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RESEARCH ON LOCAL DEFORMATION PROPERTY OF ASPHALT MIXTURE USING DIGITAL IMAGE CORRELATION

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ABSTRACT

Failure behavior of asphalt mixture is the process of local deformation accumulation, therefore, local deformation investigation is the base work of failure behavior. Digital image correlation (DIC) method is an optical method which can calculate the local deformation in the failure process. This paper aims to evaluate the precision, select the optimal test parameters for DIC deformation measurement based on natural speckle image, and investigate the local deformation characteristic in the failure process for different mixtures. The results show the image quality of AC with smaller normal maximum aggregate size (NMAS) is better for DIC deformation calculation, the relative deviation of virtual translation and shear test can satisfy the precision requirement, the optimal subset size is 31. Based on the strain distribution, the optimal local deformation measure length for indirect tensile test is set as 50mm, the critical steady strain is proposed based on R^2 obtained by the least squares fitting method, according to local deformation characteristic, the results show SMA can bear more deformation before unstable deformation stage and failure.

Keywords: Asphalt mixture; digital image correlation method; image quality evaluation; local deformation property

1 Introduction

Asphalt mixture is regarded as a multi-phase composite material system including aggregate, asphalt binder and filler, the difference of modulus between asphalt mastic and aggregate is the main reason of strain localization phenomena and complex stress state [1-3]. The failure behavior of asphalt mixture is the process of local deformation accumulation, the damage appear in asphalt mixture when local deformation is larger than that material can cope with, damage accumulation will cause failure of asphalt mixture, therefore, local deformation investigation is the base work of failure behavior.

Traditional deformation sensors such as linear variable differential transformer (LVDT) are widely used in material deformation measurement, for local deformation measurement, traditional sensor has some disadvantages:

(1) The unpredictability of local damage position make the sensor setting difficult, the sensor glued on the surface will also influence the deformation of material.

(2) The strain obtained by traditional sensor is the average strain of a line segment, no strain field information make it hard to identify the damage area which will affect the measured length selection.

Technology of optical measurement have been proposed for local deformation measurement, digital image correlation (DIC) as one of optical measurements was proposed in the 1980's [4-5], it was a direct displacement and strain fields measurement method with advantages of full-field, real-time, non-contact, flexibility and so on. Seo [6] first used DIC technique as a possible displacement/strain measurement method for asphalt mixture. Tan [7-8] compared DIC method with LVDT and determined the minimum calculation gauge length of the indirect tensile (IDT) test, according to the strain field obtained by DIC, it was found that the Poisson ratio was not a constant. Many researchers [9-12] investigated fracture phenomena in asphalt material based on DIC method.

In general, the surface of measured material for DIC should be covered by artificial speckle, the artificial spackle can improve calculation precision, however, it will also cover the aggregate and asphalt distribution information which is important for local damage identification. The natural surface of asphalt mixture includes natural speckle duo to different

material inside, if the measured result based on the natural surface can satisfy the measured precision requirement, the image of natural surface can be used for deformation measurement.

Based on the problems above, this paper aims to evaluate the precision, select the optimal test parameter for DIC deformation measurement based on natural speckle image, identify the local deformation area, propose the critical deformation determination method, finally, investigate the local deformation characteristic in the failure process for different mixtures.

2 Materials and methods

2.1 Materials property

Basalt aggregate is used in this research, two asphalt binders are used to produce the different mixtures: one neat asphalt for AC asphalt mixture and one SBS polymer-modified asphalt for SMA asphalt mixture. Details of the asphalt binder properties are listed in Table 1 and Table 2.

Table 1 Basic properties of neat asphalt.

Index	unit	Test result	Technical requirements
penetration, 25°C, 100g, 5s	0.1mm	79	60~80
ductility 15°C, 5cm/min	cm	133.0	≥100
Softening point	°C	51.4	≥44

Table 2 Basic properties of SBS polymer-modified asphalt.

Index	unit	Test result	Technical requirements
penetration, 25°C, 100g, 5s	0.1mm	66.9	60~80
ductility 5°C, 5cm/min	cm	43.3	≥40
Softening point	°C	66.5	≥50
elastic recovery (25°C)	%	82	≥60

2.2 Specimen preparation

The mix design procedure is determined as the Chinese specification JTG F40-2004, the Marshall design procedure is used in this research to optimize the asphalt content for the mixtures. Six types of asphalt mixtures (e.g., AC-10, AC-13, AC-16, SMA-10, SMA-13, SMA-16) with optimal the asphalt content are tested in this research, the grading of the mixtures are the mid-value of the grading limit based on Chinese specification JTG F40-2004.

The aggregates, the asphalt binders and mixing equipment are heated for 4 h at 155°C, the temperature for modified mixture is 175°C, cylindrical specimens are obtained by compacting the mixtures 120 times using a gyratory compactor. In order to observe the failure position, each cylindrical specimen is sawn to obtain two effective plates by discarding the top and the bottom plates for reducing density gradient effect, the height of the treated sample is 63.5mm, the specimens are used to perform the IDT test at 25°C as the Chinese specification JTG E-20-2011.

2.3 Digital image correlation method

In this research, Vic-2D 2009 produced by Correlated Solutions, Inc. is employed. This system is a complete, turn-key system for measuring the shape, displacement and strain of surfaces. A digital camera, Point Grey GRAS-20S4C/M (2.0MP, Sony ICX274 CCD, 1/1.800, resolution 1624*1224 at 30 fps), is currently employed. Accuracy of displacement is approximately 0.01 pixels on a point-to-point basis including translation and rotation measurements. Therefore, when observation area is 160 mm*120 mm, the horizontal displacement resolution of this equipment can be calculated as: $(160 \text{ mm}/1600 \text{ pixel}) * 0.01 \text{ pixel} = 1 \text{ }\mu\text{m}$, as well as the vertical displacement resolution. A sequence of digital images depicting an area of finite extent in the specimen is acquired during testing. The images are automatically processed by the software, providing accurate displacement/strain field. An iterative least-squares algorithm (least squares matching) is applied for the extraction of image correspondences based on the similarities between grey values.

The camera which focus on the whole surface of the specimen is directly connected with a personal computer. A lighting system which is used to provide adequate illumination for the specimen surface should be placed at the proper position. The test equipment is shown in Fig. 1.

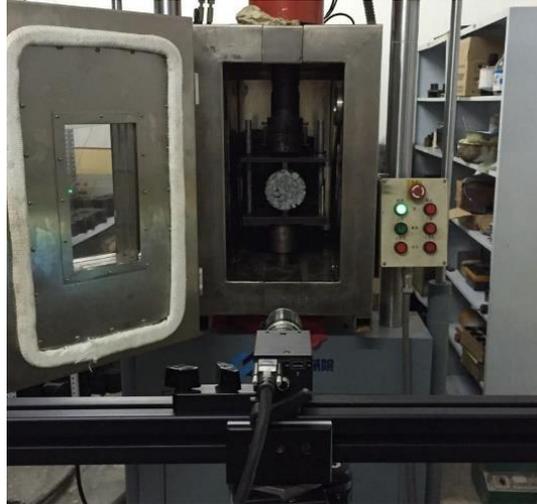


Fig.1 Test equipment in this research

3 Accuracy investigation and parameter selection

3.1 Image quality evaluation index

In order to get accurate strain field, the image obtained in the test process should include enough spackle pattern which carry the deformation information for strain calculation by DIC system. Typically, the specimen surface is covered by white color followed by spraying black painting dots on it, the aggregate and asphalt distribution in asphalt mixture is overlaid by the artificial speckle pattern, it is hard to identify the deformation of designated spot in asphalt mixture, such as the interface between asphalt and aggregate, therefore, if natural surface texture can meet the requirement of DIC algorithms, it is convenient for extracting the local deformation of asphalt mixture.

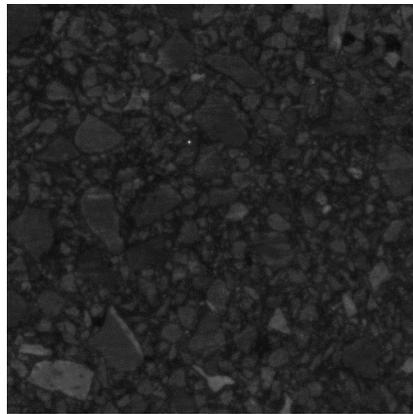
The first step for using image of asphalt mixture without artificial speckle pattern is to evaluate image quality, mixtures with different nominal maximum aggregate size (NMAS) and grading show different natural surface texture, six kinds of mixtures are employed to judge the feasibility of natural surface texture.

The images are divided into many subsets in DIC calculation process, DIC system obtain the deformation information of a point in a chosen subset size by correlation of subsets on a pair of images, therefore, the image quality of the subsets seriously influence the calculation precision. Many methods are proposed for image quality evaluation, average speckle size is used to evaluate digital laser speckle image, and this parameter is a global indicator for the whole image, it cannot reflect the local information in the image subset which affect the

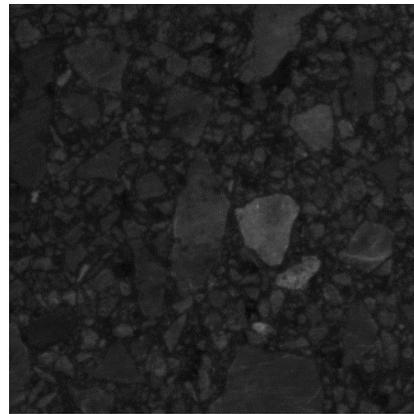
deformation calculation accuracy significantly. Subset entropy was proposed by Sun [13], this indicator aims to show the local variance of pixels in the image, the formula of subset entropy is shown as follow.

$$\delta = \frac{\sum_{P \in S} \sum_{i=1}^8 |I_P - I_i|}{2^\beta MN} \quad (1)$$

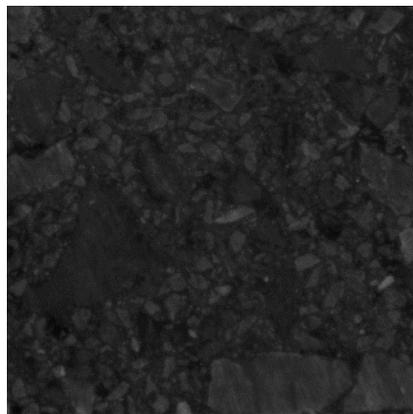
Where δ is the subset entropy, P is any pixel in a subset S with the size $M \times N$, I_P is the grey level of point P , I_i is the grey level of 8 pixel points around point P , β is the pixel depth. The equation above shows that subset entropy is the average intensity deviation of any pixel point in the subset from its neighboring points, computational accuracy increase along with the larger subset entropy. Since subset entropy has the advantage for local spackle pattern quality evaluation, it is utilized herein to evaluate the spackle pattern quality of different asphalt mixtures. During the IDT test, circle surface with the size 1000 pixels \times 1000 pixels of the cylinder specimen is photographed, due to the calculation area is a pixel matrix, a square with the size 600 pixels \times 600 pixels from the center of the circle is taken for subset entropy calculation, one image for each mixture, the images of different mixtures are shown in Fig.2.



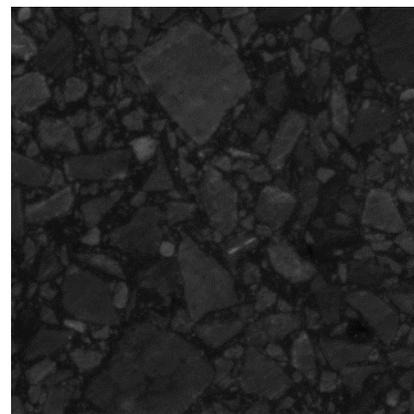
(a) AC-10



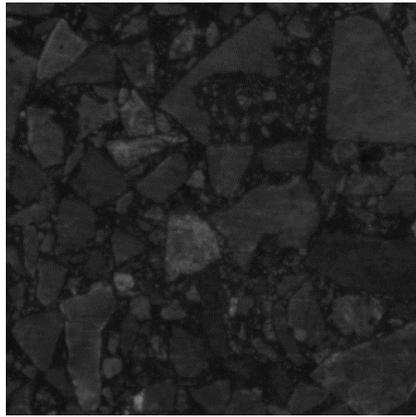
(b) SMA-10



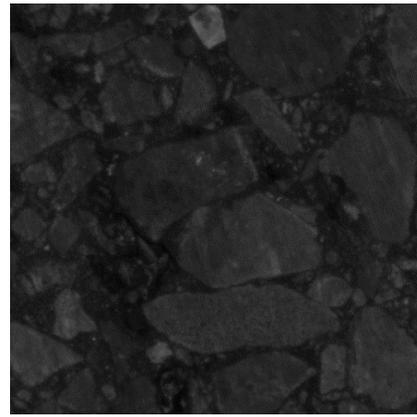
(c) AC-13



(d) SMA-13



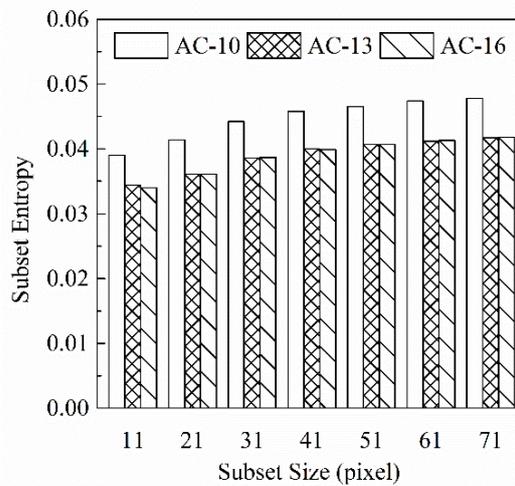
(e) AC-16



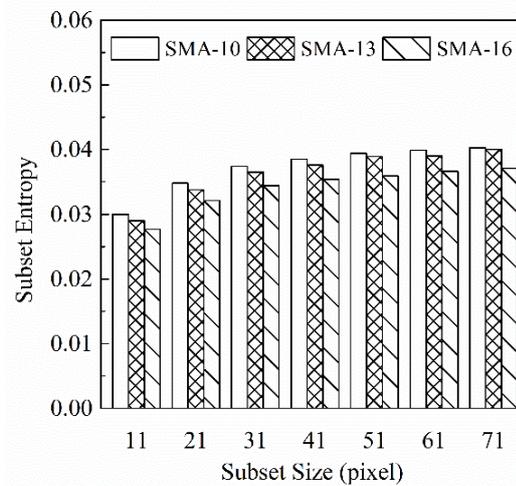
(f) SMA-16

Fig.2 Natural surface of different mixtures

Fig.2 (a)-(f) are the images of AC-10, SMA-10, AC-13, SMA-13, AC-16 and SMA-16 respectively, aggregate and asphalt mortar can be identified clearly, the exposed aggregate is seen as bright zone in the image, the other part of image is asphalt mortar with the predominant dark color. The subset entropy of images in Fig.1 with different subset size including 21, 31, 41, 51, 61, 71 pixels are shown in Fig.3.



(a) AC



(b) SMA

Fig.3 Entropy of different size subset for different mixtures

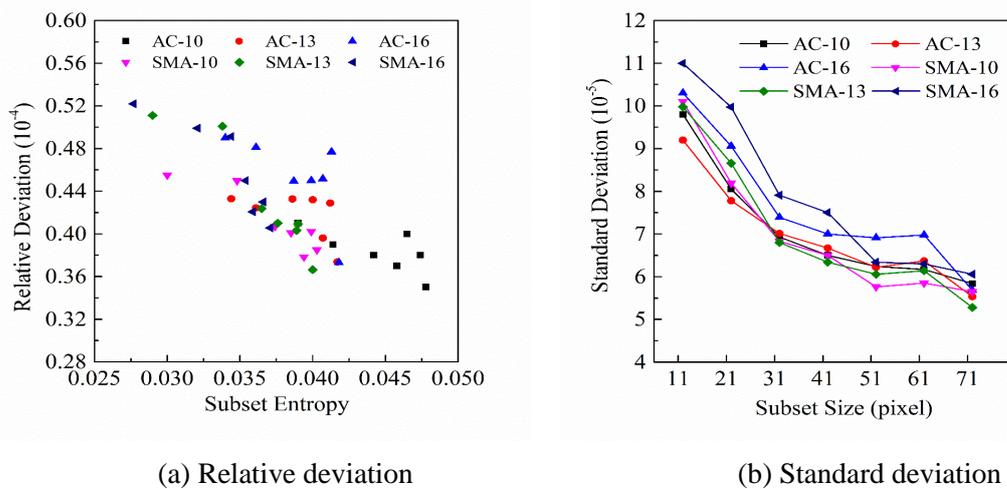
Fig.3 show that the subset entropy increase with subset size for six kinds of mixtures, it means if we select larger subset size for DIC calculation, the subset will carry more information which is beneficial for computational accuracy. Fig.3 also show the relation between subset entropy, mixture type and NMA (Nominal Maximum Aggregate Size), for the same subset size, the subset entropy decrease with the increasing NMA, subset entropy of AC is larger than that of SMA, the reason for that is the same subset of AC with small

NMAS include more small particles and interface, that will increase information content of subsets.

3.2 Calculation accuracy and optimal subset size

In order to investigate the precision of DIC deformation test with natural speckle image, virtual translation and shear test are utilized to get the deviation between theoretical value and measured value, if the deviation can satisfy the accuracy requirement, it means it is feasible to adopt natural mixture image as DIC test image. The most important test parameter of DIC system is subset size, optimal subset size will reduce the errors, the software default value is based on the standard deviation, in order to get a smaller standard deviation, the default subset size is usually very large, the strain field based on this subset size is uniform, it will lost the local deformation information, therefore, in this part, test accuracy and optimal subset size are investigated.

The virtual translation test is conducted first, the images with the size 600 pixels \times 600 pixels are moved up by one pixel, the deformation fields of six mixtures are obtained by DIC system with different subset size, the subset entropy of different mixture type and subset size is different has been mentioned before, the average relative deviation of pixels between true deformation and calculated value can be calculated, the relation between relative deviation and subset entropy is shown in fig.4 (a).



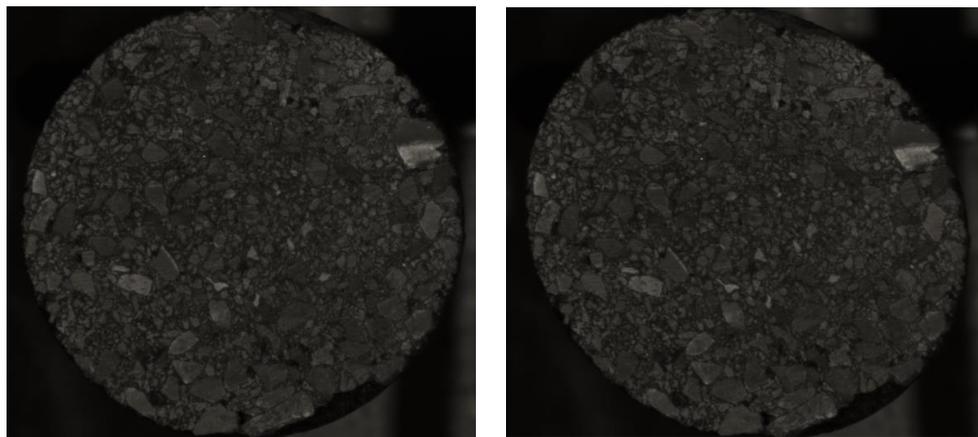
(a) Relative deviation (b) Standard deviation

Fig.4 Relation between relative deviation, standard deviation and subset size

From fig.4 (a) we can see, relative deviation linearly decrease with subset entropy, it illustrate more information in subset will improve the calculative precision, smaller NMAS

and larger subset size can reduce the relative deviation, the relative deviation is about 4×10^{-5} . The standard deviation of displacement field is shown in Fig.4 (b), we can see standard deviation decrease with subset size, it decrease rapidly at early stage, and then become steady when the subset size is larger than 31.

Virtual shear test is also conducted in the part, for the pixels matrix, from the line two, every line is applied one pixel horizontal movement related to the last line, the image before and after deformation are shown as follow.

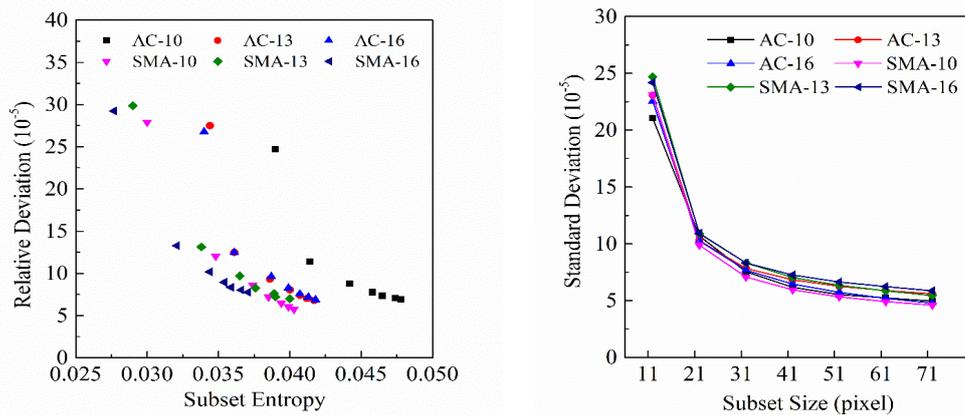


(a) Before deformation

(b) After deformation

Fig.5 Images before and after virtual shear deformation

Relative deviation and standard deviation are also analyzed, the results are shown in Fig.6.



(a) Relative deviation

(b) Standard deviation

Fig.6 Relation between relative deviation, standard deviation and subset size

Fig. 6 show the similar results with those of virtual translation test, however, relative deviation of a set of points with relative lower subset entropy is lower than that of others, the points present displacement field with 11 subset size, this phenomenon illustrate if subset size

is smaller than 11, the relative deviation increase rapidly, the reason for that is if the subset size is 11, the information in subset is insufficient for the DIC algorithm which will decrease the calculation accuracy. Except the results with 11 subset size, the relative deviation of virtual is about 8×10^{-5} .

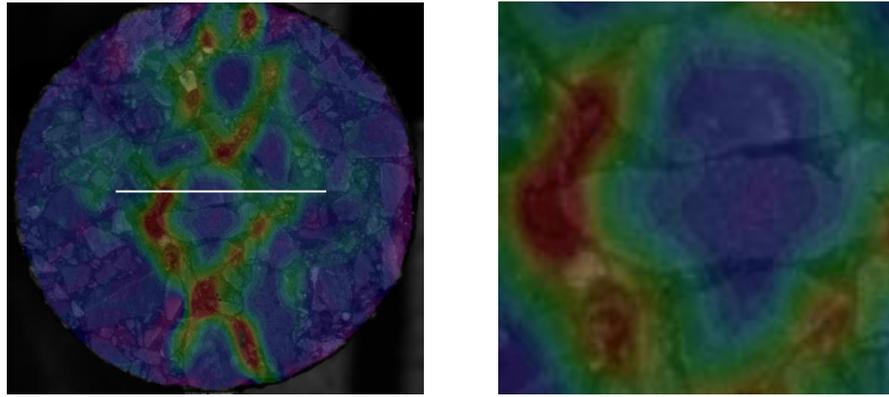
From the results above, for 1 pixel movement in virtual translation and shear test, the relative deviation is about $4 \sim 8 \times 10^{-5}$ which is enough for deformation measurement of asphalt mixture. For DIC system parameter selection, a larger subset size may result in the incorrect approximation of deformation calculated by its displacement function, while the smaller subset size can lead to the deformation is susceptible to the image quality effect. Subset size increase will reduce standard deviation, when subset size is larger than 31, the standard deviation decrease slowly, in order to balance the systematic error and random error, 31 subset size is selected as the post-processing parameter in the next work.

4 Local deformation measurement result

For IDT test, the peak deformation position of homogeneous material is in the middle of the sample, however, asphalt mixture is heterogeneous material, main crack appear around the center randomly, and it is hard to set up sensors to measure the local deformation due to position uncertainty. DIC method can obtain strain field and deformation between any two points, which is convenient to investigate the local deformation behavior, deformation process can be obtained based on image sequence collected during the test.

4.1 Peak strain position and failure region

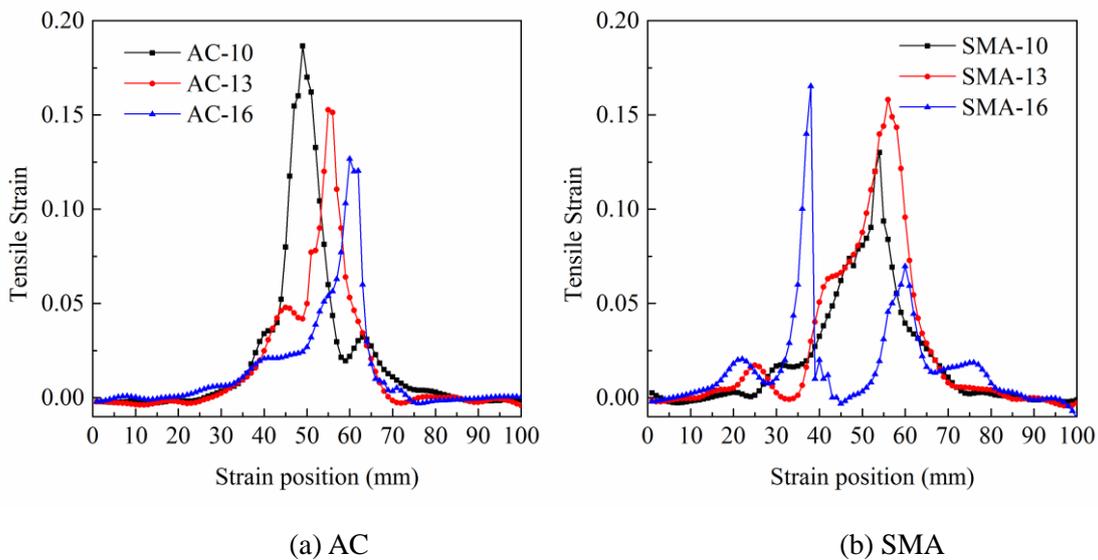
The modulus of aggregate is much larger than that of asphalt mortar, therefore, the failure position depends on aggregate distribution, generally, if there is an aggregate in the center, the failure position will deviate the center, the larger the aggregate size, the farther the failure position deviate the center. Fig. 7 (a) is typical horizontal tensile strain field of asphalt mixture in IDT test, the red area is strain concentration area, the deformation concentrate in a connected band which is also the crack position, Fig. 7 (b) is the local area of Fig. 7 (a), the local deformation band is in the asphalt mortar around the aggregate, the failure position is not in the middle of the sample.



(a) Horizontal strain field (b) Local strain field

Fig.7 Strain field of IDT test

Tan [8] investigated the horizontal tensile deformation by DIC, the results showed that the tensile strain was small beyond the centre 30 mm area, 30 mm area in the centre of the specimen was suggested as the minimum calculation (gauge) length. The centre 30mm area include most deformation, however, large aggregate will make the deformation concentration area uncertain. Therefore if we want to study the local deformation behavior, the width of deformation concentration area should defined. For the DIC test, maybe there are one or two local deformation band cross over the horizontal diameter , however, only one main crack propagate at last, when the main crack appear, the horizontal tensile strain distribution at the horizontal diameter of six kinds of asphalt mixtures are shown as follow.



(a) AC (b) SMA

Fig.8 Horizontal tensile strain distribution of different mixtures

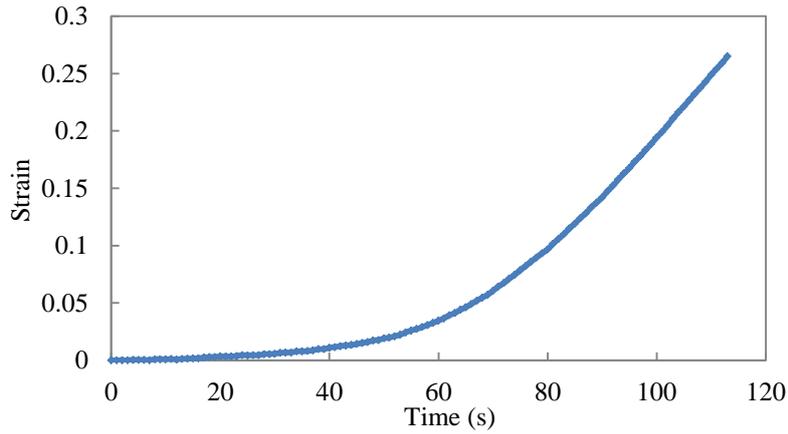
From Fig. 8 we can see, generally, there is a main peak of strain distribution, the second peak value also appear which is much smaller than main peak value, strain distribution curve

fluctuation of SMA is larger than that of AC due to more coarse aggregate inside. For LVDT deformation test, the gauge length is 25.4 mm which cannot cover all the deformation, it will cause measurement error. Previous study [8] show that the strain change out of the centre 30 mm area is minimal, this value is enough for AC, however, it cannot satisfy the measurement area requirement, especially for mixture with larger NMA. Because of the aggregate movement in the failure process, the strain at the edge of sample is compression strain which can be found in Fig.8, therefore, larger measuring length will cause error due to compression strain. In this study, 50 mm measuring length can cover almost all the deformation for mixture whose NMA is smaller than 16, the measure length is set as 50mm for the next research.

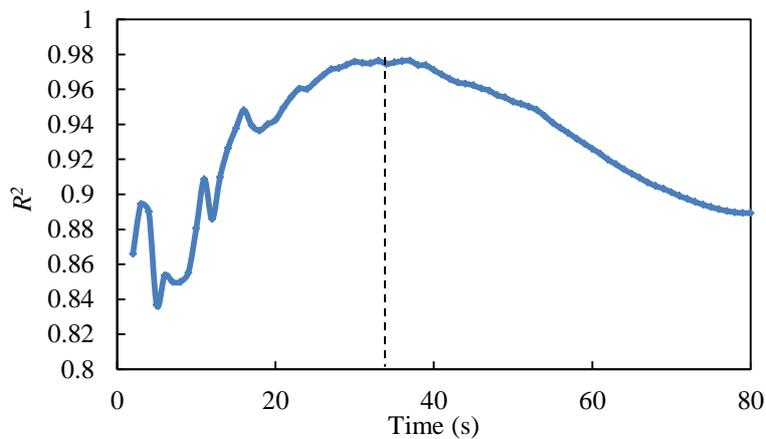
4.2 Critical strain calculation method

In order to measure the local deformation of mixtures, a 50mm line segment is set in the middle of the horizontal diameter as shown in Fig. 7 (a), the strain in the process is the deformation of the line segment. The typical strain-time curve in the test process is shown in Fig. 9 (a), from the strain-time curve we can see, the deformation increase stably at early stage, it is approximately linear, after a time point, the deformation increase rapidly, and it means internal damage accelerate mixture failure, the deformation of that time is critical deformation. In general, the cross point of two tangent lines of the early and the last stage is extracted, the deformation of that point is critical deformation, this approach relies on subjective experience and the value is imprecise.

Based on the approximate linear property of the curve at the early stage, a critical deformation identification method based on R^2 (coefficient of determination) is proposed. every deformation data per second are calculated by DIC system, if there are n data in the strain-time curve, firstly, the first 3 points are linear fitted through least squares method, the R^2 can be obtained, the next step is the first 4 points, the first 5 points...all the points are fitted respectively, R^2 getting from n-2 times fitting are utilized to make the R^2 -time curve, the typical R^2 -time curve is shown in Fig. 9 (b).



(a) Relation between time and strain



(b) Relation between time and R^2

Fig.9 Critical strain calculation method

Fig. 9 (b) shows the curve is parabolic, R^2 increased first and then decreases, and there is a peak value in the curve. Relative fluctuation of data at the early stage is large that will cause lower R^2 , with the data number increase, linearity of the curve enhance, however, when damage appear in mixture, deformation increase rapidly which will reduce the linearity, and then R^2 decrease. The strain at the moment of R^2 peak value is the critical steady deformation.

4.3 Comparison between critical steady deformation and overall failure deformation

The critical deformation above reflect the local deformation stability, after critical time, the local deformation increase rapidly, it mean local damage appears. The overall failure deformation is the strain of the test line segment when the load reach the peak, this deformation reflect the overall failure of mixture. The research in this part aim at the relation between critical stable deformation and overall failure deformation. Six kinds of mixtures including AC-10, AC-13, AC-16, SMA-10, SMA-13 and SMA-16 are investigated, there are three samples in each case and the final result is the average value of three samples.

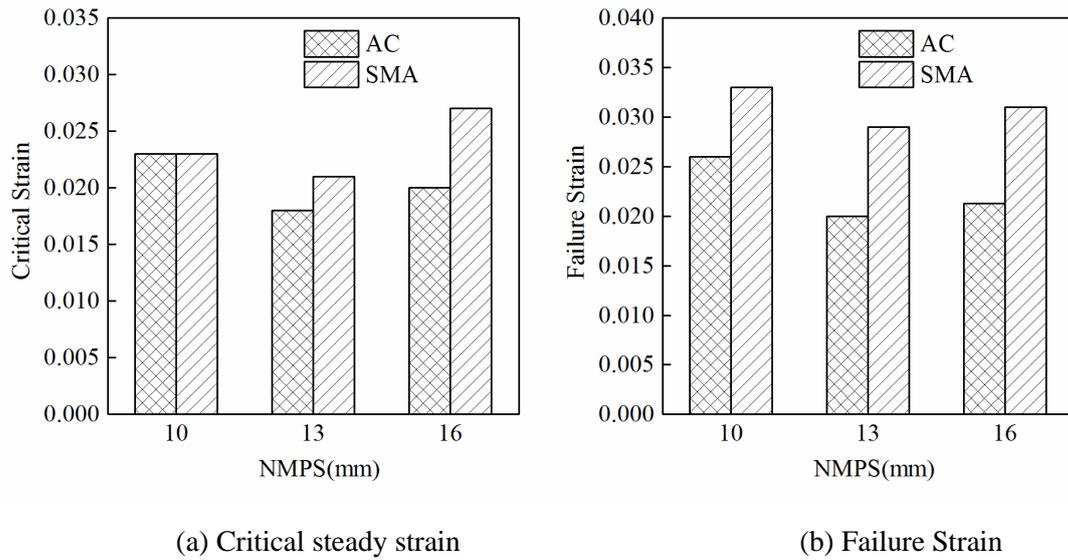


Fig.10 Critical steady strain and failure strain of different mixtures

Fig.10 (a) shows the critical strain of SMA is a little bit larger than that of AC, however, in Fig. 10 (b), the differ between AC and SMA of failure strain is larger than that of critical strain. When the strain reaches critical steady strain value, material enter the unstable deformation stage, larger strain of SMA means SMA can bear more deformation before unstable deformation stage. Also for failure strain, SMA has the ability to bear more deformation before failure. Critical steady strain and failure strain present the different stage in the failure process, the different between critical strain and failure strain is show in Fig.11.

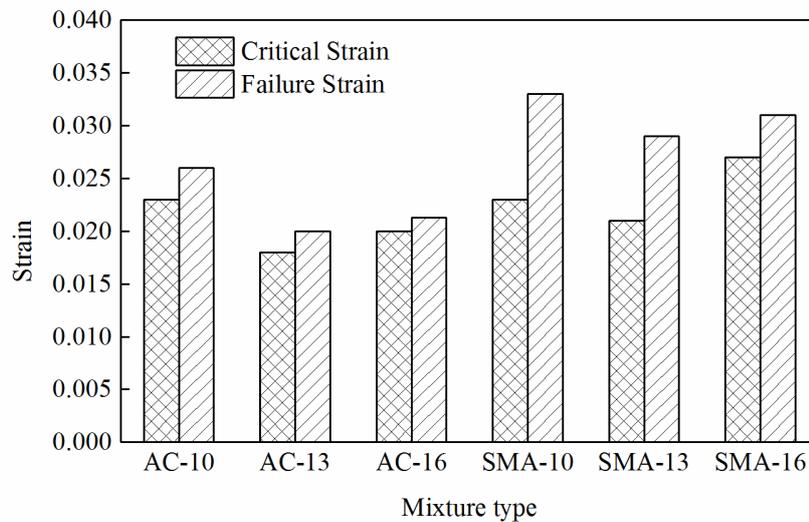


Fig.11 Comparison between critical steady strain and failure strain of mixtures

Fig. 11 shows failure strain is larger than critical strain for all kinds of mixtures, it illustrate asphalt mixtures step into accelerating deformation stage first and then lose the load bearing capacity. The differ between critical strain and failure strain of SMA is larger than that of AC, this phenomena show us after critical steady strain moment, more deformation can happen in SMA before failure, SMA show better ductility duo to the asphalt mastic including SBS modified asphalt and fiber.

5 Conclusion

This paper in detail describes local deformation property of different asphalt mixture based on DIC, research works include accuracy investigation and parameter selection for DIC with natural surface mixture image, measure length and determination method for critical strain, and the relation between critical strain and failure strain for mixtures. The findings that can be drawn from the results presented in this paper are summarized as follows.

(1) Subset entropy increase with subset size and decrease with nominal maximum aggregate size, subset entropy of AC is larger than that of SMA, it illustrates the image quality of AC with smaller NMAS is better for DIC test.

(2) Relative deviation decrease linearly with subset entropy, relation deviation of virtual translation test is about 4×10^{-5} , that of virtual shear test is about 8×10^{-5} , it illustrates the DIC deformation calculation with natural surface images can satisfy the precision requirement, at least for the mixtures with basalt aggregate. Standard deviation decrease with subset size, the curve become stable when subset size is larger than 31, subset size of 31 is selected as the test parameter.

(3) Based on the strain distribution, 50 mm measure length can cover almost all the deformation for mixture whose NMAS is smaller than 16, the optimal measure length is set as 50mm.

(4) According to the strain-time curve, the critical steady strain determined method is proposed based on R^2 obtained by the least squares fitting method in this paper. Comparing the critical steady strain and failure strain between SMA and AC, SMA can bear more deformation before unstable deformation stage and failure, it show better ductility duo to the asphalt mastic including SBS modified asphalt and fiber.

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