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Analytical Models for Artificial Dielectrics with Non-Aligned Layers

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***Abstract*—We present an analysis of artificial dielectric layers (ADLs), when a lateral shift between layers is present. The alternate lateral displacement of the layers is an important parameter to engineer the desired effective electromagnetic properties of the ADL material. More specifically, much higher equivalent dielectric constants can be realized by alternatively shifting the layers, compared to the aligned case. Closed-form expressions are given for the equivalent layer reactance that include the higher-order interaction between shifted layers. These analytical formulas can be used to design artificial dielectric slabs, as they provide the scattering parameters for generic plane-wave incidence.**

I. INTRODUCTION

ARTIFICIAL dielectrics (ADs) were introduced in [1], [2] as a light-weight alternative to real dielectrics, and obtained by embedding conducting structures in a host material according to a regular pattern. At the frequencies for which the periodicity of the pattern is much smaller than the wavelength, the structure can be assigned equivalent parameters that describe a homogeneous dielectric [3]. The effective electric parameters can be engineered by varying the size of the metal obstacles and their spatial density. This work relates to a specific type of anisotropic ADs, which are realized as a cascade of planar layers made of printed metal patches (Fig. 1). Such structures are also referred to as artificial dielectric layers (ADLs).

Recently, ADLs were exploited to improve the front-to-back ratio of integrated antennas without supporting surface waves, with a consequent enhancement of gain and efficiency [4].

Closed-form expressions to describe the scattering from artificial dielectric slabs were derived in [5], [6]. These formulas account for the higher-order interaction between layers, which cannot be neglected due to the electrically small inter-layer distance. However, while [6] only contemplates the case of aligned layers ($s_x = s_y = 0$ in Fig. 1), in this work we generalize the analysis to include the effects of alternate shifts. This new configuration is very relevant for the design of ADLs, because the shift significantly increases the effective permittivity of the slab with respect to the aligned case. Consequently, the shift between layers represents an additional important degree of freedom that can be used for the design, as it greatly extends the ranges of permittivity values that can be synthesized, given a specific fabrication technology.

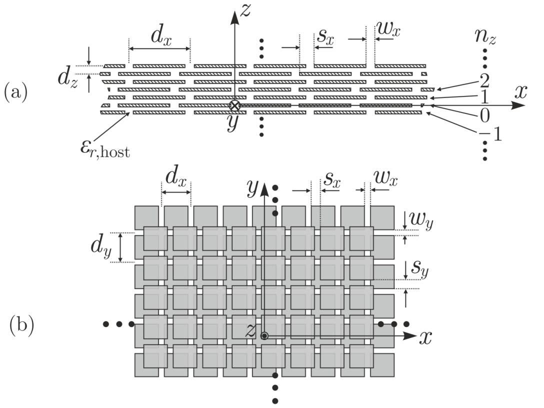


Fig. 1. Definition of the geometrical parameters characteristic of the shifted ADLs: (a) cross section and (b) top view.

II. ANALYSIS OF SHIFTED ADLS

Figure 1(a) shows an ADL medium composed by an infinite number of layers spaced by d_z and numbered with consecutive integer indexes n_z. The odd layers ($n_z = [... -3, -1, 1, 3, ...]$) are shifted with respect to the even layers ($n_z = [..., -2, 0, 2, ...]$) by s_x and s_y along x and y, respectively (see Fig. 1(b)). We assume that a plane wave with magnetic field $\mathbf{h}_i(x, y, z)$ is traveling in the negative-z direction within the ADL medium. By applying the equivalence theorem as in [6], we can define equivalent surface magnetic currents $\mathbf{m}_{n_z}(x, y)$ in correspondence of the gaps between patches in the initial problem.

The continuity of the transverse magnetic field at the layer at $z = 0$ leads to the following integral equation:

$$\sum_{n_z\,\text{even}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 4\mathbf{m}_0(\boldsymbol{\rho}')\mathbf{g}(\boldsymbol{\rho}-\boldsymbol{\rho}', n_z d_z, z=0)d\boldsymbol{\rho}' - \sum_{n_z\,\text{odd}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 2(e^{jk_{zs}2d_z}+1)\mathbf{m}_{-1}(\boldsymbol{\rho}')\mathbf{g}(\boldsymbol{\rho}-\boldsymbol{\rho}', n_z d_z, z=0)d\boldsymbol{\rho}' = 2\mathbf{h}_i(\boldsymbol{\rho}, z=0) \quad (1)$$

where $\boldsymbol{\rho} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ and $\boldsymbol{\rho}' = x'\hat{\mathbf{x}} + y'\hat{\mathbf{y}}$ refer to the observation and the source points, respectively. The function $\mathbf{g}$ represents the free-space dyadic Green's function which relates the magnetic field to a magnetic source, and k_{zs} is an unknown equivalent wavenumber describing the propagation along z.

Unlike the case of aligned layers [6], we cannot relate the magnetic currents $\mathbf{m}_0$ and $\mathbf{m}_{-1}$ using Floquet boundary condition and this complicates the analysis. However, to simplify the problem, we assume that the magnetic currents on the two layers are approximately equal in amplitude and differ only from a spatial displacement and a phase term:

$$\mathbf{m}_{-1}(\boldsymbol{\rho}') \approx \mathbf{m}_0(\boldsymbol{\rho}' - \mathbf{s})e^{-j\mathbf{k}_{\rho s}\cdot\mathbf{s}}e^{-jk_{zs}d_z} \tag{2}$$

where $\mathbf{s} = s_x\hat{\mathbf{x}} + s_y\hat{\mathbf{y}}$ is the vector indicating the shift and $\mathbf{k}_{\rho s} = k_{xs}\hat{\mathbf{x}} + k_{ys}\hat{\mathbf{y}}$ is an unknown wave vector describing the transverse propagation between adjacent layers. This approximation is equivalent to implying that the the field propagation from one layer to the next is dominated by a lossless guided phenomenon.

Under the assumption (2), and fixing $s_x = s_y$ to obtain azimuth-independent properties of the ADLs, we can solve the approximated integral equation in the spectral domain with a procedure similar to the one described in [6]. The procedure yields the following expression of the layer susceptance in the presence of the shift:

$$B_\infty = \frac{j2k_0}{\zeta_0}\sum_{m_y\neq 0}\frac{|J_0(k_{ym}w_x/2)|^2}{|k_{ym}|}\cdot \left(-\cot(k_{zm}d_z) + e^{-jk_{ym}s_y}\csc(k_{zm}d_z)\right) \tag{3}$$

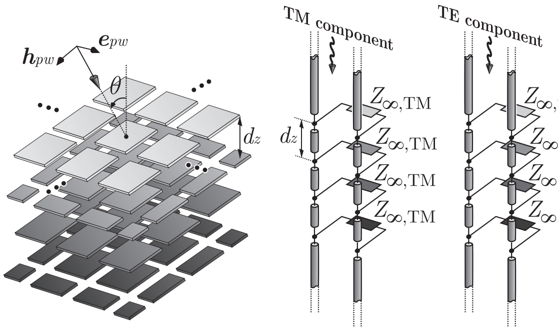
where $k_{zm} \approx -j|k_{ym}|$ and $k_{ym} \approx (-2\pi m_y)/d_y$ is the Floquet mode of index m_y. The analytical expression in (3) accounts for the higher-order coupling between layers and thus remains valid even for inter-layer distances much smaller than the wavelength. The reactance of a layer embedded in a periodic multi-layer environment as follows:

$$Z_{\infty,\mathrm{TM}} = \frac{-j}{B_\infty}, \quad Z_{\infty,\mathrm{TE}} = \frac{-j}{B_\infty(1 - \frac{\sin^2\theta}{2})} \tag{4}$$

and can be used within an equivalent circuit that describes the propagation of a generic plane wave in the ADL medium (Fig. 3). To validate the formulas we show in Fig. 4 the reflection and transmission coefficients for transverse electric (TE) and transverse magnetic (TM) plane-wave incidence (at $\theta = 60°$) and for different shifts. CST simulations are also reported and show good agreement with analytical method.

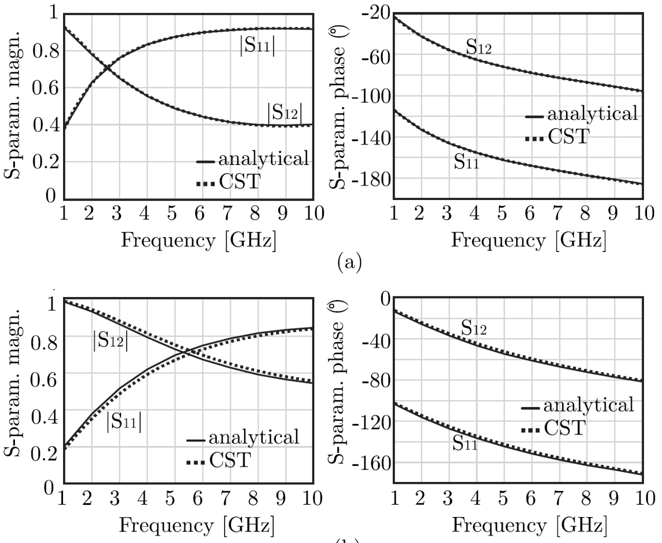
III. CONCLUSIONS

We derived closed-form formulas for the analysis of artificial dielectric layers (ADLs). The expressions of the equivalent reactance of each layer include the effect of an arbitrary shift between odd and even layers. The higher-order interaction between layers is rigorously accounted for in analytical form. The reactances can be embedded in an equivalent circuit that provides the scattering parameters for generic plane-wave incidence and for an arbitrary number of layers. The results given by our method were validated with simulations performed with commercial electromagnetic solvers. From the scattering parameters, the permittivity and permeability tensors can also be derived.



(a) (b)

Fig. 2. (a) Plane wave impinging on a cascade of four ADLs with alternate shifts and (b) equivalent circuits for TE and TM components.



(a)

(b)

Fig. 3. Amplitude and phase of the reflection and transmission coefficients of a plane wave incident on a cascade of 5 layers: (a) TE, $\theta = 60°$, $s_{x,y} = 0.25d_{x,y}$; (b) TM, $\theta = 60°$, $s_{x,y} = 0.5d_{x,y}$. The geometrical parameters are $d_x = d_y = 0.0785\lambda_0$, $w_x = w_y = 0.01\lambda_0$, $d_z = 0.012\lambda_0$, with λ_0 being the wavelength at 5 GHz.

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