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Title: Finite Elements and Isogeometric Analysis: From Shock Physics to Fluid-Structure Interaction

Author(s):
Long, Christopher Curtis
Shashkov, Mikhail Jurievich
Akkerman, Ido
Scovazzi, Guglielmo
Benson, David
Bazilevs, Yuri
Marsden, Alison

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Finite Elements and Isogeometric Analysis: From Shock Physics to Fluid-Structure Interactions

May 7, 2015

Part I: Isogeometric Analysis and Shock Physics

Chris Long[1], Mikhail Shashkov[2], Ido Akkerman[4], Guglielmo Scovazzi[5], David Benson[3], Yuri Bazilevs[3]
[1]Los Alamos National Laboratory, T-3 Fluid Dynamics and Solid Mechanics
[2]Los Alamos National Laboratory, XCP-4 Methods and Algorithms
[3]University of California, San Diego, Structural Engineering
[4]TU Delft, Mechanical, Maritime, and Materials Engineering
[5]Duke University, Civil and Environmental Engineering

Cylindrical Shocks: Numerical Issues

- Discretization errors lead to Rayleigh-Taylor type instabilities and loss of symmetry
- Can be difficult to distinguish true physics from numerical instability

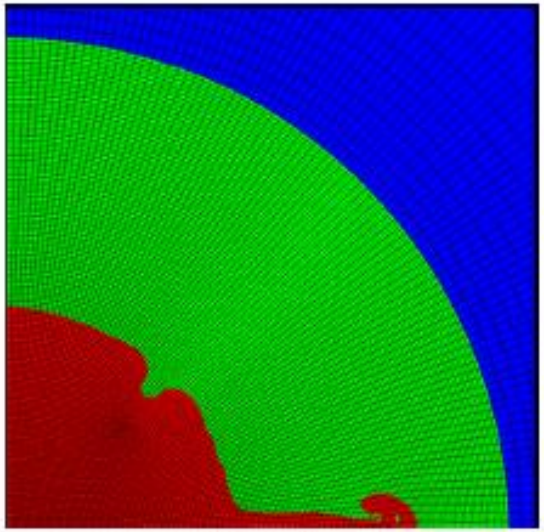


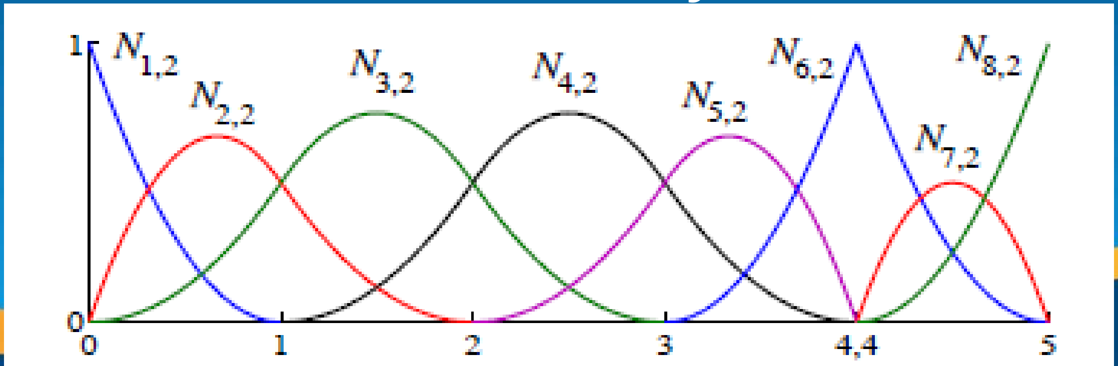
Fig. 2. Example of numerical symmetry breaking in an axisymmetric multi-material inertial confinement fusion (ICF) simulation. This is an ALE calculation where different colors are used to identify the different materials. The jet at the axis of rotation is spurious and does not disappear under mesh refinement.

- *Plot taken from:*

V.A. Dobrev, T.E. Ellis, T.V. Kolev, R.N. Rieben, High-Order curvilinear finite elements for axisymmetric Lagrangian hydrodynamics, Computers & Fluids, 2012

Isogeometric Analysis

- Use spline functions to define surface geometry and as the basis functions for finite element analysis
- Allows for modeling complex shapes with no geometric error
- Is a natural extension of (Non-Uniform Rational B-Spline) NURBS
- T-Splines and other bases may be used as well



Rayleigh-Taylor and Richtmyer-Meshkov Instabilities

- Small perturbations on a fluid interface are shocked or accelerated by body forces
- Vorticity is deposited on interface and perturbation grows
  - Can be physical or due to discretization error
- Hypothesis:
  - The exact nature of NURBS-based discretization can help to reduce this source of error

Equations and Formulation

We use a Lagrangian framework, where the following equations hold:

$$\rho_0 = \rho J$$

$$\rho \frac{d\vec{v}}{dt} = \nabla \cdot \sigma$$

$$\rho \frac{de}{dt} = \nabla^S \vec{v} \cdot \sigma$$

Formulation

We can relate the equations using the following assumptions:

$$\sigma = -p\mathrm{I} + \mu_s \nabla^S \vec{v}$$

$$p = (\gamma - 1)\rho e$$

Where “mu” is an artificial shock viscosity that is dependent on known flow parameters

Formulation

Classical definition of shock viscosity has linear and quadratic term:

$$\mu_s = \rho \left(q_1 \psi_0 \psi_1 h_s c_s + q_2 h_s^2 |\Delta_s \mathbf{v}^h| \right)$$

$$\psi_0 = \begin{cases} 1 & \text{if } \lambda_{min} < 0 \\ 0 & \text{if } \lambda_{min} \geq 0 \end{cases} \qquad \psi_1 = \frac{|\nabla \cdot \mathbf{v}^h|}{\|\nabla^s \mathbf{v}^h\|}$$

$$h_s = 2(\mathbf{e}_{min} \cdot \mathbf{G}\mathbf{e}_{min})^{-1/2} \qquad \mathbf{G} = \frac{\partial \boldsymbol{\xi}}{\partial \mathbf{x}}^T \frac{\partial \boldsymbol{\xi}}{\partial \mathbf{x}}$$

$$c_s = \sqrt{\gamma(\gamma - 1)e}$$

$$|\Delta_s \mathbf{v}^h| = |\lambda_{min}|$$

Formulation

- The weak formulation with homogeneous boundary conditions is then:

$$\int \mathrm{w} \cdot \rho \frac{\mathrm{dv}}{\mathrm{d}t} \mathrm{d}\Omega + \int \nabla^S \mathrm{w} \cdot \boldsymbol{\sigma} \mathrm{d}\Omega = 0$$

- For the axisymmetric case, we can formulate this in cylindrical coordinates:

$$\int \mathrm{w} \cdot \rho \frac{\mathrm{dv}}{\mathrm{d}t} 2\pi r \mathrm{d}r \mathrm{d}z + \int \nabla^S \mathrm{w} \cdot \boldsymbol{\sigma} 2\pi r \mathrm{d}r \mathrm{d}z = 0$$

Axisymmetry

- The factor of "2pi" cancels to get:

$$\int \mathrm{w} \cdot \rho \frac{\mathrm{dv}}{\mathrm{d}t} r\mathrm{d}r\mathrm{dz} + \int \nabla^{S} \mathrm{w} \cdot \boldsymbol{\sigma} r\mathrm{d}r\mathrm{dz} = 0$$

- The energy and continuity equations are similarly modified. The continuity equation becomes:

$$\rho_0 r_0 = \rho r J$$

- We should also note that due to axisymmetry, the gradient operator is modified.

Time Integration

- We solve the coupled system using an RK2 scheme in time.
- Each ½ time step fluid solve is solved using IGA and a conjugate gradient method to solve the resulting linear system.
- The energy update is performed.
  - Total energy is preserved exactly
  - Energy is an element level value, and updated gauss point values may be reevaluated with respect to a lower dimensional value.
  - Proof and details available in: Bazilevs, Akkerman, Benson, Scovazzi, Shashkov *Isogeometric Analysis of Lagrangian Hydrodynamics*, JCP, 2012

Time Integration

- Time step size depends on the acoustic and viscous conditions:

$$\Delta t < \frac{h_s}{2c_s} \qquad \Delta t < \frac{h_s^2 \rho}{\mu_s}$$

- RK2 time integration is carried out in two stages as follows:

$$y_{n+\frac{1}{2}} = y_n + \frac{1}{2}\Delta t f(y_n, t_n)$$

$$y_{n+1} = y_n + \Delta t f(y_{n+\frac{1}{2}}, t_{n+\frac{1}{2}})$$

Time Integration

- Step 1 of the RK2 scheme adapted for this problem is follows:

$$\int_{\Omega_n} \mathbf{w}^h \cdot \rho_n \mathbf{v}^h_{n+\frac{1}{2}} \, \mathrm{d}\Omega = \int_{\Omega_n} \mathbf{w}^h \cdot \rho_n \mathbf{v}^h_n \, \mathrm{d}\Omega - \tfrac{1}{2}\Delta t \int_{\Omega_n} \nabla^s_n \mathbf{w}^h : \boldsymbol{\sigma}^h(\mathbf{x}^h_n, \mathbf{v}^h_n, e_n) \, \mathrm{d}\Omega.$$

$$\rho_n e_{n+\frac{1}{2}} = \rho_n e_n + \tfrac{1}{2}\Delta t \nabla^s_n \tilde{\mathbf{v}}^h_{n+\frac{1}{4}} : \boldsymbol{\sigma}^h(\mathbf{x}^h_n, \mathbf{v}^h_n, e_n),$$

$$\mathbf{x}^h_{n+\frac{1}{2}} = \mathbf{x}^h_n + \tfrac{1}{2}\Delta t \tilde{\mathbf{v}}^h_{n+\frac{1}{4}},$$

- Step 2 is analogous to Step 1

Problems

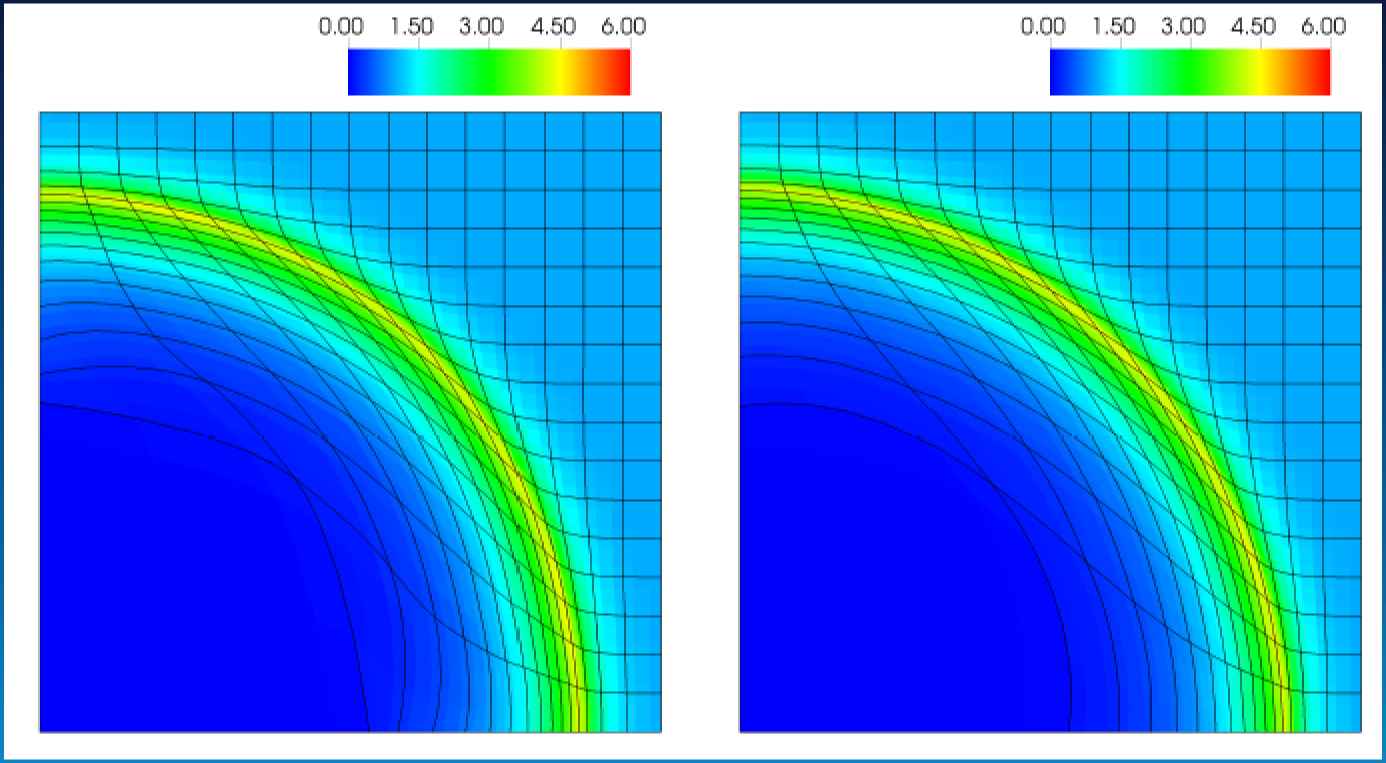
- 2-D Cartesian
  - Sedov
  - Noh
- RZ
  - Coggeshall
  - Noh
  - Multi-Material Problem
  - Sedov

Sedov ‘Blast’ Problem

- Initial condition:
  - Energy source is deposited at the origin in an area of uniform pressure and density
  - Radial expansion occurs
- An exact solution exists for comparison

2-D Sedov

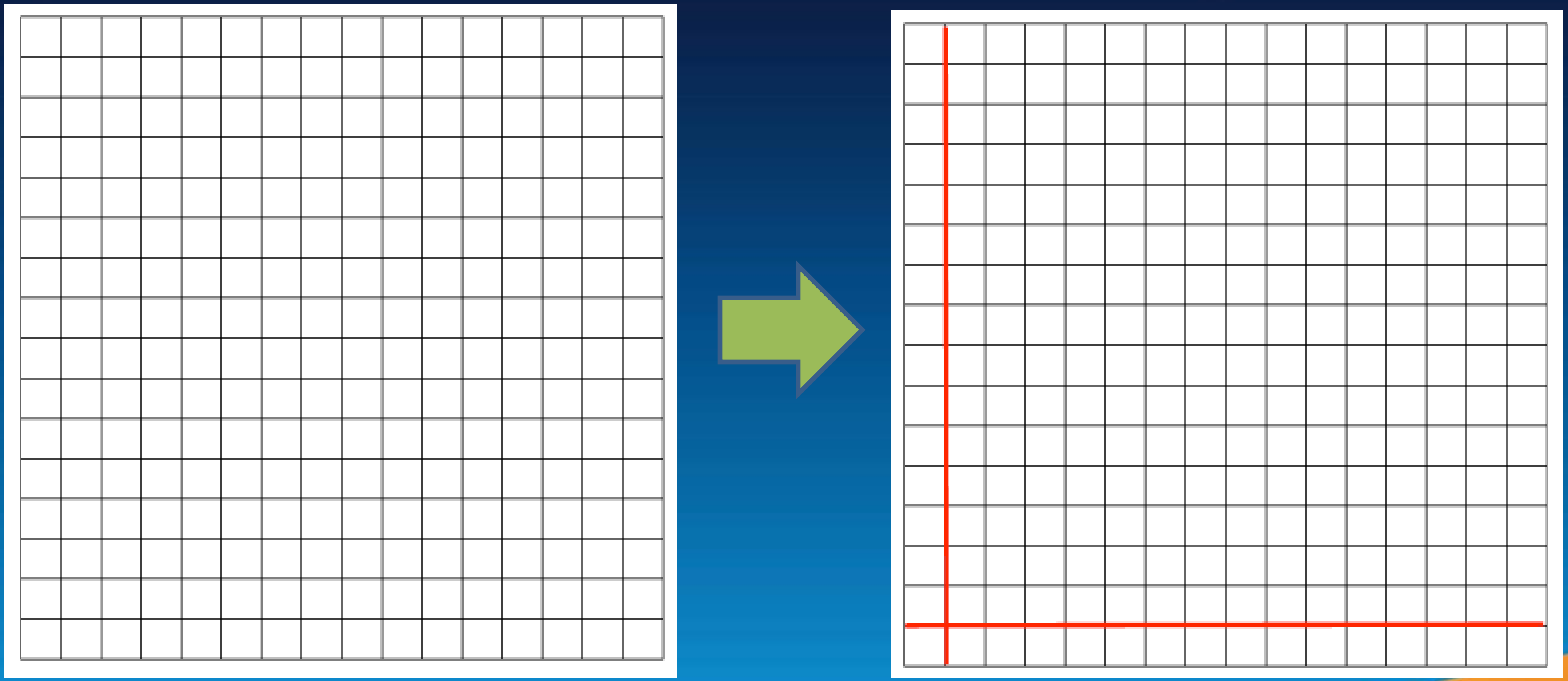
- Computations on C0 and C1 continuous IGA meshes



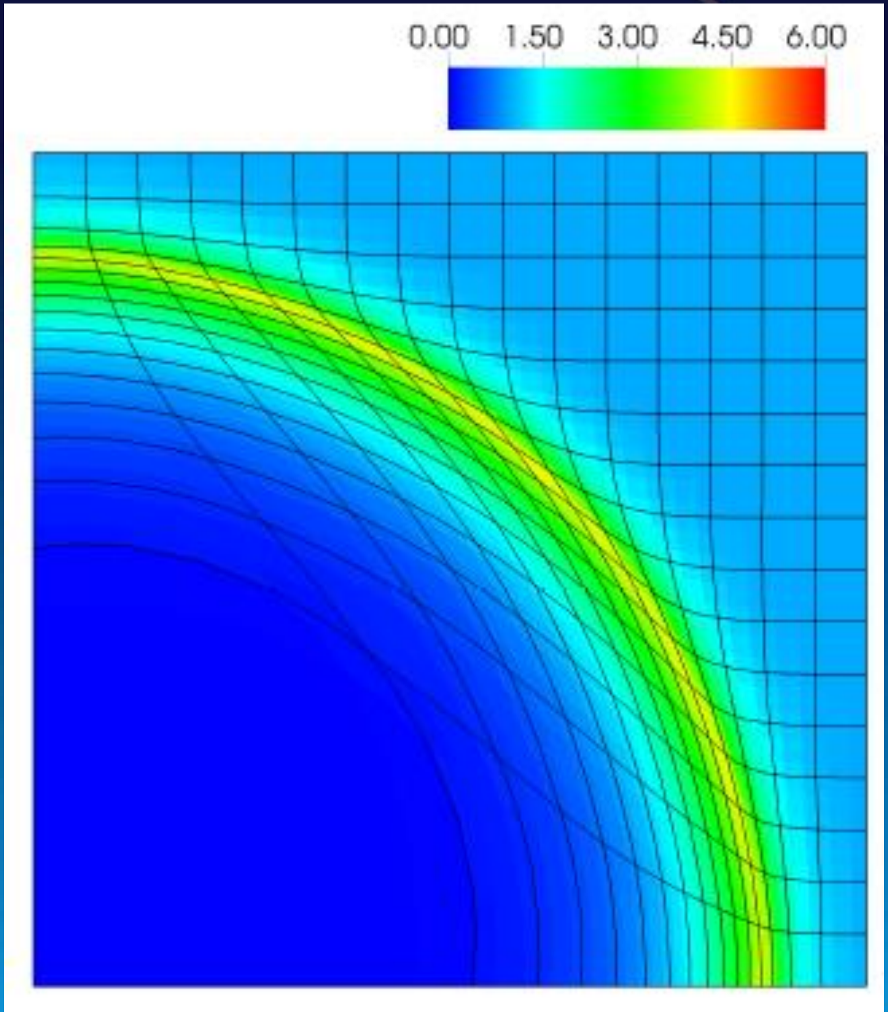
C1 (traditional IGA) C0 (quadratic FEM)

2-D Sedov

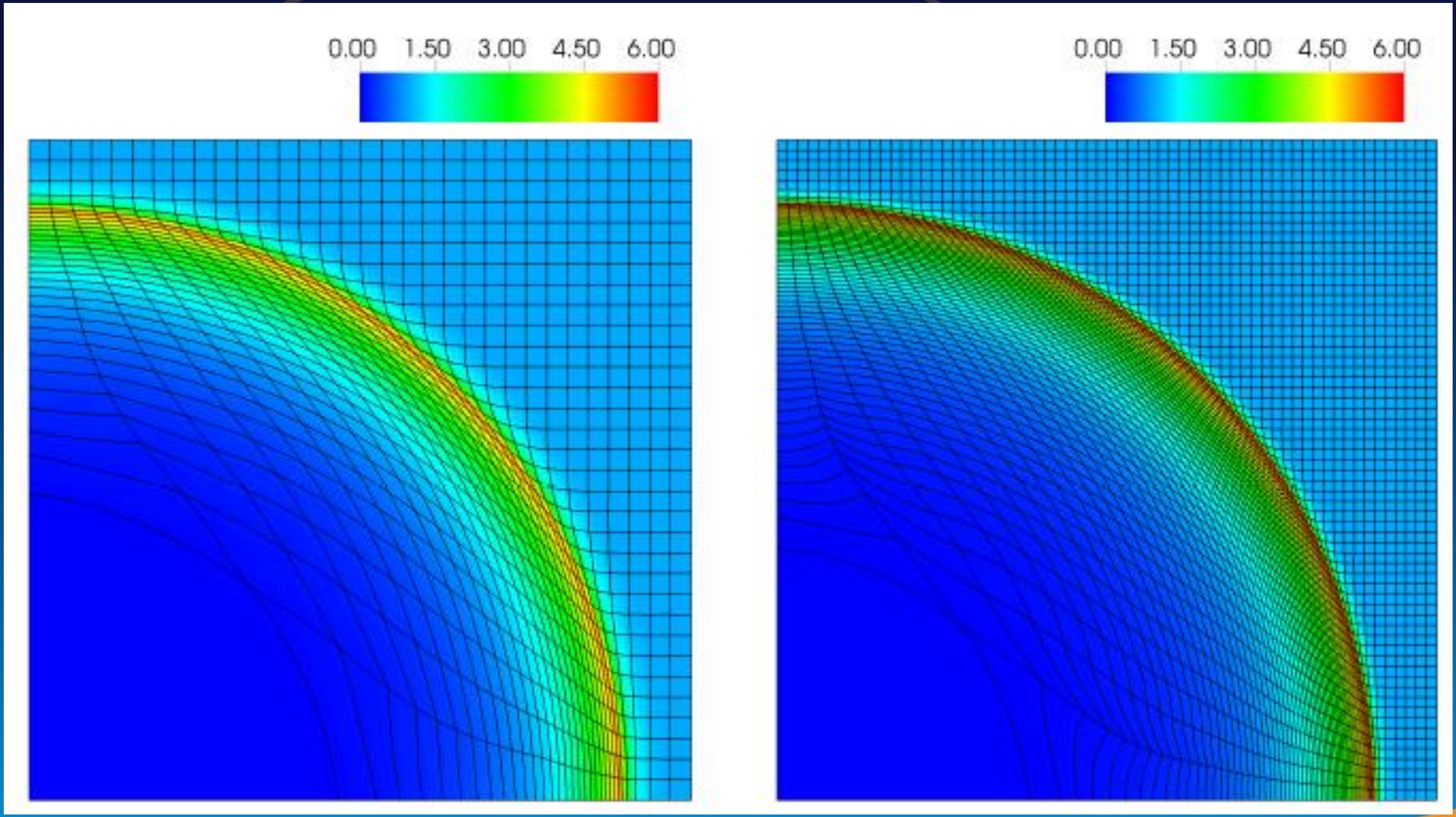
- Solution: Use mixed C0/C1 continuous boundaries



2-D Sedov



2-D Sedov



Internal Energy

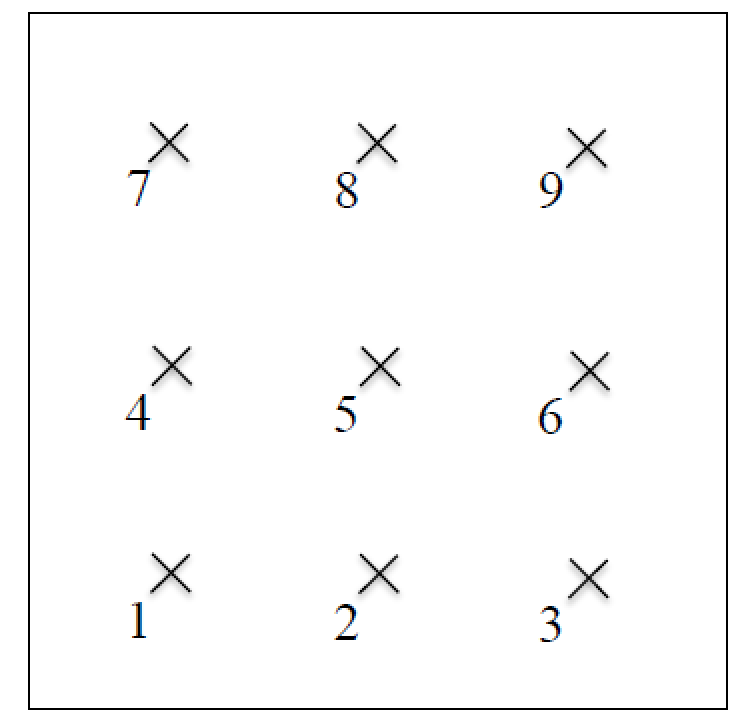
- Energy is cell-level quantity, not represented in the function space
- Redistribute energy inside the cell while respecting the cell-level quantity

$$\overline{\mathbf{e}} = \Pi \mathbf{e}$$

$$\sum_{ig=1}^{Ng} w_{ig} \overline{e}_{ig} \rho(\xi_{ig}) = \sum_{ig=1}^{Ng} w_{ig} e_{ig} \rho(\xi_{ig})$$

Internal Energy

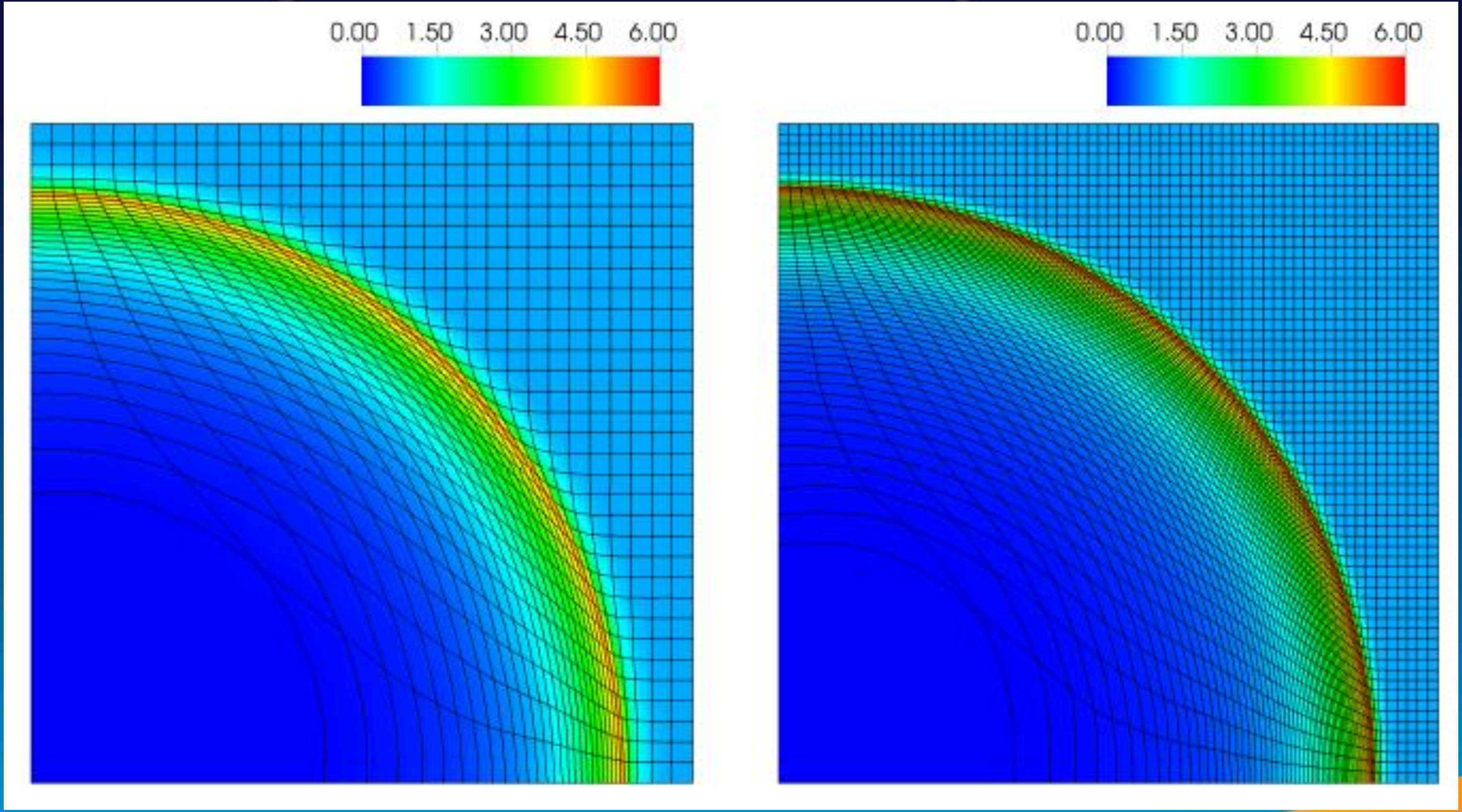
- For a 2-D element, this is accomplished as shown:



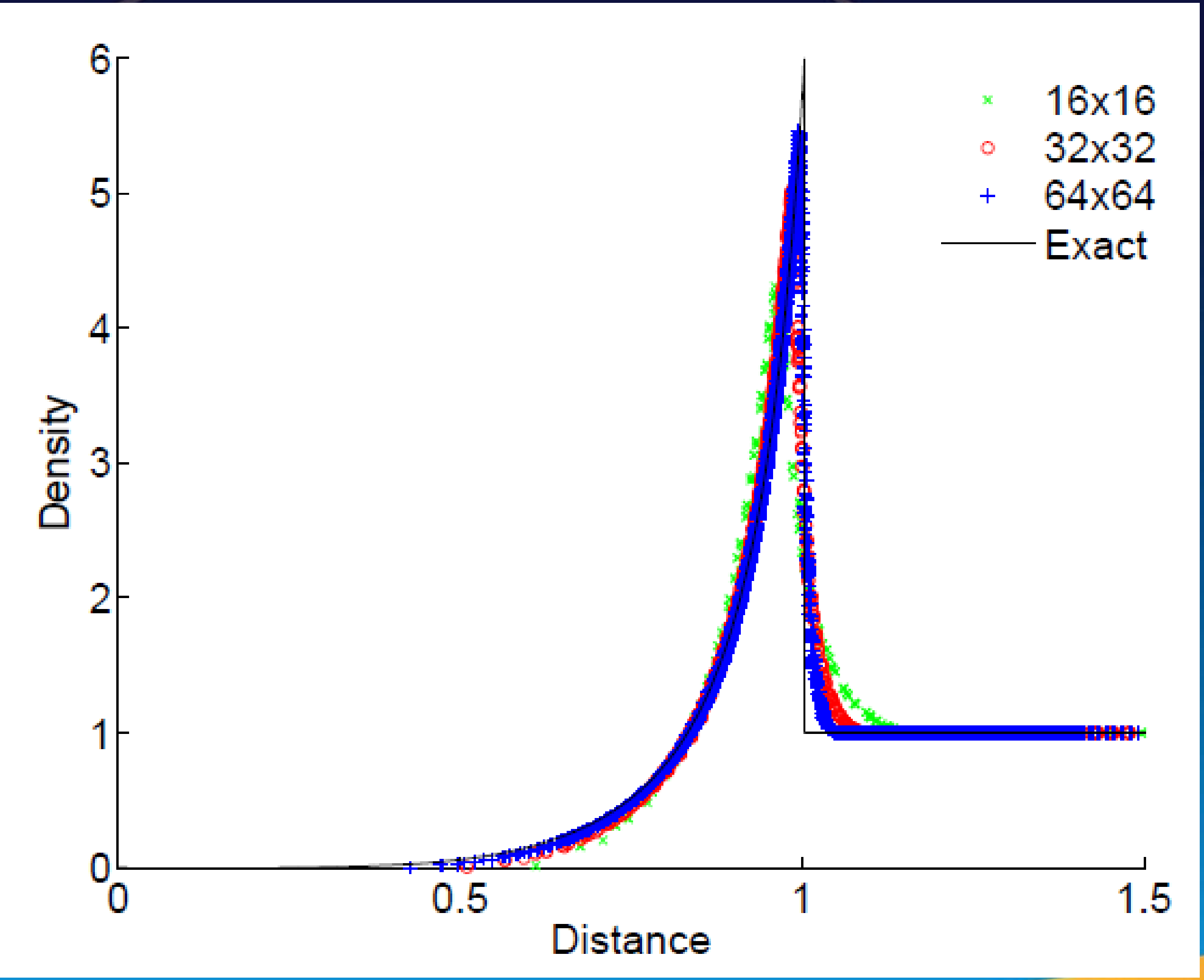
$$\overline{e}_1 = e_1,\ \overline{e}_3 = e_3,\ \overline{e}_7 = e_7,\ \overline{e}_9 = e_9,$$
$$\overline{e}_2 = 1/2(e_1 + e_3),\ \overline{e}_4 = 1/2(e_1 + e_7),$$
$$\overline{e}_6 = 1/2(e_3 + e_9),\ \overline{e}_8 = 1/2(e_7 + e_9),$$
$$\overline{e}_5 = 1/4(e_1 + e_3 + e_7 + e_9),$$

$$\sum_{ig=1}^{Ng} w_{ig}\overline{e}_{ig}\rho(\xi_{ig}) = \sum_{ig=1}^{Ng} w_{ig}e_{ig}\rho(\xi_{ig})$$

2-D Sedov



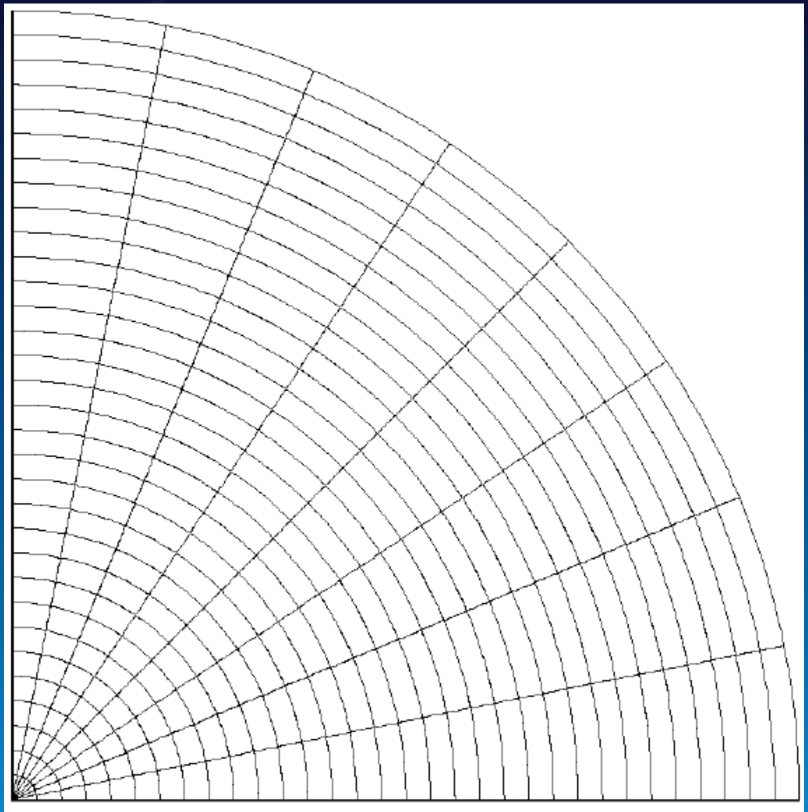
2-D Sedov

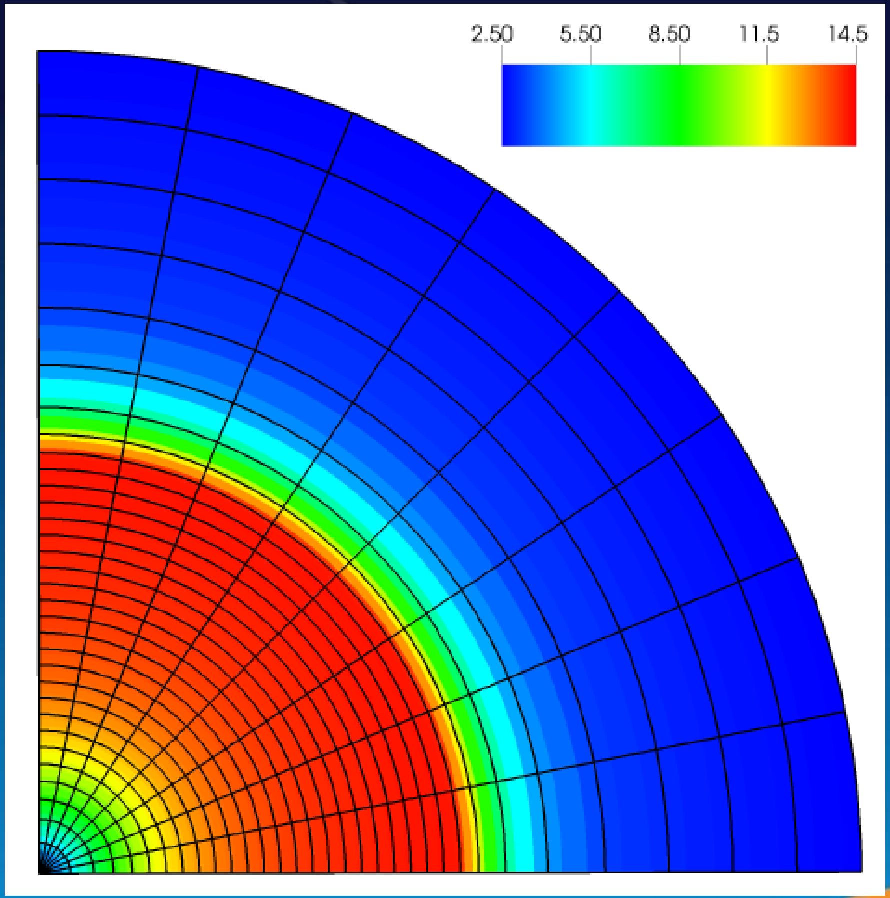


2-D Noh

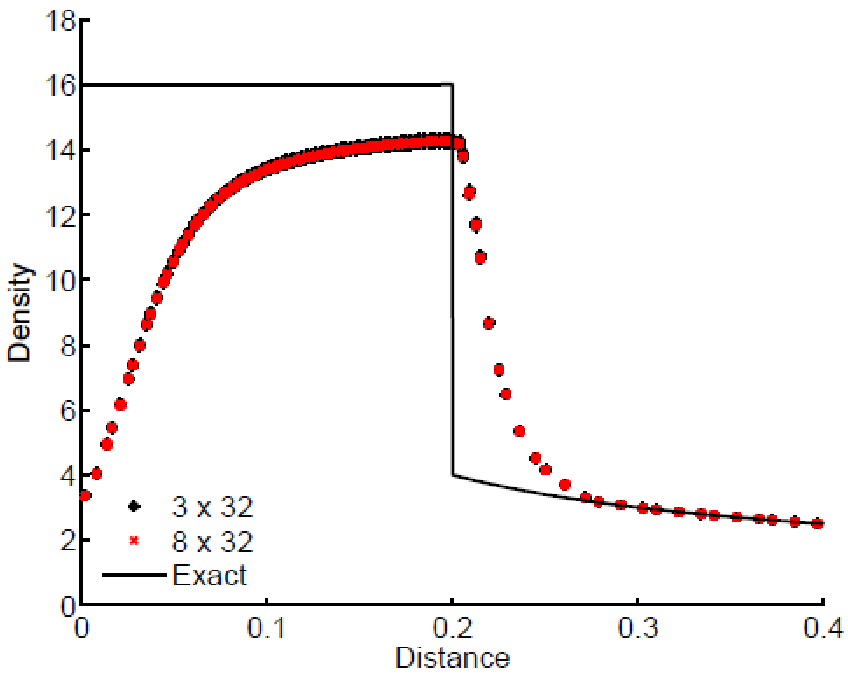
- Initial condition of uniform density, velocity field oriented toward origin,
  - Exactly symmetric initial velocity field is possible to attain and is unique to an IGA approach
- Exact solution exists

2-D Noh

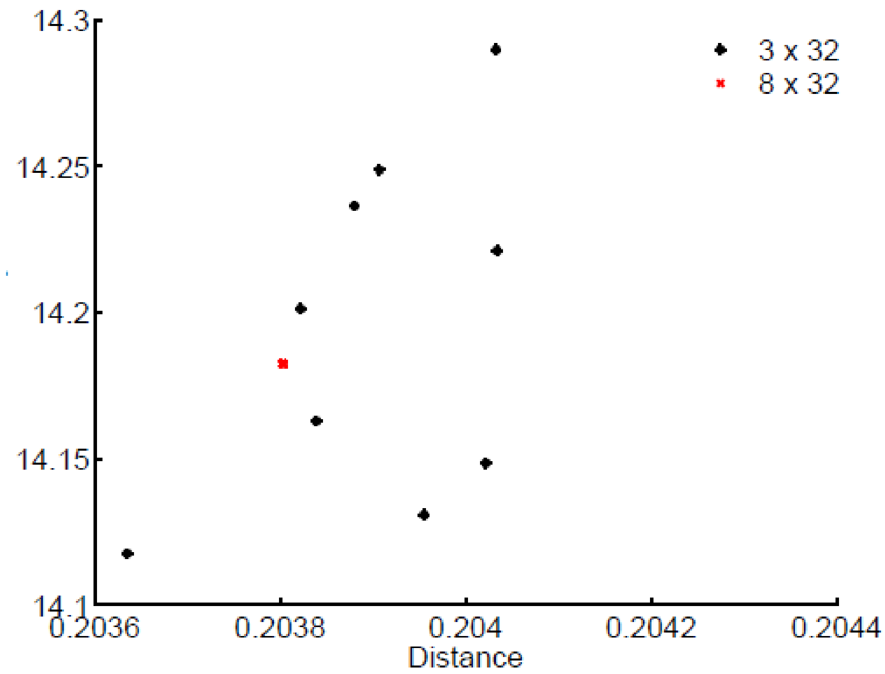




2-D Noh



Density
Distance
3 x 32
8 x 32
Exact
0
2
4
6
8
10
12
14
16
18
0
0.1
0.2
0.3
0.4



3 x 32
8 x 32
Distance
14.1
14.15
14.2
14.25
14.3
0.2036
0.2038
0.204
0.2042
0.2044

Coggeshall Problem

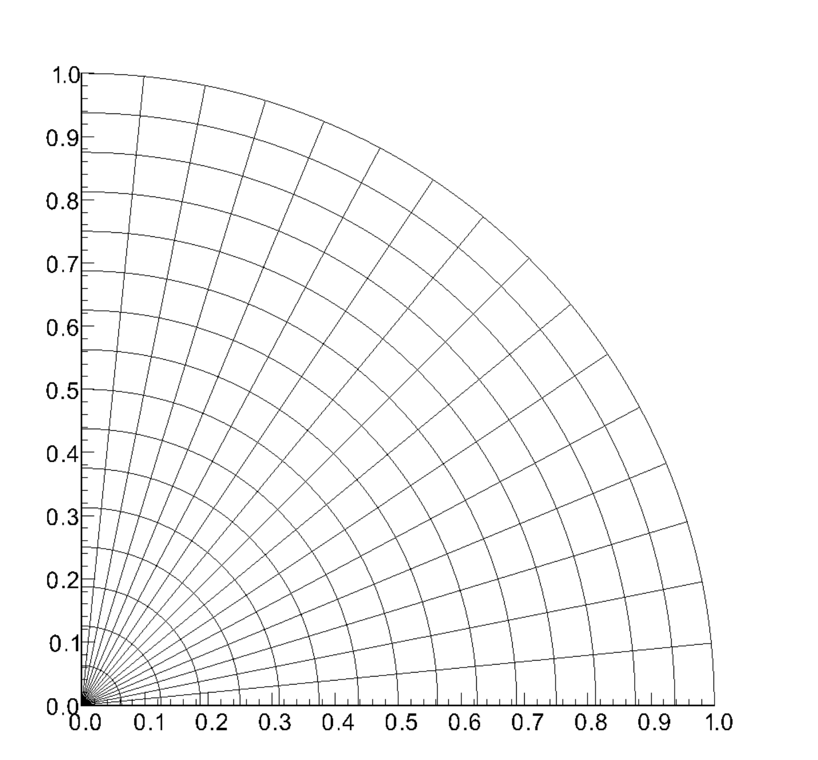
- Adiabatic compression problem with elliptical geometry
- Exact solution exists
- Initial condition is a sphere with properties of the exact solution
  - Use a pressure boundary condition on outer ring of sphere to drive problem

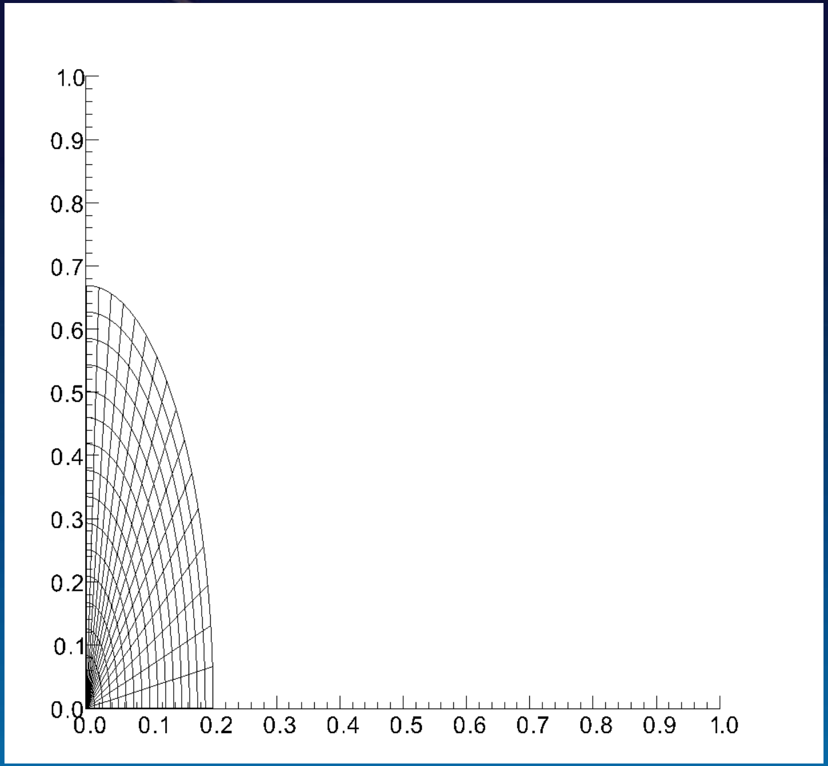
$$u = \frac{-r}{1-t}, \qquad v = \frac{-z}{4(1-t)}$$

$$\rho = (1-t)^{-9/4}, \qquad e = \left(\frac{3z}{8(1-t)}\right)^2, \qquad p = \frac{2}{3}e\rho.$$

L.G. Margolin, M.J. Shashkov, and M.A. Taylor, "Symmetry Preserving Discretizations for Lagrangian Gas Dynamics", *Proceedings of the Third European Conference Numerical Mathematics and Advanced Applications, World Scientific*. 2000

Coggeshall Problem





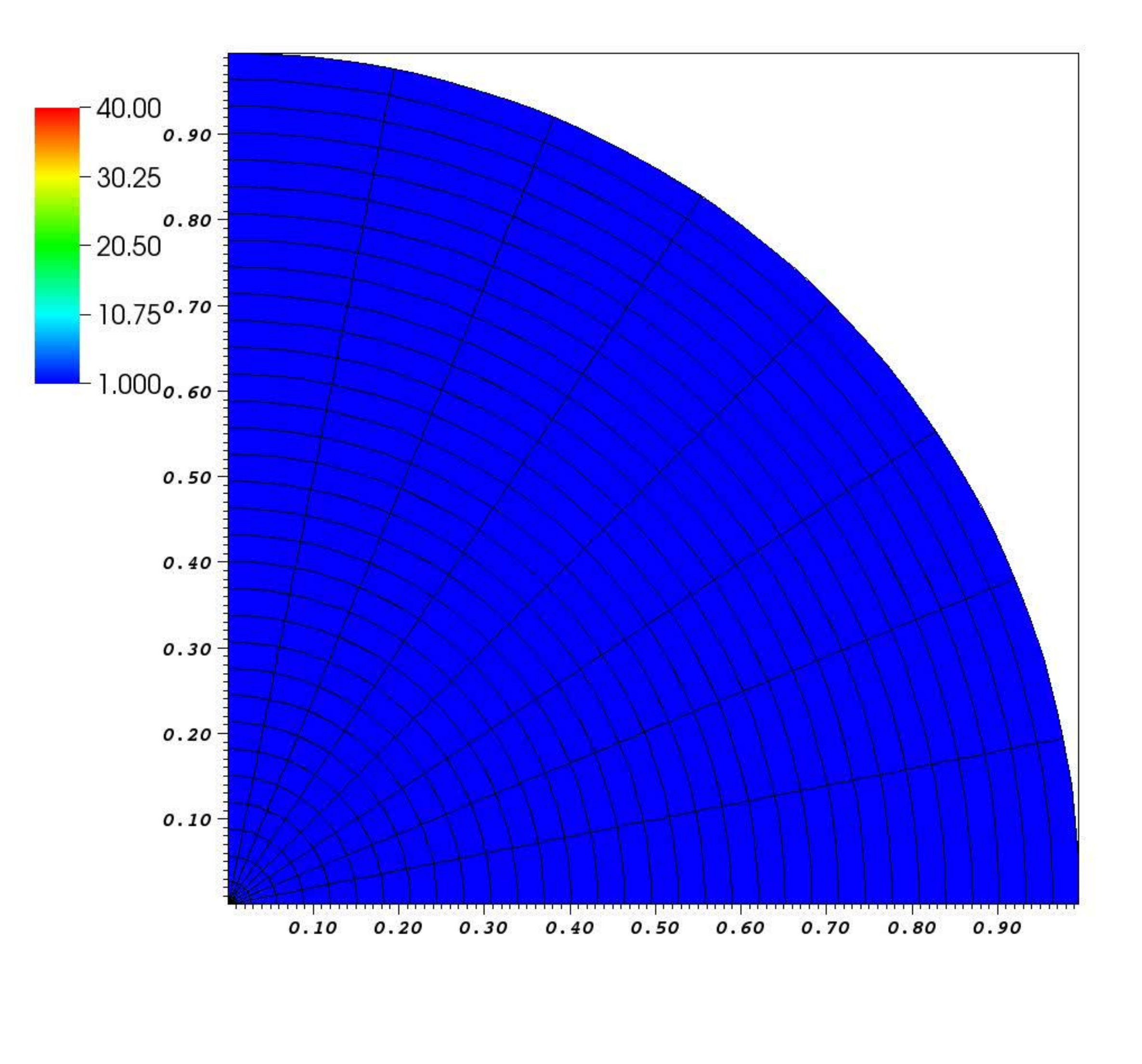
	ρ	p	v_r	v_z
3GP, Δt	7.205E-004	8.801E-005	1.676E-006	6.187E-005
4GP, $\Delta t/2$	8.630E-007	2.697E-007	3.616E-009	5.003E-008
5GP, $\Delta t/4$	5.447E-008	3.520E-008	9.118E-010	3.548E-010

Noh Implosion Problem

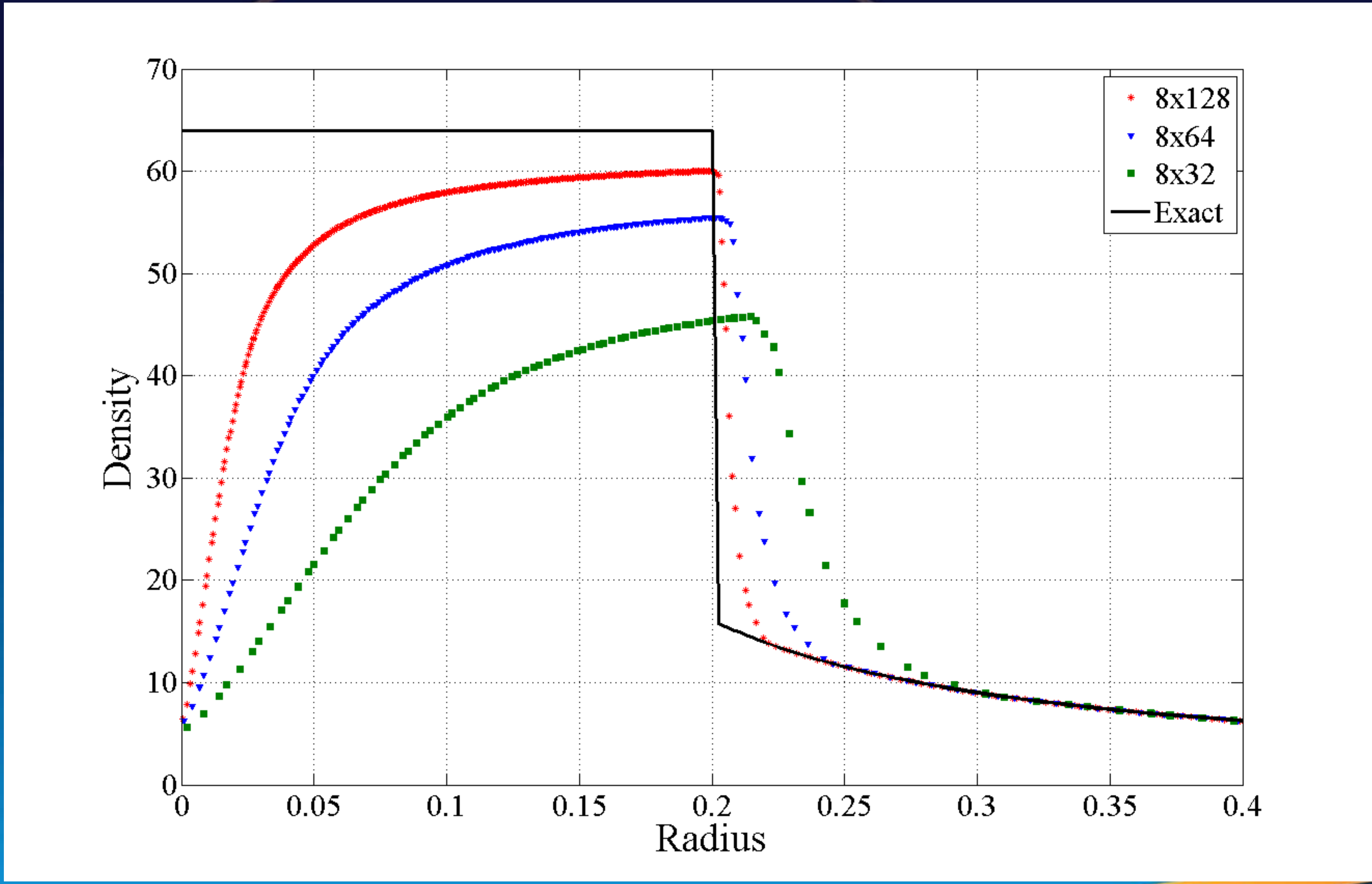
- Initial condition:
  - Velocity approaching origin in the radial direction
- Axisymmetric 3-D problem
- Has analytic solution for comparison
- Major computational problem:
  - Symmetry preservation

National Nuclear Security Administration

Noh Implosion Problem



Noh Implosion Problem

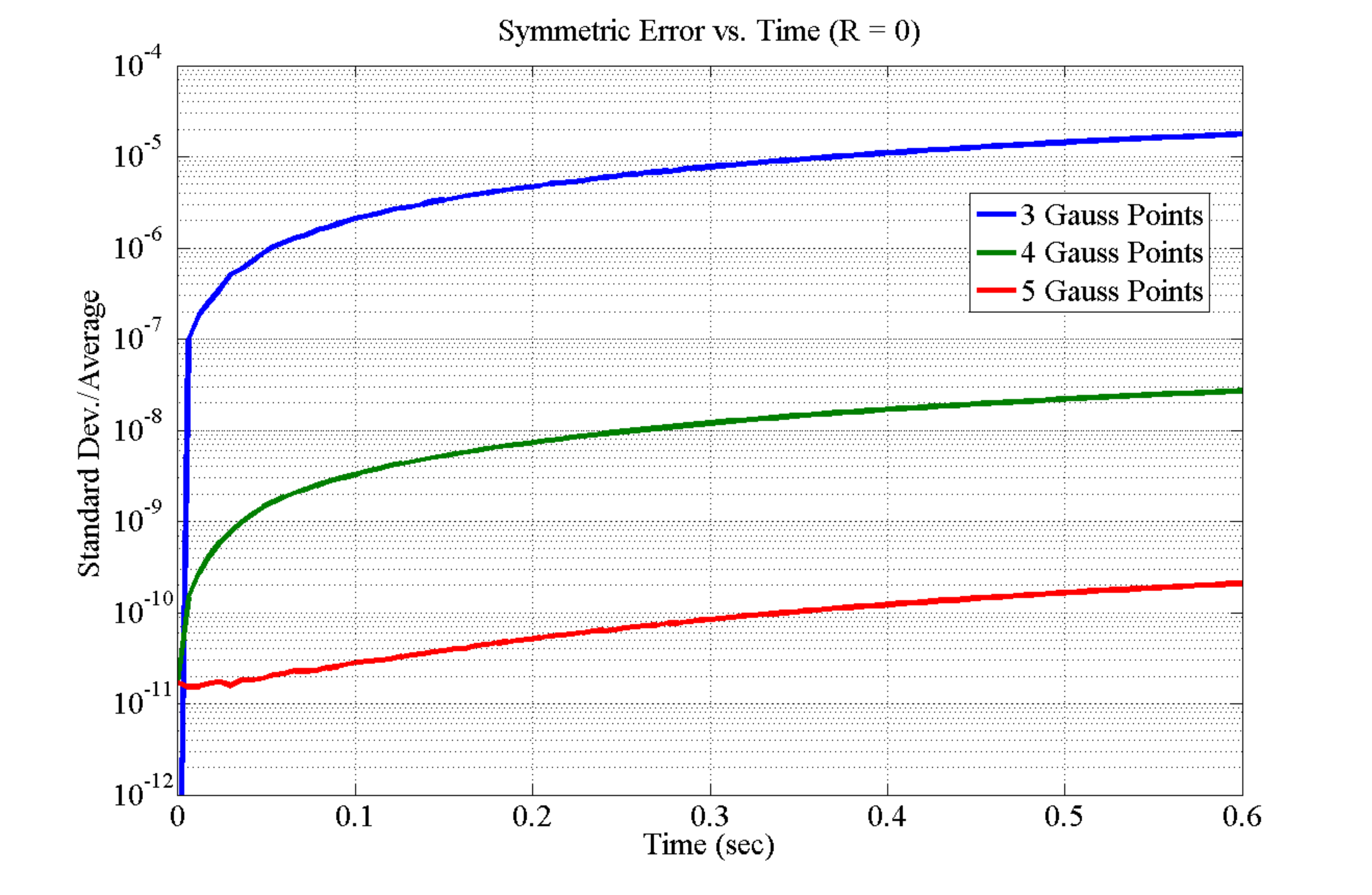


Noh Implosion Problem

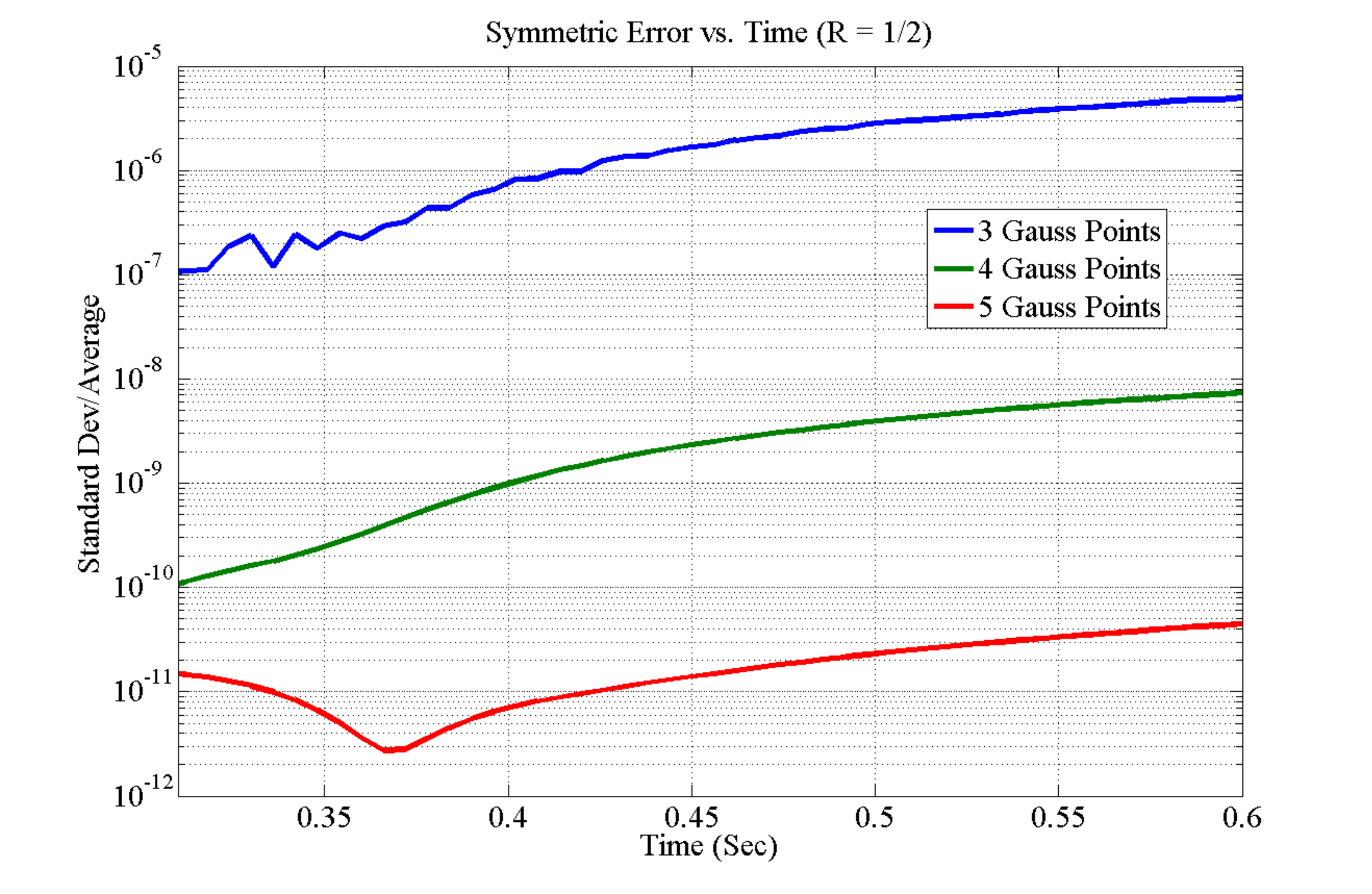
- Can define symmetric error in terms of scatter of computed data on gauss points of same radius, r

$$s = \left(\frac{1}{\bar{\rho}^2 (n-1)} \sum_{i=1}^{n} (\rho_i - \bar{\rho})^2 \right)^{\frac{1}{2}}$$

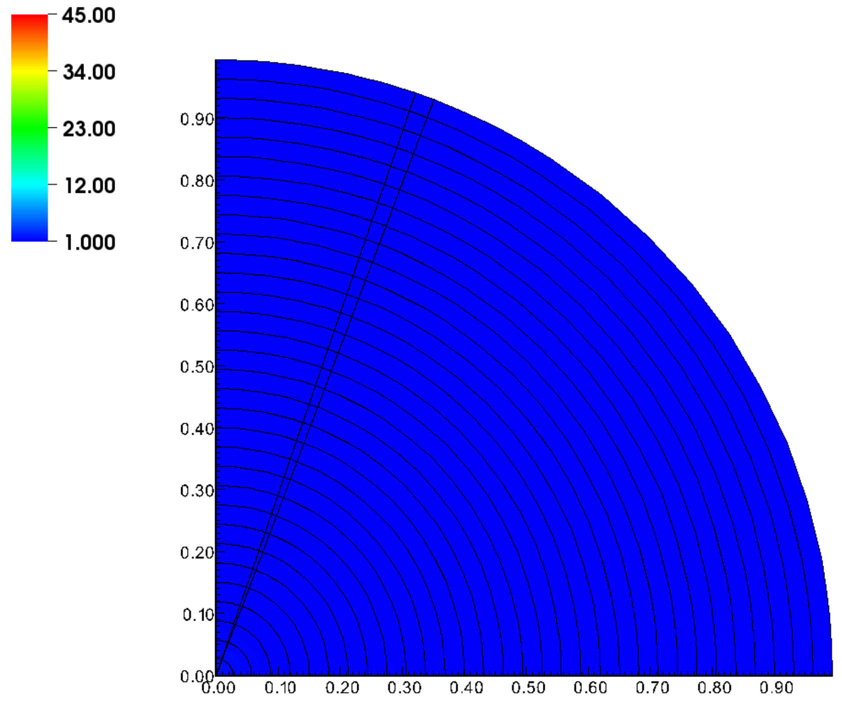
Noh Implosion Problem

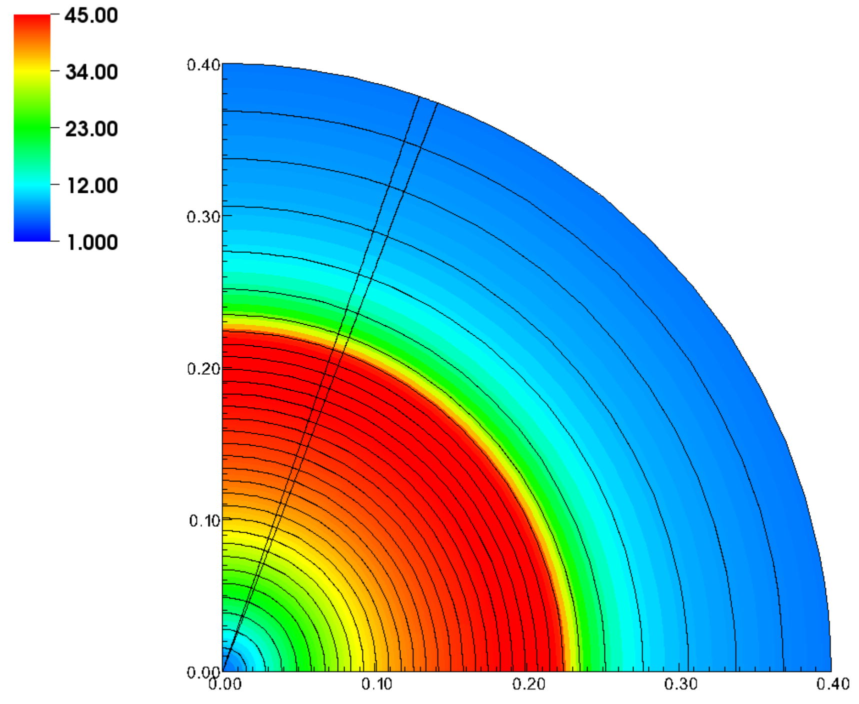


Noh Implosion Problem

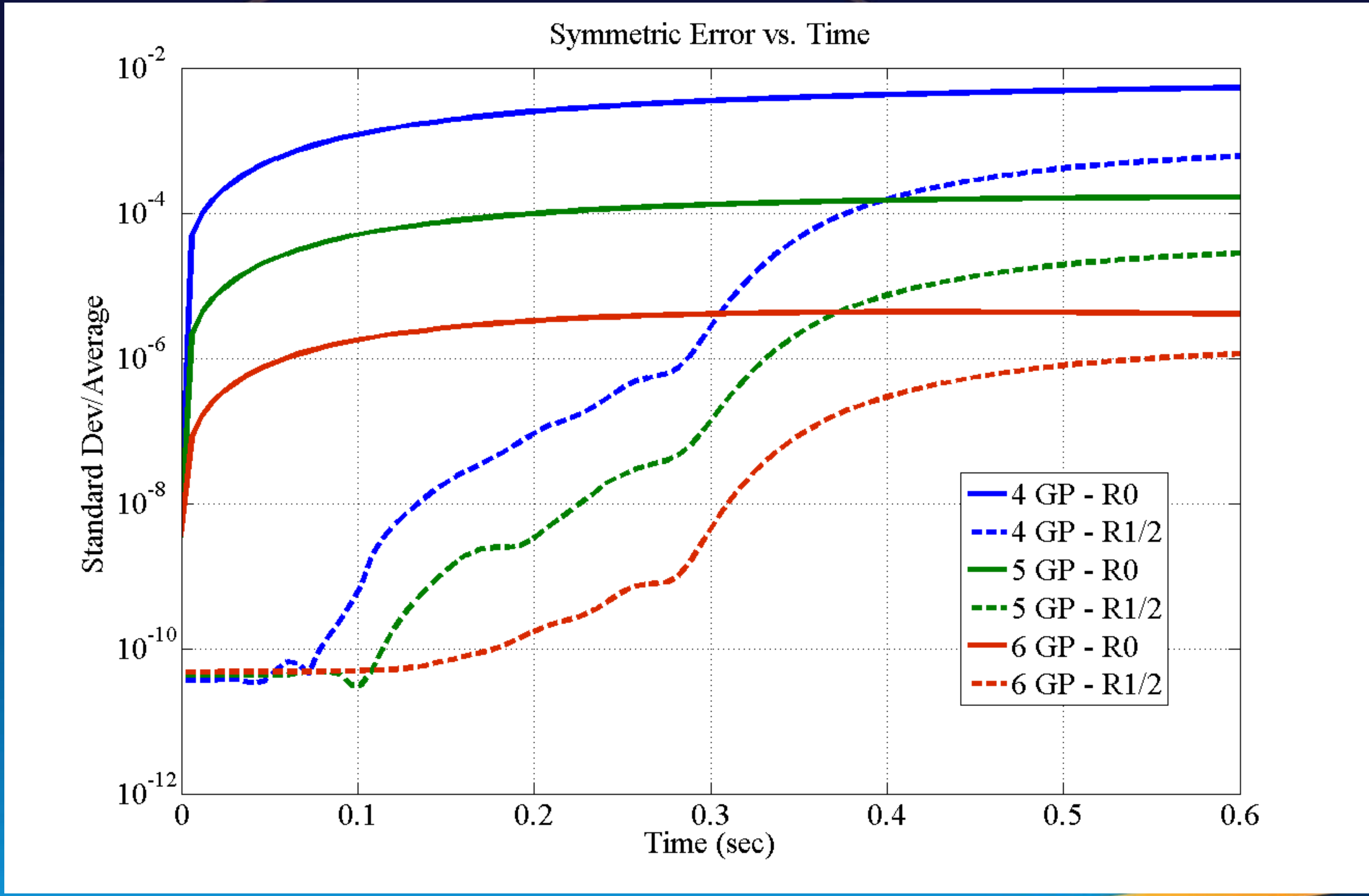


Noh Implosion Problem





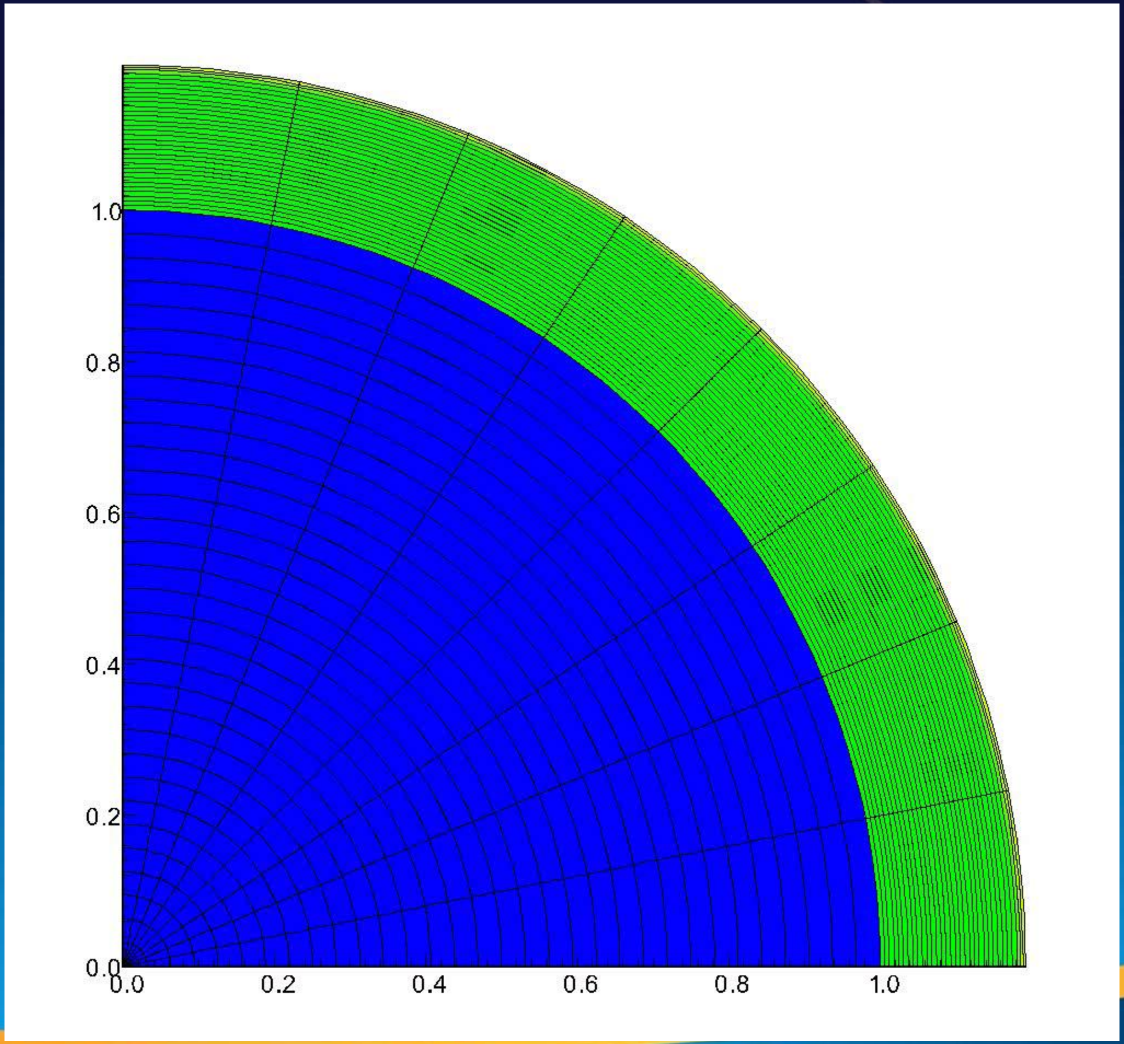
Noh Implosion Problem



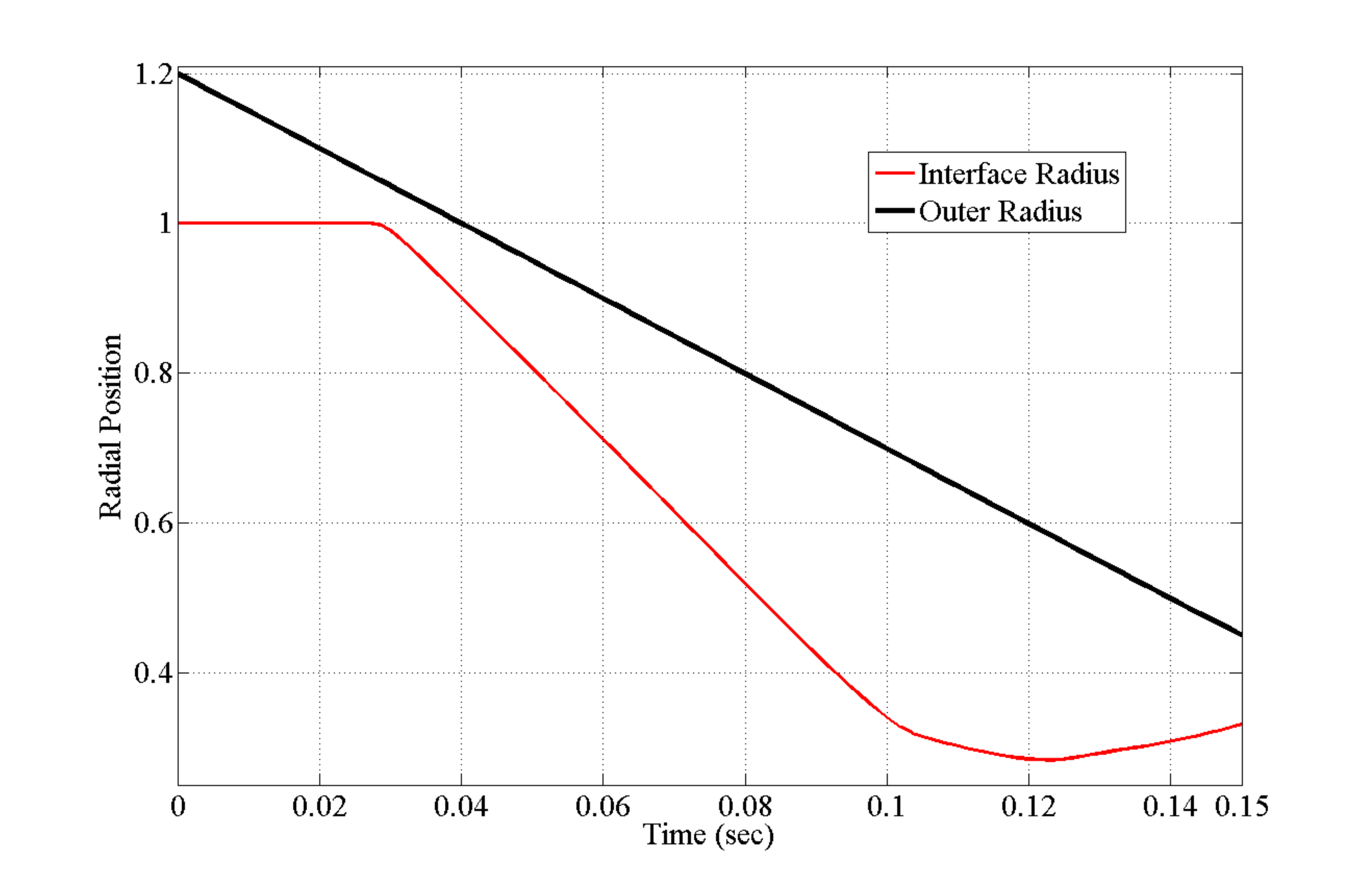
Multiple Material Problem

- Two fluids. Inner sphere of low density (0.05), and outer shell of high density (1.0)
- Initial condition:
  - Fluids are motionless, with uniform pressure.
  - Dirichlet boundary condition is imposed at the outer boundary
  - Prescribe a uniform flowrate radially toward origin
- Shock forms and strikes interface several times.
- High Atwood Number (0.905) causes fast growth of Rayleigh Taylor type instabilities.

Multiple Material Problem

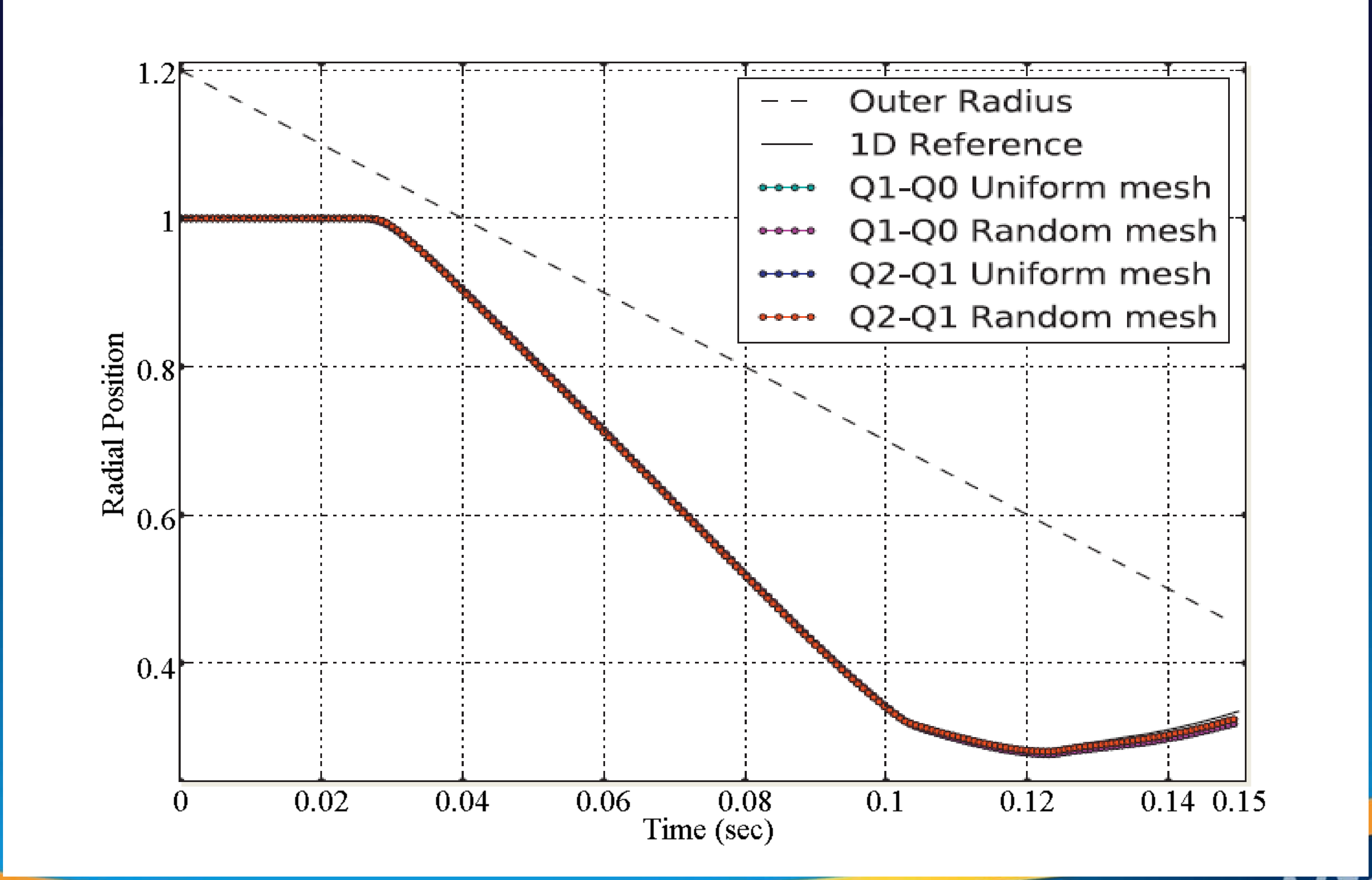


Multiple Material Problem



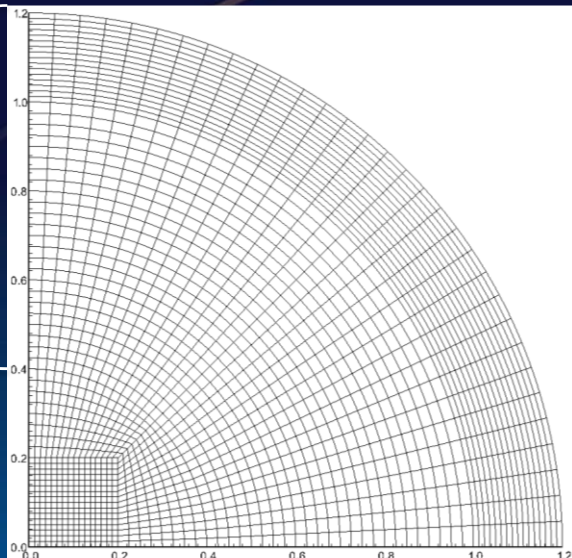
Multiple Material Problem

V.A. Dobrev, T.E. Ellis, T.V. Kolev, R.N. Rieben, *High-Order curvilinear finite elements for axisymmetric Lagrangian hydrodynamics,* Computers & Fluids, 2012

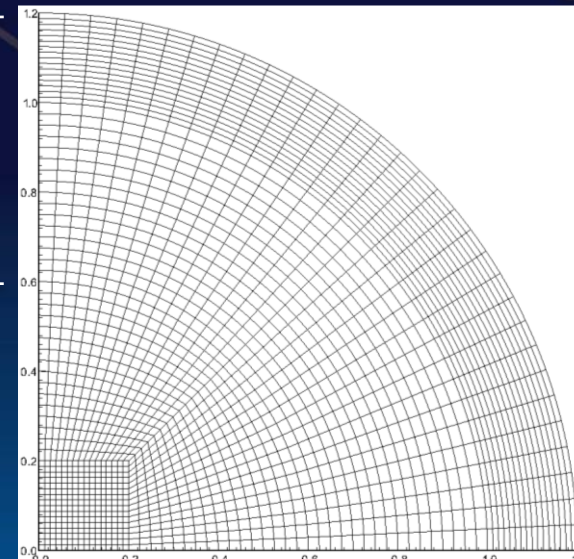


Multiple Material Problem

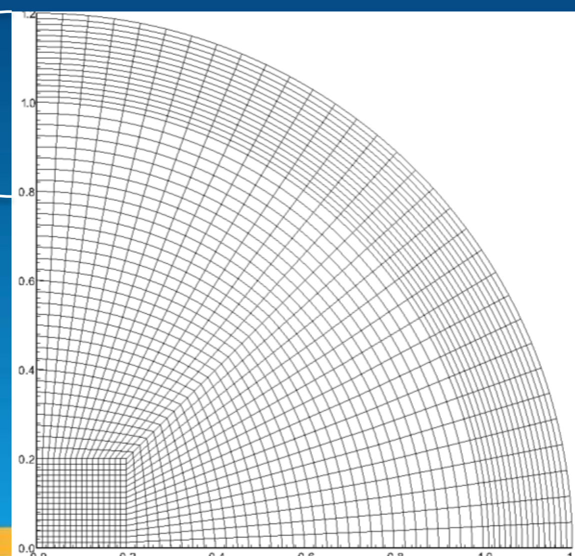
Radial symmetry region



Radial symmetry region

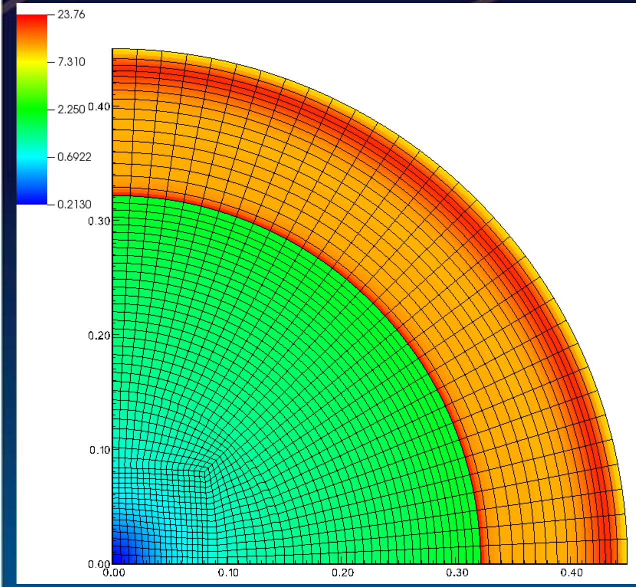


Radial symmetry region



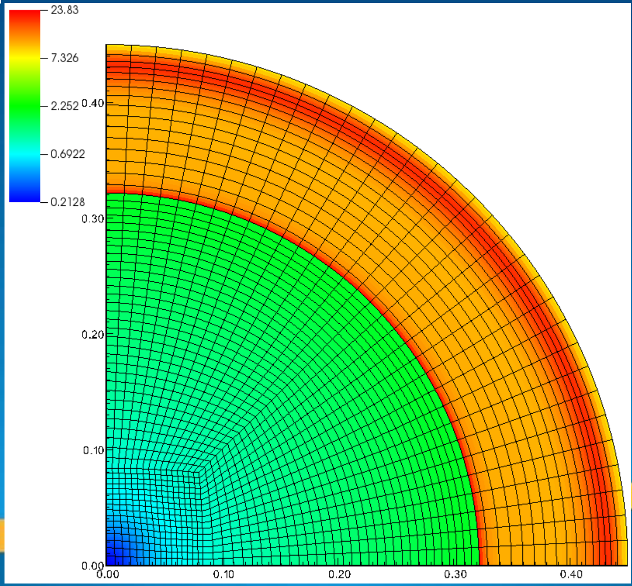
Considered three transitions from symmetric to non-symmetric region.

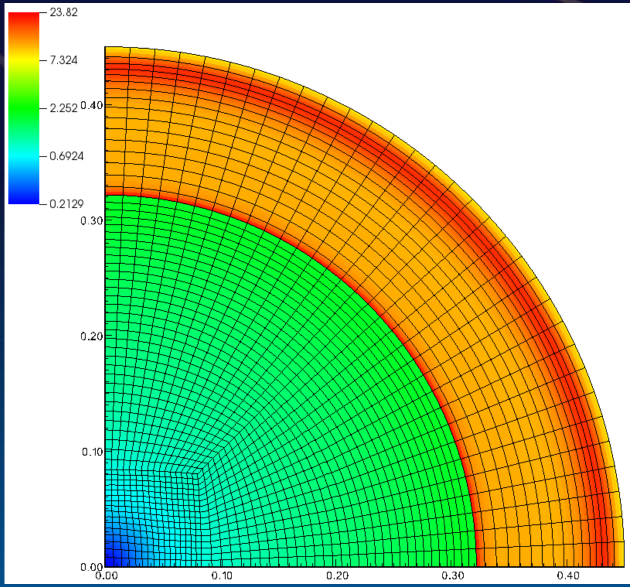
Multiple Material Problem



Symmetry Line at 0.4

Symmetry Line at 0.8

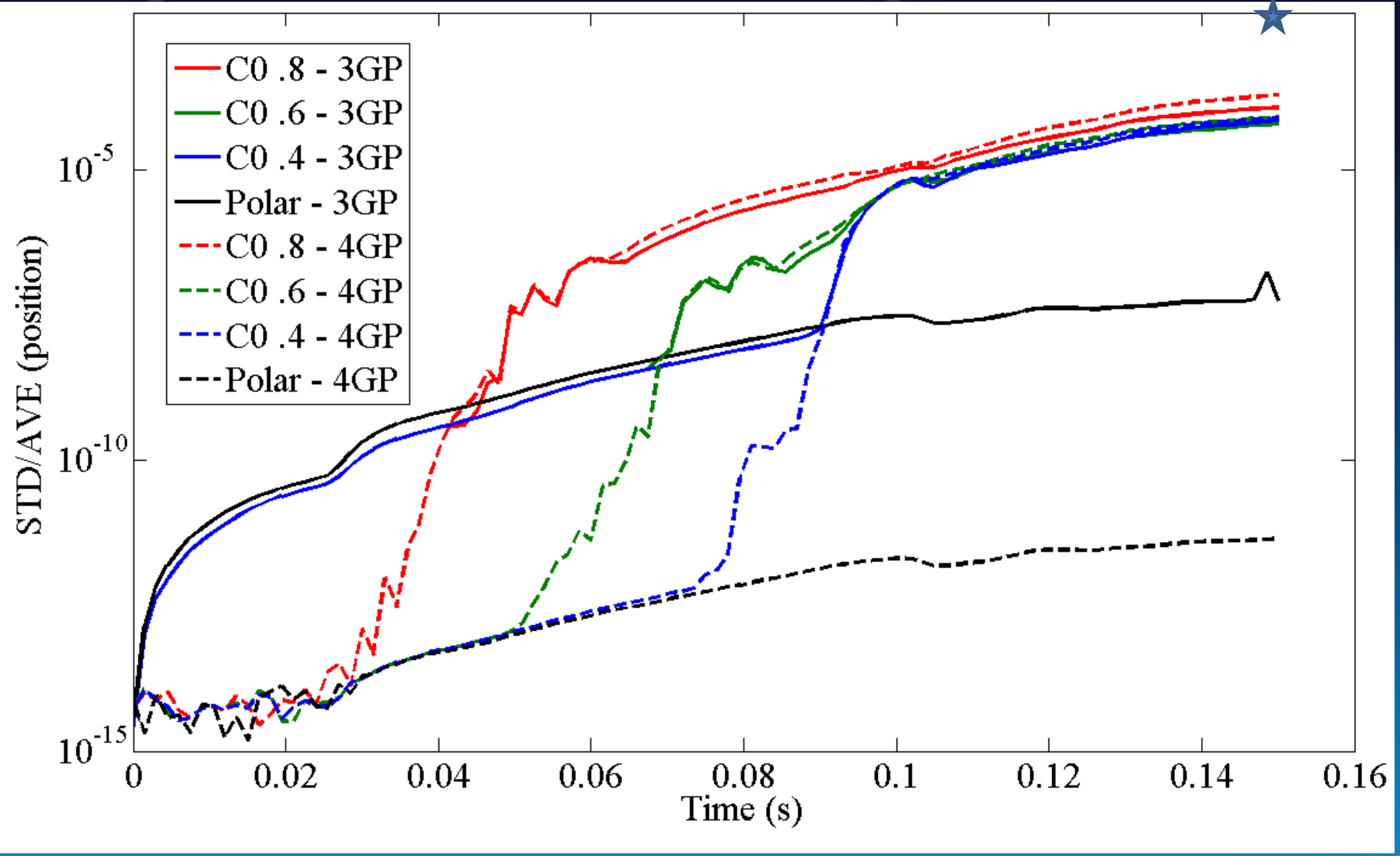




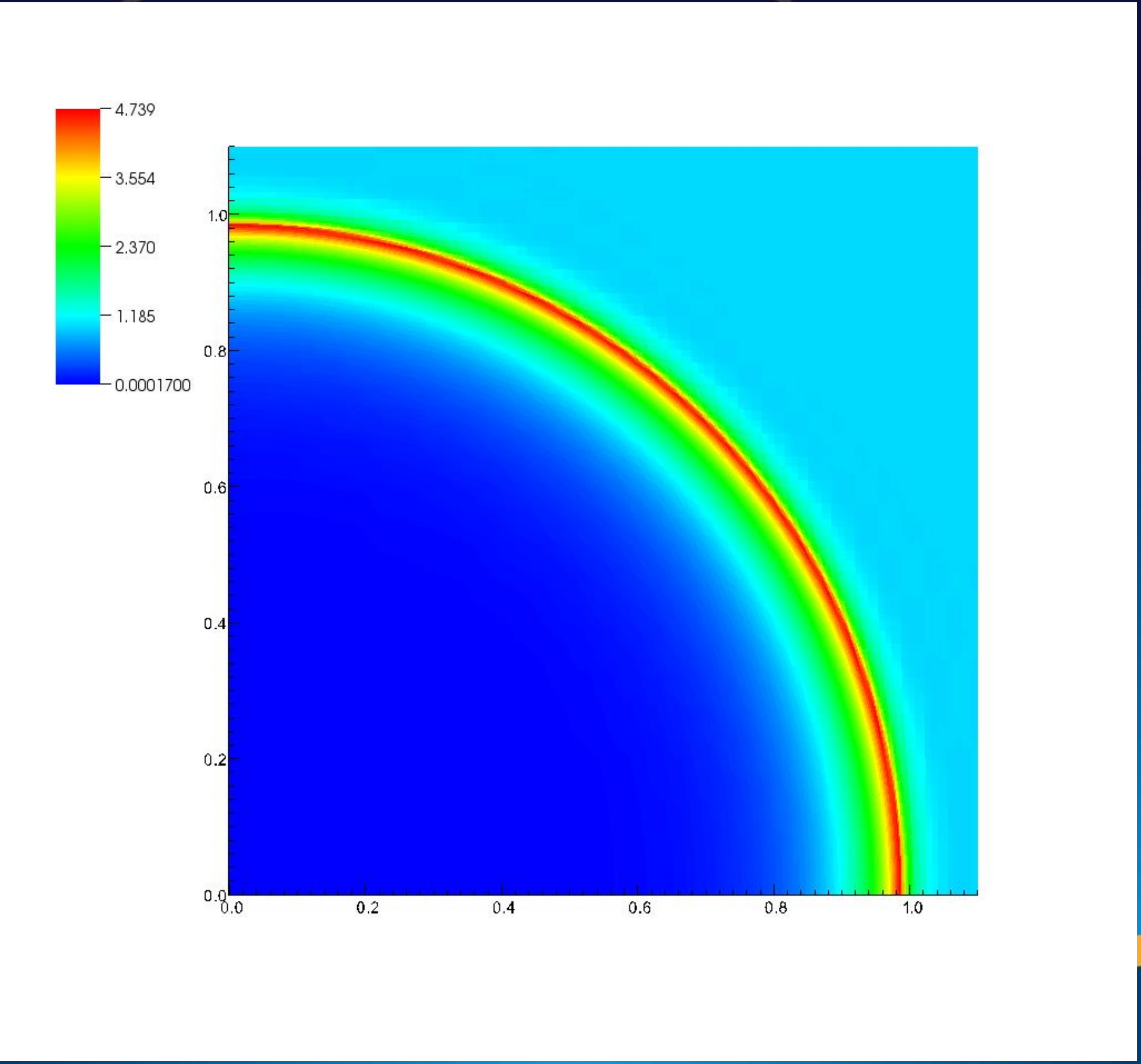
Symmetry Line at 0.6

Multiple Material Problem

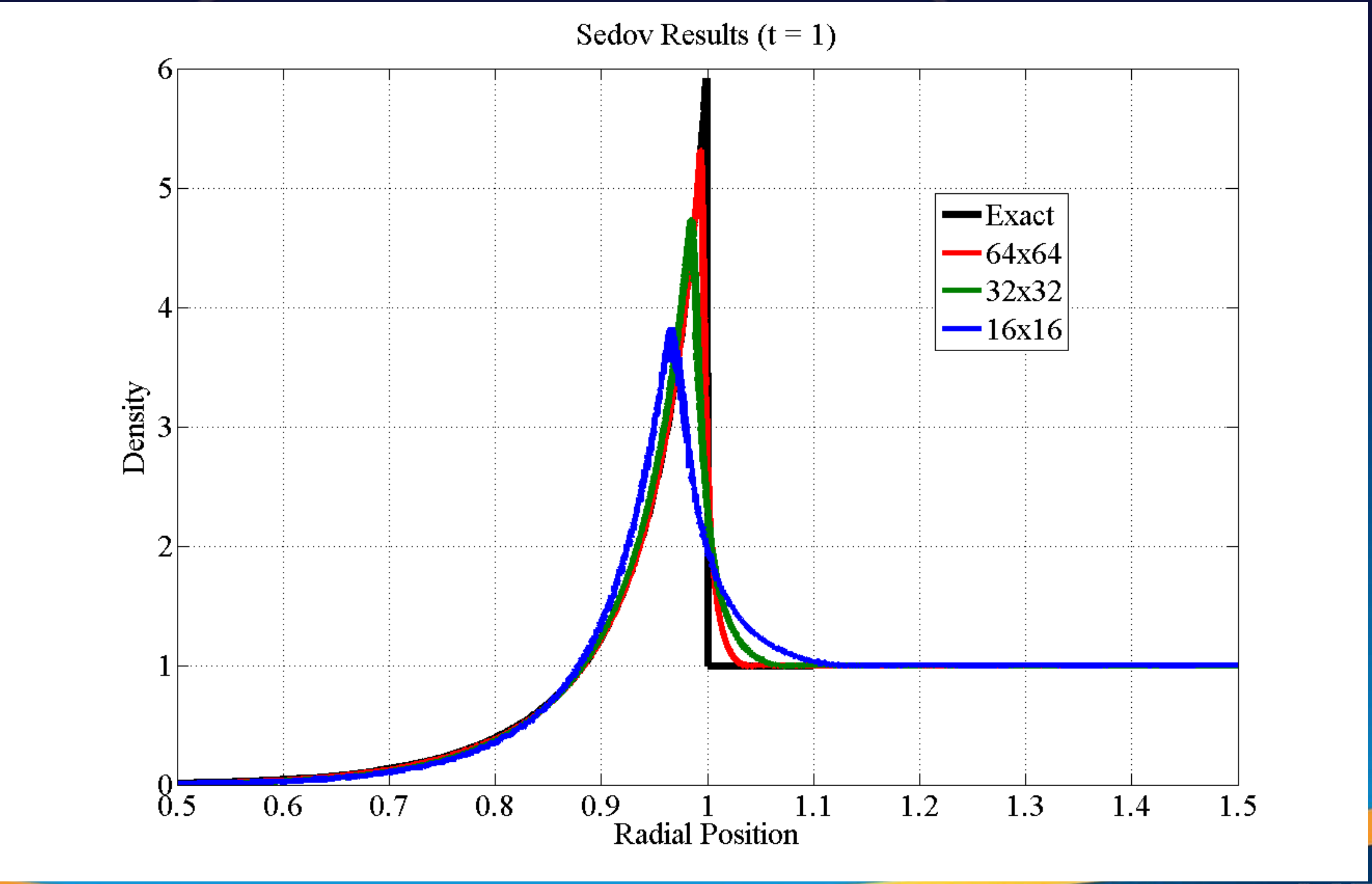
Dobrev, et al



Sedov 'Blast' Problem



Sedov ‘Blast’ Problem



Conclusions

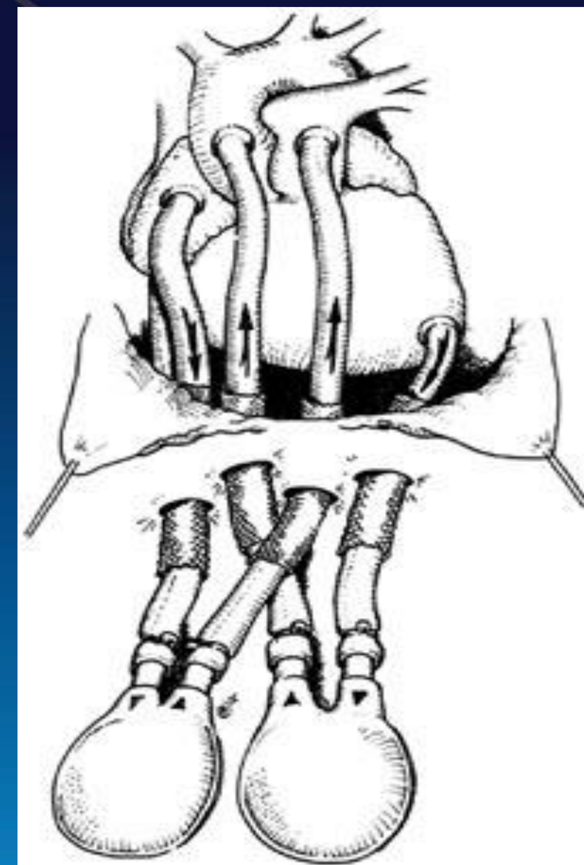
- NURBS-based IGA is a highly promising candidate in this field
- Energy can be preserved exactly
- Exact symmetry preservation and solutions are attainable in some cases, and is otherwise ‘best-in-class’.

Part II: Pulsatile Ventricular Assist Devices

Chris Long[1], Alison Marsden[2], Yuri Bazilevs[3]

[1]Los Alamos National Laboratory, T-3 Fluid Dynamics and Solid Mechanics
[2]University of California, San Diego, Mechanical and Aerospace Engineering
[3]University of California, San Diego, Structural Engineering

Pulsatile VADs



Pediatric VADs

- Thrombus formation is major clinical problem
  - ~22% of pediatric patients experience thromboembolic event
- Survival rate is 63-89%
- Bridge to recovery usage shows significant promise in children
  - Heart is able to repair itself in ways not seen in adults

T Humpl, S Furness, C Gruenwald, C Hyslop, and G Van Arsdell. The berlin heart excor pediatrics-the sickkids experience 2004-2008. *Artif Organs*, 34(12):1082–6, 2010.

SR Rockett, JC Bryant, WR Morrow, EA Frazier, WP Fiser, WA McKamie, CE Johnson, CW Chipman, M Imamura, and RD Jaquiss. Preliminary single center north american experience with the berlin heart pediatric excor device. *ASAIO J.*, 54(5):479–82, 2008.

SC Malaisrie, MP Pelletier, JJ Yun, K Sharma, TA Timek, DN Rosenthal, GE Wright, RC Robbins, and BA. Reitz. Pneumatic paracorporeal ventricular assist device in infants and children: initial stanford experience. *J Heart Lung Transplant.*, 27(2):173–7, 2008.

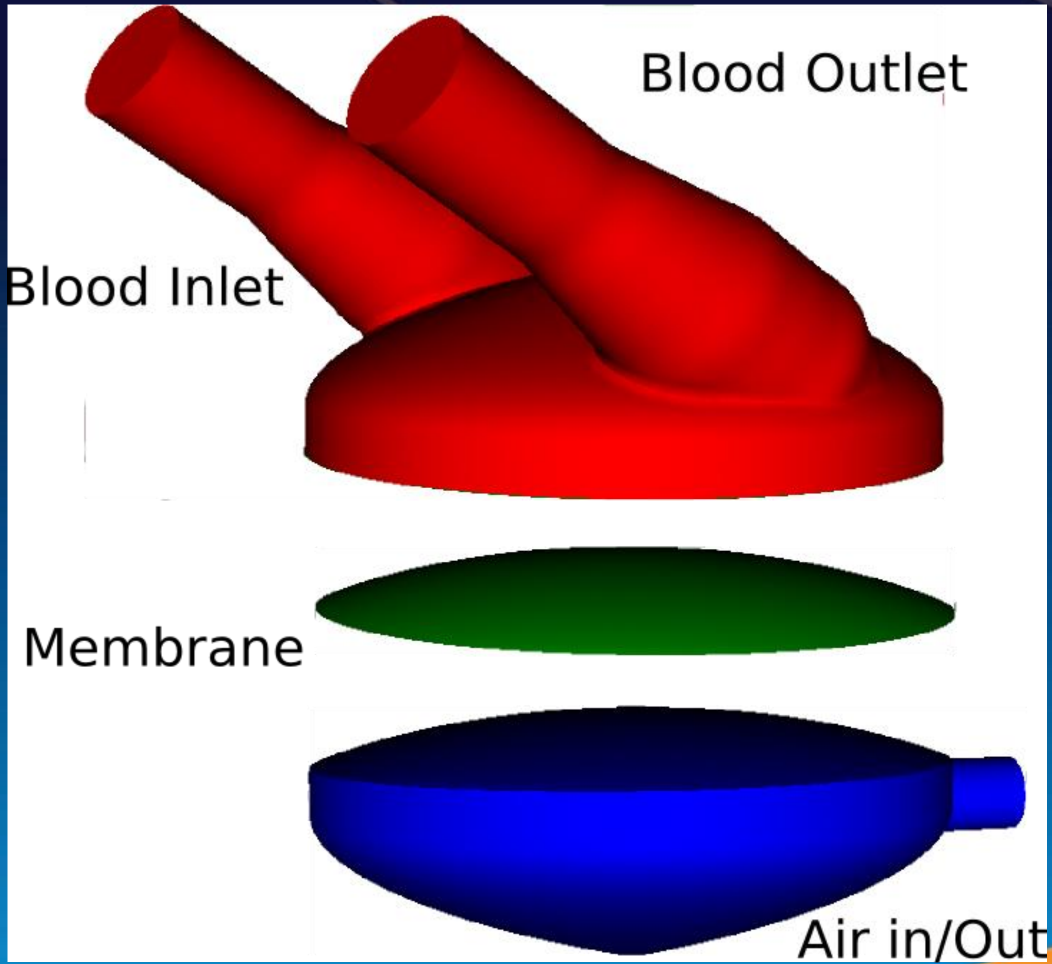
How do we simulate this?

- Problems:
  - Multiple fluids, moving domains
  - Mesh quality issues when mesh is compressed
  - Membrane dynamics are highly nonlinear
- Tools developed for:
  - Multi-domain, multi-fluid simulations
  - Isogeometric Analysis of structure
  - Ensuring good mesh quality
  - Fast and robust fluid-structure communication

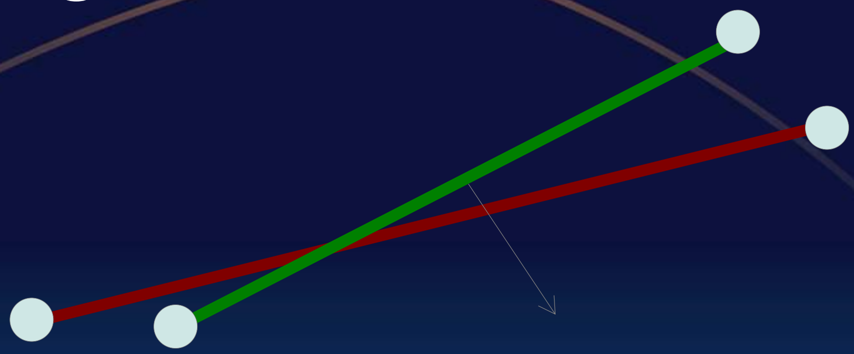
C.C. Long, A.L. Marsden, Y. Bazilevs, *Fluid-structure interaction simulation of Pulsatile Ventricular Assist Devices*, Computational Mechanics 2013(52)

Multiple Domains

- Create two distinct numerical meshes, one for each fluid domain
- Create an additional numerical mesh for the membrane (NURBS)
- Create a method for transferring boundary conditions across meshes



Non-matching discretizations



- Take normal out of Gauss point and define a line along vector path
- Find shortest distance to the line on plane defined by other mesh – store this mapping and use repeatedly

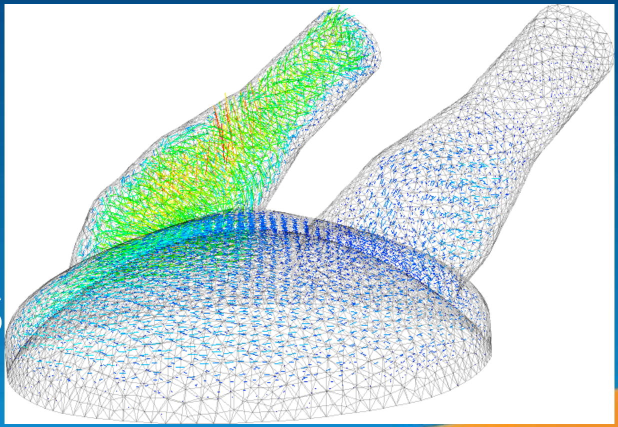
$$\int N_a N_b x_b d\Omega_g^{green} = \int N_a \bar{x} d\Omega_g^{red}$$

Mesh Quality

- When mesh becomes highly compressed, we lose mesh quality.

Pause simulation. Write out deformed mesh and nodal values to file. → Use MeshSim to read old surface mesh and create a new interior mesh. → Pass old mesh values to new mesh and restart simulation.

- Use a “Point in Polygon” method for passing mesh values
- Remeshing step takes less than 5 minutes



Non-Linear Membrane Motion

- Use a Kirchhoff-Love shell solver to solve membrane
- Transverse shear is zero

$$0 = \delta W_{int} + \delta W_{ext} = \delta W = \frac{\partial W}{\partial \mathbf{u}} \delta \mathbf{u}$$

$$\delta W_{int} = -\int_V (\mathbf{S} \circ \delta \mathbf{E}) \, dV$$

Fluid Structure Coupling

- Weak form in FSI case is:
  - Find $\boldsymbol{u} \in S_u$, $\ddot{\boldsymbol{d}} \in S_d$, and $p \in S_p$ such that $\forall\ \boldsymbol{w} \in V_u$, $\boldsymbol{w}^s \in V_d$, and $q \in V_p$:

$$\int_{\Omega_t} \mathbf{w}\cdot(\rho(\dot{\mathbf{u}}+\mathbf{u}\cdot\nabla\mathbf{u}-\mathbf{f})-\nabla\cdot\sigma)\mathrm{d}\Omega + \int_{\Gamma_E} \mathbf{w}\cdot(\sigma\hat{\mathbf{n}}-\mathbf{h}_E)\mathrm{d}\Gamma + \int_{\Omega_t} q\nabla\cdot\mathbf{u}\,\mathrm{d}\Omega$$

$$+\int_{\Omega_t^s} \mathbf{w}^s\cdot(\rho^s(\ddot{\mathbf{d}}^s-\mathbf{f}^s)-\nabla\cdot\sigma^s)\mathrm{d}\Omega + \int_{\Gamma_E^s} \mathbf{w}^s\cdot(\sigma^s\hat{\mathbf{n}}^s-\mathbf{h}_E^s)\mathrm{d}\Gamma + \int_{\Gamma_I} (\mathbf{w}\cdot\sigma\hat{\mathbf{n}}+\mathbf{w}^s\cdot\sigma^s\hat{\mathbf{n}}^s)\mathrm{d}\Gamma = 0$$

Where

$$\mathbf{u} = \dot{\mathbf{d}} \quad \text{on } \Gamma_I$$

$$\mathbf{w} = \mathbf{w}^s \quad \text{on } \Gamma_I$$

FSI

- These four matrices are difficult to assemble
  - They measure fluid/structural residual response due to variations in the structural/fluid domain
  - Have dimensions of “number of fluid nodes x number of structural nodes” or vice versa
  - Fluid and structural domains have separate discretizations and shape functions.

$$\begin{pmatrix} \mathbf{K}_{11} & \mathbf{G}_1 & \mathbf{K}_{12} \\ \mathbf{D}_1 & \mathbf{L} & \mathbf{D}_2 \\ \mathbf{K}_{21} & \mathbf{G}_2 & \mathbf{K}_{22} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{u}}_{n+1} \\ p_{n+1} \\ \ddot{\mathbf{d}}_{n+1} \end{pmatrix} = \begin{pmatrix} \mathbf{R}^{mom} \\ \mathbf{R}^{cont} \\ \mathbf{R}^{str} \end{pmatrix}$$

Matrix Free Method

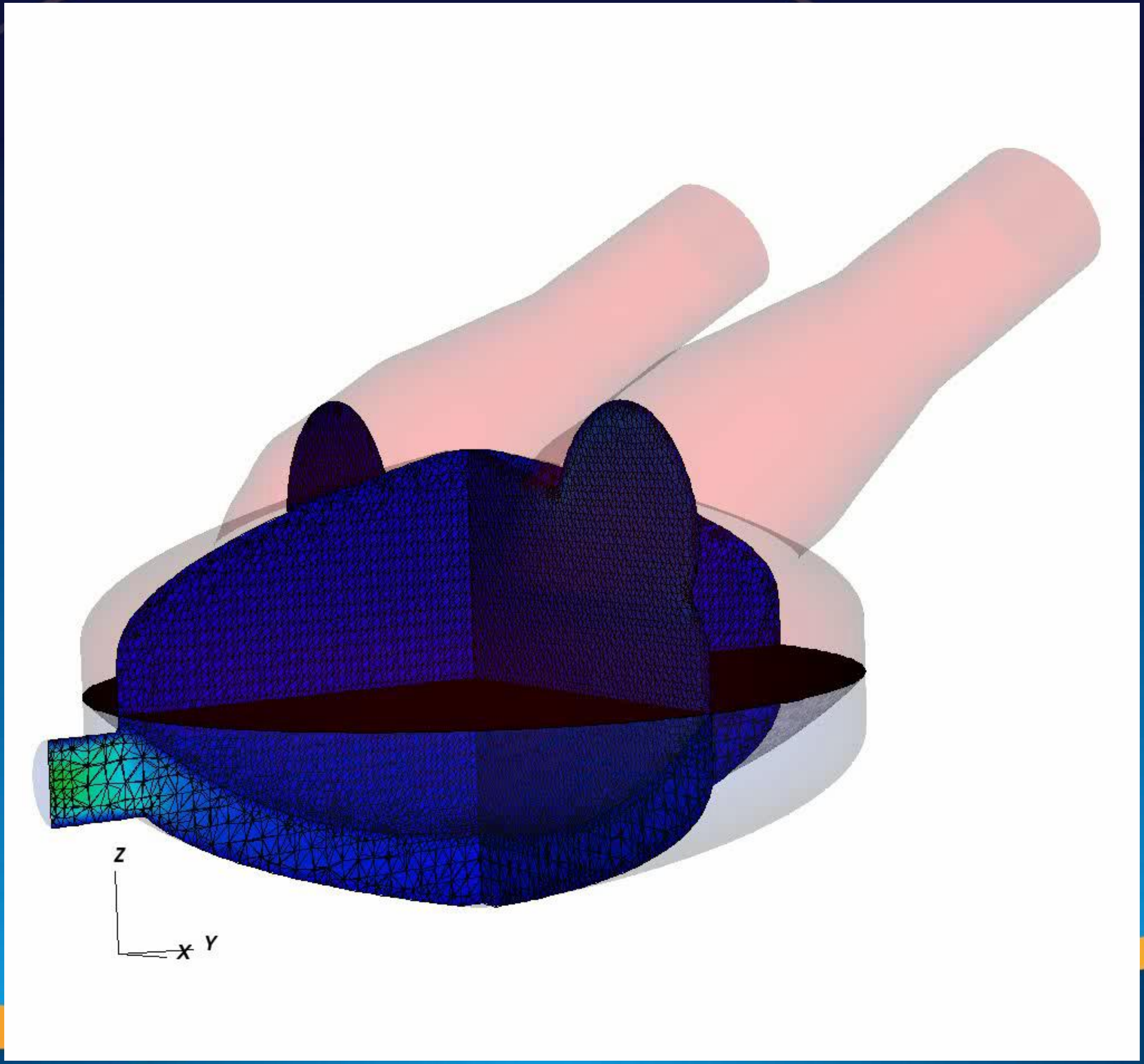
- Must approximate $\frac{\partial R_{mom}}{\partial \ddot{d}_{n+1}}$ K_{12} some other way.
- Use the definition of a derivative to achieve this:

$$\mathbf{K}_{12}\Delta\ddot{\mathbf{d}} \approx \frac{\left(\mathbf{R}^{mom}(\dot{\mathbf{u}}, p, \ddot{\mathbf{d}} + \epsilon\Delta\ddot{\mathbf{d}}) - \mathbf{R}^{mom}(\dot{\mathbf{u}}, p, \ddot{\mathbf{d}})\right)}{\epsilon}$$

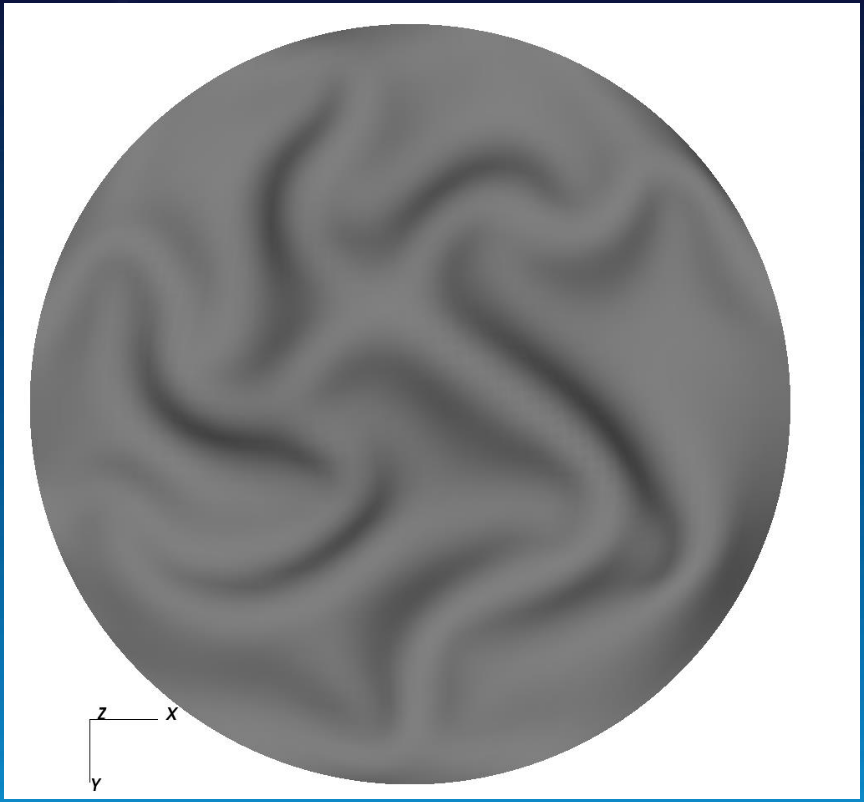
  - Mathematically very simple!
    - Downside: Requires assembly of right hand side vectors at each Krylov iteration.

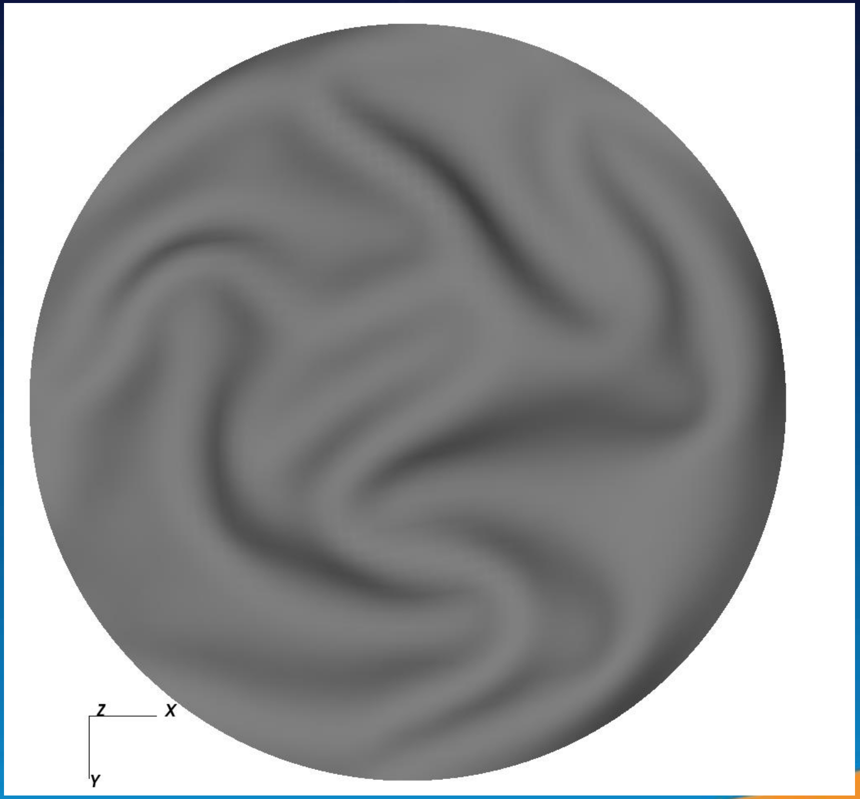
$$\begin{pmatrix} \mathbf{K}_{11} & \mathbf{G}_1 & \mathbf{K}_{12} \\ \mathbf{D}_1 & \mathbf{L} & \mathbf{D}_2 \\ \mathbf{K}_{21} & \mathbf{G}_2 & \mathbf{K}_{22} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{u}}_{n+1} \\ p_{n+1} \\ \ddot{\mathbf{d}}_{n+1} \end{pmatrix} = \begin{pmatrix} \mathbf{R}^{mom} \\ \mathbf{R}^{cont} \\ \mathbf{R}^{str} \end{pmatrix}$$

Results



Results





Quantifying Thrombotic Risk

- How can we use simulation results to make informed design choices?
- Must have a means of computing clinically relevant risk factors based off flow

Coagulation Chemistry

- Coagulation is result of complex set of chemical reactions in blood
- This area is an active field of research, and many have solved ODEs governing coagulation in blood flow
  - Most research focuses on vessel wall injury as catalyst for coagulation
  - Intrinsic or ‘contact’ pathway not well studied or modeled
- Meaningful implementation is difficult at this time

Flow risk factors

- Other flow parameters must be extracted from simulations for evaluation
- Flow stagnation and particle residence time are well-known to correlate with increased risk of thrombosis
- Several definitions of “residence time” are possible
  - Esmaily-Moghadam developed two definitions designed to measure residence time for thrombotic risk

M. Esmaily-Moghadam, T.-Y. Hsia, and A.L. Marsden. A non-discrete method for computation of residence time in fluid mechanics simulations. *Physics of Fluids*, (In Press).

Residence Time

- RT_1 and RT_2 are computed on a moving domain as:

$$RT_1 = \frac{1}{T}\int_{(N-1)T}^{NT} \frac{1}{V_t}\int_{\Omega_\tau(t)} \tau(\mathbf{x}, t)\mathrm{d}\Omega\mathrm{d}t$$

$$RT_2 = \frac{\bar{V}}{\bar{Q}}$$

Where τ is the nodal residence time, which is advected through the flow.

C.C. Long, M. Esmaily-Moghadam, A.L. Marsden, Y. Bazilevs, Computation of residence time in the simulation of pulsatile ventricular assist devices, Computational Mechanics 2013

Advection-Diffusion Solver

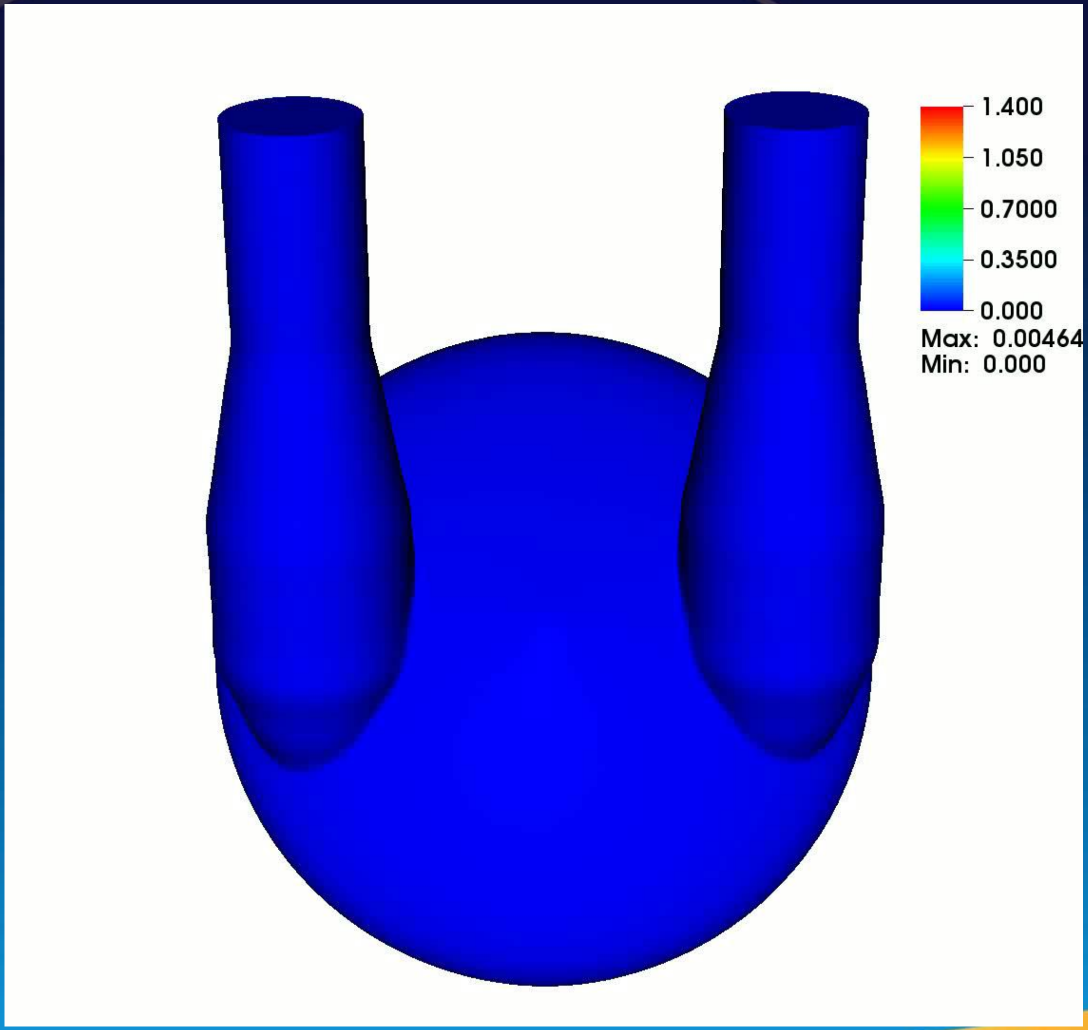
- For residence time computations, τ is initialized to 0
- We solve the advection-diffusion equation as shown, using computational results from simulations:

$$\frac{\partial \tau}{\partial t}|_{\hat{\mathbf{x}}} + \mathbf{u}_a \cdot \nabla \tau - \nabla \cdot \kappa \nabla \tau - H = 0$$

$$H(\mathbf{x}, t) = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega_\tau(t) \\ 0 & \text{otherwise} \end{cases}$$

- Use a stabilized finite-element based implementation

Results

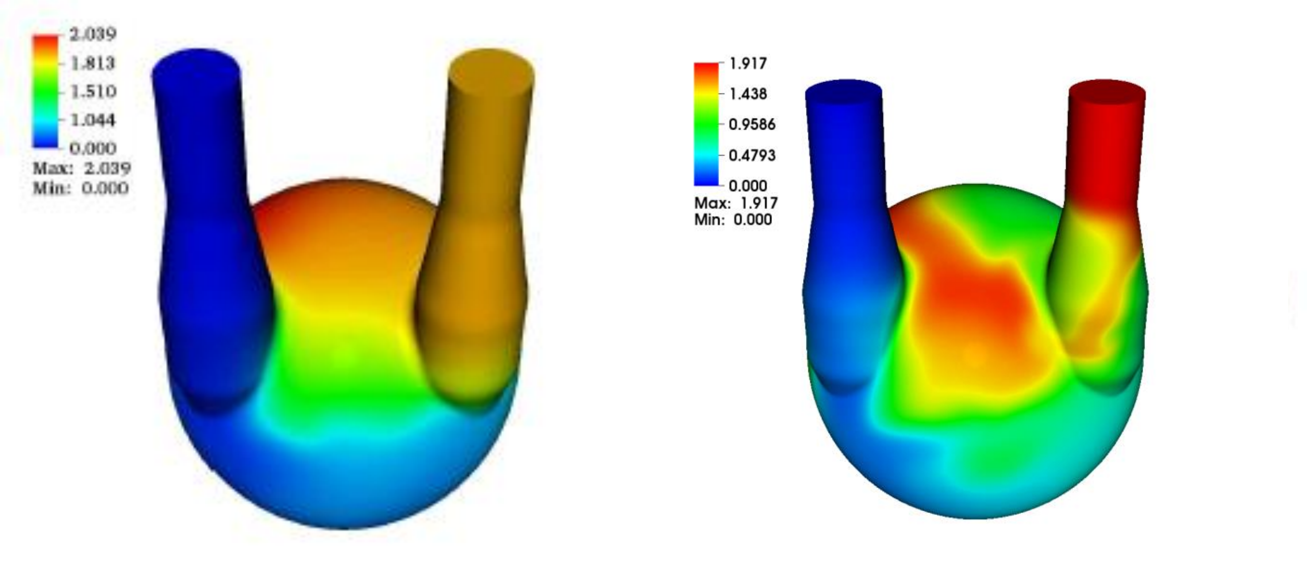


Results

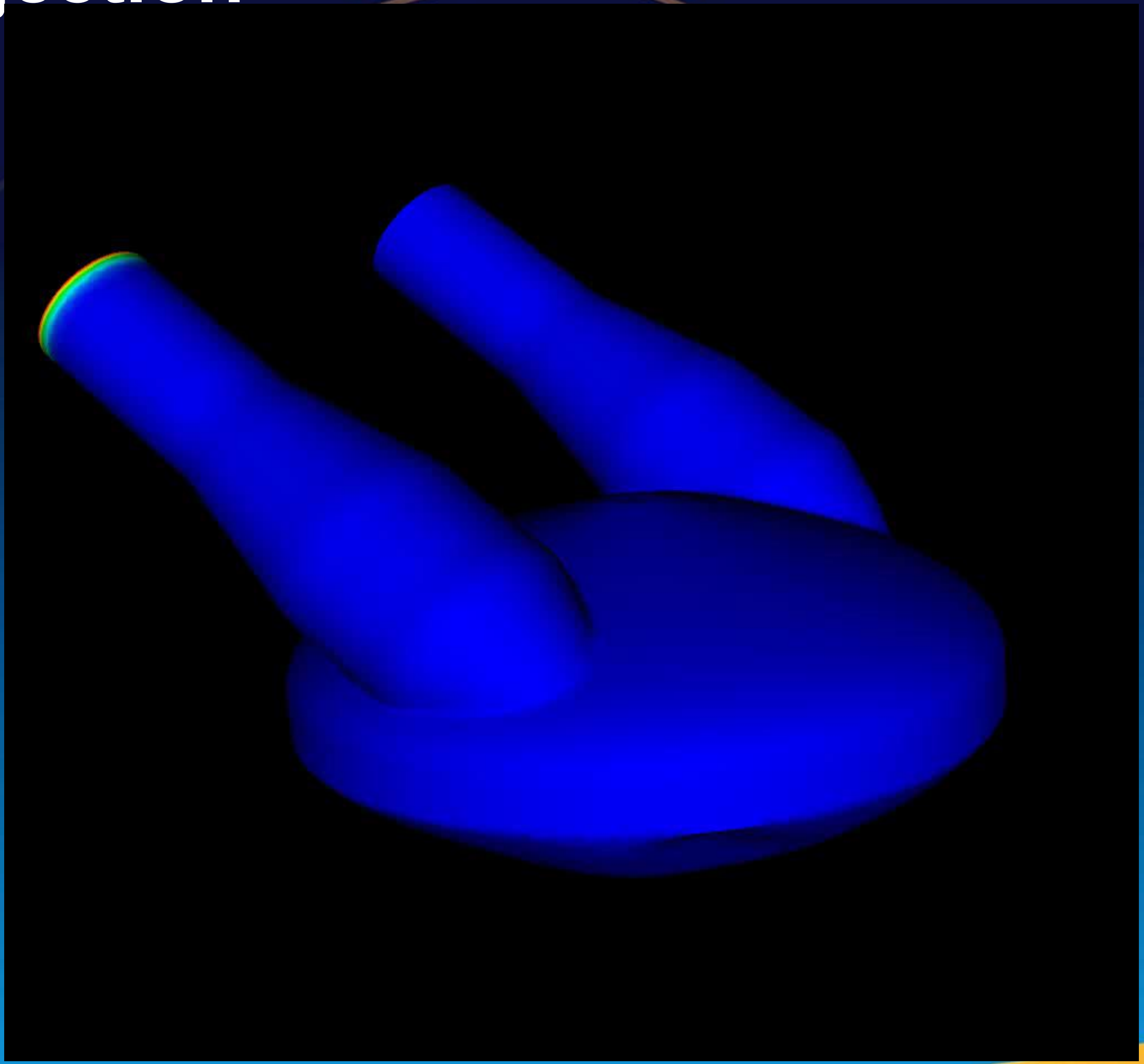
$$\bar{\tau} = \frac{1}{V_t}\int \tau(x,t)\, d\Omega$$

$$RT_1 = \frac{1}{T}\int_{(N-1)T}^{NT} \bar{\tau}\, dt$$

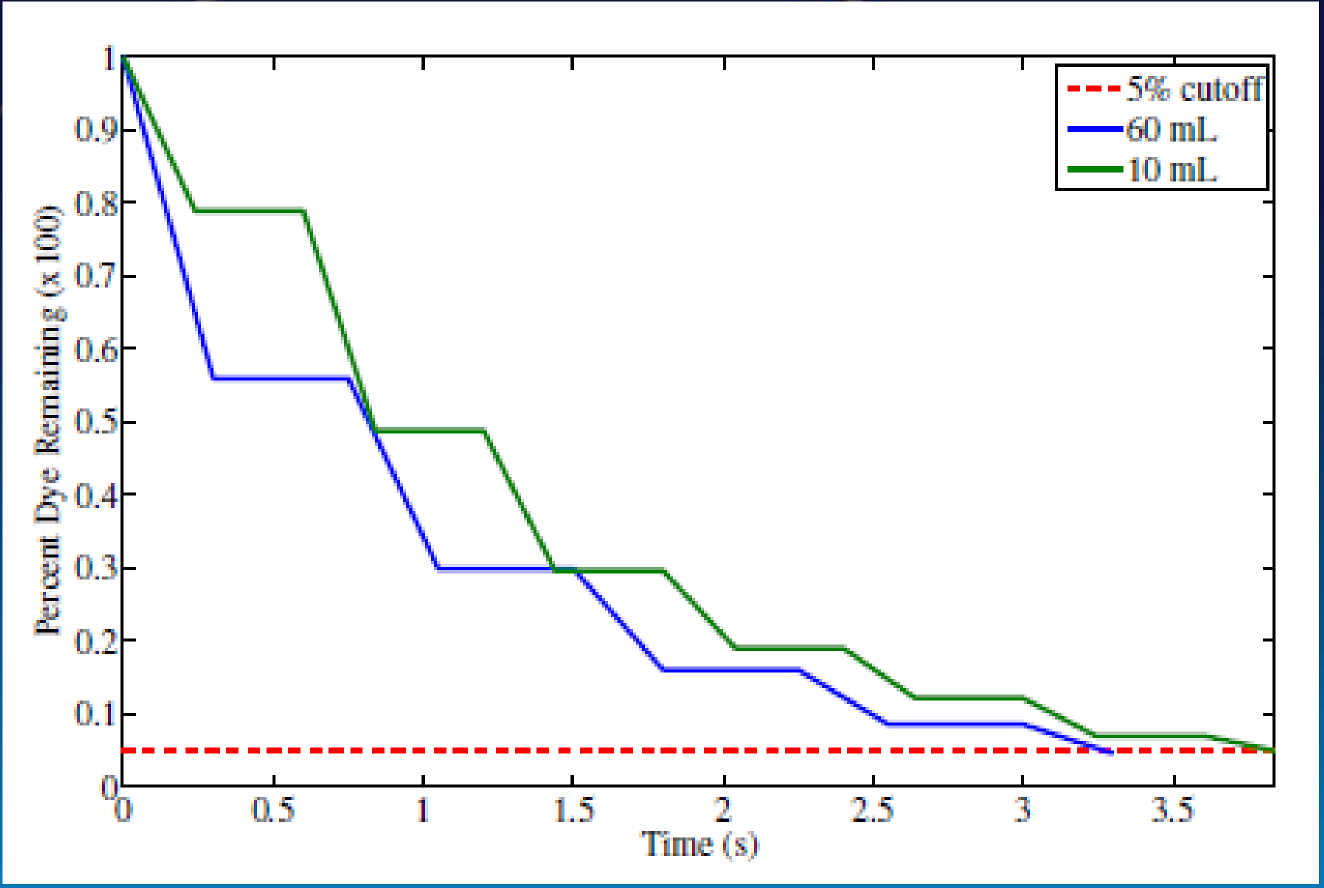
	$\bar{V}$ (cm^3)	$\bar{Q}$ ($\frac{mL}{s}$)	RT_1 (s)	RT_2 (s)	max(τ) (s)	max($\bar{\tau}$) (s)
10 mL	27.26	16.67	1.320	1.635	2.04	1.413
60 mL	107.7	80	1.216	1.346	1.917	1.379



Dye Injection



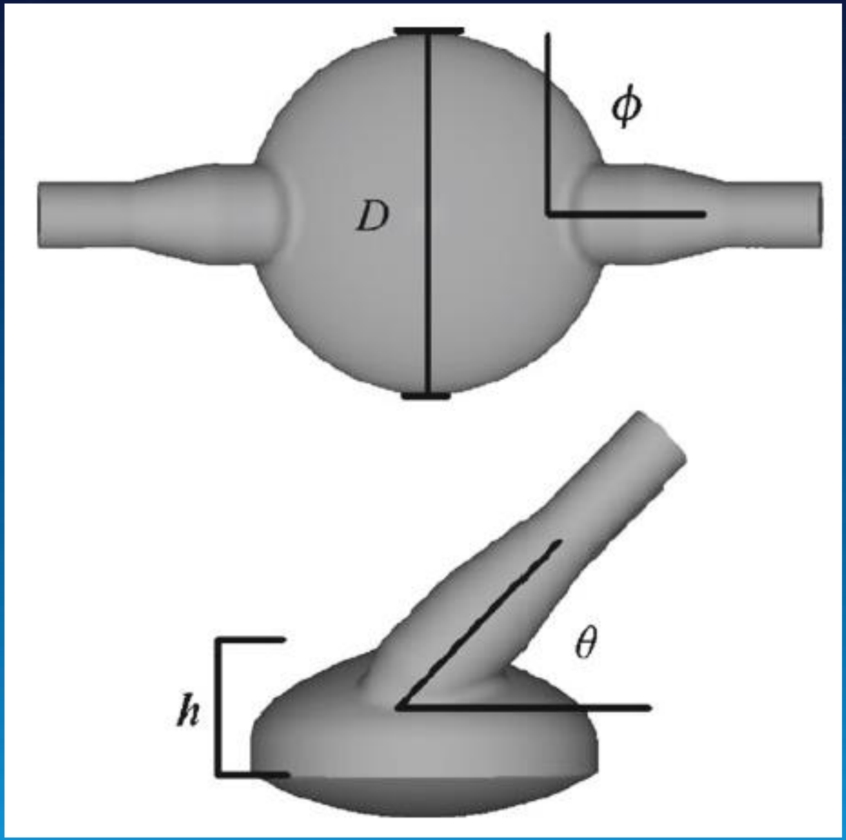
Dye Injection



Cycle No.	1	2	3	4	5	6	7	T_{95}
10 mL	78.92	48.89	29.65	19.08	12.22	7.80	4.98	3.83
60 mL	56.01	29.97	16.09	8.63	4.62	-	-	3.27

Design Space Parameterization

- Design space is created automatically based on the input of 4 key parameters



	Min	Max
D	1.9 cm	2.6 cm
h	0.6159 cm	2.1 cm
θ	$\frac{\pi}{12}$	$\frac{\pi}{2}$
ϕ	0	$\frac{\pi}{2}$

Cost Function

- Optimization requires a “cost” function, J:

$$\begin{aligned} &\text{minimize} \quad J(\mathbf{x}) \\ &\text{subject to} \quad \mathbf{x} \in \Omega \end{aligned}$$

- Evaluation of J requires a full PVAD simulation
- Should correlate with thrombotic risk

$$J = \left(\frac{RT_1 + RT_2 + \max(\tau)}{3} \right) \cdot T_{95}$$

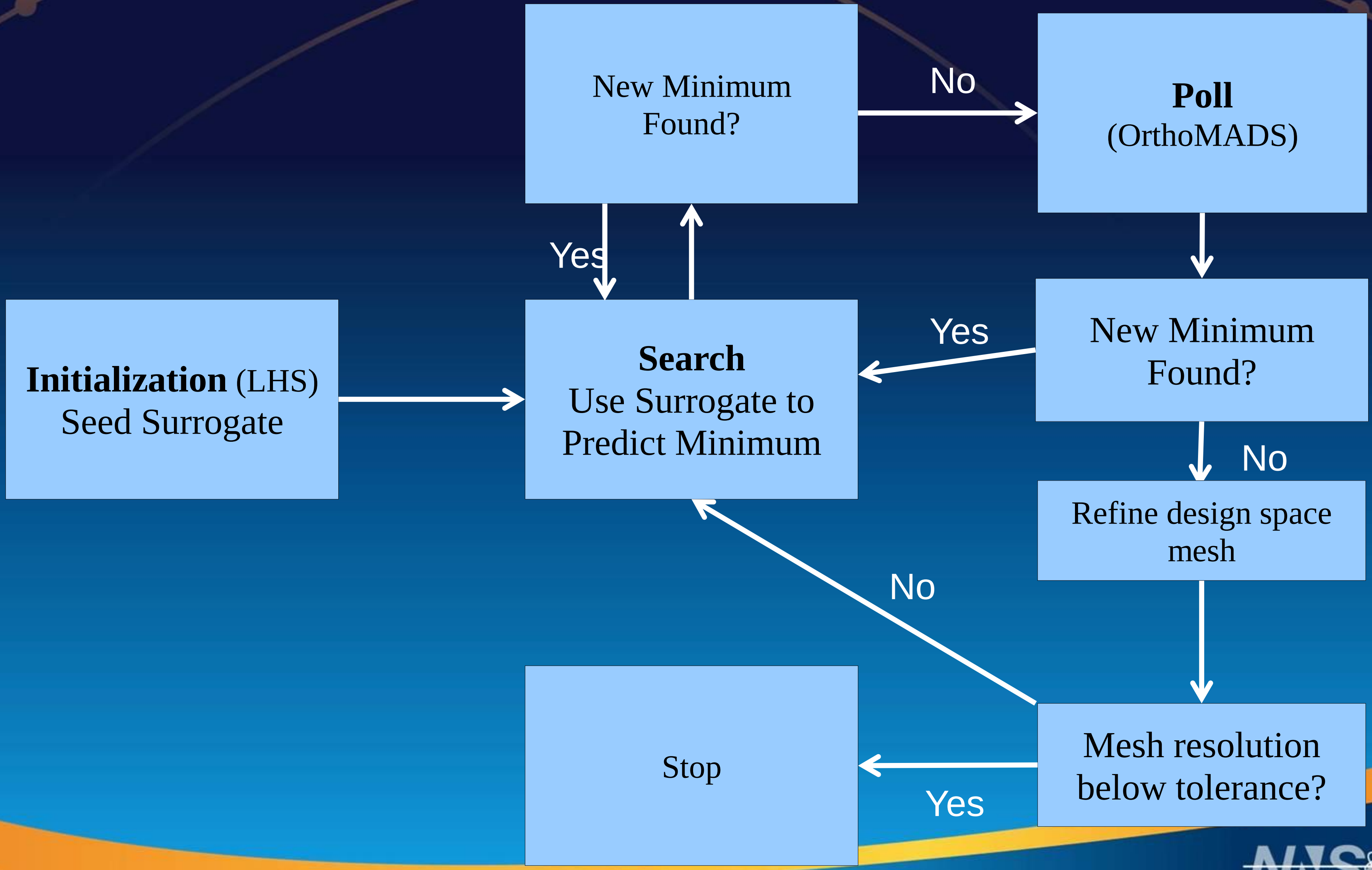
- Post-processing of results is fully automated.

SMF Framework

- Cost function evaluations are expensive! (~20,000 CPU hours)

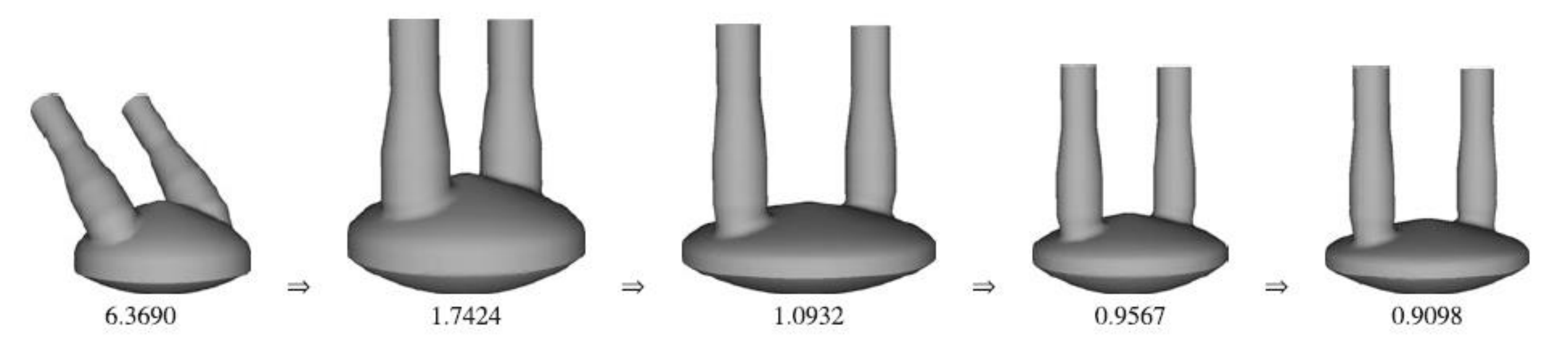
- Surrogate Management Framework (SMF)
  - Surrogate function intersects known cost function evaluations in the design space
  - Used to predict new cost function minimum
- Design space is seeded with initial computations of J

Flowchart


New Minimum Found?
No
Poll (OrthoMADS)
Yes
Initialization (LHS) Seed Surrogate
Search Use Surrogate to Predict Minimum
Yes
New Minimum Found?
No
Refine design space mesh
No
Stop
Mesh resolution below tolerance?
Yes

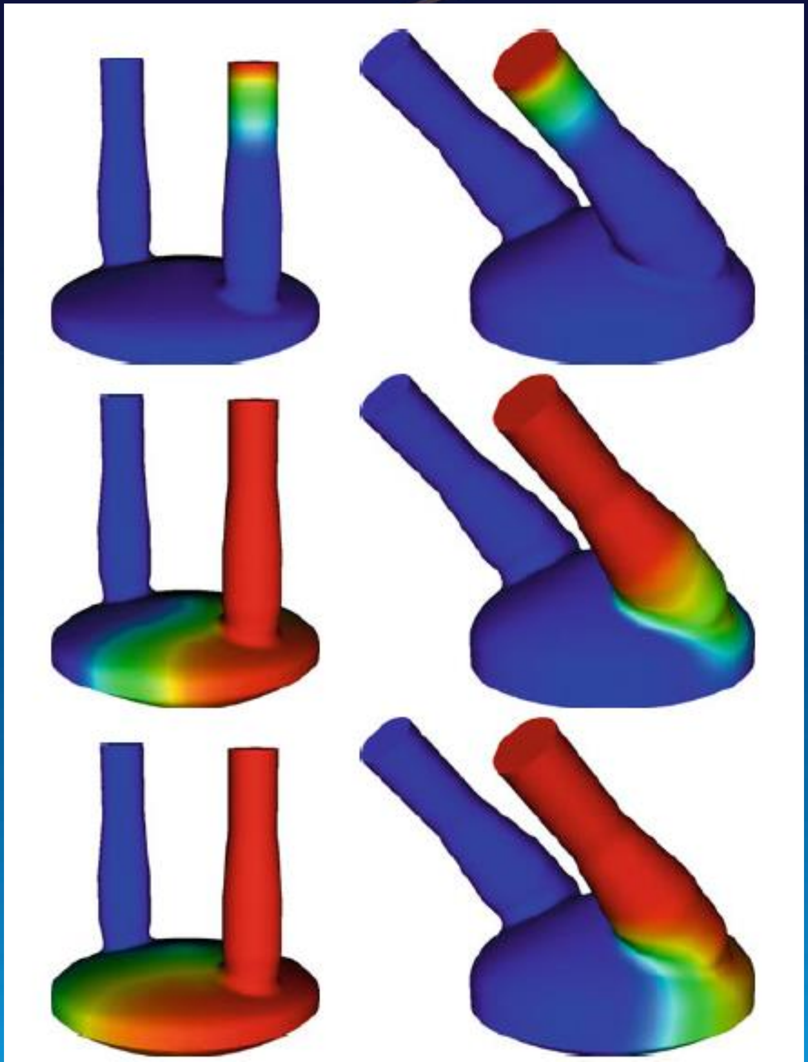
Results

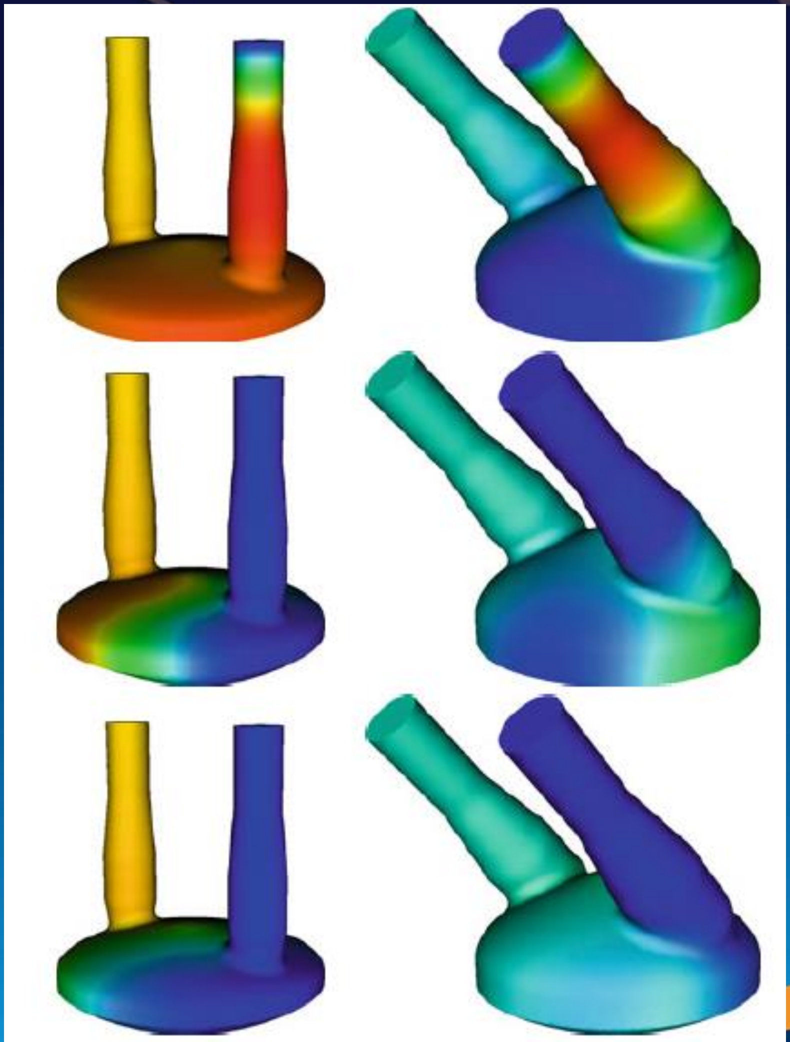
- 37 models were created over 5 Search/Poll sequences and 2 design space mesh refinements.
- Optimization scheme almost immediately settles on vertical arm orientation.



C.C. Long, A.L. Marsden, Y. Bazilevs, *Shape optimization of pulsatile ventricular assist devices using FSI to minimize thrombotic risk*, Computational Mechanics 2013

Results





Conclusions/Future Work

- Model creation, simulations, and post-processing are automated and robust
- Simple optimization with reduced number of variables suggests thrombotic risk reduction is possible
- Vertical arms are demonstrably superior in terms of residence time

- Improve cost function by implementing new risk factors – OSI, WSS are possible contributors
- Validation with experiments
- Run optimization with larger set of design parameters

Non-spatial Parameters

- Additional parameters that do not require physical changes to the device may also be used
- Constrain that a desired flowrate, Q, is achieved
  - Stroke volume, beat frequency, shape of flowrate time curve
    - Length of fill/expel phases
    - Exponent of sin function

$$q = \begin{cases} q_e \sin^{\frac{1}{2}}\left(\frac{t}{0.3}\pi\right) & \text{if } t < 0.3 \\ q_f \sin^{\frac{1}{2}}\left(\frac{t-0.3}{0.45}\pi\right) & \text{otherwise} \end{cases}$$