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# Adding a spin to Kerker's condition: angular tuning of directional scattering with designed excitation

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**We describe a method to control the directional scattering of a high index dielectric nanosphere, which utilizes the unique focusing properties of an azimuthally polarized phase vortex and a radially polarized beam to independently excite inside the nanosphere a spinning magnetic dipole and a linearly polarized electric dipole mode normal to the magnetic dipole. We show that by simply adjusting the phase and amplitude of the field on the exit pupil of the optical system, the scattering of the nanosphere can be tuned to any direction within a plane and the method works over a broad wavelength range.** © 2017

**OCIS codes:** (290.0290) Scattering; (290.4020) Mie theory; (260.5430) Polarization; (050.4865) Optical vortices.

Precise control of the scattering of subwavelength nanostructures is becoming an important research topic with applications for directional coupling of light [1, 2], displacement metrology[3, 4], optical forces[5] and directional excitation of quantum emitters[6], etc.. One way to achieve directional scattering is using high index dielectric nanoparticles which exhibit not only electric modes but also magnetic modes[7, 8]. This unique property makes them low-loss and functional building blocks for nano-antennas, compared to metallic nanostructures. The interference of the magnetic and electric dipole modes of a dielectric nanoparticle under plane wave illumination can lead to zero backward scattering[9], similar to what is theoretically predicted by Kerker et al. [10, 11]. However, for a given particle, the unidirectional scattering can only be realized along the propagation direction of the beam at a single wavelength due to the relation between electric and magnetic field of a plane wave and the properties of Mie coefficients. Independent control of linearly polarized magnetic and electric dipoles has been realized with an excitation configuration composed of orthogonally oriented focused azimuthally and radially polarized beams[12], where directional scattering over a broad wavelength range can be achieved. However, this method needs two focusing optical systems and directional scattering can only be realized along the direction normal to the plane of the electric and magnetic dipoles. A different concept, namely spin-controlled directional coupling of light has drawn lots of attentions and is considered

as an example of the spin-orbit interaction of light[13]. Several experiments have demonstrated the spin controlled directional excitation of surface plasmon and waveguide modes, utilizing the near-field interference of a rotating dipole[1, 2], the transverse spin of the evanescent waves[14, 15] and metasurfaces[16]. However, the directional propagation of light controlled by spin is binary, either to one direction or the opposite corresponding to either left or right circularly polarized light. Angular tuning is a desirable function to gain full control over the scattering of light, but rotation of the directional scattering with aforementioned methods often needs to rotate the excitation beams, which is very inconvenient in practice. In the current letter, a method is proposed to overcome this limitation and tune the scattering of a dielectric nanosphere to any direction within a plane without moving the excitation beam. The approach relies on the independent excitation of a spinning magnetic dipole and a linearly polarized electric dipole mode. Utilizing the unique focusing properties of vector vortex beams, we are able to realize such excitation and achieve arbitrary scattering direction tuning in a plane over a relatively broad wavelength range by simply adjusting the amplitude and phase of the beams.

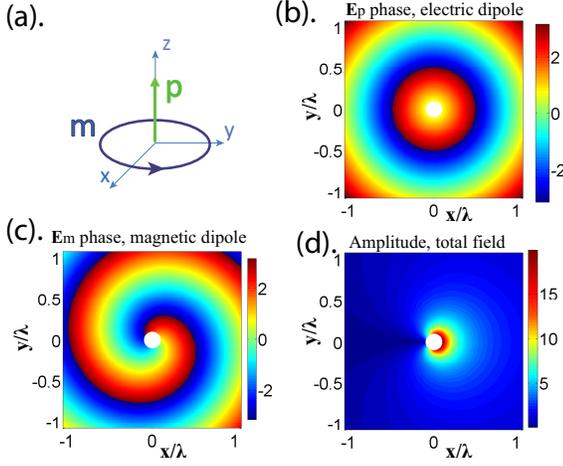
Unlike previous approaches[4, 9, 12] based on the interference of linearly polarized magnetic and electric dipoles, we propose a method to actively control the scattering to any direction in the  $z = 0$  plane using the interference of a spinning(circularly polarized) magnetic dipole  $\mathbf{m} = m_0(\hat{x} + i\hat{y})$  and an electric dipole  $\mathbf{p} = p_0\hat{z}$  linearly polarized in the  $z$  direction as illustrated in Fig.1. It is easy to show[17] that the farfield electrical field in the  $z = 0$  plane of the electrical dipole is polarized along the  $z$  direction and cylindrically symmetric around the  $z$  axis:

$$\mathbf{E}_p = \hat{z} \frac{p_0 k^2}{\epsilon_0} \frac{e^{ikr}}{4\pi r} = \hat{z} E_{zp} \frac{e^{ikr}}{4\pi r}, \quad (1)$$

whereas the farfield electric field in the  $z = 0$  plane of the spinning magnetic dipole is also polarized along the  $z$  direction with a spiral phase:

$$\mathbf{E}_m = \hat{z}(-i)m_0 k^2 Z_0 e^{i\phi} \frac{e^{ikr}}{4\pi r} = \hat{z} E_{zm} e^{i\phi} \frac{e^{ikr}}{4\pi r}, \quad (2)$$

where  $(r, \phi, \theta)$  is the spherical coordinate,  $\epsilon_0$  is the vacuum permittivity,  $Z_0 = 120\pi$  is the impedance of free space and  $k = 2\pi/\lambda$  is the wavevector in air. If the magnetic and electric dipole are modulated in such a way that  $E_{zp} = E_{zm} e^{i\phi_0}$ , the total



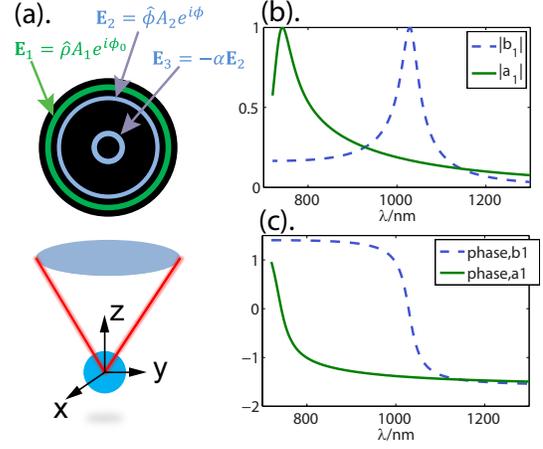
**Fig. 1.** (a). Illustration of a spinning magnetic dipole  $\mathbf{m}$  in  $z = 0$  plane and a electric dipole  $\mathbf{p}$  along  $z$  axis; (b). Radiation electric field in  $z = 0$  plane of the electric dipole is cylindrically symmetric to the  $z$  axis; (c). Radiation electric field in  $z = 0$  plane of the spinning magnetic dipole has a spiral phase; (d). Amplitude of the total electric field in  $z = 0$  plane as a result of the two dipole interference.

electric field in the  $xy$  plane is:

$$E_z = E_{zm} e^{i\phi_0} \frac{e^{ikr}}{4\pi r} + E_{zm} e^{i\phi} \frac{e^{ikr}}{4\pi r}. \quad (3)$$

As a result of the spiral phase of  $\mathbf{E}_m$ , the symmetry of the radiation field is broken and there is always a direction  $\phi = \phi_0 + \pi$  where the destructive interference occurs. By changing the phase  $\phi_0$  of the electric dipole from 0 to  $2\pi$ , the destructive interference can be tuned to any direction in the  $z = 0$  plane. This is different from the conventional Kerker's conditions where the unidirectionality can only be achieved in two directions when the linear electric and magnetic dipoles are in and out of phase. As is shown in the remaining of the letter, the scattering direction of a high index dielectric nanosphere can be controlled by focusing an appropriate pupil field. Based on Mie's solution, the high-order multipole modes of a high index dielectric nanosphere are of narrow bandwidth. In this letter, we will focus on the longer wavelength range where the electric and magnetic dipoles dominate and the higher order modes are negligible. In order to realize directional scattering with the proposed concept, one must excite the circularly polarized magnetic dipole in the  $z = 0$  plane and the electric dipole in  $z$  direction and one must be able to control the amplitude and phase of these two dipoles separately. This, however, cannot be achieved with conventional plane wave illumination. In the current letter, we utilize the unique focusing properties of designed pupil fields with unconventional polarization and phase distributions to independently generate a circularly polarized magnetic field and a linearly polarized electric field at the focal point.

As illustrated in Fig.2(a), the proposed pupil field contains three annular rings in the amplitude function. The radius of the pupil is normalized to 1. Within each ring, the amplitude is uniform. For simplicity, all three rings are chosen to have narrow width. The electrical field in ring 1 with average radius  $\rho_1$  and width  $\Delta\rho_1 \ll 1$  is radially polarized:  $\mathbf{E}_e = \mathbf{E}_1 = \hat{\rho} A_1 e^{i\phi_0}$ , where  $\phi_0$  is a constant phase and  $A_1$  is a positive real number. The focal fields can be calculated using the Richard Wolf diffraction



**Fig. 2.** (a). Illustration of the configuration of focusing the pupil field to excite a high index dielectric sphere at focus; (b). Amplitude of the electric and magnetic scattering coefficients  $a_1$  and  $b_1$  as function of the wavelength for a high index dielectric sphere of refractive index  $n = 5$  and radius  $r_0 = 100$  nm; (c). Phase of  $a_1$  and  $b_1$ .

integral[18, 19]. At the focal point  $r_f = 0$  the focused electric field is longitudinally polarized along the  $z$ - direction and:

$$\begin{aligned} E_{f,z}^{rad}(r_f = 0) &= -2e^{i\phi_0} C_0 \int_0^1 A(\rho) \frac{s_0 \rho}{(1-\rho^2 s_0^2)^{1/4}} \rho d\rho \quad (4) \\ &\approx -2e^{i\phi_0} C_0 A_1 \frac{s_0 \rho_1}{(1-\rho_1^2 s_0^2)^{1/4}} \rho_1 \Delta\rho_1 \\ &= E_f e^{i\phi_0}, \end{aligned}$$

where  $s_0$  is the numerical aperture and  $C_0$  is a constant related to the wavelength, numerical aperture and focal length. This pupil field is used to excite the pure electric dipole of the dielectric nanosphere with the magnetic dipole mode completely suppressed[12].

The pupil field  $\mathbf{E}_h = \mathbf{E}_2 + \mathbf{E}_3$  consists of two rings: ring 2 with average radius  $\rho_2$  and width  $\Delta\rho_2 \ll 1$  and ring 3 with average radius  $\rho_3$  and width  $\Delta\rho_3 \ll 1$ , where  $\rho_3 \ll \rho_2 < \rho_1$ . The electric pupil fields in ring 2 and 3 are azimuthally polarized phase vortices:  $\mathbf{E}_2 = \hat{\phi} A_2 e^{i\phi}$  and  $\mathbf{E}_3 = -\alpha \mathbf{E}_2$ , where  $\alpha$  and  $A_2$  are positive real numbers. The corresponding magnetic pupil field is a radially polarized phase vortex with  $\mathbf{H}_h = \hat{\rho}/Z_0 |\mathbf{E}_h| e^{i\phi}$ , where  $Z_0 = 120\pi$ . The corresponding fields at the focal point  $r_f = 0$  can be calculated using Richard Wolf diffraction integral[18, 19]:

$$\begin{aligned} H_{f,x}^{azm,m=1} &= \frac{C_0}{Z_0} \int_0^1 A(\rho) (1-\rho^2 s_0^2)^{1/4} \rho d\rho \quad (5) \\ &\approx \frac{C_0}{Z_0} [A_2 (1-\rho_2^2 s_0^2)^{1/4} \rho_2 \Delta\rho_2 - \\ &\quad - \alpha A_2 (1-\rho_3^2 s_0^2)^{1/4} \rho_3 \Delta\rho_3] = H_f, \end{aligned}$$

$$H_{f,y}^{azm,m=1} = iH_{f,x}^{azm,m=1}(r_f = 0) = iH_f, \quad (6)$$

$$H_{f,z}^{azm,m=1} = 0, \quad (7)$$

$$\begin{aligned} E_{f,x}^{azm,m=1} &= iC_0 \int_0^1 A(\rho) (1-\rho^2 s_0^2)^{-1/4} \rho d\rho \quad (8) \\ &\approx iC_0 \times \\ &\quad \times [A_2 (1-\rho_2^2 s_0^2)^{-1/4} \rho_2 \Delta\rho_2 - \alpha A_2 (1-\rho_3^2 s_0^2)^{-1/4} \rho_3 \Delta\rho_3], \end{aligned}$$

$$E_{f,y}^{azm,m=1} = iE_{f,x}^{azm,m=1}, \quad (9)$$

$$E_{f,z}^{azm,m=1} = 0, \quad (10)$$

It follows from Eq. (5) to Eq. (10) that the transverse components of the focal magnetic field have  $(1 - \rho^2 s_0^2)^{1/4}$  dependence in the diffraction integral while the transverse component of the focal electric field have  $(1 - \rho^2 s_0^2)^{-1/4}$  dependence. When

$$\alpha = [(1 - \rho_2^2 s_0^2)^{-1/4} \rho_2 \Delta \rho_2] / [(1 - \rho_3^2 s_0^2)^{-1/4} \rho_3 \Delta \rho_3], \quad (11)$$

all the electrical field components become zero at the focal point, but the magnetic field  $\mathbf{H}_f = H_f(\hat{x} + i\hat{y})$  is non-zero and circularly polarized, where

$$H_f = \frac{C_0}{Z_0} A_2 (1 - \rho_2^2 s_0^2)^{-1/4} \rho_2 \Delta \rho_2 \times \left[ \sqrt{1 - \rho_2^2 s_0^2} - \sqrt{1 - \rho_3^2 s_0^2} \right] \neq 0. \quad (12)$$

In this way, a pure circularly polarized magnetic field at the focal point is created and the electric field vanishes there.

Note that the condition for  $\alpha$  is independent of the wavelength. We remark that many other pupil fields can be designed to excite the electric and magnetic dipoles in the focal point. Indeed, the rings do not need to be narrow and by using broader rings, stronger dipole moments can be realized. However, the pupil field proposed in this letter has the advantage of being particularly simple to illustrate the design principles. The realization of the proposed pupil fields would be possible with the recent demonstrations in the generation of arbitrary vector vortex beams using spatial light modulators[20] and q-plates[21, 22]. Specifically the generation of azimuthally/radially polarized phase vortices has been demonstrated recently using q-plates and spiral phase plate in [22].

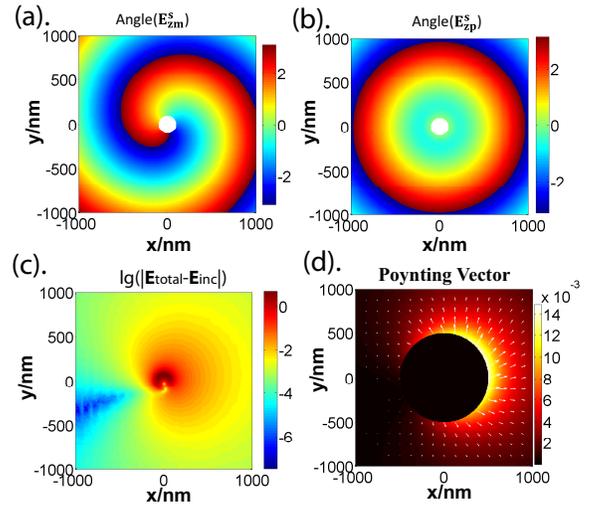
It is shown in [12, 23] that electric dipole and magnetic dipole interactions of a given dielectric nanosphere illuminated by an inhomogeneous beam depend only on the local fields at the center of the sphere. When a high index dielectric nanosphere at focus is illuminated by the focal fields of the pupil field  $\mathbf{E} = \mathbf{E}_h + \mathbf{E}_e$ , within the wavelength range where the magnetic and electric dipole modes of the sphere dominate, a circularly polarized magnetic dipole  $\mathbf{m} = i \frac{6\pi}{k^3} b_1 H_f (\hat{x} + i\hat{y})$  is excited with the pure contribution of pupil field  $\mathbf{E}_h$  while the linearly polarized electric dipole  $\mathbf{p} = i \frac{6\pi}{k^3} \epsilon_0 a_1 E_f e^{i\phi_0} \hat{z}$  is excited with the pure contribution of pupil field  $\mathbf{E}_e$ . Here  $a_1$  and  $b_1$  are the electric and magnetic dipole Mie scattering coefficients of a nanosphere, determined from the standard plane wave illumination[23]. Applying Eq.1 and Eq.2, in the  $z = 0$  plane the total scattering field of the excited magnetic and electric dipole moments is:

$$\mathbf{E}_s = \hat{z} \frac{6\pi}{k} (i a_1 E_f e^{i\phi_0} + b_1 Z_0 H_f e^{i\phi}) \frac{e^{ikr}}{4\pi r}. \quad (13)$$

This shows that when the focal fields follow the relation  $120\pi |b_1 \mathbf{H}_f| = |a_1 \mathbf{E}_f|$  which is the amplitude requirement of the Kerker's condition for directional scattering[9], there is always a direction in the  $z = 0$  plane where the phase condition for destructive interference  $\phi = -\pi/2 + \phi_0 + \text{Arg}(a_1) - \text{Arg}(b_1)$  is fulfilled.

The analytical model for angular tuning of the directional scattering is verified by a finite element method(FEM) simulation[24] of the scattering of a dielectric nanosphere of refractive index 5 and radius 100 nm under the proposed excitation. The electric and magnetic dipole scattering coefficients  $a_1$  and  $b_1$  of the nanosphere are shown in Fig.2(b) and (c). It is important to note that the same principle applies to practical high index nanospheres like Si/Ge with only different spectral dependences

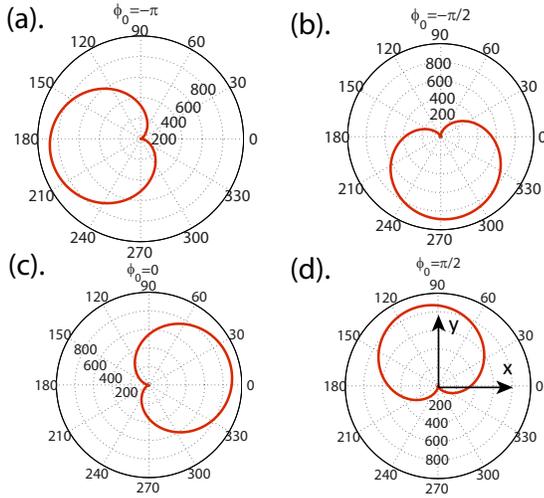
of the Mie coefficients  $a_1$  and  $b_1$ . Fig.3(a) and Fig.3(b) show the phases of the scattered electric fields in  $z = 0$  plane of the nanoparticle excited respectively by the focusing pupil fields  $\mathbf{E}_h$  and  $\mathbf{E}_e$ . When the amplitude of  $\mathbf{E}_h$  and  $\mathbf{E}_e$  are modulated so that the focal fields satisfy the condition  $120\pi |b_1 \mathbf{H}_f| = |a_1 \mathbf{E}_f|$ , there is always a direction in  $z = 0$  plane where the phase condition for directional scattering is fulfilled due to the spiral phase of the excited spinning magnetic dipole in Fig.3(a) and the cylindrically symmetric phase of the excited longitudinal electric dipole in Fig.3(b). As is shown in Fig.3(c), when the amplitude requirement of Kerker's condition is satisfied for  $\lambda = 1028$  nm, completely destructive interference is formed by these two scattering fields along the direction  $\phi = -\pi/2 + \phi_0 + \text{Arg}(a_1) - \text{Arg}(b_1)$  in the  $z = 0$  plane, and the directional scattering is thus achieved, clearly illustrated by the plot of the Poynting vector in Fig.3(d). As shown in Fig.4, by changing the phase  $\phi_0$  of the pupil field of



**Fig. 3.** FEM simulations of the scattering of a sphere with radius 100 nm and refractive index 5 with the proposed excitation at wavelength  $\lambda = 1028$  nm. (a).Phase of the scattered electric field in the  $z = 0$  plane of the spinning magnetic dipole of the sphere excited by the focal field of  $\mathbf{E}_h$ ; (b).Phase of the scattering electric field in the  $z = 0$  plane with the longitudinal electric dipole of the sphere excited by the focal field of  $\mathbf{E}_e$ ; (c). Amplitude of the total field subtracted by the incident field in logarithmic scale in the  $z = 0$  plane with both dipole excitations using pupil field  $\mathbf{E}_h + \mathbf{E}_e$  when  $120\pi |b_1 \mathbf{H}_f| = |a_1 \mathbf{E}_f|$  is fulfilled; (d). Poynting vector of the directional scattering (fields inside a circle with radius of 500 nm centered at the focal point is cropped to highlight the unidirectional scattering).

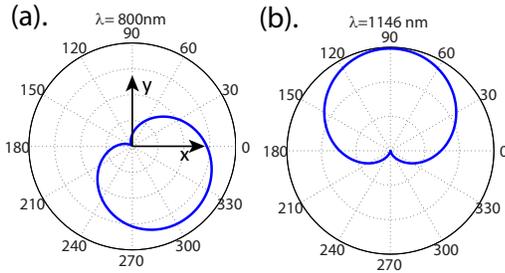
$\mathbf{E}_e = \hat{\rho} A_1 e^{i\phi_0}$ , the scattering can be tuned. When  $\phi_0$  is changed over  $2\pi$  range, the scattering direction rotates over  $2\pi$  range, making it possible to tune the scattering to any direction in the  $z = 0$  plane by only modifying the phase of the radially polarized beam.

Furthermore, this approach to control directional scattering works over a broad wavelength range as long as the dipole modes dominate and the higher orders are negligible. The directional scattering at different wavelengths can be fulfilled as shown in Fig.5 with  $120\pi |b_1 \mathbf{H}_f| = |a_1 \mathbf{E}_f|$  fulfilled by adjusting the amplitude of the pupil field  $\mathbf{E}_e$  and  $\mathbf{E}_h$ . From Fig.5 and Fig.4(c), one can see that when  $\phi_0$  is set to be zero, the direction where destructive interference happens changes at different



**Fig. 4.** Angular tuning of the directional scattering shown by the scattering pattern when  $\phi_0$  is changed over  $2\pi$  at  $\lambda = 1028$  nm (see also Visualization 1). (a).  $\phi_0 = -\pi$ ; (b).  $\phi_0 = -\pi/2$ ; (c).  $\phi_0 = 0$ ; (d).  $\phi_0 = \pi/2$ .

wavelengths. However, the scattering at each wavelength can always be tuned to the desired direction by changing  $\phi_0$  as shown in Fig. 4.



**Fig. 5.** Scattering pattern at (a).  $\lambda = 800$  nm; (b).  $\lambda = 1146$  nm,  $\phi_0 = 0$  for both wavelengths.

Note that although we focus on the interference of a spinning magnetic dipole and a linearly polarized electric dipole, the directional scattering control can also be achieved with a spinning electric dipole and a linearly polarized magnetic dipole. The corresponding excitation focal field can be realized by applying an azimuthally polarized electric pupil field in ring 1 and adjusting  $\alpha$  in ring 3 so that the magnetic field at focus is zero but the electric field is nonzero and circularly polarized. In summary, we have proposed an approach to control the scattering direction of a high index dielectric nanosphere. This approach enables the simultaneous excitation and independent control of a circularly polarized magnetic and a linearly polarized electric dipole mode with the unique focusing properties of vector vortex beams. By simply adjusting the phase and amplitude of the pupil field, the directional scattering can be tuned to any direction in the selected plane and works over a relatively broad wavelength range. With this method, we introduce a new degree of freedom, namely angular tuning, of directional scattering. Our proposal has high potentials in many applications such as the light routing/switching on a photonic chip, quantum optical networks, directional emission of quan-

tum emitters with designed excitation and optical metrology, etc.

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