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A Coalitional MPC Approach to Control of Collaborative Vehicle Platoons

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Abstract: This work presents a coalitional model predictive controller for collaborative vehicle platoons. The overall system is modeled as a string of locally controlled vehicles that can share data through a wireless communication network. The vehicles can dynamically form disjoint groups that coordinate their actions, i.e., the so-called *coalitions*. The control goals are keeping a desired reference distance between all vehicles while allowing for occasional switching of the communication topology. Likewise, the presented controller promotes a string-stable evolution of the platoon system. Numerical results are provided to illustrate the proposed approach.

Keywords: Model predictive control, vehicle platoons, multi-agent systems, string stability.

1. INTRODUCTION

Distributed model predictive control (DMPC) has proved to be a valuable technique for addressing problems involving sets of interacting subsystems. By exchanging data, local MPC controllers can coordinate their local actions and thus optimize their collective performance (Camponogara et al., 2002). One of its promising application fields is that of collaborative vehicle platoons (CVPs) (van der Sande and Nijmeijer, 2017), i.e. vehicles that use wireless communication to form a platoon and follow a leader vehicle. CVPs are heralded as a potential answer to pollution and traffic jams problems (Wang et al., 2015; He et al., 2020).

To allow vehicles in CVPs to optimize their maneuvers without compromising safety, DMPC methods such as the ones in Zheng et al. (2016) and Liu et al. (2018) have been developed. Still, as stressed by Li et al. (2020), Wang et al. (2022) and Abou Harfouch et al. (2017), there is a need for controllers that are scalable and can accommodate switching communication topologies, while providing performance and theoretical guarantees. There are relevant reasons motivating the study of switching topologies in CVPs. For instance, the communication links can fail, and there may be a limited distance range for effective information sharing. Also, vehicles entering or leaving the platoon would similarly induce changes in the communication topology.

Following this line of research, this work proposes the application to CVPs of the *coalitional* MPC strategy described in Fele et al. (2017), Baldivieso-Monasterios and Trodden (2021), and Chanfreut et al. (2021). *Coalitional* MPC can be seen as a variant of DMPC characterized by the dynamic formation of clusters of cooperative agents, the so-called *coalitions*. The underlying goal is to alleviate the computation and communication demands in comparison with fully coordinated DMPC approaches without compromising the overall performance. To this end, the communication topology is dynamically adjusted so that coordination is limited to subsets of agents. The flexibility of this strategy to accommodate changing communication topologies can also be exploited to address other issues, for example, to deal with communication failures.

In the framework of CVPs, two critical control goals are ensuring collision avoidance regardless of the uncertainties on the neighbouring vehicles actions, and guaranteeing string stability (Ploeg et al., 2014; Dolk et al., 2017). Considering this, the main contributions of this paper are:

- The design of a coalitional MPC for CVPs with safety guarantees to track reference relative distances. In particular, the proposed controller allows for switchings between different communication topologies, and the absence of collisions is proved analytically using a scenario-based approach.
- The derivation of a linear constraint that enforces strict string stability within each coalition. i.e., the control law is designed such that the control effort does not increase as we move along groups of vehicles.
- The guarantee of recursive feasibility for the MPC problem, which holds under mild assumptions regardless of the switching communication topology.

The rest of the paper is organized as follows. Section 2 describes the vehicles dynamics, the concept of coalitions, and the control goals. Section 3 presents the proposed coalitional MPC controller for CVPs, and its safety and string stability properties are proven in Section 4. Section 5

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then presents numerical results and, finally, Section 6 provides conclusions and future research directions.

2. PROBLEM FORMULATION

Consider a set $\mathcal{N} = \{1, ..., N\}$ of locally controlled vehicles which follow a leader and whose goal is to keep a reference distance from the preceding vehicle (see Fig. 1). In particular, for each vehicle $i \in \mathcal{N}$, this reference distance is defined as follows (Ploeg et al., 2014):

$$d_{r,i} \triangleq r + hv_i, \tag{1}$$

where v_i is its velocity, h the reference time headway, and r the reference distance at standstill. The continuous-time dynamics of each vehicle $i \in \mathcal{N}$ can be described as

$$\begin{cases} \dot{e}_{\mathrm{d},i} &= -ha_i + \Delta v_i, \\ \dot{d}_i &= \Delta v_i, \\ \dot{v}_i &= a_i, \\ \dot{a}_i &= \frac{1}{\tau}(u_i - a_i), \\ \Delta\dot{v}_i &= a_{i-1} - a_i, \end{cases} \tag{2}$$

where $e_{\mathrm{d},i} \triangleq d_i - d_{r,i}$ denotes its tracking error, with d_i being the distance between vehicles i and $i-1$. Also, a_i denotes the acceleration of vehicle i, $\Delta v_i \triangleq v_{i-1} - v_i$ is its relative velocity, and u_i represents the input. Furthermore, τ is the time constant of the vehicles' drive-train which, for simplicity, is assumed to be the same for all $i \in \mathcal{N}$. In (2), it should be noted that each car i is only coupled with its predecessor $i-1$ through a_{i-1}. Also, note that the leader is assumed to be independent, meaning that it sets its actions regardless of its follower vehicles.

In this work, we will use a discrete-time state-space representation of (2), i.e.,

$$\begin{aligned} x_i(k+1) =& A_{i,i}x_i(k) + B_{i,i}u_i(k) \\ &+ \underbrace{A_{i,i-1}x_{i-1}(k) + B_{i,i-1}u_{i-1}(k)}_{w_i(k)}, \end{aligned} \tag{3}$$

where $x_i(k) = [e_{\mathrm{d},i}(k),\ d_i(k),\ v_i(k),\ a_i(k),\ \Delta v_i(k)]^\top$ is the state vector, k is the time index, and $A_{i,i}$, $B_{i,i}$, $A_{i,i-1}$, and $B_{i,i-1}$ are obtained by discretizing (2). Also, $w_i(k)$ represents the coupling of vehicle i with $i-1$.

Assumption 1. The input of vehicle $i \in \mathcal{N}$ is constrained by $|u_i(k)| \leq u_{\max}$ for all $k \geq 0$, where $u_{\max} \in \mathbb{R}^+$. ◁

2.1 Communication topologies

We assume that the vehicles are interconnected by a linear wireless network with bidirectional links, i.e., any vehicles $i, i+1 \in \mathcal{N}$ can send and receive information to/from the other. Additionally, we consider multi-hop communication, meaning that information may be relayed by intermediate vehicles to others further up or down the platoon.

The links can be enabled/disabled as in Fele et al. (2017), thus imposing different communication topologies, say Λ. The latter induces a partition $\mathcal{P}_\Lambda = \left\{\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_{N_{\mathcal{C}_\Lambda}}\right\}$ of the vehicles into coalitions $\mathcal{C}_i$ as shown in Fig. 1. Note that set $\mathcal{C}_i$ contains the indices of the vehicles in the i-th coalition, and $N_{\mathcal{C}_\Lambda}$ denotes the number of coalitions under communication topology Λ.

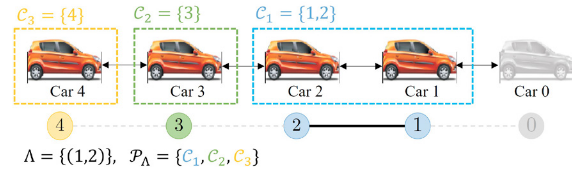

Fig. 1. Topology and resulting coalitions in an platoon with 4 vehicles following a leader.

Assumption 2. For any partition $\mathcal{P}_\Lambda = \left\{\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_{N_{\mathcal{C}_\Lambda}}\right\}$, coalitions $\mathcal{C}_i \in \mathcal{P}_\Lambda$ are non-overlapping sets of consecutive vehicles and satisfy $\cup_{i=1}^{N_{\mathcal{C}_\Lambda}} \mathcal{C}_i = \mathcal{N}$. Additionally, vehicles $j \in \mathcal{C}_i$ share their local state x_j and coordinate their control actions.◁

2.2 Coalitions model

Let us define the aggregation of the states and inputs of all vehicles in a given coalition $\mathcal{C}$ as

$$x_\mathcal{C} = [x_j]_{j\in\mathcal{C}} \quad , \quad u_\mathcal{C} = [u_j]_{j\in\mathcal{C}} \, .$$

Also, let $p_\mathcal{C} = \min_{j\in\mathcal{C}}(j-1)$ be the index of the car preceding coalition $\mathcal{C}$, hence $x_{p_\mathcal{C}}$ and $u_{p_\mathcal{C}}$ are the state and input of car $p_\mathcal{C}$, respectively. Considering this, the overall model of coalition $\mathcal{C}$ can be defined as

$$x_\mathcal{C}(k+1) = A_\mathcal{C}x_\mathcal{C}(k) + B_\mathcal{C}u_\mathcal{C}(k) + w_\mathcal{C}(k)\, , \tag{4}$$

where $A_\mathcal{C}$ and $B_\mathcal{C}$ are built by aggregating (3) to capture the dynamics of all vehicles $j \in \mathcal{C}$. Also, $w_\mathcal{C} = [w_{\min(\mathcal{C})}(k), 0, ..., 0]^\top$ represents the coupling between the first vehicle in coalition $\mathcal{C}$, i.e., $\min(\mathcal{C})$, and vehicle $p_\mathcal{C}$ preceding the coalition. Notice that, given (2), variable $w_\mathcal{C}$ depends only on the acceleration and input of the preceding car, i.e., $a_{p_\mathcal{C}}$ and $u_{p_\mathcal{C}}$. Likewise, note that $A_\mathcal{C}$ and $B_\mathcal{C}$ account for the coupling between vehicles in $\mathcal{C}$.

2.3 Control goals

We aim to minimize the tracking error while avoiding crashes even if the communication topology switches. Moreover, the controller must promote string-stability of the platoon. In this regard, based on Ploeg et al. (2014), we consider the following definitions:

Definition 1. (Strict string stability). A vehicle platoon is strictly string stable if

$$\left|\frac{v_i(k_0+k_1) - v_i(k_0)}{v_{i-1}(k_0+k_1) - v_{i-1}(k_0)}\right| < 1 \;\forall k_0, k_1, i\, . \qquad ◁$$

This definition can be relaxed to allow for bounded violations of strict string stability as follows:

Definition 2. (Relaxed string stability). A vehicle platoon is relaxed string stable if

$$\exists\, i, j < i, \text{ s.t. } \left|\frac{v_i(k_0+k_1) - v_i(k_0)}{v_j(k_0+k_1) - v_j(k_0)}\right| < 1 \;\forall k_0, k_1\, . \quad ◁$$

Hereafter, let us consider the following MPC problem, which represents the problem that each coalition $\mathcal{C}$ should *ideally* solve at each time instant k to determine its input:

$$\min_{\mathbf{u}_\mathcal{C}} \sum_{n=k}^{k+N_\mathrm{p}-1} \left(\|x_\mathcal{C}(n+1|k)\|^2_{Q_\mathcal{C}} + \|\Delta u_\mathcal{C}(n|k)\|^2_{R_\mathcal{C}}\right) \tag{5a}$$

s.t.

$$x_{\mathcal{C}}(k|k) = x_{\mathcal{C}}(k), \tag{5b}$$

$$x_{\mathcal{C}}(n+1|k) = A_{\mathcal{C}}x_{\mathcal{C}}(n|k) + B_{\mathcal{C}}u_{\mathcal{C}}(n|k) + w_{\mathcal{C}}(n|k), \tag{5c}$$

$$|u_i(n|k)| \leq u_{\max}\,,\ \forall i \in \mathcal{C}\,, \tag{5d}$$

$$d_i(n+1|k) \geq 0,\ \forall i \in \mathcal{C}\,, \tag{5e}$$

$$\left|\frac{v_i(n+1|k) - v_i(k|k)}{v_{i-1}(n+1|k) - v_{i-1}(k|k)}\right| < 1, \forall i \in \mathcal{C} \setminus \min(\mathcal{C}), \tag{5f}$$

$$\forall n = k, \ldots, k + N_{\mathrm{p}} - 1.$$

Above, $Q_{\mathcal{C}} = [Q_i]_{i\in\mathcal{C}}$ and $R_{\mathcal{C}} = [R_i]_{i\in\mathcal{C}}$ are positive definite weighting matrices defined as the block-diagonal aggregation of Q_i and R_i, respectively. Furthermore, $(n|k)$ denotes the prediction for time-step n made at k, and N_{p} is the length of the prediction horizon. Also, $\mathbf{u}_{\mathcal{C}} = [u_{\mathcal{C}}^\top(k|k), \ldots, u_{\mathcal{C}}^\top(k+N_{\mathrm{p}}-1|k)]^\top$ is the sequence of inputs of coalition $\mathcal{C}$, and $\Delta u_{\mathcal{C}}(n|k)$ is defined as $\Delta u_{\mathcal{C}}(n|k) = u_{\mathcal{C}}(n|k) - u_{\mathcal{C}}(n-1|k)$. [1] Note that (5f) is not applied to the first vehicle of every coalition, i.e., $\min(\mathcal{C})$. Therefore, it only guarantees strict string stability within each coalition and not between coalitions. It will be shown that, together with the proposed topology switching rule, it is possible to guarantee relaxed string stability over the entire platoon.

At time step k, all vehicles $i \in \mathcal{C}$ compute in a cooperative distributed manner sequence $\mathbf{u}_{\mathcal{C}}$, and implement the resulting local input $u_i(k|k)$. Note that $u_{\mathcal{C}}(k|k) = [u_i(k|k)]_{i\in\mathcal{C}}$. To optimize $\mathbf{u}_{\mathcal{C}}$, the vehicles share data through the enabled communication links interconnecting them.

The *ideal* MPC Problem (5) cannot be directly solved because (5c) is influenced by the unknown effect of $w_{\mathcal{C}}$. Moreover, this optimization problem has significant drawbacks for real-time control. Firstly, (5f) is non-linear; and, secondly, there are in principle no guarantees that (5e) and (5f) can be recursively satisfied. In what follows, we introduce a reformulation of Problem (5) which, under mild assumptions, results in a recursively feasible quadratic optimization with linear constraints that guarantees safety.

3. TOPOLOGY SWITCHING CONTROLLER

This section first introduces a scenario-based MPC approach to deal with the uncertainty on the preceding car. Subsequently, we present a linear reformulation of the safety and string-stability constraints in (5), and, finally, we describe the communication topology switching rule.

3.1 Uncertainty Scenarios

Each vehicle in coalition $\mathcal{C}$ considers a set of S realizations of the unknown input implemented by vehicle $p_{\mathcal{C}}$, i.e.,

$$\hat{\mathbf{u}}_{p_{\mathcal{C}},s} = [\hat{u}_{p_{\mathcal{C}},s}(k|k), ..., \hat{u}_{p_{\mathcal{C}},s}(k+N_{\mathrm{p}}-1|k)], \tag{6}$$

where $s \in \mathcal{S} = \{1, \ldots, S\}$. Along with this, a prediction of the acceleration of car $p_{\mathcal{C}}$ is computed by

$$a_{p_{\mathcal{C}},s}(n+1|k) = \alpha_{p_{\mathcal{C}}} a_{p_{\mathcal{C}},s}(n|k) + \beta_{p_{\mathcal{C}}} \hat{u}_{p_{\mathcal{C}},s}(n|k)\,, \tag{7a}$$

$$a_{p_{\mathcal{C}},s}(k|k) = a_{p_{\mathcal{C}},s}(k|k-1)\,, \tag{7b}$$

where $\alpha_{p_{\mathcal{C}}}$ and $\beta_{p_{\mathcal{C}}}$ are given by (3). Each of the scenarios defined by (6) and (7) leads to different predictions on variable $w_{\mathcal{C}}$ (see (5c)). Considering this, let us define the prediction model of coalition $\mathcal{C}$ in scenario $s \in \mathcal{S}$ as

$$x_{\mathcal{C},s}(n+1|k) = A_{\mathcal{C}}x_{\mathcal{C},s}(n|k) + B_{\mathcal{C}}u_{\mathcal{C}}(n|k) + w_{\mathcal{C},s}(n|k), \tag{8}$$

where $n = k, ..., k + N_{\mathrm{p}} - 1$ and $w_{\mathcal{C},s}(\cdot|k)$ is determined from (6) and (7).

The set of considered scenarios is divided into two categories. First, a subset of *extreme* scenarios, indexed by $\mathcal{S}_{\mathrm{e}}$, will be used to guarantee string stability and safety in all possible situations. In particular,

$$\mathcal{S}_{\mathrm{e}} = \{s \in \mathcal{S}\ :\ |\hat{u}_{p_{\mathcal{C}},s}(n|k)| = u_{\max} \text{ if } \ v_{p_{\mathcal{C}}}(n|k) \in (0, v_{\max}],\ |\hat{u}_{p_{\mathcal{C}},s}(n|k)| = 0 \quad \text{otherwise}\quad n = k, ..., k+N_{\mathrm{p}}-1\}\,,$$

where $v_{\max}$ is the maximum velocity that the vehicles can reach. Note that these scenarios imply that the preceding vehicle implements its maximum or minimum acceleration while staying within allowed velocity range. Secondly, $\mathcal{S}_{\mathrm{d}}$ denotes a subset of *design* scenarios defined as

$$\mathcal{S}_{\mathrm{d}} = \{s \in \mathcal{S}\ :\ |\hat{u}_{p_{\mathcal{C}},s}(n|k)| < u_{\max},\ n = k, ..., k + N_{\mathrm{p}} - 1\}\,,$$

which will be used to optimize the platoon tracking performance. These sets are defined such that $\mathcal{S} = \mathcal{S}_{\mathrm{e}} \cup \mathcal{S}_{\mathrm{d}}$.

3.2 Safety constraints

The safety constraint $d_i(k) \geq 0, \forall i \in \mathcal{N}$, and $k \geq 0$, must be recursively satisfied even if the preceding vehicle brakes abruptly. This may not be possible if the vehicles are not sufficiently spaced apart or if they approach the preceding car with a high speed and/or acceleration differential. For this reason, we introduce the following bounds on $d_{i,s}(\cdot)$ for scenarios $s \in \mathcal{S}_{\mathrm{e}}$ and for all $i \in \mathcal{C}$:

$$d_{i,s}(k+1|k) \geq 0, \tag{9a}$$

$$d_{i,s}(k+1|k) \geq -\Delta v_{i,s}(k+1|k)\delta(k+1|k), \tag{9b}$$

$$d_{i,s}(k+1|k) \geq -\Delta v_{i,s}(k+1|k)\delta(k+1|k) - \tau\Delta a_{i,s}(k+1|k)\delta(k+1|k), \tag{9c}$$

where $\Delta a_{i,s} = a_{i-1,s} - a_{i,s}$ and $\delta(n|k) = \max(\gamma(k) - (n-k)T, 0)$, with T being the sampling time and

$$\gamma(k) = \frac{v_{i,s}(k|k) + \tau(a_{i,s}(k|k) + u_{\max})}{u_{\max}}\,.$$

Note that, as used before, subscript s indicates the value of the corresponding variable in scenario s. Also, notice that $\delta(n|k)$ represents the predicted time to standstill from instant n when car i decelerates from instant k implementing extreme input $-u_{\max}$. In particular, the expression of $\gamma(k)$ has been obtained considering model (2) and the fact that at standstill the cars speed is null. Formal proofs of the safety guarantee attained by using constraints (9) are given in Section 4.1.

Remark 1. Constraints (9) are based on the physical relation $d_i(n) - d_i(k) = \sum_{l=n}^{k} \Delta v_i(l)T$. To obtain them, $\Delta v_i(l)$ is bounded assuming $u_i(l) = u_{i-1}(l) = -u_{\max}$. ◁

Assumption 3. If the system is not at standstill at time n, then $\delta(n|k) = \gamma(k) - (n-k)T$. ◁

3.3 String stability constraints

In order to rewrite the string stability condition in (5) as a linear constraint, we first introduce constraints

$$\mathrm{sgn}(\Delta v_{i,s}(n|k)) = \mathrm{sgn}(\mathrm{d}v_{i,s}(k|k))\,, \tag{10}$$

[1] For the limit case $\Delta u_{\mathcal{C}}(k|k) = u_{\mathcal{C}}(k|k) - u_{\mathcal{C}}(k-1|k)$, it will be considered that $u_{\mathcal{C}}(k-1|k) = u_{\mathcal{C}}(k-1)$.

for all $n = k, \ldots, k+N_\mathrm{p}$, where $\mathrm{d}v_{i,s}(k|k) = v_{i,s}(k+N_\mathrm{p}|k) - v_{i,s}(k|k)$ denotes the change of velocity of vehicle i over the prediction horizon.

Proposition 1. Consider an acceleration or deceleration maneuver that starts from a state such that $\Delta v_i(k) = 0^+$ or $\Delta v_i(k) = 0^-$. Then, satisfying (10) implies

$$\left|\frac{v_{i,s}(n+1|k) - v_{i,s}(k|k)}{v_{i-1,s}(n+1|k) - v_{i-1,s}(k|k)}\right| < 1,\ \forall n = k, \ldots, k+N_\mathrm{p}-1\,.$$

Proof. If $\Delta v_i(k) = 0^+$, then (10) imposes $\Delta v_{i,s}(n|k) > 0$ for all $n = k, ..., k+N_\mathrm{p}$. Likewise, $\mathrm{d}v_{i-1,s}(k|k) - \mathrm{d}v_{i,s}(k|k) \approx \Delta v_{i,s}(k+N_\mathrm{p}|k)$, and thus $\mathrm{d}v_{i-1,s}(k|k) > \mathrm{d}v_{i,s}(k|k) > 0$. Conversely, if $\Delta v_i(k) = 0^-$, then $\mathrm{d}v_{i-1,s}(k|k) < \mathrm{d}v_{i,s}(k|k) < 0$. □

Constraint (10) is still not linear, but we can use the following relation:

$$v_{i,s}(k+N_\mathrm{p}|k) - v_{i,s}(k|k) = \mathrm{d}v_{i,s}^{\mathrm{pos}} + \mathrm{d}v_{i,s}^{\mathrm{neg}}\,, \tag{11a}$$

$$\mathrm{d}v_{i,s}^{\mathrm{pos}} \geq 0,\ \mathrm{d}v_{i,s}^{\mathrm{neg}} \leq 0\,, \tag{11b}$$

where $\mathrm{d}v_{i,s}^{\mathrm{pos}}$ and $\mathrm{d}v_{i,s}^{\mathrm{neg}}$ denote positive and negative components of $\mathrm{d}v_{i,s}(k|k)$. In this regard, assume that $\mathrm{d}v_{i,s}^{\mathrm{pos}}$ and $\mathrm{d}v_{i,s}^{\mathrm{neg}}$ do not cancel each other out, that is, if $\mathrm{d}v_{i,s}(k|k)$ is positive, then $\mathrm{d}v_{i,s}^{\mathrm{neg}} = 0$ and $\mathrm{d}v_{i,s}(k|k) = \mathrm{d}v_{i,s}^{\mathrm{pos}}$. To this end, we will add a penalization on the objective function of the form $\xi(\mathrm{d}v_{i,s}^{\mathrm{pos}} - \mathrm{d}v_{i,s}^{\mathrm{neg}})$, with $\xi > 0$ being a weighting parameter. Using (11), (10) can be imposed by using the following linear constraint:

$$\gamma \mathrm{d}v_{i,s}^{\mathrm{neg}}(k|k) \leq \Delta v_{i,s}(n|k) \leq \gamma \mathrm{d}v_{i,s}^{\mathrm{pos}}(k|k), \tag{12}$$

where γ is chosen sufficiently large.

The resulting linear constraint, however, may not be recursively satisfied. In particular, if we impose (12) and $\mathrm{d}v_{i,s}(k|k)$ is positive, then $\Delta v_{i,s}(n|k)$ is constrained to be positive for all n. Furthermore, the sign of $\Delta v_{i,s}(k|k)$, which is part of the initial conditions of the problem, fixes that of $\mathrm{d}v_{i,s}(k|k)$. As a result, $\mathrm{d}v_{i,s}$ and $\Delta v_{i,s}$ cannot change sign. To circumvent this issue, we use relation $d_{i,s}(k+N_\mathrm{p}|k) - d_{i,s}(k|k) = \sum_{n=k}^{k+N_\mathrm{p}-1} \Delta v_{i,s}(n|k)T$, to propose the following reformulation:

$$\begin{aligned} N_\mathrm{p}T\gamma\, \mathrm{d}v_{i,s}^{\mathrm{neg}} - \epsilon_s &\leq d_{i,s}(k+N_\mathrm{p}|k) - d_{i,s}(k|k) \\ &\leq N_\mathrm{p}T\gamma\, \mathrm{d}v_{i,s}^{\mathrm{pos}} + \epsilon_s\,, \end{aligned} \tag{13}$$

where ϵ_s is a slack variable, turning the hard constraint into a soft one. Note that (13) is equivalent to (12) if the sign of $\Delta v_{i,s}(n|k)$ is constant over the prediction horizon and $\epsilon_s = 0$. Furthermore, note that if the sign of $\mathrm{d}v_{i,s}$ is constant, there always exists a sequence $\Delta v_{i,s}(n|k)\ \forall n$ of constant sign such that (13) holds.

3.4 Topology Switching Rule

In this paper, the communication topology and the coalitions are dynamically updated to attain a trade-off between optimal performance and coordination efforts. In particular, they are selected according to the tracking error and relative velocities as described in Algorithm 1. Note that, while Algorithm 1 could be implemented by a *central* coordinator, centralized computations are not needed. Indeed, each car decides when to enable/disable the link with the vehicle in front according to the mentioned criteria.

Algorithm 1 Topology Switching Rule

Initialize: Set auxiliary variable j to 1, define thresholds T_v and T_d, and define an initial coalition $\mathcal{C}_1 = \{1\}$. Then,
  for all vehicles $i = 2, 3, \ldots, N$ **do**
    If $|\Delta v_i(k)| > T_\mathrm{v}$ or $|e_{\mathrm{d},i}(k)| > T_\mathrm{d}$, define $\mathcal{C}_j = \{\mathcal{C}_j, i\}$.
    Otherwise, set $j = j+1$ and $\mathcal{C}_j = \{i\}$.
  end for

4. CONTROL SCHEME PROPERTIES

At each time instant k, each vehicle calculates its control input following Algorithm 2. Note that vehicles $i \in \mathcal{C}$ can solve (14) without exchanging any data with vehicles $j \notin \mathcal{C}$, and thus the coalitions can work fully in parallel.

Algorithm 2 Control scheme

At each sample time k the system proceed as follows:
1: Run Algorithm 1 to define the partition into coalitions.
2: **for all** coalitions $\mathcal{C} \in \mathcal{P}_\Lambda$ **do**
3: The vehicles in $\mathcal{C}$ define the set of scenarios on $p_\mathcal{C}$.
4: All vehicles in $\mathcal{C}$ solve jointly the MPC problem:

$$\begin{aligned} \min_{\substack{\mathbf{u}_\mathcal{C}, \mathrm{d}v_{i,\mathrm{pos}},\\ \mathrm{d}v_{i,\mathrm{neg}}, \epsilon_s}} \ & J_\mathcal{C}(y_\mathcal{C}(k), \mathbf{u}_\mathcal{C}) + \sum_{i\in\mathcal{C}} \sum_s (\xi(\mathrm{d}v_{i,s}^{\mathrm{pos}} - \mathrm{d}v_{i,s}^{\mathrm{neg}}) + \zeta\epsilon_s) \\ \text{s.t.}\ & x_{\mathcal{C},s}(k|k) = x_\mathcal{C}(k), \forall s \in \mathcal{S}, \\ & (7), (8), \forall s \in \mathcal{S}, \\ & |u_i(n|k)| \leq u_{\max}, \forall i \in \mathcal{C}, \\ & (9), (11), \forall s \in \mathcal{S}_\mathrm{e},\ \forall i \in \mathcal{C}, \\ & (13), \forall s \in \mathcal{S}_\mathrm{e},\ \forall i \in \mathcal{C} \setminus \min(\mathcal{C}), \\ & \forall n = k, ..., k+N_\mathrm{p}-1, \end{aligned} \tag{14}$$

where ξ and ζ are weighting factors. Also, $J_\mathcal{C}(y_\mathcal{C}(k), \mathbf{u}_\mathcal{C})$ is given by

$$\sum_{n=k}^{k+N_\mathrm{p}-1} \left(\sum_{s\in\mathcal{S}} p_s \|x_{\mathcal{C},s}(n+1|k)\|_{Q_\mathcal{C}}^2 + \|\Delta u_\mathcal{C}(n|k)\|_{R_\mathcal{C}}^2 \right),$$

with $p_s > 0$ the probability assigned to scenario s.
5: All vehicles in $\mathcal{C}$ define $u_i(k)$ as the corresponding element of the optimized variable $u_\mathcal{C}^*(k|k)$.
6: **end for**
7: All vehicles implement $u_i(k)$, and their state is updated according to model (3).

4.1 Safety Properties

It will be proven that, under mild assumptions, the overall platoon is safe against collisions regardless of the topology switching. In this respect, let us introduce the following assumptions and remark.

Assumption 4. The set of scenarios are defined such that if (9) holds at time instant k, then

$$d_i(k+1) \geq 0, \tag{15a}$$

$$d_i(k+1) \geq -\Delta v_i(k+1)\delta(k+1|k), \tag{15b}$$

$$d_i(k+1) \geq -\Delta v_i(k+1)\delta(k+1|k) - \tau\Delta a_i(k+1)\delta(k+1|k) \tag{15c}$$

◁

Assumption 5. For a sufficiently small sample time T, $\delta(k+2|k) \approx \delta(k+2|k+1)$. ◁

Remark 2. Regarding Assumption 4, note that (9) is considered by all vehicles $i \in \mathcal{C}$ at every step k for the *extreme* scenarios. Therefore, the vehicles consider the case in which their predecessor brakes by applying the minimum possible input, which is the most conservative situation.

Lemma 1. If Constraints (9) hold at time instant k, then there exists an input $u_i(k+1|k+1) = u_i(k+1)$ such that

$$\begin{aligned} &d_{i,s}(k+2|k+1) \geq 0, \\ &d_{i,s}(k+2|k+1) \geq -\Delta v_{i,s}(k+2|k+1)\delta(k+2|k+1), \\ &d_{i,s}(k+2|k+1) \geq -\Delta v_{i,s}(k+2|k+1)\delta(k+2|k+1) \\ &\qquad - \tau\Delta a_{i,s}(k+2|k+1)\delta(k+2|k+1). \end{aligned} \tag{16}$$

Proof. Below, we analyze the inequalities in (15) according to the signs of variables $\Delta v_i(k+1)$ and $\Delta a_i(k+1)$, showing that each of the inequalities in (15) can only be active in a subset of the state-space. Let us consider following relations for all $s \in \mathcal{S}$:

$$\begin{aligned} &d_{i,s}(k+2|k+1) = d_i(k+1) + T\Delta v_i(k+1), \\ &\Delta v_{i,s}(k+2|k+1) = \Delta v_i(k+1) + T\Delta a_{i,s}(k+1), \\ &\tau a_{i,s}(k+2|k+1) = (\tau - T)a_i(k+1) + Tu_i(k+1), \end{aligned} \tag{17}$$

where T denotes the sample time as in (9).

First, consider that $\Delta v_i(k+1) \geq 0$ and $\Delta a_i(k+1)$ can either be positive or negative. Since inequalities (15a) are satisfied, using (17), we have $d_{i,s}(k+2|k+1) \geq T\Delta v_i(k+1) \geq 0$. That is, even if $d_i(k+1) = 0$, a positive relative velocity guarantees that $d_{i,s}(k+2|k+1) \geq 0$.

Secondly, consider that $\Delta v_i(k+1) \leq 0$ and $\Delta a_{i,s}(k+1) \geq 0$. In this case, the more restrictive inequality is given by (15b). Since it must be satisfied,

$$\begin{aligned} d_{i,s}(k+2|k+1) &\geq -\Delta v_i(k+1)\delta(k+1|k) + T\Delta v_i(k+1) \\ &= -\Delta v_i(k+1)(\delta(k+1|k) - T) \\ &= -\left(\Delta v_{i,s}(k+2|k+1) - T\Delta a_{i,s}(k+1)\right)\left(\delta(k+1|k) - T\right). \end{aligned}$$

Note that if $\Delta v_i(k+1) \leq 0$, then the subsystem i cannot be at standstill, and hence by Assumption 3,

$$\begin{aligned} &d_{i,s}(k+2|k+1) \\ &\geq -\left(\Delta v_{i,s}(k+2|k+1) - T\Delta a_{i,s}(k+1)\right)\left(\gamma(k) - 2T\right) \\ &\geq -\Delta v_{i,s}(k+2|k+1)\delta(k+2|k). \end{aligned}$$

Considering that for a sufficiently small T, we can assume $\delta(k+2|k) \approx \delta(k+2|k+1)$ (see Assumption 5), we can derive

$$d_{i,s}(k+2|k+1) \geq -\Delta v_{i,s}(k+2|k+1)\delta(k+2|k+1).$$

Finally, consider that $\Delta v_i(k+1) < 0$ and $\Delta a_{i,s}(k+1) < 0$. Then, the only constraint that can be active is (15c). In particular, considering Assumption 3,

$$\begin{aligned} &d_{i,s}(k+2|k+1) \\ &\geq -(\Delta v_i(k+1) + \tau\Delta a_{i,s}(k+1))\delta(k+1|k) + T\Delta v_i(k+1) \\ &\geq -(\Delta v_i(k+1) + \tau\Delta a_{i,s}(k+1))\delta(k+1|k) \\ &\quad + (\Delta v_i(k+1) + \tau\Delta a_{i,s}(k+1))T \\ &= -(\Delta v_i(k+1) + \tau\Delta a_{i,s}(k+1))\delta(k+2|k). \end{aligned}$$

From here, it follows, using (17), that

$$\begin{aligned} &d_{i,s}(k+2|k+1) \\ &\geq -\left(\Delta v_{i,s}(k+2|k+1) - T\Delta a_{i,s}(k+1)\right)\delta(k+2|k) \\ &\quad - \left(\tau\Delta a_{i,s}(k+2|k+1) + T\Delta a_{i,s}(k+1)\right)\delta(k+2|k) \\ &\quad + T\Delta u_i(k+1)\delta(k+2|k), \end{aligned}$$

where $\Delta u_i(k+1) = u_{i-1}(k+1) - u_i(k+1)$. From the viewpoint of car i, the input of car $i-1$ can be known and compute cooperatively if they belong in the same coalition. Otherwise, it will be determined by the scenarios. Nonetheless, it is always possible to find a solution such that $\Delta u_i(k+1) \geq 0$, e.g., setting $u_i(k+1) = -u_{\max}$. Then

$$\begin{aligned} &d_{i,s}(k+2|k+1) \\ &\geq -\left(\Delta v_{i,s}(k+2|k+1) + \tau\Delta a_{i,s}(k+2|k+1)\right)\delta(k+2|k). \end{aligned}$$

Considering again that $\delta(k+2|k) \approx \delta(k+2|k+1)$, the existence of a possible input such that the third inequality in (9b) can also be satisfied is demonstrated. □

Finally, notice that constraints (9) consider only the first step of the prediction horizon to reduce conservatism in the inputs computation. Likewise, they are the only hard inequalities, together with $|u_i(\cdot)| \leq u_{\max}$ involved in problem (14). For these reasons, the proof above also implies there will exist a feasible solution of (14) at time step $k+1$, i.e. recursive feasibility is guaranteed.

4.2 String Stability Properties

Lemma 2. Using the controller in Algorithm 2 the CVP is relaxed string stable.

Proof. By design, the coalitional MPC achieves strict string stability within each coalition. This leaves to prove that the violation of string stability between coalitions is bounded. By the switching law in Algorithm 1

$$v_j(k_2) - v_j(k_1) - (v_{j-1}(k_2) - v_{j-1}(k_1)) = \Delta v_j(k_1) - \Delta v_j(k_2) \leq 2T_v$$

for all $k_1 > 0, k_2 > k_1, j \in \mathcal{N}$. □

5. SIMULATION RESULTS

In this section, we illustrate the Algorithm 2 by using a system as in Fig. 1, with four vehicles following a leader. The input of the leader evolves as shown in the bottom plot of Fig. 2 where one can see deceleration and acceleration of the leader happen at 1 and 6 seconds, respectively. The parameters used in the simulation are given in Table 1. Here the distance reference is defined such that $e_{\mathrm{d},i} = 0$ for all $i \in \{1, 2, 3, 4\}$ does not violate any constraints.

Fig. 2 shows the states of all vehicles, and Fig. 3 illustrates the evolution of the communication topology. As can be seen, the tracking error remains close to zero during the entire simulation and the behaviour of the platoon is smooth, notwithstanding the changes in the number and composition of the coalitions. The tracking error of vehicle 1 reaches higher values as it cannot communicate with the leader, which leads to greater uncertainty. The string stability condition can be verified by noting there is no overshoot in the vehicles' velocities and the vehicles' acceleration does not progressively grow along the platoon. Notice also that, as shown in Fig. 3, a decentralized operation is promoted when the tracking error and relative velocities are small, reducing the coordination burden, and thus the exchange of data between the vehicles.

Table 1. Parameters used in simulation

Parameter	Value	Parameter	Value
τ	0.1 $[s]$	Q_i, R_i	diag(10,0,0,0,1), 5
r, h	10 $[m]$, 0.5 $[s]$	ξ, ζ	0.1, 1e5
$u_{\max}$	10 $[ms^{-2}]$	N_{p}	10
T	0.05 $[s]$	S_{d}	$\{s\|\hat{u}_{p_C} = 0\}$
T_{v}	0.2 $[ms^{-1}]$	T_{d}	0.2 $[m]$

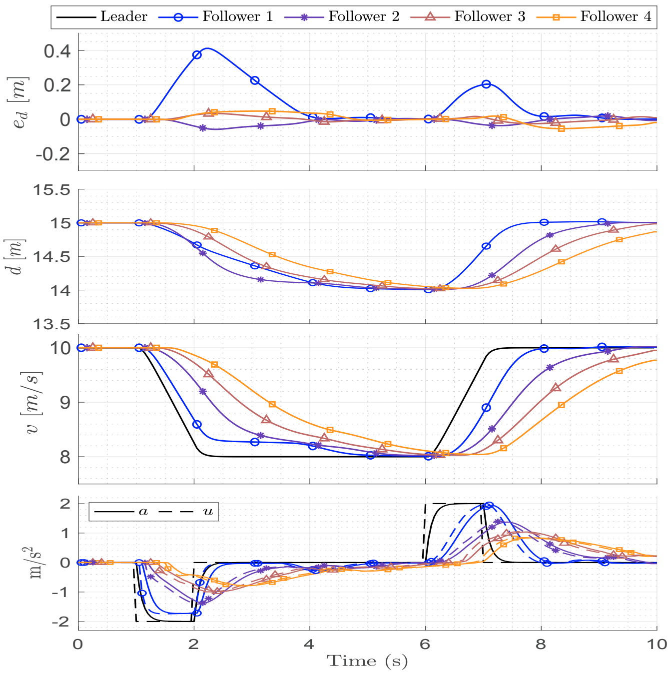

Fig. 2. Evolution of $e_{\mathrm{d},i}$, d_i, v_i, a_i, and u_i, for all vehicles $i \in \{1, ..., 4\}$. The speed, acceleration, and input of the leader is also indicated.

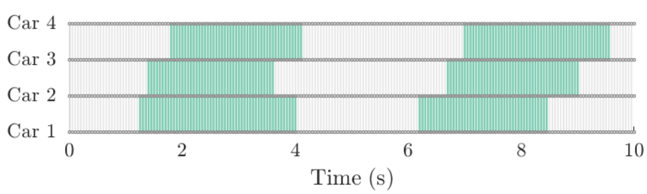

Fig. 3. Evolution of the communication topologies. Green lines indicate active communication links.

The cumulative costs with the proposed topology-switching coalitional controller and that of the decentralized and centralized approaches are respectively $9.56{\cdot}10^3$, $1.37{\cdot}10^4$, and $9.28{\cdot}10^3$. These costs are computed as $\sum_{k=0}^{T_{\mathrm{sim}}}(\|x_i(k)\|^2_{Q_i} + \|\Delta u_i(k)\|^2_{R_i})$, where T_{sim} is the simulation time length. One can see the proposed coalitional controller incurs only a 3.02% increase in the cost with respect to the centralised controller, whereas the use of the communication links is decreased by 55% (see Fig. 3). In addition, the performance was significantly improved in comparison to the complete decentralized configuration.

6. CONCLUSIONS

A coalitional MPC controller for collaborative vehicle platoons has been presented, which includes a set of constraints to guarantee safety and relaxed string stability. A topology switching law enables/disables communication between vehicles when the tracking error on the inter-vehicle distance or velocity exceeds/falls below a chosen threshold. The latter partitions the system into subsets of vehicles that coordinate their actions, thus balancing the overall performance and the use of communication and computation resources. The proposed coalitional approach has been illustrated on a platoon of four vehicles, showing that the obtained performance is still near optimal. In addition, this approach can be easily implemented in larger platoons, where partial and dynamic topologies are of interest to increase scalability and flexibility.

In future research, we will consider vehicles leaving or entering the platoon, introduce lateral dynamics in the vehicle model, and consider unreliable communication and cyber-attacks threatening safety and performance.

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