi, and A. Rosen, "On-line competitive algorithms for call admission in optical networks," *Proc. of the 4th Annual European Symp. on Algorithms*, vol. 1136, pp. 431-444, 1996.
- [4] Y. Bartal, A. Fiat, and S. Leonardi, "Lower bounds for on-line graph problems with application to on-line circuit and optical routing," *Proc. of the 28th Annual ACM Symp. on the Theory of Computing*, pp. 531-540, 1996.
- [5] R.M. Karp, "On the computational complexity of combinatorial problems," *Networks*, vol. 5, no. 9, pp. 45-68, 1975.
- [6] J. Kleinberg and É. Tardos, "Disjoint paths in densely embedded graphs," *Proc. of the 36th Annual Symp. on Foundations of Computer Science*, pp. 52-61, Oct. 1995.
- [7] K.C. Lee and V.O.K. Li, "A wavelength rerouting algorithm in wide-area all-optical networks," *J. of Lightwave Tech.*, vol. 14, pp. 1218-1229, 1996.
- [8] G. Mohan and C. Murthy, "A time optimal wavelength rerouting algorithm for dynamic traffic in WDM networks," *J. of Lightwave Tech.*, vol. 17, pp. 406-417, 1999.
- [9] J. Suurballe, "Disjoint paths in a network," *Networks*, vol. 4, no. 2, pp. 125-145, 1974.
- [10] Y. Wan and W. Liang, "Wavelength rerouting in survivable WDM networks," *Proc. of Networking*, Lecture Notes in Computer Science, vol. 3462, pp. 431-442, 2005.
- [11] G. Xue, "Optimal lightpath routing and rerouting in WDM networks," *Proc. of GLOBECOM'01*, vol. 4, pp. 2124-2128, 2001.
- [12] A.A. Beshir, F.A. Kuipers, A. Orda, and P. Van Mieghem, "On-line survivable routing in WDM networks," Technical report, available at <http://www.nas.ewi.tudelft.nl/publications/2009/>