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## Optimal Control of EGR System in Gasoline Engine Based on Gaussian Process

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**Abstract:** The contribution described in this paper is concentrated on the integration of exhaust gas recirculation (EGR) system into the process of combustion in an optimal manner. In practice, deriving a state-space model of this actuator is an energetic task as a result of involving some uncertain chemical reactions. To alleviate the effect of unobserved phenomena, which does not seem to be easy in modeling, an improved Gaussian Process (GP) is represented for identifying such dynamics. In this approach, practical modification in general formulation of GP is provided based on proportional feedback gain adjustment. Afterwards, the obtained model is considered for design of optimal model-based control strategy. The whole aim is focused on achieving a green economically gasoline engine by optimizing the trend of fuel consumption. Eventually, simulation results illustrate the effectiveness of proposed structure in EGR systems.

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*Keywords:* Automotive engineering, Diesel engines, Fractional control, EGR valve, Gaussian Process.

In recent years, Engine manufacturers and suppliers are objected to satisfy environmental regulation and customer requirements in attaining ultra-low emission vehicles with higher fuel efficiency, and improved drivability. As a consequence, a great advancement towards the structure of modern ICE engine has been made which incorporates additional devices like intake boosting, exhaust gas recirculation (EGR), and variable valve actuation. These developments would possess a superiority in providing an accurate observation of system while an inferiority can appear in an aspect of increasing the system complexity in comparison with traditional engines. As a result, the traditional approaches come up with providing engine calibration maps based on experimentally obtained lookup tables, are rendered incompetent due to a large number of tables associated with any individual control actuator. Among these subsystems, EGR process has shown to have a potential influence on the term of produced emission and fuel economy and great attraction has been provided in recent studies as in (Panni et al. (2014)).

To be able to improve the introduced goals in the newly designed engine which has evolved into an extremely non-linear coupled MIMO system, several possible methodologies can be proposed like: restating the combustion concepts, developing the system of fuel injection, making an advantage of new materials, or enhancing engine control system. The field of engine control is mostly predominant in early studies due to its flexibility in design and reliability in fabrication/implementation. One of the primary works reported in the literature on the principle of recirculating the exhaust gas in SI engines can be traced in (Stefanopoulou and Kolmanovsky (1999)) where the nonlinear behavior of an EGR system was analyzed. Afterward, (Laghrouche et al. (2010)) addressed a sophisticated overview of physical modeling and identification of this recirculation

process. These advancements in the identification of EGR loop contributes more, on the possibility of improving the control structure as in (Feru et al. (2012)). In contrast to these amount of works in relation to derive a mathematical formulation for its system dynamics, some unobservable phenomena that are not well understood or hard to model, would be problematic. Here, as an alternative way of modeling the complex behavior of the system, machine learning methods are mostly preferred.

To the best of the authors' knowledge, Gaussian Processes (GPs) are pointed out as a powerful modeling framework due to their systematic incorporation in the modeling uncertainty exist in predictions. However, the concept of system identification is not thoroughly incorporated with the advantage of GPs due to some limitations, yet it has actualized some astonishing results in literature where traditional approaches have struggled or failed. This point can be preserved by considering (Ko et al. (2007)) where the dynamics of autonomous blimp are acquired based on learning control policy and GPs. Within the assist of information regards to system dynamics, model-based controllers are then considered more reliable than the decentralized controllers in terms of numerous advantages mainly in tracking accuracy and possibility of compliance (de la Cruz et al. (2012)). Predictive control stands out in these categories due to its better robustness in adapting well to disturbance or nonlinearities, as a result of moving horizon scheme.

In this note, the EGR system within the process of combustion is identified by Gaussian Process framework and the model is then exploited with model predictive control to enhance the engine performance in both terms of exhaust emissions and fuel consumption. The main contribution of this study compared with other methods applied to torque control of SI engines is the fact that take an advantage of online learning policy, makes

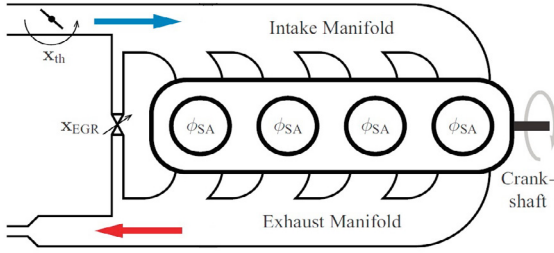


Fig. 1. Schematic description of the gasoline engine used for the study.

the design structure highly adaptive against system parameters variations in SI engines. Afterward, a simple and practical methodology to compensate the effect of offset error in the scheme of model approximation is also proposed to develop the GPs framework mainly in term of prediction error. The proposed modification is promising when the results show comparable improvement in comparison with Sparse Online Gaussian Processes in both terms of efficiency and complexity. Finally, the designed scheme provides feasible performance against any variation of torque/speed operation point in term of system constraints: engine knock and misfire, which makes the engine works in its safe operational condition.

In the remainder of paper, section 1 presents a brief description of SI engines and addresses the primary problem. Section 2 provides the basic theory beyond the Gaussian Process as well as its modification in the case of modeling the system's behavior. Afterwards, section 3 represents the designed control structures and consequently, the evaluated results are shown and compared in section 4. Eventually, conclusions are summarized in section 5.

## 1. PROBLEM FORMULATION

In this section, a brief description of the general behavior of gasoline engines shown in Fig 1 is provided. The main working principle of these classical engines is associated with three parameters: intake manifold, ignition timing, exhaust manifold. In the concept of controlling SI engines, two primary objectives are always investigated: reduction in produced emission, and efficient fuel consumption. These goals are indeed functionalized with the mentioned parameters e.g. the rate of fuel efficiency is highly affected by the way of integrating the exhaust manifold into the process of combustion. In general, the system dynamics can be summarized as  $S$  in below:

$$S : \begin{cases} \frac{dx(t)}{dt} = f(x(t), u(t)) \\ y(t) = g(x(t)) \end{cases} \quad (1)$$

where,  $u \in \mathbb{R}^{n_u}$ ,  $x \in \mathbb{R}^{n_x}$ , and  $y \in \mathbb{R}^{n_y}$  are system input, state, and output vectors, respectively. According to the principle of combustion, the control inputs are: the throttle valve  $x_{TH}$ , the spark advance  $x_{SA}$ , and the exhaust gas recirculation valve  $x_{EGR}$ . So, the vector of inputs can be rewritten as:

$$u = [x_{TH} \ x_{SA} \ x_{EGR}]'$$

Likewise, the accessible outputs are represented in  $y$ :

$$y = [\tau \ h \ q_f]'$$

where  $\tau$  is the produced engine torque in [Nm],  $h$  provides information about the occurrence of knock and misfire and  $q_f$  is

the current fuel consumption in [Kg/s]. The primary objective is to minimize the following cost function towards the predefined conditions

$$J(y, ref) = \int_{t_s}^{t_f} c_f q_f(t) + c_\tau |\tau_d(t) - \tau_{real}(t)|^2 dt \quad (2)$$

$$s.t. \ \dot{x} = f(x, u)$$

$$y = g(x)$$

$$h(x, u) < 0$$

$$u_{min} < u < u_{max}$$

which contains the total fuel consumption and torque tracking error over a whole engine cycle. The corresponding optimization problem can be restated in (3):

$$\min_u J(y, ref)$$

$$s.t. \ h(x, u) < 0$$

$$u \in U \quad (3)$$

Note that the saturation bounds on the control  $u$  have been taken into account explicitly by adding them on the output of the each control variable and  $u$  is a convex subset of  $\mathbb{R}^n$ . In this paper, we consider the following problem.

**Problem I.** Consider the dynamic behavior of engine in (1), measurable system parameters:  $u$ ,  $y$ . Derive efficient control laws with special attention on EGR system for minimizing the introduced objective function (2) over a desired torque profile  $\tau_d$ , constant engine speed  $\bar{n}$  and keeping the rate of knocking and misfiring below zero in order to satisfy the system constraints  $h(x, u)$ .

## 2. MODEL DERIVATION WITH GAUSSIAN PROCESSES

In the following, the exhaust gas recirculation process is approximated by using GPR framework. In what follows, the design of GPR algorithm in our desired application is first introduced. Meanwhile, GP's performance is boosted by presenting an adaptive modification parameter. Eventually, the task of finding a good approximation for system dynamics is accomplished by introducing two common choices of Kernel Function.

### 2.1 Bayesian System Identification

Bayesian system identification techniques stand out within the field of system identification due to their ability to assess the appearance of uncertainty in the system's dynamics. The general discrete-time model used in Bayesian system identification is stated as in (8)

$$x_{k+1} = w(x_k, u_k) + \delta_k, \quad (4a)$$

$$y_k = z(x_k) + \varepsilon_k. \quad (4b)$$

where  $x_k \in \mathbb{R}^{n_x}$  denotes the states variables as all features of the system that affects its future,  $y_k \in \mathbb{R}^{n_y}$  denotes state observations known as output vectors,  $\delta_k$  and  $\varepsilon_k$  shall be considered an independent additive noise:  $\mathcal{N}(0, \sigma^2)$ , on process and observation, respectively. Functions  $w$  and  $z$  represent the relation between the system dynamics and accordingly with the system's observed output Rasmussen and Williams (2006).

### 2.2 Gaussian Process Regression

Gaussian Processes, that are considered as stochastic processes, have shown great performance for the purposes of nonlinear regression. To this effect, Gaussian Process Regression known

as GPR is a global supervised learning method that allows us to formulate a Bayesian framework with the aim of regression. Generally speaking, we know that each collection of random variables, which possess a joint Gaussian distribution, is a Gaussian process. Under this condition, GPR is able to find the mapping function  $w \in \mathbb{R}$  which interprets the relation between inputs  $X$  and corresponding target output  $\mathcal{Y}$ , and can be formulated as

$$\mathcal{Y} \sim \mathcal{GP}(0, K(X, X) + \sigma_n^2 I_n), \quad (5)$$

where  $\sigma_n^2$  is the noise variance,  $I_n$  is the identity matrix of the size  $n \times n$ , and  $K(X, X)$  is the covariance matrix. The covariance matrix consists of covariance  $k(x, x')$  value associates with each set of training points. The mean function is often considered zero for notational simplicity.

From a practical point of view, training data set  $\mathcal{D} = \{X, Y\}$ , composed of  $n$  observations with a period of  $\Delta t$ , needs a special attention. The remarkable point is that the trend of fuel consumption is highly dependent on  $X_{EGR}$ , value of EGR valve, while the tracking error is mostly related to  $\Delta X_{EGR_k} = X_{EGR_k} - X_{EGR_{k-1}}$ , the variation of EGR valve at each sample time. This observation provides a unique mapping function in different operational regions. Note that, data sampling time should be equivalent to the controller sampling interval that will be introduced in the subsequent section. So, we have

$$Y = [\tau_{k-n\Delta t} \cdots \tau_{k-\Delta t} \tau_k]^T \quad (6a)$$

$$X = \begin{bmatrix} X_{EGR_{k-n\Delta t}} & \cdots & X_{EGR_{k-\Delta t}} & X_{EGR_k} \\ \Delta X_{EGR_{k-n\Delta t}} & \cdots & \Delta X_{EGR_{k-\Delta t}} & \Delta X_{EGR_k} \end{bmatrix}^T. \quad (6b)$$

Now, the problem is to find a prediction function  $w^*$  over a given new input  $x^*$ . According to GP prior, the joint distribution of observed values and the predicted function can be observed

$$\begin{bmatrix} \mathcal{Y} \\ w^* \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} K(X, X) + \sigma_n^2 I_n & k(X, x^*) \\ k(x^*, X) & k(x^*, x^*) \end{bmatrix}\right). \quad (7)$$

The conditional distribution of  $w^*$  at any new input location  $x^*$  is given by

$$\mu(\hat{w}^*) = k(x^*, X)(K(X, X) + \sigma_n^2 I_n)^{-1} Y \quad (8a)$$

$$\sigma^2(\hat{w}^*) = k(x^*, x^*) -$$

$$k(x^*, X)(K(X, X) + \sigma_n^2 I_n)^{-1} k(X, x^*). \quad (8b)$$

In our application, demand for a precise model of the EGR system makes us improve the performance of GPR framework. A simple manipulation in the form of an observation could be considered by finding a adaptive coefficient in order to adjust the offset of output as in (8b):

$$x_{k+1} = \alpha_k w(x_k, u_k) + \delta_k. \quad (9)$$

In this case, a proportional feedback gain  $\alpha$  is provided to eliminate the effect of offset error which is calculated iteratively based on data observed by actual system and GPR framework. The formulation can be viewed as:

$$\alpha_k = Q_{av} \bar{\Delta Y}_k (Q_{av} Q'_{av})^{-1}. \quad (10)$$

Here,  $Q_{av} = [\frac{1}{N} \cdots \frac{1}{N}]$  is the weighting matrix, helps us to compute a custom average over past horizon  $\bar{N}$ .  $\bar{\Delta Y}_k$  denotes the fraction of real engine output and predicted value over last  $\bar{N}$  observations. For clarification, it can be restated as below:

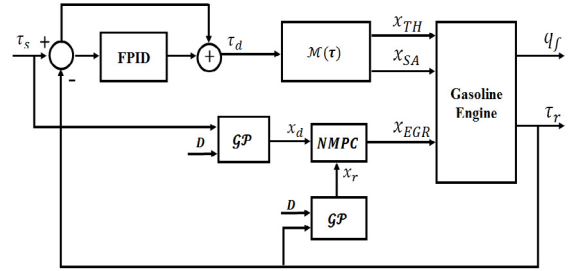


Fig. 2. Scheme of control loop.

$$\bar{\Delta Y}_k = \begin{bmatrix} y_{(k-\bar{N})\Delta t}^r & y_{(k-\bar{N}+1)\Delta t}^r & \cdots & y_{(k-1)\Delta t}^r \\ \hat{y}_{(k-\bar{N})\Delta t}^* & \hat{y}_{(k-\bar{N}+1)\Delta t}^* & \cdots & \hat{y}_{(k-1)\Delta t}^* \end{bmatrix}^T. \quad (11)$$

This manipulation can also be generalized in other applications when a stationary variation between the identified model and actual system can be calibrated by simple proportional feedback gain adjustment.

### 2.3 Kernel Functions

The principal function provided in GPR framework is covariance function that let us to know the prediction probability of input  $x^*$ . To this end, two common choices: squared-exponential (SE) and Matern (M) function are considered and analyzed, aim to minimize prediction error and maximize the probability of predicted value. SE kernel function is one widely used covariance function describes by:

$$k_{SE}(r) = \exp\left(-\frac{r^2}{2l^2}\right), \quad (12)$$

where  $l$  is characteristic length-scale, and  $r$  denotes the Euclidean distance of samples. Another class of kernel function known as a Matern covariance function can be formulated as:

$$k_M(r) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu}r}{l}\right)^\nu K_\nu\left(\frac{\sqrt{2\nu}r}{l}\right) \quad (13)$$

Here,  $\nu$ ,  $r$ , and  $l$  are corresponding hyperparameters associated with Matern covariance function. The  $K_\nu$  is a modified Bessel function. In Gaussian Process framework, model selection in the functional form of covariance function is integrated with the optimization of their hyperparameters. The learning process of these parameters is performed by maximizing the log-marginal likelihood by taking an advantage of optimization procedures like gradient-based methods. Note that, these parameters are learned off-line while the training data are updated at each sampling time.

## 3. CONTROLLER DESIGN

In what follows, we aim to derive an efficient control law to effectively regulate EGR actuator based on the observed system dynamics in previous section. To this end, the structure depicted in Fig 2 is employed in this study. The notation  $s$ ,  $d$ , and  $r$  denote the set point, desired, and real variables, respectively. This scheme considers Fractional PID controller with a sub-optimal set of linear mapping functions  $\mathcal{M}(\tau)$  for adjusting the throttle valve and spark advance that can be briefly expressed as:

$$\mathcal{M}(\tau) : \begin{cases} \mathcal{X}_{TH} = A_{TH}\tau_d + B_{TH} \\ \mathcal{X}_{SA} = A'_{SA}\tau_d + B'_{SA} \end{cases} \quad (14)$$

Here,  $A_{TH}$ ,  $B_{TH}$ ,  $A'_{SA}$ , and  $B'_{SA}$  are associated scaling parameters. Extensive description regards to the set of  $\mathcal{M}(\tau)$  is presented in (Zarghami et al. (2016)). To the best of the authors' knowledge, a nonlinear mapping function for throttle valve and spark advance has been also provided in literature while the idea beyond the use of this representation is to simplify the problem and illustrate how EGR system can affect the process of combustion when it can be well regulated. For this aim, Nonlinear Model Predictive Control (NMPC) based on the model obtained by GPR is presented.

### 3.1 Fractional PID Controller

The generalized form of Fractional PID controller is obtained by substituting  $s$  in Laplace domain with fractional powers. This manipulation can be appeared as:

$$C(s) = K_P + \frac{K_I}{s^\lambda} + K_D s^\mu \quad (15)$$

The ability to indicate fractional powers ( $\lambda$  and  $\mu$ ) for the integral and derivative parts, respectively, makes a better flexibility in scaling the control actuators. Note that, the parameters in our structure ( $\lambda$ ,  $\mu$ ,  $K_P$ ,  $K_I$ ,  $K_D$ ) are tuned by employing Genetic Algorithm. Regards to the frequency analysis of fractional controller, a vivid description is provided in (Lanusse et al. (2014)).

### 3.2 Nonlinear Predictive Control

Model Predictive Control (MPC), which is also identified as Receding Horizon Control denotes a class of algorithms that compute an optimal sequence of policies in order to enhance the future behavior of a plant. In Fig. 2, GP is considered for obtaining the actual and desired value of EGR valve. This scheme employs a classical NMPC algorithm to achieve optimum control effort with respect to the predicted future behavior of the system as well as satisfying the system constraints and achieve the best objective value introduced in (2). This process is carried out by minimizing the common choice of cost function as below:

$$J_{nmpe}(k) = \sum_{j=1}^{N_y} \|\hat{y}(k+j|k) - r(k+j)\|_{Q_j}^2 + \|\Delta u(k+j-1|k)\|_{R_{j-1}}^2 \quad (16)$$

where  $N_y$  is the prediction horizon and  $r(k+j)$  is the set-point which indicates the position of EGR valve at interval  $(k+j)$ . The other indexes:  $\hat{y}(k+j|k)$  and  $\Delta u(k+j-1|k)$  are the predicted output at instant  $k+j$  and the optimized value for the incremental control at  $k+j-1$ , respectively, calculated at time  $k$ , by employing the model composed by equations (9). The weighting matrices:  $Q_j$  and  $R_j$  are symmetric and both are positive defined with proper dimensions.

Now, the control law is given by minimizing cost function (16) in association with the control moves, that is:

$$\min_{\Delta u(k), \Delta u(k+1), \dots, \Delta u(k+N_u-1)} J_{nmpe}(k) \quad (17)$$

subject to

$$u_{mmin} \leq u_m(k+i) \leq u_{mmax} \quad (18)$$

Where  $N_u$  is the control horizon. At each sampling instant  $k$ , the optimal control sequence is yielded as the vector  $\Delta u$  by optimizing the cost function. The first element of this sequence is applied to the system to compute the control input as  $u_m(k) =$

$u_m(k-1) + \Delta u_m^*(k)$  as well as the input constraints here are considered as actuator limitation.

## 4. SIMULATION STUDIES

Sets of numerical tests are provided to evaluate the general behavior of presented structure as in three parts. First, the results of model derivation with GP's framework is represented in both conventional and modified mode. At its next stage, the designed control scheme is implemented on a high-fidelity Matlab/Simulink® model, which has been developed by Toyota as a benchmark for the JSAE-SICE “near boundary control benchmark problem” (Watanabe and Ohata (2014)). This model is able to evaluate the combustion and determine the possibility of occurring knock or misfire under the operating conditions. Finally, the optimality of control structure in EGR actuator is analyzed and compare through several possible methodologies. In all following simulations, the first five seconds are eliminated, due to effects in the model during startup and initialization.

### 4.1 Model Validation

The primary step of our simulations relies on acquiring a reliable model of system. To this end, the GP model is validated under the introduced covariance functions as long as being compared with Modified GP (MGP) framework. To analyze each modeling profile, we introduce the predicted output error defined by  $e(t) = y_r(t) - y_{GP}(t)$ ,  $t \in [t_s, t_f]$ , where this range is large enough to test the model under different conditions. The observed value for prediction uncertainty is shown by  $\sigma^2$  during the whole cycle. Other two quantitative indexes:  $\|e_m\|_2^2 = \frac{1}{t_f-t_s} \int_{t_s}^{t_f} e^2(t) dt$ , and  $\sigma_m^2 = \frac{1}{t_f-t_s} \int_{t_s}^{t_f} \sigma^2(t) dt$  calculate the mean value of previously mentioned indexes over an entire operating cycle. Table 1 contains the tracking error and correspondence uncertainty for the conventional and modified Gaussian Process based on different covariance functions. The computational effort in identifying different GP models listed in the following table shown by  $T$ .

The result obtained here illustrates that squared-exponential covariance function presents better performance within the structure of GP algorithm. In addition, the logical error reduction indicates that estimated coefficient is effective in dealing with offset error and improve the general performance of GP algorithm considerably. The maximum amplitude and average prediction error of squared-exponential covariance function have experienced a noticeable reduction by the amount of 23.1% and 38.5%, respectively. Based on this evaluation,  $K_{SE}$  is elected as GP kernel function in any later simulations. Finally, Fig. 3 shows the MGP performance under the engine complete cycle. One can propose that exploiting Sparse Online Gaussian

Table 1. Study results towards conventional and modified GP

Index	Methods				
	GP-SE	GP-Matern	MGP-SE	MGP-Matern	SOGP
$ e_{max} $	15.389	15.531	11.833	13.979	14.421
$\ e_m\ _2^2$	2.586	3.123	1.589	1.717	2.519
$\sigma_{max}^2$	443.27	668.7	~	~	192.86
$\sigma_m^2$	269.32	271.85	~	~	44.78
$T(ms)$	40	42	~	~	58

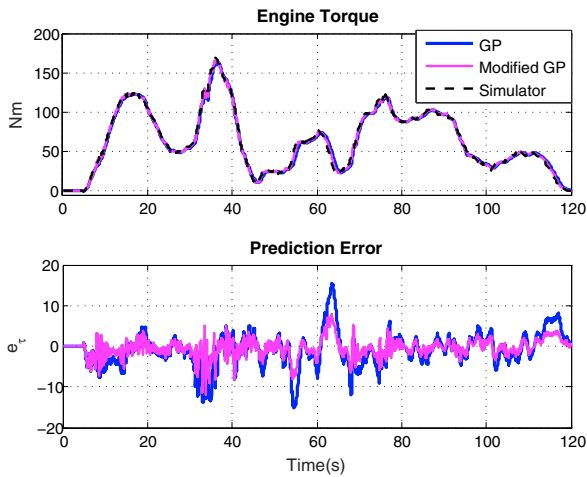


Fig. 3. Validation of Gaussian Process and Modified Gaussian Process based on squared-exponential covariance function.

Process (SOGP) can be a good alternative way to increase the GP performance. Here, it was observed that the assist of updating GP hyperparameters within SOGP framework present an absolute superiority over conventional GP while in this application it is not able to present the same attitude in terms of estimation error against the MGP approach. The last main point that needs to be mentioned is that the identified model in GP structure is determine by corresponding hyperparameters of kernel functions. In this way, the kernel hyperparameters of squared-exponential covariance function has been optimized to the value of  $r = 3.5701$  and  $l = 5.5597$ , respectively.

#### 4.2 Torque-Tracking Validation

The basic torque tracking problem of SI engines has been extensively studied over the past years. However, it was out of attention to present a structure that is able to handle the torque tracking problem for varying engine speed which is more representative for real-world driving conditions. In practice, this idea is commonly validated when the modern gasoline engines are typically operated around 2000–3000 rpm (33–50 Hz) when cruising, with a minimum (idle) speed around 750–900 rpm (12.5–15 Hz). By this consideration, the evaluation is conducted under a specific desire torque profile  $\tau_d$  and varied engine speed as shown in Fig. 4. The results depicted in Fig. 4 show that the controller tracks the desired torque profile quite

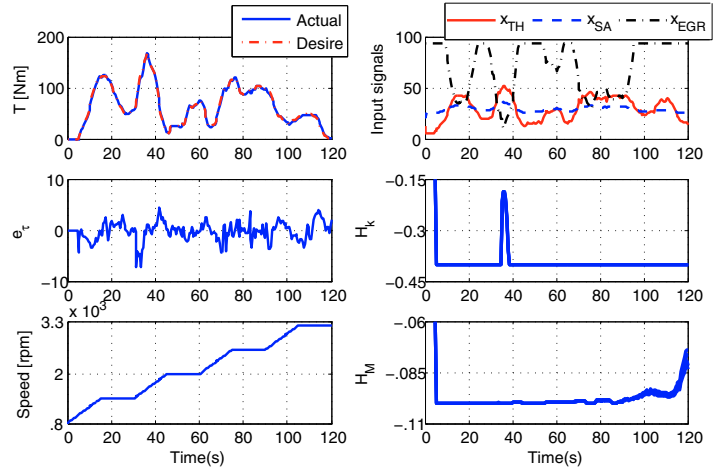


Fig. 4. System parameters generated under the desire torque profile  $\tau_d$  and varying engine speed.

well against logical variation of torque/speed. The constraints, as well as corresponding control efforts generated at this stage, is also presented in Fig. 4.

In this structure, the fractional PID controller gains are tuned by GA Matlab toolbox as listed in Table 2. The MPC controller parameters were the following: the weight matrices in cost function are defined as  $Q = 10I$  and  $R = 0.1I$ . The prediction control horizons is selected as  $N_y = 4$  (four time periods = 40 ms). Another point is that the simulation sampling time is  $\Delta t = .01ms$  while the controller runs at a higher sampling rate  $\Delta t_c = 10ms$ . The nonlinear optimization problem is solved by the interior point algorithm, provided in Matlab Optimization Toolbox.

#### 4.3 EGR Optimality Validation

In the last stage, in order to verify optimality of the designed EGR control structure, set of simulations are carried out under given torque profile  $\tau_d$  and constant engine speed  $\bar{n} = 1500(rpm)$ . Obtained results represent the best way of integrating the EGR actuator, that comes with a minimum value of the above mentioned objective function, particularly in term of fuel economy. An initial step is taken by applying a simple PID controller to regulate the EGR valve position. However, given a controller low-frequency gain or possessing a steady-state error, and some other limitations in conventional PID controller makes us put the step forward in the integration of some state-of-the-art control structures. Having such ideology, take an advantage of Fractional PID controller can eliminate much of those drawbacks come with PID controller as long as a vast improvements has been gathered at the rate of 167 (a.u.), 85 (kg) in terms of torque deviation and fuel consumption, respectively. The curves in blue and black lines in Fig. 5 show the trend of fuel consumption as well as accumulated tracking error of PID and FPID, respectively. The corresponding input waveforms are depicted in Fig. 6.

It can be easily seen that possible maximization of EGR can result in a considerable reduction in the cost function, especially in the term of fuel economy. Based on this observation, optimal model-based control structure as NMPC with Gaussian Process (GP- MPC) is designed to accomplish the optimization problem I. The obtained results shows a good improvement in terms of fuel consumption with the reduction of 27 (kg) which also

Table 2. Engine and Fractional Controller Parameters

Symbol	Description	Value
$u_{min}$	Lower input boundary	[5 -30 0]
$u_{max}$	Upper input boundary	[95 60 100]
$[t_s, t_f]$	Initial and final working time	[5s 120s]
$[c_t, c_f]$	Torque and fuel weighting	[1 8.10 <sup>5</sup> ]
$[A, B]$	Coefficients of throttle valve mapping function	[0.193 7.94]
$[A', B']$	Coefficients of spark advance mapping function	[0.134 24.9]
$k_P$	Proportional gain of fractional controller	1.925
$k_I$	Integral gain of fractional controller	5.271
$k_D$	Derivative gain of fractional controller	0.273
$\mu$	Fractional power of derivation part	0.947
$\lambda$	Fractional power of integrator part	0.921
$n$	Number of training data set for GP	100
$\bar{N}$	Backward horizon in calculating $\alpha$	20

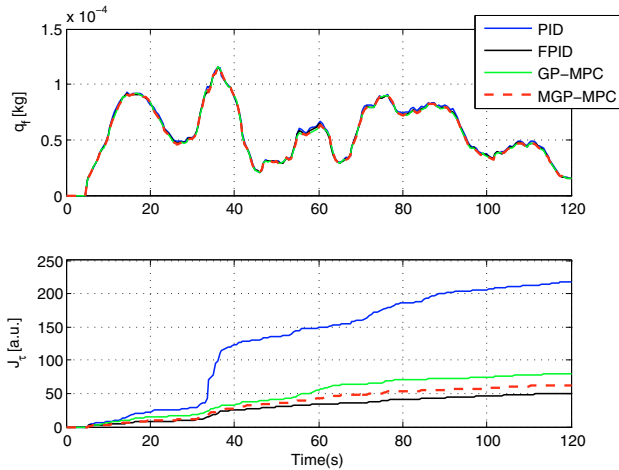


Fig. 5. The system outputs: fuel consumption and accumulated torque error, produced under introduced control methodology for EGR system over an entire engine cycle.

proves the assumption regards to working principle of EGR system. However, in order to compensate the increment of torque error that can be observed in table 3, a Modified form of GP-MPC (MGP-MPC) is exploited. As shown in subsection 4.1, deriving an accurate model of system in GP algorithm, results in a lower torque error in comparison with the conventional method by the rate of 48.9 (a. u.). The difference in performance between these two controllers is due to the difference in model of system. Comparing the prediction effect in the MGP-MPC case, one can note that the incorporating an adaptive parameter in GP framework was adjusted the control action for some critical workload in the process of combustion. Plots in green line and red dash represent the system output based on the GP-MPC and MGP-MPC methods. Finally, the optimal control commands generated by these two approaches are illustrated Fig. 6.

### 5. CONCLUSIONS

In this paper, the dynamics of exhaust gas recirculation system is approximated by Gaussian Process framework. This attitude made it possible to incorporate the information of the system by an optimal model-based control law. In this way, a classical nonlinear predictive control scheme was implemented to efficiently integrate the EGR actuator into the process of combustion. After such efforts, it was observed that regulating the EGR system needs an accurate estimation of system dynamics, which made us improve the performance of our GP structure. Further analysis was also shown a reasonable robustness behavior of designed control scheme under various torque/speed profiles. The principle proposed in the study is quite practical and the authors believe that similar attitude towards the integration of recent advances in Bayesian statistics can revitalize the industrial applications of powertrain control.

Table 3. Objectives under different control methodology in EGR system

	Torque Deviation	Fuel Consumption	Overall Objective
PID	217	5421 (kg)	5638
FPID	50	5336 (kg)	5386
GP-MPC	111.8	5307.8 (kg)	5419.6
MGP-MPC	62.9	5309 (kg)	5371.9

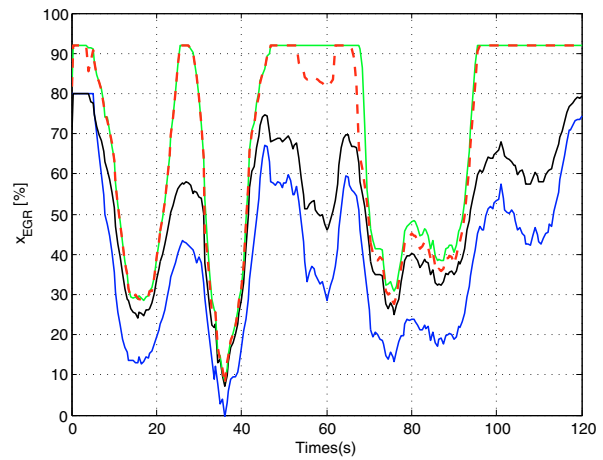


Fig. 6. The corresponding input waveforms generated for EGR actuator by employing introduced control methodologies over a complete engine cycle.

### REFERENCES

de la Cruz, J.S., Owen, W., and Kulic, D. (2012). Online learning of inverse dynamics via gaussian process regression. In *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 3583–3590.

Feru, E., Lazar, M., Gielen, R.H., Kolmanovsky, I.V., and Cairano, S.D. (2012). Lyapunov-based constrained engine torque control using electronic throttle and variable cam timing. In *2012 American Control Conference (ACC)*, 2866–2871.

Ko, J., Klein, D.J., Fox, D., and Haehnel, D. (2007). Gaussian processes and reinforcement learning for identification and control of an autonomous blimp. In *Proceedings 2007 IEEE International Conference on Robotics and Automation*, 742–747.

Laghrouche, S., Ahmed, F.S., Bagdouri, M.E., Wack, M., Gaber, J., and Becherif, M. (2010). Modeling and identification of a mechatronic exhaust gas recirculation actuator of an internal combustion engine. In *Proceedings of the 2010 American Control Conference*, 2242–2247.

Lanusse, P., Sabatier, J., and Oustaloup, A. (2014). Extension of {PID} to fractional orders controllers: a frequency-domain tutorial presentation. *{IFAC} Proceedings Volumes*, 47(3), 7436 – 7442. 19th {IFAC} World Congress.

Panni, F.S., Waschl, H., Alberer, D., and Zaccarian, L. (2014). Position regulation of an egr valve using reset control with adaptive feedforward. *IEEE Transactions on Control Systems Technology*, 22(6), 2424–2431.

Rasmussen, C.E. and Williams, C.K. (2006). *Gaussian Processes for Machine Learning*. MIT Press, USA.

Stefanopoulou, A.G. and Kolmanovsky, I. (1999). Analysis and control of transient torque response in engines with internal exhaust gas recirculation. *IEEE Transactions on Control Systems Technology*, 7(5), 555–566.

Watanabe, S. and Ohata, A. (2014). Benchmark problem for near boundary operation control for automotive engine. In –, –.

Zarghami, M., Babazadeh, M., and Hosseinnia, S.H. (2016). Performance enhancement of spark ignition engines by using fractional order controller. In *2016 European Control Conference (ECC)*, 1248–1252.