AE2203: Propulsion & Power

Part: Rocket Propulsion

By

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Preface

The rocket propulsion part of TU-Delft's Propulsion and power course (AE2203) is about the various options that exist for propelling rockets and is taught using a total of 8 lecture hours of each 45 minutes. The course builds on the material presented in earlier courses of which especially are named:

- Physics II, part on electricity (AE1204)
- Aerospace Design and System Engineering Elements (AE1201), part Spacecraft Design and Sizing, Section 6.3.
- Physics I, part on thermodynamics (AE1104)

The course introduces the main rocket systems in use on aerospace vehicles and their components to a level of detail that allows for a detailed analysis. Some typical high level learning objectives are given in the block below.

Specific Learning Objectives *(At the end of this course, the student is able to…)*

- List/describe/explain
  - The main rocket propulsion options available
  - (Main) components that make up the rocket propulsion system and their function
  - Principal figures of merit for the various components.
  - Some important limitations to rocketry.
- Apply physics to predict:
  - rocket thrust
  - propellant consumption
  - energy/power usage
- Asses effect of changes in design/operating parameters on system performance

The study material for this part of the course consists of the chapter on space propulsion from the textbook [SSE] and this document.

Pictures on front page show (left to right) rocket propelled F1 aircraft, missile, rocket assisted take-off (top), rocket-propelled car, Moon launcher, rocket back pack, Space launcher (Space Shuttle) and a sounding rocket.

Abbreviations

A-50	Aerozine 50
ACS	Attitude Control System
AN	Ammonium Nitrate
AP	Ammonium Perchlorate
EP	Electric Propulsion
ESA	European Space Agency
FVC	Fine Velocity Control
HET	Hall Effect Thruster
HV	Heating Value
IRMT	Ideal Rocket Motor Theory
ISA	International Standard Atmosphere
KE	Kinetic Energy
LOX	Liquid Oxygen
LRE	Liquid Rocket Engine
MPD	Magneto-Plasma Dynamic
MPS	Micro-Propulsion System
MMH	Mono/Methyl Hydrazine
NTO	Nitrogen Tetroxide
OCS	Orbit Control System
PE	Potential Energy
PPT	Pulsed Plasma Thruster
RATO	Rocket Assisted Take-Off
RCS	Reaction Control System
RP-1	Rocket Propellant
SL	Sea Level
SRM	Solid Rocket Motor
STP	Solar-Thermal Propulsion
UDMH	Unsymmetrical Di/Methyl Hydrazine
VAC	Vacuum

Important constants

Gravitational acceleration at sea level	g_o	9.81 m/s^2
Solar constant	S	1400 W/m^2
Avogadro’s number	N_A	6 x 10^{23} molecules/mol
Elementary charge	e	1.6 x 10^{-19} C
Permeability of free space	μ_o	4π x 10^{-7} H/m
Stefan-Boltzmann constant	σ	5.67e-8 W/m^2-K
Mass of an electron		9.11 × 10^{-31} kg
Calorie		4.1868 J

List of Symbols

Latin

Symbol	Description	Typical unit of measurement
a	Acceleration, velocity of sound	m/s^2, m/s
A	Area	m^2
B	Magnetic induction	Wb/m^2
c*	Characteristic velocity	m-s
c_p	Specific heat at constant pressure	kJ/kg/K
C_T	Specific propellant consumption	kg/(N/hr)
d	Distance	m
F	Force	N
g or g_o	Gravitational acceleration at sea level	m/s^2
h	Height, enthalpy	m, J/kg
h_c	Coefficient of convective heat transfer	
H	Magnetic field strength	A/m
HV	Heating value	MJ/kg
I	Impulse, Current	Ns, A
Isp	(Gravimetric) specific impulse	s
I_{ssp}	System specific impulse	s
j	Ion current density	A/m^2
k	Specific heat ratio, Proportionality constant	-, -
m	Mass flow rate	kg/s
M	Mass, Mach number	kg, -
N	Number	-
N_A	Avogadro´s number	-
p	Pressure	Pa
P	Power	W
q	Heat flux	W/ m^2
Q	Heat, Volume flow rate, Charge	J, m^3/s, C
r	Mixture ratio	-
R	Vehicle empty-to-total mass ratio, Specific gas constant	-, J/kg/K
R_A	Universal gas law constant	J/kmol/K
S	Surface area, Solar intensity	m^2, W/m^2
t	Time	s
T	Thrust, Temperature	N, K
v	Velocity, Specific volume	m/s, m^3/kg
V	Voltage	V
w	Jet velocity	m/s
W	Weight	N

Greek

α	Specific mass	kg/W
Δ	Increment or change	-
ε	Nozzle area ratio	-
ε_o	Permittivity of free space	$A^2s^4\ kg^{-1}m^{-3}$
E	Energy	J
γ	Specific heat ratio	-
Γ	Vandenkerckhove constant	-
η	Efficiency	-
M_W	Molar mass (also molar weight)	g/mol
ρ	Propellant mass density	kgm^{-3}

Contents

1. Introduction

1.1. General

Propulsion is associated with changing the momentum[1] of a body via a force acting on this body (action = reaction). The word propulsion is derived from two Latin words: 'pro' meaning before or forwards and 'pellere' meaning to drive. Its meaning is to push forward or drive an object forward. A propulsion system is a machine or device that produces thrust to push an object forward or more generally to change the motion, i.e. the momentum, of an object.

There are various ways of changing the momentum of an object. Consider for instance walking, bird flight, driving, and sailing. The way we change momentum depends on the environment (land, water, air and space) we are in. For example in case of land propulsion, we may use wheels to generate the propulsive force through direct contact with the solid earth. For aerospace vehicles, an important means of propulsion is jet propulsion, which acts through the generation of a high velocity (exhaust) jet. Two types of jet propulsion are generally distinguished:

- Direct and indirect reaction systems, which depend for their action on variation of the momentum of some external medium. In the case of direct reaction systems, the change in momentum of the external medium is purely obtained via energy addition to some medium, like air, ingested. This type of propulsion is also referred to as jet propulsion. Typical examples of this type of propulsion are a ramjet, and a turbojet. In the case of indirect reaction systems, the change in momentum is obtained via an engine and a propeller and is sometimes also referred to as propeller propulsion.
- The pure reaction systems, in which the propulsive effort or thrust is obtained by variation of the momentum of the system itself. These systems do not depend on some external medium for the production of the reaction effort. Rockets are systems of this type.

In rocket systems, the propulsive force (thrust) is generated by expelling mass (initially stored in the vehicle) from the vehicle at a high velocity. It differs from other engines in that it carries the mass to be expelled internally, therefore it will work in the vacuum of space as well as within the Earth's atmosphere.

1.2. Rocket applications

Practical uses of rocket systems as weapons of war, commerce and the peaceful exploration of space are discussed.

An important category of applications of rocket systems is to propel rocket weapons, like missiles and anti-tank weapons. The main purpose of using rocket propulsion in these systems is to attain high flight velocities in a very short time.

A second important application area is for space launchers, where we require high flight velocities (in excess of 7.8 km/s), but also operation at high flight altitudes well above 14-17 km and high thrust levels to overcome gravity. Important tasks are to provide propulsion for accelerated flight (ascent flight), re-entry flight (braking), and flight sustenance, but also for attitude control as well as for stage separation and propellant settling.

1 The (linear) momentum of a body is defined as the product of its mass times its velocity. It basically relates to linear motion. Analogous we have angular momentum as a measure for rotational motion. The angular momentum of a rigid object is defined as the product of the moment of inertia and the angular velocity.

For illustration, the European Ariane 5 space rocket launcher is capable of lifting a payload of about 40 ton into a low Earth orbit or 6.8 ton payload into geostationary transfer orbit. To do so, the launcher has 3 stages; a large core stage (main stage) with attached to it two booster rockets and a smaller core stage on top of the main one. The two large booster rockets assist the core stage during the initial launch phase, which takes about 130 s. After burn-out of the two boosters, they are separated from the main core stage, which continues the ascent flight. After burn-out of the main core stage after about 590 s in flight, this stage is separated and the second core stage takes over bringing the payload to its intended launch orbit. Total launcher mass at lift-off is about 746 ton of which 642 ton is propellant.

The main stage is powered by a single rocket engine (Vulcain), which engine provides for both main vehicle thrust as well as launcher vehicle yaw and pitch control. It produces 1145 kN of vacuum thrust and has a nominal burn time of 590 s. Total stage mass is ~170 tons and maximum propellant mass is ~155 tons (130 tons oxidizer and 25 tons fuel). Stage length and diameter is 29 m and 5.4 m, respectively.

Figure 1: Ariane 5 launch vehicle (Courtesy ESA/ESTEC)

The large booster rockets each provide thrust for about 130 s. During this time each booster provides a total impulse of 4.6 x 10^8 Ns. Thrust at lift-off is 5.5 MN, which reduces to about 4.0 MN at 35-55s to minimize aerodynamic loads. Maximum thrust is ~6 MN. The thrust tails off after 75 s to limit maximum launcher acceleration down to 3.5 g_o. The 2nd core stage is propelled by a single rocket engine (Aestus) producing 27.5 kN of thrust. Total propellant mass is 9. 7 tons stored in 4 propellant tanks. The EPS stage is spin stabilised. Its attitude control system consists of six thrusters that deliver a thrust of 400 N each. Of these 6 thrusters two are used for spin-up and two for spin-down. The remaining two are to allow tilting the spin axis.

A third important area is spacecraft applications, where we require propulsion for orbit transfer, orbit acquisition / trim, repositioning, de-orbit, plane changes, etc. This requires not only high flight velocities (of the order of several km/s), but also to achieve this high flight velocity in a vacuum environment. In addition, spacecraft may use rocket propulsion for:

- Orbit stabilisation or “station keeping” to compensate for disturbing forces like drag, solar wind, etc.
- Attitude control to perform 3-axis or spin stabilisation, to change the attitude of the S/C or to compensate for disturbing torques e.g. precession of spin axis
- Other: Spin-up/down, discharging/unloading of reaction or momentum wheels (typically every few days), stage separation, propellant settling to compact the bubbling propellant inside the tank, etc.

The next figure shows some features of the rocket propulsion system on a specific spacecraft.

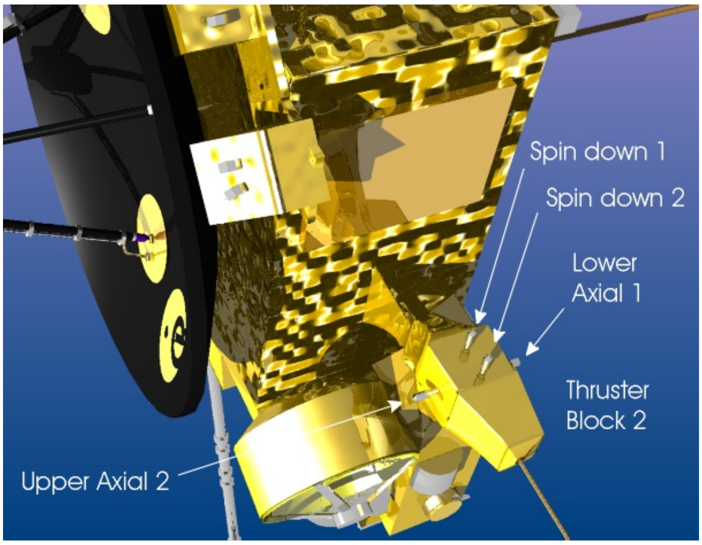


Figure 2: Ulysses rocket system features (courtesy ESA/ESTEC)

The system includes eight rocket motors that provide spin control and axial and radial delta v control. In addition, it includes a propellant storage tank and the pipes and valves necessary to regulate propellant flow from the tank to the thrusters. Instrumentation includes pressure transducers and temperature sensors. The tank is a titanium alloy shell containing hydrazine and nitrogen pressure gas separated by a membrane. Total tank volume is about 45 liters. Filters are included to filter the propellant flowing to the thruster blocks.

Rocket applications are also found in:
- Sounding rockets
- Amateur rockets
- Ejection seats
- Rocket assisted take off (JATO or Jet Assisted Take Off): Actually a rocket that is used to give heavy military transport planes an extra "push" for taking off from short airfields.
- Race cars: The world's first rocket car, the RAK2 was unveiled in 1928 by Opel. On May 23, 1928, the RAK2 was unveiled to a crowd of 3,000 people in Berlin, Germany. The car * without an engine or gears * was powered by 24 rockets and 120 kilograms of explosives. Driven by Fritz von Opel, the grandson of Opel founder Adam Opel, the crowd watched the car reach a high speed of 230 kilometres per hour in two kilometres. Rocket powered quarter mile race cars were the fastest type of race track vehicle ever built. Those cars had so much 'direct thrust' power that they could beat any conventional or jet powered racer from point A to point B, known as elapsed time. The miles per hour shown at the end of a run are interesting, but inconsequential. However, a car called 'VANISHING POINT' was reputed to have been driven at well over 640 kmph (400 mph) in the quarter mile.
- Gas-generators: Micro gas generators are used as air-bag inflators. Other generators can be used to drive or start up a gas turbine

All rocket propelled vehicles are equipped with one or more rockets (referred to as primary propulsion system) that allow(s) for adjusting the linear momentum. Some vehicles also have rockets (referred to as secondary propulsion system) that allow for 3-axis or spin stabilisation.

Table 1 provides an overview of typical characteristics for a number of primary and secondary propulsion applications.

Table 1: Characteristics of some rocket propulsion applications (adapted from [Sutton])

Application	Thrust Profile	Typical Duration	Maximum Acceleration
Large space launch vehicle booster	Nearly constant thrust	2-8 min	2-6 g
Antiaircraft or antimissile missile	High thrust boost, decreasing thrust sustain phase	2-75 sec each	Up to 100 g
Spacecraft orbit maneuvers	Restartable	Up to 10 min cumulative duration	0,2-6 g
Air launched guided missile	High thrust boost phase with low thrust or decreasing thrust for sustain phase	Boost: 2-5 sec Sustain: 10-30 sec	Up to 25 g
Variable range military ballistic missile - surface launched	Same as above	Up to 2 min each stage	Up to 10 g
Spacecraft attitude control - large vehicles	Many restarts (up to 60.000)	Up to 1 hr cumulative duration	Less than 0,1 g
Spacecraft attitude control - small vehicles	Same as above	Up to 20 min	Same as above
Reusable main engines for space shuttle	Variable thrust, many flights with same engine	8 min, over 7 hr cumulative	Up to 4 g
Lunar landing	10:1 thrust variation	4 min	Several g
Weather sounding rocket	Single burn period - often decreasing thrust	5-50 sec	Up to 15 g
Antitank	Single burn period	0,2-3 sec	Up to 20 g

Besides providing for the necessary thrust, these systems also bring some side effects. For example, for the Ariane 5 space launcher, the propulsion system:

- Increases mass: To bring about 7 ton of payload into orbit, we require a giant rocket. Ariane 5 consists for about 80% of propellant (642 ton out of a total of 746 ton) and a structure to contain the propellant and to resist the launch loads.
- Increases size (volume): To store 642 ton of propellant requires a large volume. For instance to store 642 ton of water requires 642 m^3 which comes down to a cylinder of length 50 m and diameter 4 m. For the Ariane 5 using liquid hydrogen and liquid oxygen, the density of the propellant is about 4 times lower than for water, so the effect is even more prominent.
- Increases cost: Ariane 5 propulsion systems make up about 50-70% of total launch cost. The latter stands at about 120 million Euros;
- Decreases reliability: 59% of all launch failures are caused by the propulsion system;
- Effects schedule: Initial development of Ariane 5 started in 1984 with actual development starting in 1987. First flight took place in 2000
- Effects operations: To launch Ariane 5, a launch base is required in a remote place (Kourou). Launch preparations take about 1 month including preparing and mating of the various launcher stages and the payload in special buildings and the transfer to the actual launch site on a big crawler, where the launcher is fuelled up, ready for count down.
- Etc.

Important differences of rocket propulsion with jet propulsion are given in Table 2.

An important advantage of rockets is the much higher thrust-to-weight (T/W) ratio. This allows to install the same thrust but at lower mass consequences for the total vehicle. This allows for higher acceleration rates. A second advantage is the increased thrust density, which allows to limit the size of the rocket system with about a factor 3 compared to a jet engine. A third advantage is that, because the rocket takes the mass to be expelled within, the thrust is independent of altitude, flight velocity and air temperature. A fourth advantage is that the flight velocity can be much greater than the velocity of the jet exhaust. In contrast, the flight velocity attained with turbojets is limited to maximum 1-1.5 km/s. The last advantage we mention is that the rocket has no altitude limitation since next to the fuel, it also carries the oxidizer necessary to burn the fuel. A major disadvantage is the high specific fuel consumption, which leads to a high propellant[2] mass to be carried on board of the vehicle.

Table 2: Rocket advantages over turbojet propulsion

Feature	Rocket engine or rocket motor	Turbojet engine
T/W, typical	75:1	5:1
Specific fuel/propellant consumption	0,8-1,4 kg/(Nhr)	0,05-0,15 kg/(Nhr)
Thrust density	375000 N/m^2	125000 N/m^2
Thrust versus altitude	Nearly constant	Decreases with increasing altitude
Thrust versus flight velocity	Nearly constant	Decreases with increasing flight velocity
Thrust versus air temperature	Constant	Decreases with increasing air temperature
Flight velocity versus exhaust velocity	Unrelated; flight velocity can be greater	Flight velocity always less than exhaust velocity
Altitude limitation	None; suited to space travel	14-17 km

Adapted from: Rocket Propulsion Elements

A further advantage is that a rocket device is extremely simple and often contains no moving parts, which allows for attaining a high reliability.

1.3. The rocket system

The major components of any rocket system and therefore also thermal rocket system are (figure 1):

a) Expellant or propellant, which forms the mass to be expelled;
b) Thrust generating (thruster) or accelerator system wherein the propellant is accelerated to a high exhaust velocity;
c) Feed and storage system that stores the expellant prior to its use and feeds the expellant to the (set of) accelerator(s);
d) Energy or power source that provides the energy/power necessary for thrust generation;
e) Control system that controls the working of the rocket and allows for adjusting the thrust of the accelerator(s);
f) Frame to hold the components.

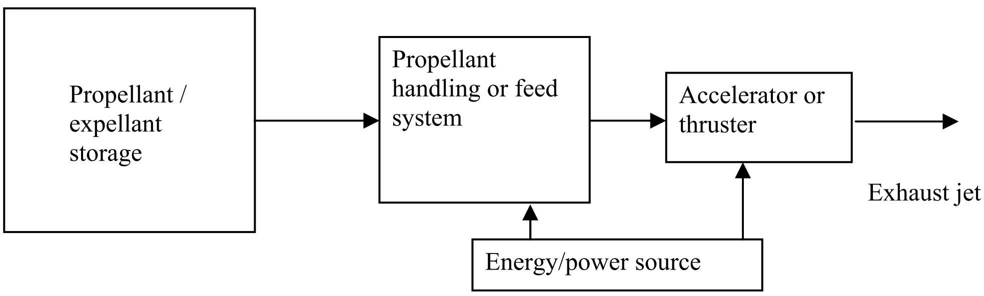


Figure 3: Main elements of rocket system

[2] A rocket propellant generally consists of a fuel and an oxidizer.

1.4. Types of rockets

Various types of rocket systems are distinguished based on how the expelled mass is accelerated to a high velocity:

a) Thermal acceleration, in which the enthalpy of the expellant is increased and converted into a high velocity jet via a nozzle. Thermodynamic expansion is the mechanism we are most familiar with. All of our chemical systems use this method to accelerate the propellants. However, we can also use nuclear or electrical energy to heat the propellant.
b) Electro-static acceleration, in which thrust is derived from the direct acceleration of positively, charged propellant ions or colloids by an electric field.
c) Electro-dynamic acceleration, in which crossed electric and magnetic fields induce a Lorentz force in plasma.

The various methods lead to differences in attainable exhaust velocity and thrust levels, see Table 3 and Table 4 taken in part from [Fortescue et al, 2003]:

Table 3: Typical attainable exhaust velocities

Propulsion type	*Exhaust velocity (km/s)*
Thermal	1 – 20
Electro-static	5 – 100
Electro-dynamic	5 – 100

Table 4: Typical attainable thrust levels

Propulsion type	Thrust acceleration (g_o)
Thermal	0.1-10
Electro-static and electro-dynamic	10^{-3}-10^{-5}

From these tables, we learn that thermal acceleration allows for limited exhaust velocity, but also for high thrust levels. In contrast, electro-static and electro-dynamic acceleration allows for high exhaust velocity, but limited thrust levels. Furthermore, the electro-static and electro-dynamic devices are much more complex to engineer than thermal systems. It is because of the relative simplicity of thermal rockets, and their high thrust levels that thermal rockets are the main type of rocket system in use for both space and earth applications including space launcher applications. Over time, it is expected that slowly electro-static and electro-dynamic devices will take over some space applications now performed by thermal systems. We mention drag compensation, and ultra-fine attitude control, but also deep space travel.

2. Fundamentals

2.1 Rocket equation

A change in momentum ΔI of a body can be determined from:

$$\Delta I = \int d(M \cdot v) \tag{2.1-1}$$

- M: body mass
- v: velocity of body.

Vice versa, we can use the above relationship to determine the change in momentum needed to accomplish a certain velocity change.

In case mass is constant, we get:

$$I = M \cdot \Delta v \tag{2.1-2}$$

The change in momentum is accomplished by an external force F (not necessarily constant) which operates on the vehicle for a certain time t_a.(action time):

$$\int_0^{t_a} F \cdot dt = \int d(M \cdot v) \tag{2.1-3}$$

In case of a non-constant system mass, analysis leads to the 'rocket equation' also referred to as 'Tsiolkowsky equation', which trades off exhaust velocity with rocket mass fraction. This equation can be derived as follows. Consider a rocket, see figure, with an instantaneous mass M traveling at an instantaneous velocity v and expelling mass ΔM at a constant velocity w relative to the vehicle. Assume <u>no</u> external forces (gravity, drag, etc.) are acting up on the vehicle.

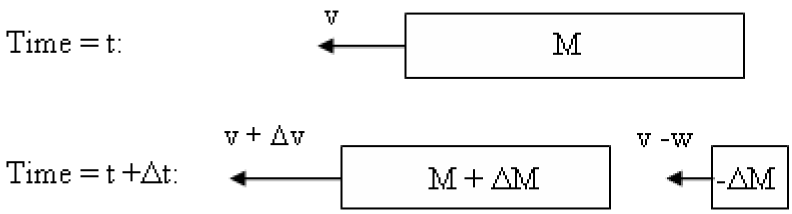


Figure 4: Rocket propulsion principle

Momentum at time t is:

$$I_t = M \cdot v \tag{2.1-4}$$

Idem at time t + Δt:

$$I_{t+\Delta t} = (M + \Delta M) \cdot (v + \Delta v) - \Delta M \cdot (v - w) \tag{2.1-5}$$

Since there are no external forces working upon the rocket, it follows that the change in momentum is equal to zero.

It follows for the momentum balance:

$$M \cdot v = (M + \Delta M) \cdot (v + \Delta v) - \Delta M \cdot (v - w) \qquad \textbf{(2.1-6)}$$

Elaboration gives (neglecting terms of second order small):

$$M \cdot \Delta v + \Delta M \cdot w = 0 \qquad \textbf{(2.1-7)}$$

For an infinitesimal change of velocity we get:

$$M \cdot dv = -dM \cdot w \qquad \textbf{(2.1-8)}$$

Separation of variables and integrating both sides leads to the rocket equation:

$$\Delta V = w \cdot \ln\left(\frac{M_{initial}}{M}\right)$$

$$\textbf{(2.1-9)}$$

$$(\Delta V)_e = w \cdot \ln(R)$$

With:
- M = instantaneous mass
- Δv = Velocity change (follows from orbit analysis)
- $R = M_{initial}/M_{final}$; M_{final} = final vehicle mass. It includes payload mass, structure subsystem mass, propulsion subsystem mass as well as the mass of all the other subsystems and the mass of propellants remaining in the vehicle. In practice, empty mass differs from dry mass in that empty mass also includes residual propellant mass (if any). $M_{final} + M_{propellant} = M_{initial}$

From this equation, we learn that to achieve certain change in flight velocity using rocket propulsion, it is best to expel the mass at the highest velocity possible. This way the empty mass and initial mass are closest, hence limiting the amount of mass to be expelled overboard. This is illustrated in the next figure.

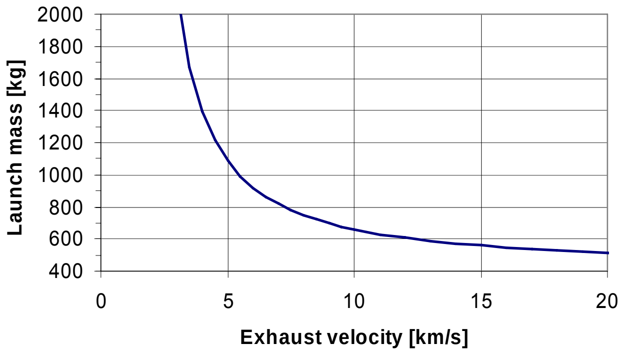


Figure 5: Results from rocket equation for vehicle with empty mass of 400 kg and mission characteristic velocity of 5000 m/s

It is noted though that in the above presented result, we have assumed that we can select the exhaust velocity without any consequence for the mass of the rocket system and hence for the vehicle carrying the rocket what so ever. It will later be shown that in reality this is not the case.

2.2. Rocket thrust (no pressure forces)

An important parameter is the thrust delivered, as it determines the acceleration that can be achieved.

From the momentum balance eq. (2.1-6) an expression can be obtained for rocket thrust. Hereto we divide the momentum balance by Δt. When taking the limit for $\Delta t \to 0$ it follows:

$$\lim_{\Delta t \to 0} M \cdot \frac{\Delta v}{\Delta t} + \frac{\Delta M}{\Delta t} \cdot w = M \cdot \frac{dv}{dt} - m \cdot w = 0 \qquad \textbf{(2.2-1)}$$

Here m is mass flow rate (m = -dM/dt).

Rewriting the equation (2.2-1) gives:

$$M \cdot \frac{dv}{dt} = m \cdot w \qquad \textbf{(2.2-2)}$$

This equation resembles the classical 2nd law of Newton:

$$M \cdot \frac{dv}{dt} = F \qquad \textbf{(2.2-3)}$$

With:

$$F_T = m \cdot w \qquad \textbf{(2.2-4)}$$

We now refer to the force F_T as the thrust force (hereafter shortly referred to as *thrust*). It is defined as the product of mass flow rate m and (relative) exhaust velocity w.

It is important to note that this equation is independent of the velocity of the rocket (v_o), because rockets are not air-breathing engines.

2.3. Rocket thrust (pressure forces included) and equivalent jet velocity

In most rocket motors, the gaseous propellants expand in a nozzle reaching a pressure p_e in the nozzle exit, see figure. This indicates that the thrust force also includes a pressure component next to the momentum component found in section 2.2. When the rocket is moving through the atmosphere, like for missiles and sounding rockets, and when the pressure in the nozzle exit is different from the pressure working on the outer surface (p_o), we should also reckon with the pressure force on the body of the rocket due to the pressure on the outer surface, which works against the thrust. This is schematized in Figure 6. Note that the pressure working on the outer surface may be different from the ambient pressure (p_a).

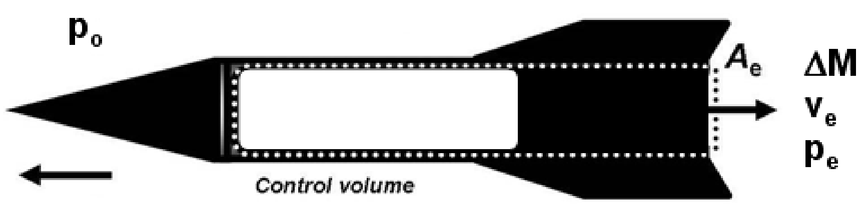


Figure 6: Rocket moving through atmosphere

Here v_e is true jet velocity (the meaning of the word "true" is to allow distinguishing this velocity from the equivalent or effective jet velocity (v_{eq}) which will be defined later) and p_e is the pressure in the nozzle exit. For now, we assume that both are constant over the exit with v_e directed opposite to the direction of motion.

For the total pressure force (F_p) on the rocket, it follows:

$$F_p = \int_{S_o} p_o ds_o - p_e A_e \tag{2.3-1}$$

At some distance from the rocket we have the undisturbed (ambient) pressure (p_a). For any closed surface around the rocket, it follows:

$$\oint p_a ds = 0 \tag{2.3-2}$$

Now taking the rocket outer surface as the closed surface, we find:

$$\int_{S_o} p_a ds_o - p_a A_e = 0 \tag{2.3-3}$$

Substitution in equation 2.3-1 gives:

$$\begin{aligned} F_p &= \int_{S_o} p_o ds_o - p_e A_e - \left(\int_{S_o} p_a ds_o - p_a A_e \right) \\ F_p &= \int_{S_o} (p_o - p_a) ds_o - (p_e - p_a) A_e \end{aligned} \tag{2.3-4}$$

This pressure force can be considered an external force working on the rocket. Combining the above result with the relation (2.2-1), we find:

$$M \cdot \frac{dv}{dt} - m \cdot v_e = -F_p = -\int_{S_o} (p_o - p_a) ds_o + (p_e - p_a) A_e \tag{2.3-5}$$

It is easy to recognize in the first term on the RHS (the integral term) the drag working on the rocket (in opposite direction to the direction of motion). Reworking gives:

$$M \cdot \frac{dv}{dt} = -F_{drag} + m \cdot v_e + (p_e - p_a) A_e \tag{2.3-6}$$

Since the only forces working on the rocket in the situation considered are the drag force and the rocket thrust, it follows for the thrust:

$$F_T = \underbrace{m \cdot v_e}_{\substack{\text{momentum} \\ \text{component}}} + \underbrace{(p_e - p_a) A_e}_{\substack{\text{pressure} \\ \text{component}}} \tag{2.3-7}$$

Here the first term on the RHS is referred to as the momentum component of the thrust and the second term as the pressure component.

It is important to note that this equation is independent of the velocity of the rocket (v_o), because rockets are not air-breathing engines. Also since the ambient pressure (p_a) decreases with altitude, there is a beneficial effect on thrust with increasing altitude. This follows from the following relation:

$$(F_T)_{altitude} = (F_T)_{vacuum} - (p_a)_{altitude} A_e \tag{2.3-8}$$

Hence, the thrust at some altitude can be determined provided vacuum thrust, nozzle exit area and ambient pressure at that altitude are known. For the latter we generally use the International Standard Atmosphere (ISA 76).

A WEB BASED ATMOSPHERIC PROPERTIES CALCULATOR CAN BE OBTAINED FROM: HTTP://WWW.AEROSPACEWEB.ORG/DESIGN/SCRIPTS/ATMOSPHERE/.

Equivalent jet velocity (v_{eq}): An imaginative velocity which when multiplied with the mass flow rate produces same total thrust as the momentum and pressure component of the thrust added together.

$$v_{eq} = \frac{F_T}{\dot{m}} \tag{2.3-9}$$

The equivalent jet velocity is …

- an important figure of merit used to compare the effectiveness of different propellants and/or one and the same propellant in different rocket motors;
- sometimes also referred to as effective jet velocity

Notice that the jet velocity as used in the derivation of the rocket equation actually equals the effective jet velocity as defined above ($w = v_{eq}$). Also note that in case of absence of pressure forces the effective jet velocity and true jet velocity are identical.

2.4. Specific propellant consumption

For jet engines, the thrust specific fuel consumption (TSFC) or sometimes simply specific fuel consumption is an important figure of merit to describe the fuel efficiency of an engine with respect to thrust output. It allows the efficiency of different sized engines to be directly compared. As for jet engines it is also wise for rockets to consider the propellant consumption per unit of thrust produced. However, since mass flow may change during the mission, it is better to use some average specific propellant consumption (C_T) defined as the ratio of propellant mass consumed and total impulse delivered:

$$C_T = \frac{M_p}{\int_o^{t_a} F_T \cdot dt} \tag{2.4-1}$$

It is typically expressed in kg/Nhr, see e.g. Table 2. For constant mass flow, it follows:

$$C_T = \frac{1}{v_{eq}} \tag{2.4-2}$$

Hence to reduce specific propellant consumption, we should strive for a high exhaust velocity.

2.5. Action time

Once we know the total impulse to be delivered by a rocket system and the thrust, the action time of the system can be determined from:

$$\int_0^{t_a} F_T \cdot dt = \int d(M \cdot v) \tag{2.5-1}$$

In case of a constant thrust and (expellant/propellant) mass flow rate, it follows:

$$t_a = {M_p}/{m} \tag{2.5-2}$$

Here the total expellant/propellant mass (from eq. 2.1-9) is determined using:

$$M_p = M_{final} \cdot \left(e^{(\Delta v / w)} - 1\right) = M_{iniitial} \cdot \left(1 - e^{-(\Delta v / w)}\right) \tag{2.5-3}$$

This relation can be derived using the rocket equation and when taking into account that final rocket mass is initial rocket mass minus propellant mass.

So propellant mass and action time can be determined when either initial or empty mass of the vehicle is known.

In case of a constant acceleration (a), operation time can simply be determined from:

$$t_a = {\Delta v}/{a} \tag{2.5-4}$$

2.6. Total impulse

By exerting a thrust on an object (spacecraft, missile, etc.) a rocket system causes the object to change its momentum. The longer the rocket system thrusts, the larger the change in momentum of the body accomplished. The product of force and the time period over which the force is applied is referred to as the impulse (I):

$$I = \int_0^{t_a} F_T \cdot dt = \int_0^{t_a} m \cdot v_{jet} \cdot dt \tag{2.6-1}$$

For constant exhaust velocity, it follows:

$$I = v_{jet} \cdot \int_0^{t_a} m \cdot dt \tag{2.6-2}$$

In case the action time is taken to be sum of all time periods that the rocket is active, we find for the total impulse delivered by the rocket propulsion system:

$$I_{tot} = F \cdot t_a = M_p \cdot v_{jet} \tag{2.6-3}$$

Hence, the *total impulse* (I_{tot}) or total change in momentum that can be accomplished by a rocket system follows from propellant mass and exhaust velocity.

To increase the total impulse delivered by a rocket propulsion system, we must either increase the thrust or the action time. From eq. (2.6-3) it than follows that either the propellant mass (M_p) or the velocity (w) at which this mass is expelled must increase.

2.7. System (gravimetric) specific impulse

The best propulsion system is generally that system which delivers the requested total impulse for the lowest propulsion system mass. An important (not much used) measure of the quality of the propulsion system is the system specific impulse defined as the impulse delivered per unit propulsion system weight:

$$I_{ssp} = \frac{I}{W_{rocket}} = \frac{I}{M_{rocket} \cdot g_o} \tag{2.7-1}$$

With:

- M_{rocket}: total rocket mass, i.e. sum of rocket hardware mass and propellant mass;
- W_{rocket}: rocket weight;
- g_o: Earth gravitational acceleration at sea level.

System specific impulse is typically expressed in seconds[3]; the higher the system specific impulse, the better the performance of the system. Note that if we use the system specific impulse to select the best propulsion system, we assume that changing the propulsion system has a negligible effect on vehicle mass.

Earth has been underlined as to stress that for reasons of comparison always Earth gravitational acceleration at sea level is used. So even when on another planet with a completely different gravitational acceleration, we still use Earth's gravitational acceleration at sea level to compute system gravimetric specific impulse.

2.8. Propellant (gravimetric) specific impulse

Another, much more used[4], performance parameter is the *specific impulse*. It is a measure of how much impulse is produced divided by the (propellant) weight that the rocket spends:

$$I_{sp} = \frac{I}{W_p} = \frac{\int_0^{t_a} F_T \cdot dt}{g_o \cdot \int_0^{t_a} m \cdot dt} \tag{2.8-1}$$

The higher the specific impulse, typically expressed in seconds, the less mass needs to be expelled to produce a given amount of thrust, so the less massive the rocket has to be. Again we note that some rocket scientists divide by mass instead by weight thereby expressing specific impulse in meters/second rather than in seconds.

At constant mass flow and exhaust velocity, we find:

$$I_{sp} = \frac{m \cdot v_{jet} \cdot t_a}{m \cdot t_a \cdot g_o} = \frac{v_{jet}}{g_o} \tag{2.8-2}$$

---

[3] Some rocket scientists define the specific impulse as total impulse divided by mass (not weight). In that case, specific impulse is expressed in meters/second.

[4] Specific impulse is much more used than system specific impulse, because most rocket systems used today are chemical rockets. Characteristic for chemical rockets is that propellant mass forms the majority of the system mass. In addition, we find that differences in dry mass for the various chemical systems exist, but in most cases are not significant.

Here again we stress that g_o by definition is Earth gravitational acceleration at sea level.

This shows that to maximize the specific impulse, we should strive for maximum exhaust velocity. This is the same result as follows from the rocket equation. It differs from the system specific impulse in that the effect of a change in propulsion system dry mass (total system mass minus propellant mass) is neglected.

Comparing eq. (2.4-2) and (2.8-2), we find that the specific impulse is proportional to the reciprocal value of the average specific propellant consumption (C_T):

$$I_{sp} = \frac{1}{g_o C_T} \qquad \textbf{(2.8-3)}$$

This shows that maximizing specific impulse is identical to minimizing the specific propellant consumption.

2.9. Volumetric specific impulse

A good measure for the size of a rocket system is the *volumetric specific impulse* (I_ρ). It is defined as the total impulse delivered per unit of propellant volume:

$$I_\rho = \frac{F \cdot t}{V_p \cdot g_o} = \rho \cdot I_{sp} \qquad \textbf{(2.9-1)}$$

The higher I_ρ, the smaller the propellant storage and hence the spacecraft; High volumetric specific impulse requires high specific impulse and a dense propellant. Nowadays, the volumetric specific impulse is not used very often. Rather one uses simply propellant density.

2.10. Input power, jet power and energy

The power required to obtain a desired thrust is given by the jet power, sometimes referred to as beam power or thrust power (ESA). *Jet power* (P_J) is defined as the kinetic power in the jet. It is related to rocket thrust and exhaust velocity by an expression of the form:

$$P_J = 1/2 \cdot F_T \cdot v_{jet} = 1/2 \cdot m \cdot v_{jet}{}^2 \qquad \textbf{(2.10-1)}$$

Rockets require high power. For example, a rocket with a thrust of 100 N and an exhaust velocity of 3000 m/s already has a beam power of 150 kW.

The efficiency with which the thruster converts input power into jet power is indicated by the *thrust or cycle efficiency* (η_{cycle}). It is defined as the (kinetic) jet power divided by the total power provided by the power source (P_{source}):

$$\eta_{cycle} = \frac{P_{jet}}{P_{source}} \qquad \textbf{(2.10-2)}$$

100% efficiency within the engine (η_{cycle} = 100%) would mean that all the energy that flows into the system (i.e. is generated on board of the rocket) is converted into kinetic energy of the jet. This is of course not possible, but some rocket designs come surprisingly close and an energy efficiency of up to 70% can be achieved, see also later in this course:. Most of the rest is lost to the environment.

For some rockets, the energy needed for propulsion stems from the propellants themselves. This is for instance the case in chemical rockets. In that case, it is interesting to find a relation between the energy converted in to jet energy and the propellant mass carried on board.
Taking thrust and exhaust velocity constant in time (constant mass flow), it follows for the total amount of energy required:

$$E = \frac{1/2 \cdot F_T \cdot t_a \cdot w}{\eta_{cycle}} = \frac{1/2 \cdot M_p \cdot w^2}{\eta_{cycle}} \qquad \textbf{(2.10-3)}$$

2.11. Thrust power and propulsive efficiency

Part of the power added to the jet is imparted as thrust power to propel the vehicle (or rocket), and part is lost as kinetic power remaining in the jet (absolute kinetic power).

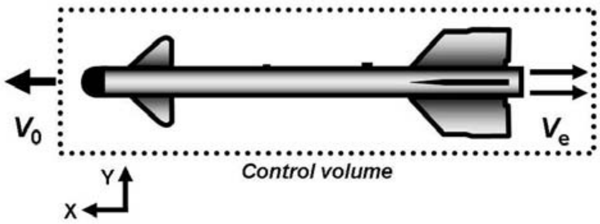


Figure 7: Rocket control volume [Ward]

The thrust power of a rocket is ($v_{jet} = v_e$):

$$P_T = F_T v_o = m v_{jet} v_o \qquad \textbf{(2.11-1)}$$

The absolute (kinetic) power in the jet equals:

$$\left(P_{jet}\right)_{abs} = 1/2 m \left(v_{jet} - v_o\right)^2 \qquad \textbf{(2.11-2)}$$

<u>Propulsive efficiency</u> (η_p): A performance indicator of how well the energy generated by the rocket is being utilized. It is defined as the fraction of total mechanical power output imparted as thrust power to propel the vehicle (or rocket), as opposed to how much is wasted.

$$\eta_p = \frac{\text{Thrust power}}{\text{Thrust power} + \text{absolute kinetic power in jet}} \qquad \textbf{(2.11-3)}$$

Substituting the earlier found relations for thrust power and absolute kinetic jet power we obtain:

$$\eta_p = \frac{m\, v_e\, v_0}{\underbrace{\tfrac{1}{2} m \left(v_e - v_0\right)^2}_{\text{energy loss}} + m\, v_e\, v_0} = \frac{2\, v_e\, v_0}{v_e^2 + v_0^2} = \frac{2}{\dfrac{v_e}{v_0} + \dfrac{v_0}{v_e}} \qquad \textbf{(2.11-4)}$$

Figure 8 shows a comparison with the propulsive efficiency for an air-breathing propulsion system.

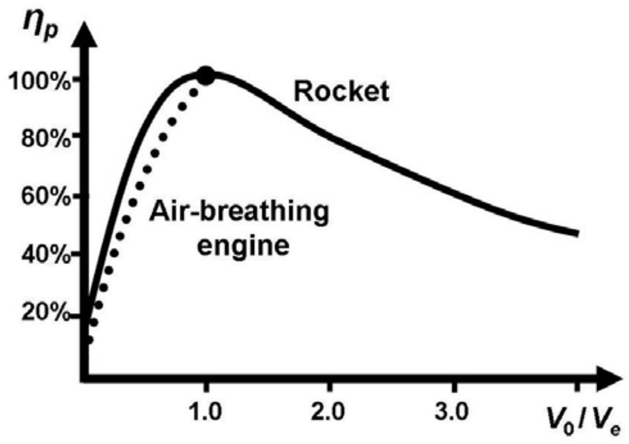


Figure 8: Comparison of propulsive efficiency of rocket and air-breathing jet engine [Ward]

The figure clearly shows that like for an air-breathing engine the propulsive efficiency of the rocket increases with increasing flight speed until the propulsive efficiency reaches 100 %. Contrary to an air-breathing engine, though the rocket can operate when $v_o > v_{jet} = v_e$ When the rocket is flying at a velocity in excess of the jet velocity, propulsive efficiency decreases with further increasing flight velocity.

2.12. Overall efficiency

The overall energy efficiency of a rocket is a figure of merit indicating how much of the energy (power) actually is used for propulsion. It is determined from the ratio between the propulsive power and the total input power.

$$\eta = \frac{\text{Thrust power}}{\text{Input power}} \tag{2.12-1}$$

2.13. Thrust control

Some missions require that the thrust is controllable for example to allow reducing acceleration loads towards the end of the flight, when the propellant tanks are almost empty. Good measures for thrust control capability of a rocket system are:

- Throttling capability or Thrust Magnitude Control (TMC): The capability to control/change the thrust of an individual rocket motor given as a ‘percentage’ (%) of nominal thrust. For example, a throttling capability of 50% means that the thrust can be reduced to 50% of its nominal value;
- Thrust Vector Control (TVC): The capability to change the thrust direction for an individual rocket motor expressed in ‘degrees’ (deg). Three rotation directions can be distinguished usually taken relative to the nominal position of a suitable body axis system. The rotations are 1 about the nozzle axis (roll direction), 1 up and down (pitch direction), and 1 left and right (yaw direction).

2.14. Pulse related parameters

The final performance parameters introduced here all relate to the pulse characteristics (on/off switching) of a rocket system. We mention:

- Impulse bit: Change in momentum per pulse.
- Minimum impulse bit: Smallest achievable impulse bit.
- Duty cycle: Nominal (single) burn time of a motor expressed in ‘second’ (s).
- Cycle life: Number representing the number of on/off cycles that a pulsed thruster is able to operate.

- Pulse duty cycle: Duration of a pulse versus time in between two pulses expressed as a 'percentage' (%).
- Thrust rise time: Time it takes for the system to go from zero thrust to full thrust.
- Thrust tail off time: Time it takes for the system to go from full thrust to zero thrust.

2.15. Example

During the first phase of the ascent flight of the European Ariane 5 rocket most thrust is provided for by two large rocket boosters working in parallel. Together they provide about 92% of the total lift-off thrust. From literature, the following performances are obtained for a single such booster:

- *Sea level thrust: 5.4 MN*
- *Propellant mass 238 (metric) tons*
- *Sea level specific impulse: 262 sec*

Determine:

(a) *mass flow rate,*
(b) *action (burn) time,*
(c) *vacuum thrust in case we have a nozzle exit diameter of 2.826 m,*
(d) *equivalent jet velocity at sea level and in vacuum,*
(e) *specific propellant consumption (in kg/Nhr) at sea level and in vacuum*
(f) *jet power, and*
(g) *propulsive power and propulsive efficiency at a flight velocity of 500 m/s (use sea level thrust conditions).*

Solution:

(a) *Mass flow rate is sea level thrust divided by product of sea level specific impulse and (Earth) gravitational acceleration at sea level. It follows: m = 5.4MN/(262s x 9.81m/s^2) = 2101 kg/s*
(b) *Action/burn time = 238 tonnes/2101 kg/s = 113.3 sec. Note that we have assumed here that the mass flow rate is constant throughout motor operation. This assumption is reasonable for most rocket motors unless the rocket motor is throttled and or thrust programming (see later in this course) is applied.*
(c) *Vacuum thrust: Sea level thrust = vacuum thrust - product of ambient pressure at sea level times nozzle exit area. Vacuum thrust = 5.4MN + 1.01325E5 x π/4 x 2.826^2 = 6.04 MN*
(d) *Equivalent jet velocity at sea level = sea level specific impulse times 9.81 m/s^2 = 2570.2 m/s. Equivalent jet velocity in vacuum conditions is vacuum thrust/mass flow rate = 6.04 MN/2101 kg/s = 2874.8 m/s*
(e) *Specific propellant consumption at sea level is $M_p/(F_T\ t_a)$ with thrust taken equal to the value at sea level. It follows 1/2570.2 = 3.9E-4 1/(m/s) = 1400.4 kg/KN-hr. In vacuum, this is 1250 kg/kN-hr.*
(f) *Jet power is ½ x mass flow rate x (equivalent jet velocity)^2 = 6.94 GW at sea level and 8.68 GW in vacuum.*
(g) *Propulsive power at sea level is thrust x flight velocity = 5.4 MN x 500 m/s = 2.7 GW Propulsive efficiency is propulsive power divided by sum of propulsive power and absolute kinetic energy in the jet. It follows η_p = 2.7 GW/(2.7GW + 0.5 2101 kg/s (2570.2m/s-500m/s)^2)*
η_p = 2.7 GW/(2.7GW + 4.5 GW) = 37.5%
Notice that in the above calculation we have determined propulsive power for a single booster. Verify that taking into account both boosters does not change the propulsive efficiency although the propulsive power goes up by a factor 2.

3. Ideal rocket motor theory

Most rockets are thermal rockets. These are rockets wherein the propellant is accelerated and shaped into a high velocity jet by a directed (thermodynamic) expansion of a high pressure gas.

A thermal rocket, see Figure 9, essentially consists of:

- ***Chamber*** – that contains the high pressure gas
  - Essentially a hollow tube, that exits in to the nozzle.
- ***Nozzle*** – that accelerates the gas flow leaving the chamber to a high velocity and provides direction to the flow
  - Essentially a pipe consisting of a convergent and divergent section (CD nozzle).

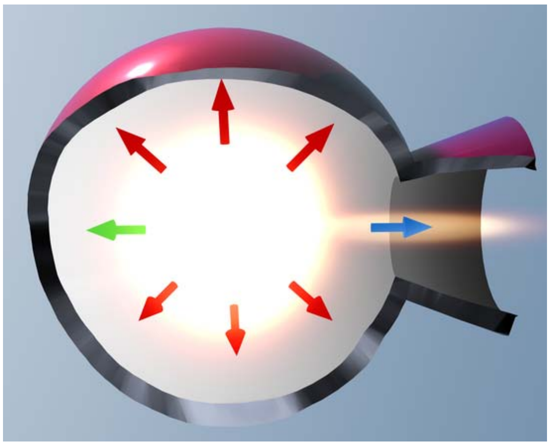


Figure 9: Schematic thermal rocket

In thermodynamic expansion a high pressure gas (contained in some chamber) is expanded in a controlled way in a nozzle to turn the thermal potential energy into directed kinetic energy, which produces thrust. The expansion occurs because of the pressure difference between the thrust chamber and the surrounding environment. The larger the pressure drop over the nozzle the higher the jet exhaust velocity will be. As the flow is expanded, the temperature of the flow decreases and hence also the internal energy of the flow. It is this energy that is used (in part) for flow acceleration.

The basic theory describing the above processes is dealt with extensively in [SSE, chapter 6.2.1]. This material together with the slides presented in class should be considered as your study material. Below we will provide two examples showing how ideal rocket motor theory can be used to analyse the performances of existing thermal rockets and/or to design your own rocket.

Example 1: Given that we have a gaseous propellant of temperature 500 K, molar mass of 20 gram/mol(molecular weight is 20) and specific heat ratio of 1.4 contained in a container at a pressure of 5 bar. In case we allow the gas to expand in a nozzle of circular cross-section and with a smallest cross-sectional area of 0.1 m^2 to an ambient pressure of 1 bar, it follows using ideal rocket theory for the velocity at which the gas leaves the motor, i.e. the jet velocity:

$$v_e = \sqrt{\frac{2\gamma}{\gamma - 1}\frac{R_A}{M}\left(1 - \left(\frac{p_e}{p_c}\right)^{((\gamma-1)/\gamma)}\right)}$$

$$v_e = \sqrt{\frac{2\times 1.4}{1.4-1}\frac{8314.32}{20}500\left(1 - \left(\frac{1}{5}\right)^{((1.4-1)/1.4)}\right)} = 732 m/s$$

Comparing this velocity with the limit velocity (nozzle exit pressure equal to 0), it follows that the velocity here obtained is roughly 60% of the limit velocity. So a substantial performance gain is still possible by further expanding the flow to a still lower pressure in the nozzle exit.

The temperature of the gas leaving the motor is 316 K (verify!!). This value again demonstrates that there is still some energy left in the jet which could be used for propulsive purposes. In the ideal case the temperature of the gas leaving the engine is 0 K. However in that case a phase change of the propellant may have occurred already.

Given the pressure of 5 bar and throat area of 0.1 m^2, we obtain for the mass leaving the engine per unit of time, i.e. the mass flow rate:

$$m = \Gamma \frac{p_c A_t}{\sqrt{\frac{R_A}{M}T_c}}$$

$$m = 0.6847\frac{(5E5)(0.1)}{\sqrt{(415.7)(500)}} = 75.1 kg/s$$

The required area ratio for the nozzle is (verify!!!): $\frac{A_e}{A_t} = 1.346$

Hence the nozzle exit diameter (D_e) is 0.414m.

Example 2: Consider the same rocket, but now we lengthen the nozzle to obtain an area ratio of 2 (nozzle exit diameter increases to about 0.50 m. This allows for a larger pressure drop over the nozzle and hence a higher (true) exhaust velocity. PLEASE NOTE THE DIFFERENCE BETWEEN TRUE AND EFFECTIVE EXHAUST VELOCITY!!!

Using ideal rocket theory, we find that the pressure ratio over the nozzle increases from 5 to 10.65. As all other parameters remain constant, we find that true exhaust velocity increases to 943.5 m/s (verify). As also mass flow remains constant we obtain for the momentum thrust a value of 75.1 kg/s x 943.5 m/s is 70.86 kN. To this we should add the pressure thrust. The pressure thrust follows from:

$$F_{pressure} = (p_e - p_a)A_e = (0.469E5Pa - 1E5Pa)0.2m^2 = -10.61 kN$$

It follows for the overall thrust a value of 60.51 kN, which is slightly more than the value that follows from the example 1 (55.0 kN).

Notice that the pressure in the nozzle exit is never allowed to be less than about 0.35 times the ambient pressure as in that case nozzle flow separation occurs and the motor will no longer be able to operate as intended.

Many different thermal rockets exist that differ in the hot gas temperatures that can be obtained and/or the molar mass of the propellants. Various types of thermal rockets are discussed in the chapters 4, 5 and 6.1.

4. Cold gas rockets

A cold gas rocket is a thermal rocket that uses a (typically inert) gas under high pressure as the reaction mass. The energy needed for the thrust generation comes from the internal energy of the gas which is expanded to form a high velocity jet (of lower pressure).

The cold gas thruster is the main working element in any cold gas rocket. A typical example is shown in the Figure 10 (left). A schematic of such a thruster is shown in the same figure on the right.

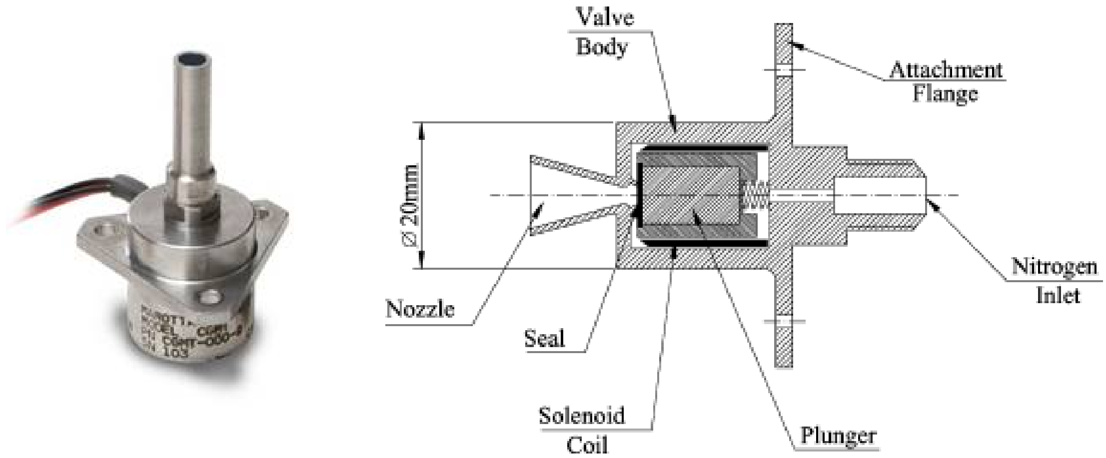


Figure 10: Cold gas thruster (left) and schematic (right)

Clearly visible in the schematic is the nozzle which in the situation shown is shut off by a valve. Gaseous propellant (nitrogen in this case) is meant to flow from right to left. Upon valve actuation though a solenoid[5] coil, the valve moves to the right and a flow path is created for the propellant to the nozzle inlet and thrust will be generated. Upon de-energizing the solenoid, the spring will force the valve back to its closed position and thrust is terminated.

Typical propellants used in cold gas rockets include: Nitrogen, Helium, Ammonia, Nitrous oxide, Butane, Argon, Xenon. These propellants are mostly characterized by that at normal conditions (1 atm., 298 K) these propellants are gaseous. Furthermore, except ammonia, nitrous oxide and butane, they are chemically inert and thus inherently safe. Typical performances of the above propellants are given in the Figure 11. The figure shows highest specific impulse for Helium with a value of about 160 s and lowest for Xenon with around 30 s. This is attributed to the difference in molar mass of these species with Helium being the lightest and Xenon the heaviest. In this respect also hydrogen with its molecular weight of 2, is a highly attractive propellant. Still in practice it is almost never used, primarily because of reasons of safety and storage volume.

[5] The term solenoid refers to a long, thin loop of wire, often wrapped around a metallic core, which produces a magnetic field when an electric current is passed through it. Solenoids are important because they can create controlled magnetic fields and can be used as electromagnets.

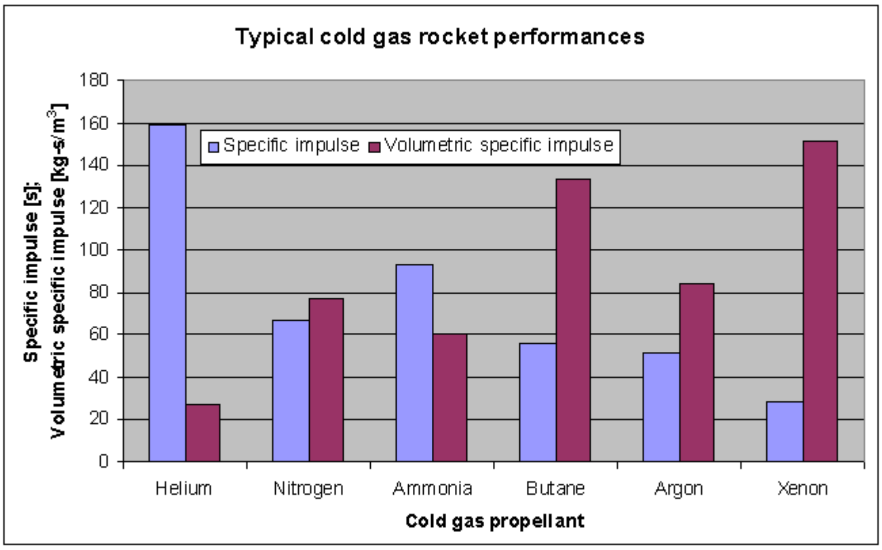


Figure 11: Specific impulse and volumetric specific impulse of various cold gas propellants at an initial temperature of 298 K, a chamber pressure of 69 bar and ideal expansion to sea level pressure (1 atm.)

An important disadvantage for cold gas systems is the storage of the gas. This generally requires high volume and/or a high storage pressure. In some cases storage pressures have been used of up to 400 -500 bar to reduce storage volume.

To evaluate the effect of mass density the figure also shows the volumetric specific impulse. Clearly now the situation is reversed showing highest volumetric specific impulse for Xenon and lowest for Helium.

The values used for calculating specific impulse and volumetric specific impulse as well as the detailed results are given in Table 5.

Table 5: Characteristics of some cold gas propellants including theoretical values of specific impulse and volumetric specific impulse

Specie	Atomic weight	Boiling point	Specific heat	γ	R	I_{sp}	ρ	I_ρ
	[-]	[°C]	[kJ-kg-K]	[-]	[J/(kg-K)]	[s]	[kg/m³]	[kg-s/m³]
Helium	4	−268.93	5.19	1.667	2008	159	0.17	27
Nitrogen	28	-195.79	1.04	1.4	297	67	1.14	77
Ammonia	17	−33.34	2.19	1.31	530	94	0.64	60
Butane	58	-0.5	1.67	1.094	143	56	2.38	133
Argon	40	−185.85	20.786 $J\cdot mol^{-1}\cdot K^{-1}$	1.667	208	51	1.63	84
Xenon	131	-108.12	20.786 $J\cdot mol^{-1}\cdot K^{-2}$	1.667	63.3	28	5.37	152

γ is specific heat ratio
R is specific gas constant
I_{sp} is specific impulse with chamber pressure is 69 atm. and optimum expansion to sea level (1 atm.)
ρ is gas density at 1 bar pressure
I_ρ is volumetric specific impulse with chamber pressure is 69 atm. and optimum expansion to sea level (1 atm.)

Characteristics given are atomic weight, boiling point (demonstrating that at normal conditions, all species are in the gaseous state), specific heat at constant pressure, specific heat ratio, gas constant, and gas density at 1 bar pressure. Notice that specific heat is given in kJ/(kgK) and in J/(molK). Consider how to convert molar heat capacity in to specific heat and vice versa.

Example cold gas system:
Using ideal rocket motor theory, we find for the limit velocity of nitrogen at an initial temperature of 298 K a value of 787 m/s; see Table 5 for gas properties of Nitrogen.
Limit exhaust velocity is hard to reach as it requires an infinitely long and hence heavy nozzle. As the length of the nozzle is limited, it means that also the pressure drop over the nozzle is limited. For a pressure drop with a factor 69 follows a true exhaust velocity of 657.3 m/s.
In case of ideal expansion, it follows a specific impulse of 67s. Multiplying this value with the gas density at 1 bar gives the volumetric specific impulse at 1 bar (= 77 kgsm^{-3} = 67 s x 1.14 kgm^{-3}). When increasing the gas storage pressure to say 20 bar (at identical temperature), the gas pressure becomes20 x 1.14 kg/m^3 = 22.8 kg/m^3 and hence also the volumetric specific impulse increases with a factor 20. Note that at 20 bar, we already need a volume of about 5 m^3 to store just 100 kg of propellant.

A generic cold gas rocket system is shown schematically in the Figure 12. It typically consists of the following major components/subassemblies:
• a high pressure-type propellant tanks,
• one or more (cold gas) thrusters each equipped with a flow control valve,
• a pressure regulator,
• a system filter,
• a pressure transducer,
• a pressurant fill/vent valve, and
• a pressure relief valve

Because of the limited number of components and that chemically inert systems can be used, the system is very simple. In the high pressure-type propellant tanks, the gas is stored under high pressure to reduce storage volume. Depending on the pressure required for the thruster the storage pressure may be regulated down to a more manageable level by a pressure regulator. A pressure transducer allows for checking the propellant available and the filter prevents any impurities from entering the delicate parts of the system. Of course a fill and vent valve are needed to allow for filling and emptying the tank on ground. The relief valve ensures, in case of failure of the pressure regulator, that the pressure in the downstream part remains within limits.

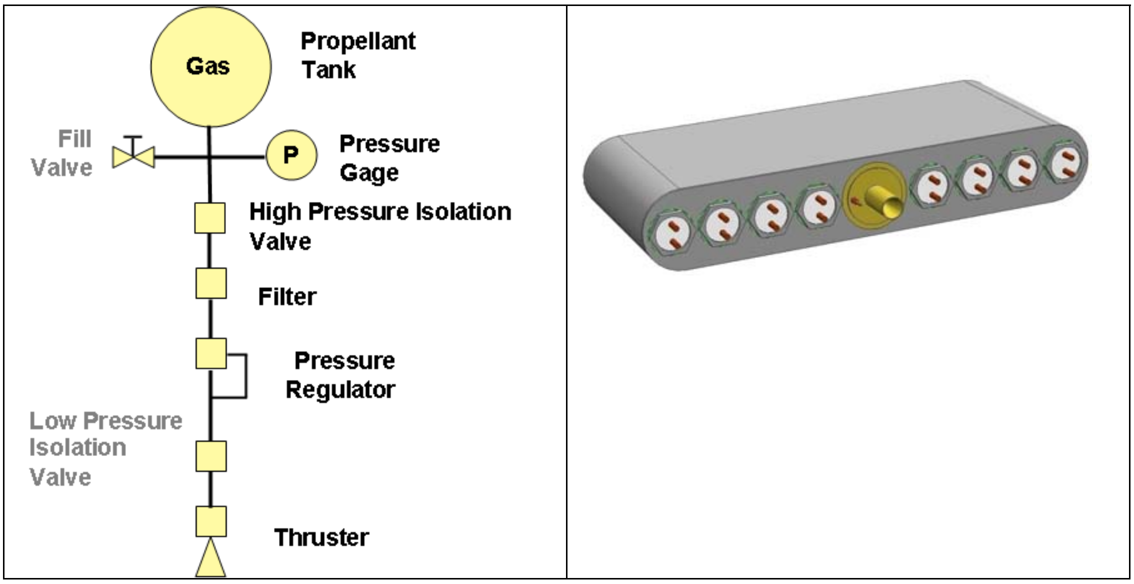


Figure 12: Cold gas system schematic (left) and schematic of TNO, TU-Delft and UTwente developed cool gas micro-propulsion system

A major issue for cold gas rockets is that even for small total impulse applications the required storage volume may be huge. One way to circumvent this is by using propellants like nitrous oxide, butane and ammonia. At high pressure these propellants can be stored in the liquid state. Another approach is shown in the Figure 12 (right). In the concept shown the gas is stored in solid form (compare how a gas is generated in an air bag) in the form of a small charge. Prior to operation the charge is activated and gas is formed. This gas is than used for propulsion. The figure shows a metallic plenum equipped with 8 solid charges. All charges can be ignited independently. After ignition of a charge, the plenum will be filled with gas. Upon valve activation the gas flows out through the nozzle (in middle in schematic). When necessary a second charge is ignited and so on. The latter system is even more simple than the system shown left as no pressure regulator is needed, which also reduces the cost of the system greatly.

In practice, we find that there are essentially two different ways of feeding the cold gas to the thruster. The first method is generally referred to as regulated pressure feeding. In this method a pressure regulator ensures a constant chamber pressure in the chamber and hence a constant thrust and mass flow rate. The second method is referred to as blow down pressure feeding. Blow down feeding is more simpler and less costly than regulated pressure feeding. However, In blow down feeding the storage pressure continuously decreases (just like in a balloon with the air escaping through its neck) and hence also the pressure in the chamber, the mass flow rate and the thrust. An example of the pressure variation in the storage tank for a blow down system is shown in Figure 13. Here initial storage pressure was 6 bar.

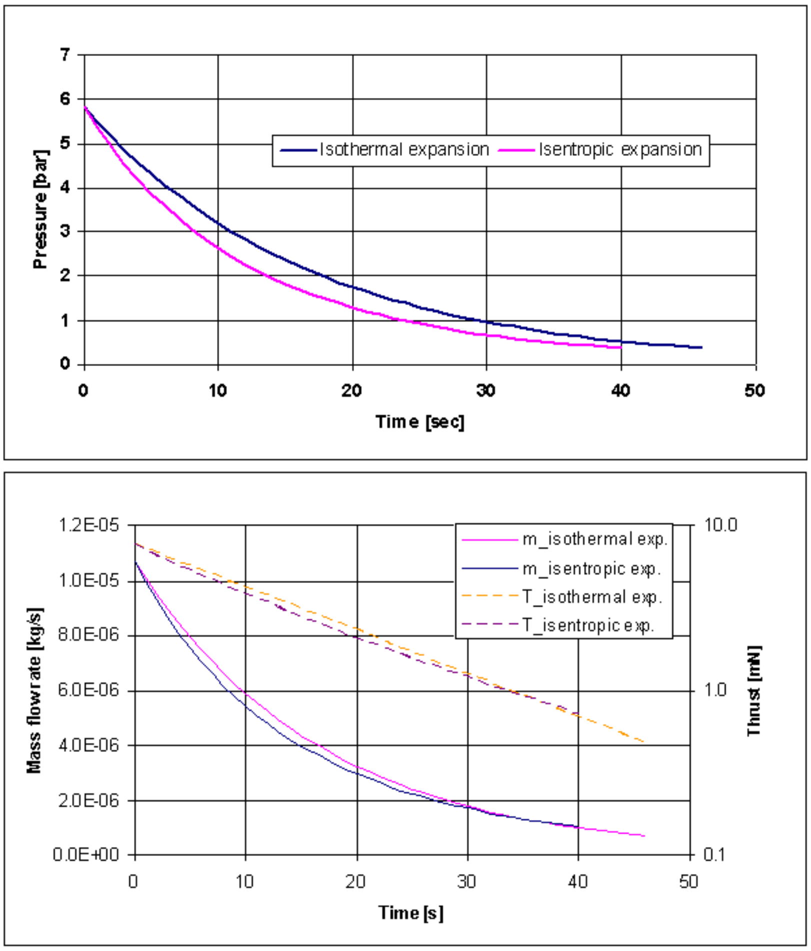


Figure 13: Simulated pressure (top) and thrust and mass flow (bottom) evolution blow down feeding using nitrogen as propellant and using a cold gas thruster with a throat diameter of 0.1 mm and a nozzle area ratio of 9.4 in vacuum (pressure losses in the feed system have been neglected)

The plots clearly show that thrust and mass flow rate vary in time. Also indicated in the figure is the effect of isothermal (constant gas storage temperature) and isentropic (adiabatic and reversible) expansion of the pressurant. In reality the expansion process is somewhere in between the two (polytropic expansion), but for this we need to determine the polytropic coefficient.

Example of regulated system: Given a tank with volume 1 m^3 filled with nitrogen gas to a pressure of 20 bar. Given that nitrogen gas behaves very much as an ideal gas, it follows that this tank contains 22.8 kg of nitrogen. Now we consider the case that the tank feeds a nitrogen thruster at a constant pressure (after the regulator) of 5 bar. Mass flow rate is 0.1 kg/s. After 10 seconds about 1 kg is used and we are left with 21.8 kg in the tank. Tank pressure is then reduced to 19.29 bar (assuming isothermal expansion). Now the system will keep on functioning until the pressure before the regulator has reduced to 5 bar (in reality pressure should be somewhat higher as losses must be accounted for. At this pressure, we have still 5.7 kg of unused propellant in the tank.

In case we have isentropic expansion of the gas in the tank, we must also take into account a temperature drop in the tank, further reducing the pressure in the tank. Consider how you would use the Poisson relations to calculate the pressure in the tank after 10 seconds of operation.

5. Chemical rockets

Chemical rockets are essentially thermal rockets, wherein the energy needed for propulsion comes from a chemical reaction or combustion of a Fuel with an Oxidizer.

Key component is the chemical rocket engine or rocket motor. A chemical rocket engine or motor essentially is a thermal rocket consisting of a combustion chamber and a nozzle. Usually at the head end, the propellant is injected after which a reaction occurs in the chamber producing hot gases. The hot gases flow downstream and enter the nozzle for expansion.

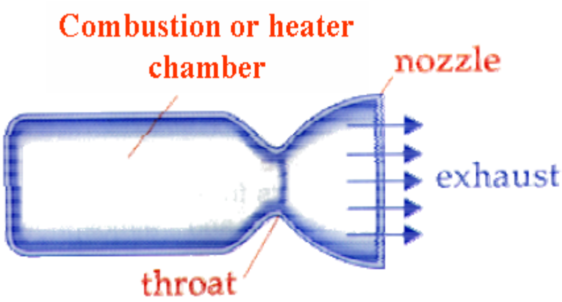


Figure 14: Schematic of chemical rocket motor or engine

Contrary to air-breathing jet and propeller engines chemical rockets must carry both fuel and oxidizer on board, which makes the system slightly more complicated. Also in contrast with air-breathing jet and propeller engines is that many different chemical fuels (and oxidizers) are in use each with their own specific properties and performances. Depending on the combination of fuel and oxidizer selected different heating values will result and because of the different products formed in the reaction, different hot gas temperatures and hot gas properties (molar mass, specific heat and specific heat ratio will result.)

The theory needed to analyze chemical rocket motors is essentially the ideal rocket motor theory, but some additions should be made to take into account the effects of the propellant chemistry, the mixture ratio of the oxidizer and fuel and the heating of the propellant by the energy that is freed in the reaction. Some essentials are described hereafter and in [SSE, chapters 6.2, except 6.2.4 and 6.2.5, and 6.3]. This material should be considered as your study material.

5.1. Chemical propellants

In a chemical rocket motor, it are the chemical propellants that provide for the required energy for thrust generation. In contrast to aircraft propulsion, rockets need to carry not only the fuel but also the oxidizer with which the fuel reacts as to provide the necessary energy. Air-breathing systems like the ones used on aircraft use the oxygen from the ambient air and hence only have to carry the fuel. Also unlike for aircraft propulsion many different fuel – oxidizer combinations exist all with their own specific properties. A list of some common fuels and oxidizers is given in Table 6.

Table 6: Typical chemical fuels and oxidizers and their state (@ 298K)

Fuels	**Oxidizers**
Liquids ➢ Ammonia ➢ Ethanol ➢ Kerosene or Rocket Propellant 1 (RP-1) ➢ Hydrazine ➢ Liquid Hydrogen ➢ Mono-methyl-hydrazine	Liquids ➢ Nitrogen Tetroxide ➢ Hydrogen Peroxide ➢ Nitric Acid ➢ Liquid Oxygen
Solids ➢ Aluminum ➢ Asphalt ➢ Plastics ➢ Rubbers	Solids ➢ Ammonium Perchlorate ➢ Ammonium Nitrate

Different fuels can be combined with different oxidizers, thereby allowing for optimizing the rocket performances.

How much energy can be obtained from the propellant depends on the fuel, the fuel amount present and the oxidizer selected. For example, Table 7 presents the lower[6] heating value (HV) of selected fuels. Here the heating value of a fuel is the amount of heat released during the combustion of a specified amount of it with oxygen at standard conditions (1 bar and 298 K).

Fuel	kJ/g
Hydrogen	141.9
Gasoline	47.0
Diesel	45.0
Ethanol	29.8
Propane	49.9
Butane	49.2
Wood	15.0
Coal (Lignite)	15.0
Coal (Anthracite)	27.0
Natural Gas	54.0

Table 7: Lower heating value of selected fuels [Wikipedia]

The total energy available now follows from:

$$E_{chemical} = M_{fuel} \times HV \tag{5.1-1}$$

[6] Lower heating value applies in case we consider water present in the reactant mixture in gaseous form. When water is present in the liquid form, it means that the heat of condensation is to be added to the heating value and hence a higher value results.

For rocket motors interest is not so much in energy available, but rather in chemical power, given by:

$$P_{chemical} = m_{fuel} \times HV \tag{5.1-2}$$

Here m_{fuel} refers to the fuel mass flow rate.

For combustion to occur, fuel has to react with an oxidizer in the proper mixture ratio. For instance, consider the reaction of RP-1 with liquid oxygen. The combustion reaction can be approximated as follows:

$$C_{12}H_{26}(l) + 37/2\ O_2(g) \rightarrow 12\ CO_2(g) + 13\ H_2O(g);$$

This reaction equation states that 1 mol of kerosene (RP-1) reacts with 37/2 mol of oxygen to form 12 mol of carbon dioxide and 13 mol of water. We furthermore find that:

- For every mol of RP-1, we need 18.5 mol of oxygen. This is an oxidizer to fuel volume mixture ratio of 18.5:1 = 18.5. Note that RP-1 is considered the fuel and oxygen the oxidizer.
- For every kg of RP-1, we need 3.5 kg of oxygen. This is an oxidizer to fuel mass mixture ratio of 3.5:1 = 3.5

The mixture ratio usually is given as a mass mixture ratio (r), but sometimes also a volume mixture ratio may be given. By convention, the oxidizer mass is given in the numerator and the fuel mass in the denominator.

The correctness of the above reaction equation can be checked by checking whether we have equal amounts of each element (C, H, O) on both sides of the arrow. This is referred to as the law of conservation of chemical elements.

Once the reaction equation is known, the properties of the gaseous products can be determined. Again considering the reaction of RP-1 with liquid oxygen:

$$C_{12}H_{26}(l) + 37/2\ O_2(g) \rightarrow 12\ CO_2(g) + 13\ H_2O(g);$$

It follows:

- Average molar mass is (12 x 44 + 13 x 18)/(12+13) = 762/25 = 30.48 mol
- Average specific heat at constant pressure is (12 x 36.94 J/(mol-K) + 13 x 75.3 J/(mol-K)/(12 +13) = 56.89 J/(mol-K)

More generically, we write:

$$M_{mixture} = \frac{\sum_{i=1}^{n} n_i (M)_i}{\sum_{i=1}^{n} n_i} \tag{5.1 -3}$$

$$(C_p)_{mixture} = \frac{\sum_{i=1}^{n} n_i (C_p)_i}{\sum_{i=1}^{n} n_i} \tag{5.1-4}$$

Here "i" refers to the various species in the mixture, n is the molar quantity, and C_p and M are molar heat capacity and molar mass, respectively.

In reality, obtaining reaction equations is slightly more complex, as also other reaction products then carbon dioxide and water may come into existence. A further complicating factor is that sometimes an excess of fuel (or oxidizer) may be used as to lower the (average) molar mass of the reaction products. This is most noticeable for hydrogen-oxygen rockets, which mostly use an excess of fuel (hydrogen).

From the available chemical power and the calculated specific heat (at constant pressure), the final (or flame) temperature of the hot propellant gases can be determined using:

$$P_{chemical} = P_{thermal} = m\ c_p(T_{final} - T_{initial}) \qquad \textbf{(5.1-5)}$$

Here it is assumed that all chemical energy is used to heat up the fuel (in practice some losses must be taken into account), that specific heat is independent of temperature and that phase transitions can be neglected, see also chapter on Advanced thermal rockets.

Example: A rocket engine uses JP4 (jet propellant 4, like RP-1 some kerosene blend) and oxygen as the propellants at a oxidizer to fuel mass mixture ratio of 2.25 (in practice, values in the range between 1.25 to 3.00 are used). The heating value (HV) of JP-4 is 20 MJ/kg and the specific heat at constant pressure (c_p) of the combustion products is 2000 J/kg-K.
From the above, it follows that for every kg of JP-4 that is combusted, we obtain 20 MJ of energy. From the mass mixture ratio it follows that to combust this 1 kg of fuel, we need 2.25 kg of oxidizer. Using conservation of mass it follows that for every kg of JP-4 that is combusted we obtain 3.25 kg of combustion products. The final temperature of the combustion mixture can now be calculated using:

$$M_{fuel}\ x\ HV = M_{total}\ x\ c_p\ x\ \Delta T$$

$$1\ kg\ x\ 20\ MJ/kg = 3.25\ kg\ x\ 2000\ J/(kg\text{-}K)\ x\ \Delta T$$

It follows ΔT = 3077 K Taking the initial temperature of the reactants as 300 K, we find a hot gas temperature of 3077 + 300 = 3377K.

In the reaction equation given earlier we have also indicated the state (l for liquid and g for gaseous) of the reactants and the products. Its importance can be illustrated as follows. Compare the reaction of liquid hydrogen (20K) and liquid oxygen (80 K) with the reaction of the same amounts of gaseous hydrogen and oxygen at say 298 K. In the former case the final temperature will be less than in the latter case as some of the heat is used to heat up the initial reactants to first 298 K. This is not needed for the gaseous propellants.

Table 8 presents various propellant combinations, their oxidizer-to-fuel mass mixture ratio, and their sea level specific impulse. These data have been taken from http://www.braeunig.us/space/propel.htm and should be considered as complementary to the data given in [Fortescue].

Table 8: Rocket propellant ideal performance [Braeunig]

ROCKET PROPELLANT PERFORMANCE					
Combustion chamber pressure, P_c = 68 atm (1000 PSI) ... Nozzle exit pressure, P_e = 1 atm					
Oxidizer	**Fuel**	**Hypergolic**	**Mixture Ratio**	**Specific Impulse (s, sea level)**	**Density Impulse (kg-s/l, S.L.)**
Liquid Oxygen	Liquid Hydrogen	No	5.00	381	124
	Liquid Methane	No	2.77	299	235
	Ethanol + 25% water	No	1.29	269	264
	Kerosene	No	2.29	289	294
	Hydrazine	No	0.74	303	321
	MMH	No	1.15	300	298
	UDMH	No	1.38	297	286
	50-50	No	1.06	300	300
Liquid Fluorine	Liquid Hydrogen	Yes	6.00	400	155
	Hydrazine	Yes	1.82	338	432
FLOX-70	Kerosene	Yes	3.80	320	385
Nitrogen Tetroxide	Kerosene	No	3.53	267	330
	Hydrazine	Yes	1.08	286	342
	MMH	Yes	1.73	280	325
	UDMH	Yes	2.10	277	316
	50-50	Yes	1.59	280	326

Table continued

ROCKET PROPELLANT PERFORMANCE					
Combustion chamber pressure, P_c = 68 atm (1000 PSI) ... Nozzle exit pressure, P_e = 1 atm					
Oxidizer	**Fuel**	**Hypergolic**	**Mixture Ratio**	**Specific Impulse (s, sea level)**	**Density Impulse (kg-s/l, S.L.)**
Hydrogen Peroxide (85% concentration)	Kerosene	No	7.84	258	324
	Hydrazine	Yes	2.15	269	328
Nitrous Oxide	HTPB (solid)	No	6.48	248	290
Chlorine Pentafluoride	Hydrazine	Yes	2.12	297	439
Ammonium Perchlorate (solid)	Aluminum + HTPB (a)	No	2.12	266	469
	Aluminum + PBAN (b)	No	2.33	267	472
Red-Fuming Nitric Acid (14% N_2O_4)	Kerosene	No	4.42	256	335
	Hydrazine	Yes	1.28	276	341
	MMH	Yes	2.13	269	328
	UDMH	Yes	2.60	266	321
	50-50	Yes	1.94	270	329

The table also shows which propellants are hypergolic (self-igniting) and which are not. Hypergolic propellants have the advantage that no igniter is needed -- propellants react on contact in engine.

Error! Not a valid bookmark self-reference. provides some additional information including the chemical formula, molecular weight (a dimensionless parameter representing the weight of a molecule; molar mass typically is equal to the molecular weight expressed in gram/mol), mass density and melting and boiling point. The latter two are important to consider whether the propellants are storable or that additional measures are needed to allow proper storage. For instance:

- *Storable* propellants can be stored as liquids under normal environmental temperatures
- *Cryogenic* propellants need to be cooled extensively to allow for storage in the liquid state

Table 9: Properties of some chemical rocket propellants [Braeunig]

PROPERTIES OF ROCKET PROPELLANTS

Compound	Chemical Formula	Molecular Weight	Density	Melting Point	Boiling Point
Liquid Oxygen	O_2	32.00	1.14 g/ml	-218.8°C	-183.0°C
Liquid Fluorine	F_2	38.00	1.50 g/ml	-219.6°C	-188.1°C
Nitrogen Tetroxide	N_2O_4	92.01	1.45 g/ml	-9.3°C	21.15°C
Nitric Acid	HNO_3	63.01	1.55 g/ml	-41.6°C	83°C
Hydrogen Peroxide	H_2O_2	34.02	1.44 g/ml	-0.4°C	150.2°C
Nitrous Oxide	N_2O	44.01	1.22 g/ml	-90.8°C	-88.5°C
Chlorine Pentafluoride	ClF_5	130.45	1.9 g/ml	-103°C	-13.1°C
Ammonium Perchlorate	ClH_4NO_4	117.49	1.95 g/ml	240°C	N/A
Liquid Hydrogen	H_2	2.016	0.071 g/ml	-259.3°C	-252.9°C
Liquid Methane	CH_4	16.04	0.423 g/ml	-182.5°C	-161.6°C
Ethyl Alcohol	C_2H_5OH	46.07	0.789 g/ml	-114.1°C	78.2°C
n-Dodecane (Kerosene)	$C_{12}H_{26}$	170.34	0.749 g/ml	-9.6°C	216.3°C
RP-1	$C_nH_{1.953n}$	≈175	0.820 g/ml	N/A	177-274°C
Hydrazine	N_2H_4	32.05	1.004 g/ml	1.4°C	113.5°C
Methyl Hydrazine	CH_3NHNH_2	46.07	0.866 g/ml	-52.4°C	87.5°C
Dimethyl Hydrazine	$(CH_3)_2NNH_2$	60.10	0.791 g/ml	-58°C	63.9°C
Aluminum	Al	26.98	2.70 g/ml	660.4°C	2467°C
Polybutadiene	$(C_4H_6)_n$	≈3000	≈0.9 g/ml	N/A	N/A

5.2. Solid rocket motors

Solid propellant rocket motors or shortly solid rocket motors (SRMs) are characterized by that the propellant is stored in the solid state in the form of a shaped solid charge, see Figure 15. Solid propellants typically consist of a rubber containing aluminium particles and a solid oxidizer, like ammonium perchlorate or ammonium nitrate. For military rockets also nitrocellulose and nitro-glycerine are used as propellants. The combustion of 1 kg of solid propellant typically delivers 4-5 MJ of energy, depending on the specific constituents used and their ratios.

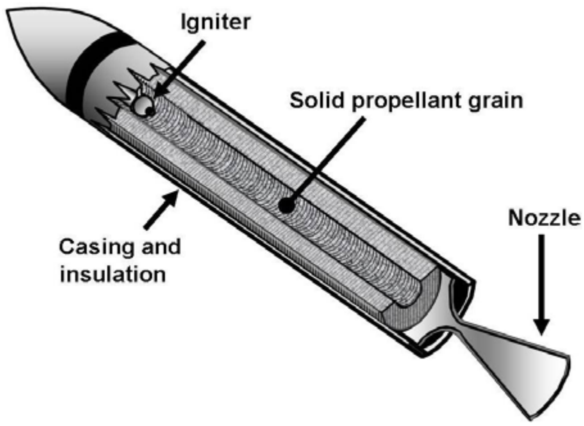


Figure 15: Solid rocket motor schematic [Ward]

SRMs need an igniter to start the burning of the solid propellant. Once the burn is started, the rocket keeps on burning until all the propellant is spent, like for fireworks. Burning occurs along the surface that is exposed to the hot flame in the combustion chamber. The centre space in the grain is referred to as the grain port and allows for a proper combustion of the initially solid propellant.

The grain or charge may be cartridge loaded or case bonded (glued to the case). After ignition, the grain burns in a direction perpendicular to the original grain surface, i.e. it is burning in parallel layers; Compare the burning of a wax candle. The velocity with which the surface regresses is referred to as the regression rate. The burn time (t_b) of a solid rocket motor is determined by the thickness (w) of the grain and the regression rate (r). Given a constant regression rate, it follows:

$$t_b = w / r \qquad \textbf{(5.2-1)}$$

Typical regression rates can be from a few mm/s up to cm/s.

Example: A solid rocket motor with a solid propellant that regresses 2 cm/s with a burn time of 100 seconds must have a thickness of (at least) 2 m in the direction of burning. For the rocket motor shown in Figure 15*, this would mean a diameter of the rocket of at least 4 m.*

The propellant mass flow rate in an SRM is determined by the regression rate. The larger the regression rate, the larger the mass flow rate will be. Besides the regression rate, the mass flow rate also depends on the solid propellant density (ρ_s) and the surface over which the grain burns, i.e. the burn surface (S):

For constant conditions along the burn surface:

$$m = \rho_S \cdot r \cdot S \tag{5.2-2}$$

The grain's geometry plays a major role in the design of solid rocket motors, since it is this geometry that determines the area and contours of the grain's exposed surfaces. It is this geometry that determines whether the mass flow rate (and hence also the thrust) increases, decreases or remains constant.

There are two basic types of geometries used. The first are end burning configurations, where the front of the flame travels in layers from the nozzle end of the block (hence end burning) towards the top of the casing. This geometry produces constant thrust throughout the burn.
The second, which is more usual, are internal burning configurations, wherein the combustion surface develops along the length of a central channel. Sometimes the channel has a star shaped, or other, geometry to moderate the growth of this surface. Most solid rocket boosters use this configuration. Some typical examples are given in Figure 16.

Figure 16: Examples of internal burning grains (left wagon wheel, right cylindrical with fins

In practice, we sometimes find combinations of axial and radial burning grains, but this is considered beyond the scope of this course.

Example: A cylindrical shaped end burning grain made of a propellant with a mass density of 1600 kg/m^3, a propellant regression rate of 8 mm/s and a diameter of 1 m generates a mass flow rate m = 1600 kg/m^3 x 8E-3 m/s x π/4 (1)^2 = 10.1 kg/s.

In case we know mass flow rate, characteristic velocity and the throat area of the nozzle, we can determine the chamber pressure using ideal rocket motor theory (critical mass flow). Verify!!

5.3. Liquid monopropellant rockets

A slightly more complicated type of chemical rocket is the liquid monopropellant rocket. In such a rocket the propellant is a liquid propellant composed of chemicals or mixtures of chemicals which can be stored in a single container with some degree of safety. The energy needed for thrust generation stems from a decomposition reaction of the propellant, which takes place under the action of a catalyst.

A catalytic thruster is the main working element in any liquid monopropellant rocket. A schematic of such a thruster is shown in Figure 17.

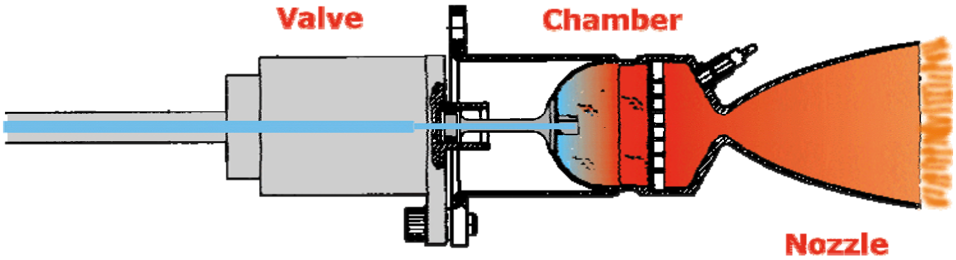


Figure 17: Schematic of catalytic hydrazine thruster (Blue=Liquid N_2H_4, Red=Hot Gaseous N_2, H_2, NH_3)

From the figure, we learn that the thruster consists of the chamber containing the catalyst and a nozzle. A valve situated upstream of the chamber controls the flow of propellant to the thruster. Since the propellants reach a high temperature, typically in the range 1000 K to 1500 K with chamber pressures up to 20 bar. Attention should be given to the selection of the proper structural materials. Typical structural materials used are either stainless steel or brass (copper alloy C3600). Sometimes also nickel alloys as Haynes 25 or 188 are used.

In practice, the most commonly used monopropellant is hydrazine with hydrogen peroxide as a good second. Some rocket cars use nitro-methane or even nitrous oxide (laughing gas). Some typical monopropellants and their characteristics are given in Table 10.

Table 10: Typical monopropellants and their characteristics

Substance	Molecular formula	Heat of decomposition	Important products	Reference
Hydrazine	N_2H_4	112 kJ/mol	Ammonia	[Zandbergen]
Hydrogen peroxide	H_2O_2	98.2 kJ/mol	Water	[Wikepedia]
Ethylene-oxide	C_2H_4O	3051 kJ/kg	Carbon dioxide	[An.]
Nitro-methane	CH_3NO_2	3920 kJ-kg	Water, nitrogen	[Bretherick]
Nitrous oxide (laughing gas)	N_2O	82.05 kJ/mol	Nitrogen, oxygen	[NIST]

Typical decomposition reactions are:

Hydrazine (1):	$3\ N_2H_4 \rightarrow 4\ NH_3 + N_2$
Hydrazine (2):	$N_2H_4 \rightarrow N_2 + 2\ H_2$
Hydrogen peroxide:	$2\ H_2O_2 \rightarrow 2\ H_2O + O_2$
Nitromethane:	$4\ CH_3NO_2 \rightarrow 4\ CO + 4\ H_2O + 2\ H_2 + 2\ N_2$
Nitrous oxide	$2\ N_2O \rightarrow 2\ N_2 + 1\ O_2$

In reality, the reactions are slightly more complicated. This will be left out of consideration.

Typical values of specific impulse as well as molar mass for hydrogen peroxide and hydrazine monopropellant can be obtained from [Fortescue]. Comparing the values of molar mass as given by Fortescue with the values as follows from the above reaction equations, one can easily see how different the values are.

A system diagram of a monopropellant system is shown in Figure 18. High pressure gas (nitrogen or helium) forces the monopropellant from the tank to the thrusters via a dedicated piping system. Valves control and regulate the propellant flow to the various thrusters.

The feed system shown in the figure is a blow down feed system. In blow down systems, the pressurant is stored in the propellant tank. Typical (initial) tank pressure is maximum 15-25 bar as to keep tank mass limited. As during operation the tank empties, the available volume for the pressurant increases and hence tank pressure decreases. To prevent a too large pressure drop, the fill ratio of the tanks is typically 50-60%, leaving 50-40% of the volume to store the pressurant gas. The feed pressure in the blow down case must remain large enough to allow proper operation of the thrust generator (thruster). Proper operation is generally ensured provided that the combustion chamber pressure is in excess of about 2-5 bar. Generally, to prevent gas bubbles entering the feed line(s), pressurant and propellant are separated by a membrane.

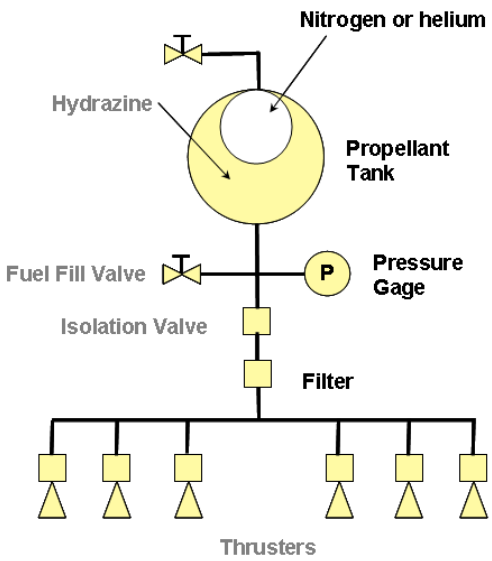


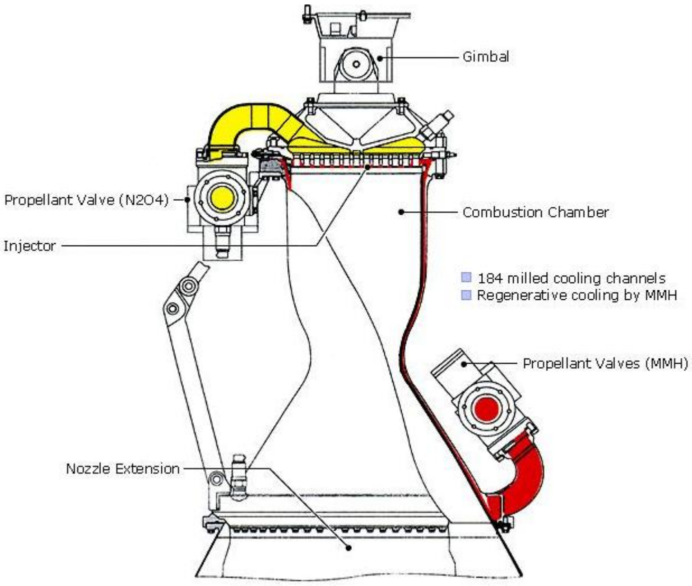
Figure 18: System schematic monopropellant rocket

An alternative feed system is a pressure regulated system. In a pressure regulated system the pressurant is stored separately from the propellant under high pressure. A pressure regulator regulates the flow of pressurant into the propellant tank, thereby maintaining a constant pressure. Propellant tank pressure is typically 15-25 bar to limit the mass of the tank. Pressurant tank pressure though is in the range 150-300 bar, thereby limiting the storage volume. To allow good operation of the pressure regulator, the pressure in the pressurant tank should be about a factor 2 higher than in the propellant tank.

5.4. Liquid bipropellant rockets

In a liquid bipropellant rocket a liquid fuel and oxidizer are combusted to generate the energy needed for propulsion.

The heart of a liquid bipropellant rocket is formed by the liquid rocket thrust chamber, which consists of a combustion chamber and nozzle, see Figure 19.



The fuel (red) and oxidizer (yellow) are injected in to the combustion chamber through an injector. This is a device that generates a fine spray of both fuel and oxidizer, thereby ensuring a proper distribution and mixing of the propellant in the combustion chamber (a bit like how a shower head distributes the water over a larger area). Before injecting the fuel, the fuel is run through a large number of coolant channels to cool the combustion chamber and part of the nozzle, thereby ensuring structural integrity.
.

Figure 19: Aestus liquid rocket thrust chamber schematic (courtesy EADS)

Thrust chambers can come in may sizes, mostly depending on the thrust level and the (combustion) chamber pressure. Thrust levels may range from a few Newton to several mega-Newton.

To feed the propellants to the thrust chamber, a feed system is needed. In practice, two types of feed systems are in use for liquid bipropellant rockets, being pressure-fed and pump-fed feed systems. The former is mostly used for low thrust, low total impulse (<15 MNs) systems and the latter for high thrust, high total impulse (> 20 MNs) systems. In between, it depends on the specific requirements which system suits best.

Principally there are two types of pressure-fed systems including self-pressurization systems and systems using a foreign gas. Self pressurization can be applied for fuels and oxidizers that have a vapor pressure in excess of the chamber pressure. Typical example of self pressurizing propellants are nitrous oxide (laughing gas) and propane and butane. For a discussion of pressurization systems using a foreign gas we refer to the chapter on monopropellant rockets.

Most large liquid rocket engines, like the Space Shuttle Main Engine (SSME), the Ariane 5 Vulcain I and II engines, the Saturn V F1 and J2 engines and the Ariane 4 Viking 4, 5 and 6 engines, use pumps to force the propellants from the storage tanks to the thrust chamber. The reason is that when using pumps higher combustion pressures can be attained (up to 200 bar for the SSME as compared to maximum about 30 bar for pressure-fed systems), thereby limiting the mass and size of the thrust chamber and/or the mass of the propellant storage tanks.

Rocket pumps are in almost all cases turbo pumps, see Figure 20. These are pumps which are driven by gas turbines. The latter are driven in turn by drive gases tapped of from the combustion chamber or a separate gas generator. Some rockets even use hot coolant gases from the motor coolant system to drive the turbo pumps.

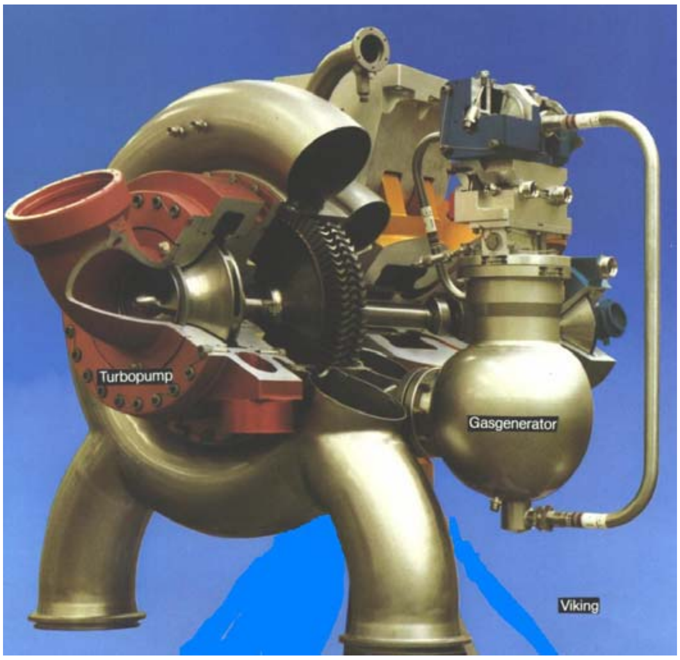


Figure 20 shows the turbo pump of the Ariane 4 Viking rocket engine. It is a single-shaft turbo pump, meaning that both fuel and oxidizer pump are driven by one and the same turbine. It is running at a speed of 10,000 rpm and has a 2.5 MW power rating. The cut out provides a view on a pump on the left, connected via an axis to a turbine (in the mid of the figure) and an axis connecting to the pump on the other side. Clearly visible is also the gas generator providing the turbine drive gas (at a rate of 1.2 kg/s) as well as the turbine exhaust ducts which guide the used turbine gases overboard.

Figure 20: Viking V turbo-pump with gas generator (courtesy Snecma)

The power required to drive a pump can be determined using:

$$P_p = \frac{m \cdot \Delta p}{\eta \cdot \rho} = \frac{Q \cdot \Delta p}{\eta} \qquad \textbf{(5.4-1)}$$

Here m and ρ are the mass flow rate and the density of the fluid being pumped, η is the pump efficiency (typically around 50%), Q is the volume flow rate and Δp is the rise in pressure needed to feed the propellants to the thrust chamber. This rise in pressure most of the times is usually substantially higher than the difference between the combustion chamber pressure and the pressure in the storage tank. For the total power we must also take into account the turbine efficiency. Turbine efficiency typically is in the range of 30%.

Pumps are usually built together with the thrust chamber to form a single assembly. Figure 21 shows the Vulcain thrust chamber (left) and the assembled engine (right) including the thrust chamber, two turbo pump (one for the fuel and one for the oxidizer), gas generator, turbine exhaust system and propellant piping and valves

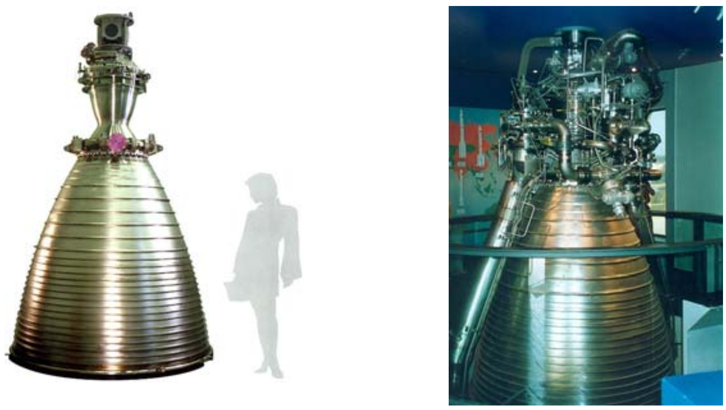


Figure 21: Vulcain engine (courtesy EADS)

After leaving the turbine, the used turbine drive gases can be exhausted separately over board (open cycle) or can be led to the main thrust chamber so that they can still be used for generating thrust (closed cycle). For details on open cycle and closed cycle systems, including their advantages and disadvantages, see [Fortescue, section 6.2.3].

Besides the thrust generation system and the feed system a liquid propellant rocket includes many other elements to ensure the proper operation of the rocket, see for instance [Fortescue, fig. 6.19]. This figure shows a typical spacecraft bi-propellant rocket system It essentially consists of:

- Thrust system
    - 1 main thruster for orbit insertion
    - 16 small thrusters for reaction wheel de-saturation
- Propellant storage
    - 1 oxidizer (NTO) tank
    - 1 fuel (MMH) tank
- Feed & distribution system
    - Four Helium high pressure pressurant tank
    - One Pressure regulator
    - Various filters, valves and pressure sensors

5.5. Hybrid rockets

Hybrid propellant rockets or just hybrid rockets (HRMs) are characterized by that the propellant constituents are stored in a different state, with usually a solid fuel and a liquid oxidizer. Like for an SRM, the solid is stored in the form of a solid shaped charge, see Figure 22. Fuels consist of coal, rubber, plastic or wood (just about anything that burns base) with or without aluminum particles embedded in the grain. Oxidizers include amongst others oxygen (stored cryogenically), hydrogen peroxide and nitrous oxide.

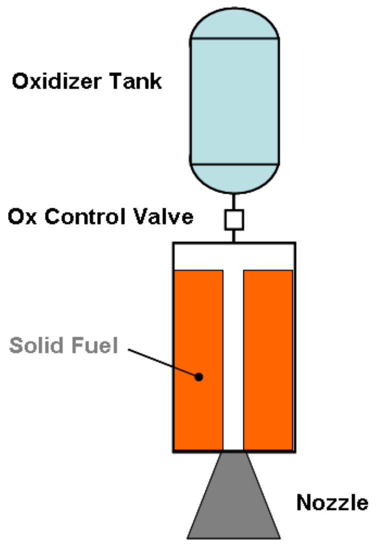


Figure 22: Hybrid rocket schematic

Typical heating values are 27.9 MJ/kg for soft coal (bituminous), 37.2 MJ/kg for rubber, and 44.2 MJ/kg for plastic.

Like for solid motors, the solid (fuel) grain or charge may be cartridge loaded or case bonded (glued to the case). The flow channels cut out in the fuel grain allow for the oxidizer to flow through and are referred to as fuel ports. These fuel ports also make up the combustion chamber where the gasified fuel reacts with the oxidizer. Start of an hybrid motor is first by injecting the oxidizer as a fine spray (fog) into the combustion chamber. Next an igniter is used to gasify the initially solid fuel, which then mixes and reacts with the oxidizer until full-fledged combustion is achieved. From that time on the ignition can be stopped and normal burning takes place. Burning occurs along the surface that is exposed to the hot flame in the combustion chamber in parallel layers (compare solid rocket motors). The velocity with which the surface regresses is referred to as the fuel regression rate. Like for a solid rocket motor, the burn time (t_b) of a hybrid rocket motor is determined by the thickness (w) of the grain and the regression rate (r). Practical regression rates for hybrid rockets are an order of magnitude smaller then for SRMs. Like for solid rocket motors, the grain geometry (next to the fuel mass density and the regression rate) determines to a large extent the fuel mass flow rate.

Hybrid rockets combine elements from solid and liquid rockets to obtain a compromise of each type's advantages and disadvantages:

- Hybrid rockets are capable of producing specific impulse levels in between those of the best performing solid and liquid (chemical) rockets.
- Similar to liquid rockets, they can be throttled, shut-down (by terminating the oxidizer flow) and restarted (multi-start capability), and
- Since the oxidizer and fuel are separated, hybrid rockets are safer to construct, store and use than solid rockets.

5.6. System considerations

In the foregoing sections, various types of chemical rockets have been described, thereby focussing on the effect of different propellants on performance, i.e. specific impulse and volumetric specific impulse and the design of the rocket. It should be clear from the foregoing that differences between the various chemical rockets may be significant. How these differences also affect other important performances is outlined in annex A.

When applying the rocket equation, we find that we should strive for maximum (effective) exhaust velocity to minimize propellant mass and hence to maximize empty mass or loaded mass. However, this is without taking into account the effect of the mass of the system itself, i.e. all the hardware (engines, motors, tanks, etc.) needed to make the rocket work. Here we will analyze the effect of the mass of the propulsion hardware on the selection of the rocket exhaust velocity. To this end, we assume that the mass of the propulsion hardware varies linearly with propellant mass, according to:

$$\alpha = \text{empty mass fraction} = \frac{(M_{rocket})_e}{M_p} = \text{constant} \qquad \textbf{(5.6-1)}$$

For solid rocket motors, this is easily verified from Figure 23. The equation indicated in the figure relates motor empty mass (indicated as y) to propellant mass (indicated as x). We find $\alpha = 0.1585$.

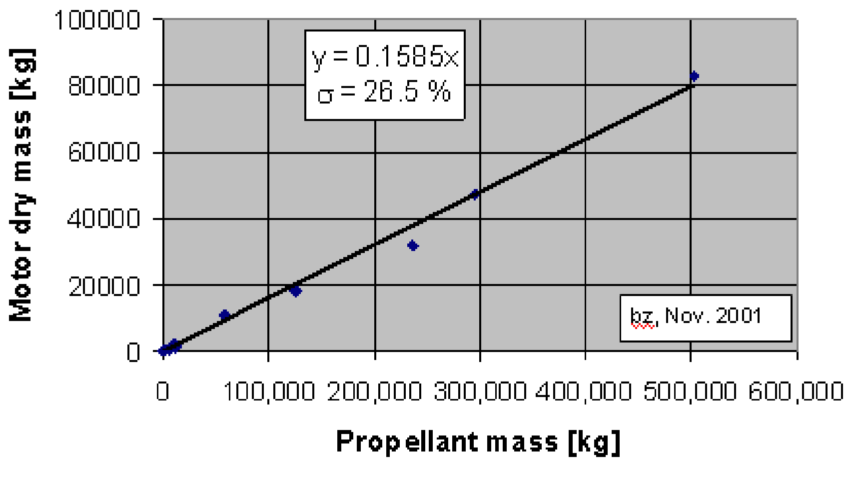


Figure 23: Solid rocket motor/stage empty mass versus propellant mass

That rocket empty mass and propellant mass are closely related is explained from that the largest part of the empty mass of a rocket motor and also a liquid propellant rocket stage/module is made up by the propellant storage system (tanks/motor casing); Hence, when propellant mass increases the size of the tanks or the motor casing increases and hence also their mass.

To evaluate the effect of dry system mass on the design, we substitute the linear relation between empty rocket mass and propellant mass in the rocket equation. This gives:

$$\Delta V = w \cdot \ln\left(\frac{M_o}{M_f}\right)$$

$$\Delta V = w \cdot \ln\left(\frac{M_f + M_p}{M_f}\right)$$

$$M_p = M_f \cdot \left(e^{\frac{\Delta v}{w}} - 1\right) = \left(M_s + (M_{rocket})_e\right) \cdot \left(e^{\frac{\Delta v}{w}} - 1\right) = \left(M_s + \alpha M_p\right) \cdot \left(e^{\frac{\Delta v}{w}} - 1\right)$$

Finally, we obtain:

$$M_p = M_{propellant} = \frac{M_s}{\left(1 - \alpha \cdot \left(e^{\frac{\Delta v}{w}} - 1\right)\right)} \cdot \left(e^{\frac{\Delta v}{w}} - 1\right) \qquad \textbf{(5.6-2)}$$

Here M_o is total vehicle mass at start, M_f is final vehicle mass, and M_s is vehicle structural mass (say vehicle mass minus empty mass of the propulsion system/rocket):

$$M_s = M_f - (M_{rocket})_e$$

In the next figure some results are given for a representative vehicle and a mission characteristic of 1800 m/s at two different net mass fractions.

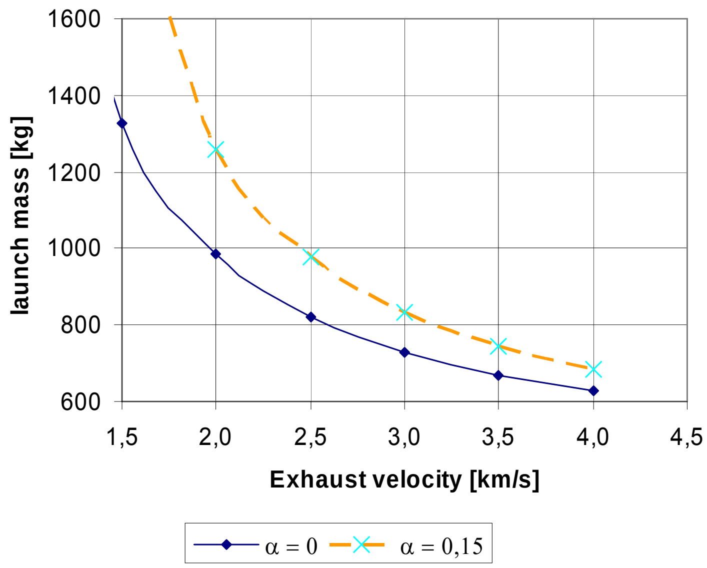


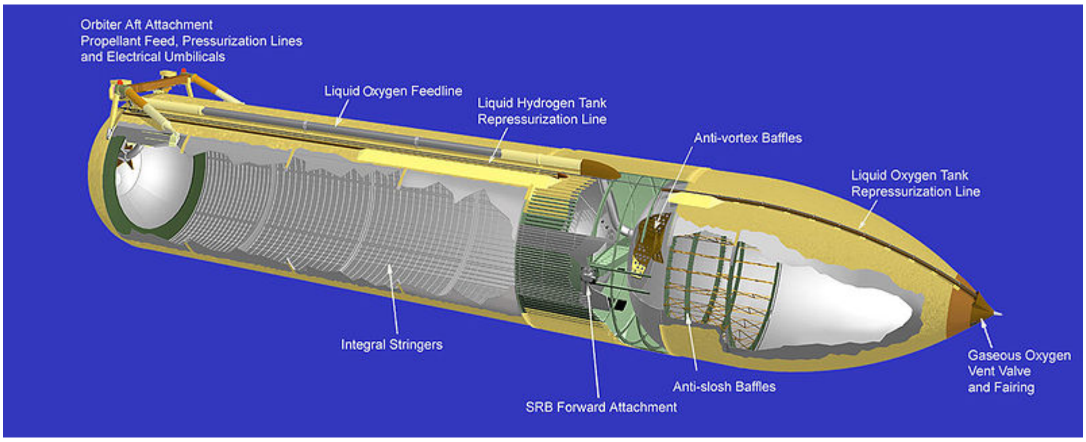
Figure 24: Vehicle (launch) mass versus exhaust velocity for two values of the net mass fraction (empty vehicle mass is 400 kg, mission characteristic velocity is 1800 m/s).

The figure shows that with increasing empty mass fraction (α) also the launch mass increases. This is mainly due to a higher propellant load. The mass of the propulsion system itself should be discounted from the empty vehicle mass. The figure also shows that we should still strive to maximize the exhaust velocity.

5.7. Example

The Space Shuttle Transportation System (SSTS) incorporates 3 Space Shuttle Main Engines (SSME) using liquid hydrogen (LH_2) as the fuel and liquid oxygen (LOX) as the oxidizer in the ratio of 1:6. The heating value of LH_2 is 120MJ/kg. The thrust produced by each engine in the near vacuum conditions of the upper atmosphere is 2.28MN at a specific impulse of 454 seconds. The typical burn time for the engines is 8.5 minutes.

Determine/calculate:
(a) the effective exhaust velocity of the engine,
(b) the mass flow rate of fuel and of oxidizer for the engine,
(c) the total mass of the fuel and oxidizer required for the cluster of 3 engines,
(d) the volume for the fuel and oxidizer tanks,
(e) the length of the fuel and oxidizer tanks assuming they are cylinders equal to the diameter, 8.4m, of that of the 46.9m long Space Shuttle External Tank,
(f) the overall efficiency η_o at a flight speed of 1.83km/s.



(a) The effective exhaust velocity of the engine is calculated by multiplying the specific impulse times the gravitational acceleration. We find a value of 454s x 9.81 m/s^2 = 4454 m/s.
(b) The mass flow rate of fuel and of oxidizer for the engine follows from the overall mass flow rate and the given mass mixture ratio. Overall mass flow rate per engine is engine thrust divided by the effective exhaust velocity = 2.28 MN/4454 m/s = 511.9 kg/s. Given the oxidizer to fuel mass ratio of 6:1, we find per engine a fuel mass flow rate of 73.1 kg/s and an oxidizer mass flow rate of 438.8 kg/s.
(c) The total mass of the fuel and oxidizer required for the cluster of 3 engines given the typical burn time of 8.5 minutes is 111.8 ton of hydrogen and 671.4 ton of oxygen.
(d) The volume for the fuel and oxidizer tanks is calculated using the previously calculated values of hydrogen mass and oxidizer mass and the mass densities of these propellants as given in **Error! Not a valid bookmark self-reference.** provides some additional information including the chemical formula, molecular weight (a dimensionless parameter representing the weight of a molecule; molar mass typically is equal to the molecular weight expressed in gram/mol), mass density and melting and boiling point.

The latter two are important to consider whether the propellants are storable or that additional measures are needed to allow proper storage. For instance:

- *Storable* propellants can be stored as liquids under normal environmental temperatures
- *Cryogenic* propellants need to be cooled extensively to allow for storage in the liquid state

Table 9. *If follows a mass density of liquid oxygen of 1140 kg/m^3 and for liquid hydrogen 71 kg/m^3 This then gives an oxygen volume of 671.4 ton/1140 kg/m^3 = 588.6 m^3 and an hydrogen volume of 111.8 ton/71 kg/m^3 = 1574.6 m^3 or about 2.5 times the oxygen volume.*

(e) The length of the fuel and oxidizer tanks assuming they are cylinders equal to the diameter, 8.4m, of that of the 46.9m long Space Shuttle External Tank. To calculate the length of the tanks, we first add 10% to the propellant volume as to allow for propellant expansion with increasing propellant temperature. Hence it follows a volume of 1732.1 m^3 for the hydrogen. Given the diameter of 8.4 m and hence a cross-sectional area of the tank of 55.4 m^2 (assuming thickness of the tank is negligible), it follows a tank height of 1732.1/55.4 = 31.3 m. For the oxygen follows: 1.1 x 588.6/55.4 = 11.7 m. This gives a total length of 43 m, which compares reasonably well with the in the literature reported value of 46.9 m especially when considering that the oxygen tank has an oddly shaped forward tank head.

(f) The overall efficiency η_o at a flight speed of 1.83km/s. Thrust power per engine is 2.28 MN x 1830 m/s = 4.17 GW and for 3 engines 12.52 GW. The power added to the flow per engine is 73.1 kg/s x 120 MJ/kg = 8.8 GW and for the three engines together 26.3 GW. The overall efficiency at the flight velocity of 1.83 km/s then is 12.52 GW/26.3 GW = 47.5%.

6. Advanced concepts

In rocketry, chemical rockets are the most widely used type of rocket. Unfortunately, the specific impulse of chemical rockets is limited to maximum about 450 s. The reason of this limitation for chemical rocket motors is that there is only limited energy available of up to ~42 MJ/kg for the most powerful chemical propellants. For some applications, this limitation leads to a very high propellant mass sometimes making the application completely unfeasible.

In this section some "advanced" concepts are being discussed that allow for higher specific impulse to be accomplished. These advanced concepts differ from chemical rockets in that they use a separate power supply, thereby allowing for a higher power input per kg of propellant and/or use other ways of accelerating the propellant. Hereafter, we first discuss non-chemical thermal rocket. Secondly we discuss electric rockets. Finally and thirdly we discuss the effect of a separate power-plant on the total mass of the rocket.

6.1 Non-chemical thermal rockets

Non-chemical or advanced thermal rockets are rockets that use other means than chemical combustion to heat the propellants. This is shown in the next figure.

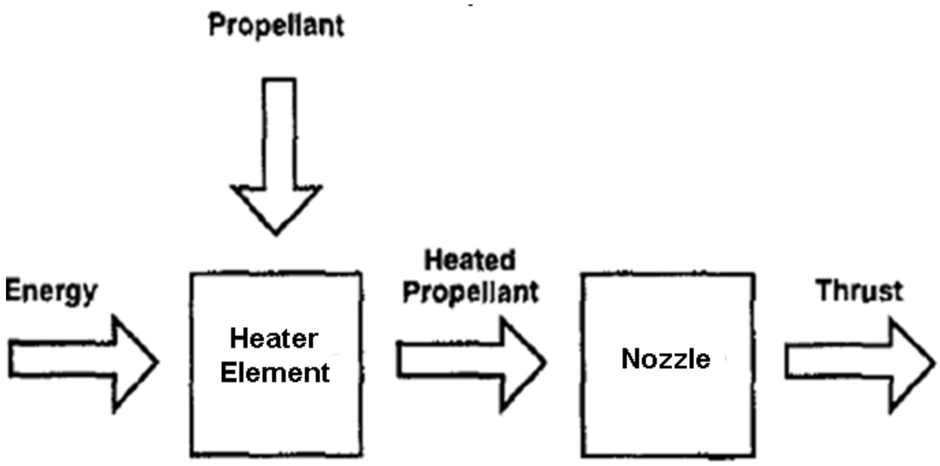


Figure 25: Schematic of non-chemical thermal rocket

The largest difference as compared to chemical rocket motors is the replacement of the chemical reaction (or combustion) chamber by a heater chamber equipped with some heater element. Another difference is in that these advanced thermal rockets require a separate power source to provide the necessary thermal power (heat) for heating the propellant.

The main advantage of advanced thermal rockets is that they are capable of achieving high specific impulse levels of up to about 1200 s, which is about three times higher then with the best performing chemical rocket. Another advantage is that also waste products, like water and carbon-dioxide can be used as propellant. It is for this reason that thermal thrusters have been investigated for use on the International Space Station. A major disadvantage is that to provide for the necessary thermal power (heat) a separate power source is needed, which adds to the mass of the system.

In practice, different thermal rockets exist of which the most important are:

- electro-thermal,
- solar/laser-thermal, and
- nuclear-thermal rockets.

In the next few sections, we will discuss the above mentioned types of advanced thermal rockets in some detail, but first we will discuss the principal relations governing the operation of such rockets.

6.1.1. Principles of operation

Non-chemical or advanced thermal rockets essentially operate based on the same principal as chemical and cold gas rockets. Like for the chemical and cold gas rockets, the thermal expansion taking place in the nozzle can be described using the earlier presented "Ideal Rocket Motor" Theory (IRMT). From this theory, it follows (ideal or optimum expansion) that specific impulse or propellant consumption is proportional to the hot gas or chamber temperature and inversely proportional to the molar mass.

$$I_{sp} = \propto \sqrt{\frac{T}{M}} \qquad \textbf{(6.1-1)}$$

Hence, to further increase specific impulse, we should aim to increase the hot gas or chamber temperature and/or to decrease the molar mass of the propellants. Some typical results calculated using IRMT are given in Figure 26.

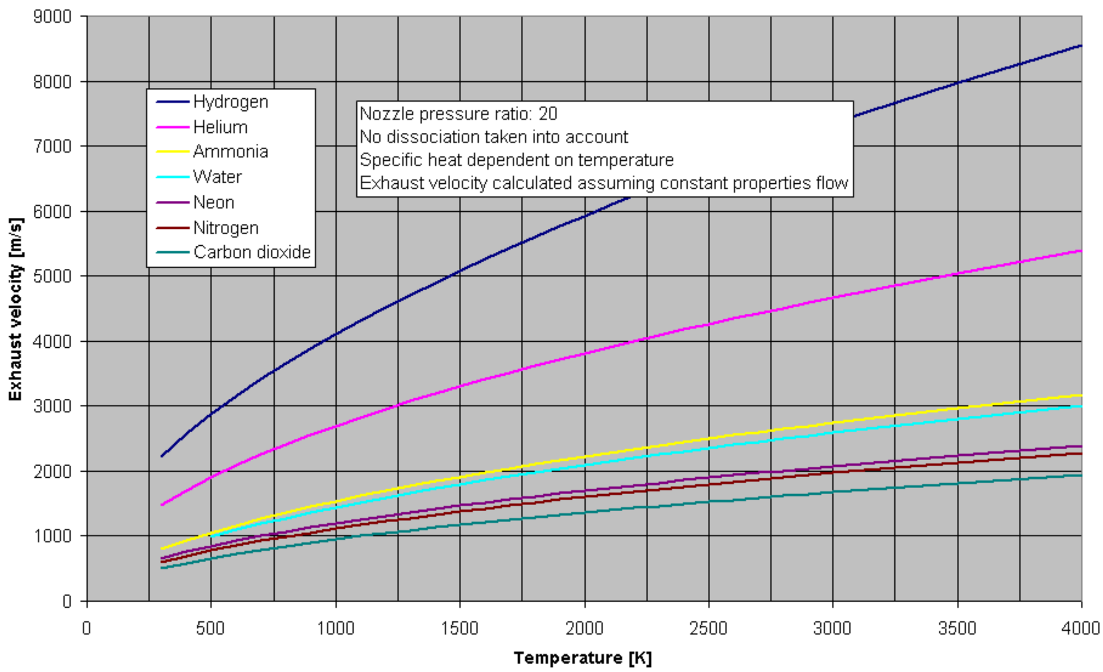


Figure 26: Theoretical exhaust velocities non-chemical thermal propellants (optimum expansion)

The figure clearly confirms that species with low molar mass have highest exhaust velocity. The figure also clearly shows that with increasing temperature the exhaust velocity increases. In reality, the results are somewhat less because of losses incurred.

In our discussion so far, the effect of the nozzle and more specific the nozzle area ratio has been neglected somewhat. However, when calculating the performance of thermal thrusters this is an important parameter to consider. For instance, the results given in the Figure 26 have been calculated a pressure drop over the nozzle of 20, which indicates a very moderate area ratio. When increasing the area ratio and hence the pressure drop over the nozzle, higher performances are feasible.

Propellant heating

From Figure 26 we learn that to attain a high exhaust velocity we need to heat the propellant to a high temperature. This requires a certain amount of heat.

From physics, we know that the amount of heat needed to increase the temperature of some specie depends on:

- the specie itself,
- how much of the specie is present,
- state of aggregation (solid, liquid,..),
- initial and final temperature, and
- for gases: conditions under which heating occurs (constant pressure or volume)

The total required heat can be considered to be the sum of the latent heat (heat associated with a state change) and the sensible heat (heat associated with a temperature change).

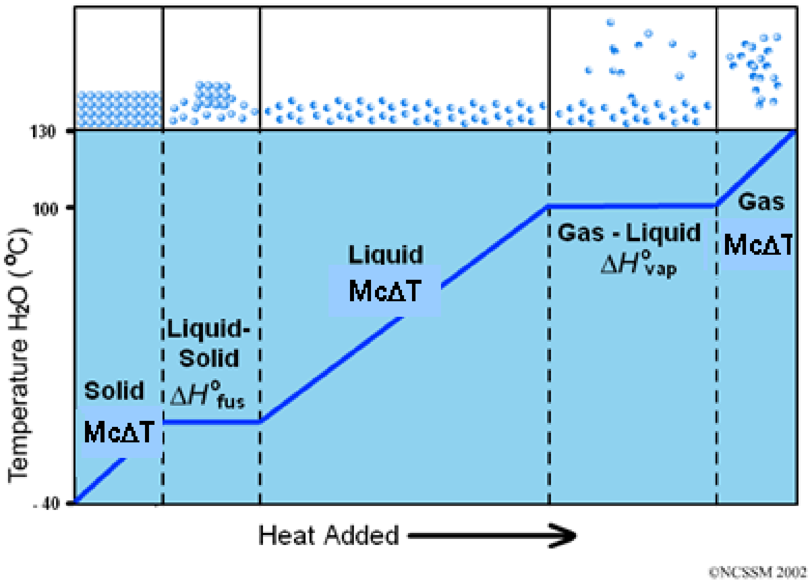


Figure 27: Effect of heat addition on water temperature and phase change

The sensible heat of may be calculated as the product of the fluid mass (M or in the above figure m) with its specific heat (c) and the change in temperature (ΔT):

$$Q_{sensible} = \sum_i M\, c_i\, \Delta T_i \qquad \textbf{(6.1-2)}$$

Here i refers to the different phases. The above relation applies in cases where the specific heat can be assumed independent of temperature. In reality, the heat capacity of most systems is not a constant. Rather, it depends on the state variables of the thermodynamic system under study. In particular it is dependent on temperature itself. Hence, the assumption of constant temperature may lead to large errors especially in the case of rockets wherein propellants are heated to a high temperature. This, however, is considered beyond the scope of this course.

When applying the above relation to the heating of gases, a distinction is necessary between heating at constant pressure and at constant volume. In rocket motors (open system) heating occurs at constant pressure and hence we should replace c by the specific heat capacity at constant pressure.

Latent heat may be calculated per phase change as the product of the fluid mass (M) and the specific latent heat (L).
*

$$Q_{latent} = M\,L \qquad \textbf{(6.1-3)}$$

Values of specific heat and specific latent heat for phase changes of some common fluids and gases can be obtained from e.g. http://en.wikipedia.org/wiki/Latent_heat and /or http://physics.info/heat-sensible/.

Typical units for specific heat capacity and latent heat are J/(kg-K) and J/kg, respectively. However, they may also be expressed relative to one mole, the unit for amount of substance. In that case we speak of the molar heat capacity and molar latent heat with units J/(mol-K) and J/mol. To convert from mol to kg, we simply divide the molar heat capacity and latent heat by the molar mass (M) of the specie considered, e.g. $c_p = C_p/M$ where C_p is molar heat capacity at constant pressure and c_p is specific heat at constant pressure.

Another important relation (from thermodynamics), valid in case the hot gas mixture in the chamber behave as an ideal gas, relates the specific heat at constant volume to the specific heat at constant pressure:

$$R_{specific} = c_p - c_v \qquad \textbf{(6.1-4)}$$

With R is the specific gas constant and c_v the specific heat at constant volume[7].

Heater efficiency

For a thermal thruster, power input to the thruster, whether this is electrical, nuclear or solar power should equal the sum of the power needed for heating the propellant (thermal power) and the power lost during heating. Consider for instance heat lost to the environment, for example, by radiation or conduction. A figure of merit for this power loss is the heater efficiency (also referred to as heating efficiency). It is defined as the thermal power divided by the total input power.

$$\eta_h = \frac{P_{heat}}{P_{in}} \qquad \textbf{(6.1-5)}$$

Typical values for the heating efficiency may be up to almost 100%, depending on the heating method applied.

Input power can be determined from jet power and cycle (or thrust) efficiency. Values of thrust/cycle efficiency are in the range of 0.3 - 0.8 depending on the thermal thruster type.

Example

From Figure 26 we learn that hydrogen allows for attaining a high exhaust velocity at a much more modest temperature than for other propellants. This is important for materials selection as most common materials used can not withstand temperatures in excess of a few hundred up to a thousand degrees centigrade. Still there might be other reasons that lead to the selection of another propellant than hydrogen.

To determine the power needed to heat the hydrogen to a high temperature, we are to consider the specific heat of hydrogen. Specific heat at constant pressure (why not at constant volume?) of hydrogen is 14.31 J/g/K (or 28.836 J/mol/K, molar mass of hydrogen is ~2 gram/mol). Taking (298K (about room temperature) as the initial temperature we find that to heat 1 kg of hydrogen from 298 to 2000 K (allowing us to reach an exhaust velocity of about 6000 m/s) we need 24.4 MJ (1 kg x 14.31 kJ/kg/K x (2000K – 298K). In case of a total mass

[7] Specific heat at constant volume is more easily measured than specific heat at constant pressure.

flow rate of 1 gram/second, i.e. we heat every second 1 gram of hydrogen, it follows a required heating power of 1 gram/s x 24.4 MJ = 24.4 kW. Hence we need 24.4 kW of power to produce a thrust of 1 g/s x 6000 m/s = 6 N.

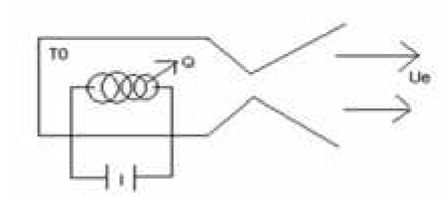
For most applications, the hydrogen is stored in the liquid state (as to limit the storage volume) at a temperature just below 20 K. In that case, we also need to take into account the heat needed to heat up the hydrogen from 20 K to 298 K as well as the heat of vaporization of hydrogen, compare Figure 27. Using a specific latent heat of hydrogen of 461 kJ/kg[8] it follows a heating power of 24.4 kW + 1 gram/s x 461 J/g + 1 gram/s x 14.31 J/g/K x (298-20) = 28.8 kW. Taking into account a heater efficiency of 80% gives a thrust input power of 28.8/0.8 = 36 kW.

6.1.2. Electro-thermal rockets

Electro-thermal propulsion involves electrically heating the working fluid and allowing the resultant hot exhaust gases to expand through a nozzle, similar to a conventional rocket. Two types of systems are distinguished, being resistojet and arcjet.

<u>Resistojet</u>

In a resistojet, see schematic, the propellant is heated by an electric heater.



This heater may be a contact heater (in contact with the flow) or a radiation heater. In the latter case heat is transferred by radiation, whereas in the former case this is by convection[9]. How the heat is transferred, determines to a great extent the design of the resistojet and its performances. Below we will present a simple model (steady state conditions only) for convective heat transfer illustrating the principle workings. Radiation heat transfer will not be dealt with as this is considered beyond the scope of this course.

Through convection heat (energy) is transferred from the heater to the propellant. For an ideal heater the energy flowing to the propellant is constant along the heater. The flow of power to the propellant can than be expressed as the product of the convective heat flux q (heat flow (in J/s) per unit of area typically expressed in W/m^2) and the contact area of the heater, i.e. the area of the heater in contact with the fluid.

$$P_{heat} = q_{convection}\ A_{contact\ surface} \qquad \textbf{(6.1-6)}$$

Here $q_{convection}$ is considered an average value for the heater. In reality this is not always the case, but this is considered beyond the scope of this lecture.

The convective heat flux can now be written as the product of the convection heat transfer coefficient h_c (some constant of proportionality, in W/(m^2-K)) and a representative temperature difference between flow and heater ΔT (in K):

$$q_{convection} = h_c\ \Delta T \qquad \textbf{(6.1-7)}$$

Note that for reasons of simplicity here the coefficient of convective heat transfer is considered an average value that holds for the heater as a whole. The temperature difference

[8] See for instance http://www.engineeringtoolbox.com/fluids-evaporation-latent-heat-d_147.html
[9] Convection heat transfer is heat transfer by fluid convection, i.e. a cold fluid flowing along a heater which thereby carries away heat from the heater.

is taken as the difference between the heater temperature and the average propellant temperature between in- and outlet of the thruster.

The *convection heat transfer coefficient* depends on the type of fluid, its temperature properties, and the specific flow properties (laminar or turbulent flow, etc.). Typical coefficients of convective heat transfer, in W/(m^2.K), are [Bejan]:
- Gases, 1 atmosphere, forced convection: 10-200
- Gases, 200 atmosphere, forced convection: 200-1000
- Organic liquids (like kerosene), forced convection: 100-1000
- Water (forced flow): 580-2300
- Boiling water: 11600

For resistojets, chamber temperature is necessarily limited by the materials of the walls and/or heater coils to some 3000K or less, and hence the exhaust velocities, even with hydrogen, cannot exceed 10,000 m/sec, which is nonetheless a factor of two or three beyond that of the best chemical rockets.

Ideally, the electric input power for the heater equals the thermal power needed to heat up the gas. Hence from the known thermal power the (electric) input power of the thruster can be determined. In turn, the electric input power can be related to the electric characteristics of the thruster using:

$$P_{electric} = V\,I = I^2 R \tag{6.1-8}$$

Here V and I are heater input voltage (V) and current (I), respectively. R is electrical resistance of the heater, which depends on heater geometry and material:

$$R = \rho \frac{L}{A} \tag{6.1-9}$$

With:
- ρ is electrical resistivity of heater material (typically in Ωm). Some typical values are given in Table 11
- L is length of heater
- A is cross-sectional area of heater

Table 11: Electric resistivity for some materials (@293 K)

	Stainless steel	Copper	Platinum	Nickel	Tungsten	Aluminum	Brass
Electrical resistivity [10^{-8} Ωm]	76.5	1.68	10.72	8.54	5.51	2.83	6.2

Once heater geometry and material have been selected, heater resistance is known and input current and voltage can be determined.

Many resistojet configurations have been conceived and developed, some to the point of routine space flight operation. Propellant gases used for resistojets include ammonia, bio-wastes, hydrazine, and hydrogen. Heating methods used include flow over wire coils, flow through hollow tubes and flow over heated cylinder or plate. Material limits attainable temperatures to less than about 2700 K, which than also limits attainable specific impulse. Structural materials used include rhenium, molybdenum, tungsten and platinum.

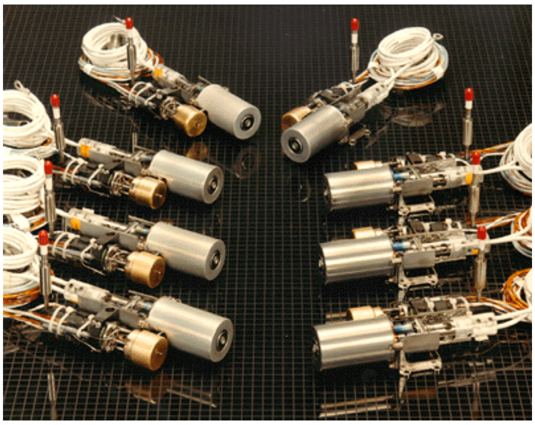


Figure 28: Resistojets (courtesy Primex)

Hydrazine as propellant is used in what is called a power augmented hydrazine thruster because the energy added by the resistojet is augmenting that obtained by the catalytic decomposition of the hydrazine. Some typical augmented hydrazine resistojets are shown in Figure 28. The resistojets depicted deliver a thrust of 0.18 to 0.36 N and a specific impulse of 280 to 304 s while requiring input power of 300 to 500 Watts. Total impulse delivered is 3.1 x 10^5 N-s, indicating a total life of minimum 239 hours or 10 days (0.36 N thrust) or double this amount if the thrust is halved.

Example: For the MBB-ERNO (know EADS) multi-propellant resistojet the following data are known [IRS]:

- *propellant: hydrogen, helium, methane, nitrogen, air, argon, and carbon dioxide*
- *thrust: 0.3 N (with CO_2 propellant)*
- *heater element power : 140-520 W*
- *propellant temperature at thruster inlet : 293K*
- *maximum hot gas temperature: 1700 K*
- *operational pressure: 1.3-2.8 bar*
- *operating voltage : 10-20 V*
- *operating current 18-23 A*
- *accomplished lifetime : 10000 h (or about 400 days of continuous operation)*

The maximum hot gas temperature of 1700K is because of design (material) limitations. Selecting nitrogen as propellant, we find that an exhaust velocity of about 1500 m/s should be feasible, see figure in text.

Total power level of this thruster using nitrogen is estimated at 350W (data indicates 140-520 W, so for now some intermediate value is selected). Assuming the thruster operates at 20V, we obtain an operating current of 17.5A and a heater resistance of 20V/17.5A = 1.14Ω..This is the resistance the heater should be designed for. This power may be provided for by for instance a photovoltaic array[10] with a power density of 100W/m^2 and a specific power of 25W/kg. It then follows a required array area of (at least) 3.5m^2, and an array mass of 350W/25W/kg = 14kg.

To compute the mass flow rate of nitrogen that can be sustained by the thruster, we first determine the heat needed to heat 1kg of nitrogen gas from 293K to 1700K. Specific heat of nitrogen is roughly 1J/g/K. Hence to heat 1g of nitrogen to 1700K we need about 1400J. Next, in case we assume a heater efficiency of 80% we find that 280W is left for heating of the propellant. So we find that the thruster can support a mass flow rate of 280W/1400J/g = 0.2g/s.

Using the earlier estimated exhaust velocity, it follows a thrust of 0.3N and a beam power of 225W. From the beam power it follows that not all heat transferred to the propellant is used to accelerate the propellant (some heat remains in the jet).

[10] See course ae1201.

The propellant is to be heated using a heater element with an electric resistance of 1.14Ω. This heating may be accomplished via convection (or by radiation). For a coefficient of convective heat transfer of 200W/m²/K and an average temperature difference of 1000K (gas temperature at thruster inlet is 298K and at outlet 1700K) we find a convective heat flux of 200kW/m². Hence, to transfer 280W of heat to the fluid requires a contact area of 280W/200kW/m² = 14cm².Hence, the heater should have a surface area in contact with the propellant of 14cm².

Finally, we calculate the diameter of the nozzle throat of the thruster using ideal rocket theory. For nitrogen at 1700K and with a specific heat ratio of 1.4, follows a characteristic velocity (see Fortescue for appropriate relation) of 1037.6m/s. Using the given operational pressure range and assuming an operational pressure of 2 bar gives a throat area $A_t = m\ c^*/p_c = 0.22g/s \times 1037.6/2E5 = 1.1E\text{-}6m^2$. *Assuming a nozzle of circular cross-section (this is the case in say 99% of the cases) gives a nozzle throat diameter of about 1.2mm.*

Selecting another propellant will lead to differences in exhaust velocity, mass flow rate, thrust and power consumption. This is left for the reader to explore him/herself. Note though that nozzle throat diameter, heater surface area, resistance, and heater temperature remain fixed as they are fixed once the design is fixed.

Arcjet
In an arcjet, the propellant is heated by leading the propellant through a high-voltage arc discharge generated in the thruster, see schematic.

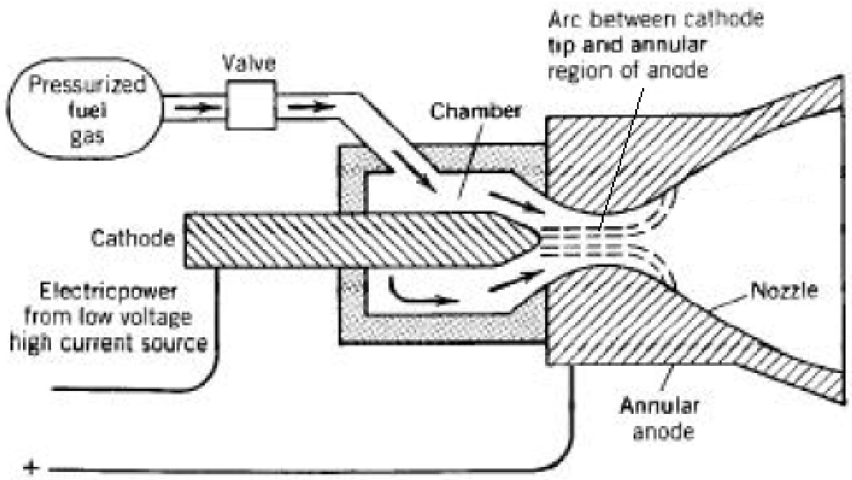


Figure 29: Schematic of arcjet

The figure shows propellant being fed into the thruster. The thruster mainly consists of two electrodes (anode and cathode)[11] between which an arc discharge is created. The propellant gas flowing through the arc discharge[12] is heated to extreme temperatures. For some thrusters temperatures of up to 10,000 deg C have been reported in the core. Fortunately temperatures close to the thruster wall are substantially less. The arcjet uses a nozzle to accelerate the hot gases. In most designs, the nozzle also serves as the anode.

The advantage as compared to the resistojet is that much higher gas temperatures can be attained, resulting in much higher exhaust velocities. A problem though is that because of the high temperatures, electrode erosion is a serious problem, severely limiting thruster life.

[11] The positively charged electrode is the anode and the negatively charged electrode the cathode.
[12] An electric arc is an electrical breakdown of a gas which produces an ongoing plasma discharge, resulting from a current flowing through normally nonconductive media.

For the arcjet, like for the resistojet, the electrical input power is given by the product of input voltage (V) and current (I). In the case of the arcjet, though, I is the discharge current running through the arc and V is the discharge voltage.

Example: For the German ATOS arcjet thruster the following data are known [ATOS]:

- *nominal power to the arcjet : 750 W*
- *propellant: Ammonia*
- *arc current : 7.7 A*
- *arc voltage : 97 V*
- *mass flow : 24 mg/s*
- *thrust : 115 mN*
- *specific impulse : 480 s*
- *accomplished lifetime : 1010 h (or 42 days of continuous operation)*
- *including 1010 ignitions*
- *thruster weight : 480 g*

From this data we learn that the arc voltage is 97 V and the arc current 7.7A. Neglecting losses, this indicates a minimum electrical input power of 747 W.

Beam power is 0.5 x 115 mN x 480 s x 9.81 m/s^2 = 271 W, indicating a thrust efficiency of maximum 36% (271 W/750 W); A typical value for arcjet thrusters. The large loss of power is associated with the amount of heat deposited into the electrodes (roughly 20%), heat radiated to the chamber walls, and a number of other processes taking place.

Based on the given (vacuum) specific impulse, we obtain an effective exhaust velocity of 4802 x 9.81 m/s^2 = 4709 m/s. Assuming that the nozzle is only 80% efficient, we determine a limit velocity of 4709 m/s / 0.8 = 5886 m/s.

Given a specific heat ratio and molar mass of the ammonia of 1.22 and 8.4 gram/mol[13], we find using the relation for the limit velocity that and assuming that the nozzle contribution is 80% of the limit exhaust velocity, we find that the ammonia should be heated to ~4000 K (a reasonably high temperature).

Verify: Limit exhaust velocity $= \sqrt{\frac{2\gamma}{\gamma-1}\frac{R_A}{M}T_c} = \sqrt{\frac{(2)(1.30)}{1.30-1}\frac{8314.32}{8.4}4000} = 5858 \text{ m/s}$

Using a specific heat at constant pressure for the ammonia of 6.9 kJ/kg/K[14], we find for the heating power needed to heat the ammonia propellant from 298 K to 4000 K:

$$m\, c_p\, \Delta T = 24E\text{-}6 \text{ kg/s x } 6900 \text{ J/kg-K x } (4000\text{-}298) = 613 \text{ W.}$$

In case we take into account the heat of vaporization for ammonia, the required power will further increase, but still quite some power (750-613 = 137 W) is lost, for instance by radiation, to the environment.

6.1.3. Solar-thermal or laser-thermal rockets

Source Wikipedia: **Solar thermal propulsion** is a form of rocket propulsion that makes use of solar power to directly heat the propellant. A system schematic of a solar thermal propelled rocket is given in Figure 30.

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[13] Values determined assuming chemical equilibrium in the arc.
[14] Notice that this value does not agree with the assumption of an ideal gas ($R = c_p - c_v$). This is because of the high gas temperatures occurring.

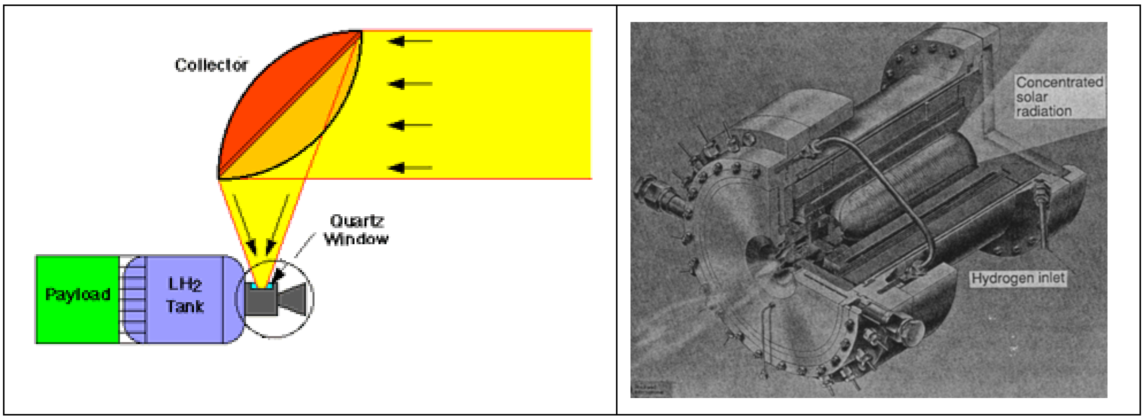


Figure 30: Solar-thermal rocket schematic (picture on right courtesy Boeing)

The Rocketdyne heat exchanger thruster shown in the figure on the right has been tested at Rocketdyne (Boeing). Using hydrogen propellant at a temperature of 2700 K, it has produced a thrust of 3.7 N and an exhaust velocity of 7900 m/s.

Solar (or laser) light is collected by a collector system (consisting of mirrors and/or lenses) and concentrated into a heat accumulator. The heat accumulated is then transferred to the propellant which subsequently is expanded to a high exhaust velocity jet in a nozzle. The latter process is identical to that for a chemical rocket engine.

Most proposed designs for solar thermal rockets use hydrogen as their propellant due to its low molecular weight which gives excellent specific impulse of 900 seconds (9 kN·s/kg). Conventional thought has been that hydrogen—although it gives excellent specific impulse—is not space storable. Recent design work has developed an approach to substantially reduce hydrogen boil-off, and to economically utilize the small remaining boil-off product for other tasks. As an alternative, Ammonia can be considered. It offers lower specific impulse than hydrogen, but is easily storable, with a boiling point of −77 degrees Celsius. The exhaust dissociates into hydrogen and nitrogen, leading to a lower average molecular weight, and thus a higher I_{sp}. On the other hand, ammonia is also considered very toxic and hence may require special precautions when using it.

Figure 31 provides a detailed view of a solar thermal thruster with the nozzle visible on the left hand side of the figure.

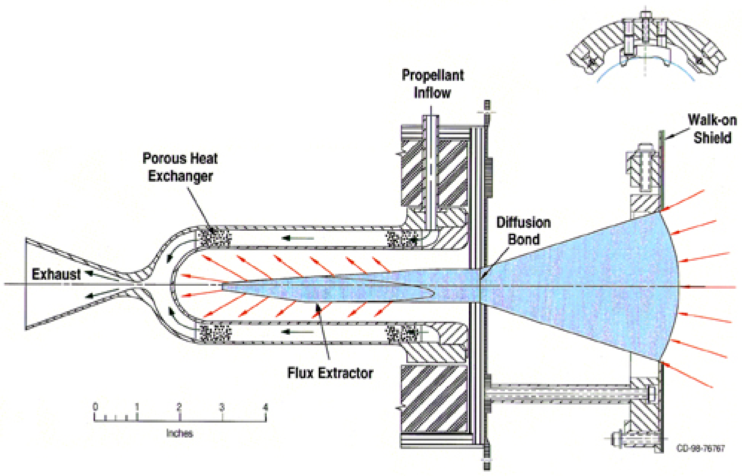


Figure 31: Schematic of solar-thermal engine (courtesy Boeing)

The schematic shows that the engine essentially is a long cylinder (the receiver or absorber cavity) with a 'window' on one end and a hemispherical cap at the other. The window may be either open or consist of a lens that acts as a secondary concentrator. The long cylindrical construction of the cavity allows most of the radiation that enters the engine to be used for heating the propellant. The latter enters the engine, flows through a foam-filled annulus around the receiver cavity, is heated to a high temperature, and expands through the nozzle.
Like for all thermal thrusters the materials' properties limit the achievable temperature of the working fluid. Typical construction materials for the engine include refractory metals like molybdenum, rhenium or tungsten. The next table shows characteristic data of two solar-thermal thrusters currently under development in Japan.

Table 12: Characteristics of selected solar thermal engines [Sanara]

Engine	#1	#2
RAC material	Molybdenum	Tungsten
Outer diameter	65 mm	8 mm
Propellant	nitrogen, helium	nitrogen
Designed propellant temperature	over 3,200 K	over 3,200 K
Specific impulse (nitrogen propellant)	750 s	750 s
Maximum chamber pressure	2 bar	2 bar

Example: Using ideal rocket motor theory and given a molar mass and specific heat ratio of hydrogen of 2 g/mol and 1.4, respectively, we find a limit velocity of 9650 m/s (@ 3200 K). This should be compared to the effective exhaust velocity of 750 s x 9.81 m/s^2 = 7357 m/s as reported for the thruster. The difference is attributed to a finite pressure drop over the nozzle and losses occurring. From ideal rocket theory it follows that adding a nozzle with a pressure drop of a factor 250 gives a true exhaust velocity of 0.79 x 9650 m/s = 7657 m/s (verify!!).

The solar power collected by the collector depends on the solar intensity S (in W/m^2) and the collector area perpendicular to the incoming solar radiation:

$$P_{solar} = S\,A_{collector} \tag{6.1-10}$$

Example continued: The above thruster receives power from a collector with a frontal diameter of 10 m. Given a solar intensity of 684 W/m^2 (average intensity at Earth surface) we find a collected power of 53.7 kW. Assuming a heating efficiency of 90%, it follows a thermal power of ~48.3 kW. Next applying a cycle efficiency of 80%, we find that the jet power is 38.7 kW. It now follows for the thrust that can be delivered by this engine a value of 2 x 38.7kW/(750 s x 9.81 m/s^2) of 10.5 N (verify!!!)

When increasing the surface area of the solar collector more power is fed to the thruster and either we can reach a higher temperature (constant mass flow rate). As a result, the exhaust velocity will increase and hence the thrust.

6.1.4. Thermo nuclear or nuclear thermal rockets

From Wikipedia, we learn that in a **nuclear thermal rocket** a working fluid, usually liquid hydrogen, is heated to a high temperature in a nuclear reactor, and then expanded through a rocket nozzle to create thrust. The nuclear reactor's energy replaces the chemical energy of the reactive chemicals in a chemical rocket engine. Due to the higher energy density of the nuclear fuel compared to chemical fuels, about 10^7 times, the resulting propellant efficiency of the engine is at least twice as good as chemical engines, leading to specific impulse values of up to 900 s.

A schematic of a nuclear system using water as propellant is shown below.

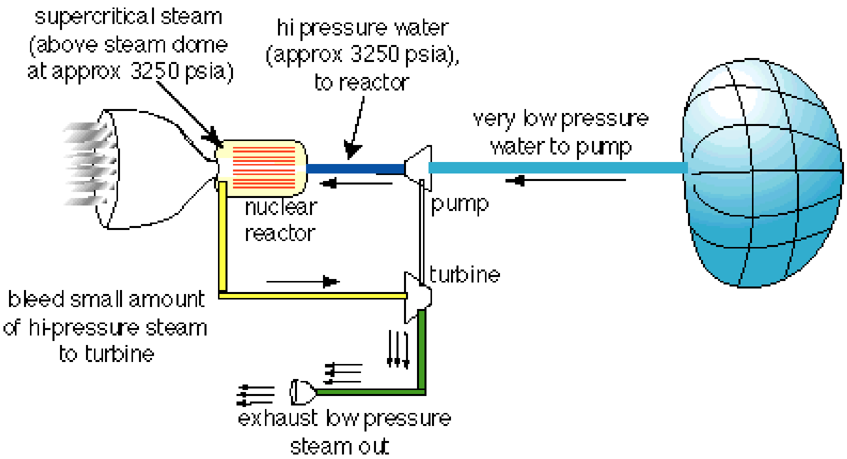


Figure 32: Schematic of a thermo-nuclear rocket

Figure 33: RD-0410

It compares well with a chemical monopropellant rocket system except that the chemical reactor is replaced by a nuclear reactor. The heating of the working fluid in the thermo-nuclear engine is either via solid heat exchanger (solid core reactor) or via particulate absorption (nuclear gas core reactor). The latter uses hot particulate seedants that travel with the working fluid to generate a hot gas. Typical propellant temperatures are 2000-3000K for solid heat exchanger concepts and 3800-5850K for nuclear gas core concepts. In the schematic, we also see very clearly the propellant feed and storage system here consisting of a water tank, a pump that feeds the water to the reactor. The pump in turn is driven by a turbine, hence the term turbo-pump feed system. The turbine drive gases, a small amount of hi-pressure steam, are exhausted separately (open cycle, see Fortescue for more details).

Typical examples of solid core nuclear-thermal rocket engines are the Nerva system of the United States of America (USA) and the RD-0410 of Russia, see Figure 33. Both systems use liquid hydrogen as the working fluid. Specific characteristics of the Nerva system and the RD-0410 are given in Table 13.

From Wikipedia we obtain that to date, no nuclear thermal rocket has flown, although several engines have been built and tested. The United States tested twenty different sizes and designs during Project Rover and NASA's NERVA program from 1959 through 1972 at the Nevada Test Site, designated Kiwi, Phoebus, NRX/EST, NRX/XE, Pewee, Pewee 2 and the Nuclear Furnace, with progressively higher power densities culminating in the Pewee (1970) and Pewee 2. Tests of the improved Pewee 2 design were cancelled in 1970 in favor of the lower-cost Nuclear Furnace (NF-1), and the U.S. nuclear rocket program officially ended in spring of 1973. Research into nuclear rockets has continued quietly since that time within NASA. Current (2010) 25,000 pound-thrust (111 kN) reference designs (NERVA-Derivative Rockets, or NDRs) are based on the Pewee, and have specific impulses of 925 seconds. In Russia (then the Soviet Union) the RD-0410 has gone through a series of tests at the nuclear test site near Semipalatinsk, but further information is sparse.

Table 13: Selected nuclear thermal rocket performances [Wade], [Humble]

	RD-0410	Nerva 2
Country	Russia	US
Propellant	LH2	LH2
Vacuum thrust [kN]	35,3	334,061
Specific impulse [s]	910	825
Nozzle expansion ratio [-]	Not Available	100
Chamber pressure [bar]	Not Available	31
Maximum propellant temperature [K]	Not Available	2361
Burn time [s]	3600	1200
Diameter [m]	1,6	Not Available
Length [m]	3,5	Not Available
Mass [kg]	2000	10138
Shield mass [kg]	Not Available	1590
Thermal power [MW]	Not Available	1570
Operational use	Experimental engine	Flight engine
Ref.	M. Wade	Humble et al

Example: For the Russian RD-0410 engine are given a vacuum thrust of 35.3 kN and a vacuum specific impulse of 910 s, see foregoing table. Assuming a hot propellant temperature of 3000 K and a thrust efficiency of 88%[15] we obtain the following data from calculations:

a) *Effective exhaust velocity (in vacuum):910 s x 9.81 m/s^2 = 8927.1 m/s*
b) *Jet power is equal to: P_j = 1/2 mw^2 = 1/2 F w = (0.5)(35000)(9000)=157 MW*
c) *Input power $P_{in} = P_j / \eta$ = 157.5/0.88 = 179 MW*
d) *The specific power of the system can be found by dividing the input power by the propulsion system mass: 179 MW / 2000 kg ≅ 90 kW/kg.*
e) *Nozzle area ratio*
The area ratio can be determined using the equation for the exhaust velocity from ideal rocket theory. This will result in a pressure ratio, which can be directly linked to a nozzle area ratio using the graphical relationship provided in the syllabus. We find:

$$v_e = \sqrt{\frac{2\gamma}{\gamma-1}\frac{R_A T}{M}\left(1-\left(\frac{p_e}{p_c}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\right)} \Rightarrow 9000 = \sqrt{\frac{2\cdot 1.4}{1.4-1}\frac{8314\cdot 3000}{2}\left(1-\left(\frac{p_e}{p_c}\right)^{\left(\frac{1.4-1}{1.4}\right)}\right)}$$

Solving this equation yields a value for the pressure ratio of 0.0001. Using the relation that exists between pressure ratio and area ratio, see ideal rocket theory, we calculate a value for the nozzle area ratio of about 140.

6.2 Electrostatic and magneto-plasma-dynamic rockets

Electrostatic and magneto-plasma-dynamic (MPD) rockets, like electro-thermal rockets, belong to the class of electrical rockets. This is a class of rockets that derive the energy needed for thrust generation from an electric power supply. Electrostatic and MPD rockets differ from electro-thermal rockets in that the propellant are not accelerated by a thermal expansion process, but by electric forces (electrostatic rockets) or by a combination of electric and magnetic forces (MPD rockets). Both types of rockets are characterised by very high values of specific impulse up from 1000 s. A disadvantage of such systems is that they require

[15] The thrust efficiency has been chosen comparable to that of the Nerva 2. Later some problems are provided for exercising upon wherein the calculation of the thrust efficiency of the Nerva 2 is just one of the problems to be solved. The reason that a comparable value has been chosen is that the two engines use the same principle to generate thrust.

very high (electrical) power in the range of kilowatts and that they have to carry on board a power supply that provides for this power.

Example: The jet power of an electrostatic rocket producing a thrust of 1 N at a specific impulse of 2000 s is 9.8 kW. Given a cycle efficiency of 50%, this than requires an input power (output power from the electrical power source) of 19.62 kW_e (the subscript "e" denotes that we are dealing with electrical power in stead of thermal power as in the preceding sections). Assuming this power is delivered by a solar array producing 100 W_e/kg (see ae1201), this leads to a mass of the solar array of about 200 kg (196.2 kg).

In this course, the above two types of electric rockets are discussed in more detail, thereby providing insight in the parameters that determine exhaust velocity, thrust and the input power needed.

6.2.1. Electrostatic or ion rockets

In an ion rocket neutral propellant is converted to ions and electrons and withdrawn in separate[16] streams. The ions pass through a strong electrostatic field produced between acceleration electrodes and are accelerated to high speeds, and the thrust of the rocket is in reaction to the ion acceleration.

Ion velocity

Coulomb's law - The fundamental equation of electrostatics is Coulomb's law, which describes the force between two point charges. From physics, we learn that the magnitude of the electrostatic force between two point electric charges (denoted Q and q) is directly proportional to the product of the magnitudes of each charge and inversely proportional to the square of the distance r between the charges (compare gravity force):

$$F = \frac{Qq}{4\pi\varepsilon_o r^2} \qquad \textbf{(6.2-1)}$$

Here ε_o is a constant called the permittivity of free space equal to about 8.854E-12 $A^2 s^4 kg^{-1} m^{-3}$.

From Physics we also know that the force exerted by a charge Q on every other charge q can be written as F = E q, where E is the electric field intensity or just electric field of the charge Q.

The intensity of the field experienced by a charge q depends of course on the charge Q (i.e. the source charge), the distance squared and the charge q (the test charge). For parallel plates with a potential difference of ΔV, the electric field is given by:

$$E = \frac{\Delta V}{d} = \frac{\sigma}{\varepsilon_o} \qquad \textbf{(6.2-2)}$$

Where d is the distance between the plates and σ is the surface charge density C/m^2.

From Physics, we furthermore know that a charge q moving between two oppositely charged plates with a voltage difference ΔV gains a certain amount of energy. The amount of energy gained is equal to the charge of the particle times the voltage difference between the two plates, E = QV. This energy is converted entirely into kinetic energy of the charged particle:

$$q\ \Delta V = 1/2\ m_{\text{charged particle}} \left(v_{\text{charged particle}}\right)^2 \qquad \textbf{(6.2-3)}$$

[16] In some other electric rockets the electrons and ions are withdrawn together as plasma.

From this relation we can determine the velocity of a charged particle that passes through a voltage difference ΔV:

$$v_{\text{charged particle}} = \sqrt{\frac{2q\,\Delta V}{m_{\text{charged particle}}}} \qquad \textbf{(6.2-4)}$$

For an ion rocket, the voltage difference is usually one of the control variables. The mass of the ions depends on the propellant specie selected and the ionic charge. Typical properties of some candidate propellants are given in the Table 14.

Table 14: Properties of candidate propellants for ion rockets

Property	Lithium	Argon	Krypton	Xenon	Cesium	Mercury
Atomic weight	6.941	40	83.8	131.3	132.91	200.5
Density [kg/m^3]	530	1.8	3.7	5.9	1930	13500
1st ionisation potential [eV]	5.39	15.7	13.9	12.1	3.89	10.4
2nd ionisation potential [eV]		27.6	24.3	21.2		18.5

In more detail, the mass of the ion can be determined from the molar mass $\hat{M}$:

$$m_{\text{charged particle}} = \hat{M}/N_A \qquad \textbf{(6.2-5)}$$

Here N_A is Avogadro's number and the molar mass is taken equal to the atomic weight (in gram/mol). Note that here we neglect the electron mass.
The ionic charge depends on the degree of ionization achieved and is usually expressed using the elementary charge e (e = 1.6 x 10-19 C); A charge of +1 means that the ion is positively charged with charge equal to 1e. Most ions will be singly ionized, with some reaching higher ionization levels. Once charge and mass of particle are known, we can determine the particle velocity as a function of the voltage difference between the two accelerating electrodes, see Figure 34.

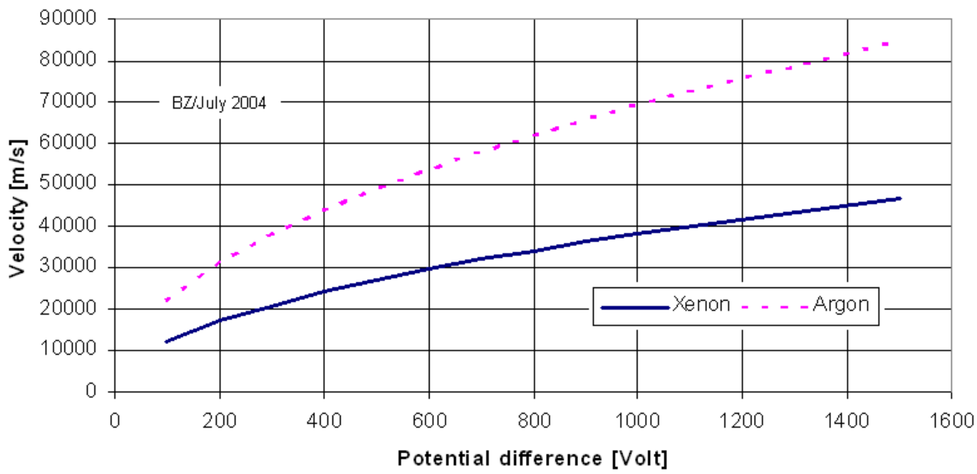


Figure 34: Ion velocity (singly charged ions) versus potential difference between accelerator grids

The figure clearly shows that high velocities of up to 90 km/s are feasible applying voltages of up to 2 kV. It also shows that, like for chemical motors, highest exhaust velocity is achieved for lowest molar mass.

So far we have mentioned that the ions are accelerated between two electrode plates. However, these plates must be of a gridded design for the ions to be able to pass through the plate, see Figure 35.

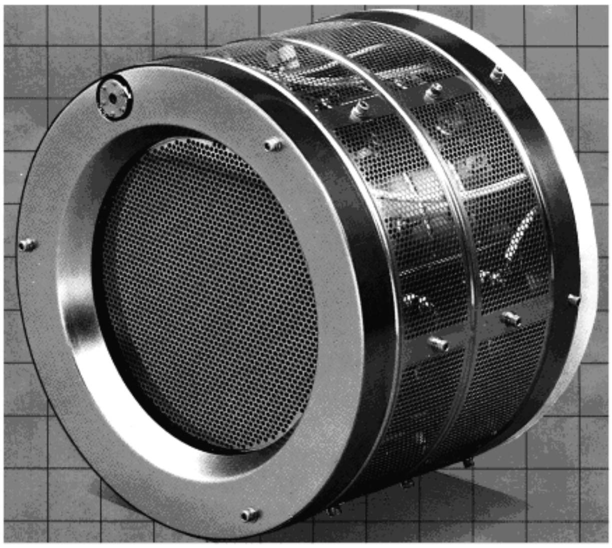


Figure 35: Gridded ion engine (courtesy Mitsubishi)

<u>Beam current</u>
Beam current is the current carried away in the ion beam. From the beam current, we can estimate the number of ionized particles in the flow given that we know the state of charge of these particles.

For a beam consisting of mono-ionized particles only, the number of ions in the beam can be calculated using

$$N_i = \frac{I_b}{q} \quad \textbf{(6.2-6)}$$

Once the number of ionized particles is known, the ion mass flow rate can be determined:

$$m_i = N_i \cdot \frac{\hat{M}}{N_A} \quad \textbf{(6.2-7)}$$

We can now define the ionization or (mass) utilization efficiency as the ratio of the flow rate of mono-ionized particles and the total mass flow rate m.

$$\eta_m = \frac{m_i}{m} \quad \textbf{(6.2-8)}$$

<u>Thrust density and mass flow rate</u>
Ion thrusters typically produce a very low thrust per unit of frontal area, i.e. grid area. This is because there is a maximum amount of charge than can be held in a certain volume. This also limits the maximum ion current density in the thruster and hence the mass flow rate. Approximating the space in between the ion accelerator grids as a one-dimensional gap with gap distance d, across which is applied a voltage V, it can be shown that the **Maximum ion current density** j (in C/s-m^3 or A-m^3) is [Jahn]:

$$j = \frac{4\varepsilon_0}{9} \cdot \left(\frac{2q}{m_q}\right)^{1/2} \cdot \frac{V^{3/2}}{d^2} \quad \textbf{(6.2-9)}$$

Here m_q refers to the mass of the charged particle.

Thrust can be computed using:

$$F_T = j \cdot \frac{m_q}{q} \cdot v_q \cdot A \qquad \textbf{(6.2-10)}$$

It follows for **thrust density**, i.e. thrust per unit of grid area:

$$\frac{F_T}{A} \approx \sqrt{\frac{m_q}{q}} \qquad \textbf{(6.2-11)}$$

From this relation, we find that for a given propellant, thrust scales about proportional to thruster grid area. From this relation, we also see that a high thrust density can be reached in case we have relatively heavy particles. It is amongst others for this reason that ion propellants in comparison with chemical and other thermal propellants have high molar mass.

<u>Input electrical power</u>
The amount of power needed for an ion rocket to operate depends on the power needed for ionization and acceleration. Actually one should also consider the power needed to operate the neutralizer, but generally this power is much less then the power used for acceleration. So we have:

$$P_T = V \cdot I \approx P_{ion} + P_{accel} \qquad \textbf{(6.2-12)}$$

Here P_{ion} is power required for ionization, given by:

$$P_{ion} = \eta_m \cdot m / \hat{M} \cdot N_A \cdot V_i \qquad \textbf{(6.2-13)}$$

V_i is ionisation potential, see for typical values Table 14. The higher the ionisation potential, the more energy is required.

P_{accel} is power required for accelerating the ions in the beam, also referred to as the beam electrical power, which is given by:

$$P_{accel} = \Delta V \cdot I_b \qquad \textbf{(6.2-14)}$$

Here ΔV is the accelerating potential difference and I_b is the beam current.

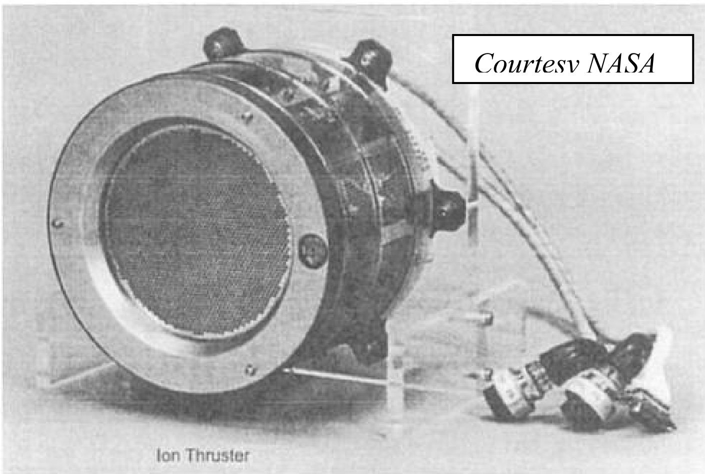
The electrical efficiency of the thruster can now be defined as the ratio of beam electrical power and input electrical power:

$$\eta_E = \frac{P_{accel}}{P_{in}} \qquad \textbf{(6.2-15)}$$

<u>Example</u>
For the ETS-8 electrostatic thruster, see figure hereafter, using Xenon propellant the following data are given:

- *Thrust: 22.05 mN*
- *Specific impulse: 2533.5 s*
- *Beam current: 0.445 A*
- *Input power: 576 W*

- *Beam voltage: 996 V*
- *Motor diameter: 0.12 m*



Photograph of the ETS-8 Kaufman ion thruster

Calculate for this thruster:

a) *Mass flow rate*
b) *Beam power*
c) *Electrical power carried in the beam*
d) *Electrical efficiency*
e) *Thrust density*
f) *Number of ions leaving the engine every second in case of singly charged ions*
g) *Ion mass accelerated*
h) *Utilization efficiency*
i) *Total or thrust efficiency*

Solutions:

a) *Mass flow rate follows from the relation between thrust and specific impulse: 22.05 mN/(2533.5 s x 9.81 m/s^2) = 0.89 mg/s*
b) *Beam power: 274 W*
c) *Electrical power carried in the beam is beam current times beam voltage = 443 W*
d) *Electrical efficiency is 274 W/443 W = 61.8%*
e) *Thrust density: 22.05 mN/(π/4x (0.12m)^2) = 1.95 N/m^2*

f) *Number of ions in the beam follows from the given beam current using (6.2-6). Here q is electric charge of particle which is equal to 1.6 x 10^{-19} C. Beam current is 0.445 A. This gives 2.78E18 ions leaving the thruster every second.*

g) *Using Avogadro's number and that the molecular weight of Xenon is 131.1, we find that the ion mass flow is 0.61 mg/s.*

h) *It follows for the utilization efficiency 0.61mg/s / 0.89 mg/s = 68.5%*

i) *Thrust efficiency or total efficiency is beam power divided by total input power = 274 W/576 W = 47.6%*

6.2.2. Magneto-plasma-dynamic rockets

In **MPD rockets** the propellant is accelerated and shaped into a high velocity beam using a combination of electric and magnetic forces. The propellant typically consists of plasma, i.e. an ionized gas containing ions and electrons in about equal numbers so that the resultant space charge is very small. Plasma can be generated by heating a gas to extreme temperatures. Because of heating, molecules turn in to their constituent atoms. Further heating leads to ionization, turning it into plasma. MPD rockets are sometimes referred to as electro-dynamic rockets to stress the interaction between electric and magnetic forces in the rocket.

A schematic of an MPD rocket is shown in the next figure.

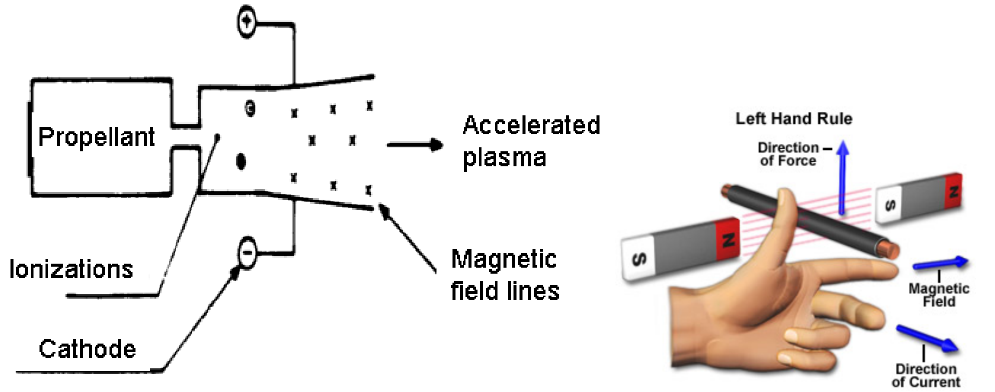


Figure 36: Schematic of MPD rocket

The figure shows propellant which is fed from the propellant storage into the MPD thruster, also referred to as plasma thruster. The thruster consists of two electrodes between which an arc discharge is created (just like in an arcjet). Note that anode is positively charged electrode and cathode is negatively charged electrode (CNAP). The propellant gas flowing through the arc discharge is heated to extreme temperatures (for some thrusters plasma temperatures of up to 50,000 deg C have been reported), thereby creating plasma. In contrast to the arcjet, we do not apply a nozzle to accelerate this plasma, but we apply a magnetic field in a direction perpendicular to the flow and the electric field generated by the two electrodes.

A magnetic field is created by the electric current returning to the power supply through the cathode, just like the magnetic field that is created when electrical current travels through a wire. This self-induced magnetic field interacts with the electric current flowing from the anode to the cathode (through the plasma) to produce an electromagnetic (Lorentz) force that pushes the plasma out of the engine, creating thrust. An external magnet coil may also be used to provide additional magnetic fields to help stabilize and accelerate the plasma discharge.

<u>Thrust force</u>

The principle method of accelerating the propellant is, as discussed earlier, by the application of the Lorentz force to electrically conductive propellant. The magnitude of the force exerted on the gas is given by the Lorentz Force Equation, written in vector notation

$$\bar{F}_T = L\,\bar{I} \times \bar{B} \qquad \textbf{(6.2-16)}$$

Here F is force, L is length of conductor, I is total current, and B is magnetic flux density or magnetic induction (in Wb/m^2)[17].

[17] 1 Wb/m^2 = 1 $kg/(A.s^2)$ = 1 T = 10^4 Gauss; Earth magnetic induction is about 0.3 Gauss.

Assuming that current density and magnetic flux density are perpendicular to each other, we can write for the force on a charge q:

$$F_T = B\,q\,v \quad \textbf{(6.2-17)}$$

Here the force F is perpendicular to the plane as defined by the magnetic flux density and the current (here represented by the product of particle charge q and particle velocity v). For illustration consider Figure 36. Here current flows from the positive electrode to the negative electrode (same direction as positively charged ions will flow). Magnetic flux density is pointing in a direction perpendicular to the plane of the paper away from the onlooker. In that case, using the left hand rule, we find that a force works on the charged particles forcing them in the direction of the thruster outlet. It is the reaction force that is generating the thrust on the rocket.

Example: Given a magnetic induction of 0.1 T (values of about a factor 10 higher are feasible using Ferro-magnetic materials) and a cross flow velocity v of the order of 1000 m/s, we find for the force on a singly charged ion:

$F_T = 10^{-1} \times 1.6 \times 10^{-19} \times 1000 = 1.6 \times 10^{-17}$ *N*

The above representation is a very simple representation of what actually happens in a thruster. For instance, because of the action of the Lorentz force, the charged particle velocity will change (both in magnitude and direction), but this is considered beyond the scope of the lectures. Here we will satisfy with representing a simple model that allows for developing a feel for the numbers and the physics.

Neglecting the effect of the Lorentz force, the particle velocity v in the thruster is determined by the voltage difference between the two electrodes, the charge of the charged particle and the particle mass, see relation (6.2-4).

Current running through a wire is known to generate its own magnetic field. The same will be for the current flowing from the anode to the cathode. For a current I running through a wire the magnetic field strength H (in A/m) at a distance r is given by (see physics):

$$H = I/(2\pi\, r) \quad \textbf{(6.2-18)}$$

It follows that the magnetic field strength increases with current and decreases with increasing distance.

From the magnetic field strength the magnetic induction can be determined using:

$$B = \mu\, H \quad \textbf{(6.2-19)}$$

Here μ is magnetic permeability given by:

$$\mu = \mu_o\, \mu_r \quad \textbf{(6.2-20)}$$

With μ_o is magnetic permeability of vacuum = $4\pi \times 10^{-7}$ H/m (H = Henry: 1 H = 1 $kgm^2/(A^2s^2)$) and μ_r = relative permeability (1-10^5 for ferromagnetic materials).

So for an MPD thruster even without an external magnetic field, the current flowing from the anode to the cathode will generate its own magnetic field. As in that case the magnetic flux density B is also a function of the total current flowing, it follows that the force exerted on the propellant is actually related by the total current squared. This provides a strong incentive to operate the thruster with the largest tolerable currents. A more correct approximation can be obtained by integrating the force density over the volume of the thruster nozzle. This yields the total force as [Jahn]

$$F_T = \frac{\mu I^2}{4\pi}\left(\ln\frac{r_a}{r_c} + \frac{3}{4}\right) \qquad \textbf{(6.2-21)}$$

Here r_a and *r_c are* the effective arc attachment radii on the anode and cathode. In [Jones], the following relations are given for the arc radii for an axis-symmetric arc and no interaction effects at the anode, follows:

$$\frac{r_a}{r_c} = 3.2 - 2.2e^{-\frac{z}{5r_c}} \qquad \textbf{(6.2-22)}$$

The arc radius, r_a, varies with the distance, z, from the cathode surface. The radius, r_c, of the cathode-spot attachment is determined by the value of the cathode-spot current density, estimated by Bowman to be around 3.5 kA/cm2.

$$r_c = \sqrt{\frac{I}{\pi\left(3500A/cm^2\right)}} \qquad \textbf{(6.2-23)}$$

Here I (in A) is current running through the arc and rc in centimeter.

The self field can be magnified by applying an external magnetic field, for instance by using coils through which a current is fed independent of the current running through the thruster.

Example: Magnetic field strength H of a coil follows by multiplying the number of turns times the current and dividing by the length of the coil. For a coil of 25 cm length, 100 turns and a coil current of 1 A follows a magnetic field strength: H = 100 (1 A) / 0.25 m = 400 A/m.

An important advantage of an MPD thruster which has not been mentioned so far is that a neutral beam is produced. So unlike for an ion thruster, no neutralizer is needed. In addition, because the plasma is electrically neutral, higher thrust per unit area (up to 10^5 N/m^2) can be produced than for an ion thruster. This has as advantage that for the same level of thrust, the thruster can be smaller.

A convenient geometry for actual MPD thrusters is shown in the next figure.

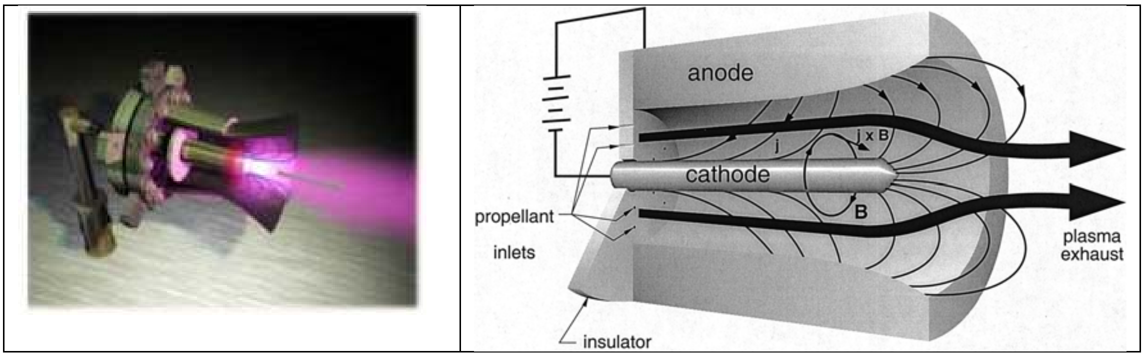


Figure 37: MPD thruster geometry (courtesy Princeton University and [Jahn])

This is a convenient geometry for several reasons, including the fact that the anode can have much more mass than the cathode. This helps with cooling, since in an arc discharge much of the thermal energy gets deposited at the anode. Care still must be taken to prevent the cathode from melting. For space engines, operating in vacuum, the cathode generally is made out of tungsten due to its ability to tolerate very high temperatures.

The plasma thruster shown in Figure 37 is a steady (or quasi-steady) plasma thruster. In this thruster a continuous electrical arc discharge is used to generate the plasma. Some typical characteristics of such a thruster are given in the Table 15.

Table 15: Typical characteristics steady plasma thrusters [Jordan]

Propulsion Type	Specific Impulse (s)	Thrust/ Weight (N/kg/g)	Specific Power (kW/kg)	Electric Power/ Thrust (kW/N)	Propellant Utilization Efficiency	Typical Thrust (mN)	Impulse Bit (mN-s)	Total Impulse (N-s)
MPD	600	0.00005	0.01	10 - 19	15 %	23	<500<	> 1,000

Here the thrust to weight ratio given applies to the thruster only as does the specific power. Also given is the electric power to thrust ratio, showing that to produce 1 N of thrust, 10-19 kW of power is needed.

Example: A plasma thruster providing a thrust of 1 N at a specific impulse of 600 s has a beam power of 2.94 kW. From the data in the table we learn that in practice about 10-19 kW of input power are needed. We obtain a thrust efficiency in the range 15-30%.

For most spacecraft the high power consumption of a plasma thruster is considered a serious disadvantage. It is because of the high power requirement that researchers started researching pulsed plasma thrusters (PPT), i.e. thrusters that only operate over very short time and use the time between pulses to power up e.g. capacitors (see also this course, part on energy generation). A typical such thruster is shown in Figure 38.

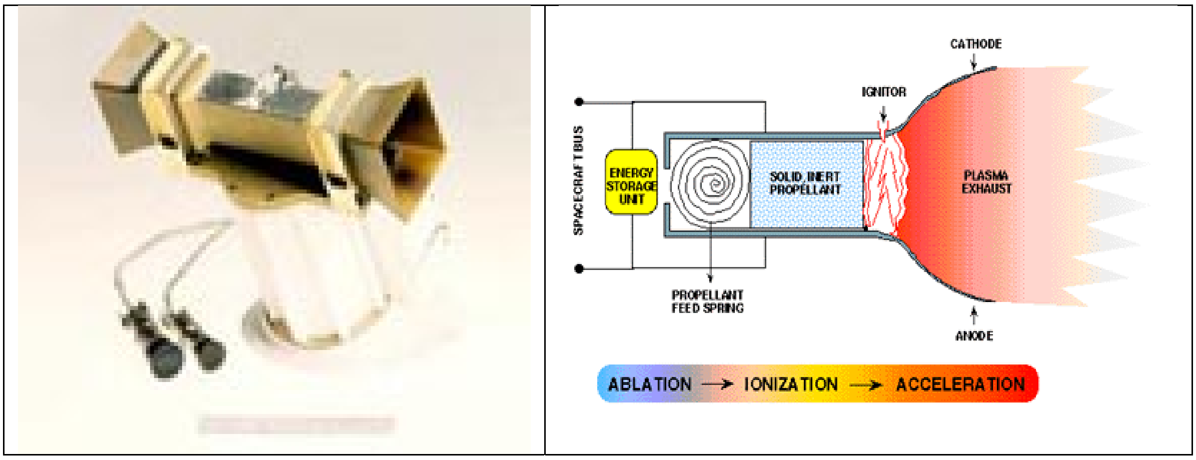


Figure 38: Pulsed plasma thruster (left, courtesy Primex) and PPT schematic (right)

Typical performance data are provided for in Table 16.

Table 16: Characteristics of some pulsed plasma thrusters [Cassady]

Table 1 Comparison of PPT Capabilities

Parameter	PRIMEX EO-1 Design	LES 8/9	NOVA
Maximum Ibit	>750 μN-sec	300 μN-sec	378 μN-sec
Minimum Ibit	<100 μN-sec	300 μN-sec	378 μN-sec
Pulse to Pulse Throttleability	Yes	No	No
Isp	1150 sec	1075 sec	~300 sec
Efficiency *(note 1)*	9.8%	~ 7%	~ 3%
Mass	5 kg *(note 2)*	7.33 kg *(Measured)*	6.35 kg
Thrust/Mass Ratio	306 μN/kg	82 μN/kg	53 μN/kg
Maximum Thrust	1.4 mN	600μN	378 μN
Total Impulse Capability	15,000 N-sec *(note 3)*	7,300 N-sec	2,224 N-sec

Note 1: Efficiency is defined as the thruster efficiency, PPU efficiency not included
Note 2: EO-1 mass driven by solar array torque environment
Note 3: EO-1 was fuel limited to 1,500 N-sec, however, key components demonstrated life to 20 million pulses ($20x10^6$ * 750mN-sec = 15,000N-sec)

The data in the table shows that very small impulse bits (see section on Fundamentals) are feasible of less than 1 mNs. For a thrust of 1 mN, this indicates a pulse time of about 1 second and a power consumption of just 8 W (Primex EO-1 Design). At an efficiency of about 10%, this limits the input power to just 80W.

6.3 System considerations

For rocket systems it follows from the rocket equation that to minimize propellant mass, we should strive for maximum (effective) exhaust velocity or specific impulse. However, for systems with a separate power plant, it follows that with increasing specific impulse (constant thrust), the jet power increases and hence also the power that should be provided to the rocket system. With increasing power also the mass of the power plant increases, thereby reducing the mass gain obtained from a reduced propellant mass.

Example: Consider a rocket providing a thrust of 10 N at an effective exhaust velocity of 10 km/s. This rocket than consumes 1 g/s of propellant. Given a thrust duration of 100,000 s (about 250 hrs), the total propellant mass used is 100 kg. Now focusing on the power plant, we find for this thruster a jet power of 50 kW. Suppose we use a photovoltaic system (solar panels) to provide for this power. Given a specific mass of 20 W_e/kg (typical range is 10-25 W_e/kg), we find a power plant mass of 50,000/20 = 2500 kg (note we have assumed a thrust efficiency of 100%). This is much more than the propellant mass and hence one could consider lowering the effective exhaust velocity. Halving the exhaust velocity leads to a doubling of the propellant mass to 200 kg. However, the jet power decreases to 12.5 kW and hence the power plant mass to 625 kg, which indicates a substantial saving.

Systems with a separate power source have an optimum (effective) jet velocity (specific impulse) [Fortescue].

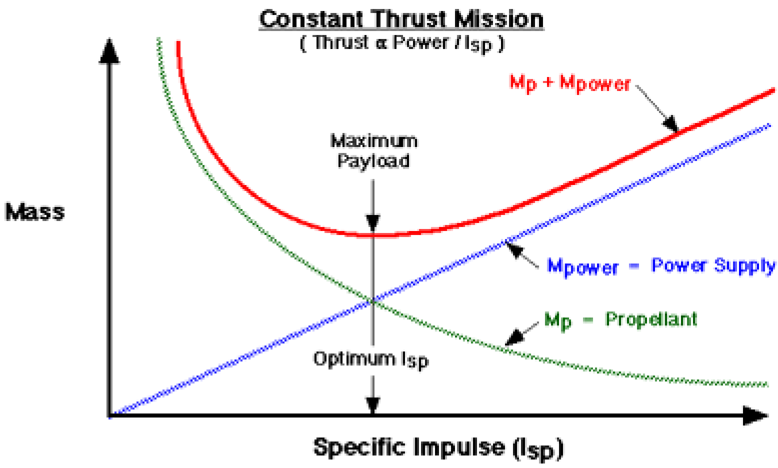


Figure 39: Plot showing propellant mass and powerplant mass as function of specific impulse

An analytical approach allowing to determine the optimum is presented below based on the assumption that the mass of power source scales linearly with the power output of this source (as for the solar panels in the foregoing example):

$$M_W = \alpha_W \; P_W \qquad \textbf{(6.3-1)}$$

With:
- $1/\alpha_W$ = specific power (W/kg) or α_W is inverse specific power (kg/W)
- P_W = power output from power source

The linear dependency of power source mass with power is evident in case of using solar energy. This is because solar panel or solar collector area increases with increasing power. In case of using nuclear energy, this assumption is less evident and one could reason that the mass of the energy source scales with the amount of energy instead of power. However, in practice again power is the dimensioning parameter. This is mostly because the mass of the energy source itself is negligible compared to the mass of the power conversion system needed to convert nuclear power into useful power. The latter again scales with power.

Typical specific power values are given in the next table.

Table 17: Typical specific power values are

Type of power system	Specific power
Thermal:	
- Radio-isotope	25-170 W_t/kg
- Nuclear-thermal	300-4000 kW_t/kg
- Solar collector-receiver at 1 AU	200-2000 W_t/kg
Electrical:	
- Photo-voltaic array	10-40 W_e/kg
- Photo-voltaic system (incl. batteries)	7-12 W_e/kg
- Nuclear-electric	2.5-100 W_e/kg

Power output required from the power source can be related to jet power[18]:

$$P_W = P_j/\eta_T \qquad \textbf{(6.3-2)}$$

[18] Notice that we assume that power output from the power source is identical to the input power of the thrust generating system. In practice, this is rarely the case.

Substitution of equation for mass of energy source gives for the system specific impulse:

$$I_{ssp} = \frac{I}{W} = \frac{F \cdot t}{\left(M_p + \alpha_W \cdot P_W\right) \cdot g_o} = \frac{m \cdot w \cdot t}{\left(m \cdot t + \dfrac{\alpha_W \cdot m \cdot w^2}{2\eta_T}\right) \cdot g_o} \qquad \textbf{(6.3-3)}$$

Reworking gives:

$$I_{ssp} = \frac{w/g_o}{\left(1 + \dfrac{\alpha_W \cdot w^2}{2\eta_T t}\right)} = \frac{w/g_o}{1 + \varepsilon w^2} \qquad \textbf{(6.3-4)}$$

With ε is specific mass of the energy source (expressed in kg/J); 1/ε is specific energy of the energy source (J/kg).

The next figure shows a plot of specific impulse versus exhaust velocity for two different values for the specific mass of the energy source.

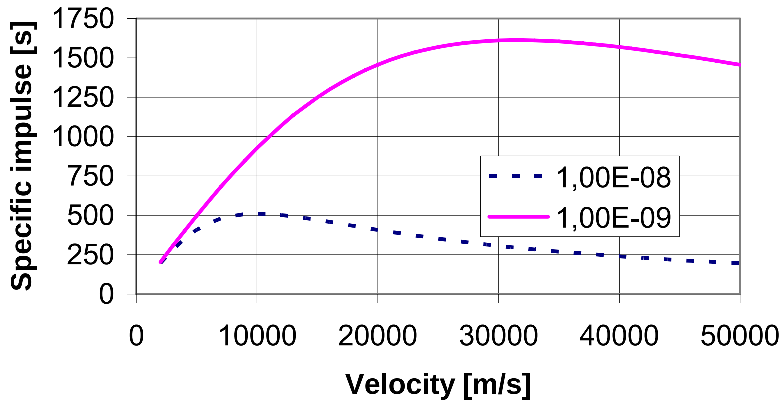


Figure 40: Optimum jet velocity for two different values of specific mass of energy source

From the figure, we learn that in this case specific impulse has some optimum value. The exhaust velocity at which this optimum occurs is referred to as the optimum jet velocity. This velocity depends on amongst others mission duration, specific power of power source, and thrust efficiency.

The value of the optimum exhaust velocity can be found by differentiating the specific impulse equation to jet velocity and setting the result equal to zero. This gives:

$$\frac{dI_{ssp}}{dw} = 0 = \frac{1/g_o \cdot (1 + \varepsilon w^2) - 2\varepsilon w \cdot (w / g_o)}{(1 + \varepsilon w^2)^2}$$

$$0 = 1/g_o \cdot (1 + \varepsilon w^2) - 2\varepsilon w \cdot (w / g_o) = 1 - \varepsilon w^2$$

$$w_{opt} = \sqrt{1/\varepsilon} = \sqrt{\frac{2\eta_T t}{\alpha_W}} \qquad \textbf{(6.3-5)}$$

For instance, in the case of $\varepsilon = 10^{-8}$ kg/J, we find that w_{opt} = 10 km/s and for $\varepsilon = 10^{-9}$ kg/J, w_{opt} = 31.6 km/s.

In practice, technical constraints may prevent us from obtaining the optimum exhaust velocity, so that we have to accept a non optimum jet velocity. In that case, we have to live with the non-optimum velocity and accept that system mass might be higher than what could be possible when using optimum values.

Verify that the mass of the power source can be written as:

$$M_W = \frac{w^2}{w_{opt}^{\ 2}} \cdot M_{propellant} \tag{6.3-6}$$

Here w is the actual jet velocity achieved. From this relation we can deduce that in case we select for the thruster at hand the jet velocity equal to the ideal or optimum jet velocity as defined in the foregoing it follows that propellant mass and power plant mass are equal.

As a final remark, we note that in case we use (excess) power from an already present power source, for example for providing power to the payload once arrived at its operational orbit, we can omit the design of the power source from our considerations and we should again strive for the highest velocity feasible.

Example: An advanced propulsion system driving an Orbital Transfer Vehicle (OTV) of total empty mass 5000 kg (including the propulsion system, but excluding the propellant) is required to transfer the vehicle from a low Earth orbit to a geostationary Earth orbit in 30 days. From mission analysis you know that this requires a thrust-to-weight ratio of 2 x 10^{-4} and a mission Δv of 5.7 km/s. Determine:

a) Optimum jet velocity given a power system specific mass of 0.026 kg/kW$_e$[19] and a thrust efficiency of 100%
b) Idem but now in case we have a thrust efficiency of 80%[20]
c) Propellant mass and OTV mass at start of mission using the answer from (b)
d) Thrust and mass flow rate given a 100% ionization efficiency, and
e) Power plant mass.

Solution:
a) Optimum jet velocity
Using the relation 6.3-5, we find for the optimum exhaust velocity:

$$w = \sqrt{\frac{2 \cdot \eta_T \cdot t}{\alpha}} = \sqrt{\frac{2 \cdot 100\% \cdot 30\text{days} \cdot 24\text{hours} \cdot 60\,\text{minutes} \cdot 60}{0.026\text{kg}/\text{kW}_e}}$$

$$w = 14.1\ \text{km/s}$$

b) We now just have to recalculate the above, but with a thrust efficiency of 80%. It follows an optimum jet velocity of 12.6 km/s.

[19] We select as power system modern lightweight solar arrays with a specific mass of 20 kg/kW or 0.02 kg/W and add 30% to this value to take into account other items of the propulsion subsystem, like thrusters and propellant storage. This gives α = 0.026 kg/W.
[20] A value more in line with what is realistically possible.

c) Propellant mass and OTV total mass
Applying the rocket equation we find for the mass ratio:

$$\Delta v = w \cdot \ln(R) \Rightarrow 5.7 = 12.6 \cdot \ln(R) \Rightarrow$$
$$\ln(R) = 0.45 \Rightarrow R = 1.57$$

Using the given dry mass (5000 kg), we find a propellant mass of about 2860 kg and a total vehicle mass of 7860 kg.

d) Thrust and mass flow rate
From the propellant mass, we calculate a mass flow of 1.10 g/s = 2860 kg/(30 days x 24 hours x 3600 seconds). Using this mass flow rate and the calculated optimum jet velocity, we find a thrust level of 13.9 N = 1.10 g/s x 12600 m/s (100% ionization efficiency).

e) Power plant mass
Based on the calculated thrust and exhaust velocity, we find for the jet power:

$$P_j = ½\, F \cdot w = ½\,(13.9) \cdot (12800) = 88.96 \text{ kW} \sim 89 \text{ kW}$$

For a thrust efficiency of 80%, this gives an overall power input of 111 kW. Using the given value for the specific mass of the power source, this leads to a mass of the power-plant of 2860 kg or 57.2 % of the dry vehicle mass and equal to the propellant mass.

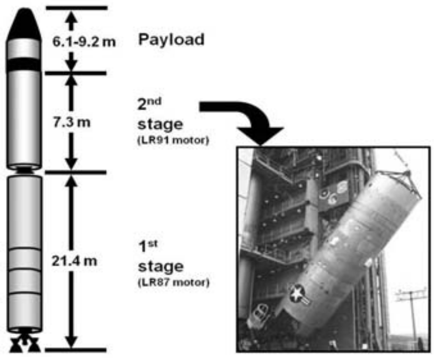
Exercise Problems

The following problems are given as to help students in exercising the material taught based on some real life problems. Answers can be found in the back.

1) For a rocket motor is given a sea level thrust of 20 kN. Determine for this rocket motor the thrust at a flight altitude of 10 km (ISA 76 atmosphere) given a nozzle exit diameter of 0.6 m.

   Hint Use Atmospheric Properties Calculator given on http://www.aerospaceweb.org/design/scripts/atmosphere/ to calculate the pressure at flight altitude.

2) The *LR91* rocket motor (used in the 2nd stage of the *Titan 23G* space launcher, see figure taken from [Ward]) uses the liquid bipropellants: nitrogen tetroxide (N_2O_4) and Aerozine 50 (A-50). Assume that the *LR-91* is an ideal rocket motor with a nozzle expansion ratio (ε) of 49.0, chamber pressure (p_c) of 5.6 MPa, and chamber temperature (T_c) of 3,400K. The exhausted gaseous products of combustion have a ratio of specific heats (γ) of 1.3 and gas constant (R) of 390.4 J/(kg·K).

   [Note: Aerozine 50 is a 50/50 mixture of hydrazine (N_2H_4) and unsymmetrical dimethyl hydrazine ($C_2H_8N_2$), also called UDMH.]

   Determine the altitude at which the nozzle is designed for optimal (full) expansion.

3) You are designing a 16 kN vacuum thrust thermal rocket engine with a vacuum specific impulse of 850s, and a maximum diameter of 0.3 m. As a starting point, you have selected Hydrogen (molar mass of 2 kg/kmol, and specific heat ratio of 1.4) as propellant, 2500 K as hot gas temperature, and 10 bar as chamber pressure. Hydrogen characteristic velocity at these conditions is about 2175 m/s. You are asked to calculate for this engine:
   a) Hydrogen mass flow;
   b) Beam power;
   c) Throat diameter;
   d) Maximum allowable (geometric) expansion ratio based on maximum diameter given;
   e) Nozzle pressure ratio;
   f) True exhaust velocity;
   g) Pressure in nozzle exit;
   h) Pressure thrust in vacuum;
   i) Effective specific impulse.

4) The following characteristics apply to the USA developed Nerva 2 thermo-nuclear rocket engine [Humble]:

- Vacuum thrust: 333.6 kN
- Vacuum specific impulse: 850 s
- Burn time: 1200 s
- Propellant: Hydrogen (stored in liquid state)
- Hydrogen gas temperature at nozzle inlet: 2560 K
- Nozzle area ratio[21]: 500
- Engine mass: 10138 kg
- Thermal power: 1570 MW

Calculate for this engine:
a) Propellant mass flow rate;
b) On board propellant mass;
c) Propellant volume given a hydrogen mass density of 71 kg/m^3;
d) Jet (or beam) power;
e) Hot gas temperature at reactor outlet given a constant hydrogen specific heat of 14 kJ/(kg-K) and 14 K, respectively. You may neglect any phase changes;
f) Cycle or thrust efficiency;
g) Limit velocity given a hydrogen specific heat ratio of 1.4 and a molar mass of 2 g/mol;
h) Nozzle pressure ratio using the earlier given specific heat ratio;
i) True exhaust velocity.

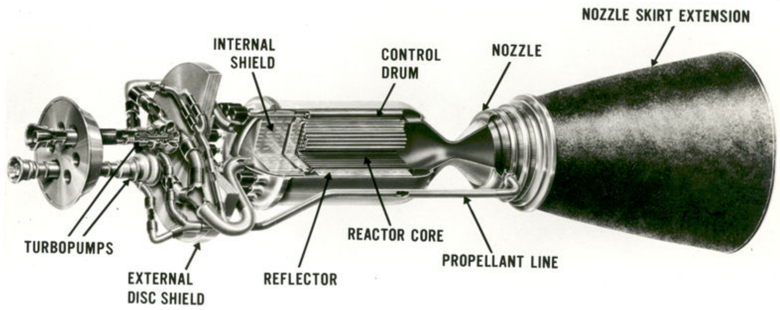


Figure 41: US Nerva 2 thermo-nuclear rocket engine (courtesy NASA)

5) The T5 electro-static thruster, see figure, using Xenon as the propellant produces a nominal thrust of 25 mN. The measured exhaust velocity of the mono-ionized particles is 32 km/s and the ionization efficiency is 80%: Determine for this thruster:

a) Mass flow rate of (mono-ionized) ions;
b) Total mass flow rate;
c) Jet power;
d) Input power given a thrust efficiency of 50%;
e) Beam voltage and beam current.

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[21] Area ratio from: http://www.astronautix.com/engines/nervantr.htm

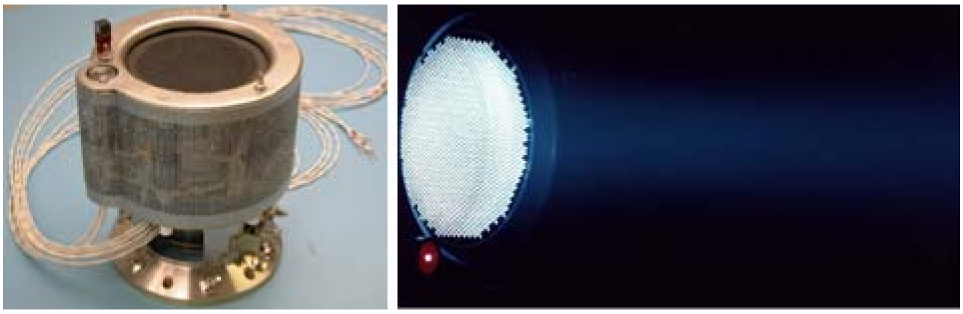

Figure 42: T5 electro-static (gridded) ion thruster

6) An advanced propulsion system consists of a separate energy source and 10 advanced motors, each with a thrust of 1 N, an exhaust velocity of 20 km/s and a thrust efficiency of 80%. The total mass of the system is 9860 kg. Determine:
   a) The required input power per thruster;
   b) The specific power of the system with all thrusters operative;
   c) The optimum exhaust velocity given a total thrust time of 180 days;
   d) The optimum required input power given that the thrust remains constant.

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Glossary

1	**Jet Propulsion** is a means of locomotion whereby thrust is produced by ejecting a high velocity jet.
2	**Rocket Propulsion** is a means of jet propulsion whereby thrust is produced by ejecting matter, which is stored in the vehicle being propelled.
3	**Propulsion system** is a machine that produces a thrusting force to drive an object forward.
4	**Rocket system** is a machine that produces rocket thrust.
5	**Thrust chamber (large rockets) or thruster** (small rockets) is the thrust generating element of a rocket system
6	**Rocket Propellant** is the stored matter that is energized and ejected by a rocket.
7	**Propellant feed and storage system** is that part of the rocket system that stores the propellants and controls the flow of propellants to the thrust generating element(s)
8	**Power plant**: The element that provides the energy needed for thrust generation. The power plant is also referred to as power source or energy source.
9	**Thermal rockets** are rocket systems where the propellant is accelerated and shaped into a high velocity jet by a directed expansion of a high pressure gas.
10	**Electric rockets:** A form of rocket propulsion in which the propellant is accelerated and shaped into a high velocity beam using electric (Coulomb) forces.
11	**Electromagnetic rockets** are rocket systems wherein the propellant is accelerated and shaped into a high velocity beam using a combination of electric and magnetic forces.
12	**Cold gas rocket systems** use a (typically inert) gas under high pressure as the reaction mass. The energy needed for the thrust generation comes from the internal energy of the gas which is expanded to form a low pressure, high velocity, jet.
13	**Chemical Rockets** are rocket systems where the energy comes from a chemical reaction or combustion of a Fuel with an Oxidizer.
14	**Nuclear rockets** are rocket systems where the energy comes from a nuclear reaction of a nuclear fuel in a nuclear power plant.
15	**Solar (or laser) rockets** refers to rocket systems where the energy needed for thrust generation is taken from the radiation energy available from the Sun or an other radiation source, like a laser.
16	**Electric rockets** refer to rocket systems that derive the energy needed for thrust generation from an electric power supply.
17	**Thrust** is a propulsive force
18	**Rocket thrust** is the propulsive force generated by a rocket system
19	**(Propellant) mass flow rate** is a measure of the propellant consumption rate.
20	**Effective jet velocity:** Same figure of merit as the mass flow rate. The higher the effective jet velocity, the lower the propellant consumption rate.

21	**Total Impulse** is the integral of thrust over the propulsion operating time. It is a measure of the total kinetic energy of the nozzle exhaust gas as released by the combustion of all the available propellant in the propulsion system. For constant thrust operation it is the average thrust multiplied by the effective propulsive operating duration or it is also the mass of the total expelled propellant multiplied by the average specific impulse.
22	**Specific Impulse** is a performance indicator of how well the propellant is used to produce thrust. It can be defined as the thrust of an equivalent rocket propulsion system (same chamber pressure, same propellant, same nozzle throat to exit area ratio) that has a propellant mass flow of unity. Higher values indicate a lower propellant consumption rate, which is usually considered to lead to a better system.
23	**Thrust duration (also referred to as action time, operating time, and burn time)**: Time that a propulsion system produces thrust
24	**Thrust (or propulsive) power** is a parameter indicating the work performed by the propulsion system
25	**Jet (or beam) power** is the total (kinetic) power in the jet.
26	**Propulsive efficiency**: A performance indicator of how well the energy generated by the rocket is being utilized. It is defined as the fraction of total mechanical power output imparted as thrust power to propel the vehicle (or rocket), as opposed to how much is wasted.
27	**Cycle (or thrust) efficiency**: A performance indicator of how effectively the energy from the energy source is converted into jet power.
28	**Overall (or total) efficiency**: A performance indicator of how effectively the energy from the energy source is converted into thrust power
29	**Chamber**: Essentially a hollow tube, that contains a high pressure gas and exits in to the nozzle
30	**Nozzle**: Usually a convergent-divergent flow tube (CD nozzle) that provides for a directed expansion of the high pressure gases that flow from the chamber.
31	**Nozzle throat** is minimum flow area between convergent and divergent nozzle section.
32	**Nozzle exit** is plane through which the exhaust gases leave the nozzle.
33	**Nozzle area ratio** is the nozzle exit area divided by the nozzle throat area. For optimum gas expansion in a nozzle the gas pressure at the nozzle exit is equal to the local ambient atmosphere pressure. Typical values of this nozzle area ratio are between 4 and 20 for expansion to sea-level pressure and between 40 and 200 for operation at very high altitude (space vacuum).
34	**Exhaust velocity**: The linear velocity of the exiting exhaust gases. It is identical to the effective exhaust velocity in case of optimum expansion. In case of non/optimum expansion, the exhaust velocity differs from the effective exhaust velocity.
35	**Critical (or choked) nozzle:** The nozzle is said to be critical or choked in case a supersonic outflow is realized. In that case the mass flow rate is independent of the ambient pressure and is commonly referred to as the critical mass flow.

36	**Chamber pressure** is the pressure in the combustion chamber of an operating rocket propulsion system.
37	**Exit pressure** is the pressure in the jet at the nozzle exit
38	**Nozzle pressure ratio** is ratio between chamber pressure and exit pressure. This ratio increases with increasing nozzle area ratio. The larger this ratio is, the higher the exhaust velocity.
39	**Chamber temperature** is temperature of propellant gases in chamber.
40	**Critical mass flow** is mass flow rate in case the nozzle is choked.
36	**Ideal (or optimum) expansion** The exit pressure is equal to ambient pressure. The flow in this case is perfectly expanded inside the nozzle and maximizes thrust given that ambient conditions remain constant.
37	**Underexpansion** occurs when the atmospheric pressure is lower than the exit pressure. In this situation the flow continues to expand outward after it has exited the nozzle. This behaviour also reduces efficiency because that external expansion does not exert any force on the nozzle wall. This energy can therefore not be converted into thrust and is lost. Ideally, the nozzle should have been longer to capture this expansion and convert it into thrust.
38	**Overexpansion**: The external pressure is higher than the exit pressure. When an overexpanded flow exits the nozzle, the higher atmospheric pressure causes it to squeeze back inward and separate from the walls of the nozzle. This "pinching" of the flow reduces efficiency because that extra nozzle wall is wasted and does nothing to generate any additional thrust. Ideally, the nozzle should have been shorter to eliminate this unnecessary wall.
39	**Chemical propellants** are propellants that through a chemical reaction also provide for the energy needed for thrust generation.
40	**Rocket engine:** The term rocket engine is mostly used for liquid (propellant) rocket system. A **Rocket Engine** usually consists of one or more **Thrust Chambers**, one or more **Tanks** for storing propellants, a **Feed Mechanism** to force the liquids into the thrust chamber, a **Power Source** to provide energy to the feed mechanism, suitable **Piping** and **Valves** to transfer the liquid propellants, a structure to transmit the thrust force to the vehicle, and **Controls** to initiate and regulate the propellant flow rates.
41	The engine **Feed Mechanism** can be a **Pressurized System**, where high pressure gas expels the liquid propellants from its tanks, or a **Turbopump System**, where pumps feed the propellants to the thrust chamber.
42	**Liquid propellants** refer to propellants that are stored in the liquid state.
43	**Monopropellant** refers to a liquid propellant composed of chemicals or mixtures of chemicals which can be stored in a single container with some degree of safety.
44	**Bipropellant** refers to a liquid propellant combination consisting of a liquid **Fuel** and a liquid **Oxidizer**, which when mixed, can react chemically to form hot combustion gas. Usually the fuel and oxidizer are stored in separate tanks on board of the vehicle to ensure safety.
45	**Storable Propellants** are propellants that can be stored in the liquid state without the need of cooling.

46	**Cryogenic propellants** are sub-cooled liquids at low temperature (such as liquid oxygen or liquid hydrogen); they are gases at ambient temperatures.
47	**Hypergolic Propellant**: If the fuel and the oxidizer react spontaneously (a chemical reaction occurs when they come in contact with each other), they are called Hypergolic
48	**Mixture Ratio** is the ratio of the liquid oxidizer flow rate divided by the liquid fuel flow rate. The best performance (highest specific impulse) is obtained at a specific optimum mixture ratio.
49	**Rocket Motor:** Term used to denote rocket systems using either a solid or a hybrid propellant.
50	**Solid Propellant** is a propellant that can be stored in the form of a solid charge, i.e. the propellant grain. It typically consists of an oxidizer (for instance ammonium perchlorate), an organic fuel (such as a rubbery polymer like polybutadiene, which also acts as the glue to hold the grain together), and various additives to improve performance, storage, thrust-time profile, manufacture, aging, etc.
51	**Hybrid Propellant** is a propellant consisting of two substances, which are stored in a different state. Usually one is stored as a solid charge (mostly the fuel) and the other as liquid, but sometimes also gaseous storage is possible.
52	**Grain**: Solid charge of a solid or hybrid rocket motor. The Grain has **Perforations**, **Slots**, **Grooves**, holes, or **Port Areas** so as to predetermine the amount of initial burning surface. Most grains are cast into and bonded to the case; some grains are bonded to a separate cartridge, which is then loaded or placed into the case
53	**Burn or Regression Rate** is the rate of regression of the burning grain surfaces as propellant is consumed or burnt (inches per second) in a direction normal to the surface. Surfaces that are bonded to the case walls or to insulators, will not burn. **Inhibitors** are layers of non-burning materials that are glued to exposed grain surfaces so that they will not burn. The propellant flow and, therefore, also the thrust are proportional to this burning rate and the exposed burning surface. The burning rate varies with chamber pressure and the initial ambient temperature of the grain.
54	**Electro-thermal Propulsion**: A form of rocket propulsion in which electrical energy is used to heat a suitable propellant after which the hot propellant is expanded through a supersonic nozzle to generate thrust.
55	**Resistojet**: A thruster, wherein the propellant is heated by passing it through a resistively heated chamber before the propellant is expanded through a supersonic nozzle to generate thrust.
56	**Arcjet**: A thruster, wherein the propellant is heated by passing it through a high voltage arc before the propellant is expanded through a supersonic nozzle to generate thrust. Because the gas is heated by an arc discharge the gas is heated to very high temperature (3000 – 4000 K), Arc temp =10,000K on average, and much greater in certain regions in the arc.
57	**Solar-thermal propulsion**: A form of rocket propulsion wherein solar energy is used to heat a suitable propellant after which the hot propellant is expanded through a supersonic nozzle to generate thrust.
58	**Solar-thermal engine (or thruster)**: The thrust generating device in a solar-nuclear propulsion system.

59	**Thermo-nuclear propulsion**: A form of rocket propulsion in which nuclear energy is used to heat a suitable propellant after which the hot propellant is expanded through a supersonic nozzle to generate thrust.
60	**(Thermo-) nuclear engine**: The thrust generating device in a thermo-nuclear propulsion system.
61	**Thermal (heat) power**: Rate at which thermal energy (heat) flows.
62	**Solar power** is the power available from the Sun. It depends on the solar intensity (solar flux) and the area of the solar collector, i.e. a device (lens or mirror) that collects the solar radiation and projects it into the solar heater chamber.
63	**Electrical power**: Rate at which electrical energy is transferred by a circuit
64	**Ion propulsion**: A form of electric propulsion in which electric energy is used to accelerate ions to generate a high velocity jet.
65	**Ion thruster**: Thrust generating device in an ion propulsion system. Ion thrusters operate on a variety of propellants, the most common being Xenon. Other propellants of interest include Krypton and Argon. Ion thrusters are able to accelerate their exhaust to speeds between 10–80 km/s (1000-8000 s specific impulse), with most models operating between 15–30 km/s (1500-3000 s specific impulse). The thrust produced by ion thrusters varies depending on the power level. High power models have demonstrated up to 3 N in the laboratory.
66	**Ion**: Electrically charged particle. The charge of an ion is usually expressed using the elementary charge ($e = 1.6 \times 10^{-19}$ C). Ions can be positively or negatively charged. For instance, a charge of 1+ means that the ion has a positive charge of 1e. Singly-ionized (or mono-ionized) ions have a charge of +/- 1. Multiple-ionized ions have a higher charge state.
67	**Beam current**: Current in the ion beam leaving an ion thruster typically expressed in A (1 A = 1 C/s). In case of mono-ionized ions the beam current is a direct measure for the number of ions leaving the thruster.
68	**Electrical efficiency**: Ratio of beam power to total input power
69	**Ionization efficiency**: Ratio of flow rate of mono-ionized ions and total mass flow rate.
70	**Propellant utilization efficiency**: Fraction of input propellant mass that is ionized. Notice the slight difference in the definition compared with the ionization efficiency.
71	**Plasma propulsion**: A form of electric propulsion in which electric and magnetic forces are used to accelerate a plasma stream to a high velocity in a directed beam.
72	**Plasma**: An ionized gas containing ions and electrons in about equal numbers so that the resultant space charge is very small. Plasma can be generated by heating a gas to extreme temperatures. Because of heating molecules turn in to their constituent atoms. Further heating leads to ionization, turning it into plasma.
73	**Electromagnetic thruster**: Thruster that generates and accelerates a plasma using electromagnetic forces
74	**MPD thruster**: Thruster that use a continuous electrical arc discharge to generate a neutral plasma which is accelerated by electromagnetic forces

75	**Pulsed Plasma Thruster**: Plasma thruster working in a pulsed mode. Plasma thruster generally requires high electric power. By pulsing the thruster, the average power needed reduces.
76	**Specific mass of power plant**: Mass of power plant (or energy source) per unit of (thermal or electric) power delivered
77	**Optimum jet (or exhaust) velocity**: Velocity at which the total propulsion system mass of an advanced rocket (having a separate energy source) is minimized.

List of Formulae

Rocket thrust (in absence of pressure forces)

$$F_{thrust} = mv_{jet}$$

Rocket thrust

$$F_T = mv_e + (p_e - p_a)A_e$$

Jet power (kinetic beam power)

$$P_{jet} = \frac{1}{2}m(v_{jet})^2$$

Propulsive efficiency

$$\eta_{propulsive} = \frac{P_{thrust}}{P_{thrust} + (P_{jet})_{abs}}$$

Cycle efficiency

$$\eta_{cycle} = \frac{P_{jet}}{P_{available}}$$

Overall or total efficiency

$$\eta_{total} = (\eta_{propulsive})(\eta_{cycle})$$

Energy equation open system

$$Q + (E_{internal} + PE + KE + PV)_{in} = W + (E_{internal} + PE + KE + PV)_{out}$$

Ideal gas law

$$\frac{p}{\rho} = RT$$

Specific gas constant

$$R = \frac{R_A}{M}$$

Isentropic relation

$$\frac{p}{\rho^{\gamma}} = \text{constant}$$

Specific heat ratio

$\gamma = c_p/c_v$.

Relation between heat capacities

$$R = \frac{R_A}{M} = c_p - c_v$$

Ideal exhaust velocity thermal rocket

$$v_e = \sqrt{2 \cdot \frac{\gamma}{\gamma - 1} \cdot \frac{R_A}{M} \cdot T_c \cdot \left(1 - \left(\frac{p_e}{p_c}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\right)}$$

Limit velocity thermal rocket

$$(v_e)_{lim} = \sqrt{\frac{2\gamma}{\gamma - 1} \cdot \frac{R_A \cdot T_c}{M}}$$

Critical mass flow rate

$$m = \frac{\Gamma \cdot p_c \cdot A^*}{\sqrt{R \cdot T_c}}$$

Vandenkerckhove function

$$\Gamma = \sqrt{\gamma} \cdot \left(\frac{2}{\gamma + 1}\right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)}$$

Characteristic velocity

$$c^* = \frac{1}{\Gamma} \cdot \sqrt{R \cdot T_c}$$

Nozzle area or expansion ratio

$$\frac{A_e}{A^*} = \frac{\Gamma}{\sqrt{\frac{2\gamma}{\gamma - 1} \cdot \left(\frac{p_e}{p_c}\right)^{\left(\frac{2}{\gamma}\right)} \left(1 - \left(\frac{p_e}{p_c}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\right)}}$$

Thrust coefficient

$$C_F = \frac{F_T}{p_c\, A^*}$$

Flow separation criterion (Summerfield)

$$p_e/p_a > 0.35\text{-}0.45$$

Flow separation criterion (Schmucker)

$$p_e/p_a > (1.88\, M - 1)^{-0.64}$$

Chemical power

$$P_{chemical} = M_{fuel} \text{ x } HV$$

Solid propellant/fuel mass flow rate

$$m = \rho_s \cdot r \cdot S$$

Burning rate law

$$r = a \cdot p^n$$

Pump power

$$P_p = \frac{m \cdot \Delta p}{\eta \cdot \rho} = \frac{Q \cdot \Delta p}{\eta}$$

Chemical rocket net mass

$$M_{\text{rocket net}} = k\, M_{\text{propellant}}$$

Sensible heat

$$Q_{\text{sensible}} = M\, c\, \Delta T$$

Latent heat

$$Q_{\text{latent}} = M\, L$$

Heater efficiency

$$\eta_h = \frac{P_{heat}}{P_{in}}$$

Thermal power

$$P_{\text{heat}} = q_{\text{convection}}\, A_{\text{contact surface}}$$

Convection heat flux

$$q_{\text{convection}} = h_c\, \Delta T$$

Electric power

$$P_{\text{electric}} = V\, I = I^2 R$$

Electric resistance

$$R = \rho \frac{L}{A}$$

Solar power

$$P_{\text{solar}} = S\, A_{\text{collector}}$$

Electric force experienced by electric particle in electric field

$$\bar{F} = q\bar{E}$$

Charged particle velocity

$$w = \sqrt{\frac{2q\Delta V}{m_q}}$$

Ion mass

$$m_{\text{charged particle}} = \hat{M}/N_A$$

Beam current

$$I_{\text{beam}} = N_{\text{ions}} \cdot q$$

Utilization or ionization efficiency

$$\eta_m = \frac{m_i}{m}$$

Maximum ion current density

$$j = \frac{4\varepsilon_0}{9} \cdot \left(\frac{2q}{m_q}\right)^{1/2} \cdot \frac{V^{3/2}}{d^2}$$

Electrical beam power

$$P_{\text{accel}} = \Delta V \cdot I_b$$

Ionization power

$$P_{\text{ion}} = \eta_m \cdot m/\hat{M} \cdot N_A \cdot V_i$$

Electrical efficiency

$$\eta_E = \frac{P_{\text{accel}}}{P_{\text{in}}}$$

Lorentz force

$$\bar{F}_T = \bar{I} \times \bar{B}$$

Magnetic induction

$$B = \mu\, H$$

MPD thrust

$$F_T = \frac{\mu I^2}{4\pi}\left(\ln\frac{r_a}{r_c} + \frac{3}{4}\right)$$

System specific impulse

$$I_{ssp} = \frac{I}{W} = \frac{F \cdot t}{\left(M_p + \alpha_W \cdot P_W\right) \cdot g_o}$$

Specific mass of power plant

$$\alpha_w = \frac{M_w}{P_w}$$

Characteristic velocity

$$w_{opt} = \sqrt{\frac{2\eta_T t_{burn}}{\alpha_w}}$$

Answers to exercise problems

In this section the answers belonging to the exercises presented earlier are given.

1) 41.2 kN
2) 19.5 km
3) Answers:
   a) Hydrogen mass flow: 1.92 kg/s;
   b) Beam power: 66.7 MW;
   c) Throat diameter: 7.3 cm;
   d) Maximum allowable (geometric) expansion ratio: 16.93;
   e) Nozzle pressure ratio: 300;
   f) True exhaust velocity: 7648 m/s;
   g) Pressure in nozzle exit: 3333 Pa;
   h) Pressure thrust in vacuum: 235.6 N;
   i) Effective specific impulse: 792.4 s.
4) Answers:
   a) Propellant mass flow rate:40 kg/s;
   b) On board propellant mass: 48 ton;
   c) Propellant volume: 676 m^3;
   d) Jet (or beam) power: 1.39 GW;
   e) Hot gas temperature at reactor outlet: 2500 K;
   f) Thruster input power: 2.32 GW;
   g) Limit velocity given a hydrogen specific heat ratio of 1.4 and a molar mass of 2 g/mol: 8533 m/s;
   h) Nozzle pressure ratio using the earlier given specific heat ratio: 3.85×10^4;
   i) True exhaust velocity: 8106 m/s.
5) Answers:
   a) Mass flow rate of (mono-ionized) ions: 0.78 mg/s;
   b) Total mass flow rate: 0.98 mg/s);
   c) Jet power: 400 W;
   d) Input power: 800 W;
   e) Beam voltage and beam current: 701 V and 0.57 A.
6) Answers:
   a) Required input power per thruster: 12.5 kW;
   b) Specific power of the system with all thrusters operative: 12.7 W/kg;
   c) Optimum exhaust velocity given a total thrust time of 180 days: 17.7 km/s;
   d) Optimum required input power given that the thrust remains constant: 110.6 kW.

Annex A: Advantages of liquid, solid & hybrid chemical rockets

In this document, the various advantages and disadvantages of liquid, solid and hybrid propellant rockets are described with as purpose to allow initial trade-offs of the three types of rocket motors against the requirements. The numbers mentioned in the text have been taken from existing engines and are considered typical values for space applications. By no means should these values be interpreted as extremes.

1. Performance: High performance liquid rockets using high-energy bipropellants offer a sea level specific impulse in the range 270-360 s. High performance solid rockets are more limited, offering a sea level specific impulse in the range 210-265 s. Hybrid rockets offer a specific impulse in the range 230-270 s, which is similar to those obtainable with bipropellant motors (apart from the very high performing ones, like liquid oxygen – liquid hydrogen). Monopropellant liquid rocket motors offer a specific impulse in the range 160-190 s. Further information can be obtained from Table 1.

Table 1: *Sea level specific impulse of some typical liquid chemical propellants*

Propellant combinations	*I_{sp} Range (sec)*
Monopropellants (liquid):	
Low-energy monopropellants	160 to 190
Hydrazine	
Ethylene oxide	
Hydrogen peroxide	
High-energy monopropellants:	
Nitromethane	190 to 230
Bipropellants (liquid):	
Low-energy bipropellants	200 to 230
Perchloryl fluoride-Available fuel	
Analine-Acid	
JP-4-Acid	
Hydrogen peroxide-JP-4	
Medium-energy bipropellants	230 to 260
Hydrazine-Acid	
Ammonia-Nitrogen tetroxide	
High-energy bipropellants	250 to 270
Liquid oxygen-JP-4	
Liquid oxygen-Alcohol	
Hydrazine-Chlorine trifluoride	
Very high-energy bipropellants	270 to 330
Liquid oxygen and fluorine-JP-4	
Liquid oxygen and ozone-JP-4	
Liquid oxygen-Hydrazine	
Super high-energy bipropellants	300 to 385
Fluorine-Hydrogen	
Fluorine-Ammonia	
Ozone-Hydrogen	
Fluorine-Diborane	

Table 2: *Sea level specific impulse of some typical solid chemical propellants*

Propellant combinations:	I_{sp} *Range (sec)*
Oxidizer-binder combinations (solid):	
Potassium perchlorate:	
Thiokol or asphalt	170 to 210
Ammonium perchlorate:	
Thiokol	170 to 210
Rubber	170 to 210
Polyurethane	210 to 250
Nitropolymer	210 to 250
Ammonium nitrate:	
Polyester	170 to 210
Rubber	170 to 210
Nitropolymer	210 to 250
Double base	170 to 250
Boron metal components and oxidant	200 to 250
Lithium metal components and oxidant	200 to 250
Aluminum metal components and oxidant	200 to 250
Magnesium metal components and oxidant	200 to250
Perfluoro-type propellants	250 and above

2. Size: High-density solid propellants have a mass density in the range of 1500 – 1900 kg/m^3 compared to about 1000 – 1350 kg/m^3 for high-density storable liquid propellants. This compares favorably to the 280 - 375 kg/m^3 attainable for the high performing liquid Oxygen – liquid Hydrogen propellant. For hybrid propellants, it is possible to obtain a density in the range 1000 – 1200 kg/m^3.
3. Flexibility: For SRM's, extinction and re-ignition is hard to realize. Hybrid and liquid rockets on the other hand are much easier to shutdown and re-start. For example for monopropellant liquid rockets and hypergolic bipropellant rockets the propellant decomposes under the action of a catalyst (monopropellant) or is self-igniting (hypergolic bipropellant) this can be accomplished through simply opening and closing of a valve. For other bi-propellants, the same advantage holds, but an igniter may be necessary to start combustion. For example, liquid propellant motors using the monopropellant hydrazine or the hypergolic propellant combination of hydrazine and nitrogen tetroxide allow for a precision pulse-mode, where thrust is produced in accurately reproducible impulse bits with a total number of pulses that can easily reach 100000.
   Thrust magnitude control for liquid and hybrid rockets is simply through controlling the flow of the liquid propellant. For solid rockets the thrust is 'pre-programmed' and difficult to change during flight. For example, the Lunar Module Descent Engine had a 10:1 throttle range. For most applications, however, a range of 3:1 seems more than acceptable.
   Liquid rockets allow for easy steering of the thrust vector by use of a gimbal. Steering of the thrust vector for hybrid and solid rockets through gimballing is more complicated as a comparable solid or hybrid motor is much larger. To achieve thrust vector control for solid or hybrid motors, we nowadays use either liquid injection or a vectorable nozzle. Typical thrust vector control angles in practice are up to 9-10 degrees for both liquid and solid rocket motors.
4. Safety: All rocket propellants are explosives, i.e. a substance (or mixture of substances),

which is capable, by chemical reaction, of producing gas at such a temperature and pressure as to cause damage to the surroundings. Liquid and hybrid propellants are more apt to external stimuli than solid propellants. For example, there have been accidents where liquid oxygen was spilled onto asphalt, which caused an explosion when a truck was driven over the spill. The small amount of heat and pressure caused by the tire was enough to trigger an explosion in that concentration of oxygen.
For solid propellants, although they require stronger stimuli to ignite, there is the added fact that fuel and oxidiser are intimately mixed so all ingredients are ready at hand. Once burning starts, it will be almost impossible to stop it. Solids also have potential for detonation of the propellant. The latter requires extensive safeguards during propellant manufacturing as well as launcher- and payload processing. For liquid propellants the risk of inadvertent ignition is limited to those cases where leakage occurs, e.g. caused by breakage or launching incidents. For hybrid rocket motors breakage or launching accidents are unlikely to result in an explosion or in involuntary ignition and operation.
Most liquid propellants, like fluorine, hydrazine, nitric acid, mono-methyl hydrazine, oxygen etc, are difficult to handle, because they are very toxic, or corrosive. This requires special pre-cautions; see e.g. Figure on liquid propellant loading. In contrast, solid propellants as well as the solid component of hybrid propellants are relatively harmless in human contact.

Figure 1: Liquid propellant loading (ESA)

5. Environmental load: The major exhaust products of various solid and liquid systems are shown in the table below.

Table 3: Major exhaust products of some typical rocket propellants

Propellant system	Major exhaust products
Ammonium perchlorate/aluminium	HCL, H_2O, Al_2O_3, CO_2, N_2
Liquid Oxygen/liquid Hydrogen	H_2O
Liquid Oxygen/hydrocarbon	CO_2, hydrocarbons, H_2O
Nitrogen tetroxide/dimethylhydrazine	NO_x, CO_2, N_2

On a single launch basis, the Space Shuttle injects the greatest mass of exhaust products into the atmosphere of any current propulsion system. Each launch vehicle consists of about 1000 tons of solid propellant and about 800 tons of liquid propellant. Typical concerns related to rocket exhaust products are toxicity, acid rain, Ozone depletion, and the ‘Greenhouse effect’. Global impact of rocket exhaust on stratospheric ozone concentration and ground level ultraviolet radiation is estimated at maximum 0.02%.Further information on the environmental effects of rocket exhaust products can be obtained from e.g. [R.R. Bennet, et al., 1992].

6. Reliability: SRMs have a simple structure containing few parts. They consist of a pressure

vessel, an igniter, a solid grain and a nozzle. Liquid propellant rocket motors are more complicated. They generally consist of one or more tanks; a propellant feed system, a hydraulic system and the engine itself. Liquid monopropellant systems are somewhat less complicated as only a single propellant must be stored and fed to the rocket engine. Hybrid rocket motors can be compared to liquid monopropellant rockets, because like monopropellant systems, they have a single feed system. [Andrews and Haberman, 1991] reported a reliability of 0,998 for SRMs and 0.985-0.989 for liquid propellant systems. More recent data [SPIAG, 1999] for launcher stages indicates an overall reliability for solid propulsion systems of 0,9946 and for liquid propulsion systems of 0,9803. Table below provides some more detail.

Table 4: US Space Flight History (1964 - May 1999)

Motor type	Failures	Attempts	Success rate
Solid propulsion			
Upper stage	10	627	0.9841
Monolithic	6	2464	0.9976
Segmented	3	402	0.9925
Liquid propulsion			
Cryogenic	9	268	0.9664
Other	28	1612	0.9826

7. Costs: Compared to liquid propulsion, solid propulsion is considered low cost. The main reason is that they are much simpler in design. For illustration, [Andrews and Haberman, 1991] compared <u>large</u> rocket systems and reported a solid propulsion cost of US$ 0.017 per Ns of total impulse and US$ 0.045 Ns for liquid propulsion More recent data [SPIAG, 1999] for launcher stages indicate a first unit cost for solid propulsion systems of 0.0171 and for liquid propulsion systems of 0.0445, see table below for more detail, and a development cost for liquid systems 6 times higher than for solids. Hybrid rocket motor costs are expected to be somewhere in between those of liquids and solids.

Table 5: First Unit Cost [SPIAG 1999]

Motor type	Approximate first unit cost		
	$/kg mass	$/N thrust	$/Ns total impulse
Solid propulsion	41.0	1.52	0.0171
Liquid propulsion	113.1	8.88	0.0445

8. Development time: Because of their simplicity, solid rockets have a shorter time to develop than liquid rockets. For example, for the Ariane 5 solid rocket booster the development time is 7 years whereas for the cryogenic main engine (Vulcain) this is 9-12 years including a 3-year long technology preparation period preceding the actual development.
9. Operability: Solid rocket motors require short preparation time since solid propellants can be stored, loaded into the solid rocket motor, over long periods of time. Liquid propellant rockets require a long preparation time due to propellant loading. In case cryogenic liquid propellants are used, extensive cooling is needed. This makes cryogenic propellants extremely difficult to store for long periods of time. Hence, they are only filled a few hours prior to launch. However, even then the propellants are constantly evaporating, causing the formation of ice on the storage tanks, which may cause damage. Also in case of a launch abort, the propellants must first be off-loaded before the rocket can be moved back to the assembly station.

References

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- Bennet R.R., et al.; Chemical Rockets and the Environment, Aerospace America, May 1991
- SPIAG; Solid Rocket Motor Briefing, June 1999.

Errata - Reader “Rocket Propulsion”

Page 19: the correct value of the true exhaust velocity in Example 2 is 845.45 m/s (not 943.5 m/s). As a consequence, the correct value of the momentum thrust is 63.49 kN (not 70.86 kN), and the correct value of the overall thrust is 52.88 kN (not 60.51 kN).

Page 56 (bottom page): Units for current density should be $C/s/m^2$ or A/m^2.

Page 64: The parameter α_w (inverse specific power) sometimes is also referred to as specific mass.