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Bakker, Mark; Bot, Bram

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The Effective Vertical Anisotropy of Layered Aquifers

by Mark Bakker[1,2] and Bram Bot[3]

Abstract

Many sedimentary aquifers consist of small layers of coarser and finer material. When groundwater flow in these aquifers is modeled, the hydraulic conductivity may be simulated as homogeneous but anisotropic throughout the aquifer. In practice, the anisotropy factor, the ratio of the horizontal divided by the vertical hydraulic conductivity, is often set to 10. Here, numerical experiments are conducted to determine the effective anisotropy of an aquifer consisting of 400 horizontal layers of which the homogeneous and isotropic hydraulic conductivity varies over two orders of magnitude. Groundwater flow is simulated to a partially penetrating canal and a partially penetrating well. Numerical experiments are conducted for 1000 random realizations of the 400 layers, by varying the sequence of the layers, not their conductivity. It is demonstrated that the effective anisotropy of the homogeneous model is a model parameter that depends on the flow field. For example, the effective anisotropy for flow to a partially penetrating canal differs from the effective anisotropy for flow to a partially penetrating well in an aquifer consisting of the exact same 400 layers. The effective anisotropy also depends on the sequence of the layers. The effective anisotropy values of the 1000 realizations range from roughly 5 to 50 for the considered situations. A factor of 10 represents a median value (a reasonable value to start model calibration for the conductivity variations considered here). The median is similar to the equivalent anisotropy, defined as the arithmetic mean of the hydraulic conductivities divided by the harmonic mean.

Introduction

Many sedimentary aquifers consist of a sequence of small layers of coarser and finer material. When flow in such aquifers is simulated as flow in a single homogeneous layer, the homogeneous hydraulic conductivity is anisotropic, where the vertical hydraulic conductivity is significantly smaller than the horizontal hydraulic conductivity. Fitts (2012, page 60) states that the anisotropy factor (the horizontal k divided by the vertical k) can range from <10 to >100 in layered soils or rocks, while Anderson et al. (2015) cite modeling studies that use even higher ratios of 1000 or more. In groundwater practice, many modelers apply an anisotropy factor of 10 (Shepley 2024). This article tries to investigate whether this is indeed a good number for flow with a significant vertical component through sedimentary aquifers consisting of many layers with hydraulic conductivities that vary over two orders of magnitude (e.g., from clay to fine sand or from coarse sand to gravel). The layering obviously also has a strong effect on transport of dissolved substances, but that is not considered in this article.

An extensive body of work exists on methods to obtain representative hydraulic conductivity values for heterogeneous hydraulic conductivity fields (see, e.g., the review by Sanchez-Vila et al. 2006), but they do not give much guidance in choosing the vertical anisotropy for groundwater modeling practice. Measurements of vertical anisotropy of layered systems in the field are rather involved as they require a set of closely spaced observation wells at different levels to capture three-dimensional head variations near partially penetrating wells or streams. Practical guidance for field measurements, including applications to field sites, are given by, for example, Weeks (1969) and Maier et al. (2022). Field measurements of vertical anisotropy give fairly local values as three-dimensional flow patterns extend horizontally over only a few aquifer thicknesses in most settings (e.g., Shepley 2024). It seems only logical that the anisotropy ratio is commonly determined during model

[1]Water Management Department, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands

[2]Corresponding author: Water Management Department, Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands; mark.bakker@tudelft.nl

[3]Olmst, The Netherlands

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calibration in groundwater modeling practice (Anderson et al. 2015, page 212), even though the calibration of spatially varying hydraulic conductivity for anisotropic systems is a complicated task (Gianni et al. 2019).

Formulas for the equivalent horizontal and vertical hydraulic conductivity of layered systems are well known (e.g., Fitts 2012; Anderson et al. 2015). The term equivalent horizontal hydraulic conductivity is used here to indicate that a horizontal head difference results in the exact same total flow in the layered system as in the equivalent homogeneous system. The equivalent horizontal hydraulic conductivity k_h for purely horizontal flow through an aquifer consisting of N horizontal, isotropic and homogeneous layers is the arithmetic mean weighted by the layer thickness

$$k_h = \frac{1}{H}\sum_{i=1}^{N} b_i k_i \tag{1}$$

where b_i and k_i are the thickness and hydraulic conductivity of layer i, respectively, and H is the total thickness of the aquifer. Similarly, the equivalent vertical hydraulic conductivity k_v for purely vertical flow is the harmonic mean weighted by the layer thickness

$$k_v = \frac{H}{\sum_{i=1}^{N} b_i/k_i} \tag{2}$$

The equivalent vertical anisotropy, α_{eq}, is computed as

$$\alpha_{eq} = k_h/k_v \tag{3}$$

Although these formulas are easy enough to apply, data on the thicknesses and hydraulic conductivities of all the layers that constitute an aquifer are often not available.

The equivalent values for k_h and k_v (Equations 1 and 2) are valid for purely horizontal or purely vertical flow, respectively. They depend on the thickness and hydraulic conductivity of each layer, but do not depend on the sequence of the layers. In practice, flow is of course never purely horizontal or purely vertical and an effective hydraulic conductivity must be used. The effective hydraulic conductivity, and thus the effective anisotropy, depends on both the order of the layers that make up the aquifer and on the flow field, as will be demonstrated in this article. It is important to note that the equivalent hydraulic conductivity (computed with Equations 1 and 2) is an aquifer characteristic, while the effective hydraulic conductivity is a model parameter that also depends on the flow field. The effective hydraulic conductivity, and thus the effective anisotropy, for a specific flow field in a specific layered system can be obtained from calibration to observations or to meet a specific condition (e.g., Anderson 2005). In this article, the effective anisotropy is obtained through numerical experiments for flow through an aquifer consisting of many small horizontal layers that extend over the entire model domain; systems where the layers are discontinuous are beyond the scope of this article.

The objective of this article is three-fold. First, it is demonstrated that the effective vertical anisotropy varies over a significant range depending on the order of the layers that make up an aquifer. Second, it is shown that the effective anisotropy is a model parameter and that a different flow field through an aquifer with exactly the same sequence of layers results in a different effective anisotropy. Third, the range of effective vertical anisotropy values is quantified for a specific layered system where the hydraulic conductivity varies over two orders of magnitude. Results are obtained through numerical experiments for flow fields with significant vertical flow such as flow to a partially penetrating canal and flow to a partially penetrating well.

Effective Anisotropy From Numerical Experiments

Numerical experiments were conducted to compute the effective vertical anisotropy. An isolated setup with a significant vertical flow component was considered; ambient (horizontal) flow may be superimposed, as it is not affected by the vertical anisotropy. In the following, the general approach is explained for confined flow to a partially penetrating canal in a layered system (Figure 1); flow is steady and two-dimensional in the vertical plane. The aquifer consists of many small layers with differing but isotropic hydraulic conductivity values. The canal penetrates the top part of the aquifer and takes water out of the aquifer at a rate Q per meter canal normal to the plane of flow. The head is fixed at the left and right sides of the domain. Note that the heads are only fixed to be able to compute the additional drawdown caused by the partially penetrating canal.

The effective anisotropy is determined by first computing the head h_c in the canal with a groundwater model that has all the small layers with differing but isotropic hydraulic conductivity values; this model is referred to as the stratified model. Next, a groundwater model is created with a homogeneous but anisotropic hydraulic conductivity. This model, referred to as the homogeneous model, simulates the same flow system and has the same number of layers, but with one homogeneous, anisotropic hydraulic conductivity for all layers. The horizontal hydraulic conductivity of the

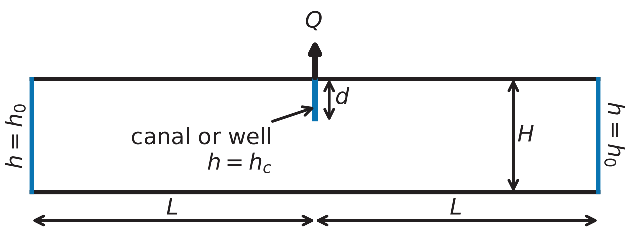

Figure 1. Cross-section of flow domain used for the numerical experiments (not to scale). Flow domain may represent two-dimensional flow towards a partially penetrating canal or radially symmetric flow to a partially penetrating well.

homogeneous groundwater model is set equal to the equivalent hydraulic conductivity (Equation 1) and the vertical hydraulic conductivity of the homogeneous model is adjusted such that the head in the canal is equal to h_c, the value computed with the stratified model. This is referred to as the effective vertical hydraulic conductivity k_v^* and the effective vertical anisotropy α_{eff} is computed as

$$\alpha_{\text{eff}} = k_h / k_v^* \tag{4}$$

Before being able to conduct the numerical experiments, it must be decided how to vary the hydraulic conductivities of the small layers that make up the aquifer. The distribution of these hydraulic conductivities may be expected to largely determine the order of magnitude of the effective anisotropy. In this article, layered aquifers are considered where the hydraulic conductivity varies over two orders of magnitude, which represents layers varying from, for example, clay to fine sand, or from coarse sand to gravel. Larger variations may be found in sedimentary aquifers with a lot of layered heterogeneity (e.g., Shepley 2024), but these are not considered here.

The reason to choose a variation of k of two orders of magnitude is that when drilling a hole in such an aquifer, the layering will be noticeable, but probably not so large that the modeler finds it necessary to simulate different layers explicitly. The layering is likely noticeable when the difference between the diameter of the largest and smallest particles is a factor of 5, that is, $D_{90}/D_{10} = 5$, where D_{90} and D_{10} are the 90% and 10% diameters of the particle size distribution, respectively; this is used as the lower limit. As an upper limit, layered aquifers are considered where $D_{90}/D_{10} = 20$. The numerical experiments are carried out for $D_{90}/D_{10} = 10$, the geometric mean of 5 and 20.

The particle size distribution of the layered system is approximated as a straight line on semi-log paper, such that the particles can be divided in groups that are linearly spaced in log space. Five equally sized groups are used with diameters that are linearly spaced in log space from diameter D for the smallest particles to $10D$ for the largest particles. Using the heuristic that the hydraulic conductivity is a function of the diameter squared ($k \sim D^2$) (Terzaghi and Peck 1967), the hydraulic conductivities of the five groups vary over two orders of magnitude, again linearly spaced in log space. The smallest value is chosen as $k = 1$ m/d so that the largest value is $k = 100$ m/d (i.e., $D \approx 0.03$ mm and $10D \approx 0.3$ mm). The hydraulic conductivity values of the five groups are presented in Table 1. Note that the specific choice of hydraulic conductivity values does not affect the anisotropy as long as the values vary logarithmically by two orders of magnitude. The equivalent hydraulic conductivity values corresponding to the k-values in Table 1 are obtained from Equations 1 and 2 as $k_h = 29$ m/d and $k_v = 3.4$ m/d for an equivalent vertical anisotropy of $\alpha_{\text{eq}} = 8.5$.

Table 1
Hydraulic conductivities of the groups used in the numerical experiments

Group	1	2	3	4	5
k (m/d)	1	3.16	10	31.6	100

Setup of Numerical Experiments

Two flow systems are considered: Flow to a partially penetrating canal and flow to a partially penetrating well (see Figure 1). The dimensions of the flow domain are chosen as follows. The aquifer is confined, the length is $L = R = 2000$ m, and the thickness is $H = 40$ m. The aquifer consists of 400 layers of 10 cm thickness which are divided in five groups of 80 layers with the hydraulic conductivity values specified in Table 1. The canal penetrates the aquifer over a distance d and runs normal to the plane of flow (Figure 1). Flow to the canal is two-dimensional in the vertical plane. The well also penetrates the aquifer over a distance d. Flow to the well is radially symmetric.

Flow is simulated with multilayer analytic elements (Bakker and Strack 2003) using the open-source TimML code (Version 6.1.2; Bakker 2023) to avoid any effects of horizontal discretization. TimML can simulate steady multi-layer flow through an arbitrary number of layers; each layer may have a different hydraulic conductivity. The Dupuit approximation is adopted for flow within a layer and vertical flow between layers is computed using a standard finite difference scheme. No horizontal discretization of the flow domain is needed. The head and horizontal component of flow are computed analytically for each layer. TimML may be used to simulate (quasi) three-dimensional flow in layered systems including wells, rivers, and inhomogeneities in the aquifer properties. TimML has a special feature to simulate flow in a layered vertical cross-section, which is used here. The boundaries at the left and right sides of the domain are constant head boundaries with $h_0 = 0$ m; the top and bottom of the aquifer are impermeable. The boundary condition along the canal is that the head is constant (but apriori unknown) along the canal, that is, equal in all the layers that are penetrated by the canal, and that the total outflow into the canal is equal to the specified discharge Q; the width of the canal is neglected. The total discharge toward the canal is set equal to $Q = 2\,\text{m}^2/\text{d}$ where half comes from the left side and the other half from the right side. Similarly, the boundary condition along the well screen is that the head is constant (but apriori unknown) along the well screen. The radius of the well is equal to r_w. Radial flow below the well screen equals zero at $r = r_w$. The total extraction of the well is $Q = 1000\,\text{m}^3/\text{d}$. Note that the specified values of Q have no effect on the computed effective anisotropy and are only given for completeness.

As an example, head contours and streamlines are shown for flow to a canal that penetrates the aquifer over a distance $d = 4$ m in Figure 2. Head contours and

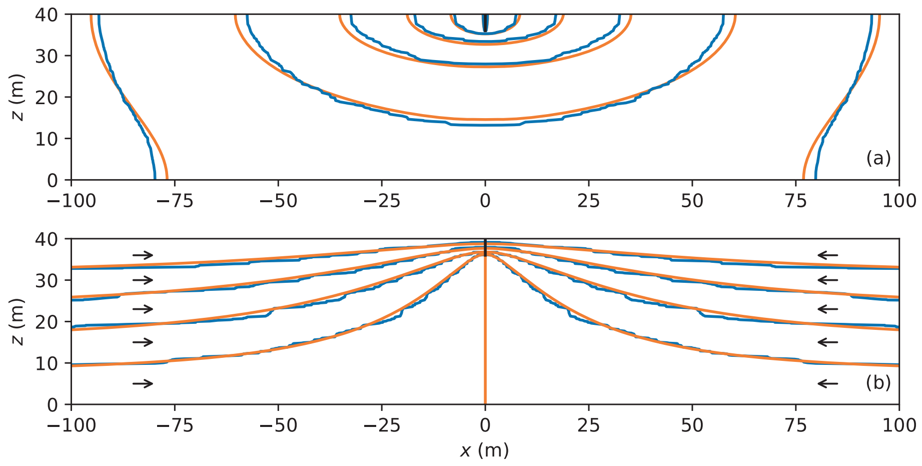

Figure 2. Cross-section of flow to a partially penetrating canal in an aquifer consisting of 400 layers for the stratified model with one random realization of layer properties (blue) and the homogeneous model with uniform k_h and k_v^* (orange); (a) head contours with contour interval 2 cm and (b) streamlines with contour interval $Q/10$.

streamlines are shown for both the stratified model with one random realization of the 400 layers (blue) and for the homogeneous model (orange). Only a 200 m section of the flow domain near the canal is shown. The head contours become vertical and the streamlines become horizontal farther away from the canal, beyond a distance $H\sqrt{k_h/k_v} \approx 3H$ for the values of anisotropy considered here (Bakker and Post 2022).

In the following, the effective vertical anisotropy is determined for 1000 random realizations of the 400 layers for three sets of experiments. First, the effective anisotropy is determined for a partially penetrating canal and for two penetration depths. Second, the effective anisotropy is determined for a partially penetrating well, for one penetration depth, but for two different radii of the well. Third, some of the experiments are repeated for a different random realization of the hydraulic conductivity such that the transmissivity of each 4 m section of aquifer is the same.

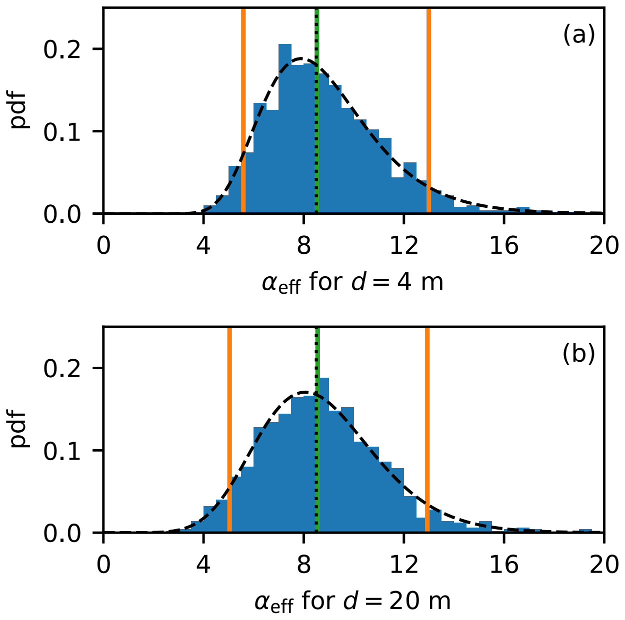

Figure 3. Histogram of the effective anisotropy for 400 randomly placed layers for flow to a partially penetrating canal in a vertical cross-section with penetration depths $d = 4$ m (a) and $d = 20$ m (b). The green line is the median, the orange lines are the 5 and 95 percentiles, and the black dotted line is the equivalent anisotropy. The black dashed line is a fitted lognormal distribution.

Flow to a Partially Penetrating Canal

The effective anisotropy is computed for 1000 random realizations of the 400 layers for two penetration depths of the canal: a penetration depth of $d = 4$ m (the top 40 layers) and a penetration depth of $d = 20$ m (i.e., 200 layers which means half the aquifer); the exact same 1000 realizations are used for both cases. A histogram of the effective anisotropy for both penetration depths is shown in Figure 3. The histograms are asymmetric for both cases. A log-normal distribution is fitted to the data for illustration purposes (black dashed line) and seems to fit fairly well. For a penetration depth of $d = 4$ m, the 5%–95% range is 5.6–13 (orange lines) with a median of 8.5 (green line). For a penetration depth of $d = 20$ m, the 5%–95% range is 5.0–13 with a median of 8.6. The median effective anisotropy for both flow fields is similar to the equivalent anisotropy ($\alpha_{eq} = 8.5$) which is shown with the black dotted line.

Although the histograms for both cases are similar, the effective anisotropy for a given layer sequence may be very different for different canal depths. This is further illustrated in Figure 4, where the effective anisotropy for $d = 20$ m is plotted versus the effective anisotropy for $d = 4$ m for all 1000 realizations. If the same effective anisotropy would have applied to both canal depths, all dots would have fallen on the black dashed line. Furthermore, if the effective anisotropy would not have been a function of the layer sequence, the effective anisotropy would be the same for all 1000 cases and all

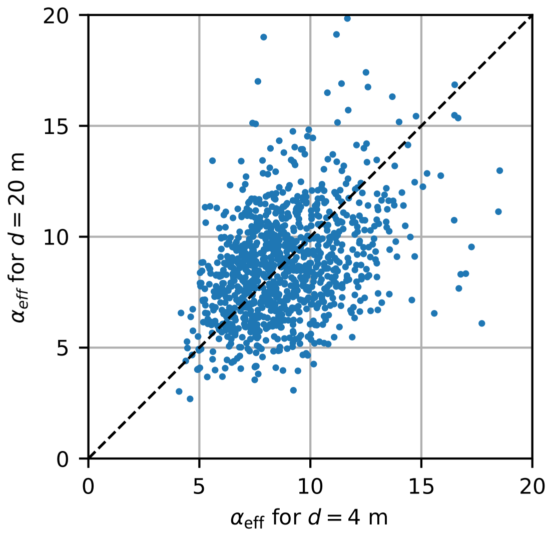

Figure 4. Correlation of the effective anisotropy for 400 randomly placed layers and $d = 20$ m versus $d = 4$ m. The black dashed line represents a perfect one-to-one relation.

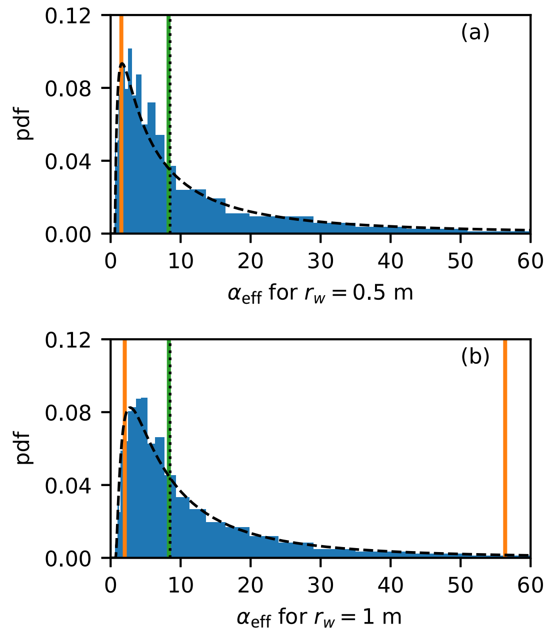

Figure 5. Histogram of the effective anisotropy for 400 randomly placed layers for radially symmetric flow to a partially penetrating well with penetration depth $d = 4$ m and well radius $r_w = 0.5$ m (a) and $r_w = 1$ m (b). The green line is the median, the orange lines are the 5 and 95 percentiles (the 95 percentile falls outside the axis limits in the top graph), and the black dotted line is the equivalent anisotropy. The black dashed line is a fitted lognormal distribution.

dots would have been on top of each other. As this is not the case, it is concluded from Figure 4 that the effective anisotropy is a function of both the order of the 400 layers and the flow field (here, the canal depth).

Flow to a Partially Penetrating Well

The effective anisotropy is computed for flow to a partially penetrating well. The exact same 1000 realizations of the 400 layers are used as for the case with the canal. The effective anisotropy is computed for all 1000 random realizations and for two radii of the well: $r_w = 0.5$ and $r_w = 1$ m, but for one penetration depth of $d = 4$ m. A histogram of the effective anisotropy for both radii is shown in Figure 5. The histograms are very asymmetric for both cases. A log-normal distribution is fitted to the data for illustration purposes (black dashed line) and seems to fit fairly well. For a well radius of $r_w = 0.5$ m, the 5%–95% range is 1.5–90 (orange lines) with a median of 8.3 (green line). For a well radius of $r_w = 1$ m, the 5%–95% range is 2.0–56 with a median of 8.3. The median effective anisotropy for both flow fields is slightly smaller than the equivalent anisotropy ($\alpha_{eq} = 8.5$) which is shown with the black dotted line.

The right tail of the histogram for the smaller well radius is significantly larger than for the larger well radius. This is further illustrated by plotting the α_{eff} for the larger well radius ($r_w = 1$ m) versus the α_{eff} for the smaller well radius ($r_w = 0.5$ m) (Figure 6a). This graph clearly shows that the effective anisotropy for a well with a smaller radius can be significantly larger than the effective anisotropy for a well with a larger radius.

As stated, the effective anisotropy of a layered aquifer is a model parameter that depends on the flow field. The effective anisotropy for flow to a canal can be vastly different from the effective anisotropy for flow to a partially penetrating well. The effective anisotropy for flow to a partially penetrating canal with $d = 4$ m is plotted versus the effective anisotropy for flow to a partially penetrating well with $d = 4$ m and $r_w = 1$ m for all 1000 realizations in Figure 6b. The effective anisotropy for flow to a canal does not exceed 15, but the effective anisotropy for flow to a well easily exceeds 50 for the exact same sequence of layers (recall that the 95 percentile for flow to a well is $\alpha_{eff} = 56$).

More Even Distribution of Layers

The range of effective anisotropy values can be large, especially for flow to a partially penetrating well. One of the reasons is that the distribution of the 400 layers is entirely random, which means that the distribution may be very uneven. For example, consider the transmissivity of the top 4 m of the aquifer. For the 1000 realizations used in the numerical experiments, the 5%–95% range is 81–156 m^2/d with a median of 117 m^2/d (note that the total transmissivity of the aquifer is 1170 m^2/d for all realizations). It may be expected that the uneven distribution of the transmissivity has a significant effect on the effective anisotropy. For this purpose, the effective anisotropy for a canal with canal depth $d = 4$ m is plotted versus the transmissivity of the top 4 m of the aquifer in Figure 7. There is a clear relationship that a larger transmissivity of the top 4 m generally results in a lower effective anisotropy, but there is still significant variability.

To investigate this further, two experiments are repeated for a more even distribution of the transmissivity.

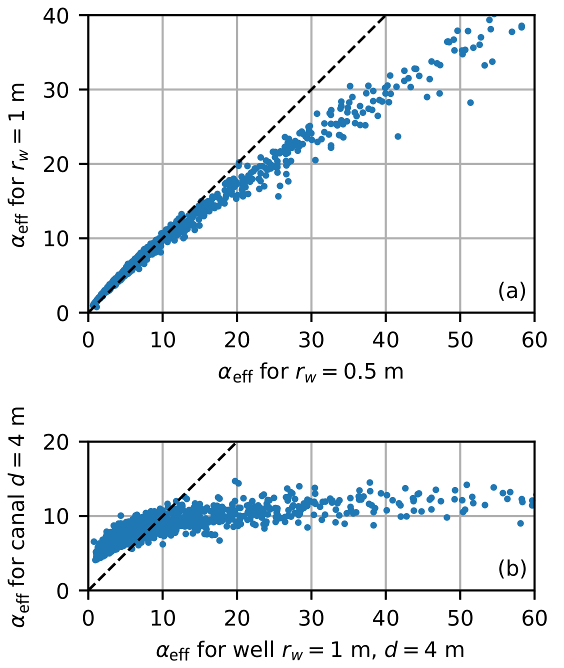

Figure 6. (a) Correlation between the effective anisotropy for a well with radius $r_w = 1$ m and the effective anisotropy for a well with radius $r_w = 0.5$ m. (b) Correlation between the effective anisotropy for flow to a partially penetrating canal and the effective anisotropy for flow to a partially penetrating well with the same penetration depth. Horizontal axis cut off at $\alpha_{\text{eff}} = 60$. The black dashed lines represent a perfect one-to-one relation.

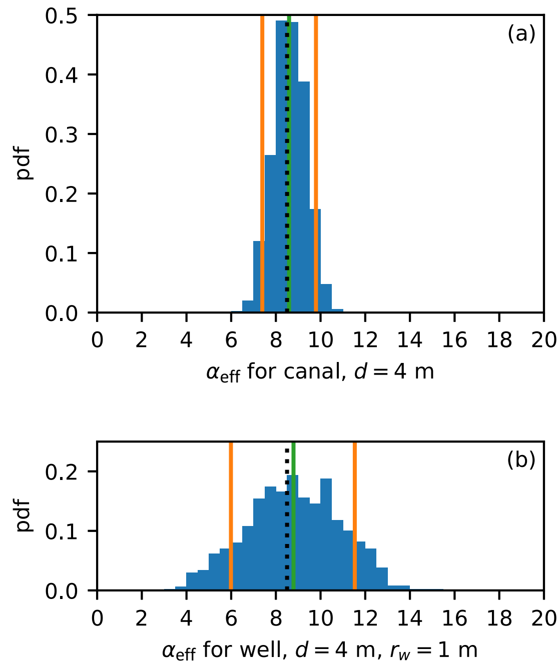

Figure 8. Histogram of the effective anisotropy for a more even distribution of 400 layers such that the transmissivity of each 4 m section is the same. (a) Effective anisotropy for a partially penetrating canal with $d = 4$ m. (b) Effective anisotropy for a partially penetrating well with $d = 4$ m and $r_w = 1$ m. The green line is the median, the orange lines are the 5 and 95 percentiles, and the black dotted line is the equivalent anisotropy.

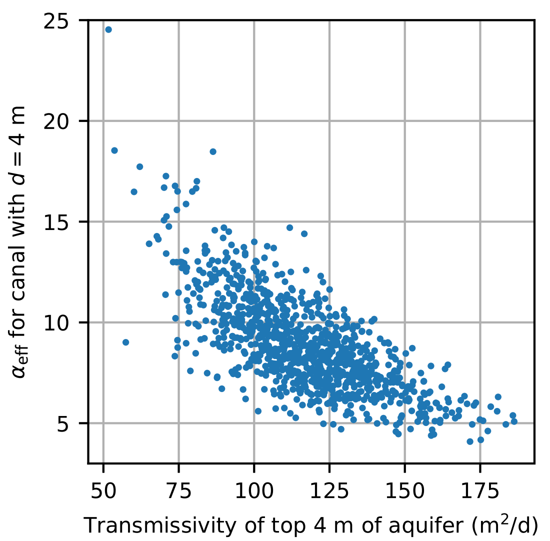

Figure 7. Correlation between the effective anisotropy for a canal with $d = 4$ m and the transmissivity of the top 4 m of the aquifer.

The aquifer is divided in 10 sections of 40 layers. Each section consists of the same set of 5 groups of 8 layers with the hydraulic conductivities of Table 1. The placement of the layers is still random and different for each section. As a result, each 4 m section of aquifer consists of the same 40 layers (albeit in different random order) so that the total transmissivity of each 4 m section is the same and equal to 10% of the total transmissivity of the aquifer. A histogram of the effective anisotropy is computed for both a partially penetrating canal and a partially penetrating well. The penetration depth is 4 m for both the canal and the well; the well radius is set to 1 m. The histograms are shown in Figure 8. Comparison of Figure 8a with Figure 3a and comparison of Figure 8b with Figure 5b show that the range of the effective anisotropy is much smaller when the layers are distributed more evenly across the aquifer thickness. For the canal, the 5%–95% range of α_{eff} has reduced from 5.6–13 for an entirely random distribution to 7.4–9.8 for a more even distribution. Similarly for a well, the 5%–95% range of α_{eff} has reduced from 2.0–56 for an entirely random distribution to 6.0–12 for a more even distribution.

Discussion

Many choices were made in the numerical experiments presented in this article. The layered aquifer consisted of 400 layers. An aquifer consisting of more layers may result in a more even distribution of the transmissivity over the aquifer and a smaller range of the effective anisotropy when the layers are divided randomly over the aquifer thickness. Conversely, fewer layers will result in a larger range of the effective anisotropy. The 400 layers were divided in five groups with different hydraulic conductivities. A larger number of groups, with a particle size distribution that is still a straight line on semi-log paper, will result in somewhat larger values for the equivalent anisotropy factor. A more S-shaped curve of the particle size distribution will result in fewer layers with hydraulic conductivity values on the low and high ends, likely resulting in smaller equivalent anisotropy values.

In the numerical experiments, the aquifer layers extended across the entire model, or at least over the distance where vertical flow is significant (three times the aquifer thickness to the left and right of the canal for the conducted experiments). More realistic subsurface models, based on geological principles, can be created with, for example, the framework developed by Bennett et al. (2019), but that may be beyond the effort that most groundwater modelers can afford in practice.

The presented numerical results are for a layered aquifer where the ratio of the diameter of the coarsest and finest layers is a factor of 10. As discussed in the Introduction, this article considers layered aquifers where the ratio is between 5 and 20, which translates to a ratio of the hydraulic conductivity of the coarsest and the finest layers between 25 and 400. For a system with five groups of layers and a distribution that is linear on semi-log paper, the corresponding range of equivalent anisotropy factors is $\alpha_{\mathrm{eq}} = 3.2 - 27$. So roughly speaking, the equivalent anisotropy varies from 3 to 30 for the systems considered in this article. The ranges presented for the effective anisotropy in this article were computed for $\alpha_{\mathrm{eq}} = 8.5$. Ranges for other values of α_{eq} may be scaled using the actual value for the equivalent anisotropy.

Conclusions

Numerical experiments were conducted to investigate the effective anisotropy factor for flow to a partially penetrating canal and flow to a partially penetrating well in a layered system. The effective anisotropy was defined such that the drawdown in a homogeneous model with an anisotropic hydraulic conductivity is equal to the drawdown in a stratified model consisting of many layers with different isotropic hydraulic conductivity values. The purpose of the presented numerical experiments was to demonstrate that the effective anisotropy is a model parameter and to quantify the range over which the effective anisotropy can vary for a set of aquifer configurations. Several flow fields were simulated for a specific layered aquifer consisting of 400 layers divided in 5 groups of 80 layers with hydraulic conductivities that varied over 2 orders of magnitude with a particle size distribution that is a straight line on semi-log paper. The equivalent anisotropy of the aquifer, defined as the arithmetic mean of the hydraulic conductivities divided by the harmonic mean, was 8.5.

At first, the 400 layers were distributed randomly over the aquifer. The effective anisotropy for different random realizations varied over a large range, but the median was approximately equal to the equivalent anisotropy of 8.5. For flow to a partially penetrating canal, the effective anisotropy varied roughly between 5 and 13. For flow to a partially penetrating well, the distribution of the effective anisotropy was much more asymmetric and varied roughly between 1.5 and more than 50. This larger variation is likely the result of the more concentrated outflow near the well screen as compared with the flow towards the canal. The effective anisotropy can be quite different for different canal depths, even in an aquifer consisting of the exact same sequence of layers. This means that the effective anisotropy is a model parameter and not a characteristic of the aquifer. The same conclusion is drawn from the experiments for flow to a well, where it was found that a smaller well radius generally resulted in a larger effective anisotropy. Additional numerical experiments were carried out for a system where the 400 layers were distributed more evenly over the aquifer thickness, such that the total transmissivity of every 10% of the aquifer thickness was the same. This resulted in a much smaller range of effective anisotropy values, but again the median was similar to the equivalent anisotropy value.

A homogeneous anisotropic aquifer is obviously an approximation of a layered system. Since the effective anisotropy is a function of the flow field, it is recommended to obtain the effective anisotropy factor from calibration, as is commonly done in groundwater modeling practice (Anderson et al. 2015). If a good field-derived estimate of the equivalent anisotropy is known, that may be a good starting point. Otherwise, an effective anisotropy of 10 seems to be a reasonable value to start the calibration process, bearing in mind that the final calibrated value can deviate significantly.

Data Availability Statement

The data that support the findings of this study are openly available in TimML at https://github.com/mbakker7/timml.

References

Anderson, E.I. 2005. Modeling groundwater-surface water interactions using the Dupuit approximation. *Advances in Water Resources* 28, no. 4: 315–327.

Anderson, M.P., W.W. Woessner, and R.J. Hunt. 2015. *Applied Groundwater Modeling: Simulation of Flow and Advective Transport*. London, UK: Academic Press.

Bakker, M. 2023. TimML version 6.1.2. https://github.com/mbakker7/timml (accessed 7 August 2023).

Bakker, M., and V. Post. 2022. *Analytical Groundwater Modeling: Theory and Applications Using Python*. Leiden, Netherlands: CRC Press.

Bakker, M., and O.D. Strack. 2003. Analytic elements for multiaquifer flow. *Journal of Hydrology* 271, no. 1-4: 119–129.

Bennett, J.P., C.P. Haslauer, M. Ross, and O.A. Cirpka. 2019. An open, object-based framework for generating anisotropy in sedimentary subsurface models. *Groundwater* 57, no. 3: 420–429.

Fitts, C.R. 2012. *Groundwater Science*, 2nd ed. Waltham, MA: Elsevier.

Gianni, G., J. Doherty, and P. Brunner. 2019. Conceptualization and calibration of anisotropic alluvial systems: Pitfalls and biases. *Groundwater* 57, no. 3: 409–419.

Maier, R., C. Leven, E. Sánchez-León, D. Strasser, M. Stoll, and O.A. Cirpka. 2022. Revealing vertical aquifer heterogeneity and hydraulic anisotropy by pumping partially penetrating wells. *Hydrogeology Journal* 30, no. 2: 463–477.
Sanchez-Vila, X., A. Guadagnini, and J. Carrera. 2006. Representative hydraulic conductivities in saturated groundwater flow. *Reviews of Geophysics* 44: 3.
Shepley, M.G. 2024. Vertical hydraulic conductivity and layered heterogeneity: From measurements to models. *Hydrogeology Journal* 32, 1017–1042.
Terzaghi, K., and R.B. Peck. 1967. *Soil Mechanics in Engineering Practice*. New York: Wiley.
Weeks, E.P. 1969. Determining the ratio of horizontal to vertical permeability by aquifer-test analysis. *Water Resources Research* 5, no. 1: 196–214.