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Publication date
2016

Document Version
Accepted author manuscript

Published in
Journal of Transport Geography

Citation (APA)
Cats, O. (2016). The Robustness Value of Public Transport Development Plans. *Journal of Transport Geography*, *51*, 236-246. http://www.sciencedirect.com/science/article/pii/S0966692316000120

Cats O. (2016). The Robustness Value of Public Transport Development Plans. *Journal of Transport Geography*, 51, 236-246. [pre-print version]

THE ROBUSTNESS VALUE OF PUBLIC TRANSPORT DEVELOPMENT PLANS

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ABSTRACT

Investments in transport are increasingly motivated by the need to improve its robustness – the capacity to absorb disturbances with a minimal impact on system performance. Nonetheless, there is lack of knowledge on how to assess and quantify the robustness value of new investments. This study investigates the robustness of alternative public transport networks by assessing the consequences of link failures on network performance. A full-scan disruption impact analysis is performed and its implications on passengers group composition and travel time losses are analyzed for a public transport development plan in Stockholm, Sweden. The results suggest that as a result of the development plan, the robustness of the case study network will improve in terms of average performance deterioration as well as worst case scenario for all performance indicators. Neglecting abnormal operations in project appraisal can potentially lead to the underestimation of its benefits. Moreover, the critical links in each network are identified and impact disparity is investigated. The analysis method presented in this study can support the consideration of development plans impacts on network robustness in the strategic planning process.

Keywords: Robustness; Public Transport; Network Design; Disruption; Link Failure.

1. INTRODUCTION

Investments in transport are increasingly motivated by the need to improve reliability and robustness and not merely travel time savings under normal operations (Mackie et al. 2014). Transport systems are subject to recurrent disruptions that may have substantial implications for network performance and society at large. Even though robustness of critical infrastructures such as mass metropolitan public transport networks is high on the planning and policy agenda (Homeland Security 2010), there is lack of knowledge on how to assess and quantify the robustness value of new investments. There is thus a growing need to develop techniques and indicators to assess the consequences of alternative network developments in terms of robustness.

Disruptions in the public transport service could result from various reasons including mechanical and technical failures, planned maintenance works or targeted attacks. This study focuses on the consequences of individual link failures, i.e. the complete breakdown of link functionality, for example in the case of disruptions caused by a signal failure, suicide attempt or physical infrastructure degradation. Such disturbances occur regularly in large urban rail networks. The impact of disruptions on network performance may vary from one link to the other, depending on the interaction between network topology and travel demand. In addition, the consequences in case of a disruption on a given link may change as a result of changes in network topology or operations such as the addition or removal of links or the increase or decrease of link capacity (Cats and Jenelius 2015). The relatively low connectivity of public transport systems makes them particularly vulnerable in case of disruptions. In contrast, the removal of random links in urban road networks will only seldom induce longer path lengths or the fragmentation of the network into disconnected sub-networks (Duan and Lu 2014). Urban rail-bound systems are particularly vulnerable to link failures because of their restricted capability to bypass link closures, and the low network density. However, these systems often comprise of several separate railway systems which could potentially offer redundancy and thus alternative routes in case of disruptions.

A systematic approach for quantifying the consequences of service disruptions is essential for the inclusion of network robustness in the network design process. In this study, system *robustness* is defined as the capacity to absorb disturbances with a minimal impact on system performance. The latter can be measured using various indicators such as total travel time loss, size of the largest sub-network that remains intact or the number of travelers affected by the disruption. Although planners and stakeholders occasionally argue that new links will increase network redundancy and hence reduce its vulnerability, the claims of increased robustness are in general not supported by evidence that has been produced using a systematic evaluation procedure. This is for example the case with a cross-radial light rail line as well as the overall public transport development plan that is currently underway in Stockholm, Sweden (Jenelius and Cats 2015). Whilst rigorous cost-benefit analyses that consider the effects on travel times and welfare under normal conditions are often conducted as part of transport planning appraisal, the implications of the development plan on network robustness remain unknown.

The primary objective of the current study is to propose a method for evaluating the robustness value of alternative network designs. The method involves a full-scan of link failure disruption scenarios and assess their implications on network integrity, passenger delays and link criticality. The full-scan approach implies analyzing the impact of a disruption on each of the network links separately for each network alternative. The robustness value of a certain network alternative is then established by comparing networks in terms of their expected disruption impact.

The evaluation approach deploys multiple performance indicators and considers both expected performance degradation, worst case scenarios and worst affected passengers. Moreover, the distribution of delays and travel disturbances over the passenger population is investigated for different disruptions scenarios under alternative networks. The analysis framework is used to evaluate the robustness implications of a substantial development plan of Stockholm multi-modal rapid urban rail network. The results shed light on the implications of using alternative performance indicators for measuring network robustness and recommendations for network planners when assessing the robustness value of new investments.

A review of the literature on evaluating transport robustness is provided in the following section. A method to model and evaluate the impact of disruptions is first presented (Section 3) followed by its application to a public transport expansion plan that is currently underway in Stockholm, Sweden (Section 4). The results include an impact analysis of disruption scenarios on travel times, passengers ability to reach their destination and link centrality (Section 5). A discussion of the main findings and their planning implications (Section 6) is followed by study limitations and recommendations for planners and future research directions (Section 7).

2. LITERATURE REVIEW

Previous studies analyzed the relation between network topology and network vulnerability. Ash and Newth (2007) studied the optimal network design to withstand link closures for a synthetic grid network. They concluded that robust network structures were characterized by the formation of a collection of hubs and cliques, i.e. clusters of nodes that are highly inter-connected, resulting with a high share of common neighbors and allowing the redistribution of flows in case of disturbances. Zhang et al. (2015) analyzed the resilience of 17 generic network structures and concluded that redundancy is a key determinant of network capability to withstand disasters. Although some insights can be gained from analyzing a taxonomy of networks structures, real-world public transport networks comprise a large number of diverse building blocks (e.g. hub-and-spoke, grid, ring, diamond etc.) which cannot be represented as direct extrapolation of their fundamental elements.

Most studies on public transport network vulnerability focused on topological indicators and how the degradation of physical links in a specific sub-network - the metro network - affects network connectivity. Roth et al. (2012) showed that the world's largest metro networks share a very similar topology, with a central core from which branches radiate. Using graph theory measures of link importance, previous studies investigated the impact of random and targeted attacks on network vulnerability for the world largest metro systems (Angeloudis and Fisk 2006), 32 metro systems worldwide (Derrible and Kennedy 2010), Shanghai (Zhang et al. 2011), London and Paris (von Ferber et al. 2012), Nanjing (Deng et al. 2013) and Madrid (Rodriguez-Nunez and Garcia-Palomares 2014). The results demonstrate that public transport networks vary in their capacity to absorb random and targeted attacks. With the exception of Rodriguez-Nunez and Garcia-Palomares (2014), previous studies discarded link labels (e.g. travel times), travel demand and passenger distribution over the network. Instead, they analyzed network vulnerability in strictly topological terms, implying uniform link labels and attributing equal importance to connections between each origin-destination. However, the impact of link failure depends not only on the availability of travel connection alternatives, but also on their attractiveness and the number of travelers that relied on the disrupted link. Indeed, Dupuy (2013) argues that by overlooking key network characteristics as well as the urban planning context, studies performed by scientists from other disciplines in the field of network geometry and urban railway systems provide very limited recommendations to network planners and thus obstruct potential implementations.

The importance of network redundancy was highlighted in most of the abovementioned studies that examined the implications of public transport network design on network vulnerability. Radial networks are highly vulnerable due to the possible fragmentation of branches from the remaining network. Increasing the number of cyclic paths, i.e. alternative ways to travel in a loop back to the origin stop without traversing the same link twice, implies adding redundancy to the network and enabling travelers to use alternative routes in case of disruptions. Derrible and Kennedy (2010) postulated that network robustness is determined by the number of cyclic paths available in the network. Interestingly, their analysis suggests that Stockholm is the third least robust system among the 32 metro systems examined.

Lines that intersect with many other lines play an important role in providing travel alternatives in case of disruptions by increasing the number of cyclic paths. The importance of circular (i.e. ring) and cross-radial (i.e. orbital services that intersect with radial lines leading to the dense core of the network) lines was therefore emphasized by Rodriguez-Nunez and Garcia-Palomares (2014) and Jenelius and Cats (2015). Rodriguez-Nunez and Garcia-Palomares (2014) and De-Los-Santos et al. (2012) conducted a full network scan for the existing metro and commuter train networks of Madrid, respectively. The former evaluated network vulnerability in terms of network performance under successive targeted attacks, whereas the latter focused on the robustness value of offering a replacement bus service. Cats and Jenelius (2014) proposed a method for identifying a subset of central links for which a detailed dynamic robustness analysis was performed using a dynamic agent-based transit assignment model. This method was then used as a component in a sequential process for identifying where should reserve capacity be allocated to best improve network robustness by Cats and Jenelius (2015). However, evaluating the robustness value of alternative investments requires analyzing the impacts of all possible link failures rather than assessing network robustness in case disruptions occur on the most central links.

Previous studies provide important principles for robust public transport design by comparing networks situated in different cities. However, none of the abovementioned studies proposed a method for assessing development plans for network evolution in terms of their network robustness. The evaluation of link failure impacts on alternative networks need to take into consideration travel impedance, travel demand levels and network flows such that the delay associated with rerouting, the number of travelers affected and ultimately the societal costs attributed to a certain link failure can be estimated. A quantitative approach that encompasses these aspects can facilitate a rigorous comparison of the effects of different network developments and patterns of travel demand on network robustness. This will allow contrasting for example two alternative investments with the same impact on indexes such as network connectivity and redundancy whilst having distinctively different implications on passenger delay, unserved passenger demand and the share of affected passengers. A comparative method for analyzing and evaluating network robustness is described in the following section.

3. METHOD

A method to analyse the robustness value of alternative public transport links by performing a full-scan of link failure scenarios is proposed. Network and demand representation, followed by the measures of system performance and link criticality and centrality deployed in this study are described in the subsequent sub-sections.

3.1 Network and demand representation

The public transport network is represented by a directed and weighted graph $G(S,E)$, where the node set S represents stops and rail stations (all called stops here for simplicity), and the link set $E \subseteq S \times S$ represents road or rail track segments between stops. The graph is fully specified by:

(1) an adjacency matrix A where cell a_{ij} equals one if nodes $i,j \in S$ are connected and zero if not;

(2) a vector of weights associated with each link $e \in E$, where each element, t_e, denotes the travel impedance induced when traversing the respective link. Travel impedance is considered deterministic at the strategic planning phase.

This representation is a variant of the L-space graph used in network theory studies. Similarly to the representation used by Derrible and Kennedy (2010), only transfer stations (served by more than a single line, except for intermediate stations on a common corridor) and terminals are included in the node set. Intermediate stations without transfer possibilities are discarded since the exact location of the disruption between interchange stations will not influence passenger rerouting alternatives.

Travel demand is assigned using the all-or-nothing approach for a given network and disruption scenario, similarly to previous studies (De-Los-Santos et al. 2012, Rodriguez-Nunez and Garcia-Palomares 2014). Ramli et al. (2014) compared the results obtained by the assignment approach applied in this study to ridership data of the rapid transit system in Singapore and concluded that it yielded reasonably accurate predictions. It is hence assumed that each passenger follows the fastest path available between the respective origin and destination stations. This approach was adopted in this study because of its computational advantage which allows assessing the large number of scenarios required for a full network scan.

Travel demand is given in the form of an OD matrix, F, where each entry, f_{ij} $(i, j \in S)$, denotes passenger demand between a pair of stops i and j. For each pair of stops the shortest path is calculated and the respective travel demand is assigned. Let δ_e^{ij} take the value one if link e is on the shortest path between nodes i and j and zero otherwise. The number of travellers traversing each link is then obtained by superposing the flows assigned for all OD pairs:

$$v_e = \sum_{i \in S} \sum_{j \in S} \delta_e^{ij} f_{ij} \qquad (1)$$

3.2 Disruption impact and robustness value

The robustness value of a network design alternative, n, is assessed by comparing network performance under a range of disruption scenarios to the corresponding performance of the base case network, n_0. The robustness analysis consists of assessing the impacts of a disruption occurring on each of the network elements. A full-scan approach is taken in this study where each scenario corresponds to the breakdown and closure of a single link in the network, independently. A scenario involving the failure of link e in network n is denoted by (n, e).

System performance can be measured by a range of indicators that are considered important by decision makers for evaluation purposes. Let y denote an indicator characterizing system performance, to be described in Section 3.3. The *impact* of the failure of link e given network n is defined as

$$\Delta y(e|n) = y(n, e) - y(n, 0) \qquad (2)$$

The impact of a link disruption for a given network, $\Delta y(e|n)$, is defined as the difference between the performance when link e is disrupted, $y(n, e)$, and the performance obtained in the undisrupted case, $y(n, 0)$. The impact corresponds thus as the deterioration in network performance inflicted by a certain disruption as compared with the undisrupted case. When the system performance indicator is the change in travel times, it becomes equivalent to the network robustness index proposed by Scott et al. (2006) for evaluating the criticality of highway segments and operationalized by Cats and Jenelius (2014) for public transport line segments.

A measure of network robustness should enable the comparison of alternative networks in respect to their capacity to absorb various disruptions. The *robustness value* of a certain network alternative, n, given a failure of link e is defined as

$$\Delta y(n|e) = y(n, e) - y(n_0, e) \qquad (3)$$

The robustness value of a network for a given link disruption, $\Delta y(n|e)$, is defined as the difference between the performance obtained in case this disruption occurs in this network, $y(n, e)$, and the performance obtained when the same disruption occurs in the base case network, $y(n_0, e)$.

The definitions of impact and robustness value given in Eq. 2-3 take into account a single disruption scenario. In order to allow the overall assessment of network robustness, the impacts of the whole range of disruption scenarios need to be considered. The robustness of a certain network alternative can be measured in terms of the change in its performance in the worst-case scenario

$$r_n^{max} = \max_{e \in E} \Delta y(n|e) \qquad (4)$$

While this measure indicates whether the network is more robust with respect to the most adverse scenario, it discards potential changes in its robustness value in withstanding all other scenarios. Alternatively, overall network robustness value, r_n, can be defined in terms of the expected disruption impact

$$r_n = E[\Delta y(n|e)] = \sum_{e\in E}[p_e \Delta y(n|e)] \quad (5)$$

where p_e is the probability that link e will breakdown. Link failure probabilities are typically unknown. By assuming equal probability to fail, Eq. 5 can be written as simply the mean value of network robustness taken over all links. The consideration of maximum impact versus mean impact can be viewed as a distinction between targeted attacks and random failures, respectively, when assuming that the perpetrator can identify the weakest link.

3.3 System performance indicators

System performance is measures in this study using several indicators. The indicators can be classified into passenger group composition and travel time losses.

3.3.1 Passenger group composition

In the event of a disruption, the population can be divided into three classes based on how the disruption influences their travel experience:

- *Cut-off* – passengers that cannot execute their trip because of network disintegration. These disconnected passengers have no path available between their origin and destination.
- *Delayed* - passengers that experience delays, i.e. longer travel times than in the undisrupted case, due to the disruption. Given the network loading approach used in this study, this implies that the disruption occurs along their shortest path and hence requires a detour. These passengers use an alternative path which is less attractive than the previously used shortest path.
- *Unaffected* – passengers that are not affected by the disruption, i.e. travel times remain the same as in the undisrupted case, because their shortest path remains intact.

The three passenger groups are denoted by F_c, F_d and F_u, respectively, where these three classes are mutually exclusive and collectively exhaustive, $F_c \cup F_d \cup F_u = F$.

The system performance indicators considered in this study are the shares of the abovementioned passenger classes as well as the extent of the impact on delayed passengers. The latter is analysed in terms of the change in total passenger travel times compared with the baseline undisrupted scenario and the distribution of delays for passengers in F_d.

Certain disruptions may result in unsatisfied demand due to the partitioning of the network. The *share of cut-off demand* that results from scenario σ is

$$c(\sigma) = \frac{|F_c|}{|F|} = \frac{\sum_{i\in S}\sum_{j\in S} f_{ij}\cdot \omega^{ij}(\sigma)}{\sum_{i\in S}\sum_{j\in S} f_{ij}} \quad (6)$$

Where $\omega^{ij}(\sigma)$ indicates whether there is no path between i and j in scenario σ. Formally, $\omega^{ij}(\sigma)$ equals one if $\sum_{e\in E}\delta_e^{ij}(\sigma) = 0$ and is zero otherwise. Alternatively, network availability can be positively defined as the share of satisfied demand, $1 - c(\sigma)$.

In order to calculate the share of delayed passengers, passengers travel times need first to be obtained. The travel impedance of travellers travelling between stops i and j under scenario σ is

$$t_{ij}(\sigma) = \sum_e \left(\delta_e^{ij}(\sigma)\cdot t_e\right) \quad (7)$$

The magnitude of the impact of a disruption could thus be measured also in terms of the *share of passengers that experience delays*, as follows

$$d(\sigma) = \frac{|F_d|}{|F|} = \frac{\sum_{i \in S} \sum_{j \in S} f_{ij} \cdot \varphi^{ij}}{\sum_{i \in S} \sum_{j \in S} f_{ij}} \quad (8)$$

Where φ^{ij} indicates whether passengers travelling between i and j in scenario σ experience delay. Formally, φ^{ij} equals one if $t_{ij}(n, e) - t_{ij}(n, 0) > \varepsilon$ and is zero otherwise, and ε is the delay threshold.

The share of passengers that are negatively affected by the disruption is then the sum of equations 6 and 8, whereas the share of unaffected travellers in scenario σ is $1 - c(\sigma) - d(\sigma)$.

3.3.2 Travel time losses

In addition to the composition of passenger groups, the impact of disruptions can also be quantified by considering the extent of the consequences for those trips that can still be performed. Similarly to the discussion in Section 3.2 above concerning mean versus maximum impact over a set of disruptions, passenger delay can be measured in average terms or by considering the worst affected passengers.

The impact of a disruption on total passenger travel time depends on the number of travellers delayed by the incident and the detours invoked by the disruption. The *total travel impedance* experienced by travellers in scenario σ is

$$tt(\sigma) = \sum_{i \in S} \sum_{j \in S} f_{ij} \cdot \left[\sum_e \left(\delta_e^{ij}(\sigma) \cdot t_e\right)\right] \quad (9)$$

Note that different disruptions may result in similar total travel time effect while differing in terms of the number of travellers delayed and the distribution of their impact over passenger population. For example, a disruption may have large negative consequences for few travellers or induce relatively minor detours to a large number of passengers.

An inherent drawback of the total travel impedance defined in Eq. 9 is that it does not allow direct comparison of scenarios that result with different network disintegration. Incomplete trips could be reflected by assigning as infinite or big M travel time value. However, this will make the comparison of $tt(\sigma)$ for such scenarios meaningless. Instead, an additional indicator that accounts only for travel times of completed trips by excluding the cut-off demand is introduced. The *average travel impedance per passenger*, $\bar{t}$, is then calculated only for the satisfied demand

$$\bar{t}(\sigma) = \frac{tt(\sigma)}{(1 - z(\sigma)) \cdot \sum_{i \in S} \sum_{j \in S} f_{ij}} \quad (10)$$

A non-compensatory approach is adopted in this study. This implies contrasting a series of performance indicators, where each of the abovementioned indictors can be assigned in Eq. 3-5 for measuring disruption impact and network robustness value. Furthermore, these system performance indicators enable the assessment of the criticality of each link for maintaining network functionality and integrity.

3.4 Link criticality and centrality

As discussed in the literature review section, it has long been recognized that the ability of transport networks to withstand degradations depends on network topology. In particular, central links, in the sense that many paths between pairs of nodes must traverse those links, are often also critical in case of disruptions (Freeman et al. 1991).

Changes in network performance in the event of a disruption on link e , $\Delta y(e|n)$, determine *link criticality*. Note that the criticality of a certain link is defined in the context of a specific network and travel demand since it may change when flow distribution is altered. Links that cause the greatest deterioration in network performance when disrupted are considered critical to maintaining network functionality.

At the same time, disruption scenarios may also alter network topology and hence *link centrality*. The impact of various scenarios on link centrality is examined in this study through the betweenness centrality measure. The latter is a network science indicator that corresponds to the share of shortest paths that traverse through a certain link. Let $g_{i,j}(e)$ denotes the fraction of shortest paths between stop i and stop j that contain link e. The *relative betweenness centrality* of link e in the public transport network is then defined as

$$b_e = \frac{1}{|S|(|S|-1)} \sum_{i\in S} \sum_{j\in S\backslash s_1} g_{i,j}(e) \tag{11}$$

This is a standardized indicator that corresponds to the share of shortest paths that traverse through a certain link when normalized by the number of all origin-destination pairs. This simple network measure has a number of limitations which may reduce its relevance for identifying central links in real-world public transport networks. In particular, it assumes that all node pairs are equally important when assessing the centrality of a link. Instead, a *weighted relative link betweenness centrality* is used in this study

$$\widehat{b_e} = \frac{1}{\sum_{i\in S} \sum_{j\in S} f_{ij}} \sum_{i\in S} \sum_{j\in S\backslash s_1} g_{i,j}(e)\, f_{ij} \tag{12}$$

Although the betweenness measures are relative, they do not necessarily sum up to one because paths consist of multiple links.

4. APPLICATION

This section describes the case study public transport network and development plan for which the abovementioned method was applied, followed by details on method implementation.

4.1 Network description

The analysis method was applied to the rapid rail-bound transport system of Stockholm, Sweden, which consists of metro, commuter and light rail trains (Figure 1). The network includes 176 stations served by 12 lines and constitutes the backbone of the public transport system in Stockholm with more than 1.5 million passenger trips per day.

Stockholm is famous for its long-term monocentric planning with a dominant central core and the planning of relatively dense satellite rail-bound towns. The inseparable urban and transport planning in Stockholm is a prime example of a radial public transport system which is primarily oriented towards suburb to center commuting (Cats et al. 2015a). The metro network is designed to provide regional accessibility rather than local coverage (Derrible and Kennedy 2010, Börjesson et al. 2013). The commuter train further extends to neighboring communities in Stockholm County and beyond. Since the turn of the century there has been a noticeable shift towards developing sub-centers intended to promote a more balanced distribution of activities. As part of this planning policy, an orbital light rail line (solid yellow line in Figure 1) was constructed to allow passengers to travel between the southern and western parts of Stockholm without going through the oversaturated corridors and transfer hubs within the city center (Jenelius and Cats 2014).

4.2 Development plan

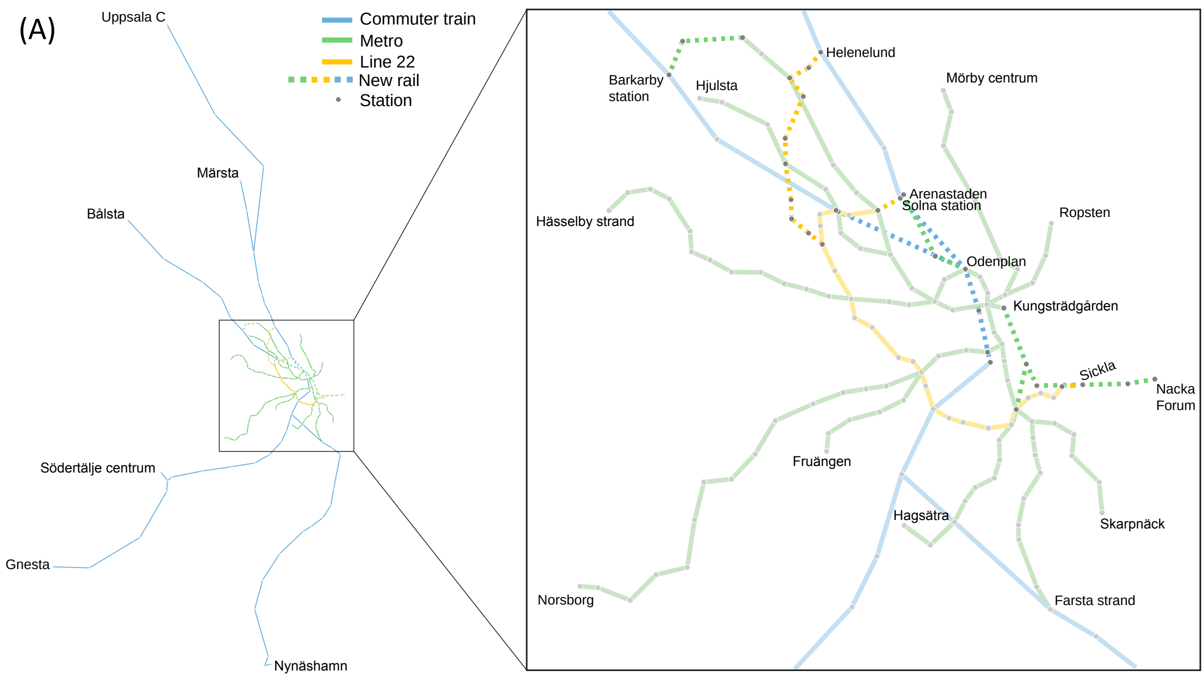
While radial commuting patterns still dominate passenger flows, a more polycentric structure is promoted and supported by the development of a corresponding transport infrastructure (Cats et al. 2015a). These developments include further extensions of the cross-radial light rail train, several expansions of the metro system and increasing the capacity of the commuter train system which are designed to support a stronger network of strategic nodes in Greater Stockholm (Stockholm City 2011).

A decision to extend this system substantially with 23 new stations and 35km of new tracks by 2025 was recently undertaken. The investment plan includes the following components (Figure 1, visualized in Gephi):

I. A new north-south commuter train tunnel (known as '*Citybanan*') six kilometres long underneath Stockholm inner-city (dashed blue line). The project will double the capacity compared with the current conditions. The construction also involves two new commuter train stations: *Odenplan* and *Stockholm City*.

II. The metro network will be extended in three locations (dashed green lines). First, Line 11 will be extended in the northward direction with two additional stations and will terminate at *Barkarby station*. Second, Lines 10 and 11 will be extended from their current end station at *Kungsträdgården* across the Baltic sea to serve the south-eastern area of *Sofia*, *Sickla* and further to *Nacka Forum*, its new end station. This extension also includes a connection to other metro lines by connecting the new station at *Sofia* to the existing *Gullmarsplan* transfer station. Third, a new metro line will be constructed from *Odenplan* north of the inner-city with two additional stations, terminating at a new station in *Arenastaden*. All of these extensions will serve areas that are undergoing significant transformation with the development of new housing and offices.

III. The light rail line, Line 22, is undergoing several developments (dashed yellow lines) including an extension towards the northern suburbs which provides connections to the commuter train and additional metro lines. The expansion plan includes the extension of the line to *Solna station* as well as the construction of a new branch towards *Helenelund*. Finally, the southwest end of the line is extended to *Sickla*, to enable interchanging with the planned metro station.



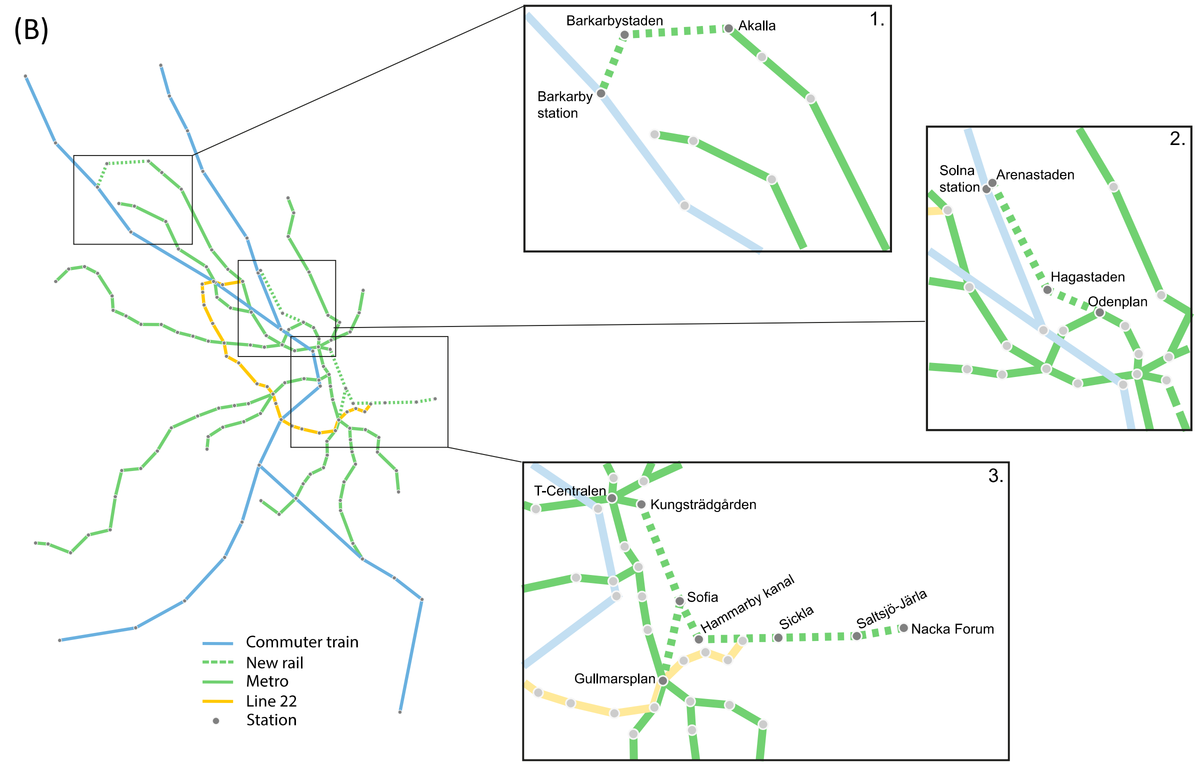


Figure 1. Planned extensions of the Stockholm rapid rail-bound network (A) and zooming-in on the metro extensions (B)

The development plan entails thus both the expansion and densification of the case study network. The development was primarily designed to increase the capacity of the most heavily saturated corridors (i.e. central parts of the north-south metro and commuter train lines) as well as extend the mass public transport network into areas where large-scale housing and office developments are ongoing or planned (i.e. *Sickla* and *Nacka* in the south-east, *Barkarby* in the north-west, and directly north of the inner-city along the new line to *Arenastaden*, see Figure 1). While planners and politicians referred to the potential value of the proposed extension plan in reducing the impact of service disruptions, these potential effects were neither systematically assessed nor quantified.

The planned extension plan will increase the number of interchange stations and end terminals from the current number of 49 to 62. Moreover, the number of network links, i.e. direct connections between interchange stations, is set to increase from 92 to 126. The extension will result in additional cross-radial connections, increase network connectivity and the number of cyclic paths. The *Extended* network yields an increase in network connectivity as measured by the gamma index – the share of links out of the maximum possible number of links in a complete graph - from 0.440 to 0.467. Furthermore, network meshedness measured using the alpha index - share of cyclic paths out of the maximum possible in a complete graph - will increase by almost 30%, from 0.151 to 0.196 in comparison to the *DoNothing* case. This indicates a considerable potential increase in the number of transfer opportunities and route alternatives in general, and in the case of service disruptions in particular.

4.3 Network analysis model implementation

Network vulnerability is evaluated in this study in terms of the capability of the *Extended* network to withstand link failures as compared with the base case *DoNothing* where the network maintains its form as of summer 2014. A directional graph of rail tracks and walking links in transfer facilities was used for representing the network and enabling the analysis of link failure disruptions. Link travel times were extracted from current timetables and detailed plans for future line extensions.

An origin-destination demand matrix for 2025 was generated based on the regional travel demand model. It is expected that more than 320,000 passenger trips will be generated and distributed over the case study network during the morning peak period (6:00-9:00) on an average weekday. The travel demand matrix projected for 2025 was assigned to both *DoNothing* and *Extended* networks. In order to allow comparing networks and disruption scenarios, the overall public transport demand is considered fixed and is assigned to the closest equivalent node. For example, the demand for the metro line that is extended to *Nacka* (Figure 1) in the *Extended* network is distributed over transfer hubs that serve as access station to the metro system in the *DoNothing* scenario (primarily *Slussen* and to a lesser extent *Gullmarsplan*) based on VISUM model assignment results for the *DoNothing* network that were available for this study. By doing so, the network and disruption impacts considered in this study are limited to topological effects on flow redistribution, discarding morphological effects on the overall distribution of travel demand (i.e. induced demand or trip distribution).

As described in Section 3, passenger demand is assigned based on an all-or-nothing assignment to the shortest path. Travel impedance was calculated based on the sum of link labels that are along each path. The assignment and the calculation of performance metrics were performed in a specially tailored MATLAB code. Gephi, a network exploration tool, was used for calculating topological indicators and visualization purposes. Model outputs include network loading results - passenger flow per link and travel time per origin-destination pair. Based on the latter, passengers can be classified into the three impact classes: cut-off (i.e. infinite travel time due to the inexistence of a connecting path), delayed (i.e. longer travel time than in the undisrupted scenario) and unaffected (i.e. identical travel time as in the undisrupted scenario).

The performance of the network planned for 2025 upon completion of the planned expansion was compared with the base case network. The full-scan approach was implemented by removing in turn each of the links for each network. Hence, a failure on each of the network links was simulated along with the undisrupted case for both the *DoNothing* and *Extended* networks. This full-scan procedure amounts to a total of 93 scenarios for the *DoNothing* network and 127 scenarios for the Extended network. The running time of the full-scan for the *Extended* network is 12 seconds on a standard PC.

5. RESULTS

A comparison between the existing and the planned network performance in case of link failures is first analysed in terms of its implications on passengers, followed by the consequences of network extension on link criticality.

5.1 Disruption Impact

For each scenario, the total passenger travel time and the share of travel demand that cannot reach its destination due to network disintegration were calculated. In addition, the share of travel demand that experiences delays (longer travel times than under the undisrupted scenario) out of those trips that could be completed was calculated. These results are presented in Table 1 for the undisrupted networks and the average, standard deviation and maximum values calculated over the respective link failure scenarios.

Table 1. Summary of performance metrics

Performance metric	**DoNothing**		**Extended**	
	Undisrupted	Disrupted	Undisrupted	Disrupted
Share of cut-off demand [%], $c(\sigma)$				
Average	0	1.33	0	0.79
Standard deviation		0.025		0.020
Maximum		14.86		14.77
Share of delayed passengers [%], $d(\sigma)$				
Average	0	3.18	0	1.49
Standard deviation		0.046		0.023
Maximum		22.43		15.83
Total travel time [min], $tt\ (\sigma)$				

Average Standard deviation Maximum	9 993 785	9,891,921 345,050 10,529,403	9 403 674	9,346,689 266,764 9,931,260
Average travel time [min], $\bar{t}(\sigma)$ Average Standard deviation Maximum	31.01	33.05 0.67 34.88	29.17	29.98 0.47 31.54

The performance of the *Extended* network was first investigated under normal operations, i.e. undisrupted case. The *Extended* network will reduce total passenger travel time by more than 9,800 passenger hours on a single peak morning period compared with the *DoNothing* network. This time savings amount to shortening travel times by 6% or 1 minute and 50 seconds shorter per passenger trip in the undisrupted case. While this may time savings may seem small, the estimated welfare benefits based on this outcome will amount to approximately €68,000 for a single peak period, using the value-of-time recommended by the Swedish Transport Administration.

For each performance indicator, disruption impacts can be derived from Table 1 by comparing each scenario with the corresponding undisrupted case. Network robustness values are calculated by comparing the performance under a given disruption for the two networks. The overall robustness is calculated based on Eq. 4 and 5, using the maximum or average values, respectively. The results are first discussed in terms of passengers group composition and thereafter travel time losses are investigated.

5.1.1 Passenger group composition

On average, a link failure in the *DoNothing* scenario cuts off 1.33% of the travel demand which corresponds to 4,762 travellers. These travellers have no travel alternative to paths that contain the disrupted link and therefore their origins and destinations belong to disconnected sub-networks. In contrast, only 0.79% (2,829) of the travellers are stranded on average when a disruption occurs in the *Extended* network. This reflects a reduction of more than 40% compared with the current level. The overall variation among disruption scenarios is low for both networks because of the high share of link closures that do not lead to any network disintegration. Notwithstanding, much higher shares of the travellers are not able to reach their destinations under certain disruptions. Every seventh traveller or more than 52,000 travellers, are unable to perform their trip in the worst case scenario, with network availability, $1 - c(\sigma)$, diminishing to 85%. This share remains almost unchanged in the *Extended* network.

In addition to those travellers that cannot execute their trip, another segment of the travel demand is subject to delays because the disruption occurred on their shortest path but rerouting is still possible, albeit with a longer travel time. The average share of passengers experiencing some delay ($\varepsilon = 0$) is more than halved in the *Extended* network compared with the *DoNothing* network, 1.49% compared to 3.18%, respectively. Moreover, the worst case scenario in the *DoNothing* network inflicts longer travel time to 22.43% of the passengers that can carry out their trip which corresponds to more than 83,000 passengers in the analysis period. The network extensions included in the *Extended* scenario also mitigate delays in the worst case scenarios, reducing the share of passengers subject to delay to less than 15.83% or 56,679 passengers.

The abovementioned differences in the shares of unsatisfied demand, $c(\sigma)$, and delayed passengers, $d(\sigma)$, between the *DoNothing* and *Extended* networks are further investigated by plotting their cumulative distribution function in Figure 2. More than 40% of all link failure scenarios result in cut-off demand in the *DoNothing* network, whereas less than 30% result in network disintegration in the *Extended* network (dashed lines). Furthermore, those disruptions that lead to cut-off demand in the *Extended* network involve fewer stranded passengers than is the case of *DoNothing*. The pattern emerging for the share of delayed passengers is considerably different (solid lines). When passenger delay is considered, the share of disruptions that does not induce any delay is higher for *DoNothing* than it is in the case of *Extended*. This result may seem counterintuitive at first but this is explained by those disruptions that result in cut-off demand rather than travel time increase. Thereafter, the *Extended* network shows better robustness with 90% of link failures causing delays to less than 15,000 passengers compared with the corresponding value of 37,500 passengers for the *DoNothing* network.

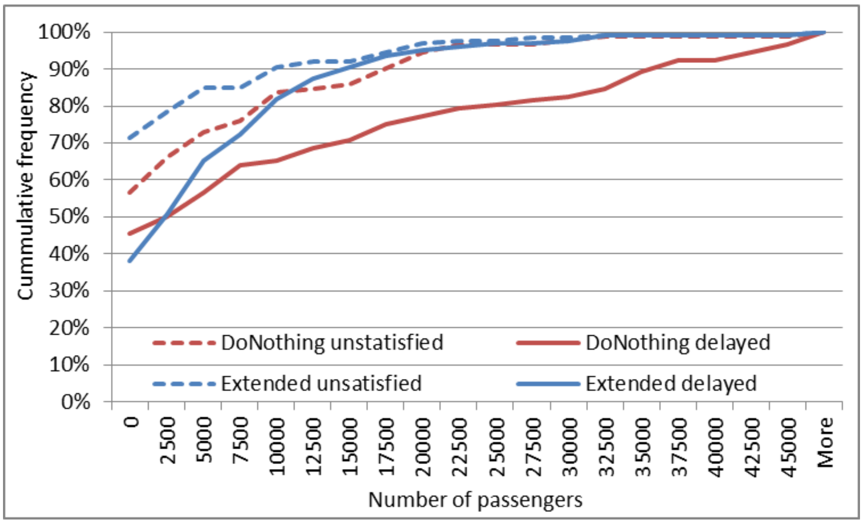


Figure 2. Cumulative density function of link failure scenarios that result in a certain number of delayed passengers and unsatisfied demand for DoNothing and Extended networks

5.1.2 Travel time losses

Total passenger travel time is lower on the average disrupted scenario than in the undisrupted case due to those trips that cannot be completed (Table 1). This is an inherent deficiency in assessing network vulnerability. In order to overcome this shortcoming, the average travel time, $\bar{t}$, was calculated as described in Section 3.3 for each disruption scenario, and is also shown in Table 1. The average time loss per passenger resulting from a disruption amounts to 6.6% of the travel time under normal conditions in the existing network, whereas it only increases by 2.8% in the case of the *Extended* network. The corresponding value for the Madrid metro network is 1.7% as reported by Rodriguez-Nunez and Garcia-Palomares (2014), reinforcing the relative vulnerability of the Stockholm network. Moreover, the impact of the most severe disruption decreases from an increase of 12.5% in average time loss per passenger to 8.1% for the *DoNothing* and *Extended* networks, respectively.

Figure 3 presents the histogram of this performance metric, $\bar{t}$, over all link failure scenarios for each network. It is evident that the average travel time is significantly shorter in the case of the *Extended* network. In fact, only two of the *Extended* network scenarios obtain an average travel time that is higher than any *DoNothing* scenarios. In other words, there is almost no overlap in the range of values obtained by the two networks where the worst performers of the *Extended* network are similar to the best performers of the *DoNothing* network. There is a small number of disruption scenarios that result in the case of the *Extended* network with shorter average travel time per passenger than the undisrupted case because those that are disconnected from the network tend to be located in the network fringes and therefore induce longer travel times.

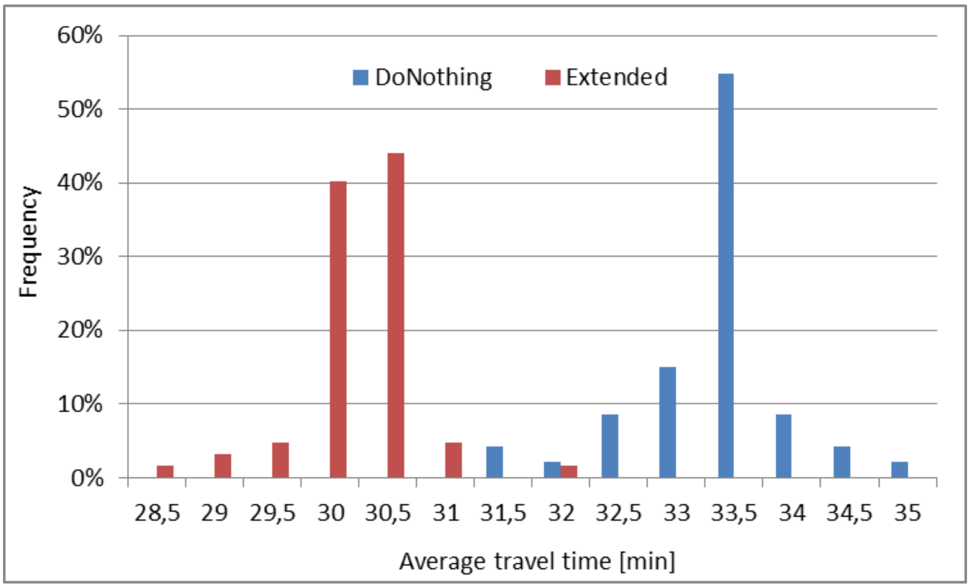


Figure 3: Distribution of average travel time over link failure scenarios

The delay caused by a service disruption may be distributed unevenly among travellers depending on the availability and characteristics of rerouting alternatives. While some disruptions impose a small delay for many travellers, others lead to long delays for few travellers. Hence, the impact of two disruptions with similar societal costs may be manifested very differently across the population, even among the minority of travellers that experience some delay due to link failure.

The distribution of delays over travellers population was further investigated by examining the difference in the share of travellers that experienced different magnitudes of increases in travel times. The differences between the *DoNothing* and *Extended* networks are presented in Figure 4. Each curve in this figure corresponds to a single link failure scenario and the graph show the change in the share of travellers experiencing a delay of one minute, two minutes and so forth in the *DoNothing* network when compared with the *Extended* network. In some rare cases, detours due to link failures prolong travel times by more than 30 minutes, showing significant disparities in the impact of disruptions. Scenarios that have positive (negative) values over the entire range of delay values indicate a higher share of delayed passengers in the current (future) network than in the future (current) network and are displayed in blue (red). As Figure 4 illustrates , most disruptions scenarios result with a higher share of delayed passengers in the *DoNothing* scenario. Notwithstanding, a large number of disruption scenarios result with a lower share of delayed passengers in the existing network compared with the planned network. However, most of these cases are associated with short delays. Overall, the results suggest that the robustness value associated with the *Extended* network is mostly yielded by reducing the number of passengers that experience relatively short delays while long delays remain largely unchanged.

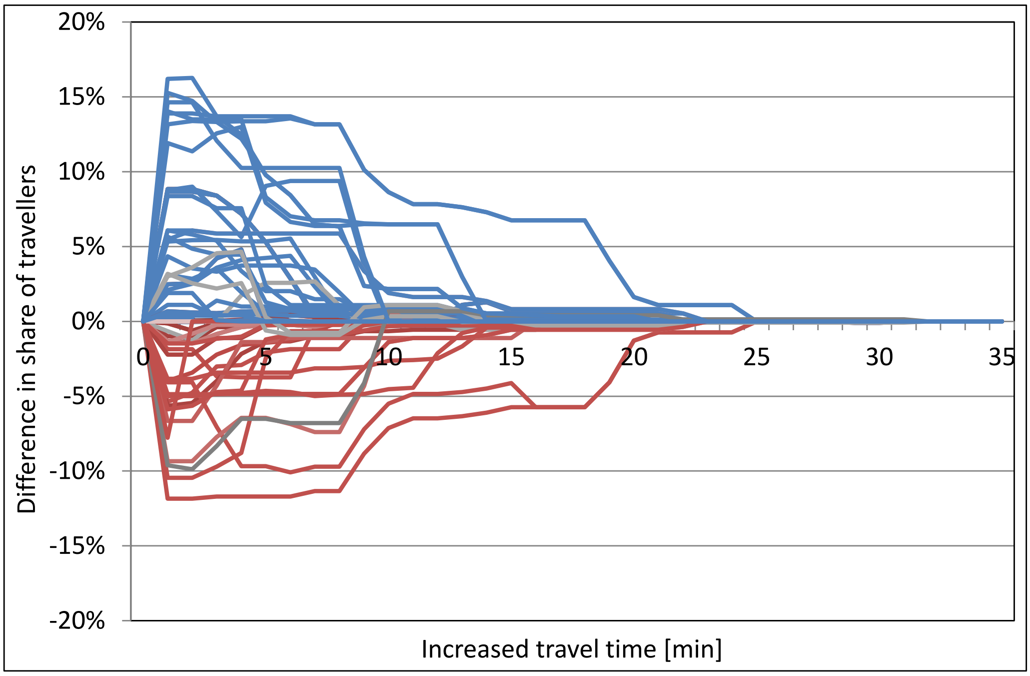


Figure 4: Difference in the share of travellers experiencing a certain increase in travel time under link failure scenarios in the DoNothing compared with the Extended network

5.2 Link Centrality and Criticality

The analysis above suggests that the *Extended* network can withstand link closure disruptions better than the *DoNothing* network does. This improved robustness is attributed to the enriched routing possibilities enabled by improved network connectivity and meshedness (Section 4.2). Figure 5 shows the cumulative distribution of the weighted link betweenness centrality, $\widehat{b_e}$, for both networks. The value of this indicator corresponds to the share of travel demand that traverses through a certain link when travelling along its shortest path. In both networks the majority of network links are on the shortest path of less than 1% of travel demand. In addition, betweeness centrality, and hence travel demand, is more evenly distributed in the *Extended* network than in the *DoNothing* scenario. Approximately 9% of links in the *DoNothing* scenario carry 3-4% of the travel demand, which corresponds to passenger loads of 10,000-14,000, amounting to 56-78% of link capacity. The more balanced distribution of passengers in the *Extended* network under normal conditions makes the network less vulnerable to link failures, whereas the performance of the *DoNothing* network is more dependent on several central links.

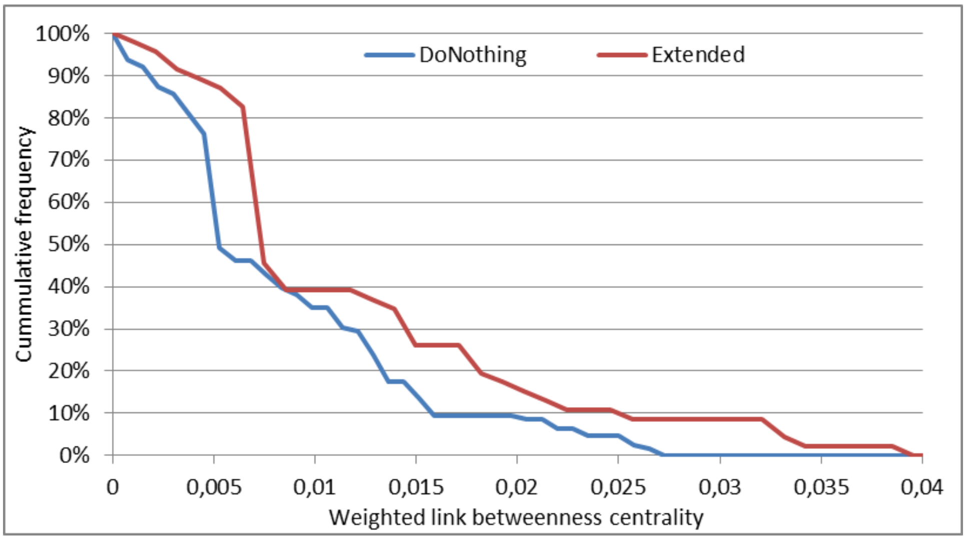


Figure 5: Distribution of weighted relative link betweenness centrality for the DoNothing and Extended networks under normal operations

The alteration of network topology changes the impacts of various link failures on network performance. Table 2 lists the five most critical links for each network and performance indicator based on respective changes in performance indicators, $\Delta y(e|n)$. While some links that are critical in the *DoNothing* network remain so in the *Extended* network, there are also considerable changes in the identification of the most critical links. In terms of share of cut-off demand, the connection between *T-Centralen – Östermalmstorg* remains critical for maintaining network integrity since the metro line branches out from *Östermalmstorg* and there are no rail-bound alternatives in this north-east section (Figure 1). Hence, all travel demand between the rest of the network and this section (or vice-versa) cannot be performed in case this link (or the opposite direction) becomes dysfunctional. Conversely, the high-demand metro branch *Solna Centurm – Akalla* becomes less critical in the *Extended* network because of the availability of new transfer connections to the commuter and light rail lines.

Table 2. Top 5 most critical links by performance indicator

Based on	DoNothing	Extended
Share of cut-off demand, $c(\sigma)$	*T-Centralen - Östermalmstorg* *Östermalmstorg – T-Centralen* *Bålsta - Sundbyberg Station* *Östermalmstorg - Ropsten* *Solna Centurm - Akalla*	*T-Centralen - Östermalmstorg* *Östermalmstorg – T-Centralen* *Bålsta – Barkarby Station* *Östermalmstorg - Ropsten* *Hässelby Strand - Alvik*
Share of delayed passengers, $d(\sigma)$	*Årstaberg - Central Station* *Fridhmesplan – T-Centralen* *T-Centralen - Fridhemsplan* *Karlberg - Central Station* *Slussen – T-Centralen*	*Årstaberg - Central Station* *Liljeholmen – Årstaberg* *Central Station – Årstaberg* *Årstaberg – Liljeholmen* *Älvsjö - Årstaberg*
Average travel time, $\bar{t}$	*Älvsjö – Årstaberg* *Årstaberg – Älvsjö* *Karlberg - Central Station* *Central Station - Karlberg* *Årstaberg - Central Station*	*Älvsjö – Årstaberg* *Årstaberg – Älvsjö* *Årstaberg - Central Station* *Central Station - Årstaberg* *Skärmarbrink - Gullmarsplan*

The redistribution of travel demand results with significant changes in link criticality in terms of the share of delayed passengers. The most critical links in the *DoNothing* network are also the most heavily loaded links in the core of the network, i.e. the metro and commuter links that lead inbound and outbound of *T-Centralen* and *Central Station*. The new commuter train station in *Odenplan* creates an attractive north-south travel alternative to the existing metro corridors (Figure 1). Consequently, disruptions on the links leading to and from *Årstaberg* induce significant delays in the *Extended* network compared with the undisrupted case. As a result, the most critical links in the *Extended* network are those connected to *Årstaberg*, south of the inner-city.

The links that cause the most devastating scenarios in terms of average passenger travel time are also listed in Table 2. These are also the links that their closure result in the highest increase in total passenger travel time while all demand can be satisfied. The worst case scenario corresponds to the same link failure (*Älvsjö – Årstaberg*) for both networks, with an increase of more than 5% in total travel costs, approximately 1 million SEK for a disruption during the peak morning period based on the Swedish value-of-time. A targeted attack may therefore be expected to induce these costs also after the development plan is realized. The robustness effect of the new metro connection between *T-Centralen* and *Gullmarsplan* (Figure 1) is especially remarkable when a disruption on the segment connecting *Gullmarsplan* to *Slussen* occurs. This disruption scenario results with an average travel time of 33.4 minutes in *DoNothing* network, an increase of 1.22% compared with normal disruptions, whereas the same disruption results with an average travel time of 30.0 min in the *Extended* network, an increase of 0.05% compared with the normal operations of the respective network. The additional connection provides redundancy for substituting the most heavily used segment in the case study network and thus contributes significantly to mitigating disruption effects.

6. DISCUSSION

The findings clearly indicate that the envisaged development plan is expected to significantly improve the capability of the case study network to withstand link breakdowns. The *Extended* network was found more robust than the existing network in terms of the average time loss induced by the average disruption. However, in the context of robustness and risk management, one should not merely plan or evaluate the typical case or a hypothetical mean value. Planning should take into consideration also 'black swans' which will inflict severe impacts on system performance. Therefore, special attention needs to be given to the analysis of the right tail of the full-scan disruption analysis. The detailed investigation of impact and disruption distributions reveal that the evaluated development plan is also more robust in terms of its capability to withstand a large range of disruptions and reduce the long tail of worst case scenarios. In contrast, the findings suggest that the development plan examined in this study is not successful in reducing the magnitude of the impact experienced by the worst affected passengers.

The robustness value of the development plan can be assessed by comparing the total time losses. This requires the estimation of the time loss experienced by those passengers who are cut-off from the network due to the disruption. If the disruption is assumed to last one hour and service is resumed instantaneously to the original service level, then the additional travel time for disconnected passengers can be approximated as equivalent to the disruption time. This conservative assumption is practical for calculating the lower bound of the time loss caused by disruption. On average, the *Extended* network reduces passenger travel times by more than 14.5 thousand pass-hours which reflect a societal cost of approximately €100,000. This is the expected robustness value, r_n, of the planned extension plan. If a worst-case scenario notion of robustness, r_n^{max}, is adopted, then the societal cost savings amount to €215,000. The development plan yields thus greater welfare benefits in case of disruption than in the case of normal operations (€68,000, see Section 5.1), suggesting that neglecting abnormal operations in project appraisal will result in the underestimation of its benefits in this case.

In addition to the impact of disruptions on passengers travel time and network integrity, it is important for transport planners and operators to identify the most critical links in the network in order to prioritize investments and allocate resources to mitigate the impacts of disruptions on these links. As networks evolve, the function that various links play changes as well, including their criticality. In the case study network, most of the links that are critical in the existing network are expected to remain so after the implementation of the development plan, with a few noticeable exceptions. Interestingly, link criticality varies greatly when using different performance indicators. In fact, there is no overlap between the five links ranked most critical by each of the performance indicators - share of cut-off demand, share of delayed passengers and average travel time. This result is consistent with previous studies of road networks which found that different links are considered most critical depending on the criteria used in the evaluation (Knoop et al. 2012, El-Rashidy and Grant-Muller 2014).

Urban and transport planners in cities worldwide need often to balance between visions of expanding and densifying land-use and network developments. The case study of Stockholm illustrates that a development plan designed primarily to elevate network connectivity can result with substantial robustness benefits. Stockholm development plan reflects a shift in its urban planning paradigm from expanding the network serving an increasingly large area to densifying the core of the network. Translated into network science terms, this strategy implies increasing network connectivity and meshedness rather than its diameter. The increase in network redundancy (as measured in terms of cyclic paths and average node degree) is key to improving network capacity to withstand link breakdown. This concurs with the results reported by Zhang et al. (2015) which investigated the resilience of various generic simple network structures. Using their network structure typology, the central part of the metro system (Gullmarsplan-T-Centralen-Fridhemsplan) transformed from a central ring to a double central rings. When designing new lines, planners are advised to generate new transfer hubs and close circuits that will relief congestion and improve network robustness.

The complexity of real-world public transport networks and their growth is the result of complicated multi-actor decision making. Network growth is therefore not determined by the direct pursue of network science objectives but is rather a manifestation of policy, planning and political interests. Furthermore, network development plans are incremental and amount to evolutionary rather than revolutionary changes in network structure. Extension plans build on historical urban and transport planning programs and their realisation. It is thus important to acknowledge the importance of path-dependency in the development of public transport networks and evaluate potential investments against the do-nothing scenario and alternative investments rather than compare them to theoretical and optimal network indicator values. This is the approach that has been adopted in this study by proposing a comparative approach to evaluating the robustness value of development plans.

7. CONCLUSION

The increasing importance of transport network resilience call for the development of methods to systematically analyze the robustness value of alternative investments. This study investigated the robustness of alternative public transport networks by assessing the impact of link failures on network performance. The full-scan of network links was performed and the impacts of each disruption were analyzed in terms of the capability of the network to maintain its integrity and guarantee that travelers can reach their destinations. In addition, the travel time consequences of each disruption were estimated by performing an all-or-nothing assignment which enabled the assessment of passengers delay and comparing the share of travelers that are subject to delays. The analysis enables examining the impact in terms of expected robustness value as well as comparing network performance under worst case scenario and the impact on passengers that are worst affected by a given disruption.

The systematic robustness analysis was applied to the rapid rail-bound network in Stockholm. This network will undergo significant investments in the coming decade. The robustness of the extended network was quantitatively evaluated against the robustness of the existing network when assigning the travel demand projected for 2025. The results demonstrate that the extended network is considerably more robust in terms of average performance deterioration as well as the worst case scenario. Moreover, the critical links in each network were identified and the equity implications were analyzed in terms of how delays are distributed over travelers' population.

In developing a resilience policy, network planners should determine which measures of performance are considered (e.g. delay, disconnected) and the dimension of interest (e.g. share, mean, maximum). A network could be considered robust if it satisfies a certain design criteria in terms of the decrease in its performance under a random failure or a targeted attack. The choice of performance indicators has implications on both the robustness value of alternative networks as well as the identification of critical links. The latter should be monitored as the network evolves in order to adequately allocate resources when preparing and recovering from disruptions.

The analysis performed in this study facilitates the consideration of expansion plans impacts on network resilience in the decision making and planning processes. Due to financial constraints, planners need to balance between potentially competitive policy objectives such as improving network accessibility and connectivity which may entail trading-off network expansion and densification, respectively. To this end, the robustness value should be ultimately monetarized and incorporated into project appraisal by using information concerning link failure probabilities. The latter allow to weight the expected consequences in the event of a disruption by the respective probabilities in order to perform a sound risk analysis. By analysing large disruption database, the frequency and duration of various disruption types could be estimated as a function of link characteristics as was demonstrated by Cats et al. (2015b). Even in the lack of detailed estimations, alternative networks can be compared and prioritized by reviewing their performance metrics in order to support transport planners as demonstrated for the case of Stockholm's development plan. The robustness value of alternative networks could be considers as part of a multi-criteria analysis framework.

The approach proposed in this study is simple, scalable and could be applied to a large range of domains, including other modal transport networks (e.g. road, air, maritime) as well as to networks other than transport (e.g. information, power transmission). Notwithstanding, the method deployed in this study has several limitations which stem from the simplifications made in the network representation and assignment. Most importantly, the assignment model could be enhanced by considering probabilistic assignment principles. In addition, travel impedance could include other travel attributes such as waiting time and transfers, albeit this will require a more complex graph representation. Unlike the static assignment performed in this study, a dynamic non-equilibrium model can represent imperfect travel information, the knock-down effects generated by an unplanned disruption and en-route decisions (Cats and Jenelius 2014). The dynamic modelling of network performance would facilitate the analysis of system resilience by considering its recovery into normal operations. However, the stochasticity and computational complexity of such a model might prohibit the full scan approach undertook in this study. Note that the simplifications made in this study are likely to result in an underestimation of travel delay as travellers are assumed to have perfect pre-trip information and no spill-over effects to neighbouring links are considered. Other avenues for future studies include the analyse of different kinds of disruptions such as the impact of partial capacity reduction, multiple link failures and node closures.

ACKNOWLEDGEMENTS
The author would like to thank two anonymous reviewers and the editor for their insightful and helpful comments. The author also thanks Mitzie Molin for her assistance with the case study.

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