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# Estimation and reduction of random noise in mass anomaly time-series from satellite gravity data by minimization of month-to-month year-to-year double differences

Pavel Ditmar, Natthachet Tangdamrongsub<sup>1</sup>, Jiangjun Ran<sup>2,\*</sup>, Roland Klees\*

*Delft University of Technology, Stevinweg 1, 2628 CN, Delft, The Netherlands*

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## Abstract

We propose a technique to regularize a GRACE-based mass-anomaly time-series in order to (i) to quantify the Standard Deviation (SD) of random noise in the data, and (ii) to reduce the level of that noise. The proposed regularization functional minimizes the Month-to-month Year-to-year Double Differences (MYDD) of mass anomalies. As such, it does not introduce any bias in the linear trend and the annual component, two of the most common features in GRACE-based mass anomaly time-series. In the context of hydrological and ice sheet studies, the proposed regularization functional can be interpreted as an assumption about the stationarity of climatological conditions. The optimal regularization parameter and noise SD are obtained using Variance Component Estimation. To demonstrate the performance of the proposed technique, we apply it to both synthetic and real data. In the latter case, two geographic areas are considered: the Tonlé Sap basin in Cambodia and Greenland. We show that random noise in the data can be efficiently (1.5 – 2 times) mitigated in this way, whereas no noticeable bias is introduced. We also discuss various findings that can be made on the basis of the estimated noise SD. We show, among others, that knowledge of noise SD facilitates the analysis of differences between GRACE-based and alternative estimates of mass variations. Moreover, inaccuracies in the latter can also be quantified in this way. For instance, we find that noise in the surface mass anomalies in Greenland estimated using the Regional Climate Model RACMO2.3 is at the level of 2 – 6 cm equivalent water heights. Furthermore, we find that this noise shows a clear correlation with the amplitude of annual mass variations: it is lowest in the north-west of Greenland and largest in the south. We attribute this noise to limitations in the modelling of the meltwater accumulation and run-off.

**Keywords:** Mass transport, GRACE, Tikhonov regularization, Variance Component Estimation, Tonlé Sap, Greenland Ice Sheet

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\*Corresponding author

Email address: [jiangjunrangmail.com](mailto:jiangjunrangmail.com) (J. Ran)

<sup>1</sup>Currently at: University of Newcastle, Australia

<sup>2</sup>Currently at: State Key Laboratory of Geodesy and Earth's Dynamics, Institute of Geodesy and Geophysics, Chinese Academy of Sciences, Wuhan, China

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## 1. Introduction

The Earth's system is characterized by on-going large-scale mass transport. In most of land areas, it is associated with various hydrological processes. An exception are the polar regions, where the dominant contributors are ice sheets and Glacial Isostatic Adjustment (GIA).

An accurate quantification of large-scale mass-transport is of major importance in various applications, including water management, climate science, and solid Earth geophysics. Satellite Gravimetry (SG) is a powerful tool to monitor large-scale mass transport. The first satellite mission suitable for that purpose – Gravity Recovery and Climate Experiment (GRACE) – was launched in 2002 (Tapley et al., 2004). In the first instance, SG data are used to compute time-series of the Earth's gravity field solutions. Typically, one solution per month is obtained. Each of them consists of a set of spherical harmonic coefficients complete to some maximum degree (usually between 60 and 120). After appropriate processing (see, e.g., Wahr et al., 1998; Ditmar, 2018), such solutions may yield a time-series of mass anomalies within a region of interest, i.e., the differences between the instantaneous amount of mass at (or near) the Earth's surface and the corresponding long-term mean value. Currently, the GRACE mission is not operational anymore, but its successor – GRACE Follow-On (GFO) – is scheduled for launch in early 2018 (Flechtner et al., 2014, <https://gracefo.jpl.nasa.gov>).

Mass anomalies extracted from SG data suffer from inaccuracies. A part of the error budget consists of random noise propagated from the original satellite observations via spherical harmonic coefficients. Such noise is not time-correlated and may be quite strong, especially if the target region is small. The estimated mass anomalies may suffer also from systematic disturbances. For instance, various filters are typically used to reduce noise in spherical harmonic coefficients (Wahr et al., 1998; Han et al., 2005; Wouters and Schrama, 2007; Swenson and Wahr, 2006; Kusche, 2007; Klees et al., 2008; Siemes et al., 2013). Unfortunately, filters also distort the signal of interest, introducing among others leakage errors.

The random and systematic errors mentioned above may complicate the usage of SG-based mass anomaly estimates in practice. For instance, these errors make it more problematic to estimate the quality of a geophysical model describing mass transport of a certain type when SG is used as a source of independent information. This is because the differences between the geophysical model and SG-based estimates will be contaminated by errors in the latter estimates themselves. This may be particularly harmful if errors in SG-based estimates are comparable to or exceed errors in the geophysical model.

With this article, we present a novel procedure that allows for: (i) quantifying the level of random noise in a mass anomaly time-series based on SG data; and (ii) reducing this level. The basic properties of the proposed procedure are as follows:

- It is based on the Tikhonov regularization concept (Tikhonov and Arsenin, 1977) and does not require an explicit parameterization of the signal in the time domain, which makes the procedure very flexible
- A new variant of the regularization functional is proposed, which minimizes the month-to-month year-to-year double differences in order to keep seasonal variations and linear trends (the dominant features of many mass transport processes) untouched, so that the bias introduced by the regularization is reduced.
- Known stochastic properties of random noise (e.g., time-dependent standard deviation or full error variance-covariance matrix) can be accounted for in the statistically optimal way

- 45 • The optimal regularization parameter is computed by Variance Component Estimation  
46 (VCE) (Koch and Kusche, 2002), which makes the procedure not only flexible, but also  
47 fully automatized.
- 48 • VCE allows also for a re-estimation of the random noise level in the original SG-based  
49 estimates.

50 The ability of the procedure to quantify the level of random noise in a mass anomaly time-  
51 series from the time-series itself makes it particularly valuable when SG is used for the validation  
52 of a geophysical model. Knowledge of this level allows for a quantification of the contribution of  
53 random noise in SG-based estimates to their differences with respect to the time-series subject to  
54 validation. Then, it is easy to estimate the Standard Deviation (SD) of remaining noise, which is  
55 composed of systematic errors in SG estimates and noise in the geophysical model assuming that  
56 remaining noise is not correlated with random noise in SG estimates. This opens the door for  
57 the quantification of noise in the geophysical model alone (since the contribution of systematic  
58 errors in SG estimates can be assessed by, e.g., a numerical experiment).

59 The proposed procedure has been already successfully used in a number of studies: to assess  
60 the performance of a novel variant of a so-called mascon approach in the context of Green-  
61 land Ice Sheet monitoring with SG (Ran et al., 2017); to calibrate the error covariance matrices  
62 of degree-1 and  $C_{20}$  spherical harmonic coefficients estimated from a combination of GRACE-  
63 based monthly solutions and an ocean bottom pressure model (Sun et al., 2017); as well as to  
64 demonstrate the added value of a novel technique for GRACE data processing by considering the  
65 estimated mass anomalies in Mississippi River basin and in Greenland (Guo et al., 2018). In this  
66 article, we present an in-depth analysis of the proposed techniques, including an open discus-  
67 sion of its strong points and limitations. We focus on two geographical areas as representative  
68 examples. The first one is the Tonlé Sap basin (Cambodia), which is subject to large seasonal  
69 and inter-annual mass variations of hydrological origin. The other area is Greenland, where a  
70 combination of snow fall and ice mass loss results in strong seasonal mass variations combined  
71 with large negative long-term trends. The two examples were deliberately chosen to demonstrate  
72 that the proposed methodology has a broad spectrum of potential applications. Among others,  
73 we discuss how the aforementioned "remaining noise" can be quantified and how this informa-  
74 tion can be used to know more about a mass anomaly time-series alternative to the SG-based  
75 one. In addition, we isolate the "remaining noise" in the differences between *regularized* SG  
76 estimates and the alternative time-series. This allows us to quantify the level of random noise in  
77 SG estimates after regularization and, therefore, to assess how efficiently that noise is damped  
78 by the proposed procedure.

79 The structure of the article is as follows. Sect. 2 contains a description of the proposed reg-  
80 ularization procedure. In Sect. 3, we apply the developed procedure to mass anomaly time-  
81 series based on simulated and real GRACE data. Among others, we discuss in detail how the  
82 SD of "remaining noise" and the reduction of random noise by regularization can be quantified  
83 (Sect. 3.1.2). Furthermore, realistic numerical simulations are conducted in order to support real  
84 data processing and make a comprehensive assessment of performance of the proposed regular-  
85 ization scheme. Sect. 4 contains a discussion and conclusions.

## 86 2. Theory

87 Mass anomaly time-series  $H_i^{(\text{obs})}$  based on SG data may contain gaps and strong noise. The  
88 proposed technique allows for a quantification and reduction of the noise level, as well as for fill-

89 ing in data gaps, if they are present. To that end, the Tikhonov regularization concept (Tikhonov  
 90 and Arsenin, 1977) is used. To simplify the presentation of the method, we assume that the reg-  
 91 ularized mass anomaly time-series is a continuous function  $\hat{H}(t)$ , where  $t$  is time in years. The  
 92 corresponding equations for discrete time-series are provided in Appendix A. In the actual imple-  
 93 mentation of the proposed technique, the discretization of the original and regularized time-series  
 94 is always one month.

95 We postulate that the regularized time-series  $\hat{H}(t)$  minimizes the penalty functional

$$\Phi[H] = \sum_i \left( H(t_i) - H_i^{(\text{obs})} \right)^2 + \alpha \Omega[H], \quad (1)$$

96 where  $t_i$  is the time of the  $i$ -th observation,  $\alpha$  is the regularization parameter, and  $\Omega[H]$  is the  
 97 regularization functional. The latter depends on the function  $H(t)$  and its derivatives up to a given  
 98 order. For simplicity, we assume here that noise in the input data is white. A generalization to  
 99 arbitrary Gaussian noise is straightforward (see Appendix A).

100 The highest order of the derivatives of  $H(t)$  used in the definition of the regularization func-  
 101 tional defines the order of that functional. A commonly-used Tikhonov regularization functional  
 102 is the zero-order functional

$$\Omega[H] = \int (H(t))^2 dt, \quad (2)$$

103 which requires that the target function  $\hat{H}(t)$  is as close to zero as possible. As an alternative, the  
 104 first-order functional

$$\Omega[H] = \int (H'(t))^2 dt \quad (3)$$

105 (where  $H'(t)$  is the time-derivative of  $H(t)$ ) is used frequently. This functional tries to make  
 106 the unknown function the smoothest possible one. In the context of GRACE data processing, a  
 107 somewhat similar idea was applied in the computation of mascon solutions (see, e.g., Luthcke  
 108 et al., 2006, 2013). Both zero- and first-order functionals inevitably bias the solution, since  
 109 they penalize all signals (an exception is a constant, which is not penalized by the first-order  
 110 functional). This makes their application to mass anomaly time series sub-optimal.

111 Many mass anomaly time-series typically show a pronounced annual periodicity; the temporal  
 112 behaviour of mass anomalies in neighboring years is rather similar. This applies to, e.g., most  
 113 signals of hydrological origin, as well as to signals related to the part of an ice sheet that is  
 114 subject to summer melt. Therefore, we believe that a regularization functional that takes this  
 115 periodicity into account would be a more natural choice when estimating mass anomalies. The  
 116 most straightforward way to design such a regularization functional is to minimize the year-to-  
 117 year differences of mass anomalies:

$$\Omega[H] = \sum_{k=1}^{K-1} \int_0^1 (h_{k+1}(t) - h_k(t))^2 dt, \quad (4)$$

118 where  $K$  is the total number of years considered and  $h_k(t)$  is by definition the mass anomaly in  
 119 the  $k$ -th year ( $t \in [0; 1]$ ;  $h_k(1) = h_{k+1}(0)$  due to the continuity of  $H(t)$ ; we remind that  $t$  is time in  
 120 years).

121 Unfortunately, the regularization functional of Eq. (4) penalizes an inter-annual variability of  
 122 mass anomalies. This is a weak point whenever such a variability is present. This holds true, for

123 instance, for many hydrological processes (particularly, in areas where a long-term depletion of  
 124 groundwater stocks takes place), as well as for ice sheets and mountain glaciers, many of which  
 125 are subject to a long-term mass loss nowadays. Furthermore, a GIA-related signal may also be  
 126 responsible for inter-annual mass variations (namely, long-term nearly-linear trends). Therefore,  
 127 we propose to minimize the year-to-year difference not between mass anomalies themselves but  
 128 between their time-derivatives:

$$\Omega[H] = \sum_{k=1}^{K-1} \int_0^1 (h'_{k+1}(t) - h'_k(t))^2 dt. \quad (5)$$

129 After discretization, this reduces to a minimization of Month-to-month Year-to-year Double Dif-  
 130 ferences (MYDD). Obviously, such a functional does not penalize year-to-year differences in  
 131 the presence of an arbitrary (but constant) offset between mass anomalies in neighbouring years.  
 132 The regularization functional of Eq. (5) is exploited here.

133 In the context of hydrological and ice sheet studies, the regularization functional of Eq. (5) has  
 134 a physical interpretation. According to the mass balance equation, the rate of mass change in a  
 135 particular river basin or ice drainage system is equal to the difference between mass gain (i.e.,  
 136 precipitation) and mass loss (e.g., due to evaporation, transpiration, sublimation, water run-off,  
 137 or ice discharge). Thus, the proposed regularization functional of Eq. (5) does not penalize the  
 138 mass anomaly signals that reflect stationary climatological conditions (i.e., when the mass gains  
 139 and mass losses per calendar month do not change from year to year).

140 To find the optimal regularization parameter  $\alpha$ , we propose to use Variance Component Es-  
 141 timation (VCE). A brief description of this method, adapted from (Koch and Kusche, 2002), is  
 142 provided in Appendix A. An advantage of VCE is that it not only provides the optimal regular-  
 143 ization parameter, but also allows the level of noise in the input data to be quantified.

144 To illustrate the behaviour of the regularization functional of Eq. (5), we consider a simple  
 145 numerical example. Let the true time-series  $H(t)$  covering a 3-year time interval be analytically  
 146 defined as

$$H(t) = A \sin 2\pi t + Ct, \quad t \in [0; 3], \quad (6)$$

147 where  $A = 1$  cm in terms of Equivalent Water Height (EWH) and  $C = 0.5$  cm/yr, see Fig. 1.  
 148 Furthermore, the observations are assumed to be noise-free and cover only the first and the sec-  
 149 ond year of the considered time interval, where the sampling rate is one month. The adopted  
 150 regularization scheme allows the full 3-year time-series of mass anomalies to be restored. Since  
 151 the seasonal variability of the considered function does not change, the proposed regularization  
 152 scheme fully recovers it on the basis of the available data, without introducing any bias (Fig. 1).  
 153 In particular, the linear trend is fully recovered, which is due to the fact that the requirement of  
 154 similarity in successive years is applied to the time-derivatives of mass anomalies rather than to  
 155 mass anomalies themselves.

156 It can be proven analytically that any function  $H(t)$  not penalized by the regularization func-  
 157 tional of Eq. (5) is a combination of arbitrary seasonal variations and a linear trend (see Appendix  
 158 B). This means that the class of functions that can be processed with the proposed regulariza-  
 159 tion without suffering from a bias is relatively wide. This may also have a negative effect. If a  
 160 time-series is too short or noise is too strong, the regularized time-series may contain pronounced  
 161 periodic features that are purely noise-driven and do not represent a real signal. To illustrate this,  
 162 we consider a true function  $H(t)$ , which comprises only a linear trend over a 3-year time interval:

$$H(t) = Ct, \quad t \in [0; 3], \quad (7)$$

163 where  $C = 0.5$  cm/yr (EWH). The observations simulated with a one-month sampling rate cover  
164 the entire time interval. They are artificially contaminated with a relatively strong white noise  
165 of 1-cm EWH standard deviation (Fig. 2, top plot). By chance, the simulated observations in  
166 November of each year suffer from a positive noise value. As a result, the regularized time-  
167 series shows a strong peak in this month. It is worth adding that the VCE estimate of the data  
168 noise SD remains reasonable: 1.020 cm EWH. Thus, the estimation error is only 2%.

169 Next, we repeat the previous experiment, using a two-times longer set of synthetic observa-  
170 tions: six years instead of three. All the other parameters of the experiment are kept as before. In  
171 that case, the regularized time-series still suffers from data noise, but its impact is dramatically  
172 reduced (Fig. 2, bottom plot). Remarkably, the VCE estimate of the noise SD of 0.988 cm EWH  
173 is even more accurate than in the previous experiment. This differs from the actual noise SD by  
174 only 1.2%.

Fig.  
2

### 175 3. Application

176 In this section, we apply the proposed regularization procedure to mass anomaly time-series  
177 in two geographical areas: (i) the Tonlé Sap basin in Cambodia and (ii) Greenland. In both  
178 cases, the processed time-series are based on real GRACE data. In Sect. 3.1, we provide general  
179 information about the GRACE data, and the data analysis approach (particularly, about quanti-  
180 fying the reduction of random noise in GRACE data after regularization). In Sections 3.2 and  
181 3.3, we present the results for the Tonlé Sap basin and Greenland, respectively. The structure of  
182 both sections is similar. First, we discuss the data processing aspects specific for the considered  
183 geographical area. Second, we discuss the results of a numerical study, where the behaviour of  
184 actual mass anomalies is reproduced. Third, we consider the results of real data processing.

#### 185 3.1. General information

##### 186 3.1.1. Input data

187 The space segment of the GRACE mission consisted of two twin satellites, which followed  
188 each other in a nearly the same polar orbit with a 200-km separation. The satellites were  
189 equipped, among others, with a K-Band Ranging (KBR) system, which allowed temporal varia-  
190 tions in the inter-satellite separation to be measured with micrometer-level precision. A number  
191 of research centres process GRACE observations to produce a time-series of monthly gravity  
192 field solutions, which form the core of the so-called level-2 data product of the GRACE mission.  
193 In our study, we make use of the solutions produced at the Center for Space Research (University  
194 of Texas at Austin) (Bettadpur, 2012). Each of these solutions is formed by a set of spherical har-  
195 monic coefficients complete either to degree 60 (this variant was used to estimate mass anomalies  
196 over the Tonlé Sap basin) or to degree 96 (this variant was used for Greenland). Degree-1 coef-  
197 ficients are absent in the GRACE level-2 data product. Therefore, an independently computed  
198 time-series of these coefficients (Swenson et al., 2008) was exploited. Furthermore, the spherical  
199 harmonic coefficient  $C_{2,0}$  was replaced in each GRACE monthly solution by the one estimated  
200 from satellite ranging data (Cheng and Tapley, 2004) due to an insufficient accuracy of the former  
201 one.

202 Mass anomaly estimates based on GRACE data are contaminated by random noise. The noise  
203 level increases rapidly with decreasing size of the area of interest. This noise is not correlated  
204 in the time domain, but shows a strong spatial correlation, which reflects, among others, the  
205 anisotropic sensitivity of GRACE KBR observations. They sense the along-track (North-South)

206 component of the mass anomaly gradient much better than the cross-track (East-West) compo-  
 207 nent. As such, random noise in mass anomaly estimates depends also on the shape of the area  
 208 of interest: an area elongated in the East-West direction is a much more favourable study object  
 209 than an area elongated in the North-South direction. In addition, random noise increases towards  
 210 the equator due to a lower density of satellite groundtracks, as well as due to small intersection  
 211 angles of ascending and descending tracks, which makes the sensitivity of measurements partic-  
 212 ularly anisotropic. State-of-the-art data processing in the spatial domain was applied to produce  
 213 mass anomaly estimates with the lowest possible noise level. Further details are provided in  
 214 sections 3.2.1 (Tonlé Sap basin) and 3.3.1 (Greenland).

### 215 3.1.2. Analysis of results

216 For both study areas, GRACE-based mass anomaly time-series are compared with reference  
 217 ones, which are obtained with other techniques. The points of our special attention are: (i) quan-  
 218 tification of random noise in GRACE data; (ii) the bias introduced into the data by the proposed  
 219 regularization procedure; and (iii) reduction of noise in GRACE data after regularization. In a  
 220 simulated experiment, an estimation of the noise SD after regularization is straightforward. In  
 221 an experiment with real data, a reference dataset is needed. Doing so, we follow a two-step  
 222 procedure. In the first step, we analyse the difference between the original GRACE data set  
 223 (i.e., the data set not subject to any interpolation or regularization) and the reference one. These  
 224 differences reflect (i) random noise in GRACE data and (ii) "other" errors, which may include in-  
 225 accuracies of the reference data, as well as systematic errors in GRACE data (for instance, those  
 226 due to signal leakage). We assume that random noise and "other" errors are not cross-correlated,  
 227 so that

$$\Delta_{\text{orig}}^2 = \sigma_{\text{GRACE-orig}}^2 + \sigma_{\text{other}}^2, \quad (8)$$

228 where  $\Delta_{\text{orig}}$  is the rms difference between GRACE and reference data,  $\sigma_{\text{GRACE-orig}}$  is the SD of  
 229 random noise in the original GRACE data (which is estimated using VCE) and  $\sigma_{\text{other}}$  is the SD of  
 230 the other errors. This allows the SD of "other" errors to be estimated as

$$\sigma_{\text{other}} = \sqrt{\Delta_{\text{orig}}^2 - \sigma_{\text{GRACE-orig}}^2}. \quad (9)$$

231 In the second step, we analyze the difference between the regularized GRACE data and the  
 232 reference data. Assuming that the effect of regularization on the systematic errors in GRACE  
 233 data is negligible, we can state that

$$\Delta_{\text{reg}}^2 = \sigma_{\text{GRACE-reg}}^2 + \sigma_{\text{other}}^2, \quad (10)$$

234 where  $\Delta_{\text{reg}}$  is the rms difference between the two data sets and  $\sigma_{\text{GRACE-reg}}$  is the SD of random noise  
 235 in the regularized GRACE data. Eq. (10) allows the latter noise to be quantified as

$$\sigma_{\text{GRACE-reg}} = \sqrt{\Delta_{\text{reg}}^2 - \sigma_{\text{other}}^2}. \quad (11)$$

236 We use the quantity

$$\frac{\sigma_{\text{GRACE-reg}}}{\sigma_{\text{GRACE-orig}}} \times 100\% \quad (12)$$

237 to describe the reduction of random noise in a particular GRACE dataset due to regularization.  
 238 Finally, knowledge of "other" errors imposes an upper limit for possible errors in the reference  
 239 data and in systematic errors in GRACE data. If there are reasons to believe that the contribution  
 240 of the latter errors is minor, the estimate  $\sigma_{\text{other}}$  can be used to quantify the accuracy of the reference  
 241 data themselves.



242 3.2. Tonlé Sap basin

243 Tonlé Sap basin located in Cambodia has an area of  $82 \times 10^3$  km<sup>2</sup>. It surrounds the Tonlé  
244 Sap Lake, which is the largest freshwater lake in Southeast Asia. The region is characterized by  
245 monsoon climate, the rainy season lasting from May to September or early October. As a result,  
246 a flood event takes place in the second half of each year, usually reaching the peak in October.

247 3.2.1. Data preparation

248 In this study, we use two time-series of mass anomalies over the Tonlé Sap basin: a GRACE-  
249 based and a reference one. Both time-series were prepared by one of the co-authors and exploited  
250 earlier in (Tangdamrongsub et al., 2016).

251 The time-series of GRACE-based mass anomalies is based on monthly gravity field solutions  
252 pre-processed as explained in Sect. 3.1.1. At the next step, the solutions were cleaned from along-  
253 track artefacts by means of the de-stripping procedure (Swenson and Wahr, 2006) and smoothed  
254 with a Gaussian filter of 350-km half-width (Jekeli, 1981; Wahr et al., 1998). After that, the  
255 smoothing effect of the Gaussian filter was mitigated by a signal restoration technique (Chen  
256 et al., 2014). Finally, the (unregularized) time-series of monthly mass anomalies within the Tonlé  
257 Sap basin was computed (Wahr et al., 1998). Mass anomalies in the months without GRACE data  
258 were obtained by means of a cubic polynomial interpolation, using the Matlab function *interp1*.  
259 Further details regarding the adopted data processing scheme can be found in (Tangdamrongsub  
260 et al., 2016).

261 The reference estimates of mass anomalies in Tonlé Sap basin were obtained on the basis  
262 of surface reflectance data collected by the Moderate-Resolution Imaging Spectroradiometer  
263 (MODIS) instrument on board Terra and Aqua satellites (Vermote et al., 2011). The reflectance  
264 data were used to estimate the mean inundated area within the Tonlé Sap basin in each month.  
265 A comparison of those estimates with GRACE-based mass anomalies allowed an empirical rela-  
266 tionship between the two time-series to be established:

$$H(x, t) = a_0 + a_1 x(t) + a_2 e^{-\frac{x(t)}{1000}} + a_3 \cos 2\pi t + a_4 \sin 2\pi t, \quad (13)$$

267 where  $t$  is time in years (zero time being at the beginning of a year),  $H(t)$  is mean mass anomaly  
268 within the basin in cm EWH,  $x$  is inundated area in km<sup>2</sup>, and  $a_0, \dots, a_4$  are constant coefficients  
269 obtained by means of the linear regression:  $a_0 = -0.54$ ,  $a_1 = 1.4 \times 10^{-3}$ ,  $a_2 = -16.2$ ,  $a_3 =$   
270  $-4.8$ , and  $a_4 = -9.2$ . The last two terms in Eq. (13) were needed to take into account seasonal  
271 variations in the soil moisture content (Tangdamrongsub et al., 2016).

272 In our study presented below, we use as input unregularized mass anomaly estimates in the  
273 time-interval (Jan. 2003 – Oct. 2014). To improve the consistency between the GRACE- and  
274 MODIS-based mass anomalies, we have estimated their mean values in the considered time inter-  
275 val (the months with no GRACE data being excluded in both cases). After that, the corresponding  
276 mean value has been subtracted from each data set. The resulting GRACE- and MODIS-based  
277 time-series can be seen in Fig. 3 as blue dots and red lines, respectively. They both show a  
278 clear seasonal variability, with the maximum in October. In the first half of the considered time  
279 interval (i.e., 2003 – 2008), about the same annual pattern is visible with a peak amplitude in  
280 the range 25 – 30 cm EWH. In the second half of the considered time interval (2009 – 2014), a  
281 strong inter-annual variability is observed. In odd years (2009, 2011, and 2013), the peak mass  
282 anomaly reaches 40 cm EWH, which is substantially above the average peak level observed in  
283 2003 – 2008. In even years (2010, 2012, and 2014), the peak anomaly reaches only about 20 cm.  
284 Such an inter-annual variability poses a challenge for the proposed procedure, since the latter is  
285 tailored to scenarios when seasonal variations in neighbouring years are similar.

Fig.  
3

286 3.2.2. Numerical study

287 The time-series of mass anomalies in the Tonlé Sap basin is mimicked by a quasi-periodic  
 288 function  $H(t)$  that reaches minimum and maximum in April and October of each year, respec-  
 289 tively. In 2003 – 2008, the signal amplitude stays at the same middle level  $A_m$ . In 2009-2014, the  
 290 signal amplitude is year-dependent: it stays at a high level  $A_h$  in odd years and at a low level  $A_l$   
 291 in even years. More specifically:

$$H(t) = c + A[1 - \cos(2\pi t - \varphi)], \quad (14)$$

292 where

$$A = \begin{cases} A_m & \text{in Jan.2003–Mar.2009,} \\ A_h & \text{in Apr.2009–Mar.2010,} \\ & \text{Apr.2011–Mar.2012,} \\ & \text{Apr.2013–Mar.2014;} \\ A_l & \text{in Apr.2010–Mar.2011,} \\ & \text{Apr.2012–Mar.2013,} \\ & \text{Apr.2014–Oct.2014.} \end{cases} \quad (15)$$

293 The phase  $\varphi$  is set equal to 1.8326, which corresponds to the mid of April (the month when  
 294 the mass anomalies are the lowest). The numerical values of the coefficients  $c$ ,  $A_m$ ,  $A_h$ , and  
 295  $A_l$  are estimated from the mass anomalies based on real GRACE data with a linear regression:  
 296  $c = -21.42$  cm;  $A_m = 20.53$  cm;  $A_h = 27.28$  cm; and  $A_l = 17.31$  cm. The simulated time-  
 297 series is contaminated by pseudo-random zero-mean Gaussian white noise with a SD of 4.2 cm,  
 298 which is consistent with our estimation of noise in real data processing (see Sect. 3.2.3). To  
 299 make the results more representative, each numerical experiment is repeated with 1000 different  
 300 noise realizations. The major outcome of each experiment is: (i) an estimate of the noise SD in  
 301 the original data time-series; (ii) the noise SD after regularization; and (iii) the bias introduced  
 302 by regularization. Noise after regularization is defined as the difference between the regularized  
 303 noisy time-series and the true one. It is a combination of regularized random noise and the  
 304 bias of the true signal introduced by regularization. To quantify the latter, we re-estimate the  
 305 signal amplitudes from the regularized time-series with the linear regression, and then subtract  
 306 the true amplitudes. For each estimate, we report the mean over the 1000 realizations and the  
 307 corresponding SD.

308 In the first experiment, the time interval 2003 – 2008 is considered. In this time interval, the  
 309 true signal is exactly periodic, which is an ideal case for the proposed regularization procedure.  
 310 In this experiment, the estimate of the random noise SD is very close to the true value, whereas  
 311 the bias introduced by the regularization is negligible (Table 1). The reduction of data noise is  
 312 quite substantial: the noise SD after regularization is only 44% of the original one.

313 In the second experiment, we consider the time interval 2009 – 2014, when the true signal  
 314 shows a substantial inter-annual variability. As a result, the SD of noise in the original data is  
 315 estimated less accurately (in average, it is under-estimated by about 15%: see Table 1). Further-  
 316 more, a moderate bias is introduced (about 5% of the difference between the high amplitude  $A_h$   
 317 and the low amplitude  $A_l$ ). The noise reduction due to regularization is still substantial (though  
 318 more modest than in the first experiment): the SD of noise after regularization is 73% of the  
 319 original one.

320 The third experiment covers the entire time interval 2003 – 2014. In this experiment, the be-  
 321 havior of the signal component that does not follow the annual periodicity (and, therefore, is

*Table 1*

penalized by regularization) is different over the years: it is absent in the first half of the considered time interval and relatively large in the second half. As a result, regularization introduces a bias into the  $A_h$  and  $A_l$  signal amplitudes, which is larger than in the second experiment: in average, about 14% of the difference  $A_h - A_l$  (see Table 1). On the other hand, the noise SD of the original data is estimated much more accurately than in the second experiment: in average, it is underestimated by only 2%. We see two factors that may lead to that improvement. First, it is a longer duration of the considered time-series, which makes the VCE procedure more robust (there is a less chance that a part of random noise shows a periodic behaviour and, therefore, escapes the quantification; see also the discussion at the end of Sect. 2). Second, it is the absence of a non-annual signal in the first half of the considered time interval. As a result, at least half of the considered data set offers the ideal conditions for the quantification of random noise. To separate the contribution of these two factors, we conduct another numerical experiment.

The time interval considered in the fourth experiment is the same as in the third one: 2003 – 2014. The true signal, however, experiences inter-annual variations over the entire time interval, i.e. the expression Eq. (15) describing the signal amplitude is modified as follows:

$$A = \begin{cases} A_h & \text{in Apr.2003–Mar.2004,} \\ & \text{Apr.2005–Mar.2012,} \\ & \text{Apr.2007–Mar.2008,} \\ & \text{Apr.2009–Mar.2010,} \\ & \text{Apr.2011–Mar.2012,} \\ & \text{Apr.2013–Mar.2014;} \\ A_l & \text{in Jan.2003–Mar.2003,} \\ & \text{Apr.2004–Mar.2005,} \\ & \text{Apr.2006–Mar.2007,} \\ & \text{Apr.2008–Mar.2009,} \\ & \text{Apr.2010–Mar.2011,} \\ & \text{Apr.2012–Mar.2013,} \\ & \text{Apr.2014–Oct.2014.} \end{cases} \quad (16)$$

It turns out that now, the noise SD is estimated more accurately than in the second experiment: the under-estimation is reduced from 15% to 10% (Table 1). Still, this estimate is much less accurate than the one obtained in the third experiment. This means that the accurate estimation of the noise SD in the third experiment is mostly explained by the absence of an inter-annual signal in 2003 – 2008.

Finally, we note that the level of random noise in all the numerical experiments presented so far is relatively high: 4.2 cm EWH or 42% of the difference between the high amplitude  $A_h$  and the low amplitude  $A_l$ . One may ask how the performance of the proposed procedure depends on the signal-to-noise ratio. In order to shed light on this issue, we conduct the fifth numerical experiment. It is identical to the third one, but the noise SD is reduced from 4.2 to 2.0 cm. The reduction of the noise level makes its estimation with the proposed procedure more difficult: the resulting estimate is, in average, about 20% lower than the actual noise level (see Table 1). Furthermore, the reduction of the noise level due to regularization is more modest than in any of the previous experiments: the resulting noise SD is 77% of the original one. On the other hand, the bias is lower than before: less than 3% of the difference  $A_h - A_l$ .

### 3.2.3. Regularization of mass anomalies based on real GRACE data

353 Here, we use the time-series of GRACE-based mass anomalies excluding the months when  
354 original GRACE data do not exist (that is, the results produced by interpolation are ignored).  
355 In line with the findings of the numerical study, the obtained results look satisfactory, including  
356 the time interval 2009 – 2014 (black lines in Fig. 3). A closer inspection still reveals some bias  
357 introduced by the regularization: the peak values in the year of extreme flood events (2009, 2011,  
358 and 2013) become smaller, whereas the peak value in the dry year 2010 becomes larger. This  
359 effect is, however, minor. At the same time, regularization clearly reduces random noise in the  
360 original GRACE-based estimates.

361 The statistics related to GRACE and reference mass anomaly estimates, as well as to their  
362 differences, is summarized in column 3 of Table 2. Just like in the numerical study, we also split  
363 the entire time interval under consideration into two sub-intervals: (I) 2003 – 2008 and (II) 2009  
364 – 2014. Table 2 reports the results both for the individual sub-intervals and for the total interval  
365 (I+II).

366 The rms difference between the GRACE (non-regularized) and reference mass anomalies is  
367 about 6 cm EWH, the results for sub-intervals I and II being very similar. At the first glance, this  
368 could be interpreted as an evidence of a similar accuracy of the time-series within the entire time  
369 interval under consideration. A further analysis shows, however, that this is not the case. VCE  
370 reveals that the noise SD of the un-regularized GRACE time-series changes in time substantially:  
371 it exceeds 5 cm EWH in the first sub-interval but drops more than two times in the second time  
372 interval. According to the findings of the numerical study, this difference can be partly explained  
373 by the presence of inter-annual signal variations in 2009 – 2014. In the case of real data, however,  
374 this difference is much larger. A discussion of this reduction in the estimated noise level is  
375 continued in Sect. 4.

376 The SD of "other" errors estimated with Eq. (9) also shows a temporal variability. Unlike  
377 random noise, "other" errors increase: from about 3 cm EWH to more than 5 cm EWH. We  
378 explain this by a limited performance of the empirical link given by Eq. (13), particularly when  
379 the behaviour of mass anomalies deviates from a "regular" behaviour. For instance, GRACE  
380 shows that extreme flood events, like those in 2011 and 2013, are followed by an increased mass  
381 level in the course of the next dry season, as compared to other years (Fig. 3). Most probably,  
382 this is because extreme flood events cause an accumulation of large ground water stocks, which  
383 are not fully depleted in the course of the next year. The reference data, which are based only on  
384 the extent of open water bodies, cannot observe this process.

385 Application of regularization reduced the contribution of GRACE to the differences between  
386 GRACE-based and reference mass anomalies. As a result, the dependence of the differences on  
387 time increases: the rms difference increases from 4.6 cm EWH in the first sub-interval to 5.5 cm  
388 EWH in the second one.

389 Finally, the noise SD after regularization is estimated with Eq. (11). It turns out that regular-  
390 ization reduces random noise rather substantially: to 60 – 66% of the original level. Remarkably,  
391 the reduction is similar for both sub-intervals and for the entire time interval under consideration.  
392 Furthermore, the result is consistent with the findings of the numerical study. All this increases  
393 the confidence in the results obtained.

### 394 3.3. Greenland

395 The area of Greenland exceeds 2 million km<sup>2</sup>. Most of it is covered by the Greenland Ice Sheet  
396 (GrIS) – the second largest ice sheet on Earth. GrIS contains enough ice to rise global mean sea  
397 level by 7.4 m (Vaughan et al., 2013). The GrIS mass balance is primarily a sum of two com-  
398 ponents: the Surface Mass Balance (SMB) and ice discharge. The SMB reflects the relationship

*Table*  
2

399 between the surface mass gain and mass loss processes, which are predominantly represented by  
400 snowfall and meltwater runoff, respectively (Van den Broeke et al., 2009). Seasonal GrIS mass  
401 variations are usually attributed to SMB only; the variations in ice discharge are believed to be  
402 slow (Van den Broeke et al., 2009). In our study, we rely on this assumption, in spite of recent  
403 evidences that ice discharge may contribute to the GrIS mass balance at inter-annual (Moon et al.,  
404 2012) and seasonal time scales (Moon et al., 2014). We address mass variations both over the  
405 entire Greenland and over individual drainage systems. In the latter case, the territory of Green-  
406 land is split into 5 regions: North (N), North-West (NW), North-East (NE), South-West (SW),  
407 and South-East (SE) (see Fig. 4), which is consistent with previous studies (e.g., Van den Broeke  
408 et al., 2009; Ran et al., 2017).

Fig.  
4

### 409 3.3.1. Data preparation

410 Since a mass re-distribution caused by GIA is present in the study area, the model of A et al.  
411 (2013) was used to clean GRACE monthly sets of spherical harmonic coefficients from that sig-  
412 nal. Next, each monthly solution was converted into a set of mass anomalies using the mascon  
413 approach of Ran (2017); Ran et al. (2017). This leads to a higher spatial resolution and reduced  
414 signal leakage, as compared to a direct conversion of spherical harmonic coefficients into mass  
415 anomalies. In particular, the signal leakage from Greenland to the surrounding ocean can be pre-  
416 vented, while preserving the in-land signal from damping. The lateral mass anomaly distribution  
417 within each mascon was assumed to be homogeneous. Importantly, the inversion of spherical  
418 harmonic coefficients into mass anomalies per mascon was performed without any filtering or  
419 regularization, in order to mitigate the signal leakage between the mascons. Of course, this could  
420 result in a higher noise level, as compared to a spatially-filtered or regularized solution. However,  
421 that noise can be mitigated by applying a regularization in the time domain, as is discussed be-  
422 low. The territory of Greenland was split into 28 mascons. The obtained mass anomalies (in Gt)  
423 were summed up to give the total mass anomaly per drainage system and for entire Greenland,  
424 respectively.

425 The set of reference mass anomalies was extracted from daily SMB estimates based on the  
426 Regional Atmospheric Climate Model, version 2.3 (RACMO 2.3) (Ettema et al., 2009). The  
427 original SMB estimates (in terms of EWH) were integrated over time and then averaged in space  
428 and time to produce the total mass anomaly per region per month. To restore the ice discharge  
429 signal, the differences between GRACE- and RACMO-based mass anomaly time-series were  
430 approximated by a quadratic algebraic polynomial. After that, those polynomials were added  
431 back to the corresponding RACMO-based time-series.

432 As an example, we present the obtained results for the NW drainage system and entire Green-  
433 land in Fig. 5. The unregularized GRACE-based time-series and RACMO-based time-series are  
434 shown there as blue dots and red lines, respectively. In the NW drainage system, seasonal mass  
435 variations are hardly visible. The dominant signal is a long-term negative trend, which increases  
436 in the course of time. As far as entire Greenland is concerned, an accelerated mass loss is also  
437 visible, but that long-term behaviour takes place in the presence of a clear seasonal cycle: mass  
438 accumulates in winter and diminishes in summer. Particular large mass loss is observed in the  
439 year 2012, which is notorious for an extensive summer melt over the entire GrIS (Nghiem et al.,  
440 2012).

Fig.  
5

### 441 3.3.2. Numerical study

442 We use the time-series shown in Fig. 5 to set up two numerical experiments. In each experi-  
443 ment, we reproduce the behaviour of actual mass anomalies (represented in terms of EWH). As

444 in the numerical experiments discussed in Sect.3.2.2, the "true" signals are defined analytically  
 445 and contaminated by pseudo-random zero-mean Gaussian white noise. The noise SD was de-  
 446 fined consistently with the corresponding estimate based on real data (see Sect. 3.3.3). In each  
 447 experiment, 1000 noisy time-series realizations are synthesized and analyzed.

448 In the first experiment, we reproduce mass changes in the NW drainage systems. The corre-  
 449 sponding time-series is approximated by a parabola:

$$H(t) = \frac{a(t - t_0)^2}{2} + b(t - t_0) + c. \quad (17)$$

450 The reference time  $t_0$  is in the middle of the considered time interval, i.e., the beginning of July  
 451 2008. This is needed to avoid the absorption of the trend estimate  $b$  by the acceleration term in  
 452 a linear regression analysis. The constant coefficients  $a$ ,  $b$ , and  $c$  are defined on the basis of real  
 453 GRACE-based time-series:  $a = -1.82 \text{ cm/yr}^2$ ,  $b = -16.29 \text{ cm/yr}$ , and  $c = -45.64 \text{ cm}$ . The noise  
 454 SD is set equal to 3.4 cm.

455 In this experiment, the proposed regularization procedure shows an excellent performance  
 456 (Table 3). The SD of actual noise is only 3% below the true value, whereas the noise SD after  
 457 regularization is reduced to the level of 38% of the original one. It is also remarkable that  
 458 the bias introduced by regularization is negligible in both the trend estimate and the estimated  
 459 acceleration. This is in spite of the fact that the acceleration term does not belong to the class  
 460 of functions exempt from penalization. We explain this by the fact that the "local" impact of the  
 461 acceleration term in each particular set of neighbouring months is minor, so that the simulated  
 462 function is still close to the ideal one.

463 In the second numerical experiment, we mimic the behaviour of mass anomalies of entire  
 464 Greenland. To that end, we extend the signal of Eq. (17) with an annual term:

$$H(t) = \frac{a(t - t_0)^2}{2} + b(t - t_0) + c + A [1 - \cos(2\pi(t - t_0) - (\varphi - \varphi_0))]. \quad (18)$$

465 In line with the real mass anomaly time-series, the phase  $\varphi$  is set equal to 1.8326, which implies  
 466 that the seasonal mass accumulation would reach a maximum in the middle of April if a long-  
 467 term-trend were absent. The additional phase shift  $\varphi_0$  is included to reflect the fact that the  
 468 reference time  $t_0$  does not coincide with the beginning of a year:  $\varphi_0 = 2\pi(t_0 - \text{int}[t_0])$ . The  
 469 amplitude  $A$  of the annual signal is set equal to a certain "normal" level  $A_n$  in almost all the  
 470 years. The only exception is the year 2012, when it is defined differently. More specifically:

$$A = \begin{cases} A_n & \text{in Jan.2003–Mar.2012,} \\ & \text{Apr.2013–Dec.2013;} \\ A_{2012} & \text{in Apr.2012–Mar.2013.} \end{cases} \quad (19)$$

471 All the constant coefficients are estimates by a linear regression from the real GRACE-based  
 472 time-series shown in the bottom plot of Fig. 5:  $a = -1.13 \text{ cm/yr}^2$ ,  $b = -13.21 \text{ cm/yr}$ ,  $c = -23.40$   
 473  $\text{cm}$ ,  $A_n = -8.57 \text{ cm}$ , and  $A_{2012} = -14.30 \text{ cm}$ . The SD of the noise added to the synthetic signal  
 474 is set equal to 1 cm, which makes the experiment set-up consistent with real data processing (see  
 475 Sect. 3.3.3). Such a noise level is rather low. For instance, it is only 17.5% if the difference  
 476 between the normal annual amplitude  $A_n$  and the annual amplitude in 2012  $A_{2012}$ . In that sense,  
 477 this set-up is close to the set-up of the fifth (low-noise) numerical experiment considered in  
 478 Sect. 3.2.2.

*Table*  
3

479 The results obtained after applying regularization are, in general, better than those of the fifth  
480 experiment in Sect. 3.2.2. The original noise SD is underestimated by only 10%, whereas the  
481 noise SD after the regularization is reduced to the level of 69%, as compared to the original one  
482 (Table 3). Furthermore, the biases introduced into the linear trend, acceleration, and the normal  
483 annual signal amplitude are negligible. For instance, the bias in the annual signal amplitude does  
484 not exceed, in average, 1% of the difference  $A_n - A_{2012}$ . A good performance of the regularization  
485 procedure in this experiment is explained by the fact that the signal is close to the ideal one: the  
486 annual signal stays most of the time at a constant level, whereas the impact of the acceleration  
487 term apparently remains minor. On the other hand, it is worth noticing that the bias introduced  
488 into the annual signal in 2012 reaches 8% of the difference  $A_n - A_{2012}$ . Though we still consider  
489 such a bias as minor, it is definitely larger than those observed in the fifth (low-noise) numerical  
490 experiment considered in Sect. 3.2.2. This is a clear indication that "unusual" signals (e.g., a  
491 larger mass loss in a particular summer than in average) are subject to larger distortions. This is  
492 an expected result, since the regularization tends to make such signals similar to the signals in  
493 neighboring years.

### 494 3.3.3. Regularization of mass anomalies based on real GRACE data

495 Finally, we apply the proposed regularization procedure to mass anomalies extracted from real  
496 GRACE data. As in Sect. 3.2.3, we split the considered time interval into two sub-intervals in order  
497 to make the analysis more representative and to facilitate a consistency check of the results:  
498 (I) 2003 – 2007 and (II) 2008 – 2013. The results both for the individual sub-intervals (I, II)  
499 and for the total interval (I+II) are analyzed. In the latter case, two variants of the recovered ice  
500 discharge signals are considered. In both variants, those signals are approximated by quadratic  
501 polynomials, as explained above. The only difference is that in the first variant, a single poly-  
502 nomial is computed for the entire time interval 2003 – 2013. We consider it as the "primary"  
503 variant; it is used, in particular, to compute the reference mass anomalies shown in Fig. 5. In  
504 the alternative variant, on the other hand, the best-fitting quadratic polynomials are found for the  
505 sub-intervals 2003 – 2007 and 2008 – 2013 independently. Thus, the reference mass anomaly  
506 time-series in the alternative variant is nothing but the result of merging the reference time-series  
507 for sub-intervals (I) and (II). A comparison of the results of these two variants allows some con-  
508 clusions to be drawn regarding their robustness with respect to long-term uncertainties associated  
509 with ice discharge.

510 Regularized GRACE time-series for the NW drainage system and entire Greenland are shown  
511 in Fig. 5 as black lines. In columns 4 – 7 of Table 2, we present further information about the  
512 outcome of the regularization for the drainage systems N, NW, NE, and the combined region  
513 "SW&SE". The last column reports the obtained results for entire Greenland.

514 The estimated SD of random noise in GRACE-based mass anomalies for the northern drainage  
515 systems (N, NW, and NE) is quite similar: 3 – 4 cm EWH. This is in spite of the fact that the  
516 area of the drainage system N is more than two times smaller than that of the other regions.  
517 Most probably, this can be explained by the northern location of the drainage system N, so that  
518 its small size is compensated by a high density of GRACE ground tracks. The region SW&SE  
519 shows a relatively low noise level: 1 – 2.5 cm. We explain this by the shape of that region: unlike  
520 the regions NW and NE, it is not extended in the meridional direction, which implies a higher  
521 accuracy of GRACE-based mass anomaly estimates. The lowest noise level (0.8 – 0.9 cm) is  
522 observed for entire Greenland, which is definitely due to the large size of this region. The noise  
523 levels estimated for the entire time interval (I+II) and the sub-intervals (I) and (II) show a good  
524 agreement. The only exception is the SW&SE region, where a substantial reduction in the noise

525 level is observed. As similar reduction was observed earlier in the analysis of mass anomalies in  
526 the Tonlé Sap basin (Sect. 3.2.3). This issue is further discussed in Sect. 4.

527 The rms differences between the non-regularized GRACE-based mass anomalies and the refer-  
528 ence ones show less variability than the random errors in GRACE estimates discussed above:  
529 they stay at the level of 3 – 5 cm EWH, except for the southern region SW&SE, where the RMS  
530 difference reach 5 – 7 cm. In two cases (N and entire Greenland), the rms differences computed  
531 over the entire time interval (I+II) are larger than the errors computed for both sub-intervals I and  
532 II, if the first variant of ice discharge correction is exploited. When the alternative variant of ice  
533 discharge correction is applied (i.e., when the corresponding quadratic polynomials are estimated  
534 for the two sub-intervals individually), the rms differences obtained for the entire interval I+II  
535 are always between the rms differences obtained for the sub-intervals I and II, as expected.

536 By subtracting the contribution of random noise from the obtained rms differences in line with  
537 Eq. (9), we estimate the SD of "other" noise. "Other" noise for the entire GrIS likely reflects  
538 errors in the SMB estimates produced by the RACMO model, as well as the processes not related  
539 to the ice sheet surface, such as the meltwater retention inside the ice layer and the residual ice  
540 discharge signal. The contribution of a multi-year time-scale to "other" noise can be assessed by  
541 a comparison of the estimates obtained with the two variants of ice discharge correction in 2003  
542 – 2013: 3.1 cm EWH for the first variant versus 2.6 cm EWH for the alternative one. Thus, the  
543 contribution of a multi-year time-scale is at the level of only 15%; the rest of "other noise" is  
544 likely associated with a relatively short time scale (2 – 3 years or less). "Other" noise estimates  
545 for individual drainage systems show a substantial variability. Those estimates, however, must  
546 be interpreted with some caution. The fact is, all of them are obtained by subtracting two close  
547 numbers. Thus, the observed variability may reflect inaccuracy of the obtained error estimates.  
548 An extreme example is the drainage system NE in time interval II. "Other" noise cannot be  
549 quantified in that case at all, since the rms difference between GRACE (original) and reference  
550 time-series is smaller than the estimated error SD of GRACE-based mass anomalies. However,  
551 in spite of these uncertainties, the "other" errors show a consistent behaviour. They stay at a mid  
552 level (2.5 – 3 cm EWH) for the drainage systems N and NE, as well as entire Greenland; they  
553 reduce to ~2 cm for the drainage system NW, and increase to 5 – 6 cm for the region SW&SE.  
554 This behavior shows an excellent correlation with the mean amplitude of annual signals in the  
555 considered regions: 7 – 9 cm EWH in the regions with the mid level of "other errors", ~4 cm in  
556 the drainage system NW with a low error level, and ~17 cm in the region SW&SE, where the  
557 level of "other" errors is relatively high (see the last row in Table 2). We believe, therefore, that  
558 the observed errors reveal deficiencies associated with modelling the summer ice melting (the  
559 primary cause of seasonal mass variability).

560 The rms differences between the regularized GRACE-based mass anomalies and the refer-  
561 ence mass anomalies are also computed. Then, Eq. (11) allows us to quantify random noise in  
562 GRACE-based mass anomalies after regularization. It turns out that the regularization typically  
563 reduces the random noise SD to 40 – 60% of the original value. This outcome is an agreement  
564 with the results of the numerical studies. In a few cases, an even more substantial reduction of  
565 random noise seems to be achieved. For instance, the SD of random noise for entire Greenland is  
566 estimated for some time intervals as only ~ 20% of the original level. However, these estimates  
567 are likely caused by an underestimation of the original noise SD due to its low level, as it is  
568 discussed in Sect. 3.3.2. If, for instance, the true noise SD is originally equal to 1 cm (i.e., if this  
569 underestimation is 10%, which is not impossible according to the conducted numerical study),  
570 the estimate of noise SD after the regularization should be increased from ~20% to ~50% of the  
571 original level, which is consistent with the other results.



572 **4. Discussion and conclusions**

573 In this study, we developed a statistically-optimal regularization technique that allows one to  
574 smooth and interpolate a mass anomaly time-series based on satellite gravimetry data, as well as  
575 to estimate the level of random noise in it. The proposed regularization functional minimizes the  
576 MYDD (month-to-month year-to-year double differences) of mass anomalies. As we showed  
577 theoretically, this functional does not introduce any bias into two types of signals, which com-  
578 monly occur in the Earth’s system: arbitrary signals with an annual periodicity and long-term  
579 linear trends.

580 We conducted a number of numerical simulations, in which actual signals and errors in  
581 GRACE-based mass anomaly time-series were reproduced. In all the considered experiments,  
582 the bias introduced into the actual signals was minor and did not exceed, in average, 14%. The  
583 largest bias was observed in the cases when the level of random noise was high and when the  
584 signal in a given year was substantially different from the signal in the neighbouring years. At  
585 the same time, the developed regularization scheme effectively reduces random noise. In the  
586 considered numerical experiments, for instance, the noise SD was typically reduced to 40 – 70 %  
587 of the original level. The factors that facilitate an efficient noise reduction are high level of noise  
588 in the original time-series and minimal inter-annual variability of signals.

589 Another important outcome of the proposed regularization methodology is the assessment of  
590 random noise in mass anomaly time-series; such estimates are provided by the VCE procedure,  
591 which is a part of the regularization technique. Conducted numerical experiments showed that  
592 the obtained estimates of noise SD are close to the true values or slightly less. However, this  
593 under-estimation did not exceed 22% in the conducted experiments. The factors that facilitate an  
594 accurate estimation of noise SD are a long duration of the analyzed time-series and a relatively  
595 high noise level, as compared to the penalized signal (the signal that shows neither an annual  
596 periodicity nor a long-term linear behaviour).

597 The proposed technique can be considered as a handy tool to quantify the accuracy of various  
598 mass anomaly time-series in general. As such, it can be applied, for instance, to estimate the  
599 performance of a particular methodology designed for SG data processing, to compare the accu-  
600 racy of alternative mass anomaly estimates, to demonstrate and compare the impact of various  
601 supporting data used in SG data processing, etc. Examples of such applications can already be  
602 found in (Sun et al., 2017; Ran et al., 2017; Guo et al., 2018).

603 In our study, we applied the developed procedure to analyze GRACE-based time-series of  
604 mass anomalies in the Tonlé Sap basin in Cambodia and Greenland. In this way, we showed how  
605 some more findings can be extracted from the estimates of random noise SDs.

606 First, the noise SD estimates allow for a separation of the contribution of random noise and  
607 ”other” errors when GRACE mass anomalies are compared with mass anomalies derived from  
608 other data and/or models. The ”other” errors comprise systematic errors in GRACE data (e.g.,  
609 due to signal leakage) and errors in the reference data. In the study of Greenland, for instance,  
610 we found that the SD of ”other” errors stays at the level of 2 – 6 cm EWH and strongly correlates  
611 with the amplitude of the annual signal. From this, we concluded that the revealed errors are  
612 likely associated with modelling of summer ice melting. The most probable cause of these errors  
613 is meltwater accumulation and run-off. On the one hand, the signal related to meltwater may  
614 be quite significant, since it takes meltwater, in average, about two weeks to leave GrIS (van  
615 Angelen et al., 2014). On the other hand, this signal is not fully taken into account by the  
616 RACMO2.3 model: it implies that the run-off process is instantaneous. A further analysis of this  
617 signal in GRACE-based mass anomalies can be found in (Ran, 2017). Speaking more generally,

618 the conducted study opens the door for a more accurate quantification of noise in reference  
619 mass anomalies when the latter are compared with those based on SG data. This concerns any  
620 application area of SG, such as the study of ice sheets, hydrology, oceanography, and others.

621 Second, the quantification of "other" errors allowed us to estimate the SD of random noise  
622 in GRACE-based mass anomalies after regularization. It turned out that regularization typically  
623 reduces noise to 40 – 66% of the original level (i.e., about 1.5 – 2 times). This is in a good  
624 agreement with the results of numerical experiments.

625 A division of the considered time interval into two allowed us to check the internal consistency  
626 of the noise SD estimates obtained for a given region. The estimates obtained for entire Green-  
627 land and for its northern regions turned out to be in a reasonable agreement (the differences were  
628 within 20%, cf. Table 2). However, the estimates obtained for the combined SW&SE region of  
629 Greenland and for the Tonlé Sap basin turned out to be quite different: they show that noise in  
630 2009 – 2014 or 2008 – 2013 is noticeably (more than 2 times) lower than in 2003 – 2008 or 2003  
631 – 2008. To shed more light on this issue, we considered, among others, the SW and SE regions  
632 of Greenland separately. For the SE region (the area is 398,000 km<sup>2</sup>), the obtained estimates of  
633 noise SD were 9.2 cm, 8.3 cm, and 9.3 cm for the time intervals (I), (II), and (I+II), respectively.  
634 For the small SW region (the area is 214,000 km<sup>2</sup>), the corresponding estimates were 18.0 cm,  
635 14.3 cm, and 16.6 cm, respectively. Thus, even though some noise reduction is observed, it stays  
636 with the 20-% limit. From this and other evidences, we conclude that a very large reduction in  
637 the noise level observed for the the combined SW&SE region of Greenland and the Tonlé Sap  
638 Basin is likely an evidence of an insufficient robustness of the proposed technique when short  
639 ( $\leq 6$  years) are concerned. Thus, it is advised to consider results obtained for such time intervals  
640 with a caution.

641 Another caveat concerns the temporal behaviour of the signals in the time-series under consid-  
642 eration. The proposed regularization functional minimizes a variant of signal double-differences,  
643 which implies that the signal must change smoothly in the time domain. Obviously, a signal that  
644 rapidly change from month to month may be over-regularized, whereas the level of random noise  
645 may be overestimated. In the extreme case, when the stochastic behavior of signal is not distin-  
646 guishable from that of white noise, the separation of the time-series into signal and noise is,  
647 naturally, impossible.

648 It is also worth mentioning that the proposed regularization technique may be used to fill in  
649 gaps in mass anomaly time-series. Since the year 2011, the GRACE data time-series suffers from  
650 multiple gaps, which are frequently filled in by means of interpolation. We found, however, that  
651 the obtained results are not necessarily better than those produced with a simple interpolation  
652 scheme (e.g., cubic splines). A typical example in the estimation of mass anomalies for entire  
653 Greenland in August-September 2013, when no GRACE data were available (see the inset in the  
654 bottom plot in Fig. 5). Unfortunately, this is exactly the time interval when rapid mass loss due  
655 to summer melting occurred. By chance, a particularly large summer mass loss took place one  
656 year earlier – in 2012. Then, the proposed regularization technique uses that mass change pattern  
657 to fill in the gap in 2013. However, the RACMO model shows that mass loss in summer 2013  
658 was minor. Then, a cubic interpolation, which ignores the behaviour of mass anomalies in other  
659 years, apparently yields better results. This illustrates a conceptual problem associated with data  
660 gaps. The presence of such gaps means some loss of information in the collected data. If the  
661 behavior of a target process in that time interval is "non-typical" in whatever sense, a reliable  
662 recovery of such behavior becomes conceptually impossible: no mathematical technique can  
663 replace a collection of field measurements.

664 The proposed regularization technique has space for further improvements. For instance, we

665 assumed so far that noise in the mass anomaly time-series is stationary. In reality, this may not be  
666 the case because the accuracy of GRACE-based mass anomalies may change in time. There are  
667 several reasons for that. First, the altitude of GRACE satellites rapidly decreased after year 2011  
668 ([http://www2.csr.utexas.edu/grace/operations/orbit\\_evolution/semiB.png](http://www2.csr.utexas.edu/grace/operations/orbit_evolution/semiB.png)), which must have had  
669 a positive impact on the accuracy and spatial resolution of the mass anomaly estimates; Second,  
670 the attitude control of GRACE satellites was relatively poor at the beginning of the mission,  
671 which may reduce the quality of the resulting estimates (Inácio et al., 2015). Third, GRACE  
672 orbits enter the periods of a short repeat cycle from time to time, which also deteriorates the  
673 quality of the resulting estimates (Wagner et al., 2006). In addition, switching from GRACE  
674 to GFO data in the future will also likely change the accuracy of mass anomaly estimates due  
675 to a higher accuracy of the onboard instruments. Finally, it is not unlikely that gaps in GRACE  
676 time-series, as well as the gap between the GRACE and GFO missions will be somewhat filled in  
677 by the usage of GNSS data from various other satellite missions. Though the accuracy of GNSS  
678 data is relatively low, still they definitely can capture some mass transport signals (Ditmar et al.,  
679 2009; Gunter et al., 2011; Weigelt et al., 2013; Guo et al., 2017). Thus, the picture of future  
680 mass anomaly time-series will not be "black-and-white" (mass anomaly is either provided or not  
681 provided). Instead, the time-series will likely be continuous, but of rather heterogeneous quality:  
682 more accurate in the months when GRACE or/and GFO data are available and much less accurate  
683 otherwise. As it is shown in Appendix A, the proposed regularization technique can be easily  
684 adjusted to such a situation. Then, this technique may become a tool to homogenize future mass  
685 anomaly time-series by exploiting all available information in the statistically optimal sense (i.e.,  
686 taking into account the accuracy of each particular monthly estimate).

687 Another direction of further developments is the optimal estimation of the regularization pa-  
688 rameter, taking into account the dependence of the actual mass anomaly signal on time. Cur-  
689 rently, the adopted regularization procedure makes use of time-invariant soft constrains (cf.  
690 Eqs. (A.3-A.5) in Appendix A). In reality, the expected deviations of the actual signal from a  
691 regular behaviour may show a variability in time (an example is the mass anomalies in the Tonlé  
692 Sap basin in 2009 – 2014). By taking this variability into account in the construction of the  
693 regularization functional, one may further improve the quality of the regularization.

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700 The SLR-based time-series of  $C_{2,0}$  coefficients was downloaded from webpage  
701 [ftp://ftp.csr.utexas.edu/pub/slr/geocenter/GCN\\_RL05.txt](ftp://ftp.csr.utexas.edu/pub/slr/geocenter/GCN_RL05.txt). The MODIS-based mass anoma-  
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Table 1: Results of the numerical study that reproduces mass variations over the Tonlé Sap Basin. The following information is provided for each of the experiments: (1) Considered time interval; (2) Reference to the analytic expression used to define the "true" signal; (3) Actual SD of pseudo-random noise added to the synthetic signal; (4) Noise SD estimated with the VCE technique; (5) Actual noise SD after the regularization; (6) Same as the previous item but in percentages of the original noise SD; and (7-9) Bias introduced by the regularization into the signal amplitudes. The shown error bars reflect the variability of the obtained estimates after the consideration of 1000 different noise realizations. Units are cm EWH.

Experiment	1	2	3	4	5
Time interval	2003–2008	2009–2014	2003–2014	2003–2014	2003–2014
True signal	Eq. (15)	Eq. (15)	Eq. (15)	Eq. (16)	Eq. (15)
Random noise SD before regularization (true)	4.20	4.20	4.20	4.20	2.00
Random noise SD before regularization (VCE-based estimation)	4.16±0.19	3.57±0.54	4.10±0.38	3.76±0.69	1.57±0.20
Noise SD after regularization	1.83±0.32	3.07±0.30	2.70±0.30	2.90±0.35	1.53±0.10
Noise after regularization (% of original noise)	44±8	73±7	64±7	69±8	77±5
Bias in the middle-amplitude signal ( $A_m$ )	0.01±0.70	N/A	0.08±0.58	N/A	0.01±0.27
Bias in the high-amplitude signal ( $A_h$ )	N/A	-0.53±0.88	-1.44±0.95	-0.68±0.63	-0.29±0.37
Bias in the low-amplitude signal ( $A_l$ )	N/A	0.49±0.87	1.36±0.88	0.69±0.63	0.26±0.35

Table 2: Results of two case studies (over the Tonlé Sap Basin and over Greenland) based on real data. In the second case, the results are shown both for individual drainage systems (columns "N" – "SE&SW") and for entire Greenland. The following estimates are provided for each of the considered regions: (1) SD of random noise in the original GRACE-based mass anomalies (i.e., before regularization), estimated with the VCE technique; (2) the rms difference between the GRACE-based (original) and reference mass anomalies; (3) SD of "other" errors (consisting of systematic errors in GRACE-based mass anomalies and errors in reference data); (4) the rms difference between the GRACE (regularized) and reference mass anomalies; (5) SD of random noise in GRACE-based mass anomalies after regularization; (6) same as the previous item but in percentages of the original random noise SD. The results are shown both for the individual sub-intervals (I and II) and the entire time interval (I+II). In the Tonlé Sap Basin case study, the sub-intervals are: (I) 01.2003 – 12.2008 and (II) 01.2009 – 10.2014. In the study of Greenland, the sub-intervals are: (I) 01.2003 – 12.2007 and (II) 01.2008 – 12.2013. Bold font is used for the estimates based on the time-series for the entire time interval (I+II) when the primary variant of ice discharge correction is exploited (i.e., when the RACMO-based time-series are corrected for ice discharge in the entire time interval at once). Italic font is used for the estimates based on the time-series for the entire time interval (I+II) and the alternative variant of ice discharge correction (i.e., when the RACMO-based time-series are corrected for ice discharge in the sub-intervals (I) and (II) individually). The last line shows the amplitude of annual variations in 2003 – 2013 in different Greenland regions estimated on the basis of the original GRACE data. Units are cm EWH.

	Time interval	Tonlé Sap basin	Greenland regions				
			N	NW	NE	SW & SE	Entire Greenland
Area (km <sup>2</sup> × 10 <sup>3</sup> ) <sup>a</sup>		82	256	686	601	612	2154
1. Random noise in GRACE-based mass anomalies before regularization (VCE-based)	I+II	<b>4.16</b>	<b>3.61</b>	<b>3.34</b>	<b>3.42</b>	<b>1.89</b>	<b>0.880</b>
	I	5.42	3.44	3.60	3.81	2.48	0.802
	II	2.57	3.42	2.85	3.89	0.90	0.869
2. RMS difference from reference data before regularization	I+II	<b>6.05</b>	<b>4.69</b>	<b>3.93</b>	<b>4.52</b>	<b>6.62</b>	<b>3.215</b>
	I+II <sup>b</sup>	-	<i>4.52</i>	<i>3.75</i>	<i>4.40</i>	5.22	2.785
	I	6.25	4.40	4.02	5.12	5.30	2.834
	II	5.81	4.62	3.49	3.64	5.15	2.741
3. "Other" noise, including noise in reference data	I+II	<b>4.40</b>	<b>2.99</b>	<b>2.08</b>	<b>2.95</b>	<b>6.34</b>	<b>3.093</b>
	I+II <sup>b</sup>	-	<i>2.72</i>	<i>1.72</i>	<i>2.76</i>	<i>4.86</i>	<i>2.642</i>
	I	3.12	2.74	1.78	3.42	4.68	2.718
	II	5.22	3.11	2.02	-	5.07	2.599
4. RMS difference from reference data after regularization	I+II	<b>5.06</b>	<b>3.55</b>	<b>2.71</b>	<b>3.32</b>	<b>6.38</b>	<b>3.096</b>
	I+II <sup>b</sup>	-	<i>3.35</i>	<i>2.49</i>	<i>3.14</i>	<i>4.92</i>	<i>2.650</i>
	I	4.60	3.44	2.57	3.89	4.85	2.751
	II	5.48	3.56	2.64	2.91	5.07	2.607
5. Random noise in GRACE-based mass anomalies after regularization	I+II	<b>2.50</b>	<b>1.91</b>	<b>1.74</b>	<b>1.51</b>	<b>0.72</b>	<b>0.151</b>
	I+II <sup>b</sup>	-	<i>1.97</i>	<i>1.80</i>	<i>1.49</i>	<i>0.77</i>	<i>0.199</i>
	I	3.39	2.08	1.84	1.84	1.28	0.427
	II	1.68	1.74	1.70	-	0.29	0.203
6. Random noise in GRACE-base mass anomalies after regularization (% of original noise)	I+II	<b>60%</b>	<b>53%</b>	<b>52%</b>	<b>44%</b>	<b>38%</b>	<b>17%</b>
	I+II <sup>b</sup>	-	<i>54%</i>	<i>54%</i>	<i>44%</i>	<i>41%</i>	<i>23%</i>
	I	62%	60%	51%	48%	52%	53%
	II	66%	51%	60%	-	32%	23%
Amplitude of annual mass variations			7.6	3.8	7.5	17.4	9.0

<sup>a</sup> – Shown areas reflect the geometry of regions used in GRACE data inversion (see Fig. 4). Those areas may somewhat deviate from the actual area of Greenland or the areas of individual drainage systems.

<sup>b</sup> - The alternative variant of ice discharge correction is applied.

Table 3: Results of the numerical study that reproduces mass variations in the NW drainage system of GrIS and in entire Greenland. Information provided is similar to that reported in Table 1. Units are cm EWH (except for the linear trend and acceleration).

Region considered	NW	Entire Greenland
True signal	Eq. (17)	Eq. (18)
Random noise SD before regularization (true)	3.4	1.0
Random noise SD before regularization (VCE-based estimation)	$3.30 \pm 0.10$	$0.90 \pm 0.08$
Noise SD after regularization	$1.29 \pm 0.18$	$0.69 \pm 0.05$
Noise after regularization (% of original noise)	38	69
Bias in the linear trend (cm/yr)	$0.00 \pm 0.09$	$-0.02 \pm 0.03$
Bias in the acceleration signal (cm/yr <sup>2</sup> )	$0.01 \pm 0.07$	$-0.01 \pm 0.02$
Bias in the "normal" annual signal ( $A_n$ )	N/A	$-0.04 \pm 0.13$
Bias in the annual signal in 2012 ( $A_{2012}$ )	N/A	$0.44 \pm 0.29$



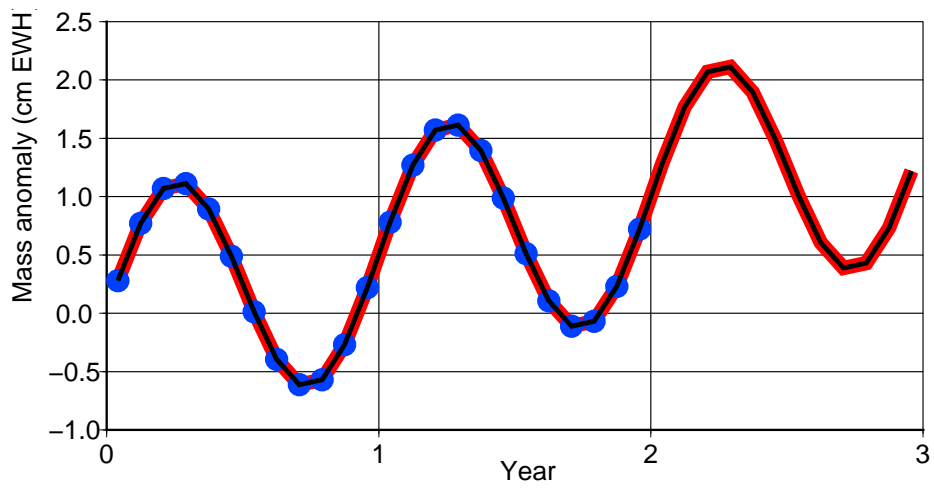


Figure 1: Simulated noiseless observations of mass anomalies (blue dots) and the time-series recovered on their basis with the proposed regularization technique (black line). The "true" time-series is shown in red.

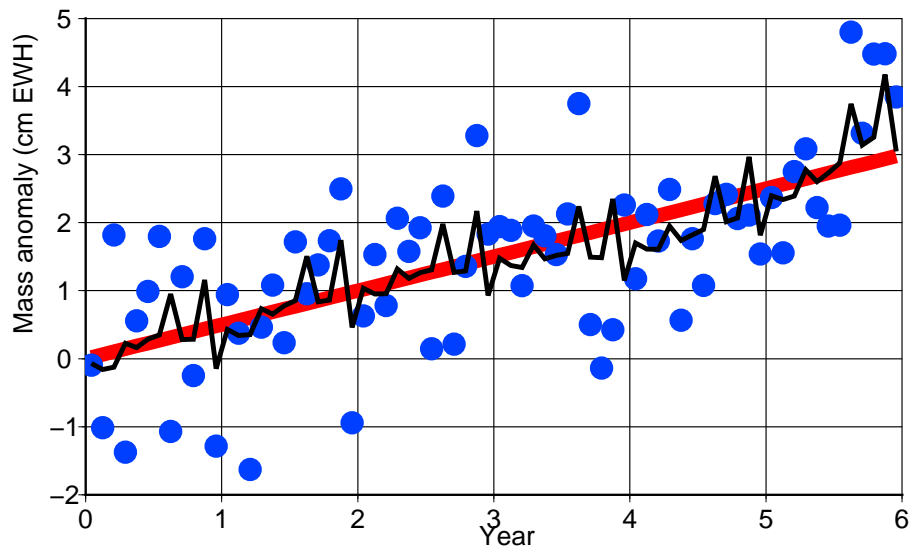
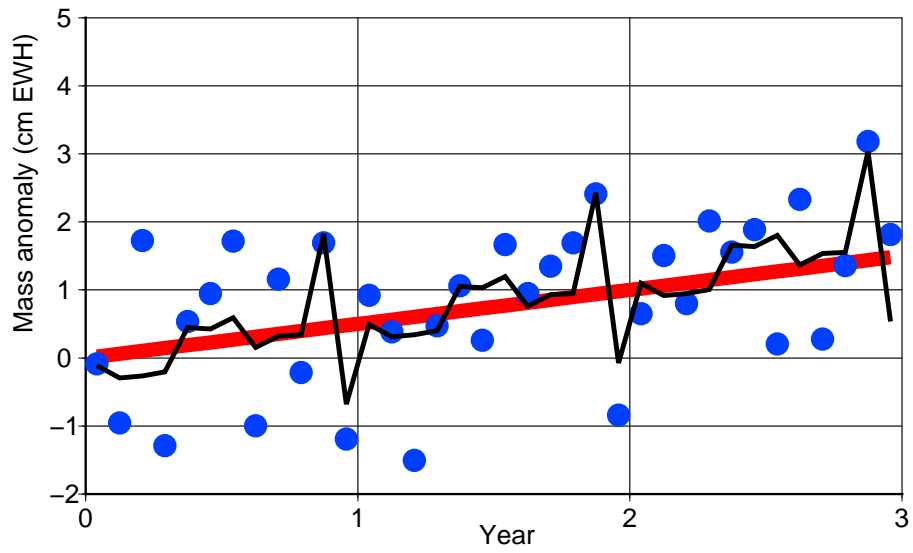


Figure 2: Simulated noisy observations of mass anomalies (blue dots) and the regularized time-series computed on their basis with the proposed technique (black line). The "true" time-series is shown in red. The considered time intervals are 3 year (top plot) and 6 years (bottom plot).

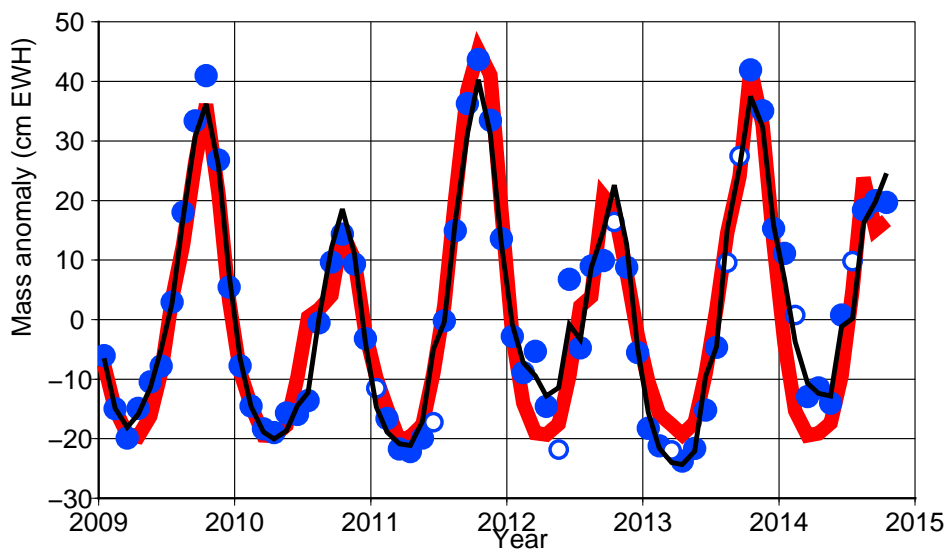
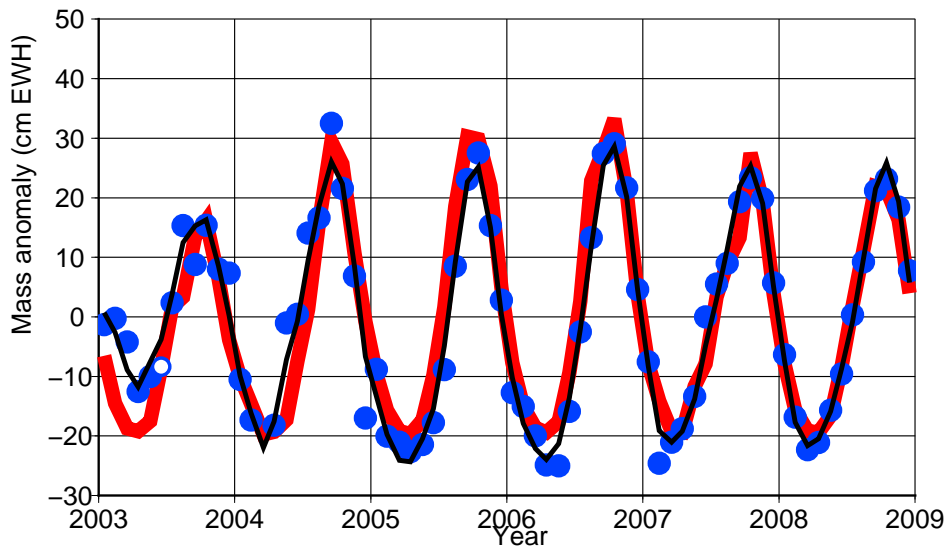


Figure 3: Mass anomalies at the Tonlé Sap basin: directly extracted from GRACE data without a regularization (blue circles) and obtained on their basis by cubic interpolation (open circles), as well as those obtained after applying the proposed regularization procedure (black line). Reference mass anomaly estimates based on MODIS data are shown in red. To make the illustration better readable, the entire time interval under consideration is split into two parts: 2003 – 2008 (top plot) and 2009 – 2014 (bottom plot).

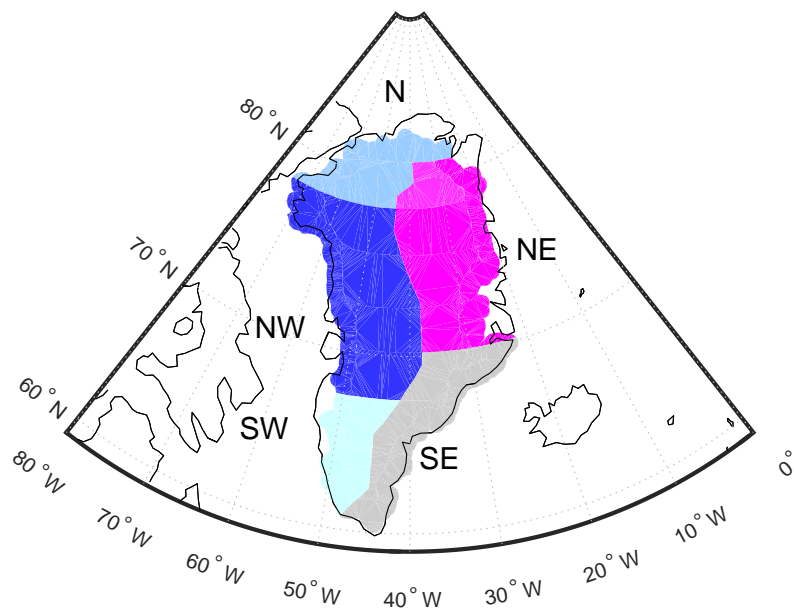


Figure 4: Adopted division of the territory of Greenland into individual drainage systems.

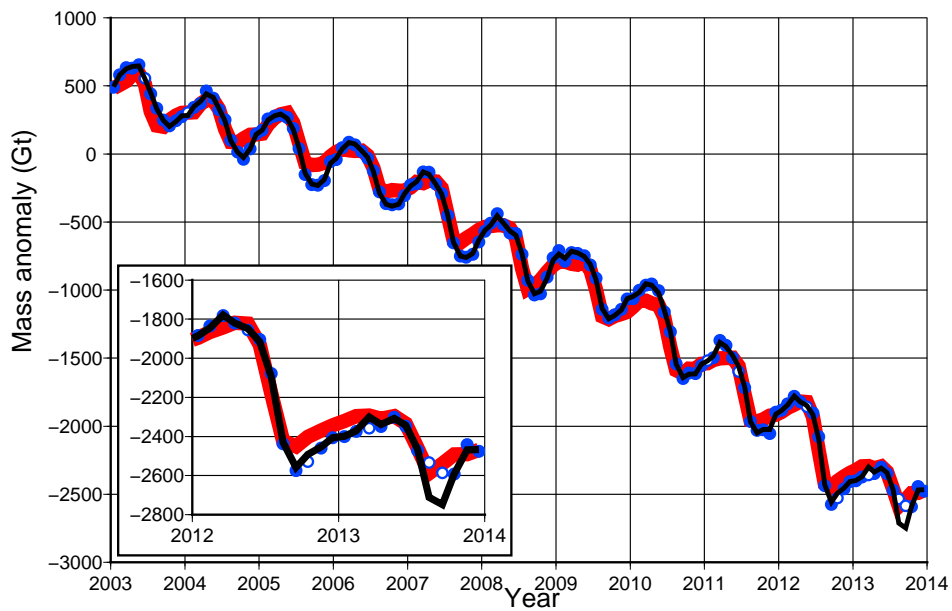
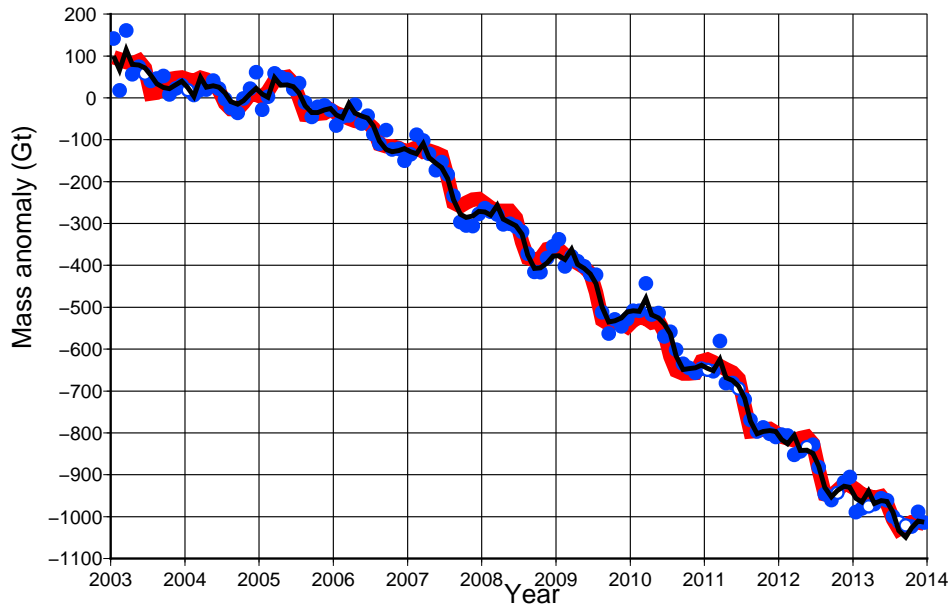


Figure 5: Mass anomalies in the NW drainage system (top plot) and entire Greenland (bottom plot). The plots show mass anomalies directly extracted from GRACE data without a regularization (blue circles) and obtained on their basis by cubic interpolation (open circles); as well as those obtained after applying the proposed regularization procedure (black line). Reference mass anomalies estimated on the basis of RACMO2.3 model are shown in red. The inset in the bottom plot zooms in on the time interval 2012 – 2013.

807 **Appendix A. Regularization of the discrete time-series and the optimal choice of the regu-**  
808 **larization parameter using Variance Component Estimation**

809 The description of Variance Component Estimation (VCE) is adapted from (Koch and Kusche,  
810 2002). We present VCE in the context of an arbitrary linear functional model, i.e. the model that  
811 links the data vector  $\mathbf{d}$  and the vector of unknown parameters  $\mathbf{x}$  with linear observation equations

$$\mathbf{Ax} = \mathbf{d}, \quad (\text{A.1})$$

812 where  $\mathbf{A}$  is an arbitrary matrix (frequently called "design matrix"). In our special case, matrix  $\mathbf{A}$   
813 is unit.

814 The data can be contaminated by arbitrary correlated Gaussian noise. The noise covariance  
815 matrix  $\mathbf{C}$  is assumed to be known up to a scaling factor, i.e., it can be represented as

$$\mathbf{C} = \sigma_d^2 \mathbf{P}^{-1}, \quad (\text{A.2})$$

816 where  $\mathbf{P}$  is a known matrix (called "weight matrix") and  $\sigma_d^2$  is an unknown constant.

817 The system Eq. (A.1) is to be solved under soft constraints, which are introduced by means of  
818 an additional set of linear equations

$$\mathbf{Dx} = \mathbf{0}, \quad (\text{A.3})$$

819 where  $\mathbf{D}$  is an arbitrary matrix. For instance, setting the matrix  $\mathbf{D}$  to the unit one reduces the  
820 soft constraints to the classical zero-order Tikhonov regularization. In our special case, matrix  
821  $\mathbf{D}$  is defined such that the expression  $\mathbf{Dx}$  is the finite-difference analog of the double-difference  
822 expression  $h'_{k+1}(t) - h'_k(t)$ , cf. Eq. (5). This means that all the non-zero elements of matrix  $\mathbf{D}$   
823 are equal, up to a constant scaling factor, to 1 or  $-1$ . To prevent a jump at the beginning of each  
824 year, we apply the similar constraints also onto "December-January" pairs of months, i.e. by  
825 definition  $x_{(k+1,1)} = x_{(k,13)}$ , where the first lower index stands for the year and the second one for  
826 the calendar month of a year.

827 The set of soft constraints given by Eq. (A.3) can be interpreted as a system of additional  
828 observation equations with zero "observations" in the right-hand side. In what follows, those  
829 "observations" are called pseudo-observations.

830 Assuming that the pseudo-observations are contaminated by white noise, one can find the  
831 least-squares solution  $\hat{\mathbf{x}}$  of the combined system composed of linear equations (A.1) and (A.3)  
832 by minimizing the penalty function

$$\Phi[\mathbf{x}] = \frac{1}{\sigma_d^2} (\mathbf{d} - \mathbf{Ax})^T \mathbf{P} (\mathbf{d} - \mathbf{Ax}) + \frac{1}{\sigma_x^2} \mathbf{x}^T \mathbf{R} \mathbf{x}, \quad (\text{A.4})$$

833 where  $\sigma_x^2$  is the error variance of the pseudo-observations and  $\mathbf{R}$  is the regularization matrix,  
834 which is defined as

$$\mathbf{R} = \mathbf{D}^T \mathbf{D}. \quad (\text{A.5})$$

835 The multiplication of the penalty function (A.4) with  $\sigma_d^2$  yields the equivalent penalty function

$$\tilde{\Phi}[\mathbf{x}] = (\mathbf{d} - \mathbf{Ax})^T \mathbf{P} (\mathbf{d} - \mathbf{Ax}) + \alpha \mathbf{x}^T \mathbf{R} \mathbf{x}, \quad (\text{A.6})$$

836 where  $\alpha$  is the regularization parameter defined as

$$\alpha = \frac{\sigma_d^2}{\sigma_x^2}. \quad (\text{A.7})$$

837

838

839 The penalty functional (1) introduced at the beginning of the article is a continuous analog of the  
840 penalty function (A.6), provided that matrix  $\mathbf{P}$  is unit.

841 Obviously, the explicit expression for the vector  $\hat{\mathbf{x}}$  minimizing the penalty function (A.4) (and,  
842 therefore, the penalty function given by Eq. (A.6)) is:

$$\hat{\mathbf{x}} = \frac{1}{\sigma_d^2} \mathbf{N}^{-1} \mathbf{A}^T \mathbf{P} \mathbf{d}, \quad (\text{A.8})$$

843 where  $\mathbf{N}$  is the normal matrix defined as

$$\mathbf{N} = \mathbf{N}_d + \mathbf{N}_x \quad (\text{A.9})$$

844 with

$$\mathbf{N}_d = \frac{1}{\sigma_d^2} \mathbf{A}^T \mathbf{P} \mathbf{A} \quad (\text{A.10})$$

845 and

$$\mathbf{N}_x = \frac{1}{\sigma_x^2} \mathbf{R}. \quad (\text{A.11})$$

846 The goal of the VCE method is to estimate the level of noise in each input data set. In the  
847 context of actual data, this means that the scaling factor  $\sigma_d^2$  is to be found. If data noise is  
848 assumed to be white, so that matrix  $\mathbf{P}$  is unit, this task reduces to the estimation of the noise  
849 variance. Furthermore, both the actual observations and the pseudo-observations are treated by  
850 the VCE method equally. Since noise in pseudo-observations is assumed to be white, the VCE  
851 method can estimate the noise variance  $\sigma_x^2$  of the pseudo-observations as well.

852 The VCE method is iterative. It starts from certain initial values  $(\hat{\sigma}_d^2)_0$  and  $(\hat{\sigma}_x^2)_0$ , which allow  
853 the initial solution  $\hat{\mathbf{x}}_0$  to be found with Eqs. (A.8 – A.11). Then, an updated estimate of factor  $\sigma_d^2$   
854 is found as

$$\hat{\sigma}_d^2 = \frac{1}{n - \hat{\tau}_d} (\mathbf{d} - \mathbf{A}\hat{\mathbf{x}})^T \mathbf{P} (\mathbf{d} - \mathbf{A}\hat{\mathbf{x}}), \quad (\text{A.12})$$

855 where  $n$  is the number of data (i.e., the length of the vector  $\mathbf{d}$ ) and

$$\hat{\tau}_d = \text{trace} [\hat{\mathbf{N}}_d \hat{\mathbf{N}}^{-1}]. \quad (\text{A.13})$$

856 The noise variance of pseudo-observations – factor  $\sigma_x^2$  – is estimated similarly:

$$\hat{\sigma}_x^2 = \frac{1}{m - \hat{\tau}_x} \hat{\mathbf{x}}^T \mathbf{R} \hat{\mathbf{x}}, \quad (\text{A.14})$$

857 where  $m$  is the number of pseudo-observations (i.e., the length of the zero vector in the right-hand  
858 side of Eq. (A.3)) and

$$\hat{\tau}_x = \text{trace} [\hat{\mathbf{N}}_x \hat{\mathbf{N}}^{-1}]. \quad (\text{A.15})$$

859 The improved estimates of the factors  $\sigma_d^2$  and  $\sigma_x^2$  are used for an improved estimate of the solution  
860  $\mathbf{x}$ , etc. The iterations are repeated until convergence.

861 **Appendix B. Proof that any function not penalized by the proposed regularization is a com-**  
862 **bination of seasonal variations and linear trend**

863 Let us demonstrate analytically that any function  $H(t)$  not penalized by the regularization  
864 functional from Eq. (5) is a combination of arbitrary seasonal variations and a linear trend. Let  
865 function  $H(t)$  in the first and second year be equal to arbitrary functions  $h_1(t)$  and  $h_2(t)$ , re-  
866 spectively. Obviously, function  $H(t)$  escapes penalization if and only if  $h_2'(t) = h_1'(t)$ , i.e., if  
867  $h_2(t) = h_1(t) + C_1$ , where  $C_1$  is an arbitrary constant. Since the time-series of mass anomalies  
868 is a continuous function,  $h_2(0) = h_1(1)$ . Therefore, constant  $C_1$  can be represented as  $C_1 =$   
869  $h_2(0) - h_1(0) = h_1(1) - h_1(0)$  or, alternatively,  $C_1 = h_2(1) - h_1(1) = h_2(1) - h_2(0)$ . Thus, constant  
870  $C_1$  is nothing but the yearly mass change, which is equal in the first and the second year. In the  
871 third year, a non-penalized function  $H(t)$  must be equal to  $h_3(t) = h_2(t) + C_2$ , where the constant  
872  $C_2$  can be defined, in line with the derivation above, as  $C_2 = h_3(1) - h_3(0) = h_2(1) - h_2(0) = C_1$ .  
873 Therefore,  $h_3(t) = h_1(t) + 2C_1$ . By considering the further years, we readily find that function  
874  $H(t)$  avoids penalization if it is defined in the  $k$ -th year as

$$h_k(t) = h_1(t) + (k - 1)C_1. \quad (\text{B.1})$$

875 The first term in Eq. (B.1) describes an arbitrary seasonal variability; the second term is a linear  
876 function of time and represents a long-term linear trend.