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Kolechkina, Alla; van Nooijen, Ronald

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Behaviour of a series of reservoirs separated by drowned gates

A. G. Kolechkina and R. R. P. van Nooijen

Delft University of Technology, Netherlands

Introduction

Modern control systems tend to be based on computers and therefore to operate by sending commands to structures at given intervals (discrete time control system). Moreover, for almost all water management control systems there are practical lower limits on the time interval between structure adjustments and even between measurements. The water resource systems that are being controlled are physical systems whose state changes continuously. If we combine a continuously changing system and a discrete time controller we get a hybrid system. We use material from recent control theory literature to examine the behaviour of a series of reservoirs separated by drowned gates where the gates are under computer control.

System

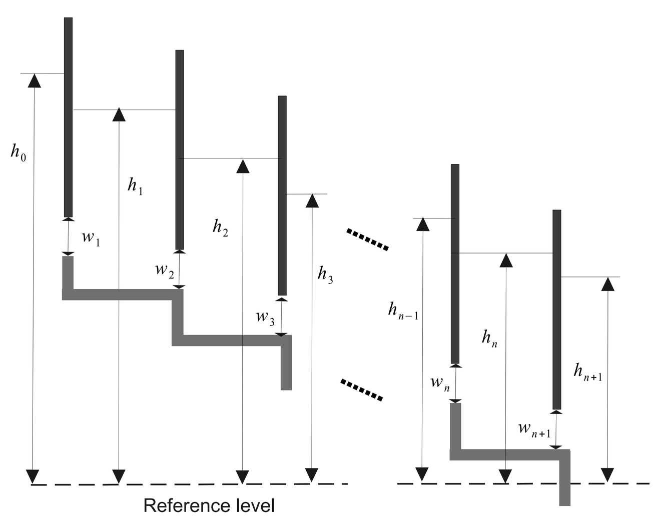


Figure 1: The system

Drowned gate

$$q_{\mathrm{dg}}(h_{k-1},h_k,w_k) = c_k b_k w_k \sqrt{2g}\frac{h_{k-1}-h_k}{|h_{k-1}-h_k|^{1/2}} \qquad (1)$$

with gate width b_k and gate constant c_k.

Mass balance per pool

For $k = 1,2,\ldots,n$

$$\frac{dh_k}{dt} = \frac{q_{\mathrm{dg}}(h_{k-1},h_k,w_k) - q_{\mathrm{dg}}(h_k,h_{k+1},w_k) - q_{\mathrm{dist},k}(t)}{a_k}$$

with a_k water surface area of pool k and $q_{\mathrm{dist},k}(t)$ ("dist" stands for disturbance) unmeasured seepage, normally zero.

Reformulation

Move normal operational situation to zero state, zero input and scale all quantities appropriately.

$$u_{\mathrm{u},0}(t) = \frac{h_0(t\cdot\tau_c+t_0) - h_{\mathrm{design},0}}{h_{\mathrm{scale}}}$$

$$j=2,3,\ldots,n+1 \,:\, u_{\mathrm{u},j}(t) = \frac{q_{\mathrm{dist},j}(t\cdot\tau_c+t_0)}{q_{\mathrm{scale}}}$$

$$u_{\mathrm{u},n+2}(t) = \frac{h_{n+1}(t\cdot\tau_c+t_0) - h_{\mathrm{design},n+1}}{h_{\mathrm{scale}}}$$

$$j=1,2,\ldots,n \,:\, x_j(t) = \frac{h_j(t\cdot\tau_c+t_0) - h_{\mathrm{design},j}}{h_{\mathrm{scale}}}$$

$$j=1,2,\ldots,n+1 \,:\, u_{\mathrm{c},j}(t) = \frac{w_j(t\cdot\tau_c+t_0) - w_{\mathrm{design},j}}{w_{\mathrm{scale}}}$$

where τ_c is control time step, h_{scale} is the level scale factor $h_{\mathrm{design},k}$ is the design level for h_k and so on.

Non-linear hybrid system

$$\dot{\vec{x}}(t) = \vec{f}(\vec{x}(t)) + B\vec{u}(k),\, t\in[k,k+1) \qquad (2)$$

$$\vec{u}(k+1) = C\vec{x}(k) + D\vec{u}(k) \qquad (3)$$

$$\vec{x}(k) = \lim_{t\uparrow k}\vec{x}(t)$$

Linearized version

$$\dot{x}(t) = A\vec{x}(t) + B\vec{u}(k),\, t\in[k,k+1) \qquad (4)$$

$$\vec{u}(k+1) = C\vec{x}(k) + D\vec{u}(k) \qquad (5)$$

$$\vec{x}(k) = \lim_{t\uparrow k}\vec{x}(t)$$

$$A = \frac{\partial\vec{f}}{\partial\vec{x}}$$

Theorem 1

The equilibrium $(x,u) = (\vec{0},\vec{0})$ of system given by (2), (3) is uniformly asymptotically stable when (4), (5) is uniformly asymptotically stable or equivalently when the matrix

$$H = \begin{bmatrix} e^A & \int_{\tau=0}^{1} e^{A(1-\tau)}d\tau B \\ C & D \end{bmatrix} \qquad (6)$$

is Schur stable. (Theorem 8.3.1 of [1])

Theorem 2

If the matrix H given in (6) has no eigenvalues on the unit circle and has at least one eigenvalue outside the unit circle in the complex plane then the equilibrium $(x,u) = (\vec{0},\vec{0})$ of the nonlinear digital feedback control system (2), (3) is unstable. (Theorem 8.3.2 of [1])

Proportional control

The principle of proportional control is that the control action is linearly proportional to the deviation from design value, $u = c_P(x - x_{\mathrm{design}})$.

Important point

As we will show, even simple water systems with simple proportional control are not "automatically" stable.

Magnitude of eigenvalues for two reservoirs

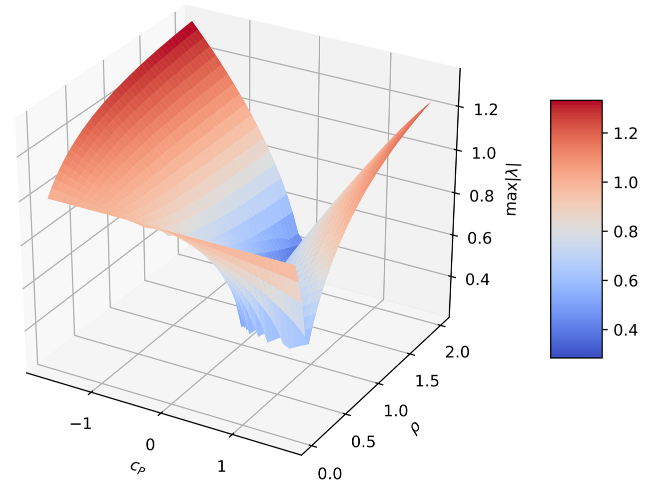


Figure 2: Magnitude of eigenvalues

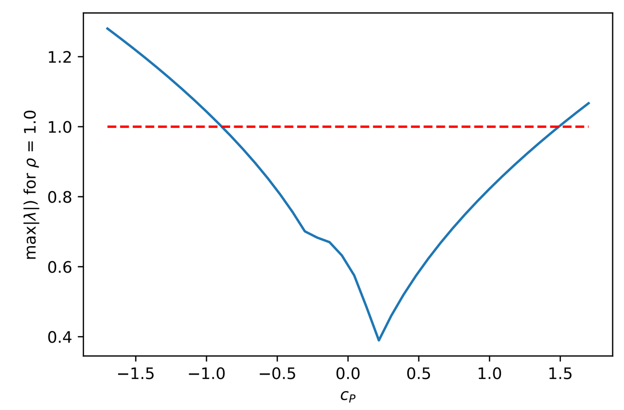


Figure 3: $\rho = 1$

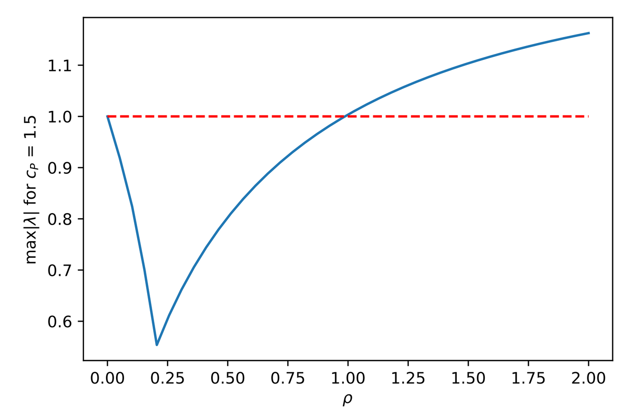
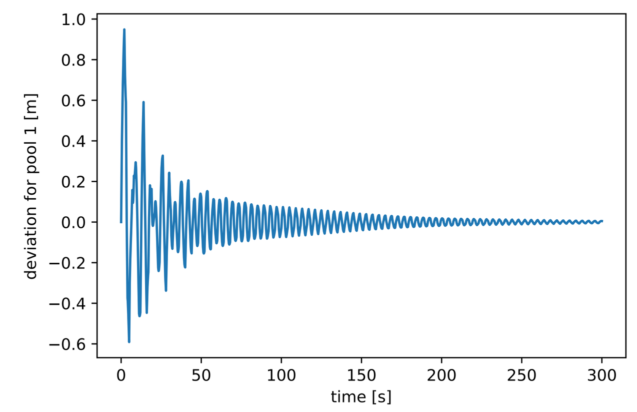


Figure 4: $c_P = 1.5$

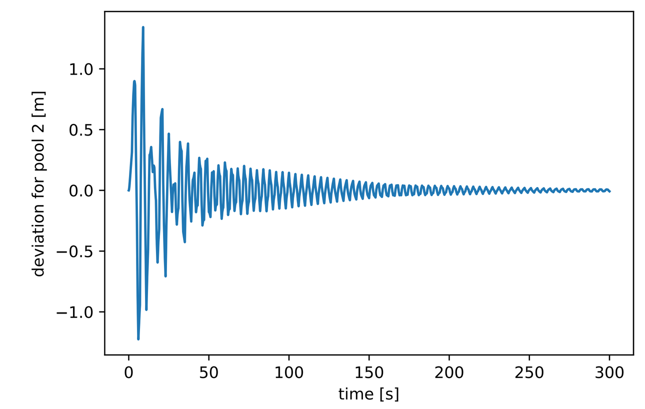
Asymptotically stable versus unstable

Computer simulation results.

$$\rho = \frac{q_{\mathrm{scale}}\tau_c}{a h_{\mathrm{scale}}}$$

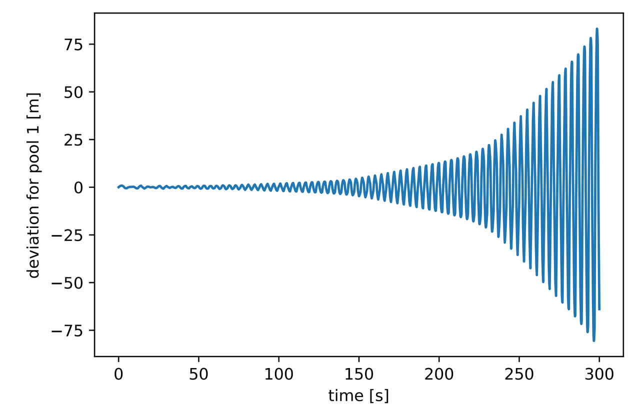


(a) Pool 1

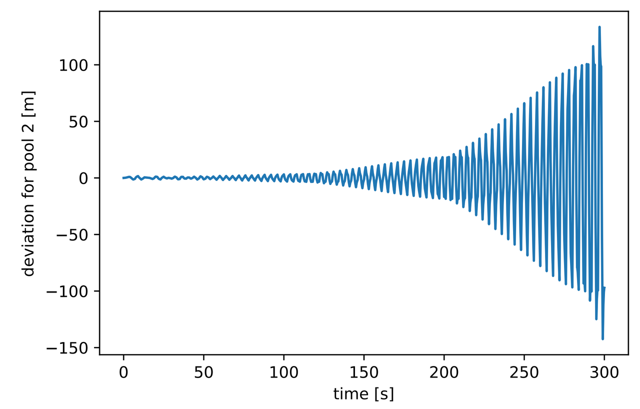


(b) Pool 2

Figure 5: $c_P = 1.45; \rho = 1$



(a) Pool 1



(b) Pool 2

Figure 6: $c_P = 1.55; \rho = 1$

Literature

[1] Anthony N. Michel, Ling Hou, and Derong Liu. *Stability of dynamical systems.* Systems & Control: Foundations & Applications. Birkhäuser/Springer, second edition, 2015.