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ABSTRACT

Public transport agencies need to leverage on emerging technologies to remain competitive in a mobility landscape that is increasingly subject to disruptive mobility services ranging from ride-hailing to shared micro-mobility. Customized bus (CB) is an innovative transit system that provides advanced, personalized, and flexible demand-responsive transit service by using digital travel platforms. One of the challenging tasks in planning and operating a CB system is to efficiently and practically schedule a set of CB vehicles while meeting passengers' personalized travel demand. Previous studies assume that CB passengers' preferred pickup or delivery time is within a pre-defined hard time window, which is fixed and cannot change. However, some recent studies show that introducing soft flexible time windows can further reduce operational costs. Considering soft flexible time windows, this study first proposes a nearest neighbour-based passenger-to-vehicle assignment algorithm to assign CB passengers to vehicle trips and generate the required vehicle service trips. Then, a novel bi-objective integer programming model is proposed to optimize CB operation cost (measured by fleet size) and level of service (measured by passenger departure time deviation penalty cost). Model reformulations are conducted to make the bi-objective model solvable by using commercial optimization solvers, together with a deficit function-based graphical vehicle scheduling technique. A novel two-stage human–machine collaborative optimization methodology, which makes use of both machine intelligence and human intelligence to collaboratively solve the problem, is developed to generate more practical Pareto-optimal CB scheduling results. Computation results of a real-world CB system demonstrate the effectiveness and advantages of the proposed optimization model and solution methodology.

1. Introduction

1.1. Background and motivation

New mobility services, such as car-sharing, bicycle-sharing, and ride-hailing, have transformed the way people access and utilize transport service. Public transport agencies need to leverage on emerging technologies to remain competitive in a mobility

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landscape that is increasingly subject to new and disruptive mobility services (Shaheen and Cohen, 2020; Berrada and Poulhès, 2021; Ceder, 2021; Militão and Tirachini, 2021; Azadeh et al., 2022; Filippi et al., 2023). A demand-responsive transit system, named customized bus (CB), has been launched in a large number of cities ranging from Beijing to Shanghai in the past decade (Liu and Ceder, 2015; Liu et al., 2016; Tong et al., 2017; Zhang et al., 2017; Lyu et al., 2019; Vansteenwegen et al., 2022; He et al., 2023a; Zhen et al., 2023). As of 2022, more than 400 CB lines are in operation in Beijing. CB provides a transfer-free mobility service, and thereby may potentially help shift private car users to public transport. CB services could be tailored for specific user groups based on a common trip purpose and destinations, such as customized commuter bus, customized school bus, customized community bus, customized feeder/shuttle bus, and customized business bus (Errico et al., 2013; Liu and Ceder, 2015; Lee et al., 2021; Lee et al., 2022).

The planning and operation of CB services differs in several important aspects from that of traditional transit. In traditional transit, the transit operator designs service route, determines stop location, develops service frequency, timetable and vehicle schedule. Passengers are not involved in the decision-making process. However, in a CB system, aided by digital travel platforms, such as smartphone app, WeChat applet, and web applications, passengers and operators are both involved in the service planning and operation process, and can interact in real-time. In this way, a CB system is tailored to better cater for the specific requirements of passengers, as well as further reducing the total operational cost of operators. Fig. 1 shows a typical CB vehicle scheduling process. Initially, CB passengers submit their travel requests, including desired departure location, arrival location, and departure time or arrival time through a digital travel platform. Then, the CB operator collects travel requests, and assigns passengers to CB vehicles, considering passengers' spatio-temporal constraints, vehicle capacity constraint, as well as operator's cost and profit constraints. It results in a passenger-to-vehicle assignment scheme and a set of vehicle service trips. Based on the set of vehicle trips, the operator creates an optimal vehicle schedule that results in a minimum operational cost (fleet size). The passenger-to-vehicle assignment scheme and vehicle schedule are provided to CB passengers via the digital travel platform. If CB passengers are satisfied with the results, then the CB vehicle scheduling process is completed; otherwise, they can provide their feedback to the operator. The operator then further modifies the results to make the vehicle scheduling scheme better cater for passenger needs while guaranteeing all travel requests are served. The interactions between CB passengers and operators can further improve the vehicle scheduling scheme.

CB scheduling plays a very important role in providing a cost-effective and attractive CB service. When scheduling CB services, the scheduler needs to make a trade-off between operational cost, which is usually measured by the required minimum fleet size (Winter et al., 2016; Winter et al., 2018; Militão and Tirachini, 2021; Gkiotsalitis, 2022), and level-of-service, which is usually measured by passenger travel time (Chen et al., 2021a; Wu et al., 2023; Zhao et al., 2023a). The interactions between passengers and operators offer new opportunities to optimize CB scheduling by further considering soft and difficult-to-quantify constraints, such as non-fixed departure time windows. These constraints are very difficult to be formulated as closed-form mathematical equations or convex functions. Thus, it is impossible to solve the problem by only using commercial optimization solvers. However, they can be easily dealt with by human (schedulers) using their knowledge, wisdom and experience. This calls for the design of new mathematical models and novel solution approaches to conduct better and more practical CB scheduling.

The study addresses the CB scheduling problem considering non-fixed flexible departure times. We propose a novel human–machine collaborative optimization methodology which makes use of both machine intelligence and human intelligence to collaboratively solve the problem. The effectiveness of the methodology is demonstrated through a case study of a multi-terminal CB system in Chengdu, China. It is anticipated that the human–machine collaborative optimization methodology can serve as a useful optimization framework and tool in supporting daily CB operations and further exploring the trade-off between operational cost and level-of-service.

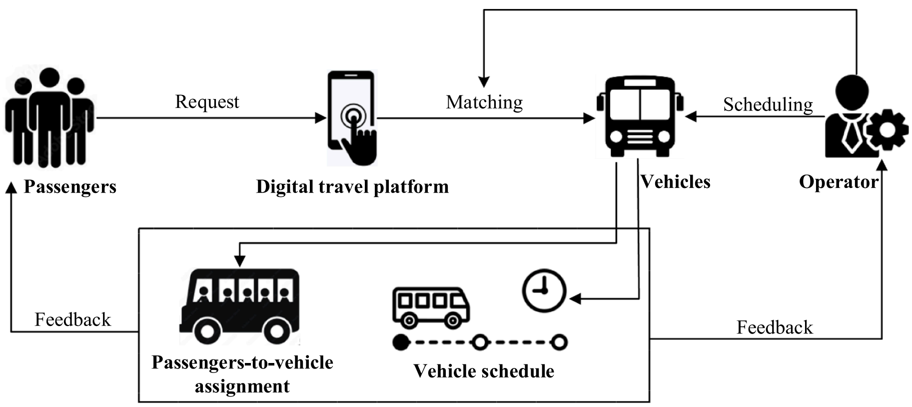

Fig. 1. The customized bus scheduling process.

1.2. Literature review

1.2.1. Customized bus scheduling

In recent years, there is an increasing number of studies on CB. Past studies have mainly focused on four areas: (i) CB demand and passenger behaviour analysis (e.g., Qiu et al., 2018; Li et al., 2019; Li et al., 2021a; Liu et al., 2021; Wang et al., 2021; Wang et al., 2022; He et al., 2023b; Gu and Chen, 2023; Wang et al., 2023a), (ii) CB network route design (e.g., Ma et al., 2017; Tong et al., 2017; Lyu et al., 2019; Guo et al., 2019; Guo et al., 2020; Huang et al., 2020a; Huang et al., 2020b; Ma et al., 2020; Chen et al., 2021a; Chen et al., 2021b; Dou et al., 2021; Wu et al., 2022a; Guo et al., 2023a; Ma et al., 2023a; Ma et al., 2023b), (iii) CB fare and service pricing (e.g., Liu and Ceder, 2015; Chen, 2021; Li et al., 2021b; Yue et al., 2022; Wang et al., 2023b), and (iv) CB vehicle scheduling (e.g., Wang et al., 2018; Han and Fu, 2020; Liu et al., 2020a,b; Sun et al., 2020; Sun et al., 2021; Zhou et al., 2021; Liu et al., 2022; Wu et al., 2023). At the early development stage of CB, researchers are more interested in strategic-level problems, such as demand analysis, network route design, stop location determination, fare and pricing schemes. Both passenger-submitted demand data and passively collected data, such as traditional bus passenger trip data (Qiu et al., 2018), smartphone-based mobile internet data (Liu et al., 2020a, b), and vehicle trajectory data (Ma et al., 2023a), have been utilized in previous studies. With the successful implementation and ongoing expansion of CB, operational problems, such as vehicle scheduling, attract increasing research attention.

The CB scheduling problem aims to assign CB vehicles to conduct all the scheduled vehicle trips with the objective of minimizing operational costs, which is usually measured in terms of the required minimal fleet size, and maximizing the level of service. Some optimization models have been proposed to optimize the CB scheduling problem. For example, considering a fixed departure time window, Wang et al. (2018) formulated the CB scheduling problem as a 0–1 integer programming (IP) model aiming to minimize the total travel distance of CB vehicles. A greedy algorithm was proposed to generate an initial feasible solution; then, a genetic algorithm (GA) was employed to further optimize the solution. Numerical computation results indicate that the total travel distance of CB vehicles can be reduced by using the GA-based solution method. Han and Fu (2020) also studied CB scheduling considering a fixed departure time window. They formulated the CB scheduling problem as a two-stage 0–1 integer programming model with the objective of minimizing the required CB fleet size, passenger delay cost and refused passenger travel request penalty cost, and maximizing the CB operator profit. The model was solved by a GA at the first stage to obtain the minimal fleet size; then, a non-dominated sorting genetic algorithm II with an elite strategy (NSGA II) was employed to further optimize the passenger delay cost, the refused travel request penalty cost, and the operator profit. Computation results show that the two-stage optimization model and GA and NSGA II solution algorithms can generate a reasonable CB scheduling scheme with a fixed departure time window. Liu et al. (2020a,b) proposed a visual analytics approach to scheduling CB service without considering departure time windows. They considered different optimization metrics, including CB driving distance and duration, passenger walking distance and duration, passenger walking reachability ratio within an area, and number of car-hailing records around the CB routes. They did not formulate the CB scheduling problem as a mathematical programming model, but rather employed visual figures to find a better and acceptable CB scheduling scheme. Sun et al. (2020) proposed a mixed integer non-linear programming model to optimize CB scheduling with a heterogeneous fleet considering a fixed departure time window. The model was solved using a hybrid genetic algorithm (HGA) that integrates simulated annealing algorithm procedures into the GA. Computation results from a case study of the CB network in Xi'an, China showed that the total cost, including both operator and passenger costs, can be reduced. Sun et al. (2021) further extended their model to consider stochastic CB vehicle arrival times. In addition, a new HGA with adaptive destroy-and-repair (HGA-ADAR) was proposed to solve the probabilistic optimization model to generate CB vehicle routes and schedule. Zhou et al. (2021) developed an integer programming model to optimize CB scheduling considering using multiple CB vehicle types. The model was solved by using a GA. Simulation analysis and computation results showed that the use of multiple CB vehicle types has the potential to reduce the CB vehicle operation cost and passenger waiting time cost. Liu et al. (2022) proposed a mixed integer linear programming model to optimize the vehicle cost, including both fixed and variable operational costs, and driver costs. The model was solved by using a heuristic multi-objective optimization algorithm based on preemptive scheduling. Computational results showed that the proposed solution algorithm has better performance than the GA and simulated annealing algorithms. Wu et al. (2023) studied the CB scheduling problem considering variable time windows. They formulated the problem as an integer programming model with the objective of minimizing CB operator operating cost, passenger time cost, and number of unserved passengers. The model was solved with a branch-and-cut algorithm with grid-density-based clustering. Computational results showed that the algorithm is scalable and efficient.

Previous studies on CB scheduling are summarized in Table 1, alongside the properties of this study. These studies are compared in terms of optimization model features, solution methods, and departure time window characteristic. It can be seen that almost all previous studies, except for the study of Wu et al. (2023), adopted a fixed departure time window or did not consider a flexible departure time window. Some recent studies showed that introducing flexible time windows can further reduce operational costs (Guo et al., 2020; Chen et al. 2021a; Wu et al., 2023). Almost all previous studies formulated the CB scheduling problem as a single-objective IP model, and solved it using heuristic or *meta*-heuristic algorithms. CB vehicle operational cost and passenger-related costs were the major considerations. It is clear that the CB scheduling problem considering variable and flexible departure time window and multiple optimization objectives has recently gained research interest. In this regard, new efficient and practical solution methods are required.

Table 1
Comparisons of previous and our studies on CB scheduling.

Authors (year)	Optimization model			Solution method	Departure time window
	Optimization objective	Decision variable	Model characteristic		
Wang et al. (2018)	Min total travel distance of CB vehicles	Vehicle chains (routes)	0–1 IP model	Greedy algorithm, genetic algorithm	Fixed
Han and Fu (2020)	Min fleet size, passenger delay cost, refused request penalty cost, Max profit	Vehicle chains (routes) and arrival times at service stations	Two-stage 0–1 IP model	Genetic algorithm, non-dominated sorting genetic algorithm II with an elite strategy	Fixed
Liu et al. (2020b)	Min driving distance and duration, walking distance and duration, Max walking reachability, number of car-hailing records	CB departure times	N.A.	Visual analytics approach	Not consider
Sun et al. (2020)	Min operator and passenger costs	Vehicle schedule and routes	MINLP model	Hybrid genetic algorithm	Fixed
Sun et al. (2021)	Min operator, in-vehicle cost, late and early arrival penalty costs	Vehicle schedule and routes	IP model	Hybrid genetic algorithm with adaptive destroy-and-repair	Fixed
Zhou et al. (2021)	Min operational cost, passenger waiting cost	Vehicle routes and fleet size	IP model	Genetic algorithm	Not consider
Liu et al. (2022)	Min vehicle operations and driver costs	Vehicle and driver assignment	MILP model	Heuristic algorithm	Fixed
Wu et al. (2023)	Min operating cost, passenger time cost, number of unserved passengers	Vehicle departure time, passenger-to-vehicle assignment	Single-objective IP model	Branch-and-cut algorithm with grid-density-based clustering	Variable
This study	Min fleet size, Min passenger departure time deviation penalty cost	Vehicle departure time, passenger-to-vehicle assignment	Bi-objective IP model	Human-machine collaborative optimization, deficit function, optimization solvers	Variable

Note: IP=integer programming, MILP=mixed integer linear programming, MINLP=mixed integer non-linear programming, N.A. = not applicable.

1.2.2. Human-machine collaborative decision-making in transport research

Human-machine collaborative decision-making refers to a decision-making process in which both human and machine work together to analyse information, evaluate options, and make decisions (Crandall et al., 2018). In this collaborative decision-making model, humans and machines complement each other's strengths, combining human creativity, intuition, and contextual understanding with machine intelligence, efficiency, and computational efficiency. This human–machine collaboration is suitable for solving complex decision-making problems that involve conflicting objectives and difficult-to-quantify or non-quantifiable constraints. (Nakamura and Salvendy, 1994; Crandall et al., 2018; Haesevoets et al., 2021; Ren et al., 2023). It has been widely used in various fields, such as business, healthcare, finance, manufacturing, logistics and transportation systems.

Many transport planning and operations problems involve conflicting objectives and non-quantifiable constraints. Therefore, human–machine collaborative decision-making is a very suitable approach for solving these challenging problems in transport research, such as transport network and infrastructure planning, timetable development, vehicle scheduling, and automated driving. In earlier studies, human–machine collaborative decision-making was employed to solve the travelling salesman problem (Krolak et al., 1971) and generalized truck-dispatching problem (Krolak et al., 1972). Rapp (1972) already described the use of human–machine interaction to optimize the transit network design. First, a computer is used to generate a set of alternative transit network configurations with graphic displays, along with their evaluations. Then, the transit planner further makes some modifications based on his or her knowledge and judgement. Through human–machine interactive optimization, the resulting transit network is more applicable in practice. Human-machine collaborative decision-making was also used to create coordinated timetables to optimize passenger transfers (Rapp and Gehner, 1976), and design service timetables and vehicle schedules for intercity passenger transportation companies (Lardinois et al., 1992). Fisher (1985) made an earlier comprehensive survey of successful human–machine collaborative decision-making systems that have been applied in the areas of vehicle scheduling, location problems, job shop scheduling, and course scheduling. Recently, human–machine collaborative decision-making was adopted to improve the safety and the performance of automated vehicle systems (Chen et al., 2021c; Huang et al., 2021; Lv et al., 2021; Zhao et al., 2023b).

An important application area of human–machine collaborative decision-making, which is closely related to this study, is solving complex scheduling problems. Godin (1978) made an earlier survey and discussion on human–machine interactive scheduling. He highlighted the importance of using graphic displays in designing a powerful interactive scheduling system, which was also emphasized by DeSanctis (1984). Fisher (1985) surveyed some graphic vehicle scheduling systems. He emphasized that graphics can help visualize soft and non-quantifiable constraints to which the objective function value is sensitive, and thereby facilitate solution improvement by violating some soft constraints. Nulty and Ratliff (1991) described a human–machine collaborative optimization methodology for ship fleet scheduling by using flexible human–machine graphics interface. Human-machine collaborative optimization is also used in transit vehicle scheduling. Some graphical human–machine interactive transit vehicle scheduling software packages have already been developed, e.g. VAMPIRES, HOT, and more recently, OPTIBUS (Ceder, 2016).

The multi-objective CB scheduling problem with non-fixed flexible departure times involves non-quantifiable time window constraints, which cannot be solved by relying solely on computers, and makes the human–machine collaborative optimization methodology a suitable and promising solution approach.

1.3. Research gaps, contributions and organization

The above literature review clearly indicates that the CB scheduling problem considering variable time window and flexible departure times has both theoretical and practical significance. However, insofar there is no readily available solution method for this. To this end, this study proposes a new solution approach that combines machine intelligence and human intelligence to collaboratively solve the problem.

The contributions of the paper are fourfold. First, we study a new variant of the CB scheduling problem, by considering soft time windows that allow flexible vehicle departure times, to further save operational cost. Second, we develop a nearest neighbour-based passenger-to-vehicle assignment and vehicle trip generation algorithm so as to better assign passengers to CB vehicles and generate the required vehicle service trips. Third, we propose a new bi-objective integer programming model to optimize both operation cost (measured by fleet size) and level of service (measured by passenger desired departure time deviation penalty cost). Model reformulations are provided to make the model solvable by using commercial optimization solvers, together with a deficit function-based graphical vehicle scheduling technique. Fourth, a novel two-stage human–machine collaborative optimization methodology is developed to solve the bi-objective integer programming model so as to generate better and more practical CB scheduling results.

The remaining of this article is organised as follows. In section 2, we describe the nearest neighbour-based passenger-to-vehicle assignment and vehicle trip generation algorithm. Section 3 presents the bi-objective integer programming model, together with model reformulations. The two-stage human–machine collaborative optimization methodology is provided in Section 4. In Section 5 we provide the details of a case study to demonstrate the effectiveness of the model and solution methodology, together with the policy and practical implications. Section 6 concludes the paper, discusses limitations, and offers some possible directions for future research.

2. Passenger-to-vehicle assignment and vehicle trip generation

After collecting passenger travel requests through digital travel platforms, one challenging task is to assign passengers to CB vehicles in order to generate the required vehicle trips. The objective of this assignment is to minimize the number of required vehicle trips while ensuring that passenger travel requests are satisfied with time, space and vehicle capacity constraints. An algorithm based on the nearest neighbour time periods is developed to conduct the passenger-to-vehicle assignment and generate the required vehicle trips considering the possibility of slightly violating passengers' preferred departure time windows. The algorithm for assigning passengers to vehicles and generating the required vehicle trips is described in Algorithm 1 in a step-by-step manner.

Algorithm 1: Nearest neighbour-based passenger-to-vehicle assignment algorithm	
Input:	A scheduling horizon T; time period τ; index of time period k; set of CB terminal stations D; CB terminal station $T_i, i \in D$; number of passenger travel requests at terminal T_i with a preferred departure time window $[k\tau, (k+1)\tau]$, $N_{T_i,k}$; number of seats on a CB vehicle N_v; desired minimum vehicle load factor $\lambda, 0 < \lambda \leq 1$.
Output:	Passenger-to-vehicle assignment and vehicle trips in each time period
Step 1:	For each time period $[k\tau, (k+1)\tau]$, check $N_{T_i,k}$
Step 2:	If $0 < N_{T_i,k} < \lambda N_v$, then one vehicle is used with $N'_{T_i,k} = N_{T_i,k}$ passengers assigned to the vehicle, and go to Step 7.
Step 3:	If $\lambda N_v \leq N_{T_i,k} \leq N_v$, then one vehicle is employed and the $N_{T_i,k}$ passengers are assigned to the vehicle.
Step 4:	If $(n-1)N_v \leq N_{T_i,k} \leq nN_v, n = 2, 3, 4\cdots$, then n vehicles are used; among them, $(n-1)$ vehicles are each assigned with N_v passengers, and one vehicle is assigned with $N'_{T_i,k} = N_{T_i,k} - (n-1)N_v$ passengers.
Step 5:	If $\lambda N_v \leq N'_{T_i,k} \leq N_v$, then one vehicle is employed and the $N'_{T_i,k}$ passengers are assigned to the vehicle.
Step 6:	Otherwise, if $0 < N'_{T_i,k} < \lambda N_v$, go to Step 7.
Step 7:	Check the number of passengers $N'_{T_i,k-1}$, and $N'_{T_i,k+1}$ in the nearest neighbour time periods $[(k-1)\tau, k\tau]$ and $[(k+1)\tau, (k+2)\tau]$.
Step 8:	If $0 < N'_{T_i,k-1} + N'_{T_i,k} \leq N_v$ or $0 < N'_{T_i,k} + N'_{T_i,k+1} \leq N_v$, then bundle these passengers into a single group with the minimum passenger time window deviation and assign the grouped passengers to one vehicle; by doing so, one CB vehicle trip can be saved.
Step 9:	If $N_v < N'_{T_i,k-1} + N'_{T_i,k} \leq 2N_v$ or $N_v < N'_{T_i,k} + N'_{T_i,k+1} \leq 2N_v$, then one vehicle is employed and N_v passengers are assigned to the vehicle, and check whether the rest number of passengers $(N'_{T_i,k-1} + N'_{T_i,k} - N_v)$ or $(N'_{T_i,k} + N'_{T_i,k+1} - N_v)$ can be assigned to other vehicles that are not full in the nearest neighbour time periods $[(k-1)\tau, k\tau]$ and $[(k+1)\tau, (k+2)\tau]$ or not. If this is possible, one CB vehicle trip can be saved; otherwise, keep the original assignment; that is, one vehicle is assigned with $N'_{T_i,k}$ passengers, and the other is assigned with $N'_{T_i,k-1}$ or $N'_{T_i,k+1}$ passengers.
Step 10:	Output the passenger-to-vehicle assignment results and vehicle trips.

Example 1: Consider a small illustrative example with a scheduling horizon of [7:45, 8:30], and a time period $\tau = 15$ min. Then, the schedule horizon can be divided into three time periods (windows), i.e., [7:45, 8:00], [8:00, 8:15], and [8:15, 8:30]. For a given CB terminal, the number of requested passengers for these three time periods are 19, 42, 36, respectively. The number of seats on a CB vehicle is 20. The desired minimum load factor $\lambda = 0.5$. Without violating the three time windows, six CB vehicle trips are required, as shown in Fig 2(a). That is, one vehicle trip for time period [7:45, 8:00] with 19 passengers assigned to the vehicle, three vehicle trips for time period [8:00, 8:15] with two vehicle trips assigned with 20 passengers and one vehicle trip assigned with two passengers, and two vehicle trips for time period [8:15, 8:30] with one vehicle trip assigned with 20 passengers and the other vehicle trip assigned with 16 passengers. By implementing Algorithm 1, the two passengers assigned to the third vehicle trip in time period [8:00, 8:15] can be grouped with the 16 passengers assigned to second vehicle trip in time period [8:15, 8:30]. That is, the two passengers can be assigned to the second vehicle trip in time period [8:15, 8:30], together with the 16 passengers, by violating the time window [8:00, 8:15], as shown in Fig 2(b). By doing so, the third vehicle trip in time period [8:00, 8:15] can be saved. This contributes to reducing the total number of vehicle trips, which may in turn reduce the minimum fleet size required and thereby reduce the total operational cost. Thus, there is a trade-off between the number of vehicle trips (related to fleet size) and the passenger desired departure time deviation penalty.

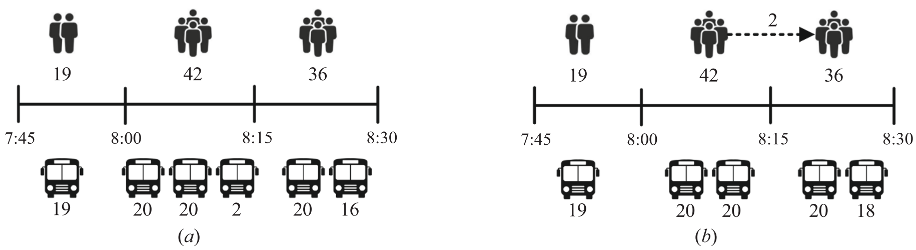

Fig. 2. Illustration of the nearest neighbour-based passenger-to-vehicle assignment and vehicle trip generation process: (*a*) initial assignment, (*b*) optimized assignment.

3. Bi-objective integer programming model

By solving the CB scheduling problem with flexible vehicle departure times we aim at determining the optimal assignment of CB vehicles to carry out all the generated vehicle trips while allowing for flexible vehicle departure times. In the following, we formulate the problem as a bi-objective integer programming model. The first objective, from the perspective of CB passengers, aims at maximizing the level of service, which is measured by the passenger departure time deviation penalty cost. The second objective, from the perspective of operators, aims at minimizing the operation cost, which is measured by the required minimum fleet size. The decision variables are the CB vehicle departure times from terminal stations.

Let us consider a scheduling horizon T, which is discretized in minutes, i.e., T=[1, 2, 3,…, N_t]. Let I denote the set of all generated CB vehicle trips. Consider two CB vehicle trips p and q. The desired departure times for these two trips are denoted as $t_{p,d}$ and $t_{q,d}$, respectively. When CB passengers submit their travel requests, they will be asked to choose a departure terminal station T_i, together with a desired departure time window $[k\tau,(k+1)\tau]$. In practice, the departure time window $[k\tau,(k+1)\tau]$ indicates that vehicle trips scheduled within it will depart at time $(k+1)\tau$. If a passenger chooses a desired departure time window $[k\tau,(k+1)\tau]$, then it indicates that his or her desired departure time is $(k+1)\tau$. Thus, if vehicle trip p is assigned to the time window $[k\tau,(k+1)\tau]$, then we have:

$$t_{p,d} = (k+1)\tau \tag{1}$$

CB vehicles are assumed to depart from terminal stations with flexible departure times. The departure time flexibility is reflected in the following constraint

$$t_{p,d} \in [(k+1)\tau - \delta_p^-, (k+1)\tau + \delta_p^+] \tag{2}$$

where δ_p^- and δ_p^+ are early and late departure time deviation parameters, respectively. These two parameters δ_p^- and δ_p^+ are not fixed here in advance. Hence, the departure time window $[(k+1)\tau-\delta_p^-, (k+1)\tau+\delta_p^+]$ is *not fixed*. In this situation, the time window is defined as a soft departure time window, compared to the hard time window in which the two parameters δ_p^- and δ_p^+ are fixed.

Let $t_{p,r}$ denote the vehicle running time from the departure terminal of trip p to the arrival terminal of trip p plus the terminal turnover time, and $t_{p,e}$ denote the deadheading (empty-vehicle) running time from the arrival terminal of trip p to the departure terminal of trip q. Let M be a very large positive constant. The required minimum fleet size can be determined by firstly solving the below 0–1 integer programming model.

$$MaxN_1 = \sum_{p\in I}\sum_{q\in I} x_{p,q} \tag{3}$$

s.t.

$$t_{q,d} - \left(t_{p,d} + t_{p,r} + t_{p,e}\right) \geq -M\left(1 - x_{p,q}\right), \qquad \forall p, q \in I \tag{4}$$

$$\sum_{q\in I} x_{p,q} \leq 1, \forall p \in I \tag{5}$$

$$\sum_{p\in I} x_{p,q} \leq 1, \forall q \in I \tag{6}$$

$$t_{p,d} \in [(k_p+1)\tau - \delta_p^-, (k_p+1)\tau + \delta_p^+], \forall p \in I \tag{7}$$

$$t_{q,d} \in [(k_q+1)\tau - \delta_q^-, (k_q+1)\tau + \delta_q^+], \forall q \in I \tag{8}$$

$$x_{p,q} \in \{0,1\}, \forall p, q \in I \tag{9}$$

The model is a vehicle trip-connection-based max-flow model that maximizes the number of feasible vehicle trip connections. Constraint (4) indicates that if two CB vehicle trips p and q can be conducted by the same vehicle, then the variable $x_{p,q}$ can be 0 or 1; otherwise, it equals to 0. Constraint (5) guarantees that each CB vehicle trip may be connected with no more than one successor trip. Similarly, Constraint (6) ensures that each CB vehicle trip may be connected with no more than one predecessor trip. Constraints (7) and (8) are decision variables. Constraint (9) is the intermediate binary variable. The objective function (Eq. (3) maximizes the number of feasible CB vehicle trip connections N_1. The model intends to calculate the maximum flow in a unit capacity bipartite network, which has a computational complexity of $O(n^{1/2}m)$ with n nodes (number of vehicle trips) and m arcs (number of feasible trip connections).

Then, the required minimum fleet size N_{min}^{fs} can be calculated as

$$N_{min}^{fs} = |I| - MaxN_1 \tag{10}$$

where $|I|$ is the number of CB vehicle trips. The proof of this formula can be found in Ford and Fulkerson (1962).

Let $t_{j,d}$ and $t_{j,a}$ denote the desired and actual departure times of a CB passenger j. Then, the deviation from the preferred departure time Δt_j can be calculated by

$$\Delta t_j = \left| t_{j,a} - t_{j,d} \right| \tag{11}$$

The departure time deviation penalty cost is then defined as

$$\Delta t_j = \alpha(t_{j,a} - t_{j,d}), if t_{j,a} \geq t_{j,d} \tag{12}$$

$$\Delta t_j = \beta(t_{j,d} - t_{j,a}), if t_{j,a} < t_{j,d} \tag{13}$$

where α and β are late and early departure time penalty parameters, respectively. If passenger j is assigned to vehicle trip p, then

$$t_{j,a} = t_{p,d}, \quad if\ j \in G_p \tag{14}$$

where G_p is the set of passengers assigned to vehicle trip p. Next, the total passenger desired departure time deviation penalty cost can be calculated by

$$\Delta t = \sum_{j \in J} \Delta t_j \tag{15}$$

where J is the set of all the served CB passengers. Then, the overall bi-objective integer programming model can be formulated as the following model:

[M1]

$$Min\, \Delta t = \sum_{j \in J} \Delta t_j \tag{16}$$

$$Min N_{min}^{fs} = |I| - Max N_1 \tag{17}$$

s.t.

$$N_1 = \sum_{p \in I} \sum_{q \in I} x_{p,q} \tag{18}$$

Constraints (4)-(9), (12)-(14)

The model can be transformed into the below equivalent bi-objective integer linear programming model.

[M2]

$$Min\, \Delta t = \sum_{j \in J} \Delta t_j \tag{19}$$

$$Max N_1 = \sum_{p \in I} \sum_{q \in I} x_{p,q} \tag{20}$$

s.t.

Constraints (4)-(9), (12)-(14)

Since the early and late departure time deviation parameters δ_p^- and δ_p^+ are not fixed in advance, the model [M2] cannot be directly solved by using exiting solution methods or commercial optimization solvers. In the next section, a novel solution approach that combines machine intelligence and human intelligence is proposed to collaboratively and practically solve the CB scheduling problem with flexible departure times.

4. Solution approach

We propose a two-stage human–machine collaborative optimization methodology to solve the bi-objective optimization model detailed in the previous section. The optimization methodology combines the advantages of machine (computer) and human (scheduler) to collaboratively solve the model. At the first stage, a computer is employed to efficiently solve a single-objective integer programming model to generate a CB scheduling scheme. At the second stage, a scheduler is employed to practically further improve the CB scheduling scheme by using a graphical vehicle scheduling technique that is based on a deficit function-based graphical scheduling method. In this section, we first provide a concise description of the deficit function-based graphical vehicle scheduling method; then, the two-stage human–machine collaborative optimization methodology is described.

4.1. Deficit function-based graphical vehicle scheduling method

The deficit function (DF)-based graphical scheduling method was initially proposed by Linis and Maksim (1967) for flight scheduling. A DF is defined as a step function that is associated with a transportation terminal. Its value increases by one at the time of each vehicle trip departure and decreases by one at the time of each vehicle trip arrival. To construct a DF, the only information needed is a set of scheduled vehicle trips. Let $d(k,t,S)$ denote the DF for terminal k at time t for schedule S. The value of $d(k,t,S)$ represents the total number of departing vehicles minus the total number of arriving vehicles at terminal k, up to and including time t. The maximum value of $d(k,t,S)$ over the schedule horizon $T=[T_1, T_2]$, designated as $D(k,S)$, depicts the deficit number of vehicles required at terminal k. It indicates that at least $D(k,S)$ vehicles need to be allocated to terminal k in order to conduct the scheduled vehicle trips. Usually, the notation S will be omitted when it is clear which underlying schedule is being considered. The sum of all the maximum values of DFs corresponds to the required minimum fleet size, which is described in the following minimum fleet size theorem.

Minimum Fleet Size Theorem: If for a set of terminals K and a set of vehicle trips I all vehicle trips start and end within the schedule horizon $T=[T_1, T_2]$, then the minimum number of vehicles required, i.e., the minimum fleet size, to service all vehicle trips in I is equal to the sum of the maximum deficit function values across all terminals.

$$MinN^{fs}(S) = \sum_{k\in K} D(k,S) = \sum_{k\in K} \underset{t\in[T_1,T_2]}{Max}\, d(k,t,S) \tag{21}$$

The proof of this formula can be found in Ceder (2016).

Example 2: Figure 3 shows how to use the DF-based graphical scheduling method to conduct vehicle scheduling and calculate the required minimum fleet size. As shown in the upper part of Fig 3, the small example problem has four vehicle trips within a scheduling horizon of T=[8:00, 10:30]. Trip 1 runs from terminal a to terminal a, and trip 2 runs from terminal b to terminal a, and trips 3 and 4 run from terminal a to terminal b. According to the definition of DF, we can draw the DF figures for the two terminals, as shown in the lower part of Fig 3. It shows that the maximum value of $d(a,t)$ is $D(a) = 3$, while for terminal b, it is $D(b) = 1$. According to the minimum fleet size theorem, the required minimum number of vehicles, i.e., minimum fleet size, is $D(a) + D(b) = 3 + 1 = 4$. We can further obtain the four vehicle chains; that is, [1], [2], [3], and [4]. This indicates that four vehicles are required to conduct the four vehicle trips, and each vehicle conducts one vehicle trip.

The main advantage of a DF is its graphical and visual nature. With a DF figure, the scheduler can easily observe the maximum value of a DF. Thus, the minimum fleet size can be easily obtained. More importantly, the scheduler can further optimize the fleet size by observing DF figures and making some small adjustments of vehicle trip departure times. Taking the illustrative vehicle scheduling problem shown in Fig. 3 as an example, the scheduler observes that the maximum value of d(a,t) can be reduced from three to two if the departure time of trip 4 can be slightly shifted; that is, shifting the departure time of trip 4 from 9:27 to 9:30. Then, the vehicle conducting trip 1 can continue conducting trip 4 after finishing serving trip 1. Fig. 4 shows the new DF figures of the illustrative example after slightly shifting the departure time of trip 4 from 9:27 to 9:30. We can see that the maximum value of d(a,t) has

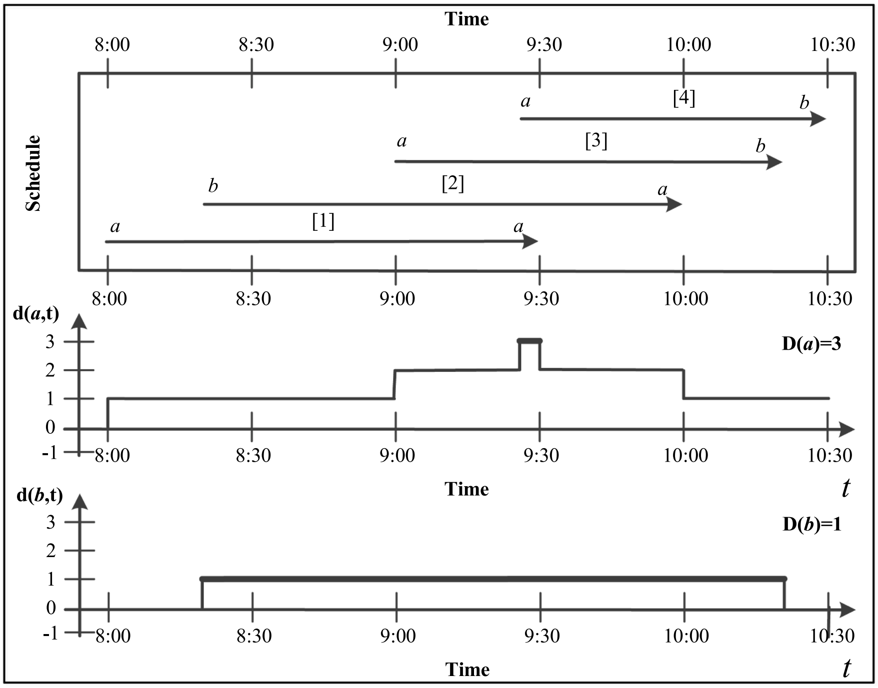

Fig. 3. Illustration of a deficit function used for the purpose of vehicle scheduling.

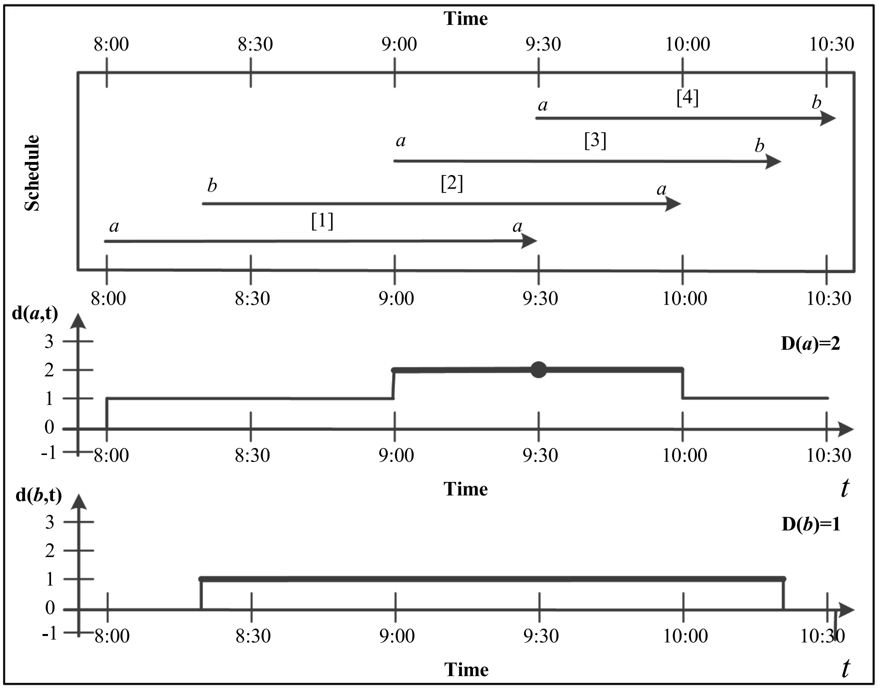

Fig. 4. New deficit functions of the illustrative example after slightly shifting the departure time of trip 4.

consequently decreased from three to two, which means one vehicle has been saved for terminal a. The sum of the maximum values of the two new DFs is D(a) + D(b) = 2 + 1 = 3. It indicates that the required minimum fleet size is three. Totally, three vehicles are required to conduct the four vehicle trips. The three vehicle chains are: [1]-[4], [2], and [3].

This illustrative example shows how to use the DF-based graphical scheduling method to determine the required minimum fleet size. In addition, the graphical feature of DFs can facilitate schedulers to further optimize fleet size by making slight adjustments of vehicle trip departure times. The proposed two-stage human–machine collaborative optimization methodology makes use of these advantages of the DF-based graphical scheduling technique to deal with the flexible vehicle departure time constraints.

4.2. Human-machine collaborative optimization approach

The proposed two-stage human–machine collaborative optimization methodology uses the DF figures as the graphical user interface between a computer (machine) and a scheduler (human). The overall solution procedure is graphically shown in Fig. 5. At the first stage, computerized optimization software packages are employed to efficiently solve an integer linear programming model without considering the soft time window constraint. Initially, the departure time for a vehicle trip p, $t_{p,d}$ is set as $(k_p+1)\tau$, which means all vehicle trips depart from the terminal stations at the desired departure times. Thus, there is no passenger departure time deviation penalty, i.e., $\Delta t = 0$. The soft time window constraints (7) and (8) become

$$t_{p,d} = (k_p+1)\tau,\ \forall p \in I \tag{22}$$

$$t_{q,d} = (k_q+1)\tau,\ \forall q \in I \tag{23}$$

By replacing constraints (7) and (8) with the new constraints (22) and (23), the M2 model becomes the M3 model shown below, which is linear and does not include soft time window constraints. This integer linear model can be solved to global optimality by using commercial optimization solvers. In this situation, there is no passenger departure time deviation, i.e., $\Delta t_j = 0$, and $\Delta t = 0$; thus, there is no need to have constraints (12)-(14) and objective function $Min\,\Delta t$. The M3 model degenerates into a single-objective integer programming model [M4], which maximizes the number of feasible trip connections without considering soft time window constraints.

[M3]

$$Min\,\Delta t = \sum_{j\in J}\Delta t_j \tag{24}$$

$$MaxN_1 = \sum_{p\in I}\sum_{q\in I}x_{p,q} \tag{25}$$

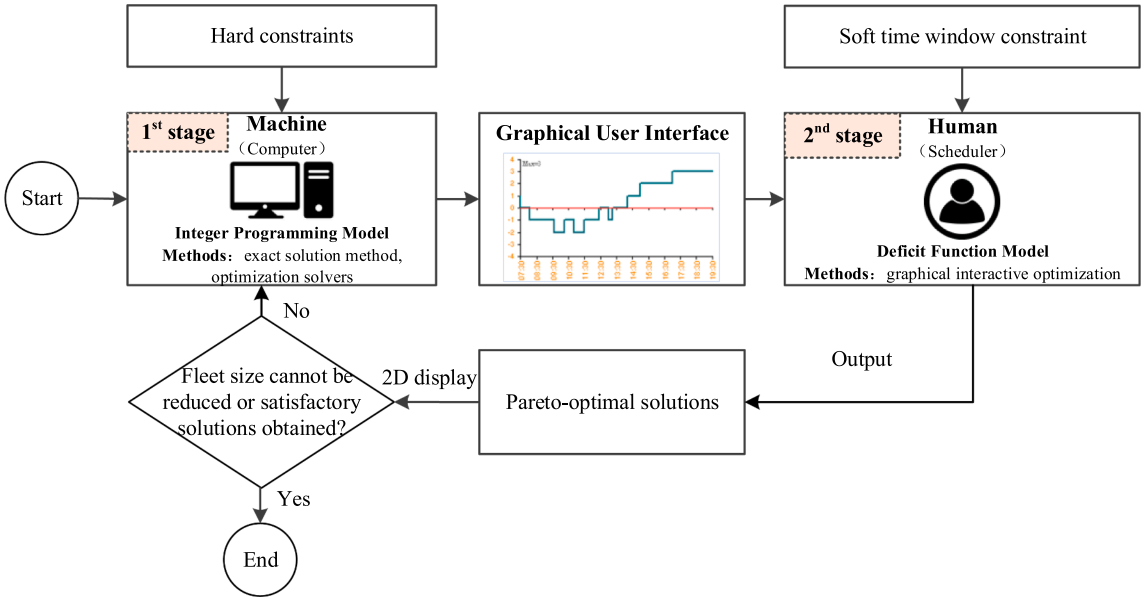

Fig. 5. Procedure of the two-stage human–machine collaborative optimization methodology using deficit function figures as the graphical user interface.

s.t.

Constraints (4)-(6), (9), (12)-(14), (22)-(23) .
[M4]

$$MaxN_1 = \sum_{p\in I}\sum_{q\in I} x_{p,q} \tag{26}$$

s.t.

Constraints (4)-(6), (9), (22)-(23) .

The M4 model can be easily solved by using commercial optimization solvers to get the maximum number of trip connections $MaxN_1$, and then an initial fleet size $N_{min}^{fs,0}$ can be obtained using Eq. (10). By doing so, we can obtain the first Pareto-optimal solution, i.e., $s_0 = \left\{\Delta t, N_{min}^{fs}\right\} = \{0, N_{min}^{fs,0}\}$. With the vehicle trip schedule resulted from the initial solution, the related DF figures can be drawn. Then, the scheduler can further optimize the fleet size by using the DF-based graphical vehicle scheduling model at the second stage. In each human–machine interaction iteration, the scheduler aims at further reducing the fleet size by one through shifting vehicle trip departure times. Note that there may be different vehicle departure time adjustment schemes that all can contribute to reducing the fleet size. In this situation, the scheduler should choose the scheme that results in the minimum passenger departure time deviation, i.e., $Min\,\Delta t$. This can be easily done by using the graphical feature of the DF and the number of passengers assigned to each vehicle departure. In addition, there are usually not too many feasible vehicle departure time adjustment schemes which can be easily checked by the scheduler. By doing so, a new Pareto-optimal solution, i.e., $s_m = \{\Delta t_m, N_{min}^{fs,m}\}$, can be obtained. Thus, in each human–machine interaction iteration process, a new Pareto-optimal solution is generated. The iteration process will not stop until the fleet size cannot be further reduced or the scheduler is satisfied with the current solutions.

When the iteration process stops, a set of Pareto-optimal solutions, i.e., $S = \{s_0, s_1, \cdots, s_m, \cdots\}$, can be obtained. These Pareto-optimal solutions are displayed using a two-dimensional (2D) space, corresponding to the two objective functions. The graphical display of the Pareto-optimal solutions can facilitate the scheduler to make a trade-off between the two objectives and choose a desired solution (CB vehicle scheduling scheme) for practical implementation.

It can be seen that the proposed two-stage human–machine collaborative optimization methodology combines the powerful computation capacities of computers, graphical features of DFs, and the knowledge and experience of schedulers to better solve the CB scheduling problem with flexible departure time constraints. The resulting solutions are better and more practical than the solutions generated by only using computerized optimization software packages.

5. Model application

The optimization model and solution methodology described in the previous sections are applied to solve a real-world CB scheduling problem with flexible departure times. In the following we present the case study results to better understand the performance of the two-stage human–machine collaborative optimization methodology.

5.1. Case study description

The model is applied for the airport express CB system in Chengdu, China. It was launched on June 27, 2021, and provided a customized shuttle transit service between the suburban Chengdu Tianfu International Airport and the downtown area. It is a reservation-based CB service. CB passengers need to buy tickets through a smartphone app (Chengdu Bus App) to reserve seats before taking buses. Fig. 6(a) shows the ticketing interface of the Chengdu Bus App. The airport express CB system adopts a time-differentiated flat fare scheme; that is, CB passengers are charged with a fixed fare of 15 CNY during the operation period of 6:00–23:30, and 25 CNY during the operation period of 23:30–6:00, for a single trip. Fig. 6(b) shows the two bi-directional CB lines considered in the case study. Line 1 operates between the Chengdu Tianfu International Airport and Chengdu East Railway Station. Line 2 operates between the Chengdu Tianfu International Airport and Jinsha Transport Hub Station. The two CB lines have a tentative service headway of 30 min, but both allow for flexible departure times.

5.2. Data collection and parameter setting

The two CB lines are visualized on the road street map, as shown in Fig. 6(b), by using ArcMap10.8. Initial line headways and service trip departure times are collected from the Chengdu Bus App. CB passenger demand data are also collected for the studied lines, including the departure location, arrival location, departure time of each CB passenger. Vehicle trip running times between departure and arrival terminal stations are collected from two digital maps, namely AutoNavi (Gaode) map and Baidu map, for different operation periods, including morning peak hours (7:00–9:00), evening peak hours (17:00–20:00), and off-peak hours. The average value is taken as the vehicle trip running time. In addition, the terminal layover time and passenger boarding and alighting times are also included in a vehicle trip time. Table 2 lists the average trip time between terminal stations for the two CB lines. With the trip departure time and trip time data, the trip arrival time can be obtained. Vehicle deadheading (empty-vehicle running) times between the three terminal stations are also collected. These data are processed and stored in Microsoft Excel Spreadsheets, and are subsequently accessed by optimization solvers.

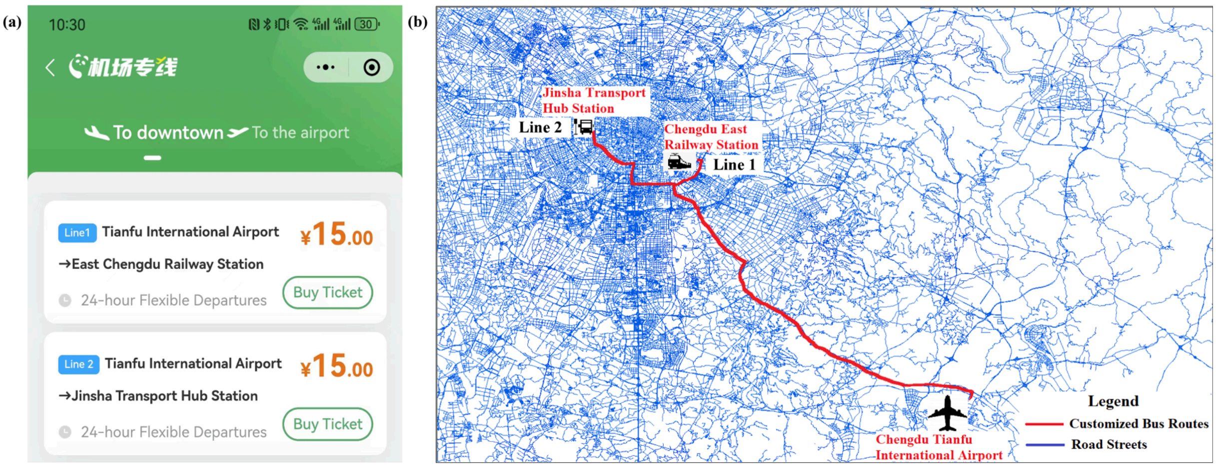

Fig. 6. The airport express customized bus service in Chengdu, China: (a) The Chengdu Bus App ticketing interface, (b) customized bus routes configuration.

Table 2
Average trip time (min) between CB terminal stations.

	Chengdu Tianfu International Airport (T1)		
	Off-peak hours	Morning peak hours	Evening peak hours
Chengdu East Railway Station (T2)	70	83	85
Jinsha Transport Hub Station (T3)	95	110	108

The CB fleet is homogenous with a capacity of 49 passengers. The desired minimum vehicle load factor λ is set as 0.5 to guarantee service profitability. Following the research findings of Noland and Polak (2002), the late and early departure time penalty parameters α and β are set as 2.04 and 1, respectively.

5.3. Results and analysis

The implementation of Algorithm 1 is performed in a Microsoft Excel Worksheet and all computation experiments were conducted in a personal computer (PC) with an Intel Core TM i7-7600U CPU @ 2.80 GHz, 8.00 GB RAM, and a 64-bit Windows 10 operating system. The integer linear programming models were coded and solved with the commercial optimization solver 'LINGO', version 18.0. The computation results show that all the integer linear programming models in each iteration can be solved very quickly, i.e. within less than one second.

Without implementing the nearest neighbour-based passenger-to-vehicle assignment algorithm, the demand data results in 286 vehicle service trips. Using these trip information as input data, we solve the vehicle trip-connection-based max-flow model [M4], and obtain a maximal number of 253 feasible vehicle trip connections. According to the minimal fleet size calculation formula (Eq. (10), a minimal number of 33 CB vehicles is required. Since all passengers depart from the three terminal stations at their desired departure times, the passenger departure time deviation penalty cost is $\Delta t = 0$. Thus, we obtain the first initial Pareto-optimal solution $s_0 = \left\{\Delta t, N^{fs}_{min}\right\} = \{0, 33\}$. After implementing the nearest neighbour-based passenger-to-vehicle assignment algorithm, the required number of vehicle service trips is further reduced to 223. With the trip information of these 223 trips, the vehicle trip-connection-based max-flow model [M4] is solved again, which results in a minimal fleet size of 23 CB vehicles. In total, 27 CB vehicle deadheading trips are employed to achieve the minimal fleet size. Some passengers are assigned to the near neighbour time periods, which is not his or her desired departure time period. After implementing the nearest neighbour-based passenger-to-vehicle assignment algorithm, it results in a passenger departure time deviation penalty cost of $\Delta t = 17179.88$ passenger-minutes. Thereby, we obtain the second Pareto-optimal solution $s_1 = \left\{\Delta t, N^{fs}_{min}\right\} = \{17179.88, 23\}$. After using a computer to solve the integer linear programming models at the first stage, DF figures with respect to the minimal fleet size are constructed and displayed using the graphical user interface. By doing so, CB schedulers can be involved to further optimize the results by considering flexible departure times at the second stage. Fig. 7 displays the DF figures of the three terminal stations with respect to the minimal fleet size of 23 CB vehicles. As can be seen, 11, 2, and

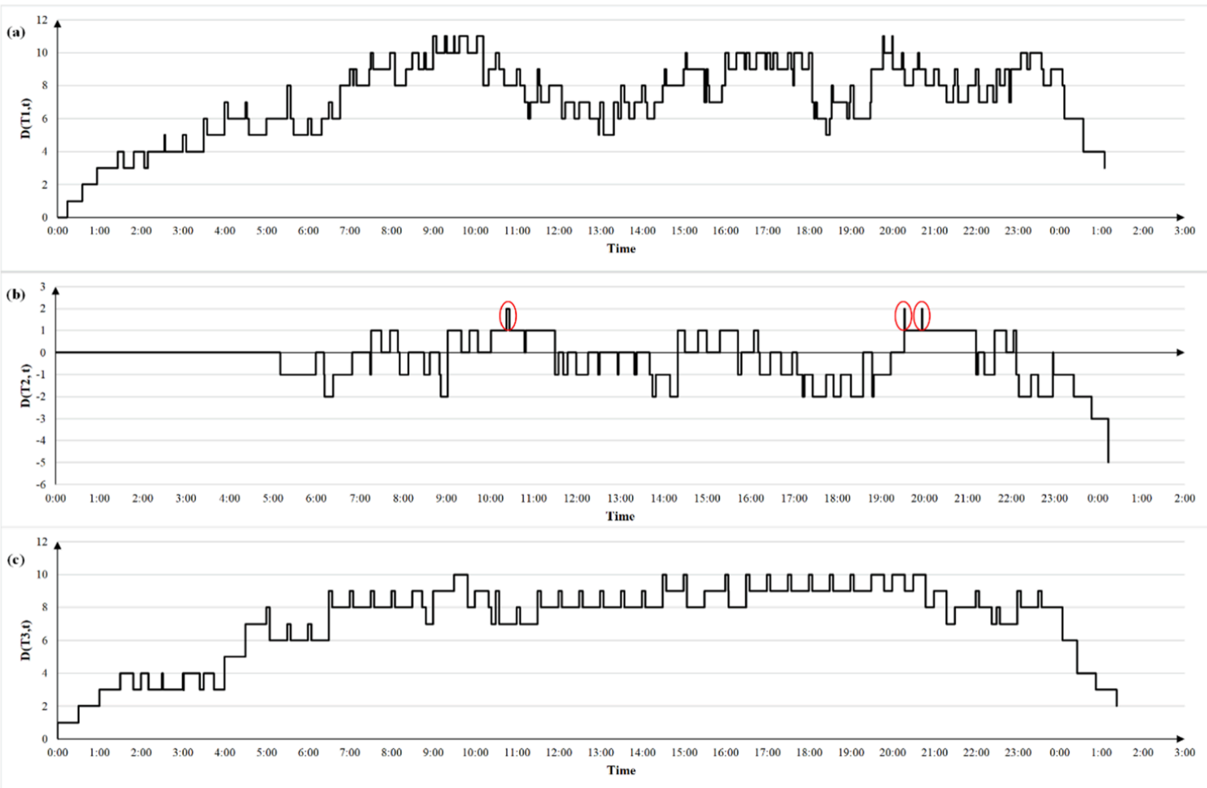

Fig. 7. Deficit functions of the three terminal stations: (a) Chengdu Tianfu International Airport, (b) Chengdu East Railway Station, (c) Jinsha Transport Hub Station.

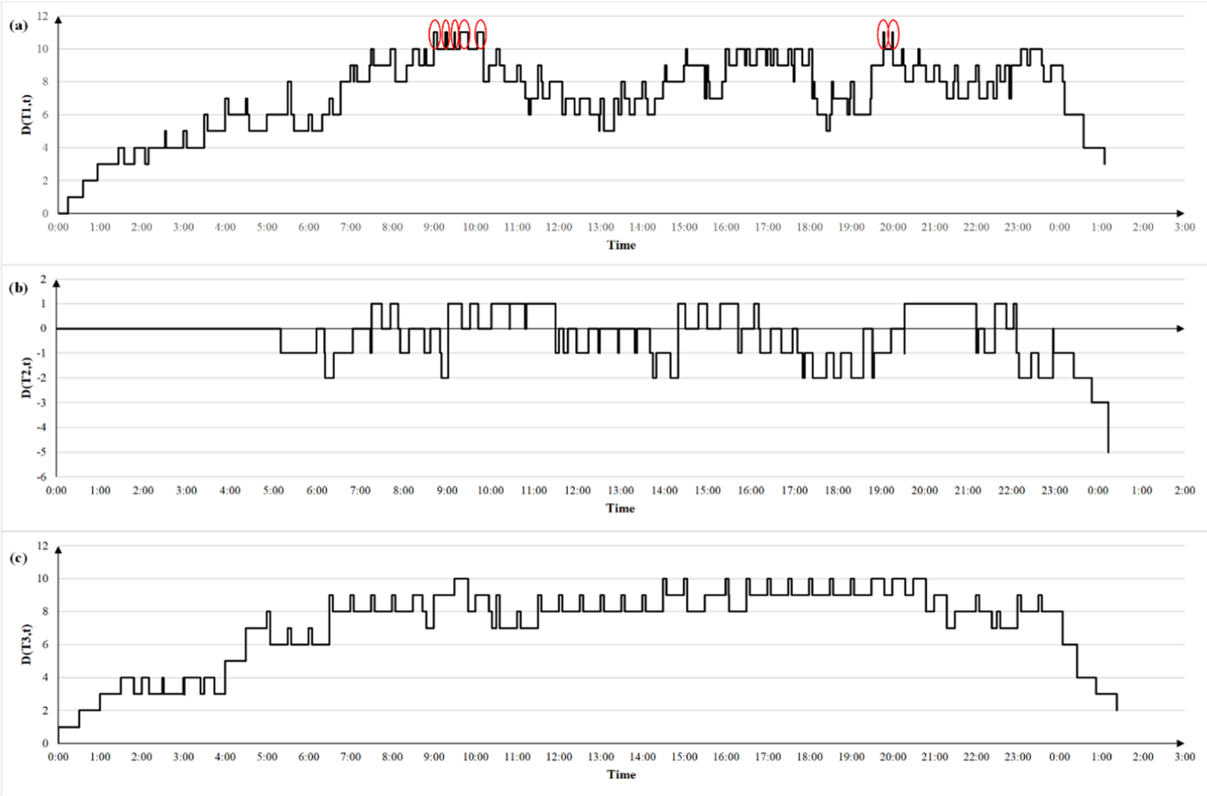

Fig. 8. Deficit functions of the three terminal stations after the first departure time adjustment: (a) Chengdu Tianfu International Airport, (b) Chengdu East Railway Station, (c) Jinsha Transport Hub Station.

10 CB vehicles are required for the Chengdu Tianfu International Airport, Chengdu East Railway Station, and Jinsha Transport Hub Station, respectively.

The CB scheduler then observes the maximal values of the three DF figures and checks whether the maximal values of the DFs can be reduced by slightly changing the departure times of some CB vehicle trips or not. Fortunately, the scheduler observes that the maximal value of the DF of the Chengdu East Railway Station can be reduced from 2 to 1, after slightly adjusting the departure times of three vehicle trips (the red circle areas highlighted in Fig. 7(b)). That is, one trip departs one minute earlier, one trip departs one minute later, and one trip departs four minutes later. After making these trip departure time adjustments, the new trip information is used as input data to the vehicle trip-connection-based max-flow model [M4]. Solving the model again with the computer optimization solver, we can obtain the new minimal fleet size of 22 CB vehicles. We use the adjusted trip departure time information to calculate the passenger departure time deviation penalty cost using Eqs. 12–13. This results in a total passenger departure time deviation penalty cost of 17598.12 passenger-minutes. Then, we can obtain the third Pareto-optimal solution $s_2 = \left\{\Delta t, N_{min}^{fs}\right\} = \{17598.12, 22\}$. The DF figures are updated with the adjusted trip information, which is shown in Fig. 8.

The CB scheduler continues observing the maximal values of the three DF figures and checks whether the maximal values of the DFs can be further reduced by slightly changing the departure times of some CB vehicle trips or not. Yet again, the scheduler observes that the maximal value of the DF of the Chengdu Tianfu International Airport can be reduced from 11 to 10, after slightly adjusting the departure times of seven vehicle trips (the red circle areas in Fig. 8(a)). That is, three trips depart one minute earlier, one trip departs five minutes earlier, one trip departs nine minutes earlier, and two trips depart two minutes later. With the adjusted trip departure time information, Model M4 is solved again, which results in a minimal fleet size of 21 CB vehicles. The associated total passenger departure time deviation penalty cost is calculated again, which is 19522.52 passenger-minutes. Then, we can obtain the fourth Pareto-optimal solution $s_3 = \left\{\Delta t, N_{min}^{fs}\right\} = \{19522.52, 21\}$. The DF figures are updated again with the new trip information, which is shown in Fig. 9.

The CB scheduler then continues observing the maximal values of the three DF figures and checks whether the maximal values of the DFs can be further reduced by slightly changing the departure times of some CB vehicle trips or not. Unfortunately, the scheduler observes that the maximal values of the three DFs is exceeded more than ten times. It means that more than ten trips need to be considered in the departure time adjustment, which is not easy and will result in a considerable increase of the total passenger departure time deviation penalty cost. Thus, The CB scheduler decides to stop the interactive optimization process.

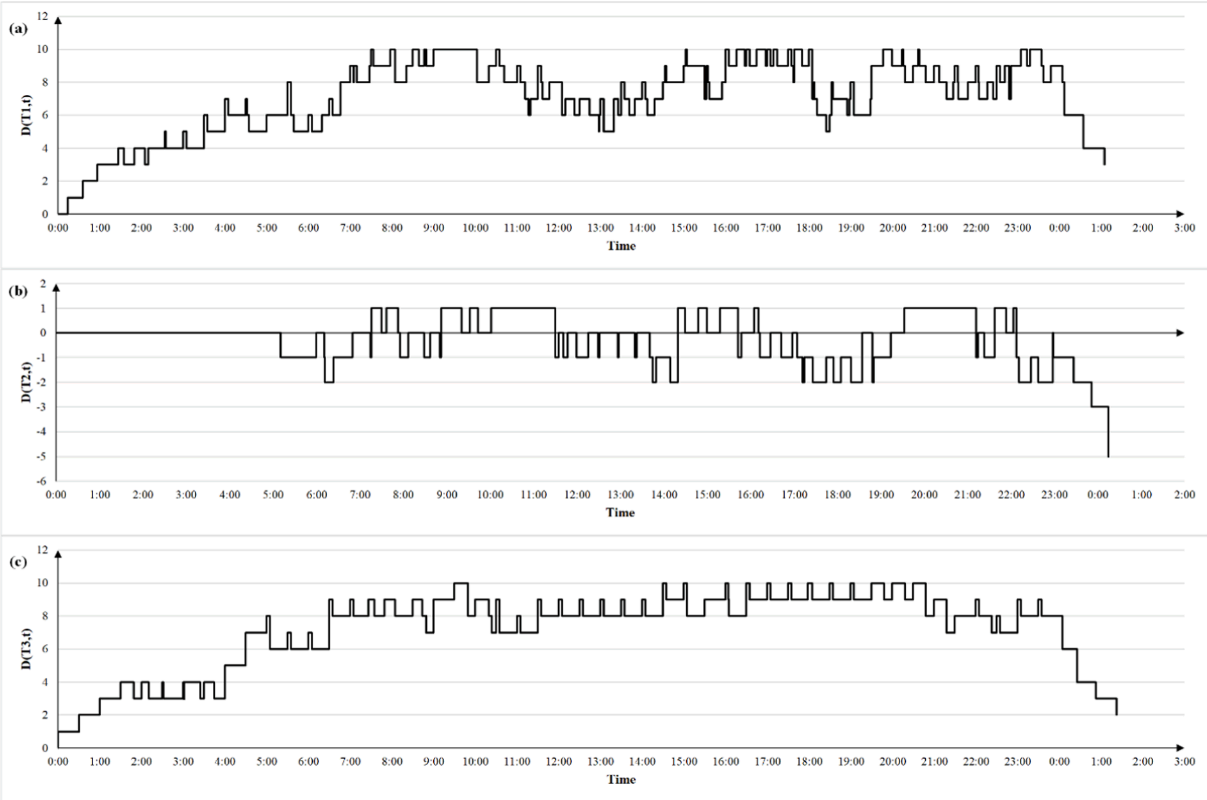

Fig. 9. Deficit functions of the three terminal stations after the second departure time adjustment: (a) Chengdu Tianfu International Airport, (b) Chengdu East Railway Station, (c) Jinsha Transport Hub Station.

The final CB vehicle scheduling results are shown in Fig. 10 in the form of a Gantt chart. In the chart, each vehicle trip is represented by a horizontal bar. The length of the bar corresponds to the duration of the trip, and its position along the timeline indicates its start and end times. Different vehicle trips, i.e., service trips or deadheading trips, are represented by different color bars. Fig. 10 indicates that a minimal number of 21 vehicles are required to serve the 223 CB service trips, and 27 deadheading trips are included in the 21 vehicle trip chains so as to achieve the minimal fleet size.

Finally, at the end of the human–machine collaborative optimization, the set of Pareto-optimal solutions, i.e., $S = \{s_0, s_1, s_2, s_3\}$, are displayed in a 2D space with respect to the two optimization objectives, as shown in Fig. 11. With this graphical information in hand, the CB scheduler can make an explicit trade-off between the required minimal fleet size and the total passenger departure time deviation penalty cost. For example, the scheduler can use some of the saved cost resulted from reduced fleet size as compensation or reward to CB passengers whose desired departure times have been deviated, so as to increase the acceptance for the new CB vehicle schedule amongst CB passengers. The new CB vehicle schedule and compensation or reward scheme can be easily provided to CB passengers through the digital travel platform. Using passengers' feedback, the CB scheduler can decide to select a preferred solution or a set of preferred solutions for practical implementation.

6. Discussions on implications for policy and practice

A series of policy and practice implications and managerial insights are identified based on our model application. First, model application results demonstrate that CB service operators could make use of both machine (computer) intelligence and human (scheduler) intelligence to better solve complex CB scheduling problems with flexible and non-quantifiable constraints. CB schedulers' knowledge, experience and preferences play an important role in the human–machine collaborative scheduling process, and it is therefore important to offer relevant training sessions and coach their collaborative work and interface with the computerized scheduling system.

Second, CB schedulers are advised to use soft and flexible CB vehicle schedules, rather than rely on hard and fixed ones when designing CB vehicle schedules, since soft and flexible vehicle schedules have the potential of reducing the required fleet size and saving operational costs. However, soft and flexible vehicle schedules may lead to schedule deviations from the passengers' desired departure times. Thus, it is important that CB schedulers need to make a trade-off between the operational cost and level of service. One promising solution to keep a balance between operational cost and level of service is to use some of the saved operational cost, resulting from reduced fleet size, to compensate for level-of-service loss in the form of discounted fares (Singh et al., 2023). This

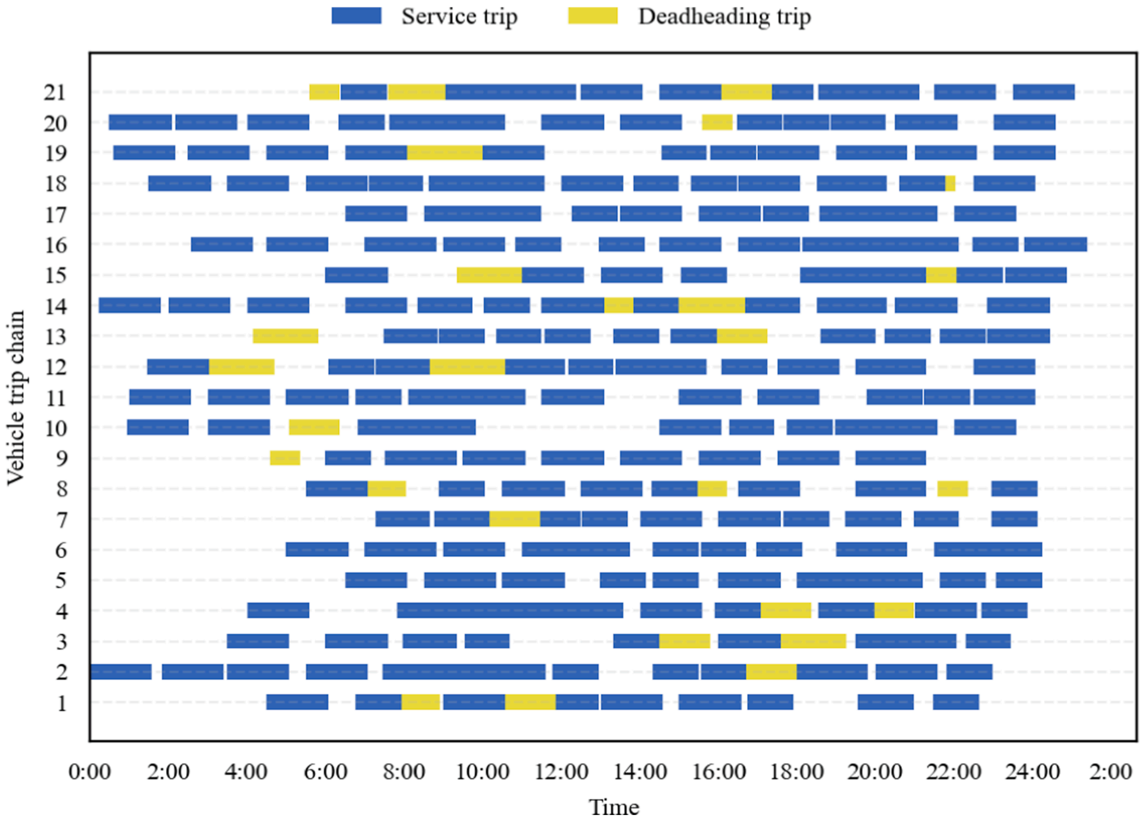

Fig. 10. Gantt Chart of the final vehicle schedule.

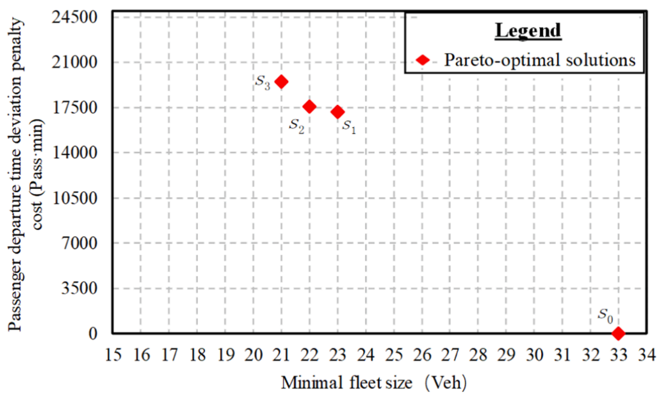

Fig. 11. Pareto-optimal solutions with respect to the two optimization objectives.

approach can enhance the attractiveness and applicability of flexible CB vehicle schedules.

Third, the graphical representation of the deficit function (DF) plays an important role in user interfaces, and can thereby facilitate effective interactions between machine (computer) and human (scheduler) and help schedulers deal with soft and non-quantifiable constraints. DF-based CB scheduling software packages should be developed so as to assist CB schedulers in conducting human–machine collaborative CB scheduling to solve complex CB scheduling problems with soft constraints and practical considerations.

7. Conclusion

Customized bus (CB) is an emerging form of demand-responsive transit systems. The use of digital travel platforms in CB offers the possibility of having real-time interactions between CB passengers and schedulers, and also provides the opportunities to consider soft and non-quantifiable constraints in optimizing CB planning and operations. This study examines the CB vehicle scheduling problem with flexible departure times. The problem is formulated as a bi-objective integer linear programming model with the objectives of minimizing the required minimal fleet size and total passenger departure time deviation penalty cost. A novel human–machine collaborative optimization methodology is proposed to solve this problem by making use of the graphical features of the deficit function (DF)-based vehicle scheduling model. The model is applied for the case study of the airport express CB system in Chengdu, China demonstrating the effectiveness and advantages of the human–machine collaborative optimization methodology. The proposed bi-objective optimization model and graphical human–machine collaborative optimization methodology are very useful for solving complex CB scheduling problems that involve soft and non-quantifiable constraints.

A shortcoming of this study is that the CB passenger-to-vehicle assignment and vehicle scheduling activities are conducted in a sequential way. Preferably, and a potential avenue for further research, these two activities will be conducted simultaneously in order to exploit the CB system capability to the greatest extent and maximize system productivity and efficiency (Tong et al., 2017; Wu et al., 2022b).

Subsequent and future research may include: (i) extending the optimization model to consider multiple vehicle types since CB operators may use different vehicle types to better cater for passenger demand; (ii) further exploring the collaboration and interaction mechanism between humans (scheduler) and machines (computer) to increase the efficiency and effectiveness of the human–machine collaborative optimization methodology (Crandall et al., 2018); (iii) extending the optimization model and solution approach to consider real-time CB travel requests, and integrating flexible CB with other transport systems to create more cost-effectives and convenient mobility service (Calabro et al., 2023; Guo et al., 2023b); (iv) considering different pricing strategies and fare structures to maximize operator profit; (v) developing computerized scheduling software packages using the optimization models and DF-based two-stage human–machine collaborative optimization methodology, and testing with large-scale real-world CB scheduling problems considering different features, such as uncertain passenger demand, stochastic vehicle running time, and flexible vehicle routing.

CRediT authorship contribution statement

Tao Liu: Writing – review & editing, Writing – original draft, Supervision, Project administration, Methodology, Data curation, Conceptualization. **Hailin You:** Investigation, Formal analysis, Data curation. **Konstantinos Gkiotsalitis:** Writing – review & editing, Methodology, Conceptualization. **Oded Cats:** Writing – review & editing, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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