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A MOOC for Exam Training Mathematics using Intelligent Tutoring

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ABSTRACT

Many high school students require costly private lessons to prepare for their final exams. These lessons go beyond what regular schools can provide. As a proof of concept, a distance learning program for exam preparation has been created using Moodle. It offers study materials that incorporate gaming and real-life applications. Each lesson begins with a diagnostic test and follows a specific training program to address students' weaknesses. An AI-based intelligent tutoring system has been developed to take on the role of a teacher and assist students with their work. After the system analyzes errors, the tutor provides specific hints and guidelines to help students solve problems accurately. The developed study material was tested on a group of students preparing for their final exams in mathematics.

CCS CONCEPTS

• **Applied computing** → Education; E-learning; • **Human-centered computing** → Human computer interaction (HCI); Interactive systems and tools; • **Mathematics of computing** → Mathematical software.

KEYWORDS

Distance learning, Intelligent tutoring, MOOCS, Mathematics

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1 INTRODUCTION

In the Netherlands, all students are required to pass a central written exam, which covers various topics. While there are many training materials available, some students, who are at risk of failing the exam, are not capable or willing to train individually for the final exam. Previously, schools focused on the entire student cohort and did not offer additional training for individual struggling students. There are specialized institutes and independent teachers who provide exam training, but these lessons are not free. Our developed MOOC will be offered free of charge and will utilize a special didactic approach to motivate students through gaming and applications.

In math lessons, small classes of 25-30 students work on assignments supervised by teachers. The lessons cover introduction, orchestration and examples of assignments presented by the teachers. While the focus is on students working on assignments individually or in groups, many students struggle and require more individual supervision. However, in a classroom setting, providing individual support to all students is challenging due to time and resource constraints.

Additional private math lessons are available from teachers or institutes, either during or after school hours. Typically, these lessons come at a cost, which may be difficult for financially struggling parents to afford. On the other hand, schools are seeking free online courses to support their students' math education without overburdening teachers. In this paper, we introduce an online math course for first-grade secondary school students, designed using Moodle. The course emphasizes independent study. A special digital tutor has been designed to encourage student engagement in learning activities, monitor their progress and provide feedback.

The goal of this project is to develop a MOOC (Massive Open Online Course) to train students for their final school exam. The MOOC will incorporate interactive assignments and gamification elements. It will focus on practical applications. A digital tutor called HAVI will monitor students' progress, support them, and engage and motivate them in a funny way.

2 RELATED RESEARCH

According to Wikipedia, a computer system that simulates human tutors and provides immediate and customized instruction or feedback to students without requiring intervention from a human teacher is known as an intelligent tutoring system (ITS). ITSs aim to enable meaningful and effective learning through various computing technologies.

In a recent review paper by Chien-Chang Lin et al. [1], the role of AI in designing intelligent tutoring systems within sustainable education is discussed. This may involve providing personalized learning experiences for students based on their learning styles and preferences. Additionally, AI may offer data-driven insights into students' performance, emotions and engagement levels for teachers. However, the use of AI in sustainable education presents challenges related to privacy and data security.

An important aspect of intelligent tutoring systems is the use of intelligent learning diagnosis to evaluate current learning status and predict future progress. The authors in [2] discuss the commonly used assessment methods, including psychometric-based and deep learning methods. The authors also introduce a new assessment method.

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Figure 1: Mathematical assignments of real-life applications.

In another paper [3], the authors explore the impact of AI on online learning platforms. They compare the learning outcomes of two popular learning platforms: a traditional MOOC platform with video lectures and multiple-choice quizzes and a personalized, active and practical learning experience provided by the Korbit learning platform. The Korbit platform offers significantly better learning experiences, which is similar to our model.

Additionally, in paper [4], the authors review intelligent tutoring systems in education by analysing 36 papers on their impact on student learning. Another paper, [5], discusses the use of expert systems in the design of intelligent authoring systems, suggesting that the knowledge should be extracted from teachers as experts.

Furthermore, in the context of the European project FETCH on distance learning, we have focussed on the development of MOOCs [6–8,9]. We have developed a special didactic model using MOOCs with a strong emphasis on emotions as a key factor for students learning through MOOCs. In the current project, an intelligent tutor design can serve as a strong motivator for students by making MOOCs more appealing to their motivation. As part of this effort, we have also investigated the use of social media in MOOCs.

3 LEARNING MATERIAL

3.1 Background

The conventional approach to exam preparation typically involves providing students with a comprehensive overview of the study material and presenting them with previous exam questions as examples. This method includes summarizing and explaining key points from the exam materials. It necessitates the presence of a tutor to guide and encourage students through the learning process. However, it is not suitable as a self-study material for students with learning disabilities, often causing them to give up the training quickly. Additionally, many students cannot afford a tutor, which is the primary motivation behind our development of a MOOC that offers exam preparation materials.

Unlike traditional methods, our MOOC presents the study materials in a different manner. Instead of starting with theory, we begin with assignments at the level of the final exam. We utilize these assignments as testing and diagnostic tools. Analyzing the student solutions for the problems given provides insight into their weaknesses and knowledge gaps. We offer additional hints, explanations and training materials to support their learning accordingly. To support this approach, an intelligent tutor trained in exam preparation analysis is required.

In order to enhance student motivation, assignments are framed as games or puzzles with real-life applications, as examples shown in Figure 1. The first assignment involves designing an algorithm as a labyrinth, while the second assignment requires students to work with different shapes of vases and create a graph showing the increasing water level in the vases without using the content formula. The students also need to address questions related to the increase and decrease of the speed of the rising water. The third assignment involves working with a mobile phone keyboard, which only has eight keys and where each key represents 3-4 characters. One of the tasks is to decode a string of key numbers using a specific dictionary. The fourth assignment involves launching two drones and analyzing their trajectories to determine whether the drones will collide, based on their speed and angle of elevation. In the fifth assignment, students are tested on their knowledge and abilities in algebra and calculus as they solve a crossword with algebraic expressions instead of words. The particular assignment will be analyzed in more detail in the next section.

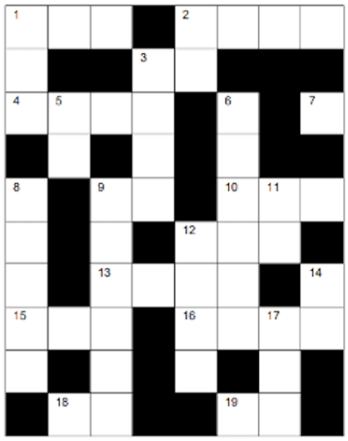
One of the authors and many of his colleagues provide individual lessons to prepare for exams. Based on their experience, they are aware of the areas where students need extra help. Due to the personalized nature of these lessons, it is not feasible to offer them on a large scale. Students have expressed dissatisfaction with having to attend these lessons after regular school hours. They prefer to choose their own time and pace.

A digital tutor in distance learning via MOOCs can facilitate a preliminary test of a student, and then guide their participation after enrollment. The test can reveal the students' current state of knowledge. In case of making mistakes, students may ask for help anonymously. It is a well-known fact that mathematics can only be mastered through practice. Furthermore, as it is commonly acknowledged that students value the type of personalized and anonymous support, the MOOC can provide students with the opportunity for individual training and the ability to track their progress anonymously.

3.2 Assignments by Crosswords

One of the assignments in our MOOC includes a crossword puzzle shown in Figure 2. In this assignment, the students generate strings of the following symbols and digits, i.e., +, -, :, ×, 0-9, and a-z. One symbol in every field of the puzzle starting at numbered fields 1 until 19. The computed strings are a reduction of the horizontal and vertical strings of symbols in the crossword.

Many people enjoy solving crosswords as a recreational activity. We believe that our approach to mathematics by incorporating



Horizontal

1	$5x - 8y - (5x - 3y)$
2	$\sqrt[2]{25x^2} \times (\sqrt[2]{6y})^2$
3	$-3x^4 + (2x^2)^2 - x(x^3 + 1)$
4	$-4x^2 + (2x + 1)^2$
9	$x^6 \div x^2 - (x^2)^2 + 5x^2 \div x$
10	$\sqrt[2]{25x^2y^2}$
12	$(-24xyz) \div -12z$
13	$y^2(z + 1) \div y$
15	$-24a \div (12a)$
16	$2z\sqrt{4p^2q^2}$
18	$18q = 12q + \ldots$
19	$18x = \ldots - (-16x)$

Vertical

1	$(-56a) \div (4a)$
2	$12 + 3(-x - 4) - 3 \times -2x$
3	$6(2xy \div 4x) - 15x$
4	$(x + y)^2 - x^2 - y^2 - xy$
6	$-5x \times 3yz$
7	$5x \times -3z \times 0$
8	$\sqrt{a}(2\sqrt{a} - 4\sqrt{a}) - \sqrt{b} \times \sqrt{b}$
9	$\sqrt{9x^2y^2 + 16x^2y^2} \times \sqrt{4p^2q^2}$
11	$\sqrt{x^2y^2}$
12	$\sqrt{(2 + 2x)^2}$
14	$(p^2 \div p) \times x$
17	$-pqxy = 4y \times (- \ldots)$

Figure 2: A mathematical crossword puzzle.

Figure 3: The homepage of the MOOC focused on math exam preparation for secondary school students in the Netherlands.

games, such as crosswords, can challenge and motivate students while also providing enjoyment. We have observed that subjects such as algebra and calculus are not fully developed in secondary schools, which poses a difficulty for many students, not just those with dyscalculia.

There are heuristics and algorithms available for automatically generating crosswords by providing lists of horizontal and vertical strings of symbols [9]. Moodle also offers a tool for creating crosswords.

3.3 Distance Learning Tool

The MOOC was developed using Moodle, a well-known distance learning tool. Figure 3 shows the homepage of the MOOC [10]. The authors had previous experience in developing MOOCs using Moodle [6–8]. The main reason for choosing Moodle is that many secondary schools in the Netherlands use it to develop and distribute distance learning materials. Teachers and students are familiar with using Moodle. It also offers puzzles and games plugins that can be used without additional programming.

In the 1980s, a group of mathematics teachers developed a new didactic approach[11]. This approach focused on self-learning, games, puzzles and applied mathematics. It was incorporated and implemented into textbooks and referred to as the "De Wageningse methode" [12]. This innovative teaching method has inspired us and is partially used as educational materials in our MOOC.

3.4 Database of School Exams

Every year in May, the final grade of secondary school students in the Netherlands take part in the Central Written Exam [13, 14]. Last year, 51,381 HAVO – secondary school students participated in the exam. The math exam, in particular, consists of various parts corresponding to different learning topics. The exams are graded by the students' teachers and an independent teacher to ensure consistent and fair grading. Detailed guidelines including assignments, potential solutions and grading instructions are published to standardize the correction process. The database containing previous years' exams, solutions and comments is a valuable resource for research purposes.

The assignments in our MOOC share similarities with several assignments in the written exam, providing a representative training set for the final exam. We use the guidelines for the written exam to create a knowledge base around the assignments. For the crossword assignment, we have generated a list of possible answers, including correct solutions, potential incorrect solutions and incomplete answers. Correct answers can be obtained by applying a set of learning rules to the initial problem. Some of the incorrect answers are derived from common errors found in the database. Additionally, there might be incorrect answers due to writing errors or the misapplication of knowledge rules. Table 1 displays examples of strings of correct knowledge rules that are included in the intelligent tutoring tool.

3.5 Mathematical Input Interface

Students need to learn on how to write mathematical symbols, but recognizing handwritten characters is too complicated for our MOOC. As a result, students have to choose from multiple-choice questions or select from a list of symbols displayed on the screen. While Moodle supports keyboard input, using equations requires knowledge of LaTeX, which may be challenging for secondary students. Therefore, MOOC designers should consider using a manual keyboard for assignments or exploring alternative tools such as DragMath (Figure 4), an equation editor in Moodle.

3.6 Diagnostic

An important aspect of intelligent tutoring systems is how to assess assignment solutions. Fortunately, the primary author has eight years of experience as a mathematics teacher at a secondary school. He is part of the team responsible for grading yearly written exams and contributes to the development of exam correction guidelines. The assignments in our online courses are modeled after those given on the central written exam, providing not only correct solutions but also a list of common errors. Most student answers to assignments are listed in a table along with the applicable solutions rules (see examples in Table 2).

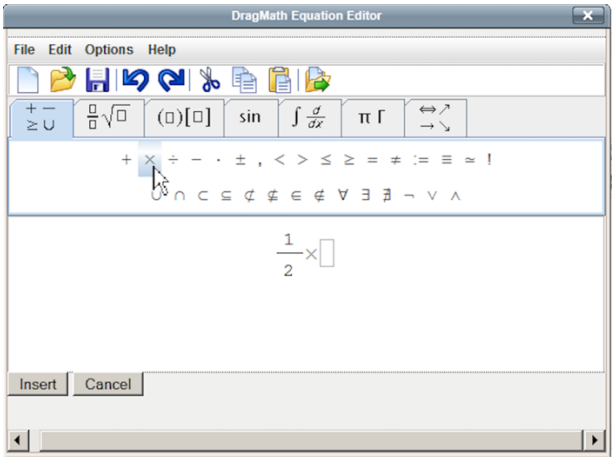


Figure 4: The interface of DragMath in Moodle.

Students may provide answers not listed in the table, such as random or incorrect answers. In the event of potential typing errors or missing symbols, the tutor responds with "The answer should be a string of n symbols", while "Consider symbol x again" for an incorrect response without a specific diagnosis.

To aid the tutor in assisting, the assignments correspond to sequential rules that must be followed to arrive at a correct solution (see Table 1). If a student continues to make errors, one solution may involve providing references to the relevant theory.

3.7 Error Analysis Past School Exams

The previous school exams are documented and detailed in [13, 14]. The exams consist of six learning topics. For each topic, we have defined an assignment as noted above. The exam documentation includes common errors made by students, which are recorded and graded. These errors have been organized into six sets corresponding to the learning topics. In particular, we will focus exclusively on the learning topics related to Algebra and Calculus. We have summarized the common errors made by the students below and utilized them in designing the assignments. We anticipate that some students may continue to make these errors while working on our teaching materials allowing an opportunity for correction and additional training.

Based on the feedback from school exams, students tend to make the following mistakes:

- Failing to apply the calculation rules in the correct order of operation, i.e., start by removing brackets, continue working on powers or roots, then multiplication or division before addition or subtraction
- Not performing calculations from left to right
- Neglecting to include negative signs
- Copying the assignment incorrectly or making writing errors
- Using the wrong application of special products.

3.8 Algorithm Intelligent Tutoring Tool

In this section, we will describe how the intelligent automated tutor HAVI supports students in solving the crossword puzzle. HAVI

Table 1: Used rules in algebra and calculus.

Computation order	Computation with roots	Properties of powers and exponentials
1. Remove brackets	7. $\sqrt[p]{a} + \sqrt[p]{a} = 2\sqrt[p]{a}$	15. $a^p \times a^q = a^{p+q}$
2. Combining similar terms	8. $\sqrt[p]{a} \times \sqrt[p]{b} = \sqrt[p]{a \times b}$	16. $a^p \div a^q = a^{p-q}$
3. Compute what is between brackets	9. $\sqrt[p]{a} \div \sqrt[p]{b} = \sqrt[p]{a \div b}$	17. $(a^p)^q = a^{p \times q}$
4. Compute powers and squares, from the left to the right	10. $\sqrt[p]{a^p} = \lvert a \rvert$, if p is an even number	18. $a^p \times b^p = (a \times b)^p$
5. Multiply and divide from the left to the right	11. $\sqrt[p]{a^p} = a$, if p is an odd number	19. $a^p \div a^p = (a \div b)^p$
6. Addition and subtraction from the left to the right	12. $\sqrt[p]{\sqrt[q]{a}} = \sqrt[p \times q]{a}$	20. $a^{-p} = 1/a^p$
	13 $\sqrt[p]{a} = a^{\frac{1}{p}}$	21. $a^1 = a$
	14. $\sqrt[p]{0} = 0$	22. $a^0 = 1$
		...

Table 2: Assignments in crosswords and applied rules.

Hor.	Assignment	Solution	Applied Rules	Vert.	Assignment	Solution	Applied Rules
1	$5x - 8y - (5x - 3y)$	$-5y$	R1, R2	1	$(-56a) \div (4a)$	-14	R5
2	$\sqrt[2]{25x^2} \times (\sqrt[3]{6y})^2$	$5x * 6y$	R10, R4, R10	2	$12 + 3(-x - 4) - 3 \times -2x$	$3x$	R1, R2
3	$-3x^4 + (2x^2)^2 - x(x^3 + 1)$	$-x$	R4, R2	3	$6(2xy \div 4x) - 15x$	$3y - 15x$	R3, R5
4	$-4x^2 + (2x + 1)^2$	$4x + 1$	R4, R2	4	$(x + y)^2 - x^2 - y^2 - xy$	xy	R4, R2
9	$x^6 \div x^2 - (x^2)^2 + 5x^2 \div x$	$5x$	R5, R4, R5	6	$-5x \times 3yz$	$-15xyz$	R5
10	$\sqrt[2]{25x^2y^2}$	$5xy$	R2, R4	7	$5x \times -3z \times 0$	0	R5
12	$(-24xyz) \div -12z$	$2xy$	R5	8	$\sqrt{a}(2\sqrt{a} - 4\sqrt{a}) - \sqrt{b} \times \sqrt{b}$	$-2a - b$	R7, R3, R5, R1
13	$y^2(z + 1) \div y$	$yz + y$	R5, R5	9	$\sqrt{9x^2y^2 + 16x^2y^2} \times \sqrt{4p^2q^2}$	$10xypx$	R10, R2, R4, R5, R2
15	$-24a \div (12a)$	-2	R5	11	$\sqrt{x^2y^2}$	xy	R10, R4
16	$2z\sqrt{4p^2q^2}$	$4pqz$	R4, R5	12	$\sqrt{(2 + 2x)^2}$	$2 + 2x$	R10
18	$18q = 12q + \ldots$	$6q$	R6	14	$(p^2 \div p) \times x$	px	R3, R1
19	$18x = \ldots - (-16x)$	$2x$	R1	17	$-pqxy = 4y \times (- \ldots)$	pq	R5

operates similarly in other assignments due to the special structure of mathematical assignments and the knowledge extracted from the mathematics teachers, which enables the design of an intelligent tutor.

Students have the option to start an assignment without the help of the tutor. They can enter a reduced string of symbols into the crossword and then click the verification button. When submitting their answers, the student's responses can be categorized into four groups: correct answer, not correct answer, undefined answer, or no answer. If the answer is correct, the tutor responds with "Congratulations, go on". If the answer is not correct, the tutor looks up in the database to identify the mistake and may respond in the following ways: "Your answer is not correct, apply rule *xx*", "The answer is not correct, please try again", or "The answer is almost correct, please go on". In the case of an undefined answer, the tutor may reply with "Unfortunately, your answer is not recognized, please try it again" or "We didn't detect a reply for 30 seconds, you will be logged off after 15 seconds".

From the outset, students have the option to ask for help. The tutor recommends starting with one of the easiest assignments from the following list of items: {Hor. 1, Hor. 18, Hor. 19, Vert. 6, Vert. 7}. By solving the initial problems, students will gain confidence and motivation. If a student makes mistakes or asks for help again, the correct knowledge rule is provided by the tutor.

Once the simple questions are solved, the tutor produces a list of strings intersecting the correct string. The tutor may ask the students to try again, provide a hint, or even offer a solution. The tutor must be able to vary its responses.

Finally, the student can move on to the remaining questions. The tutor's responses generate positive, negative, or neutral scores. Subsequently, a final score is computed. If the final score is below a defined threshold, the student is advised to review the theory again. The students' interaction with the system is facilitated by Moodle after the necessary data has been implemented in Moodle.

3.9 Experiments

The developed MOOC has been tested in parts:

- The MOOC designers have experience developing MOOCs using Moodle as a development tool [6–8].
- A team member is a mathematics teacher with years of experience in designing and grading school exams. Exams are graded by a pair of teachers who discuss and agree on the final grades.
- School exams are well documented and come with grading guidelines designed by experienced teachers based on student work.

- Two of the authors provide voluntary individual exam tutoring for students
- There are numerous organizations offering exam help and support for secondary school students. While the materials are documented, they can only be accessed after paying a fee. The development team had access to most of these materials and reviewed them during the design of the current MOOC.

4 CONCLUSIONS

This paper presents a prototype of a MOOC designed to assist secondary school students in the Netherlands in preparing for their math exams. The prototype was developed using Moodle, a platform that is commonly used in secondary schools in the country. The MOOC is available free of charge [10]. The learning materials are based on past written school exams, which are documented and freely available on the Internet. These materials are presented in the form of games to engage students and increase their interest and motivation. Similar materials have been successfully tested in a classroom setting. The development of the MOOC involved a team of experienced secondary school teachers. Additionally, an intelligent tutor was designed based on knowledge extracted from experienced educators and an analysis of the database of past school exams.

A recent report from the Dutch School Inspectorate of Education highlighted the inadequacy of students' calculus abilities in the first three degrees of secondary schools. As a result, the request for the new (online) training materials was made to address the mathematical abilities of these students.

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