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## Line planning problem in a dense High-Speed Rail corridor

Fei Yan · Rob M.P. Goverde · Nikola Besinovic

**Abstract** To satisfy the growing passenger transportation demands and improve the service quality in a railway system, a high-quality line plan needs to be designed. Line planning is an initial optimization problem in the process of railway transportation management, which includes the origin and destination (OD) of trains, routes, stop patterns and frequencies. Aiming to find a optimal line plan for a dense high-speed railway corridor, this paper proposes a optimization model with objectives of minimizing passenger's total travel time and empty-seat-hour. Considering the problem is NP-hard, we introduce a novel matheuristic approach that combines metaheuristic and mathematical programming technique. Genetic algorithm (GA) is developed for providing possible combination of frequencies, and integer linear program (ILP) is applied for optimization of passenger assignment model. With integration of both, we produce a optimal line plan with frequencies. Finally numerical experiments of Chinese case are applied to verify the proposed model and approach.

**Keywords** line plan · stop pattern · service frequency · genetic algorithm

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## 1 Introduction

Railway systems are highly complex systems, which consist of several planning stages: line planning, timetabling, train platforming, rolling stock circulation, and crew scheduling. Line planning is the initial step in the strategic planning process, and it aims to find a set of lines or paths in the network with corresponding stop schemes and operation frequencies that satisfies travel demand in a given network. As the output of line planning is the input for timetabling, the line plan has a huge impact to the timetable design. Therefore, determining a better line plan plays an important role on achieving a stable and robust timetable with good service quality.

The line planning problem has been extensively studied in the literature. There are two main conflicting objectives, minimization of operation costs from operator's perspective (cost-oriented) and minimization of generalized travel cost from passengers' perspective (customer-oriented). Researchers have developed a series of integer programming models to achieve both objectives. For a cost-oriented approach, it aims to find a line plan serving all passengers and minimizing the costs of operators. It is always modeled concerning the lines and corresponding frequencies, or train types and capacity presented in Claessens et al (1998), Goossens et al (2004), and Goossens et al (2006). For the customer-oriented approach, maximizing the number of direct travelers is proposed in Bussieck (1997), and minimizing the total travel time in Schöbel and Scholl (2005) and Borndörfer et al (2007). Branch and bound, branch-and-cut, and column generation are used for solving these mixed integer linear programs. Borndörfer et al (2007) considers a novel multi-commodity flow method in which passenger paths can be freely routed and lines are generated dynamically. Schöbel and Scholl (2005) used a change-and-go graph to model travel and transfer times. However, for large scale instances the model tends to be computationally intractable. Therefore, Borndörfer and Neumann (2010) proposed a compact integer programming approach to deal with transfer minimization problems even for large instances. They incorporated penalties for transfers that are induced by "connection capacities" and compared a direct connection capacity model with a change-and-go model. Bussieck et al (2004) proposed a fast solution combining nonlinear techniques with integer programming, and a game-theoretic model is introduced in Schöbel and Schwarze (2006), where each line acts as player to minimize cost. As a huge rail network in China, Fu et al (2015) describes a integrated hierarchical approach to optimize line planning problem, with a classification of stations and trains, and bi-level optimization model. The stop patterns are predefined by a enumeration of higher-classified stations for higher level trains and limiting maximal number of stops for lower level trains. And minimization of passenger's travel time in upper level is used to estimate passenger route; maximization of served passengers in lower level is designed for optimization of frequencies. Schöbel (2012) gave a review of different line planning models from a mathematical and algorithm approach. Goerigk et al (2013) conducted a comprehensive experimental study

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to evaluate the quality of line plans from four different models by travel times and robustness.

From discussion above, we found that almost all of these researches need a predefined set of possible lines, which is also called a *line pool*. *Line pool* is a set of lines with defined train OD, train route and stop pattern, which could simplify the problem due numerous combinations of stop patterns and route choice. Meanwhile, most of these models are formulated to optimize line plan which is specific for a cyclic timetable pattern or acyclic timetable pattern. Cyclic and acyclic timetables are widely implemented in rail networks. To correspond, we categorized line plans with a cyclic and an acyclic pattern. The features of a line plan with a cyclic pattern are fixed stop patterns with high frequencies which provide a regular service pattern for passenger. However, a line plan with high frequency might result in a low seat occupation, while mixed operation of fast trains and all-stop trains might result in a low capacity utilization of the infrastructure, both of which would increase operational costs. And existing line plans corresponding to a cyclic nature tend to prioritize passengers transfers instead of the defining direct train lines, which would not be convenient for passengers if the transfer conditions are not good or for long travel distances. The acyclic line plans have flexible stop patterns with low frequency (almost one train per day) considering the the passenger distribution, which could use capacity effectively. But this would lead to irregular timetables, which not only increase the complexity of the operator's management, but also brings more waiting time in stations for passengers. Due to the advantages and disadvantages of line plans with cyclic or acyclic nature, we propose a line planning model which optimize a integrated line plan with both features in order to improve the efficiency of the railway system. For big passenger OD demand, lines with high frequencies are provided, while to small passenger demand stations, few trains with acyclic pattern is available. This way not only reduce capacity loss but also provide frequent service for some passengers. The remainder of this paper is organized as follows. In Section 2, the line plan problem in a dense High-Speed Rail corridor is defined. Section 3 describes the model with the proposed objectives and constraints in detail. The method and algorithm to find an optimal solution are introduced in Section 4. The results of numerical experiments are analyzed and evaluated based on data from a Chinese railway network. Finally, conclusions and future research work are presented.

## 2 Problem Description

High-speed railways are developing fast all over the world, which calls for speed improvements. Due to the time loss and capacity loss by dwelling, acceleration and deceleration, trains with too many stops are not preferred by both passengers and operators. This holds in particular for some lines in interregional corridors of the Chinese high-speed railway network, with relatively shorter length of lines and station spacing. And the passenger features

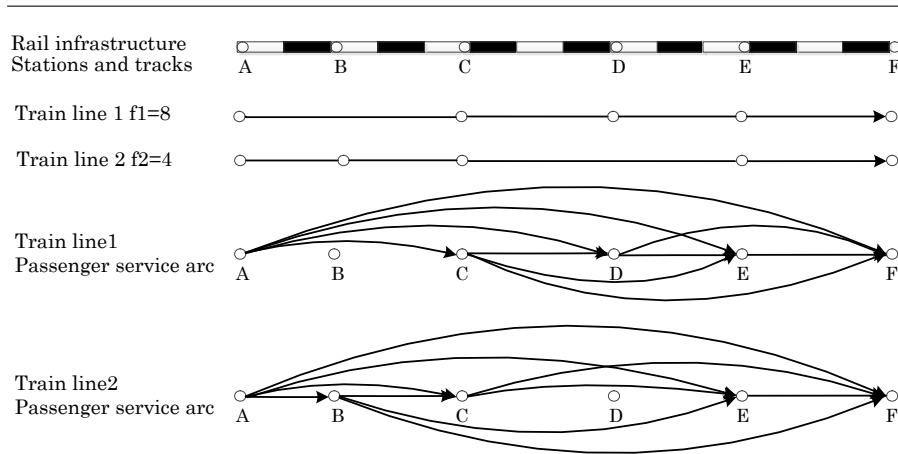
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are huge demand, unbalanced distributions and a big amount of commuters. From the characteristics of lines and passengers, requirements of high speed and high frequencies are important for both passengers and operators. Meanwhile, proper transfer facilities are not equipped in stations of China, which means a direct service is also required for passengers of short distance due to the time-consuming transfer. Therefore, the line planning model for such a network is designed with objectives of empty seat-hours and passenger's total travel time considering both operators and passengers.

Normally, profit or cost is proposed as objective from an operator's point. However, because the monetary value from an train operating company is not always that accurate and the empty seat-hours could give a direct view of capacity loss, we choose a more straightforward way to minimize the cost. For the second objective, waiting time and in-vehicle journey time play a primary role when passengers select a train line for a trip. Waiting time on a platform depends highly on the service frequency for passengers travelling without planning their trip, see details in 4. In-vehicle travel time consists of dwell time and running time, including acceleration time and deceleration time. The main difference between different train lines is not only the dwell time, but also the acceleration time and deceleration time which have even more impact on the total journey time. Hence, we use the more precise journey time rather than average travel time (distance divided by average speed), on the other hand, this could also enhance the complexity of model. Nevertheless, both empty-seat-hour and total travel time could also be transformed into monetary value using time values if necessary.

In order to calculate objectives, passenger assignment model is embedded inside with consideration of travel time. In passenger assignment model, each line has a corresponding travel time of each passenger. When a line plan is given, this model finds the best train line for each passenger with minimal of total travel time. With this result, the value of objective function could be obtained.

A line plan is a set of train lines with train ODs, routes, train types, stop patterns and frequencies. Hence, the optimization model needs to be built from networks on three levels. The first level is the railway infrastructure, which is called the physical network, including stations and rail tracks connecting the stations. In the second level, the train lines with stop pattern and frequency are provided to passengers, which gives for each OD an opportunity to travel. With the line plan scheduled, the service arcs to the passengers can be depicted. The passengers travel choice is influenced by the total travel time over different train lines which have the same service arc in the third level. This third level determines how the passenger are assigned on each train, which is the network for passenger assignment model. Figure 1 illustrates a simple network with six stations along the corridor. One train line with train OD from A to F, stopping at C,D and E; and the other with the same OD, stopping at B, C and E. It can be observed that both lines have ten service arcs and they also share some arcs with the same OD. For passengers from C to E, both lines provide this service. Line 2 offers a direct service, and line 1 has a higher



**Fig. 1** Networks for the line planning optimization

frequency. Therefore, which train line the passenger needs to be determined by a passenger assignment problem. For passengers from B to D, no direct service is available, so a transfer node and connecting train needs to be selected.

### 3 Optimization Model

#### 3.1 Assumptions and notation

In order to simplify the complexity of the line planning problem, we use some basic assumptions.

- (1) All passengers have a direct service to the destination without transfer.
- (2) The passenger demand is symmetric from both directions and therefore only one direction is taken into account for optimization.
- (3) Only one train type is considered and all trains have the same seat capacity.

The notation of parameters and variables is summarized as follows.

Parameters:

- $L$  : Set of train lines.
- $l_k$  : Train line from  $L$  with  $k$ -th stop pattern.
- $S$  : Set of stations in the corridor.
- $S_k$  : Set of stations on line  $l_k$ .  $N(S_k)$  is the number of stations, and  $i, j$  indicates the  $i$ -th or  $j$ -th station counted from the origin station.
- $C(l_k)$  : Seat capacity of train line  $l_k$ .
- $\mu$  : Maximal seat occupancy rate in the passenger assignment model.
- $N$  : Maximal number of train lines in the line plan.
- $r_{i,i+1}$  : Scheduled running time between route section  $i$  to  $i + 1$  discarding acceleration and deceleration.

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$d_i$ :	Scheduled dwell time of a stop in station $i$ .
$P_{ij}$ :	Passenger demand between stations $i$ and $j$ .
$\lambda^v, \lambda^w$ :	Weights of in-vehicle time and waiting time.
$ac, de$ :	Average acceleration time and deceleration time.
$T$ :	Length of operation period per day.
$F_{min}$ :	Critical frequency where waiting time is half the headway.
$T_{aw}$ :	Average waiting time of passenger.
$f_{min}, f_{max}$ :	Lower and upper bound of frequency for each train line.
$L_{min}$ :	Lower bound of total number of trains operated in this corridor. It is the minimal service frequency when all passenger are served in the section with the biggest accumulative passenger flow.
$L_{max}$ :	Upper bound of total number of trains operated in this corridor. it can be capacity constraint of track section and/or fleet size .

Decision Variables:

$f(l_k)$ :	Frequency of train line $l_k$ .
$x_{ij}^{l_k}$ :	Binary variable, equal to 1 if line $l_k$ has stops in both station $i$ and $j$ , and 0 otherwise.

Other Variables:

$N(l_k)$ :	Binary variable, equal to 1 if line $l_k$ is selected in the line plan, and 0 otherwise.
$q_{ij}^{l_k}$ :	Number of passengers from station $i$ and $j$ assigned to line $l_k$ .
$F_{ij}^v$ :	Total service frequency from station $i$ to $j$ .
$Q_{i,i+1}^{l_k}$ :	Cumulative passenger flow assigned to line $l_k$ between two successive stations $i$ and $i+1$ .
$y_i^{l_k}$ :	Binary variable, equal to 1 if line $l_k$ stops in station $i$ , and 0 otherwise.
$T_{ij}^{l_k}$ :	Generalized cost of passengers from station $i$ to $j$ on line $l_k$ .
$t_{ij}^v(l_k)$ :	In-vehicle time from $i$ to $j$ on line $l_k$ , including dwell times and running time with acceleration and deceleration times.
$t_{ij}^w(l_k)$ :	Waiting time in station $i$ for passengers from $i$ to $j$ .

Note that the binary decision variable  $x_{ij}^{l_k}$  indicates whether or not line  $l_k$  offers a transport service from  $i$  to  $j$ .

### 3.2 Model formulation

Using the notations explained above, the mathematical model of the line planning is formulated as follows. The cumulative passenger flow can be computed as

$$Q_{i,i+1}^{l_k} = \sum_{m=1}^i \sum_{n=i+1}^{N(S_k)} q_{mn}^{l_k}, \quad i \in S_k \quad (1)$$

and the intermediate binary variables  $y_i^{l_k}$  are computed from given  $x_{ij}^{l_k}$  as

$$y_i^{l_k} = \begin{cases} 1, & \text{if } \sum_{j=i+1}^{N(S_k)} x_{ij}^{l_k} + \sum_{j=1}^{i-1} x_{ji}^{l_k} > 0 \\ 0, & \text{if } \sum_{j=i+1}^{N(S_k)} x_{ij}^{l_k} + \sum_{j=1}^{i-1} x_{ji}^{l_k} = 0. \end{cases} \quad (2)$$

Then the line planning optimization problem is given as follows.

$$\min \sum_{l_k \in L} \sum_{i=1}^{N(S_k)-1} (C(l_k) \cdot f(l_k) - Q_{i,i+1}^{l_k}) \cdot (r_{i,i+1} + ac \cdot y_i^{l_k} + de \cdot y_{i+1}^{l_k}) \quad (3)$$

$$\min \sum_{l_k \in L} \sum_i \sum_j (\lambda^v \cdot t_{ij}^v(l_k) + \lambda^w \cdot t_{ij}^w(l_k)) \cdot q_{ij}^{l_k} \quad (4)$$

Subject to

$$\sum_{l_k \in L} q_{ij}^{l_k} = P_{ij}, \quad \forall i, j \in S \quad (5)$$

$$\sum_{l_k \in L} \mu \cdot C(l_k) \cdot x_{ij}^{l_k} \cdot f(l_k) \geq P_{ij}, \quad \forall i, j \in S \quad (6)$$

$$f_{min} \leq f(l_k) \leq f_{max}, \quad \forall l_k \in L \quad (7)$$

$$L_{min} \leq \sum_{l_k \in L} f(l_k) \leq L_{max} \quad (8)$$

$$\sum_{l_k \in L} N(l_k) \leq N \quad (9)$$

$$x_{ij}^{l_k} \in \{0, 1\}, \quad \forall i, j \in S, l_k \in L \quad (10)$$

$$f(l_k) \in \mathbb{N}, \quad \forall l_k \in L \quad (11)$$

$$q_{ij}^{l_k} \in \mathbb{N}, \quad \forall i, j \in S, l_k \in L. \quad (12)$$

Objective function (3) describes the maximum capacity utilization by minimizing the total empty seat-hours. The travel time of each section is influenced by whether the line has a stop on one or both stations, in which case the acceleration and/or deceleration time is added to the travel time. Objective function (4) minimizes the passengers' total travel time, which contains in-vehicle time and waiting time at the origin station. For each passenger OD corresponding to a selected line, the in-vehicle time and waiting time can be estimated as

$$t_{ij}^v(l_k) = \sum_{m=i}^{j-1} r_{m,m+1} + \sum_{m=i+1}^{j-1} (y_i^{l_k} \cdot (d_m + ac + de)) \quad (13)$$



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and

$$t_{ij}^w(l_k) = \begin{cases} \frac{T}{2F_{ij}} & \text{if } F_{ij} \geq F_{min} \\ T_{aw} & \text{otherwise,} \end{cases} \quad (14)$$

with

$$F_{ij} = \sum_{l_k \in L} x_{ij}^{l_k} \cdot f(l_k). \quad (15)$$

The average waiting time at the origin station depends highly on the frequency. If a certain OD has a relatively high operation frequency and regular intervals (headway), the passenger arrivals follow a uniform distribution (Furth, 2006). Therefore, the average waiting time is half the headway. Since the timetable is still unknown, we use half the average headway as estimated average waiting time when the frequency is higher than  $F_{min}$ . For line ODs with lower frequency, passengers tend to arrive near the scheduled departure time. Constraint (5) specifies that each passenger OD demand needs to be satisfied and is equal to the sum of passengers assigned to the lines. Constraint (6) makes sure that the service frequency between each OD is sufficient to meet passenger demand. Constraints (7) and (8) guarantee that the frequency of each train line could only vary in a certain range. If the frequency is zero, that means this line is not selected in line plan. The total number of trains should be lower than the infrastructure capacity, and not be lower than minimal frequency of the section with biggest accumulative passenger flow. As numerous stop patterns are available, a maximum number of train lines is forced in constraint (9) in order to get a relatively regular stop pattern, where  $N(l_k)$  can be calculated from the line frequencies as

$$N(l_k) = \begin{cases} 0, & \text{if } f(l_k) = 0 \\ 1, & \text{otherwise.} \end{cases} \quad (16)$$

Finally, the variables are restricted to be binary in (10), or nonnegative integers in (11) and (12). Note that we denote  $\mathbb{N} = \{0, 1, 2, \dots\}$ .

In order to find a way to deal with both objectives, we sum up both objectives together with certain weights. Expressing the objective (3) and (4) as  $f_1$  and  $f_2$ , respectively, the combined objective becomes

$$\min \quad \alpha \cdot f_1 + (1 - \alpha) \cdot f_2, \quad (17)$$

with  $0 \leq \alpha \leq 1$ . As the weight has a great impact on the final decision, it needs to be calibrated for a certain case. If a cost-oriented line plan is requested, it is better to assign a value to  $\alpha$  close to 1; and if a customer-oriented line plan is required, the value is better close to 0. A direct way to calibrate  $\alpha$  is to vary its value between 0 to 1 and compute the corresponding objective values. Then the objective value as function of  $\alpha$  could help to determine which value to choose for a certain case.

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## 4 Genetic Algorithm

The line planning optimization problem is a multi-objective, discrete and non-linear program, which is very difficult to solve by traditional optimization techniques. It is proved also that the line planning problem is NP-hard (Bussieck, 1997). Therefore, we introduce a new matheuristic approach for solving it. In general, matheuristics are optimization algorithms made by the interoperation of metaheuristics and mathematical programming techniques (Boschetti et al, 2009). An essential feature is the exploitation in some part of the algorithms of features derived from the mathematical model of the problems of interest. For our line planning problem we develop an approach that combines a well known metaheuristic GA (Goldberg, 1989) with an ILP formulation. The aim of integrating metaheuristics is to construct a promising neighborhood of good solutions. In particular, a GA is applied to give a recommendation of frequencies per line. Then, the ILP for the passenger assignment model is solved for a fixed line frequencies. After multiple iterations, better solutions are obtained.

GA is a search heuristic that mimics the process of natural selection. In a GA, a population of candidate solutions, i.e., *individuals* to an optimization problem is evolved toward better solutions. Each individual has a set of properties like a genetic representation and a fitness function. The former consists of single *genes* that construct an individual, while latter defines the quality of the individual.

For the line planning problem we define an individual as one possible solution (i.e. line plan) where a single gene defines a frequency of one line. Thus, a gene is a nonnegative integer value between 0 and  $f_{max}$ . The length of an individual equals to the number of lines in a line pool. The fitness function of an individual is evaluated by solving the ILP formulation (3)-(12) for the fixed frequencies defined by that individual.

Also, several operators are introduced within GA to combine and transform individual in order to produce a better ones. Namely, it includes selection, crossover and mutation. *Selection* is a process where a proportion of the existing population is selected to breed a new generation. *Crossover* is a process of taking two "parent" individuals and producing a "child" individuals from them. In order to preserve a genetic diversity the *mutation* operator is used. Mutation alters one or more genes in an individual from its initial state. Thus, GA may find a better solution by using mutation.

The developed matheuristics steps are as follows:

1. In the initial step, we: a) set the GA parameters like a number of individuals in the population  $N_{pop}$ , number of generations  $N_g$ , probabilities of crossover  $pc$  and mutations  $pm$ , as well as stopping criteria. Also, we generate an initial population of  $N_{pop}$  individuals randomly subject to constraints (13) and (14).

2. *Fitness function evaluation*. It may happen that the fitness of an individual results in a infeasible solution of the ILP due to nonexisting path(s) for certain passengers (see constraint (6)). In order to keep feasible individuals within the population, if an individual is infeasible we substitute it with a

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new individual and recompute the fitness. We do the same with all infeasible individuals until the population consists only of feasible ones.

3. *Reproduction.* An elitist strategy is used for reproduction. Individuals with lower fitness values are more desirable; hence, the  $N_{elite}$  individuals with the lowest fitness values are automatically copied to the next generation.

4. *Selection operation.* In the developed algorithm several selection rules are implemented. We apply the roulette wheel selection for crossover operation to generate  $N_{pop} - N_{elite} - N_{rand}$  individuals.

5. *Crossover operation.* We use a one-point crossover to select  $pc$  of the individuals for the new population.

6. *Mutation operation.* Two point mutation is implemented. The position of a gene to be mutated is chosen based on the following rules. First, one mutation point is chosen based on the structure of the population. We select a gene with the most often repeated frequency value. Second, the other mutation point is determined randomly. Third, if a value of a gene is too low, e.g., smaller than 5, it is set to zero. Finally, the mutation operator is applied with the probability of  $pm$ .

7. New individuals are created randomly in order to maintain the diversity of the population. Therefore, the algorithm finally generates new  $N_{rand}$  individuals.

8. *Termination check.* We introduce two stopping criteria. First, the maximal iterations or computation time is reached. Second, the successive iterations no longer produce better results the model detects the convergence of a solution. If any of two criteria is met, terminate the algorithm. Otherwise, go to Step 2.

## 5 Numerical Experiments

In this section, a numerical case from the real world is studied to verify the effectiveness of the proposed model and algorithm. A dense high-speed rail corridor is selected as a case study. Table 1 shows the OD matrix of the passenger flow with rough estimation but satisfy the purpose of the experiments. Observe that the distribution of passenger flow is uneven with a huge amount between the first station and the last station, and a small amount between intermediate stations, such as station 2 to 6. The design speed is 300 km/h and the whole distance is around 300 km. Stations 1, 2, 3 and 8 have technical facilities to be origin and final stations. With consideration of the existing line plan, passenger flow and station conditions, a line pool is generated with 19 lines. For the sake of safety and a homogeneous pattern, a train line without any stop is not recommended.

The values of the parameters are listed in Table 2. The number of population and generation in the genetic algorithm are chosen as 50 and 500, respectively. It needs a relatively long time to obtain a good solution. Therefore, only several values are chosen in this paper. The model is solved using

**Table 1** OD matrix of passenger flow

OD matrix	1	2	3	4	5	6	7	8
1	0	6320	12405	9205	8128	2013	1991	14782
2	6320	0	2235	945	857	151	590	2268
3	12405	2235	0	3370	2220	582	697	4998
4	9205	945	3370	0	3376	533	857	5135
5	8128	857	2220	3376	0	739	832	3996
6	2013	151	582	533	739	0	614	879
7	1991	590	697	857	832	614	0	3208
8	14782	2268	4998	5135	3996	879	3208	0

**Table 2** Input parameters of the model

Parameter	Value or description of calculation
Maximal number of train lines $N$	13
Seat capacity $C(l_k)$	620 seats/train-set
Seat occupancy rate $\mu$	0.85
Dwell time $d_i$	2 min
Acceleration and deceleration time $ac\ de$	2.5 min, 1.5min respectively
Time period $T$	18 h with 6 h of maintenance time
Critical point of totally frequency $F_{min}$	30 trains
Weights $\lambda^v, \lambda^w$	1, 2, see details in Wardman (2004)
Average waiting time $T_{aw}$	30 min
Line frequency lower bound $f_{min}$	0
Line frequency upper bound $f_{max}$	18 (maximal one train a period)
Total frequency lower bound $L_{min}$	106
Total frequency upper bound $L_{max}$	140

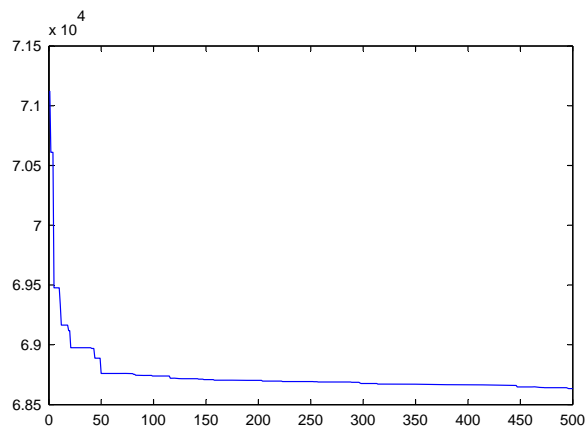
MATLAB R2013b and Gurobi 604 (GUROBI (2015)) on a Dell PC with 8 GB RAM and a four-core 3.7 GHz CPU.

Both passenger's convenience and operator's cost are considered in this model. Therefore,  $\alpha = 0.5$  is assigned first. Table 3 depicts the optimized stop pattern, frequency and empty-seat percentage. Most of the lines are almost fully utilized with a seat occupancy rate of 0.85. Lines with many stops are not that attractive, but have a comparable high frequency. The total number of frequencies is the same as the lower bound, which means that the service frequency just meets the demand for the most busy corridor. Figure 2 illustrates the convergence with the maximal iterations. For the last 100 iterations, the objective are without too much improvement, which means it could be a optimal solution. Figure 3 indicates the relation between service frequency (stop times) and passenger demand of the corresponding station. As a cyclic pattern is proposed, the service frequency of most stations are much higher than its passenger demand except for station 1.

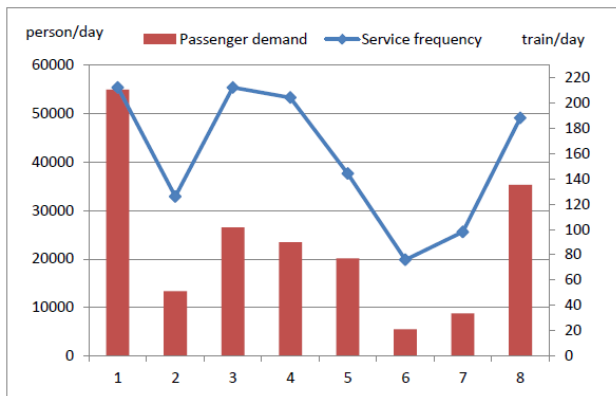
For  $\alpha = 0$ , Table 4 displays the selected stop pattern with corresponding frequency and empty-seat percentage. The frequency of most lines is relatively higher than the previous case, and as a result the empty-seat percentage is also high, especially for the all-stop train with 98%. But lines with less stops are still occupied well. The total number of frequency is with 139 almost equal to the upper bound, as the cost objective is not considered here. And Table

**Table 3** Line plan with minimum total travel time and empty seat-hour

Train line	Frequency	Empty-seat percentage
1-2-3	4	15%
1-3-4	4	15%
1-3-4-8	17	15%
1-3-4-5-8	6	15%
1-3-4-5-7-8	8	15%
1-3-4-5-6-8	8	15%
1-2-3-4	4	18%
1-2-3-4-7-8	5	15%
1-2-3-4-5-8	2	15%
1-2-3-4-5-7-8	18	19%
1-2-3-4-5-6-8	12	43%
1-2-3-4-5-6-7-8	18	45%



**Fig. 2** The number of iterations for total travel time and empty seat hour



**Fig. 3** Passenger demand with corresponding service frequency

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**Table 4** Line plan with minimum total travel time

Train line	Frequency	Empty-seat percentage
1-3-4-8	20	15%
1-3-4-7-8	5	33%
1-3-4-5-8	9	16%
1-3-4-5-7-8	12	64%
1-3-4-5-6-8	4	53%
1-3-4-5-6-7-8	20	58%
1-2-3-4-8	17	21%
1-2-3-4-7-8	3	54%
1-2-3-4-6-7-8	7	37%
1-2-3-4-5-8	13	32%
1-2-3-4-5-7-8	4	45%
1-2-3-4-5-6-8	5	77%
1-2-3-4-5-6-7-8	20	98%

**Table 5** Line plan with minimum empty-seat-hour

Train line	Frequency	Empty-seat percentage
1-2-3	4	15%
1-3-4	8	24%
1-3-4-8	18	17%
1-3-4-7-8	15	18%
1-3-4-6-8	7	28%
1-3-4-5-8	15	28%
1-3-4-5-6-7-8	3	15%
1-2-3-4-8	18	15%
1-2-3-4-7-8	2	33%
1-2-3-4-6-8	4	23%
1-2-3-4-5-8	12	55%

5 shows a line plan and seat occupancy rate when only empty-seat-hour is considered as objective, where the empty seats are much more lower than in Table 4. Meanwhile, the total total number of frequency is the same as the frequency lower bound. Figure 4 depict the convergence curve of both objectives with the maximal iteration times, which shows a relatively effective result. From Figure 5, we could observe that both service frequency could meet passenger demand and with much more high frequencies in some stations. Service frequency 1 represents the optimal result of total travel time; and service frequency 2 stands for empty-seat-hour. Even though with conflicting objective, both service frequency curves have almost the same display, meaning passenger demand influence a lot for the final result.

## 6 Conclusion and future work

From the experiments, it can be concluded that the approach is feasible to obtain an integrated line plan with a cyclic and acyclic nature. Some trains with a high frequency providing frequent service for high passenger OD demand,

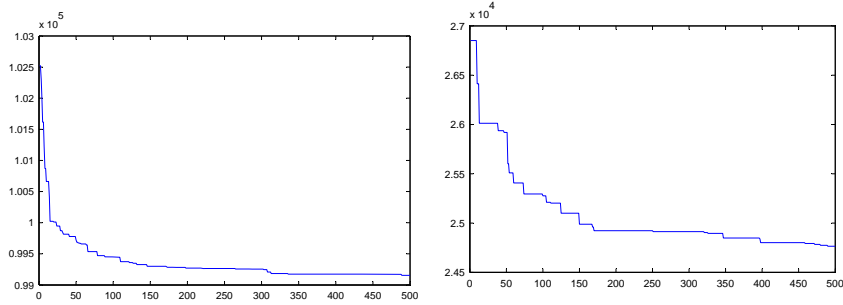


Fig. 4 Iterations of optimizing total travel time(left) and total empty-seat-hour(right)

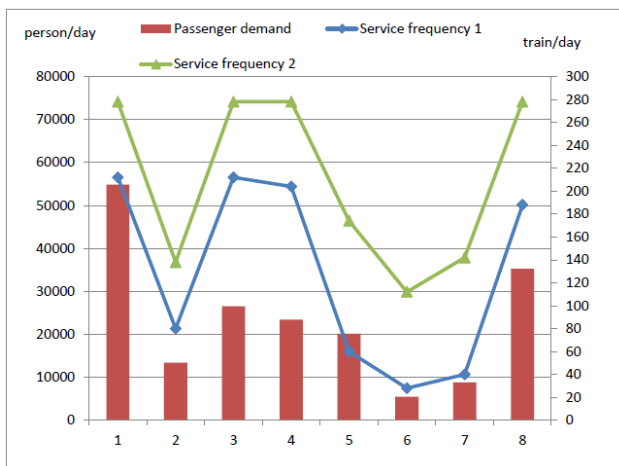


Fig. 5 Passenger demand with corresponding service frequency

and some trains with a lower frequency serving stops with low passenger demand. Trains with lots of stop always have high frequency and low capacity utilization. This can be explained by objective function (4), since waiting time has a high dependency on frequency in (14). So train lines such as all-stop pattern lines may not be a good travel choice, however, they play an important role for reducing the waiting time of the average passenger. Therefore, considering the waiting time, the passenger assignment model needs to be modified in order to have a smaller influence in the next step. And an exact algorithm is taken into consideration to solve the problem in the future.

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