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Volatility in Electrical Load Forecasting for Long-term Horizon – An ARIMA-GARCH Approach

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***Abstract*— Electrical load forecasting in long-term horizon of power systems plays an important role for system planning and development. Load forecast in long-term horizon is represented as time-series. Thus, it is important to check the effect of volatility in the forecasted load time-series. In short, volatility in long-term horizon affects four main actions: risk management, long-term actions, reliability, and bets on future volatility. To check the effect of volatility in load series, this paper presents a univariate time series-based load forecasting technique for long-term horizon based on data corresponding to a U.S. independent system operator. The study employs ARIMA technique to forecast electrical load, and also the analyzes the ARCH and GARCH effects on the residual time-series.**

Keywords—ARIMA; ARCH; GARCH; long-term load forecast; volatility.

I. INTRODUCTION

Long-term load forecasting (LTLF) plays an important role in power systems for system planning, scheduling expansion of generation units by construction and procurement of generation units. It spans from a few years (> 1 year) to 10-20 years [1]. Because it takes several years and requires a huge investment for construction of power generation and transmission facilities, accurate and error-free forecasting is necessary for an electric utility. Accuracy of LTLF has a direct impact on development of future generation and transmission planning, and hence it is a crucial instrument for planning and forecasting future conditions of the electricity network. Based on the forecast, electric utilities coordinate their resources to meet the forecasted demand using a least-cost plan. In general, LTLF is subjected to a large number of uncertainties and ample amount of research indicates that load predication in presence of uncertainties is required for future capacity resource needs and operation of existing generation resources.

Based on time-scale, load forecast can be broadly classified into three main categories [2]:

- Short-term load forecast (STLF): The time-period of STLF lasts for few minutes, hours to one-day ahead or a week. STLF aims at economic dispatch and optimal generator unit commitment, while addressing real-time control and security assessment.
- Mid-term load forecast (MTLF): The time-period of MTLF is a month to a year or two. MTLF aims at maintenance scheduling, coordination of load dispatch and price settlement so that demand and generation is balanced.
- Long-term load forecast (LTLF): The time-period of LTLF is few years (> 1 year) to 10-20 years ahead. LTLF aims at system expansion planning, i.e., generation, transmission and distribution. In some cases, it also affects the purchase of new generating units.

Forecasting for the mid-term and specially for the long-term horizon is a whole different problem from forecasting for the short term. It cannot be done by simply fitting a model (either statistical or computational) over a dataset, and then extrapolating from it. It is evident from refs. [3-4], that MTLF or LTLF is usually ignored because of the complications. Makridakis et al. [5] clearly stated that long-term forecasting 'requires a different approach', and suggests that these forecasts should be based on (an) identifying and extrapolating mega-trends going back in time as far as necessary (as an example, they discuss the variations in the price of copper, since the year 1800); (b) analogy and (c) constructing scenarios to consider future possibilities. Hong [6] performed a study on past, current and future trends in energy forecasting. The article showcased the trend in spatial, STLF, LTLF and energy price forecasting in a lucid manner. Ref. [7] can be referred for a detailed review of MTLF and LTLF studies.

Load forecasting is usually tied to reliability analysis [8-10], and very recently in European projects (for e.g., GARPUR (www.garpur-project.eu)). Preciseness of long-term forecast significantly affects the development of future generation systems. For example, construction of a new generation plant takes approximately 5-10 years, and involves huge amount of capital investment. In order to need the demand and make the economic growth continuous, load forecasting is required for the related electricity utilities. Utilities do not want a huge investment going in vain.

In the long run, it has become more important for planners and forecasters to study movements of load time-series and its fluctuations. The movements are usually measured by the

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volatility (or conditional standard deviation) of the load time-series. One of the biggest problem with modelling the volatility is one of its features, it has periods with low movements and then suddenly periods with high movements. Traditional time-series models such as ARIMA model have been extended to essentially analogous models for the variance. Autoregressive conditional heteroskedasticity (ARCH) [11] was developed in order to model and forecast the variance of economic time series over time. Later, ARCH models have been generalized to become the generalized ARCH or GARCH models. ARCH and GARCH models have become common tools for dealing with time series heteroskedastic models, considering the moments of a time series as variant. They have proven to be a useful means for empirically capturing the momentum in conditional variance and had been used intensively in academic studies. Literature survey did not reveal any study on this aspect of power systems. Hence, this paper aims at a maiden approach to study the volatility in forecasting electrical load for long-term horizon.

The rest of the paper is organized as follows: Section II discusses long-term load forecast and the ARIMA methodology. Section III presents an in-depth learning of volatility in time-series for long-term horizon, while addressing the various statistical terminology used to assess it. Finally, section IV concludes the paper.

II. Long-term Load Forecasting

Long-term load forecasting is much more complex than simply fitting a mathematical model to some data, and it requires a lot more knowledge about the "substantive" problem. Compared to STLF that uses a sort of exercise on data modeling (for e.g., fitting models to datasets and extrapolating from them, without really understanding much about the way an electrical system works), MTLF/LTLF, on the other hand, depends less on the analyst's expertise on modeling, and more on experience with power systems, and a thorough understanding of the way the system works, and how the electricity market may be affected by the changes in a country's economy throughout the years, or by changes in technology, etc.

The MTLF/LTLF takes into account some explicit factors like historical load and weather data, economic indicators like gross domestic product (GDP) and their forecasts, and demographic data which includes consumer data like population, appliances in use, etc. Influence of weather follows a hierarchy in MTLF/LTLF as compared to STLF where all weather variables are treated with equal importance. Ref. [12] indicated that the weather variables follow a decreasing order of importance starting with temperature, humidity, wind and precipitation being the last on the list. To tackle this large number of factors for forecasting problem, the three methods suitable for MTLF/LTLF are [13]: Time-series approach, econometric approach, and end-use approach. Literature survey suggests another classification theory of MTLF/LTLF methods based on load impacting factors taken into consideration. The methods can be classified to two methods [14]: Conditional modeling approach, and autonomous approach.

In this study, a univariate regression-based model, called Autoregressive Integrated Moving Average (ARIMA) method is employed. It is a model based on a history of itself and it's moving average. And, the term "integrated" in ARIMA refers to the differencing process which is explained later in this section. Literature survey reveals that ARIMA technique has been successfully implemented in short-term load forecasting [15-16], and forecasting electricity prices [17-18]. Features of this approach are:

- Only looks at a single variable to model. In this study, the model only works with load data from an U.S. Regional Transmission Organization.
- ARIMA model can be used to model stationary time-series. The term "stationary time-series" in this study indicates the fact that both mean and standard deviation of data are finite and constant over time. If the time series exhibits variations that violate the stationary assumption, then there are specific approaches that could be used to render the time series stationary. The most common one is what is often called the "differencing operation", which is dealt later in this section.

A. *ARIMA Modeling*

ARIMA models are based on the theory that the behavior of the variable itself answers for its future dynamics [3], and are used to remove serial correlation. The parameters of ARIMA consist of three components: p (autoregressive parameter), d (number of differencing), and q (moving average parameters). In general, the ARIMA (p, d, q) model is expressed as [19]:

$$\phi_p(B)[\nabla^d y_t - \mu] = \theta_q(B)a_t \text{ for } t = 1, \dots, N \qquad (1)$$

where, ϕ_p is the autoregressive polynomial of B for order p and is given as $\phi_p(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p$. In the representation, φ_p refers to the autoregressive estimated parameters.

B is the backward shift operator, a simplified version to represent lag values. For e.g., the time-series y_t under investigation can be represented as $By_t = y_{t-1}$ and the white noise a_t normally distributed with mean zero and variance σ_t^2 can be represented as $Ba_t = a_{t-1}$.

∇ is the differencing operator ($\nabla = 1 - B$)

μ is the stationary series mean, assuming the series is stationary after differencing

θ_q is the moving average polynomial of B for order q and is given as $\theta_q(B) = 1 - \vartheta_1 B - \vartheta_2 B^2 - \cdots - \vartheta_q B^p$. In the above representation, ϑ_q refers to the estimated moving average parameters.

N is the number of samples

Next important thing is choosing the right model or in other words, order of ARIMA model. The autocorrelation function (ACF) and partial autocorrelation function (PACF) are the core of ARIMA model. Box-Jenkins method [3] provides a way to identify ARIMA model according to autocorrelation and partial autocorrelation graph of the series. In order to achieve this, there are three rules to identify ARIMA model:

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- If ACF (autocorrelation graph) cut off after lag n, PACF (partial autocorrelation graph) dies down: ARIMA(0, d, n) -> identify MA(q)
- If ACF dies down, PACF cut off after lag n: ARIMA(n, d, 0) -> identify AR(p)
- If ACF and PACF die down: mixed ARIMA model, need differencing

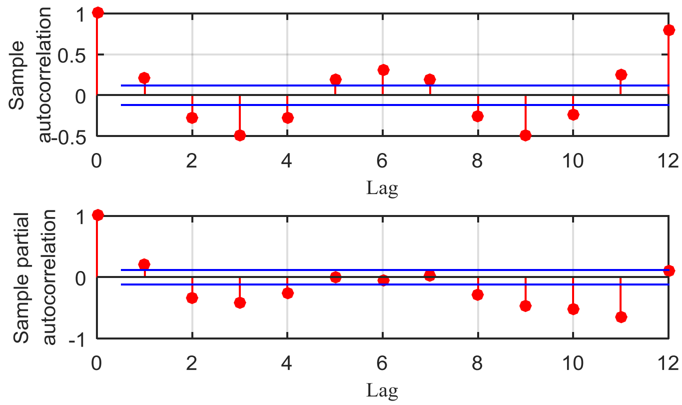

Fig. 1. ACF and PACF of residuals without seasonality

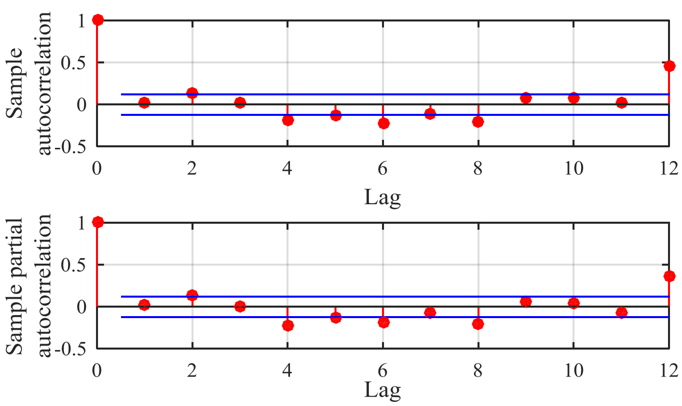

Fig. 2. ACF and PACF of squared residuals without seasonality

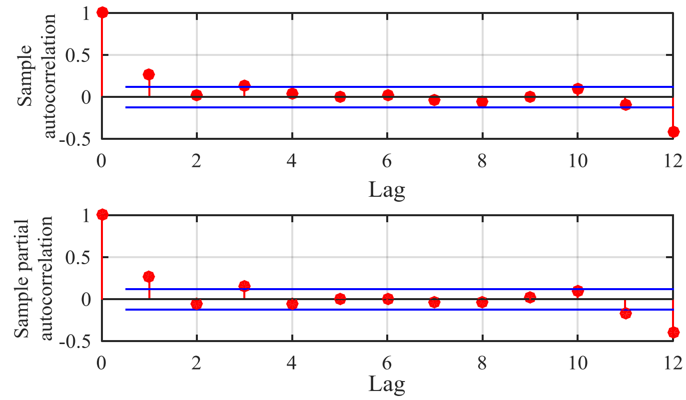

Fig. 3. ACF and PACF of residuals with seasonality

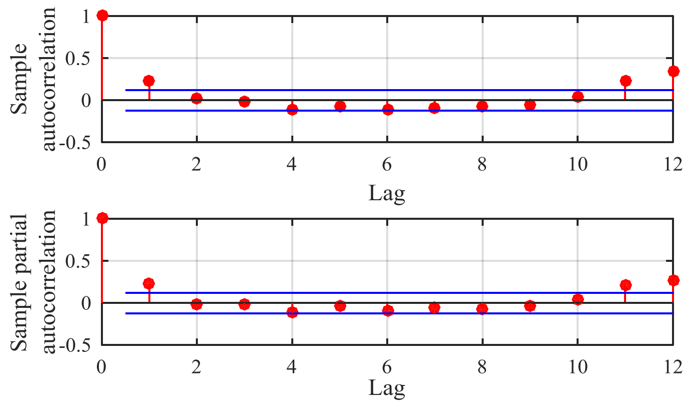

Fig. 4. ACF and PACF of squared residuals with seasonality

If ACF & PACF of the model residuals show no significant lags, the selected model is appropriate. The model selected in our study is ARIMA (1,1,1), i.e., a model with one autoregressive term, a first difference, and one moving average term. Since seasonality is one of the impacting factors in load forecasting for long-term horizon, we analyzed the time-series with and without seasonality. Fig. 1 and Fig. 2 illustrates ACF and PACF of residuals and squared residuals without seasonality, and Fig. 3 and Fig. 4 illustrates ACF and PACF of residuals and squared residuals with seasonality. A first

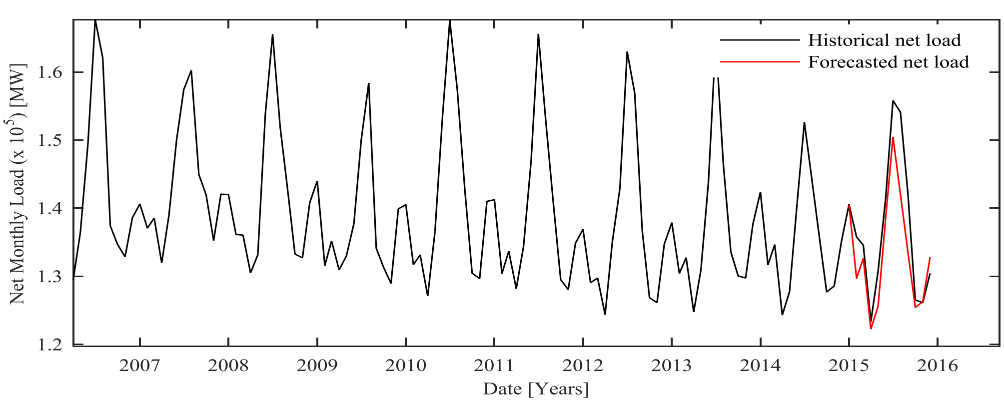

Fig. 5. Forecasted load for the year 2015

difference is used to account for a linear trend in the data. We avoid using higher orders i.e. > 1, as we prefer the model to refer to the most recent events since the more distant the historical events from the present, the less confident we can argue that the relationship is linear. The Box-Jenkins methodology is employed to fit linear time series models [5]. To assess the effectiveness of our proposed forecast methodology, MATLAB [20] is used. Fig. 5 shows the forecast load for known year (2015) using the historical data for years (1993-2014).

B. Stationarity Testing

As we see from the simulated results, forecasted load is a time-series and it is important to analyze data set to study the characteristics of the data and extract meaningful statistics in order to predict future values of the series. Time-series analysis is performed by two methods: time-domain and frequency-domain. The former closely investigates the autocorrelation of the series and is of great use of Box-Jenkins and ARCH/GARCH methods to perform forecast of the series while the latter is based mostly on Fourier Transform. In time-domain method, the first step in modelling time-series data is to convert the non-stationary time-series to stationary one. This is important because a lot of statistical and econometric methods are based on this assumption and can only be applied to stationary time series. Non-stationary time-series are erratic and unpredictable while stationary process is mean-reverting, i.e., it fluctuates around a constant mean with constant variance. One can difference N times the data to create a stationary process from a non-stationary time-series, which is described in the next sub-section.

It is generally believed that the method is relatively rough that the stationarity of time series is tested by autocorrelation analytic map. The unit root test is considered to be a more official method. Unit root test includes DF test, ADF test, PP test, Said-Dickey test, DF-GLS test, etc. [21]. Our study used ADF test for unit root test to check for stationarity in the load time-series.

C. Differencing

Differencing is a better way to remove locally varying trends to make it stationary than explicitly subtracting a fitted trend. Any trend observed in the time series (say, through curve fitting, regression analysis or first principles) can be subtracted out to leave what remains to be modelled. Evaluating a forecasting model requires quantifying the prediction error. Differencing transforms a time series X into another series Y where $y_t = x_t - x_{t-1}$, trying to find a better fitting model. Differencing does not require estimating a parameter, although it costs one series point per difference. The first difference accounts for a trend that impacts the change of the mean of the time series, the second for a change in the slope.

III. Volatility in Long-term Horizon

In long-term horizon, utmost care should be taken while designing the model. The model should account for future upward and downward trends from affecting factors (for e.g., economic factors such as energy prices, GDP, unemployment), so that it can self-tune over time. The trend initiates volatility, and it not only spikes up during a crisis, but it eventually drops back to approximately the same level of volatility as before the crisis. The designed model should be able to address this. Another observation in time-series modeling is that the error series usually have the characteristic of fat tail assembly, once it happens, if we still regard the error series as the independent identical distribution (IID) variable according to the classical assumption of least-square method, it is unreasonable. Actually, this time-series model can only explain time-series fluctuation partly and there is still a part of information exists in the error term of the regression equation. In this study, the forecasted load series is treated as error series. From Figs. 1 and 2, it is evident that although ACF & PACF of load series have no significant lags, the time-series is still checked for any cluster of volatility. It is important to remember that ARIMA is a method to linearly model the data and the forecast width remains constant because the model does not reflect recent changes or incorporate new information. In other words, it provides best linear forecast for the series, and thus plays little role in forecasting model nonlinearly. Consequently, the error terms do not satisfy the homoscedastic assumption of constant variance. As a reminder, heteroscedastic means a set of statistical distributions having different variances and heteroskedasticity means non constant volatility.

From the ARIMA model result in Fig. 5, we look for a residual series with zero mean and constant variance and which is non-autocorrelated. If this condition is satisfied, the ARIMA model is able to remove the autocorrelation effect from the data. However, if the quadratic residual series is correlated, the residual series is denoted as heteroskedastic and a non-linear model should be used to represent it because the series has non-pure random behavior. The residuals are obtained by absorbing the content of time-series. This is obtained by using the *curvefitting* toolbox in MATLAB. Even though the residuals are white noise, the ARCH-test is performed to verify the homoskedasticity. If this supposition is not met, it is necessary to fit an ARCH model to estimate the variability behavior. This becomes important because the heteroskedasticity presence is a signal that the process may have large variability and can influence the mean trajectory as the control limit estimation. The ARCH test conformed that there still remains serial correlation in the series and the model needs modification.

An autoregressive conditional heteroskedastic (ARCH) model with order $p \geq 0$ is defined as:

$$Z(t) = \sqrt{h(t)}e(t) \tag{2}$$

$$h(t) = \alpha_0 + \sum_{i=1}^{p} \alpha_i Z_{t-i}^2 \tag{3}$$

where $e(t) \sim IID(0,1)$, $\alpha_0 > 0$, and $\alpha_i \geq 0$ are constants and $e(t)$ is independent of $Z_{t-k}, k \geq 1$. A stochastic process Z_t defined by the equations (2) and (3) is called a ARCH (p) process. Here h_t is the variance of Z_t and e_t is the innovation (error) variable. For simplification, we set $p = 1$.

The time-varying variance (i.e., volatility or heteroskedasticity), which depends on the observations of the immediate past, is called conditional variance, and the ARCH model can describe this problem well [22]. Firstly, check if load series displays any cluster of volatility. Our results confirmed the presence. Next, observe the squared residual plot. If there are clusters of volatility, ARCH/GARCH should be used to model the volatility of the series to reflect more recent changes and fluctuations in the series. At this stage, it is important to understand the GARCH model. The basic ARCH model has been transformed and developed into more sophisticated models, such as GARCH, EGARCH, TGARCH and GARCH-M. Many of the different models have different features, which makes the forecast accuracy better. To keep it simple for our study, the scope of this paper restricts us to use the Generalized-ARCH or GARCH model as well as the ARCH model to study volatility.

In 1986, Bollerslev proposed a transformation of the ARCH model, called the generalized ARCH or GARCH model [23]. The aim was to overcome several restrictions in ARCH model that has to be fulfilled so that the model can sufficiently estimate the volatility. The GARCH model is an extension of the ARCH model. A generalized autoregressive conditional heteroskedastic (GARCH) model with order $p \geq 0$ and $q \geq 0$ is defined as:

$$Z(t) = \sqrt{h(t)}e(t) \tag{4}$$

$$h(t) = \alpha_0 + \sum_{i=1}^{p} \alpha_i Z_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2 \tag{5}$$

where $e(t) \sim IID(0,1)$, $\alpha_0 > 1$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ are constants with

$$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1 \tag{6}$$

and $e(t)$ is independent of $Z_{t-k}, k \geq 1$. A stochastic process Z_t defined by the equations (4) and (5) is called a GARCH (p, q) process. Here h_t is the conditional variance of Z_t, given $\{Z_s, s < t\}$ and e_t is the innovation (error) variable. For simplification, we set both $p = 1$ and $q = 1$.

GARCH is not only used to forecast the conditional variances, it is also used to provide unconditional variances. The term *conditional* implies explicit dependence on a past sequence of observations whereas the term *unconditional* is more concerned with long-term behavior of a time series and assumes no explicit knowledge of the past. The conditional variance plot for load series is shown in Fig. 6. Both conditions are crucial in providing an essential piece of information about risk forecasting. If our forecast horizon lead time is short, conditional volatility forecasts should be used to refine the return estimate. On the contrary, if the lead time is significant, unconditional volatility forecasts may be the only feasible estimate. This is because the forecast lead time is large enough to question the validity of the variance forecasts [24].

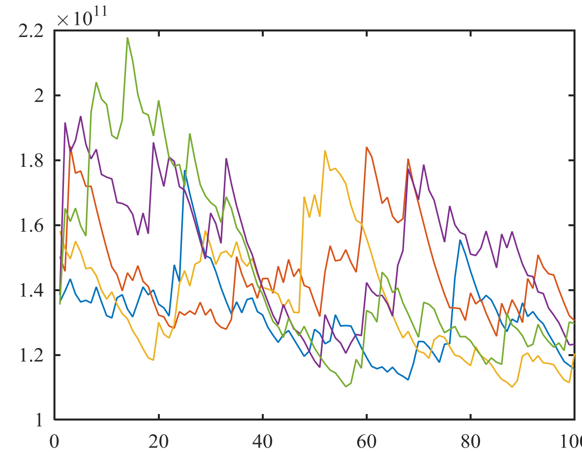

Fig. 6. Conditional variance plot of forecasted load series

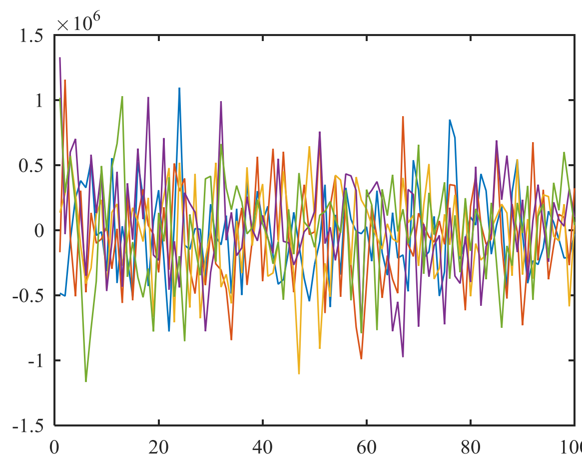

Fig. 7. Innovation plot for the forecasted load series

Fig. 7 illustrates the innovation plot for the load series. The innovation plot is obtained by choosing 5 paths of length 100 from the GARCH(1,1) model, without specifying any pre-sample innovations or conditional variances. The model being simulated does not have a mean offset, so the response series is an innovation series. Another important observation from Fig. 6 and Fig. 7 is that the starting conditional variances are different for each realization because no pre-sample data was specified. Our basic understanding is that the load series follows a stationary time-series model with a stochastic volatility structure. The presence of stochastic volatility implies that the load series is not necessarily independent over time. Hence the model considers the unconditional and conditional distribution where the conditioning is on the current volatility. The estimated GARCH and ARIMA-GARCH parameters are shown in Table I and Table II. In our study, the ARCH and GARCH function in MATLAB is used. One of the assumptions in the functions is that a t-statistic > 2 in magnitude corresponds to approximately a 95% confidence level. The t-statistic column is the parameter value divided by the standard error, and is normally distributed for large samples. It measures the number of standard deviations the parameter estimate is away from zero.

TABLE I. GARCH (1,1) PARAMETERS

Parameter	GARCH values	Standard Errors	t-Statistic
α	-67.9842	245.17	-0.277294
α_1	-0.361694	0.0246818	-14.6543
β_1	-0.361694	0.0246818	-14.6543

TABLE II. ARIMA-GARCH PARAMETERS

Parameter	ARIMA-GARCH values	Standard Errors	t-Statistic
α	1.37368e+07	0.000162653	8.44549e+10
α_1	0.812068	0.019251	42.1832
β_1	0.0381873	0.0212819	1.79435

IV. CONCLUSION

In conclusion, it is important to highlight three important aspects. The first aspect is how to forecast electrical load for long-term horizon using univariate technique, when adequate data is not available (only load data is used in this study). The second aspect is related to the modelling techniques presented. Although ARIMA-ARCH models are not so straightforward, the parameter estimates using the joint methodology are efficient, providing a better fit to the process. Finally, the ARCH and GARCH models was able to quantify the impact of an external effect that can occur in the process. ARCH-GARCH incorporates new information and analyses the series based on conditional variances where users can forecast future values with up-to-date information. Another observation is that ARIMA model focuses on analyzing time-series linearly and it does not reflect recent changes as new information is available. Therefore, in order to update the model, users need to incorporate new data and estimate parameters again. And, this coins the term dynamic modelling. The variance in ARIMA model is unconditional variance and remains constant. ARIMA is applied for stationary series and therefore, non-stationary series should be transformed, as we did to the load series in this study.

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