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Evaluation of recursive Bayesian filters for modal contribution estimation in high-tech compliant mechanisms

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Abstract: This study evaluates three recursive Bayesian input and state estimation algorithms, as introduced in the field of Structural Health Monitoring, for estimating modal contributions for high-tech compliant mechanisms. The aim of estimating modal contributions is the use for active vibration control. High-tech compliant motion stages allow for different sensor configurations, making new and interesting performance evaluations of these filters possible. The algorithms used, namely, the Augmented Kalman Filter (AKF), Dual Kalman Filter (DKF) and Gilijns de Moor Filter (GDF) are implemented on a compliant motion stage for guidance flexure deformation estimation. Our results show the GDF performs overall best, with good estimation performance and real-world tuning capability.

Keywords: Vibration Control, Application of mechatronic principles, Motion control systems, Smart structures

1. INTRODUCTION

Recent advances towards high-throughput high-precision manufacturing in the high-tech industry have led to increasing interest in compliant motion systems. When operating these systems at high accelerations, however, higher frequency vibrational modes can reduce overall accuracy and thus throughput. One possible solution for this is to actively add damping to the system often referred to as Active Vibration Control (AVC).

Most common methods of AVC require collocated measurements, where a piezo sensor and an actuator are placed at the same location on the flexible structure. These sensor-actuator pairs are then connected through a control algorithm like Positive Position Feedback (PPF) as sketched in Caughey (1985) to achieve active damping in a decentralised way. A different, more centralised, approach is Independent Modal Space Control (IMSC), where every vibrational mode can be dampened independently as introduced by Meirovitch and Oz (1979) and Balas (1978). This class of methods allows for the damping of specific higher order vibrational modes without spending control effort on modes of less interest by decoupling the MIMO AVC problem for all modes into a SISO problem for every controlled mode independently.

IMSC methods generally require information about the modal contribution of each mode in a compliant system. This information is traditionally obtained by employing a modal filter as introduced by Meirovitch and Baruh (1983, 1985). These static filters, however, do not take measurement and process noise into account, limiting performance of the overall AVC system (Meirovitch and Baruh, 1985).

To obtain an estimate about the modal contribution based on noisy measurements, recursive Bayesian filters for state estimation can be implemented instead of static modal filters. These recursive Bayesian filters assume stochastic system modelling, and can therefore inherently use a noise-corrupted system model formulation. A problem with more traditional recursive Bayesian filters, like the original linear Kalman Filter (KF), is they require information about all inputs to the system to make a prediction about the modal contributions (the system state). For compliant systems, however, the input is often not known since it includes not only known control inputs, but also disturbances. While some disturbances can be measured or modelled, most can not.

Similar problems can be found in the field of Structural Health Monitoring (SHM) for civil structures, where state estimates of structures like bridges and buildings are required for strain and lifetime analysis. Here, knowledge about system input is also scarce. As a solution, recursive Bayesian filters were introduced for simultaneous input and state estimation.

Among other approaches, the Augmented Kalman Filter (AKF) introduced into SHM by Lourens et al. (2012b,a), Dual Kalman Filter (DKF) introduced by Eftekhar Azam, Saeed et al. (2015) and Gilijns de Moor Filter (GDF) introduced by Gillijns and De Moor (2007a,b) are often evaluated in SHM for simultaneous input and state estimation (Moradi et al., 2021). These three methods thus represent state of the art in civil engineering, and are

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evaluated for use in high-tech compliant mechanisms in this work.

The implementation of these algorithms into high-tech compliant mechanisms instead of civil structures presents a significantly different environment, allowing for interesting new sensor configurations. In civil structures for instance, strain measurements are hard to obtain, whereas acceleration measurements are cheap and easy. This unavailability of strain measurements reduces estimation performance significantly as sketched by Naets et al. (2015); Eftekhar Azam, Saeed et al. (2015) and Tatsis and Lourens (2017). For high-tech compliant mechanisms, however, these strain measurements are much easier to obtain due to a small system size and highly controlled environment. In addition to this, high-tech systems often benefit from in-depth system knowledge allowing for accurate modelling, which also benefits recursive Bayesian filter implementation.

To the best of the authors' knowledge, the possibilities of implementing recursive Bayesian filtering solutions from SHM into compliant mechanisms have not been previously studied. This forms a research gap which we attempt to address in this work. For the first time, the recursive Bayesian filters for modal contribution estimation are implemented on a compliant motion stage. The estimation performance of filters is studied for both external stage disturbances and closed loop position control. Since closed loop position control introduces high accelerations into the system, the dependence of the recursive Bayesian filter's performance on the magnitude of system acceleration is evaluated, in addition to their overall estimation performance.

The remainder of the paper is organized as follows. To provide a system model, introduce notation and the desired modal contribution states, section 2.1 first introduces the necessary Finite-Element modelling approach. Section 2.2 elaborates further on the three recursive Bayesian filters to be evaluated. The implementation on an actual compliant motion stage guidance flexure is presented in section 3 whereafter the design of experiments is presented in section 4 followed by the results, discussion and conclusions.

2. BACKGROUND

2.1 Finite Element modelling

The dynamics of a compliant flexure discretised by 1D Euler-Bernoulli beam elements is described by

$$M\ddot{\bar{x}} + C'\dot{\bar{x}} + K\bar{x} = B'\bar{f}, \tag{1}$$

where the matrices M, C' and $K \in \mathcal{R}^{2n\times 2n}$ denote the mass, damping and stiffness matrices of the system, with n representing the number of nodes used for the finite element approximation. The system state $\bar{x} \in \mathcal{R}^{2n\times 1}$ represents a collection of the nodal degrees-of-freedom (DOFs). For the Euler-Bernoulli finite element assumption, each node is attributed a lateral (horizontal) displacement, and rotational DOF. The location of the external input forces $\bar{f} \in \mathcal{R}^{n_u\times 1}$ is determined by the selection matrix $B' \in \mathcal{R}^{2n\times n_u}$ with n_u the amount of external input forces acting on the system.

For displacement measurements, the interpolation matrix $S_d \in \mathcal{R}^{n_d\times 2n}$ is introduced as $\bar{y}_d = S_d\bar{x}$, where S_d merely interpolates the nodal DOFs through a third order shape function $N(y)$ as presented in Cook et al. (2001). The number of displacement measurements is denoted by n_d.

For acceleration measurements, (1) is used to obtain an expression for the state acceleration $\ddot{\bar{x}}$, and is again premultiplied by an interpolation matrix $S_a \in \mathcal{R}^{n_a\times 2n}$ as:

$$\bar{y}_a = S_aM^{-1}B'\bar{f} - S_aM^{-1}C'\dot{\bar{x}} - S_aM^{-1}K\bar{x}. \tag{2}$$

The strain measurement is obtained using a similar approach as mentioned by Preumont (2012) and Aktas and Esen (2020). The strain ϵ_i over element i is defined by

$$\epsilon_i = \underbrace{h\int_0^L \frac{d^2}{dy^2}N(y)dy}_{S_{s,i}}\,\bar{x}_i, \tag{3}$$

where h denotes the distance away from the centre line of the flexure and L is the length of the element. The system matrix $S_s \in \mathcal{R}^{n_s\times 2n}$ is then constructed by applying this elemental selection matrix $S_{s,i}$ for every element the strain is measured over, such that $\bar{y}_s = S_s\bar{x}$. To reduce system order, and introduce the desired modal contribution η in the state vector, the system is modally decomposed by solving the un-damped eigenvalue problem $(K - \omega^2M)\bar{v} = 0$, hereby obtaining the eigenvector matrix $\Phi \in \mathcal{R}^{2n\times N}$ with N the amount of modelled eigenmodes $\bar{v}$. The corresponding system eigenfrequencies are captured in the matrix $\Omega = \text{diag}\,[\omega_1 \dots \omega_N]$. Assuming the system is spanned by its eigenvectors, the new state is defined as $\bar{x} = \Phi\bar{z}$, with $\bar{z} = [\eta_1 \dots \eta_N]^T$. Applying this state transformation and mass normalising Φ such that $\Phi^TM\Phi = \mathcal{I}$ and $\Phi^TK\Phi = \Omega^2$, the full state space system can be formulated in modal space with $\bar{q} = \left[\bar{z}, \dot{\bar{z}}\right]^T$ as

$$\dot{\bar{q}} = \underbrace{\begin{bmatrix} 0 & I \\ -\Omega^2 & -\Phi^TC'\Phi \end{bmatrix}}_{A_{c,m}}\bar{q} + \underbrace{\begin{bmatrix} 0 \\ \Phi^TB' \end{bmatrix}}_{B_{c,m}}\bar{f} \tag{4}$$

$$\bar{y} = \underbrace{\begin{bmatrix} S_d\Phi & 0 \\ -S_a\Phi\Omega^2 & -S_a \\ S_s\Phi & 0 \end{bmatrix}}_{C_{c,n}}\bar{q} + \underbrace{\begin{bmatrix} 0 \\ S_a\Phi\Phi^TB' \\ 0 \end{bmatrix}}_{D_{c,n}}\bar{f}. \tag{5}$$

The objective of the recursive Bayesian filters considered in this work is to obtain an estimate of these modal contributions η. For this, the LTI state space model introduced in (4) and (5) is discretised by making a Zero-Order-Hold (ZOH) assumption resulting in a discrete-time linear state space system. To represent model uncertainty and process disturbances, process noise $\bar{w}$ is assumed in addition to measurement noise $\bar{v}$, to model measurement uncertainty. The full discrete-time system model can thus be summarised as

$$\bar{q}_{k+1} = A\bar{q}_k + B\bar{f}_k + \bar{w}_k, \tag{6}$$

$$\bar{y}_k = C\bar{q}_k + D\bar{f}_k + \bar{v}_k. \tag{7}$$

The process and measurement noise terms $\bar{w}$ and $\bar{v}$ are assumed independent and identically distributed (IID) with a Gaussian distribution, zero mean and covariance matrices Q and R respectively.

It is noted that both the input force $\bar{f}$ and state $\bar{q}$ are assumed unknown in this work, and need to be estimated.

2.2 Recursive Bayesian simultaneous input and state estimation

As mentioned in section 1, three recursive Bayesian filtering approaches for simultaneous input and state estimation are evaluated in this work, namely the Augmented Kalman Filter (AKF), the Dual Kalman Filter (DKF) and the Gilijns de Moor Filter (GDF).

The first approach is the AKF as introduced into structural mechanics by Lourens et al. (2012b). Its formulation only requires an augmented system model, whereafter the regular linear Kalman filtering framework as introduced in Kalman (1960) can be used. For an augmented system model, the input is modelled as a random walk process. This however requires the additional tuning of the input variation covariance (hereafter referred to as Q_f). Since little information about the variation of the input is known, this can limit real-world application significantly (Lourens et al. (2012b)).

The second approach, the DKF, as presented by Eftekhar Azam, Saeed et al. (2015) has been introduced as a remedy to instability of the AKF for acceleration-only measurements. Here, an additional Kalman filtering stage is introduced to obtain an estimate of the input before the second Kalman filtering stage estimates the state. This approach, however, still assumes a similar random walk process for the input as the AKF. Thus also limiting real-world tuning capability (Tatsis and Lourens, 2017).

The third approach is the GDF (Gillijns and De Moor, 2007a,b). It is first introduced into structural mechanics by Lourens et al. (2012a). For the original filter formulation as proposed in Gillijns and De Moor (2007b), a similar two-stage estimation structure is set up as in the DKF. Instead of a Kalman filter estimate for the input however, a recursive Weighted Least-Squares estimation is used. For this, no random walk model is assumed for the input dynamics, thus eliminating the need for the additional tuning of an input covariance matrix.

3. METHOD

3.1 Experimental setup

The system used for filter evaluation is a compliant motion stage flexure. The setup is presented in Figure 1, where in addition, the measurement and sensing equipment is also introduced.

For reference, general mechanical properties of the stage flexures are presented in Table 1.

Table 1. Mechanical properties of compliant flexure.

Material	AISI 1095 Spring Steel
Length	100 mm
Width	12.7 mm
Height	0.4 mm

On this setup, a strain measurement at the base of the flexure, an acceleration measurement and a position measurement of the stage are implemented. It is assumed that the stage is rigidly connected to the flexure tip, and dynamics of the stage itself are neglected. The stage is actuated by a current controlled race-coil actuator, providing closed loop position control using the position measurement.

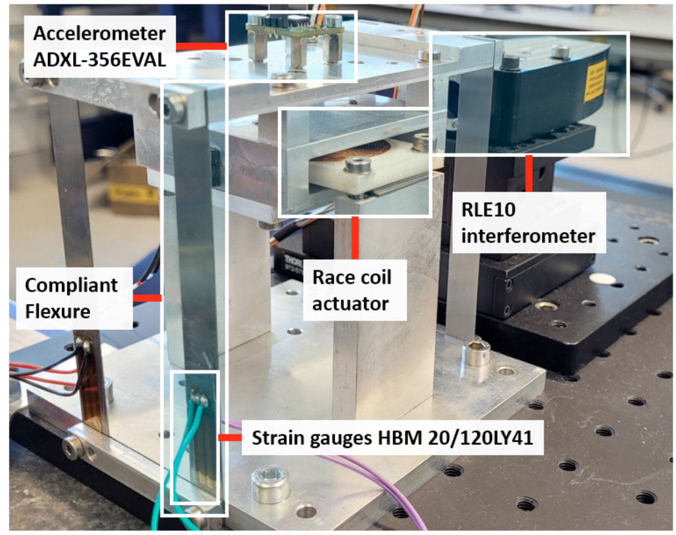

Fig. 1. Compliant motion stage setup with a race coil actuator and acceleration, strain and position sensors.

The strain measurements are implemented in full Wheatstone bridge configuration. The two actual measurement strain gauges are placed at the second mode strain maximum at the base of the flexure in order to increase observability of this mode.

Data is collected by a TI f28379d Launchpad, also implementing closed-loop position control of the motion stage. This board allows for a sampling frequency of 10kHz.

3.2 System modelling

To obtain an actual model of the system, the finite element modelling approach introduced in section 2.1 is used. It is assumed that the tip of the flexure is fixed in rotation, and only one interaction force f exists in the lateral direction on this tip, containing all interaction between the stage and the flexure. This also allows for the application of the methods in this work to become invariant to stage loading, or unknown disturbances on the stage itself since this is all captured in this interaction force, which is estimated. The stage is also assumed fully symmetrical, allowing for the modelling of only one flexure at a time.

The full system model is truncated to the first two modes using the modal decomposition as introduced in section 2.1, since higher modes can not be measured. This thus results in the state vector $\bar{q} \in \mathcal{R}^{4\times 1}$ containing the modal contributions η and modal velocities $\dot{\eta}$.

As model validation, the measured frequency response from the racetrack coil input force to the measured position output is presented in Figure 2. A mass representing roughly 1/4 of the stage body mass is added to the tip of the modelled flexure, for validation. In addition, the modal damping of the model is tuned to match the measured system, since this is not directly obtained from material properties. As can be seen, the location and amplitude of the first modelled mode coincides relatively well with identified dynamics. The two independent identification sequences for the second mode show some additional dynamics compared to the modelled response. It is assumed this is due to additional remnant modes occurring from flexure interaction, since the true system does not consist of merely 1 flexure (as assumed in modelling), but 4.

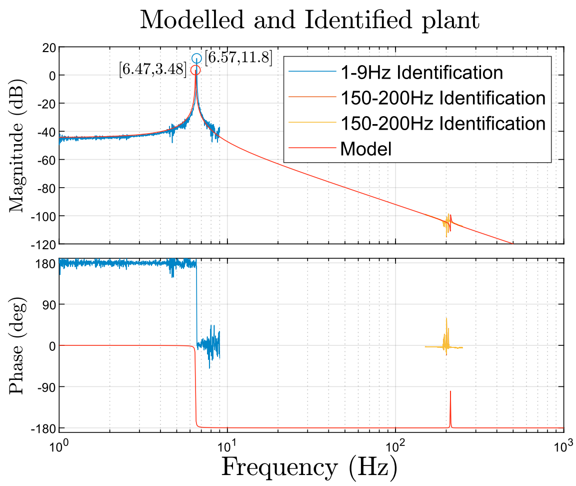

Fig. 2. Modelled (red) and identified (blue, yellow and orange) plant dynamics. Bode plot from input force to flexure tip position.

For the recursive Bayesian filter implementation, it is important to note that only the strain and acceleration measurements are used for estimation. The position measurement is only used for validation of the model and evaluation of estimation performance.

3.3 Filter Implementation

For the recursive Bayesian filter implementation, all states and covariance matrices require initialisation as well as parameter tuning.

For this initialisation, the filter states $\bar{q}_0$ for all three filters are initialised at 0. No information is available about the variance of this estimate, which is assumed large, by initialising all three state covariance matrices P_0 at $10^3\ \mathcal{I}_{n_q \times n_q}$ with n_q the amount of states.

For filter tuning, the AKF and DKF require the tuning of the process noise, measurement noise and input covariance matrices Q, R and Q_f respectively. The GDF only requires tuning of the Q and R matrices. To simplify this tuning process, and to reduce tuning parameter space dimension, the Q and R matrices are assumed diagonal as $Q = \sigma_q^2 \mathcal{I}_{n_q \times n_q}$ and $R = \sigma_r^2 R_0$ with scalar tuning parameters σ_q and σ_r similar to Petersen et al. (2019). Since only one input force is assumed, Q_f a scalar. The tuning parameter space is therefore $\mathcal{R}^3$.

The initial measurement covariance matrix R_0 is constructed using measured noise covariance values as $R_0 = \text{diag}\left[\text{cov}(y_a)\ \text{cov}(y_s)\right]$, with $\text{cov}(y_a) = 0.0587 \text{m}^2/\text{s}^2$ and $\text{cov}(y_s) = 1.095 \cdot 10^{-16} \text{m}^2/\text{m}^2$. This ensures relative covariance between the sensors is maintained when only tuning σ_r.

The first two parameters, σ_q and σ_r, are tuned using a non-linear Nelder-Mead simplex algorithm for all three filters following initial empirical tuning and a parameter space grid search. Here, the cost function is set up as a minimisation of the Normalised Root-Mean-Squared Error (NRMSE) of the estimated tip position versus the measured tip position for an independent training data set of 20s. It is important to note that no global optimality is guaranteed, as this tuning process is sensitive to local minima.

After convergence of this tuning, the AKF and DKF still require tuning for Q_f, which is done using an L-curve regularisation similar to Tatsis and Lourens (2017). Since this L-curve depends on the values chosen for σ_q and σ_r, the process of optimisation followed by L-curve regularisation is iterated until parameters converge. These final filter parameters are presented in Table 2.

Table 2. Tuned filter parameters.

	AKF	DKF	GDF
q_{tune}	$1.86{\cdot}10^{-9}$	$3.03{\cdot}10^{-13}$	$2.24{\cdot}10^{-5}$
r_{tune}	$2.26{\cdot}10^{4}$	$1.11{\cdot}10^{4}$	$2.29{\cdot}10^{14}$
Q_f	$1{\cdot}10^{10}$	$1 \cdot 10^{1}$	-

4. DESIGN OF EXPERIMENTS

To be able to draw relevant conclusions about the evaluation of these filters on high-tech compliant motion stages, their performance is evaluated on two performance metrics: system acceleration dependency and overall estimation performance.

Since high-tech compliant mechanisms experience high accelerations and velocities, the filters' acceleration dependency is an important performance metric for this specific application. Overall NRMSE fit score between the estimated and measured stage position provides a metric to evaluate and compare the filters' overall performance.

Four independent data sets are gathered to evaluate the two performance metrics mentioned above.

The first two data sets aim at evaluating filter acceleration dependency by introducing different acceleration regimes. This is done by initialising in open loop (first regime) whereafter a reference position is tracked using closed loop PID control (second regime). The system is allowed to dampen out after closed loop control is turned off (third regime), resulting in 3 distinct acceleration regions (initialisation, closed loop control and residual vibrations) per data set. For these in total 6 regions, the mean absolute acceleration of the stage is computed per region. The first data set employs a square wave as reference alternating between $\pm 2{\cdot}10^{-4}$ m with a period of 2 seconds, whereas the second data set requires the tracking of a 10Hz sinusoidal reference of the same amplitude.

To evaluate overall estimation performance, two additional data sets are used. Here, an impulse force is provided, whereafter the stage is allowed to dampen out. These data sets are used in addition to the first two to evaluate the filters on both control and disturbance inputs.

The performance of the filters for these metrics is determined using the tip location estimate, where the estimated displacement of the tip is compared to the stage measurement. In this work, it is assumed this tip displacement is proportional to the filter's modal amplitude estimation performance since no reference signal for this modal amplitude is available.

5. RESULTS

The results for the filter implementation on the experimental setup are sub-divided into system acceleration and overall estimation performance evaluation.

For the acceleration performance evaluation, the mean absolute error between the estimated and measured tip location is computed for every section of the first two data sets (6 sections in total). These values are plotted in Figure 3 on the left y-axis. The second y-axis presents the mean absolute acceleration measurement per section.

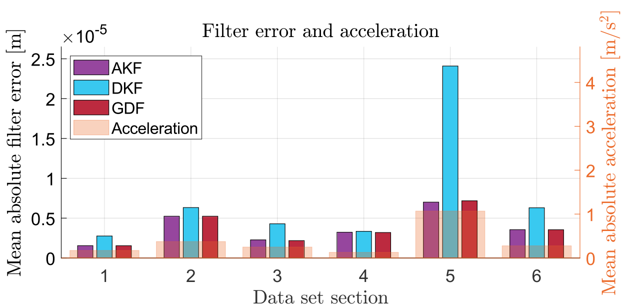

Fig. 3. Mean absolute filter tip position estimation error for all 6 data set sections. A second y-axis presents the mean absolute acceleration measured.

As can be seen, the three filters perform significantly different with lower measured acceleration in the system compared to higher acceleration. To show a possible correlation, the mean absolute filter error for all 6 data set sections is plotted against the mean absolute measured acceleration in Figure 4. The Pearson correlation coefficient is computed as well as a linear least squares fit to show the possible difference in correlation.

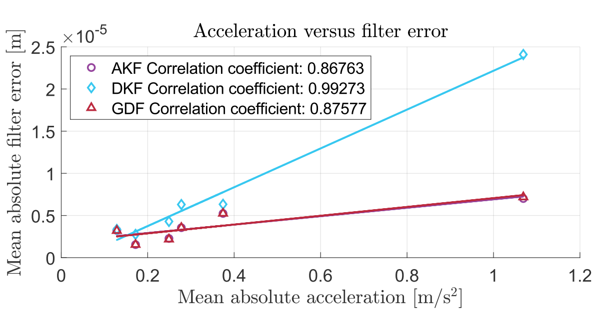

Fig. 4. Mean absolute filter error versus mean absolute acceleration for all 6 data set sections. The AKF and GDF closely overlap.

As can be seen, a relatively large correlation coefficient for the DKF in conjunction with the steep gradient of the least squares fit suggests the DKF performs poorly on high acceleration signals. The AKF and GDF perform very comparably.

For overall estimation performance, the NRMSE between the estimated and measured tip position for all four data sets is presented in Figure 5. Here, it can again be seen the AKF and GDF perform similarly, where the DKF builds up considerably more error. The GDF outperforms the AKF slightly overall.

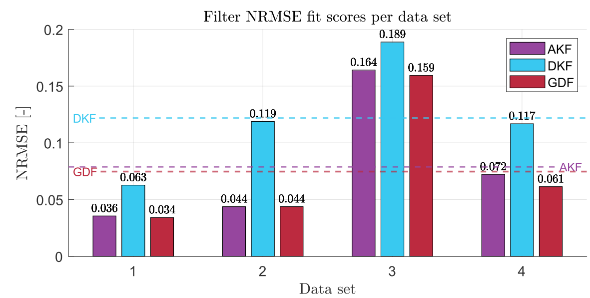

Fig. 5. Overall NRMSE fit performance between the estimated and measured tip position. The dashed line represents the mean filter performance on all data sets.

6. DISCUSSION

The results presented in the previous section sketch an early view on the recursive Bayesian filters performance on high-tech compliant motion stages. There are, however, significant remarks to be made about the implementation and validation process used.

The first is filter tuning. This process severely impacts the performance of these algorithms, and can therefore influence their relative comparison. In this work, an attempt is made to remedy this by tuning all filters using the same optimisation algorithm and training data set. Dependence on tuning parameters varies, however, thus resulting in significantly different final values.

The second remark is the validation process used for these filters. As they are all validated based on the flexure tip position, no hard conclusions can be drawn about the modal amplitude estimation performance as the true value for these signals is not known. The assumption that the tip location estimate performance is proportional to the modal contribution estimation performance might only hold for the first eigenmode. This thus limits possible conclusions about estimation performance for higher frequency vibrational modes.

The third and final remark is the limited amount of data used to study the filter acceleration dependence. This limitation is somewhat mitigated by choosing a variety of scenarios representing realistic working conditions of a compliant motion system. More operating points could however strengthen this analysis.

7. CONCLUSIONS & RECOMMENDATIONS

This work evaluates recursive Bayesian Filtering solutions, initially introduced for applications in SHM in civil engineering, for estimating modal contribution states of high-tech compliant mechanisms. Specifically, three filters are evaluated on acceleration dependence and overall estimation performance. All considered filters are able to reconstruct modal contributions. Their performance, however, varies.

From the results on acceleration dependence, it can be carefully concluded that the DKF performs poorly on this experimental setup for higher accelerations, whereas the AKF and GDF both perform relatively well.

On overall estimation performance, a similar trend is observed, with the DKF performing relatively poorly. The AKF and GDF perform very comparably when looking at NRMSE estimation performance.

It is recalled that the GDF does not assume any dynamics for the input, which relieves the need for L-curve tuning of the input covariance matrix. This significantly increases real world applicability since L-curve tuning is only possible when a validation signal is available. This is in line with results obtained by Tatsis and Lourens (2017).

Based on the acceleration dependence, overall estimation performance and applicability, it can thus be concluded that the GDF shows the most promising results for implementation on high-tech compliant mechanisms.

An interesting direction for future work is the use of the obtained estimated modal contributions in a full AVC scheme to dampen higher order modes for high-tech compliant systems, and evaluate the performance over more conventional AVC methods.

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