From flat sheets to curved geometries
Origami and kirigami approaches

Callens, Sebastien; Zadpoor, Amir A.

DOI
10.1016/j.mattod.2017.10.004

Publication date
2017

Document Version
Final published version

Published in
Materials Today

Citation (APA)
Callens, S., & Zadpoor, A. A. (2017). From flat sheets to curved geometries: Origami and kirigami approaches. *Materials Today*, *21 (April 2018)*(3), 241-264. https://doi.org/10.1016/j.mattod.2017.10.004

ELSEVIER

From flat sheets to curved geometries: Origami and kirigami approaches

Sebastien J.P. Callens *, **Amir A. Zadpoor**

Department of Biomechanical Engineering, Delft University of Technology (TU Delft), Mekelweg 2, Delft 2628CD, The Netherlands

Transforming flat sheets into three-dimensional structures has emerged as an exciting manufactur paradigm on a broad range of length scales. Among other advantages, this technique permits the us functionality-inducing planar processes on flat starting materials, which after shape-shifting, resul a unique combination of macro-scale geometry and surface topography. Fabricating arbitra complex three-dimensional geometries requires the ability to change the intrinsic curvature of initi flat structures, while simultaneously limiting material distortion to not disturb the surface featu The centuries-old art forms of origami and kirigami could offer elegant solutions, involving o folding and cutting to transform flat papers into complex geometries. Although such techniques limited by an inherent developability constraint, the rational design of the crease and cut patte enables the shape-shifting of (nearly) inextensible sheets into geometries with *apparent* intrin curvature. Here, we review recent origami and kirigami techniques that can be used for this purp discuss their underlying mechanisms, and create physical models to demonstrate and compare t feasibility. Moreover, we highlight practical aspects that are relevant in the development of advan materials with these techniques. Finally, we provide an outlook on future applications that co benefit from origami and kirigami to create intrinsically curved surfaces.

Introduction

The many developments in additive manufacturing (AM) over the last decades have significantly increased the attractiveness of this manufacturing technique to fabricate arbitrarily complex three-dimensional (3D) geometries at the nano-, micro-, and macro-scales. Examples include bone-substituting biomaterials [1], penta-mode mechanical metamaterials [2,3], triply periodic minimal surfaces [4], and energy-absorbing cellular architectures [5]. Despite many advantages of AM, one major limitation is the incompatibility with planar surface patterning and imprinting processes, which are crucial for imbuing surfaces with specific functionalities such as hydro- or oleophobicity [6], integration of electronic circuits [7], or control over cell interaction in the case of biomaterials [8]. The ability to combine arbitrarily complex surface features with arbitrarily complex geometries c enable development of advanced materials with an unp dented set of functionalities.

A potential solution to this deadlock is provided by anc manufacturing paradigm, which has been of growing int to the scientific community during recent years: the sh shifting of thin, planar sheets (which we consider 2D) into structures [9–12]. The planar sheets could first be decorated u planar patterning processes, after which they are transfor into complex 3D geometries. Additional advantages are the and inexpensive production methods of the 2D sheets and t efficient packing for storage and transportation [11,13]. M over, the 2D-to-3D paradigm is particularly interesting for development of micro- or nanoscale 3D constructs (e.g. m and nanoelectromechanical devices [14]) since conventi macroscale techniques cannot easily be scaled down to allow rication of such small structures [15,16]. As such, the ou plane transformation of 2D sheets opens up new opportur

* Corresponding author.
E-mail address: Callens, S.J.P. (s.j.p.callens@tudelft.nl).

https://doi.org/10.1016/j.mattod.2017.10.004

for the development of complex 3D structures with functionalized surfaces, especially at small length scales.

An important parameter governing the complexity of 3D structures is the surface curvature and the variation thereof throughout the structure. In order to create arbitrarily complex 3D structures from 2D sheets, the curvature of the initially flat sheets should be altered in a controllable manner. The simplest curved shapes could be obtained through bending or rolling of flat sheets. However, more complex target shapes are characterized by “double curvature” and exhibit spherical (dome-shaped) or hyperbolic (saddle-shaped) geometries, which cannot be realized with inextensional deformations of a flat sheet (this is readily understood when attempting to wrap a sphere or saddle with paper). Instead, the flat sheet would need to be subjected to in-plane distortions in order to achieve double-curved parts. At the macro-scale, for example, flat sheet-metal is plastically stretched to create double-curved shells (e.g. using (multi-point) stretch-forming [17]), and fiber-reinforced composite laminates are subjected to in-plane shearing deformations [18]. At smaller scales, researchers have recently used stimulus-responsive materials that exhibit in-plane distortions in the form of differential shrinkage [13,19] or swelling [20,21] to achieve complex curved shapes from initially flat sheets, which is closely related to non-uniform growth processes in initially planar shapes observed in nature, resulting in wavy patterns at the edges of plant leaves [22,23] and enabling the blooming of the lily flower [24].

Subjecting 2D sheets to in-plane distortions is, therefore, a feasible strategy to achieve complex curvature in 3D. Significant downsides are that the strategy is primarily applicable to soft elastic materials (such as gel sheets) and requires complicated programming of the shape-shifting or complex external stimuli to achieve the target shapes. Moreover, the in-plane distortions are likely to disturb any of the surface features that were imprinted on the 2D sheets, hence partially eliminating one of the major advantages that the 2D-to-3D shape-shifting offers. Fortunately, an alternative strategy that is more compatible with rigid materials and delicate surface features exists at the intersection of art and science: the use of *origami* (traditional Japanese paper folding) and *kirigami* (extended version of origami, also allowing cuts) to create, or at least approximate, complex curved shapes. Simply by imposing specific fold patterns, extended with cuts in the case of kirigami, initially flat sheets could be transformed into 2D or 3D geometries. Owing to their predictability, controllability, and scalability, origami and kirigami techniques have gained traction among scientists and engineers to develop deployable structures [25–27], reconfigurable metamaterials [28–32], self-folding robots [9,33], biomedical devices [34–37], and stretchable electronics [38–40]. Fig. 1 presents some examples of the potential applications of origami and kirigami across a range of length scales. By folding or cutting along the right patterns, origami and kirigami could transform planar sheets to approximate complex curved geometries, without the need for in-plane distortions.

In this review, the different origami and kirigami approaches to approximate surfaces with “double” (or “intrinsic”) curvature are discussed. We begin by providing a closer look at differential geometry and its links to origami, providing a more formal definition of the concepts “surface curvature” and “flat sheets”. T several origami techniques proposed to approximate curved s faces are reviewed in the following section, followed by a secti on recent advances in kirigami. We conclude by comparing t different techniques in terms of their suitability to approxim curved surfaces and discussing the practical aspects as well as p viding an outlook on future directions and applications.

Geometry of surfaces and origami

The notion of curvature could be somewhat ambiguous and m be applied to a range of geometrical objects, such as curves, s faces, or higher dimensional manifolds (i.e. higher dimensi generalizations of surfaces). Here, we are interested in the cur ture of surfaces, meaning how much the surface deviates from tangent plane at a certain point. We briefly introduce the c cepts that are crucial for understanding surface curvature in g eral and its relation to origami/kirigami.

Defining surface curvature

It is useful to start the discussion of surface curvature by int ducing the *principal curvatures*, κ_1 and κ_2, at a given point the surface. The principal curvatures are the maximum and m imum values of all the normal curvatures at that point (Fig. 2 The directions corresponding to these principal curvatures the *principal directions* [41]. The principal curvatures and dir tions are a convenient way of indicating how the surface cur in the vicinity of a point on the surface. It is important to n that the principal curvatures cannot be uniquely determined points where the normal curvatures are all equal. Such a po is called an *umbilical point*. The plane and the sphere are the o two surfaces that are entirely composed of umbilical poi [41,42]. The principal curvatures can be combined to obt two well-known measures of the curvature at a given point the surface. The first measure is the *mean curvature* H, which simply the mean of both principal curvatures:

$$H = \frac{1}{2}(\kappa_1 + \kappa_2)$$

A flat plane has $H = 0$ since all normal curvatures are ze However, when the plane is bent into a wavy shape (Fig. 2b), the mean curvature becomes non-zero at certain lo tions since one of the principal curvatures becomes non-zero

The second useful measure of curvature is the *Gaussian cur ture* K, defined as the product of the principal curvatures:

$$K = \kappa_1 \cdot \kappa_2$$

This concept was introduced in Gauss’ landmark paper on *Theorema Egregium* (“remarkable theorem”), considered by so to be the most important theorem within differential geomet It is clear from this definition that the Gaussian curvature a certain point vanishes as soon as one of the principal curvatu is zero, in which case the point is called a “parabolic poir When the principal curvatures are both non-zero and positi the Gaussian curvature is positive and the point is termed “el tic”. Finally, when the principal curvatures are non-zero and opposite signs (i.e. the surface curves upwards in one directi and downwards in the other), the Gaussian curvature is negati which corresponds to a “hyperbolic” point [41,42].

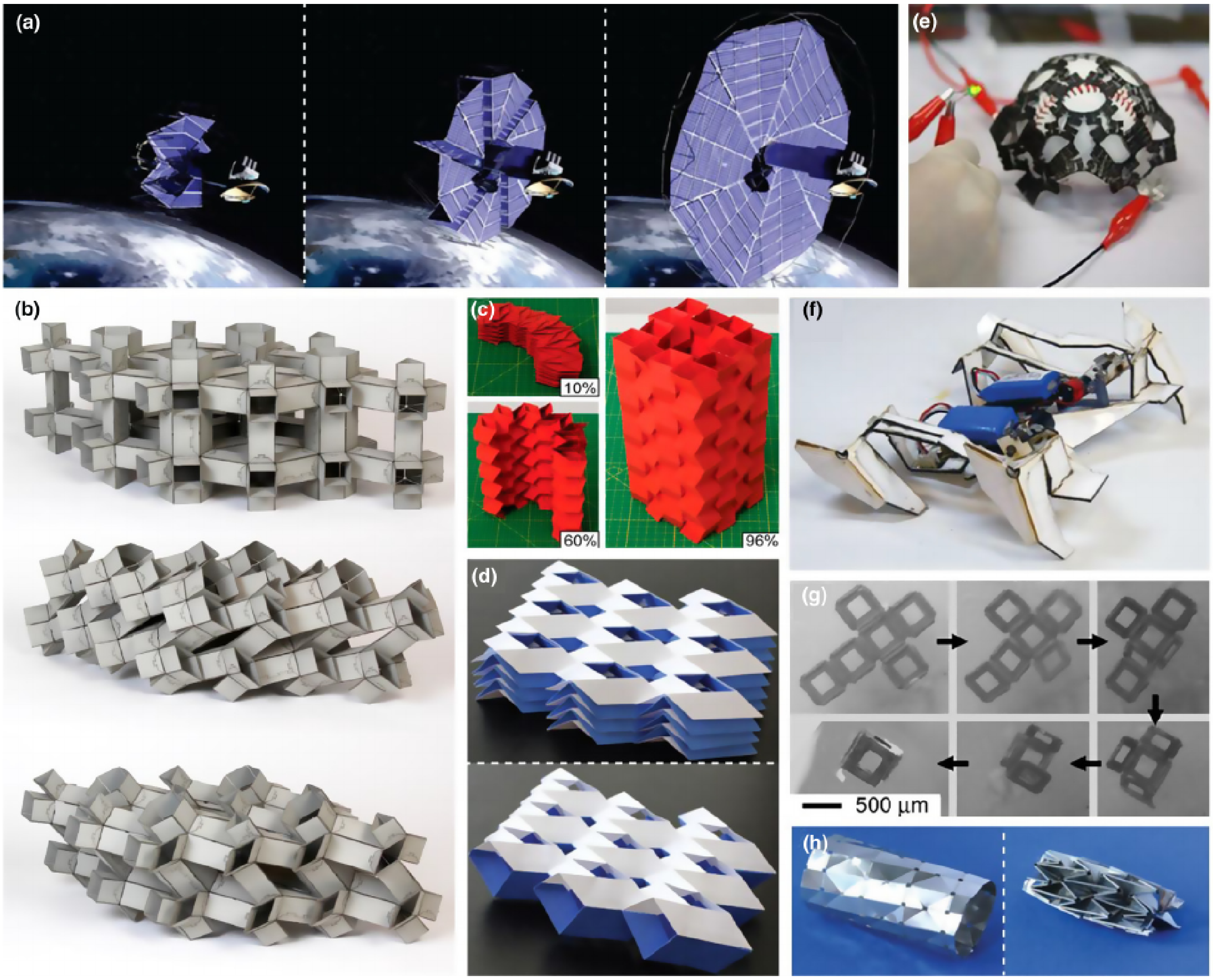

FIGURE 1

Various examples of scientific and engineering applications of origami and kirigami. (a) An artist's impression of an origami-based deployable solar arra space satellites (adapted with permission from ASME from Ref. [27]). (b) A cardboard prototype of a reconfigurable origami-based metamaterial (top), w has two degrees of freedom (middle and bottom). Adapted by permission from Macmillan Publishers Ltd: Nature [31], copyright 2017. (c) A deployable p structure based on origami zipper tubes. Reproduced from Ref. [29]. (d) Paper versions of cellular metamaterials combining aspects from origam kirigami. Reproduced from Fig. 3G and Fig. 3H of Ref. [32]. (e) A stretchable electrode based on a fractal kirigami cut-pattern, capable of wrapping arou spherical object while lighting an LED. Reproduced from Ref. [40]. (f) A centimeter-scale, crawling robot that is self-folded from shape-memory compo Reproduced with permission from AAAS from Ref. [33]. (g) A biomedical application of origami: a self-folding microscale container that could be use controlled drug delivery. Reproduced from Ref. [35] with permission from Elsevier. (h) Another biomedical application: a self-deployable origami stent based on the waterbomb pattern, developed by K. Kuribayashi et al. [34] (Reproduced with permission from Ref. [37]).

While both the mean and Gaussian curvatures could be defined in terms of the two principal curvatures, they represent a fundamentally different perspective on surface curvature. The mean curvature is an *extrinsic* measure of the surface curvature. This means that it depends on the way the surface is embedded in the surrounding three-dimensional space (which is Euclidean 3-space within the context of this paper). On the other hand, the Gaussian curvature is an *intrinsic* measure of the surface curvature, meaning that it is independent of the surrounding space and can be determined solely by measuring distances and angles within the surface itself [42–44]. In other words, the mean (extrinsic) curvature of the surface could only be determined by an observer outside of the surface that has knowledge of its surroundings, while the Gaussian (intrinsic) curvature of the surface could be also determined by a 2D resident living on the face that has no perception of the surrounding 3D space.

The distinction between these two types of curvatu important, as some surfaces might be extrinsically curved remain intrinsically flat. For example, it was already mentio that bending of a flat plane into a wavy shape gives the su a non-zero mean curvature. However, the Gaussian curva of the surface is still zero since one of the principal curvat is zero (Fig. 2b). Therefore, while the extrinsic curvature of flat plane could be changed by bending it, its intrinsic curva remains zero everywhere. Such a surface, having zero Gau curvature everywhere, is called a *developable surface*. In addi to the plane, three fundamental types of developable sur exist in 3D: the generalized cone, the generalized cylinder,

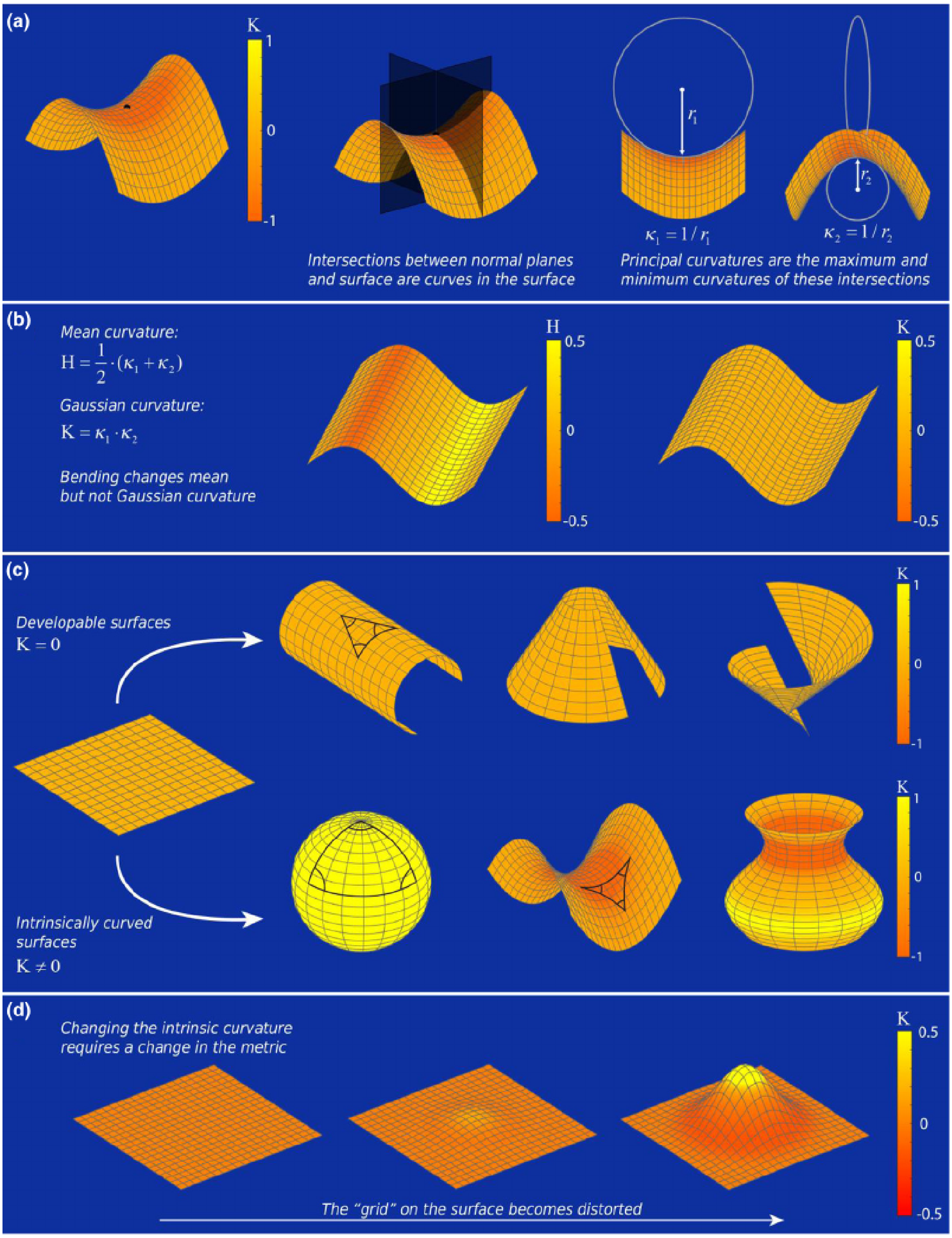

FIGURE 2

Measures of surface curvature. (a) The principal curvatures are calculated from the intersections between the normal planes at a point and the surface. intersections form curved lines in the surface, with a certain "normal curvature". The maximum and minimum values of all possible normal curvatures are principal curvatures, which are of opposite sign in this case given the fact that the surface curves "upward" in one direction and "downward" in the other. color bar indicates the Gaussian curvature. (b) The mean and Gaussian curvatures could be calculated from the principal curvatures. Bending a planar surf could change the mean, or extrinsic, curvature (left figure, color bar indicates mean curvature), but not the Gaussian, or intrinsic, curvature (right figure, co bar indicates Gaussian curvature). (c) Transforming a planar surface (color bar indicates the Gaussian curvature). Top row: three types of developable surfa which could be flattened onto the plane through bending. From left to right: a cylindrical surface, a conical surface, and the tangent developable surface space curve (a helix in this case). Bottom row: three types of intrinsically curved surfaces. From left to right: a sphere with $K > 0$, a saddle with $K < 0$, ar vase surface with varying K. The sum of the internal angles of a triangle drawn on an intrinsically curved surface does not equal π. (d) Interpreting relationship between the metric and the Gaussian curvature (color bar indicates Gaussian curvature). Creating the bell-shaped surface from an initially plane requires distortion of the grid on the plane.

the tangent developable to a space curve (Fig. 2c) [42,45,46]. The key feature of developable surfaces is that they could be constructed by bending a planar surface, without requiring extensional deformations. The observation that the Gaussian curvature of a flat plane does not change when bending the plane also holds more generally and forms the essence of Gauss' remarkable theorem: the Gaussian curvature is bending-invariant [42,47,48]. Arbitrary bending of any surface could therefore change its extrinsic (mean) curvature, but it cannot change the intrinsic (Gaussian) curvature of the surface. Consequently, a flat plane cannot be transformed into a spherical or saddle-shaped surface by bending deformations alone since these surfaces have non-zero intrinsic curvature (Fig. 2c).

We have stated that the Gaussian curvature is an intrinsic property of the surface, yet the classical definition that we have given above relies on extrinsic concepts, namely the principal curvatures. This is in fact the "remarkable" aspect in the Theorema Egregium of Gauss [42,47]. However, Gauss showed that the Gaussian curvature could also be defined on the basis of angle and distance measurements within the surface itself (i.e. intrinsically). A first understanding of this intrinsic description is obtained when considering a triangle drawn on the various surfaces, as shown in Fig. 2c. A triangle on the surface of the plane or any other developable surface ($K = 0$) will always have the sum of its internal angles, α_i, equal to π. However, on the surface of a sphere ($K > 0$), the sum of the angles is larger than π, while on the surface of a saddle ($K < 0$), the sum of the angles is smaller than π [43,47]. This angle measurement clearly relates to the intrinsic geometry of the surfaces, as a 2D resident of the surface that has no knowledge of the space in which the surface is situated could determine whether the surface has positive, negative, or zero Gaussian curvature, simply by measuring the angles of a triangle [43]. However, the resident would not be able to distinguish, for example, a flat plane from a cylinder surface since they both have the same (zero) intrinsic curvature.

A more formal "intrinsic" description of the Gaussian curvature requires the introduction of another important concept within differential geometry: the metric tensor, or simply *metric*. The metric of a surface describes the distances between the neighboring points on a surface which could be given as follows (in Einstein summation convention) [49]:

$$ds^2 = g_{ij} dx^i dx^j$$

where ds represents the distance between points and g_{ij} represents the metric components. In the case of a flat plane, the metric tensor (which is called a "Euclidean metric" in this case) is simply represented in Cartesian coordinates as:

$$g = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In which case ds^2 reduces to the standard expression:

$$ds^2 = dx^2 + dy^2$$

Physically, the metric could be interpreted as a grid on the surface [50,51]. On a flat plane, this would be a regular grid consisting of equally spaced, perpendicular lines. When the flat plane is subjected to pure bending (see Fig. 2c), the grid is not distorted and all distances and angles are preserved. For this reason, b ing is called an *isometric* deformation, i.e. it leaves the m unaffected. However, if the plane is deformed into, for exan a bell-shaped surface with regions of positive and negative G sian curvatures (Fig. 2d), the grid becomes distorted, i.e. the ric changes and becomes "non-Euclidean". This si interpretation of the metric as a grid on the surface, tho not mathematically rigorous, does provide the important ins that we are aiming for: changing the Gaussian curvature req a change in the surface metric (this leads to the inherent c lenge that map-makers face: any map of the Earth will s some level of distortion [52]). Moreover, this change in m (and thus in Gaussian curvature) cannot be achieved thro bending alone but requires stretching or shrinking of the sur

Gauss showed that the Gaussian curvature could be def entirely in terms of the components of the metric tensor its derivatives, thereby proving the intrinsic character of this vature measure [47]. As a simple example, consider a Euclidean metric defined in Cartesian coordinates of the fo

$$g = \begin{pmatrix} 1 & 0 \\ 0 & \gamma(x) \end{pmatrix}$$

The function $\gamma(x)$ describes the distances between points direction as a function of x-position ($ds^2 = dx^2 + \gamma(x) dy^2$). In case, the Gaussian curvature is indeed defined entirely in t of the metric as [47,49,50]:

$$K = -\frac{1}{\sqrt{\gamma}} \frac{\partial^2 \sqrt{\gamma}}{\partial x^2}$$

Following the above definition, the Euclidean, "flat" m introduced earlier would indeed result in $K = 0$, or zero in sic curvature. The general definition of the Gaussian curva in terms of the metric components and their derivatives not be stated here, as this requires more advanced conc from differential geometry which are outside the scope of review. The reader interested in more detailed mathema accounts of Gauss' results is referred to several excellent sou [47,48,52].

The direct relation between the surface metric and Gaus curvature has been harnessed by several researchers to con lably transform flat sheets into intrinsically curved geome [19–21,53,54]. As explained by Klein et al., this could be achi by prescribing a non-Euclidean "target metric" in the flat sh which essentially means that a non-uniform expansion or traction distribution is "programmed" into the sheets [19]. U activation by an external stimulus, differential swelling/shr ing occurs, which is accommodated by deforming into a cur 3D geometry in accordance with the newly imposed me Hence, these "metric-driven" [20] approaches represent a cessful application of Gauss' results to the shape-shiftin advanced materials. However, an important remark is that t approaches deal with real sheet materials of a small but f thickness, while our discussion thus far has only consid mathematical surfaces of zero thickness. The presence of thickness forces researchers to consider the elastic energy o curved sheets, consisting of a stretching component (E_s) a bending component (E_b), both of which depend on the s thickness [19,53,55]:

$$E = E_s + E_b$$

When a non-Euclidean target metric is prescribed in the flat sheet with finite thickness, the sheet will adopt a shape that minimizes its elastic energy E. This leads to a competition between both components of the energy: the bending energy E_b is zero when the sheet remains flat, while the stretching energy E_s is zero when the sheet achieves the curved, 3D geometry with the prescribed target metric [19,49,53]. The final shape corresponds to a balance between both contributions, which is determined by the sheet thickness t. Since the stretching energy scales with t and the bending energy scales with t^3, there will be a certain thickness that marks a transition between bending energy domination and stretching energy domination [49,56]. Consequently, the thinner a sheet becomes, the more energetically favorable it becomes to bend than to stretch [49,51,53]. In other words, the bending energy decreases more rapidly with decreasing thickness than the stretching energy does, meaning that when given the choice between bending or stretching to accommodate local shrinking/swelling, it will "cost" much more energy for the thin sheets to stretch than to bend (which is why very thin sheets are often considered inextensible membranes [55]). The sheets will thus bend in 3D to adopt the target metric (if a suitable embedding of the target metric exists), although the exact target metric will not be achieved for finite thickness since there will always be some energetic cost to bending a sheet [19,53].

In summary, the concept of surface curvature could be discussed from an extrinsic and an intrinsic perspective, using the mean and Gaussian curvature respectively. Some surfaces might be curved from an extrinsic view, yet intrinsically remain flat (a developable surface). When the aim is to achieve extrinsic curvature from a flat surface, this could be easily achieved by an inextensional bending (isometric) deformation of the surface, which leaves the Gaussian curvature unaffected. However, achieving intrinsic (Gaussian) curvature from a flat surface is more complicated, as it requires the distances between points on the surface to change (i.e. the metric should change). This cannot be achieved through bending alone, but requires in-plane stretching or shrinking of the surface. The geometrical aspects of origami introduced in the following section are better understood within the context of the ideas presented here.

Geometrical aspects of origami

Origami has inspired artists for hundreds of years to transform ordinary sheets of paper into intricate yet beautiful 2D or 3D geometries. Recently, engineers and scientists have also become attracted to origami and have studied the paper-folding art from a more mathematical perspective, giving rise to the field of *computational origami* [57]. Origami offers many interesting mathematical challenges, such as the folding of an arbitrary polyhedron from a flat piece of paper [58] or the question of flat foldability, i.e. whether a crease pattern results in a folded state having all points lying in a plane [59]. Another aspect that has received broad attention and that is of greater relevance to the folding of 3D engineering structures is the question of *rigid-foldability*. An origami design is rigid-foldable if the transition from the flat to the folded state occurs smoothly through bending at the creases only, thus, without bending or stretching the faces in between the creases. In other words, a rigid origa design could be folded from rigid panels connected with hing which is desirable for deployable origami structures made fr rigid materials, such as solar panels, medical stents, or rob [60,61].

Classical origami starts with a flat sheet of paper, which co be considered a developable surface and by definition has z Gaussian curvature. Folding this flat sheet along predefin crease lines essentially means bending the sheet at a very h radius of curvature. Since bending does not change the me of the sheet, the Gaussian curvature will remain zero at (nea all points on the folded sheet. In other words, no matter how sheet is folded, it remains intrinsically flat. It must, howev be noted that some degree of stretching is involved in the fold of paper. More specifically, Witten [55] has explained that sh folds must involve some stretching, as they would otherw result in an infinitely high bending energy. Nevertheless, t stretching is only confined to the small fold lines and it is ty cally neglected, i.e. a non-stretchable sheet with idealized sh folds is assumed [62].

When discussing origami and polyhedral surfaces, it is use to introduce yet another definition of the Gaussian curvatu known as Gauss' spherical representation. Gauss introduced t description in his original paper on the Theorema Egregium. T concept has been used, for example, by Miura [63] and Huffm [64] in the analysis of origami. Gauss' spherical representati could be obtained by first considering a closed, oriented conto Γ around a point P on an arbitrary surface (Fig. 3a). Let us coll the unit vectors on Γ that are normal to the surface and transl them to the center of a unit sphere (the Gauss sphere), effectiv tracing out a new oriented contour Γ' on the surface of t sphere and obtaining the "Gauss map" of the original conto Both contours enclose a certain area on their respective surfac say Γ encloses F and Γ' encloses G. The Gaussian curvature K then defined as the ratio of G to F, in the limit that Γ approach P [42,65]:

$$K = \lim_{\Gamma \to P} \frac{G}{F}$$

While calculating the Gaussian curvature on an arbitrary s face might not be trivial using the above definition, Gauss' sph ical representation does provide an additional interpretation the intrinsic curvature. For example, the Gauss map of a clo contour on a developable surface encloses zero area on the Ga sphere ($G = 0$), indeed corresponding to zero Gaussian curvat following the above definition (Fig. 3). On the other hand, closed contours on spherical or saddle surfaces map into clo contours with non-zero enclosed areas on the Gauss sphere, in cating the non-zero Gaussian curvature of these surfaces. N that $G < 0$ when the orientation of Γ' is opposite to that of resulting in a negative Gaussian curvature (Fig. 3a) [65].

Applying Gauss' spherical representation to the simplest ty of origami, a single straight crease crossing a flat sheet of pap once again proves that folding has no intrinsic effect on the s face. The normals on each face map into a single point on Gau sphere, while the normals on the crease between the faces are n uniquely defined and map into an arc connecting both poin

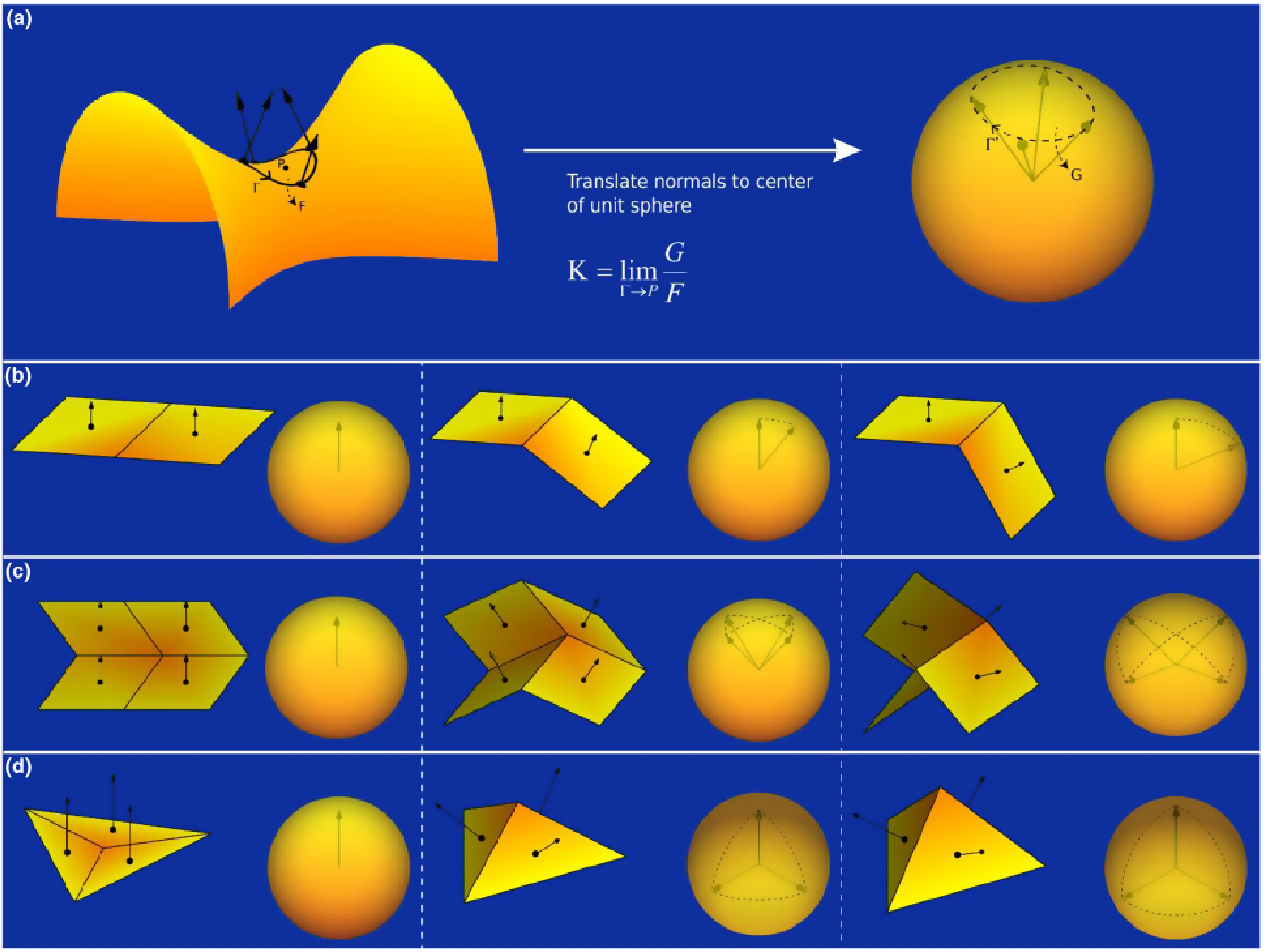

FIGURE 3

Gauss' spherical representation of the Gaussian curvature. (a) Definition of the Gauss map using a closed, oriented contour on a point on an intrins curved surface. The unit normals on Γ are translated to the center of a unit sphere and trace out a new contour Γ. (b) Folding along a simple straight c does not change the Gaussian curvature (note the zero enclosed area on the unit sphere). In the flat state (left pane), all the normals point in the direction, resulting in no enclosed area on the unit sphere. In the folded configuration, both are connected on the unit sphere through a zero-area arc. (c Gauss map applied to a unit cell of the rigid-foldable Miura-ori pattern. In the partially folded configurations (middle and right pane), the normals trace bowtie contour on the Gauss sphere, with one-half of the bowtie classifying as "positive" area (clockwise tracing) and the other half as "negative" (counter-clockwise trace), resulting in zero net area. (d) The Gauss map applied to the three-valent vertex of a tetrahedron. The transformation of the flat (left pane) to the folded state (middle and right pane) induces a change in the Gaussian curvature (non-zero area on the unit sphere), showing that a t valent vertex cannot be achieved in rigid origami.

resulting in $G = 0$ and, thus, no Gaussian curvature (Fig. 3b) [44]. Miura [63] used Gauss' spherical representation to analyze different configurations of fold lines joining at a common vertex and showed that some combinations cannot be folded rigidly [63]. For example, a vertex of valency three (three creases joining at the vertex) is never rigid-foldable: the three faces surrounding the vertex have normals in different directions, tracing out a spherical triangle on the Gauss sphere with non-zero area (Fig. 3d). This would imply that $K \neq 0$, which is not possible when rigidly folding a flat surface. Similarly, Miura showed that a four-valent vertex with all mountain ("upwards") or valley ("downwards") folds cannot be folded rigidly, while a four-valent vertex with three mountain folds and one valley fold (and vice versa) could be rigidly folded [63,65]. It must be, however, emphasized that a three-valent vertex or a four-valent vertex with all mountain folds could be folded when the rigid folding requirement is relaxed, i.e. when the faces are allowed to bend.

Based on the above insights, it might be argued that orig is not a suitable approach to create intrinsic curvature from sheets as origami deals with isometric deformations. Howe applying the right fold and cut patterns could alter the " bal" or "apparent" Gaussian curvature, without the need in-plane stretching or shrinking of the flat sheet. In esse origami and kirigami techniques allow to *approximate* intr cally curved surfaces through developable deformation many small faces connected through fold lines. The spe origami and kirigami techniques that have been used by o researchers for this purpose are described in the next sect of this review. Note that we will restrict to the traditional f of origami in which flat (Euclidean) sheets are folded. I however, also possible to apply origami to non-Euclid paper, as shown by Alperin et al. [66] who folded an orig crane from hyperbolic paper, i.e. paper with constant nega curvature.

Origami approaches

In this section, we will review different origami approaches that have been used to approximate intrinsically curved surfaces. The four different approaches discussed here, namely origami tessellations, tucking molecules, curved-crease origami, and concentric pleating, all start from a flat, uncut sheet that is folded along predefined creases.

Origami tessellations

Origami tessellations, characterized by a periodic crease pattern or "tiling" of a flat sheet, have inspired artists for many decades [67] but have also found their way into scientific and engineering applications such as compliant shell mechanisms [68] and mechanical metamaterials [30,69]. Moreover, the rational design of the tessellation pattern allows for changing the apparent curvature of flat sheets without requiring local stretching of the faces.

Miura-ori

The most widely studied origami tessellation is the herringbone pattern known as *Miura-ori*, originally introduced as an efficient packing of solar sails [25] but also observed in spontaneous wr kling of thin, stiff films on thick, soft substrates subjected to bi ial compression [70]. A Miura-ori unit cell consists of a fo valent vertex connecting four parallelograms using three mou tain folds and one valley fold (Fig. 4). An important property this origami design is that it is rigid-foldable, as indicated Gauss' spherical representation [63,65] (Fig. 3c). Schenk a Guest studied the geometry and kinematics of Miura-ori a showed that purely rigid Miura-ori has only a single degree freedom, i.e. in-plane folding and unfolding [28]. Based on analysis of a single unit cell, they concluded that a Miura- sheet is an auxetic material, characterized by a negative plane Poisson's ratio. In a later study, however, Lv et al. show that some specific configurations could exhibit a positive plane Poisson's ratio as well [71]. While rigid-foldability Miura-ori allows only in-plane deformations, experiments w simple paper models reveal that the folded sheets could a deform out-of-plane (Fig. 4d-e). Indeed, Schenk and Guest id tified saddle and twist deformation modes and showed that th are only possible in non-rigid Miura-ori, where the individ faces are allowed to bend [28]. This leads to an interest

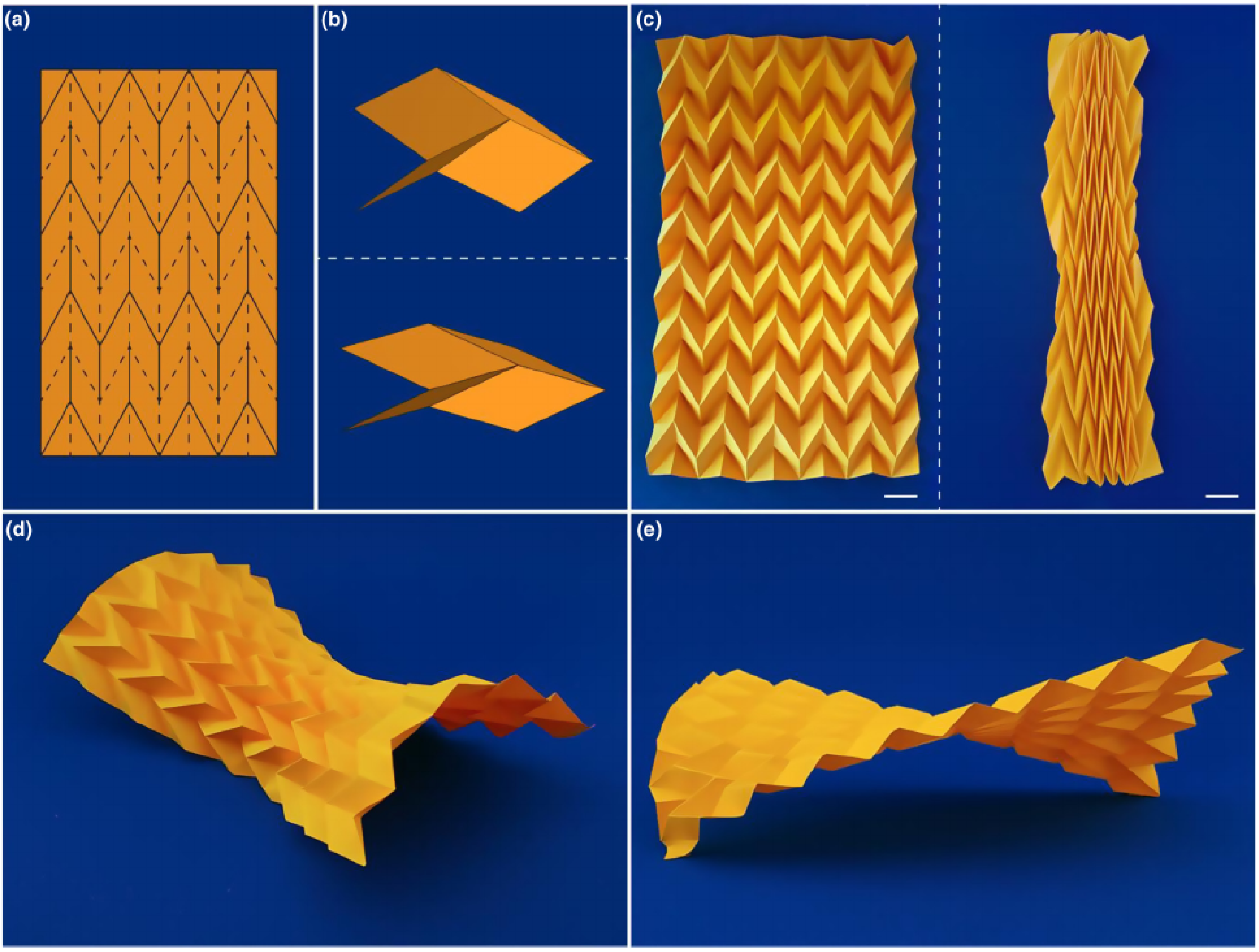

FIGURE 4

Miura-ori tessellation. (a) A crease pattern of a Miura-ori tessellation. Solid lines are mountain folds and dashed lines are valley folds. (b) Two different cells for a Miura-ori tessellation, consisting of four parallelograms connected through three mountain folds and one valley fold. (c) A paper model of Miura-ori tessellation in a partially folded state (left) and collapsed state (right), showing the flat-foldability of this origami pattern (Scale bar is 2 cm). Saddle-shaped and (e) twisted out-of-plane deformations of the (non-rigid) Miura-ori sheet.

property of Miura-ori (and origami tessellations in general): through developable deformations at unit cell level, the *global* Gaussian curvature of the sheet could be changed, making this origami tessellation an interesting candidate for compliant shell mechanisms [68,72].

Several researchers have sought for generalizations or variations of the Miura-ori pattern that approximate a curved surface when folded, without requiring out-of-plane deformations. Tachi investigated quadrilateral mesh origami consisting of quadrilateral faces joined at four-valent vertices and established rules for rigid-foldability. Starting from a regular Miura-ori pattern, he explored variations that could fit a freeform surface (e.g. a dome-shape), while remaining rigidly foldable [73]. Gattas et al. parameterized the Miura-ori to be able to systematically compare different pattern variations. They investigated five rigidly foldable "first-level derivatives" obtained by changing a single characteristic such as the crease orientation [74]. Depending on the derivative, geometries with an overall single or double curvature could be achieved, although the latter seemed to be limited to a non-developable crease pattern [74]. Sareh and Guest considered Miura-ori as one of the seventeen plane crystallographic or "wallpaper" groups (a *pmg* group) and provided a framework to obtain flat foldable symmetric generalizations of the Miura-ori, some of which could result in globally curved geometries when folded [75]. More recently, Wang et al. proposed a design method to obtain Miura-ori generalizations that approximate cylindrical surfaces upon folding [76]. While their method takes into account rigid folding and the thickness of the faces, which is useful for practical applications, it is restricted to cylindrical geometries and hence single curvature [76]. Perhaps the most complete and successful approach to approxi arbitrarily curved surfaces with Miura-ori generalizations is one recently proposed by Dudte et al. (Fig. 5) [77]. Dudte e employed constrained optimization algorithms to solve inverse problem of fitting an intrinsically curved surface w generalized Miura-ori tessellation. They showed that the sur of generalized cylinders could be approximated using flat able and rigid-foldable tessellations, which could not be gua teed for intrinsically curved surfaces. In the latter case, snap transitions were required during the action of folding or un ing, although the final configuration was strain-free [77]. researchers also showed that a higher number of unit cells re in a more accurate fitting of the surface but at the cost of a hi folding effort [77].

Other periodic tessellations

In addition to Miura-ori, several other tessellations are known among origami artists and scientists. Examples are tessellations obtained when tiling the plane with a six eight-crease waterbomb base, the former of which has been to create an origami stent [34,78]. Other famous origami tess tions were developed by Ron Resch [79,80] and have inspired entists to create origami-based mechanical metamaterials [7 freeform surface approximations [81]. Origami tessellations are rigid-foldable are of particular interest for engineering a cations. Evans et al. used the method of fold angle multip to analyze the existing flat foldable tessellations and ide those that were rigid-foldable [61]. Furthermore, the researc presented rigid-foldable origami "gadgets", local modificat to a crease pattern, to develop new rigidly foldable tessellat

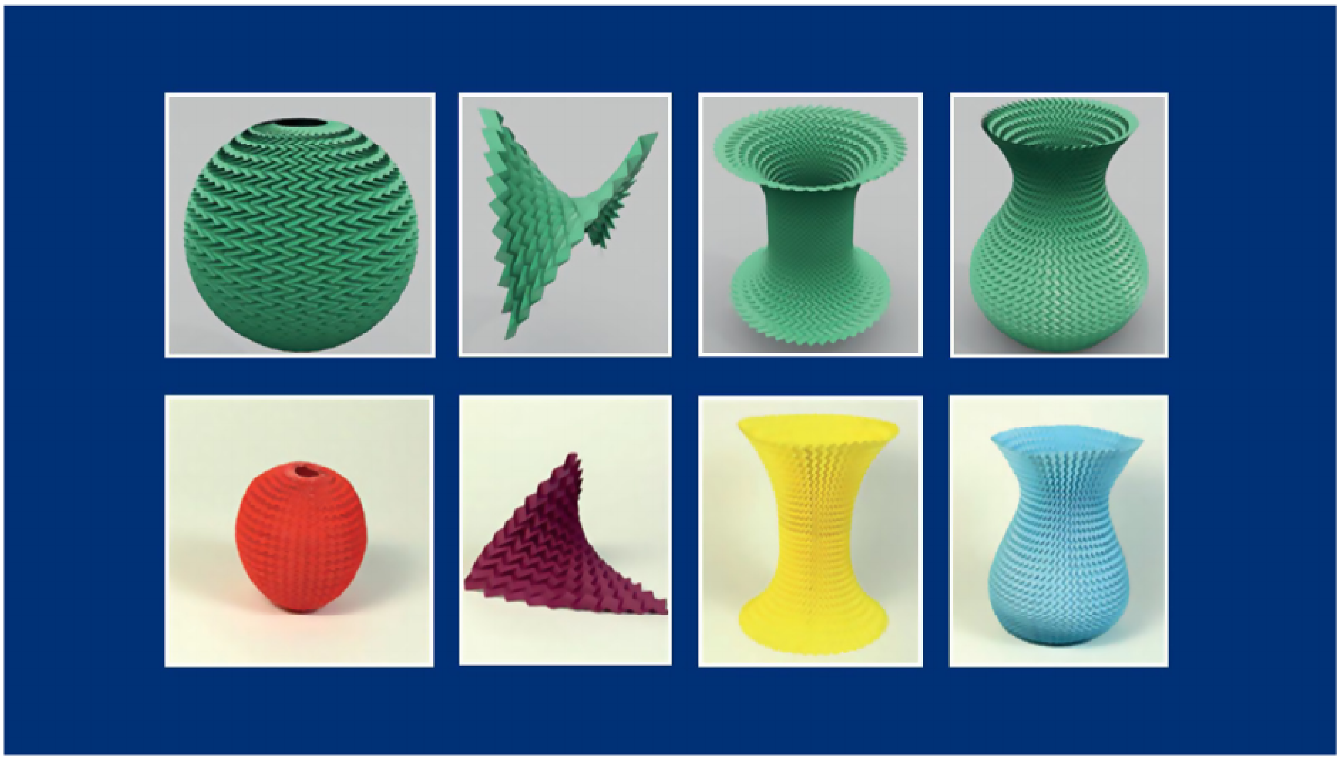

FIGURE 5

Generalized Miura-ori tessellations fitting curved surfaces. The top row depicts simulations, while the bottom row shows physical models. Adapte permission from Macmillan Publishers Ltd: Nature Materials [77] copyright 2016.

[61]. Tachi studied the rigid foldability of "triangulated" origami tessellations, in which the quadrangular faces are divided into triangles (essentially capturing the bending of faces of non-triangulated origami) [82]. Through numerical simulations (by means of a truss model), it was observed that most periodic triangulated origami tessellations exhibit two (rigid) degrees of freedom, a folding/unfolding motion and a twisting motion, with Miura-ori being an exception as it only shows the folding/unfolding motion [82].

Applying a periodic tessellation such as Resch's triangular pattern to a flat sheet essentially means texturing the sheet with small-scale structures that give rise to unusual properties on a global scale [72]. Due to the strong interaction between the local kinematics and global shape, these tessellated sheets have earned the name "*meta-surfaces*", in analogy with 3D metamaterials [68,72]. An interesting property of the textured sheets is that folds may partially open or close locally, effectively simulating local stretching or shrinking. As a consequence, the sheets could undergo large deformations and change their global Gaussian curvature, without stretching of the actual sheet material [68,72] (Fig. 6). However, the opportunities to approximate curved surfaces using this approach limited despite the ease with which small paper models co be manipulated. That is because approximating saddle sha might involve some facet and crease bending that make difficult, if not impossible, to achieve anticlastic geometr (negative Gaussian curvature) through rigid folding [81]. Mo over, for large tessellated sheets (with many unit cells), ev synclastic geometries (positive Gaussian curvature) might r be rigid-foldable. Tachi showed that smooth rigid fold of (triangulated) periodic tessellations in dome shapes obstructed once the tessellated sheet becomes too large, mak only cylindrical surfaces feasible [82]. A similar conclusion v reached by Nassar et al. [83]. Indeed, the tessellation shown Fig. 6c naturally adopts a cylindrical shape in the partia folded configuration.

Although several standard origami tessellated sheets co conform to curved surfaces, the achievable geometries limited. In order to obtain more complex 3D shapes, Ta developed the *Freeform Origami* method to generalize exist tessellations, essentially building further on his work quadrilateral mesh origami [84]. He provided mathemati

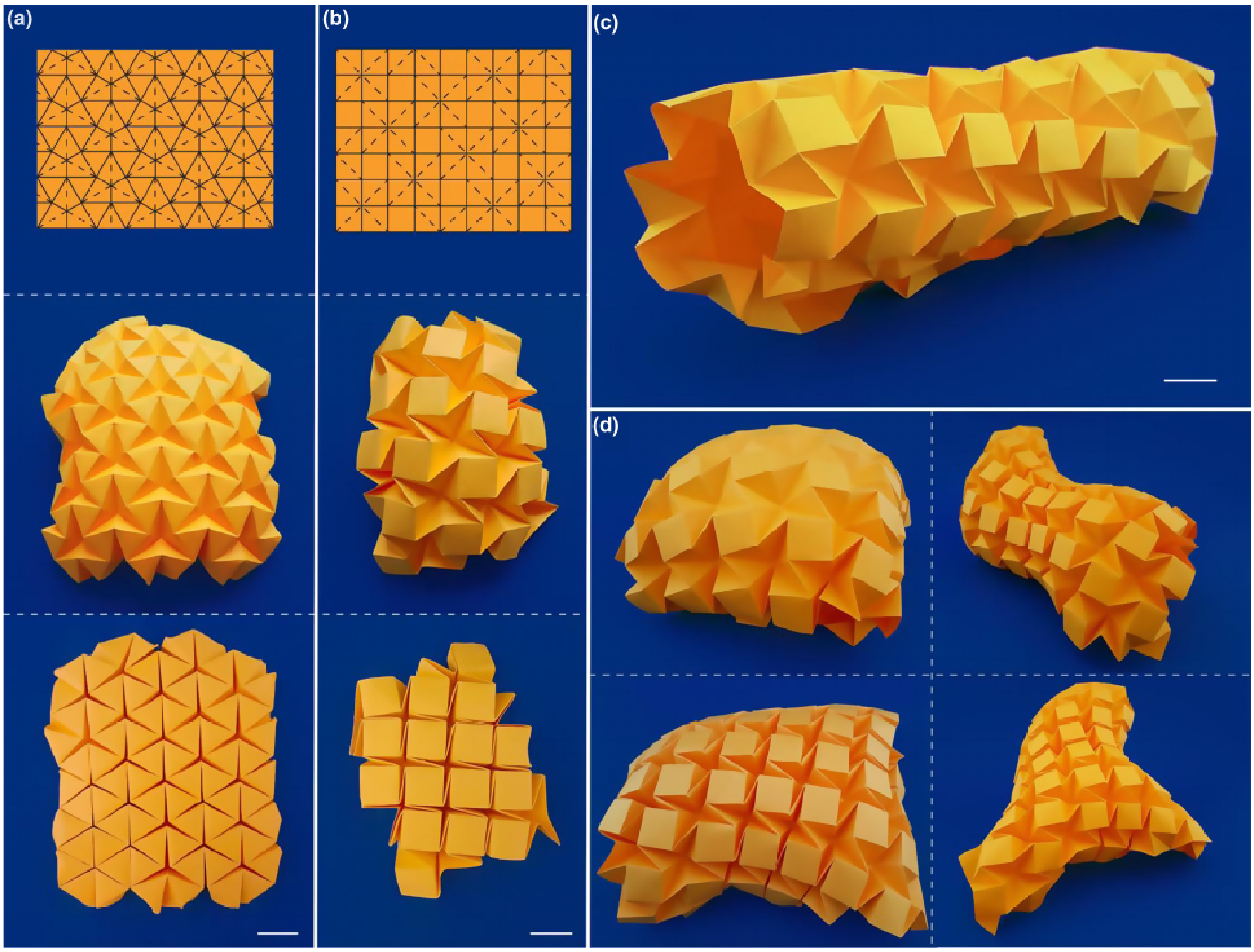

FIGURE 6

Origami tessellations. (a and b) Triangular Ron Resch and square waterbomb tessellations, respectively. Top: crease pattern (solid lines are mountain fo dashed lines are valley folds), middle: partially folded state, bottom: fully folded state (Scale bar is 2 cm). (c) Natural resting state of the partially folded squ waterbomb tessellation (for a large enough sheet), adopting a cylindrical shape (Scale bar is 2 cm). (d) Various configurations with global intrinsic curvatur the same square waterbomb tessellated sheet, obtained through locally opening and closing the unit cells.

descriptions of the conditions that apply to the traditional tessellations such as developability and flat-foldability, and numerically calculated perturbations of these tessellations while preserving those conditions. The algorithm was implemented in a software package that allows the user to actively disturb an existing folded origami tessellation and observe the corresponding changes to the crease pattern in real-time [84]. However, the software is not capable of solving the inverse problem of finding the crease pattern that belongs to a given 3D surface. Other researchers have also used mathematical methods to calculate new tessellations that could fold into 3D geometries. Zhou et al. proposed the "vertex method" to inversely calculate a developable crease pattern based on the Cartesian coordinates of a given 3D geometry [85]. While their method is versatile enough to develop the crease pattern for a structure that fits between two (single-) curved surfaces, a major limitation is that it is not applicable to intrinsically curved surfaces [85]. More recently, Song et al. built further on this work and developed a mathematical framework to create trapezoidal crease patterns that rigidly fold into axisymmetric double-curved geometries [86]. More specifically, their method calculates the crease pattern that fits both an inner and outer target surface with the same symmetry axis. While intrinsic curvatures could in this way be achieved, the proposed method is limited to very specific ring-like geometries, possessing rotational symmetry and having relatively small curvatures [86].

Tucking molecules

Other approaches to approximate intrinsically curved surfaces could be obtained from the field of computational *origami design*, in which one searches for the crease pattern that belongs to a given shape, typically a 3D polyhedron. The first well-known computational tool facilitating origami design was proposed by Lang and is based on tree-like representations of the desired shapes ("stick-figures") [87]. However, the method is restricted to calculating origami bases that need to be shaped afterward into the desired geometry. To enable the construction of crease patterns for arbitrary 3D polyhedrons, Tachi developed his well-known "origamizing" approach based on *tucking molecules* [59,88]. The starting point is a polyhedral representation of an arbitrary surface, which is made topologically equivalent to a disk (such that it is not a closed polyhedron but has a cut that allows it to open). The basic idea of the approach is to map all the surface polygons of the polyhedron onto a plane and to fill the gaps in between with tucking molecules, which are flat foldable segments, creating a 2D crease pattern in doing so [88]. The resulting crease pattern is not considered an origami tessellation in the context of this review as it is highly non-periodic and comprises polygons of different shapes and sizes. The tucking molecules connect adjacent surface polygons and are tucked away behind the visible surface upon folding (Fig. 7a). Tachi defined edge-tucking and vertex-tucking molecules, respectively bringing edges or vertices together in the folded configuration. In order to fit the desired 3D shape with the surface polygons, crimp folds are also employed to locally adjust the tucking angle [88]. The entire procedure was implemented in a software package for which the input is a polygon mesh and the output is a 2D crease pattern, allowing the creation of complex origami structures that were never folded before, such as the origami Stanford bu (Fig. 7c) [59]. Tachi attributed the increased practicality of approach compared to earlier origami design methods to t reasons: multi-layer folds rarely occur, the crimp folds offer s tural stiffness by keeping vertices closed, and the method h relatively high efficiency, defined by the ratio of polyhedral face area to required paper area [59]. Despite its versatility generality, the origamizing method has the drawback that s 3D polyhedrons cannot be mapped into a 2D pattern, a prob that was recently addressed by Demaine and Tachi [89], or the proposed pattern is inefficient. Moreover, the flat fol requirement of the tucking molecules significantly reduces applicability of this method to the folding of stiff, thicker m rials [81]; and the presence of crimp folds obstructs smooth ing [59], making this method intractable for indus applications.

Tachi also proposed a more practically applicable metho approximate curved surfaces, combining aspects from his ea work on freeform origami [84] and the origamizing appro [88]. The basic idea is that generalizations of Resch's tessellat are calculated that fit a given polyhedral surface [81]. Contra the origamizing approach, the surface polygons do not hav be mapped onto the plane; but a first approximation of the sellation is directly obtained from the 3D structure, after w it is numerically optimized to become developable and to a collisions of faces. As such, the implementation of this met is related to the earlier work on *Freeform Origami* [84], but it is considered in this section since (simple) tucking molecules used. Tachi defined a "star tuck" and variations thereof as b ing blocks to tessellate the given 3D surface [81]. The fundar tal difference between conventional tucks and the star tuc that the latter could also exist in a semi-folded state and do have to be folded flat (Fig. 7b). The surface polygons coul arranged to locally fit the desired shape through partial ope of the star tucks, while this required crimp folds in the origa ing approach. The algorithm was implemented in a soft package, allowing users to interactively design the genera Resch tessellations corresponding to a given surface (Fig. However, for highly complex surfaces, which would require nificant stretching and shrinking to become developable, c patterns cannot always be generated. Moreover, smooth folding of the tessellations is not guaranteed for all c Nonetheless, the proposed method has significant potentia the practical folding of advanced materials into intrinsi curved surfaces.

Curved-crease origami

When discussing curvature and origami, it is natural to also sider curved-crease origami. While this variant of traditional gami has interested artists for several decades, the mathem of curved-crease folding is underexplored and practical app tions of origami have been primarily limited to straight cre [90]. Straight-crease and curved-crease origami is fundamen different since both faces adjacent to a curved crease *always* to bend in order to accommodate the folding motion [91,92 other words, curved-crease origami is never rigid-foldable. consequence, curved-crease origami cannot be reduced to a ter of tracking vertex coordinates as in the case of rigid orig

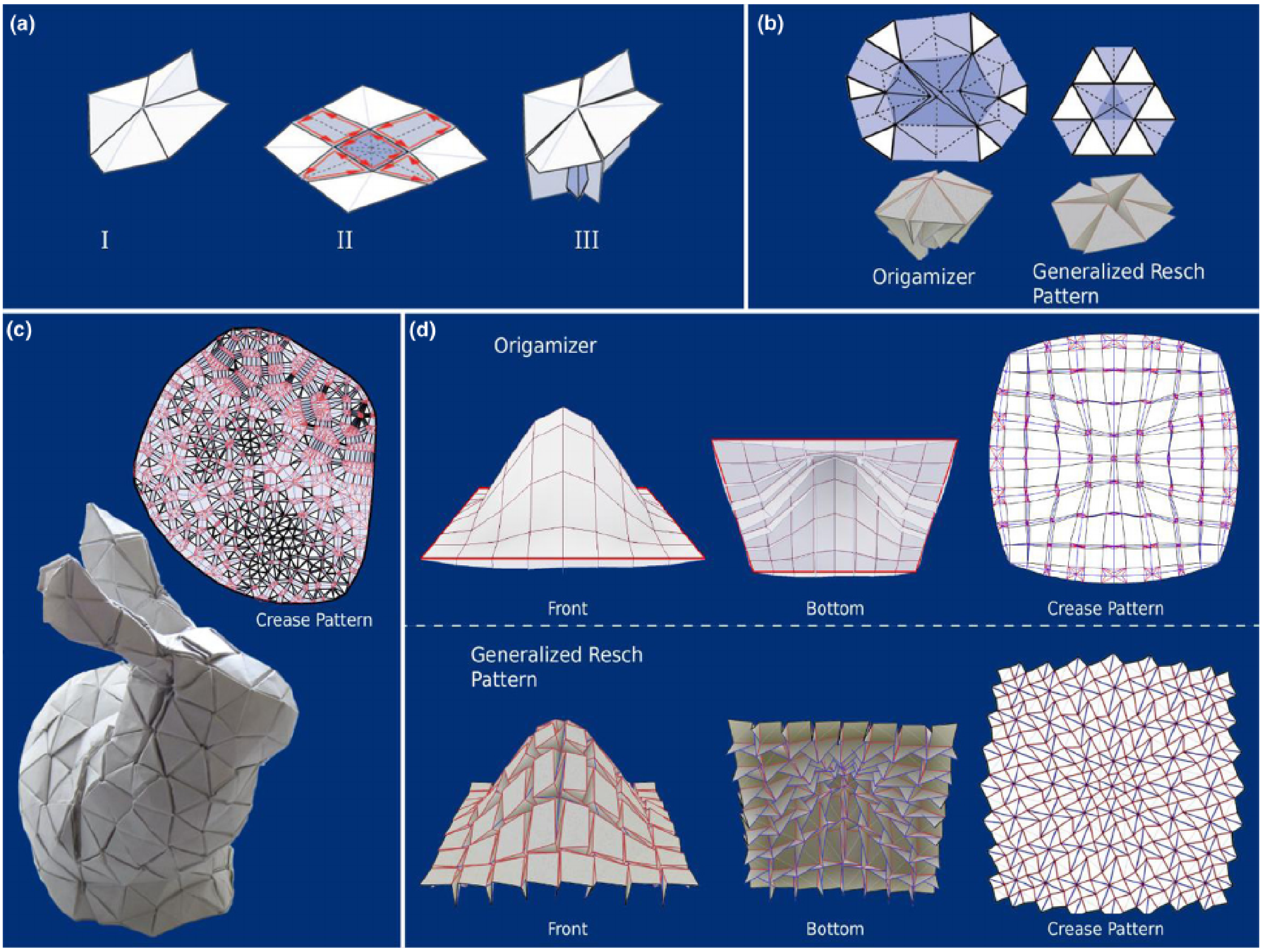

FIGURE 7

Origami involving tucking molecules. (a) The definition of a tucking molecule used in the "Origamizer" approach. (I) A section of a polyhedral surface, (II) flattened polyhedral surface, showing the surface polygons connected through edge- and vertex-tucking molecules. (III) The folded configuration, show the excess material being tucked away behind the outer surface. Reproduced with permission from IEEE from Ref. [59]. (b) Comparison between tuck molecules in the "Origamizer" approach and the generalized Ron Resch tessellation approach. Reproduced with permission from ASME from Ref. [81]. Origami Stanford bunny, folded from a single-sheet crease pattern created using the "Origamizer" software. Reproduced from Fig. 1 from Ref. [89]. Comparison between the surface approximations of the "Origamizer" approach (top) and the generalized Ron Resch pattern approach (bottom) to intrinsically curved bell-shaped surface. Note the partially opened tucking molecules in the latter approach. Figures and crease patterns were obtained us "Origamizer" [59] and "Freeform Origami" [73] (red lines are mountain folds, blue lines are valley folds).

and the bending stiffness of the folded sheet becomes an important parameter [90]. Moreover, the bending induced in the faces necessitates simultaneous folding along multiple creases, complicating automated folding processes [93].

One of the first and most influential analyses of curved creases was performed by Huffman, using Gauss' spherical representation to examine the local folding behavior [64]. Indeed, the Gauss map of a closed contour crossing a straight-crease maps into a zero-area arc while the map of a contour encompassing a curved crease has non-zero area due to the facet bending, which is indicative of its non-rigid-foldability. The geometry of curved-crease folding has been further explored by Duncan and Duncan [91] and Fuchs and Tabachnikov [94]. They presented several theorems relating the properties of the curved crease to those of the adjacent faces, which are outside the scope of this review. The most important take-away is that folding along a curved crease satisfies the developability of the sheet, meaning that a curved-folded origami consists of developable patches of eit a generalized cylinder, a generalized cone, or a tangent dev opable to a space-curve [91,95,96]. Hence, folding along curv creases cannot alter the intrinsic curvature of the sheet, as in-plane distortion of the faces occurs. However, curved-cre origami does provide means to alter the *global* intrinsic curvat of flat sheets, in a manner similar to straight-crease folding. M specifically, we identify two approaches to approximate non-z Gaussian curvature: the use of curved-crease couplets and fold along concentric curved creases. In the current sub-section, o the first approach is discussed, as the latter fits within the broa concept of concentric pleating that is treated in the next s section.

Curved-crease couplets, a term introduced by Leong [97], pairs of curved and straight creases that have been employ by origami artists to create 3D origami with apparent posit and negative intrinsic curvatures. Typically, axisymme

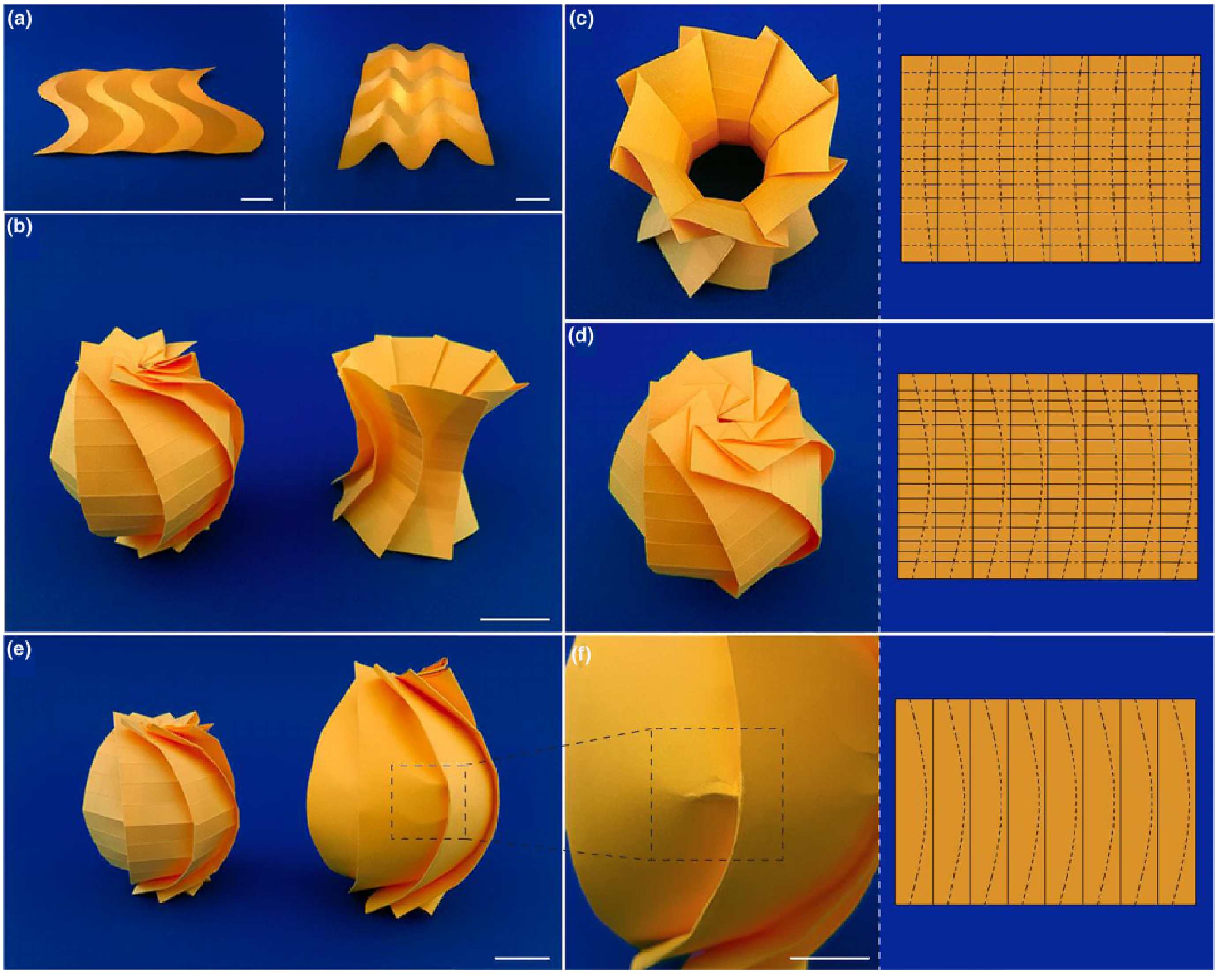

FIGURE 8

Curved-crease origami. (a) Simple examples of curved-crease origami (Scale bar is 2 cm). (b) An origami sphere (positive Gaussian curvature) and an ori hyperboloid (negative Gaussian curvature) created with the method described by Mitani [98] (Scale bar is 2 cm). (c and d) Top views of the ori hyperboloid and sphere (left panes) and the associated crease patterns (right panes, solid lines are mountain folds and dashed lines are valley fold Comparison between the standard origami sphere obtained with Mitani's method [98] and the "smooth" variant (Scale bar is 2 cm). (f) Closer view wrinkle in the smooth origami sphere, indicative of the frustration between curved and straight creases (left) and the associated crease pattern wi horizontal creases (right) (Scale bar is 1 cm).

structures are created from relatively simple crease patterns (Fig. 8). A design method and software tool to generate crease patterns based on "rotational sweep" was proposed by Mitani [98] and a very similar tool was created by Lang [99]. The basic idea is that a flat sheet is "wrapped" around the desired cylindrical or conical geometry and that the excess material is folded into *flaps*, sections of material that are only connected at one edge [100], on the outside surface. This is different from Tachi's origamizing approach [59] in which excess material is tucked away inside the geometry, resulting in more complicated crease patterns [98]. A flap in Mitani's method consists of a kind of curved-crease couplet, containing a straight mountain crease and a piecewise linear valley crease, approximating a curved line. The latter crease represents half of the vertical cross section of the desired shape and, when revolved around the vertical axis, traces out this shape [97,98]. To create the crease pattern, the curved-crease couplets are simply repeated N times and arranged on a rectangular sheet (for cylindrical geomet or on an N-gon (for conical geometries), with higher valu N resulting in higher rotational symmetry. Using this met double-curved shapes could be approximated, as show Fig. 8. More recently, Mitani also proposed a variant of method in which the flaps are replaced by "triangular p protrusions" [101]. Again, the excess material that results w wrapping the desired geometry is placed on the outside sur this time in a slightly different manner involving four cre instead of two.

Although recent efforts have been made to unders curved-crease origami from a more mathematical perspe [102], it remains a field that is primarily reserved for artist such, only limited work has been done that explores the cap ities of curved-crease folding to approximate intrinsically cu surfaces. Nonetheless, it is expected that the use of curved cre has a significant potential in practical origami not onl

achieve complex geometries but also for kinetic architectures [96] and shape-programmable structures [103].

Concentric pleating

As a final category of folding strategies to approximate intrinsically curved surfaces, we consider origami based on concentric pleating, i.e. alternately folding concentric shapes into mountains and valleys. Geometries with apparent negative Gaussian curvature spontaneously result after folding the remarkably simple crease patterns (Fig. 9). The original crease patterns for this origami consisted of equally spaced concentric squares or circles, although variants with ellipses or parabolas have also been folded [90]. One might classify the concentric pleating as a type of origami tessellation, however, we consider it separately due to its remarkable properties.

The classical model with the concentric squares is called the pleated hyperbolic paraboloid or simply *hypar*, after the negatively curved surface it seems to approximate. As explained by Demaine et al., the 3D shape naturally arises due to the paper's physics that balances the tendency of the uncreased paper to remain flat and that of the creased paper to remain folded [104]. Seffen explained the geometry by considering the pleating as a "corrugation strain" toward the center without causing an axial contraction of the hinge lines, thereby forcing the model to deform out-of-plane [68]. Thus, the pleating introduces a d tortion of the flat sheet that is relieved by settling on an ener minimizing 3D configuration. This principle of anisotropic str (shrinking) has been recently used by van Manen et al. to p gram the transformation of flat shape memory polymer she into an approximation of a hypar, using thermal activati [105].

Although paper models of the pleated hypar are ubiquito and well-known among origamists, mathematicians have qu tioned whether the standard crease pattern of concentric squa could actually result in the pleated hypar. More specifically: d a *proper folding* (folding angles between 0 and π) along exac these creases exist? Demaine et al. proved the surprising fact t it does not, hence the folding along the standard crease patt cannot result in the hypar without some stretching or additio creasing of the paper [104,106]. The problem lies in the twist of the interior trapezoidal faces of the hypar. While a stand piece of paper could be effortlessly twisted and curled in spa Demaine et al. proved that this twisting should not occur the interior faces of the hypar. Using aspects of differential geo etry, such as the properties of torsal ruled surfaces, the resear ers proved two theorems: straight creases must remain strai after folding and a section of the paper bounded by strai creases must remain planar and cannot bend or twist [10

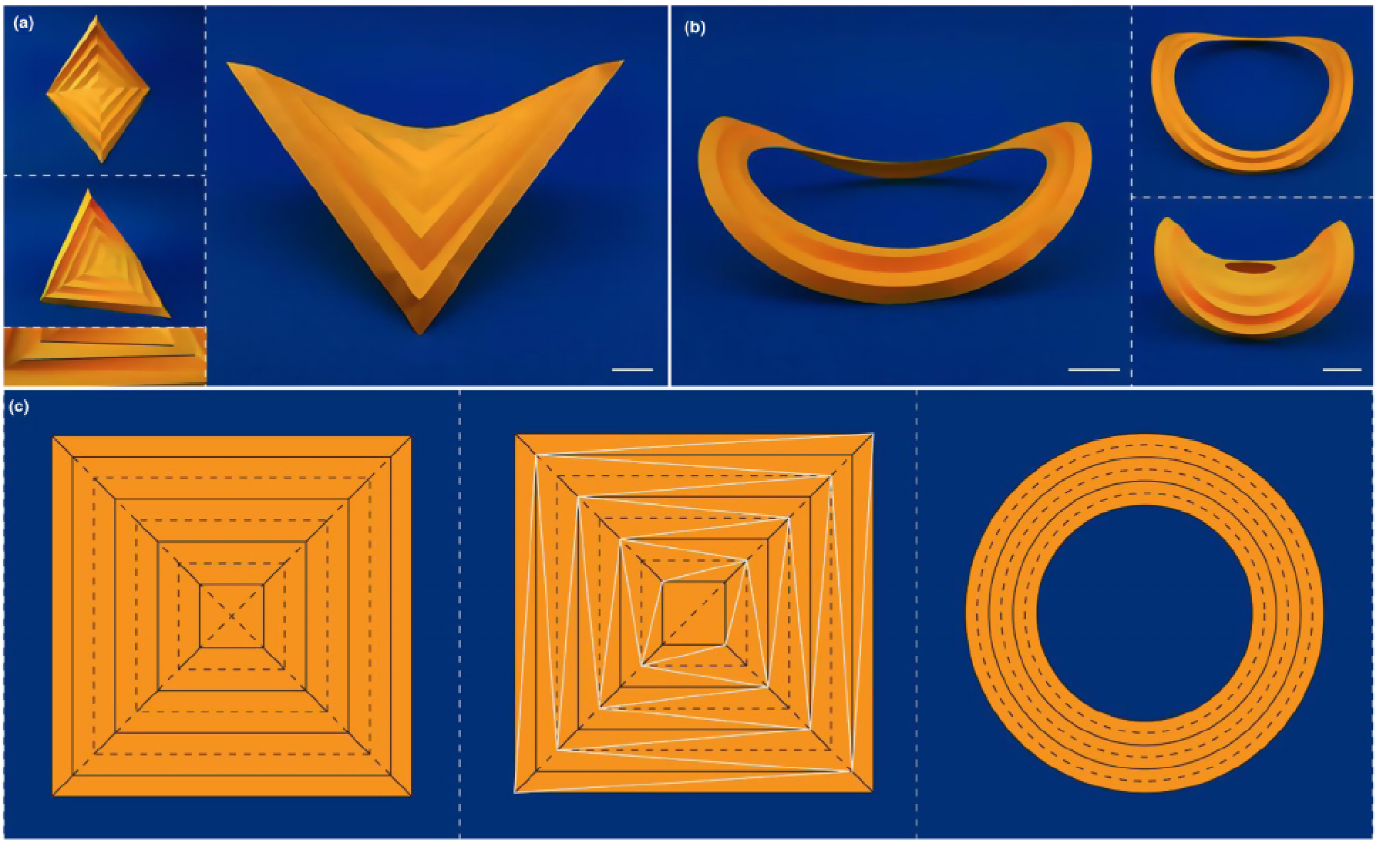

FIGURE 9

Concentric pleating origami. (a) Origami hyperbolic paraboloid ("hypar"), obtained by pleating concentric squares. Top left: top view, middle left: side vi bottom left: Closer view of the twisted faces in the standard hypar model, right: side view showing the global saddle-shaped geometry of the hypar (Scale is 2 cm). (b) Circular variant of the hypar, obtained by pleating concentric circles (with center hole). Left: side view of an annulus with three creases (Scale is 2 cm), top right: top view of an annulus with three creases, bottom right: side view of an annulus with eight creases (Scale bar is 1 cm). (c) Different cre patterns. Left: standard crease pattern for the square hypar, middle: triangulated crease pattern for the square hypar, white lines represent the additio creases that enable proper folding of the hypar (adapted from Ref. [104]), right: crease pattern for the circular hypar (solid lines are mountain folds, dash lines are valley folds).

Demaine et al. conjecture that the actual folding of the hypar from the standard crease pattern is enabled through additional creases in the paper, potentially many small ones. Alternatively, some stretching at the material level might occur [106]. In any case, it is clear that the folding of the standard crease pattern into the pleated hypar is highly non-rigid, as could also be intuitively understood when drawing a closed curve in one of the square rings: the curve crosses four folds with the same mountain or valley assignment, which cannot fold rigidly according to Gauss' spherical representation [65].

The theorems of Demaine et al. [104] have implications for straight crease origami that exhibits non-rigid behavior, such as the Miura-ori discussed above. It was explained that a (partially) folded Miura-ori could deform out-of-plane through facet bending, which should not be possible according to the above theorems. However, facet bending may be enabled by an additional spontaneous crease in the quadrangular facets, making the facet piecewise planar and satisfying the theorems in doing so. This "triangulation" of the facets has been employed by researchers to capture facet bending in mathematical origami models [69,72,77,107,108]. Moreover, Demaine et al. [104] proved that a triangulation of the standard hypar crease pattern also renders the pattern rigid-foldable, making it possible to create hypars from more rigid materials such as sheet metal [104]. The insights into the hypar crease pattern are also relevant for the curved-crease couplets that were introduced in the previous section, consisting of alternating curved and straight creases. The curved creases require the adjacent faces to bend, while the straight creases do not allow bending. Furthermore, the straight creases do not remain straight in the folded 3D model. Following the theorems of Demaine et al. [104], it thus seems that these curved-crease couplets cannot be folded. This problem is alleviated in Mitani's method [98] by including horizontal creases and representing the curved crease as a piecewise linear crease (Fig. 8c and d), thereby ensuring that the model remains piecewise planar. It must also be noted that the theorems of Demaine et al. [104] are applicable to interior faces (not at the boundary) and fold angles between 0 and π, while the curved-crease couplets end at the paper boundaries and the straight creases seem to be folded at an angle of π. Indeed, Mitani states that the horizontal creases can be omitted when the straight creases are folded close to π [98], resulting in a smooth folded geometry (Fig. 8e). However, closer inspection of such models still reveals the occurrence of small, spontaneous kinks or wrinkles, indicative of the frustration between the straight and curved crease (Fig. 8f).

In addition to the pleated hypar, another classical model is obtained by pleating concentric circles with a hole in the middle (Fig. 9b). Similar to the hypar, this pleated annulus deforms into a saddle, with the degree of curvature depending on the fold angles. Mouthuy et al. attributed this specific deformation to the "overcurvature" of the ring, which is a measure of how much the curvature of the ring exceeds that of a circle with the same circumference [109]. Indeed, the pleating causes the curvature of the concentric creases to increase while their length is preserved, resulting in overcurvature. While Demaine et al. proved that the standard hypar crease pattern cannot be folded without additional creases or stretching, it remains unknown whether this is also the case for the pleated annulus [104]. However researchers conjecture that the annulus could be folded f exactly the given crease pattern and that additional creases stretching are required. Dias et al. investigated the mecha of the simplest type of pleated annulus: a paper strip with a gle circular crease [110]. The researchers provided analy expressions for the elastic energy of the annulus, to which l the faces and the crease contribute. While the incompatib between the pleating and the resistance to in-plane stretc forces the model to buckle out-of-plane, the actual shape it se on is determined by the minimization of this elastic en [110]. Later, Dias and Santangelo extended the work to a ple annulus with several concentric circles and investigated po tial singularities that might arise when attempting to fold model from the given crease pattern [111]. The researchers not prove that the crease pattern is exactly foldable, but t results indicated that singularities do not occur for a sufficie narrow crease spacing, supporting the conjecture of Dem et al. [104,111].

Concentric pleating is a captivating type of origami, as simple crease patterns result in geometries with apparent in sic curvature. Although the mechanisms of this technique not yet fully understood, particularly for curved creases, con tric pleating could offer an interesting pathway to achieve in sic curvature. Especially when extreme values of overcurva are induced or when different types of hypars or annuli are c bined, complex geometries may arise, examples of which are "hyparhedra" proposed by Demaine et al. [112].

Kirigami approaches

Kirigami is an art form that is closely related to origami but involves cutting the paper at precise locations. Kirigami has been explored to the same extent as origami but has rece gained traction among scientists as a promising paradigm tov stretchable electronics [38,39,113,114] or advanced honeyc structures [115–117]. In this section, we review two distinct gami approaches that could be employed to approximate in sically curved surfaces, namely lattice kirigami and kirig engineered elasticity.

Lattice kirigami

Lattice kirigami is a relatively novel and promising cutting folding technique that was introduced by Castle et al. [62 essence lies in removing some areas from the sheet through ting, after which the resulting gaps are closed through fol along prescribed creases. Lattice kirigami has its roots in cry lography, particularly in the defects arising in crystal latt Understanding the essentials of this technique therefore req some terms and concepts from crystallography.

The starting point is the honeycomb lattice, represented 2D tessellation of regular hexagons. An infinite flat plane (Gaussian curvature) could be tiled using only hexagons, but is not possible for intrinsically curved surfaces such as sphere saddles [118]. For example, a soccer ball cannot be tiled hexagons entirely but requires twelve pentagons to confor the spherical shape. The insertion of a pentagon or a hepta within a tiling of hexagons is known as a lattice *disclina*

which is a type of topological defect that disrupts the orientational order of the lattice [118,119]. These local lattice distortions cause the surfaces to deform out-of-plane to relieve in-plane strains, reminiscent of the metric-driven principles discussed before. The disclinations themselves form concentrated sources of Gaussian curvature: pentagons result in positive Gaussian curvature and heptagons in negative Gaussian curvature [118]. Fig. 10 illustrates the effect of disclinations in a hexagonal weave when a single hexagon is replaced by a pentagon or a heptagon, a technique which has long been employed by basket weavers to create complex shapes [120]. Another type of topological defect is a *dislocation*, which disturbs the translational symmetry of the lattice and is formed by a dipole of disclinations (with opposite topological charge) [118,119]. While disclinations and dislocations are considered defects in a topological sense, they are often necessary distortions of the crystal lattice in natural processes. For example, Sadoc et al. showed that these defects are crucial elements in phyllotaxis, the efficient packing algorithm that nature uses in self-organizing growth processes, such as the spiral distribution of florets in flowers [119,121]. The work of Sadoc et al. [119] was in fact a direct inspiration for Castle et al. to develop lattice kirigami [62].

The basic idea behind lattice kirigami is to strategically remove areas from a honeycomb lattice, paste the newly formed edges together and fold along prescribed creases to create disclination dipoles, resulting in a stepped 3D surface with local co centrations of Gaussian curvature. Fig. 11a provides a sim example showing two disclination dipoles at the ends of cut [62]. Inspection of a single disclination dipole reveals t the cutting and pasting transforms one hexagon into a pentag (by removing a wedge of $\pi/3$) and combines two partial he gons into a heptagon (i.e. adding a wedge of $\pi/3$), see a Fig. 11c. By systematically exploring cutting and pasting on honeycomb and its *dual lattice*, Castle et al. established the ba rules for lattice kirigami that satisfy a no-stretching conditi and preserve edge lengths on the lattices [62]. The research identified the basic units of lattice kirigami: i.e. a 5–7 disclinat pair and a 2–4 disclination pair, with the values indicating coordination number of the vertices (Fig. 11c and d). The ga that are left after cutting are closed through "climb" or "glid moves, or a combination of both, in order to result in a stepped surface (Fig. 11a–d). Additionally, the researchers inv tigated the "sixon" (Fig. 11e), which is obtained by remov an entire hexagon from the honeycomb and closing the g using appropriate mountain and valley folds in the adjac hexagons. Through their basic rules, Castle et al. [62] construc the foundations for an elegant and new approach tow stepped approximations of freeform surfaces.

Sussman et al. built upon these foundations and demo strated that lattice kirigami is well-suited to obtain stepp

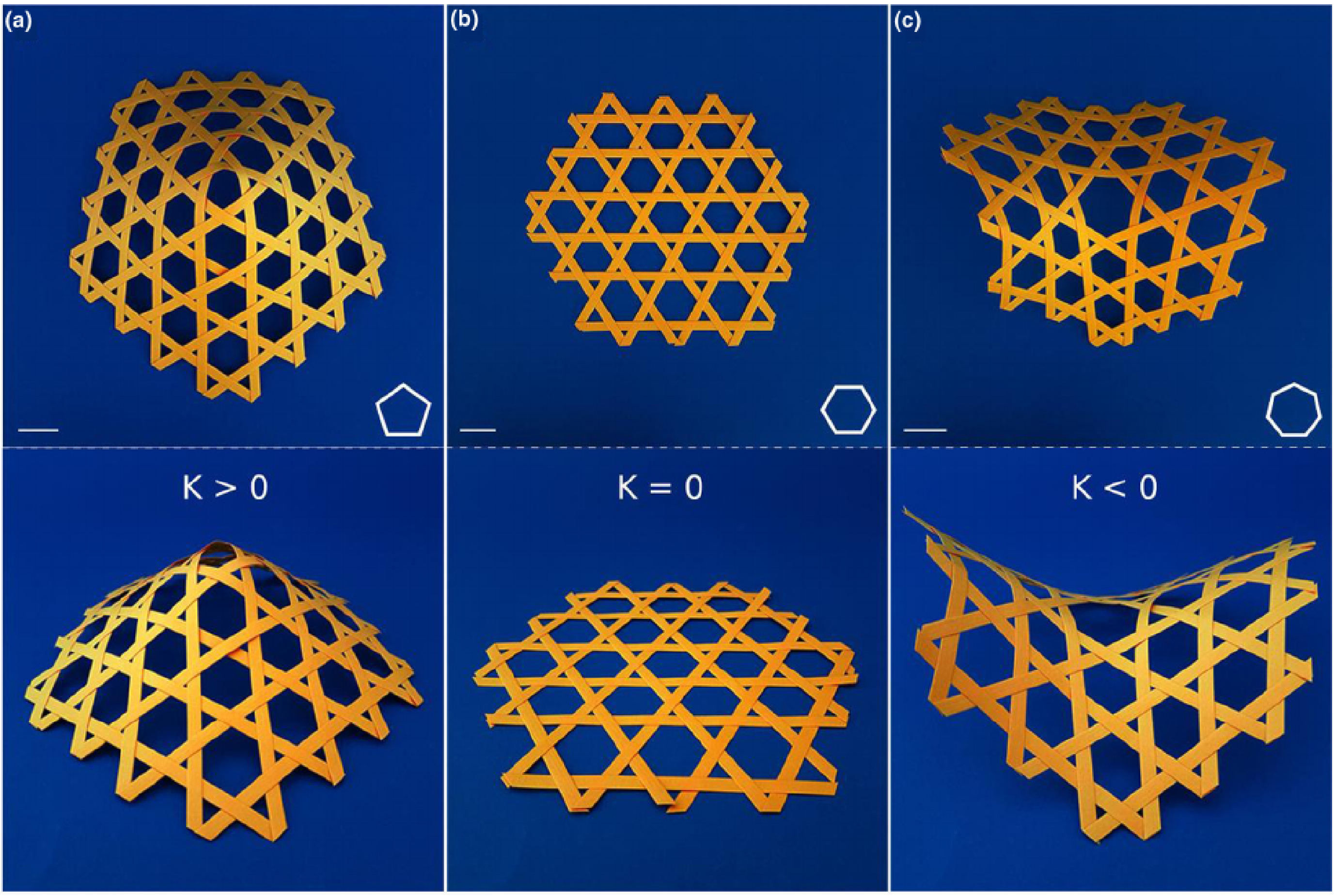

FIGURE 10

Lattice disclinations in a hexagonal lattice. (a) Inserting a single pentagon in a hexagonal weave induces positive Gaussian curvature (Scale bars are 2 cm). A hexagonal weave without lattice disclinations remains flat. (c) Inserting a single heptagon in the hexagonal weave induces negative Gaussian curvature (physical models photographed were made based on the work presented in Ref. [120]).

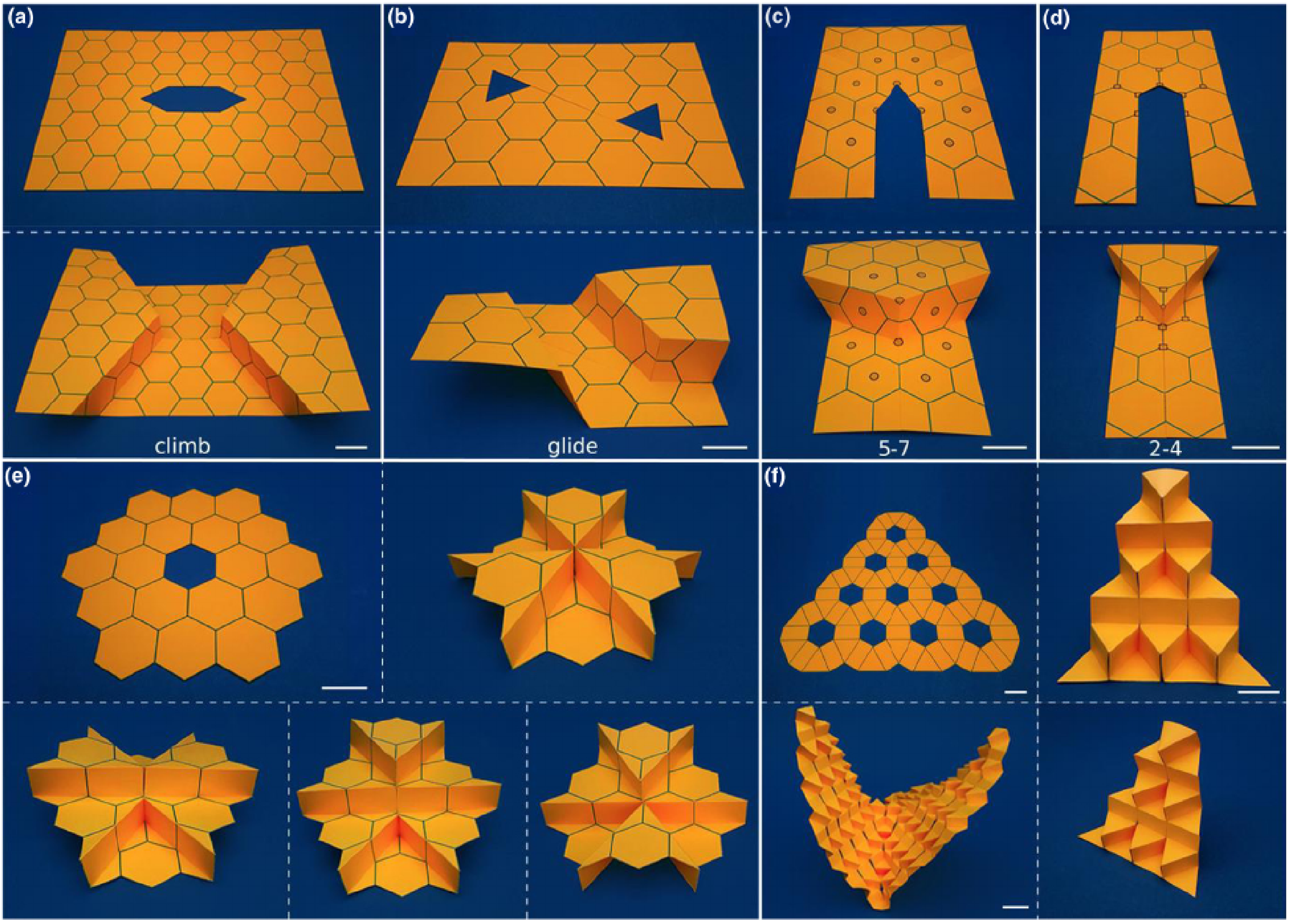

FIGURE 11

Lattice kirigami. (a) The basic principle of lattice kirigami: a wedge is removed from a honeycomb lattice (top), the edges are brought together and the p is folded along prescribed fold lines (known as a "climb" move). (b) Another basic move, the "glide", in which the gaps are closed through folding and sl along the slit connecting the two excised triangles. (c) A "5–7" disclination dipole, characterized by one vertex surrounded by five hexagon centers anc vertex surrounded by seven hexagon centers (represented with the solid circles). d) A "2–4" disclination dipole: one vertex has two neighboring hex corners, the other vertex has four neighboring corners (solid squares). (e) Excising an entire hexagon results in a "sixon", which could be folded into diff configurations by popping the plateaus up or down. (f) Folding a pluripotent "sixon sheet", i.e. a tessellation of sixons on a triangular lattice, enables step approximations of curved surfaces (e.g. left bottom pane). All scale bars are 2 cm.

approximations of arbitrarily curved surfaces, using a relatively simple inverse design algorithm [122]. Key to their approach is that the kirigami motifs presented by Castle et al. [62] could be folded into several configurations by "popping" the plateaus up or down (Fig. 11e). By connecting many of these motifs together in admissible configurations and properly assigning the plateau heights, complex stepped structures could be obtained. Sussman et al. first considered the use of standard 5–7 climb pairs, but this approach has the important limitation that every target structure requires a new fold and cut pattern [122]. In order to obtain a truly *pluripotent* kirigami pattern that could fit several target shapes, the researchers used sixon motifs. These sixons could be conveniently arranged on a triangular lattice with the centers of the excised hexagons on the lattice points (Fig. 11f). The hexagonal gaps are then closed by folding the remaining hexagons in either one of their allowed configurations (i.e. popping the plateaus upward or downward). As a consequence, a myriad of stepped surfaces could be achieved from this basic kirigami tessellations, simply by varying the mountain/valley assignment of the fold lines while making sure adjacent plateaus only differ by one sidewall height (i.e. one at a time), as shown in Fig. 11f for a saddle-like geometry [1 The mountain/valley assignment for every fold line coul easily determined from a "height map" of the target surface. S man et al. showed that surfaces with arbitrary curvature coul approximated, provided that the gradient with which the su rises or falls is not too steep (depending on the ratio of pla width to plateau height). Their results showed that la kirigami constitutes a very versatile approach to approxir intrinsically curved surfaces, and has great potential self-folding due to its simplicity as compared to conventi origami techniques [122].

The most recent progress into lattice kirigami has been m by Castle et al. [123] who generalized their earlier work by re ing some of the initially imposed rules and restrictions. Na generalizations included the removal of larger wedges from sheet and cutting and folding along different angles tha the original method. Additionally, the researchers demonstr

lattice kirigami on other Bravais lattices and arbitrarily complex lattices with a basis [123]. However, the most extensive generalizations came in the form of area-preserving kirigami, in which only slits are made and no material is removed, and additive kirigami, in which new material is actually inserted in the slits, reminiscent of natural growth of cells. Furthermore, Castle et al. showed that complex cuts could be decomposed into the general basic kirigami operations they presented [123]. The researchers envision that the generalized lattice kirigami framework provides more opportunities to create arbitrary shapes from initially flat sheets than the original method, due to the increased freedom in distributing local sources of Gaussian curvature along the sheet. However, a drawback is that the generalizations are not yet suitable with inverse design algorithms, which inhibits the use of such kirigami techniques in practical applications [123].

Lattice kirigami has not received the same attention as traditional origami by the scientific community, the great steps undertaken by the abovementioned researchers notwithstanding. However, it is clear that lattice kirigami offers an exciting and promising paradigm toward 3D structures. By strategically removing material or creating incisions, this technique could alleviate some of the traditional origami issues such as interlocking folds or the cumbersome tucking of excess material (which is non-existent in kirigami), thereby offering higher design freedom and simplicity [62,122,124].

Kirigami-engineered elasticity

The second kirigami technique we consider here involves cutting the paper at many locations without folding it afterward. The basic idea is that specific cut patterns imbue flat sheets with a high "apparent" elasticity, or stretchability, which does not arise from stretching the actual material but rather from the geometric changes enabled by the cuts, which is why we term this technique kirigami-*engineered* elasticity [39]. Owing to the high stretchability and the scale-independent nature, this technique has recently been proposed as an interesting avenue toward stretchable electronic devices [38,39], small-scale force sensors [113], macro-scale sun-shading [125], and solar-tracking photovoltaics [114]. However, the kirigami-engineered elasticity technique is also useful for wrapping flat sheets on intrinsically curved surfaces since the cuts allow the sheet to locally stretch in-plane, thereby permitting the required metric distortions to conform to the curved surfaces.

We distinguish two approaches toward kirigami-engineering elasticity in the available literature, one involving out-of-plane buckling of cut struts and one involving in-plane rotation of polygonal units, as shown in Fig. 12 for standard paper models. The former approach involving the out-of-plane buckling was first used by Shyu et al. to create highly stretchable nanocomposite sheets with predictable deformation mechanics [39]. The researchers enriched the nanocomposite sheets with a cut pattern consisting of straight slits in a rectangular arrangement such as the one shown in Fig. 12a. Upon tensile loading perpendicular to the slits, the struts formed by the cutting operation buckle out-of-plane, allowing the overall sheet to reach an ultimate strain of almost two orders of magnitude higher than that of the pristine material (from 4% to 370%) [39]. Around the same time, Blees et al. demonstrated that the same technique is applicable to graphene since this one-atom-thick material behaves similar to paper in terms of the Föppl–von Kármán number, a meas for the ratio of in-plane stiffness to out-of-plane bending stiffn [113]. Although Fig. 12a shows the struts buckling all in the sa direction, this is not necessarily the case and struts might r domly buckle upward or downward, resulting in unpredicta and non-uniform stretching behavior. In order to control a program the tilting of the struts in the desired direction, Ta et al. recently introduced "kiri-kirigami" in which additio notches are etched into the material between the cuts [12 Those notches are geometrical imperfections in the context buckling and serve as cues to guide the tilting in the desi direction. By implementing the appropriate notch pattern, t tilting orientation of the struts could be programmed befo hand, and could be varied throughout the same kirigami sh [125]. All of the abovementioned works used the kirigami te nique solely for imparting greater elasticity to flat sheets. Ho ever, the out-of-plane buckling that this kirigami technic entails could also be used to efficiently create textured metar terials from flat sheets, as was recently demonstrated by Rafs jani and Bertoldi [126]. These researchers perforated thin she with a square tiling of orthogonal cuts with a varying orientati with respect to the loading direction. Uniaxial tensile load results in out-of-plane buckling of the square units, and t could be made permanent by increasing the load beyond t plastic limit of the hinges between the squares. The results textured "metasheets" which are flat-foldable and could sh similar deformation characteristics as Miura-ori sheets, such negative Gaussian curvature upon non-planar bending [126].

The second method to achieve kirigami-engineered elastic involves in-plane rotation rather than out-of-plane buckling. this approach, the imposed cut pattern divides the sheet in (typically square or triangular) units connected through sm "hinges" (Fig. 12b). Upon stretching the sheet, the units rot (almost) freely around the hinges, resulting in a deformation t is driven mostly by the rigid unit rotation instead of stretching the units themselves. Cho et al. used this principle and dev oped a "fractal" kirigami technique in which the sheet is hier chically subdivided into ever smaller units that could rotate a contribute to the overall extension of the sheet (Fig. 1 [40,127]. While an increased level of hierarchy (i.e. more subd sions) could increase the expandability, there is a limit which dictated by the allowable rotations of the units. Cho et showed that the maximum expandability may be increased alternating the cut motif between levels, allowing larger rotati angles for the individual units [40]. The researchers demo strated the fractal kirigami technique at different length sca and achieved an areal expandability of up to 800%. Furthermo they showed that the kirigami sheets could conform to an obj of non-zero Gaussian curvature (a sphere in this case) throu non-uniform stretching of the pattern [40], as also shown Fig. 12b. Further research into fractal kirigami was aimed understanding the complex mechanics of the hinges, prone stress concentrations, as well as the influence of material prop ties [128]. Based on experiments and numerical simulatio Tang et al. proposed dog bone-shaped cuts and hinge widths t vary with the hierarchy level in order to increase the streng and ultimate expandability of the fractal cut patterns (even wh applied to brittle materials) [128]. Despite the impress

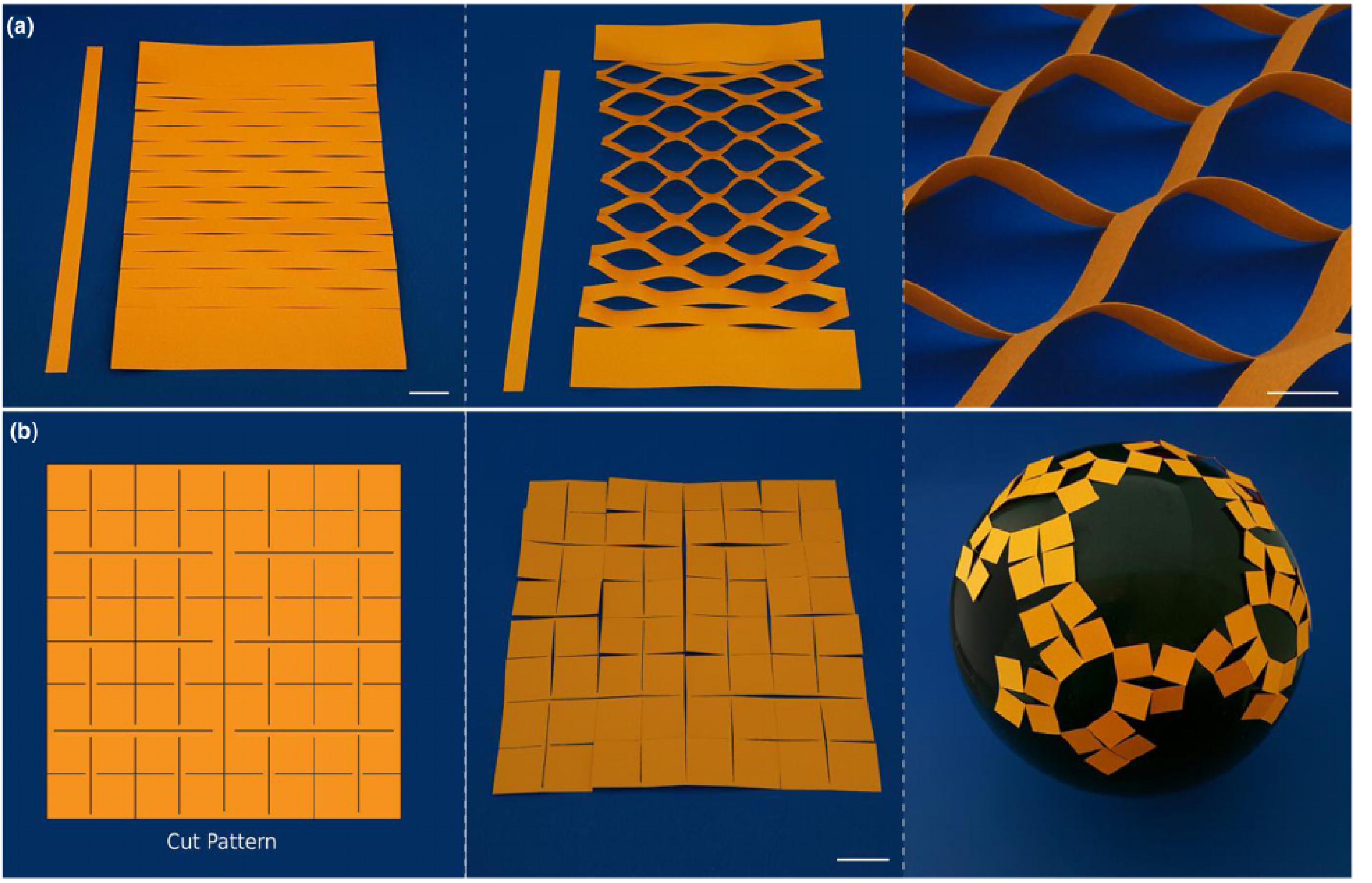

FIGURE 12

Kirigami-engineered elasticity. (a) A parallel arrangement of slits (left) causes the small struts in between to buckle out of the plane upon tensile lo (middle), thereby providing the sheet with higher elasticity (Scale bar is 2 cm). Right: a closer view showing the struts buckling in the same direction (Sca is 1 cm). (b) Fractal cut kirigami. Left: a three-level cut pattern with motif alternation, according to Ref. [40]. The small amount of material between adj cuts serves as hinge between the rigid square units. Middle: the cut pattern applied to a standard piece of paper (Scale bar is 2 cm). Right: the cut she paper shows a high degree of expandability and could conform to a spherical geometry through rotation of the square units.

expandability that could be achieved with these standard cut patterns, a drawback impeding the adoption in real applications was the lack of compressibility. Therefore, Tang and Yin recently proposed to extend the standard cut pattern, consisting of only slits with actual cut-outs [129]. By introducing circular pores in the original square units, sheet compressibility could be obtained through buckling of the pore walls, while stretchability was still guaranteed by the straight cuts [129].

Comparing the two kirigami-engineered elasticity approaches discussed above, it seems that the fractal cut method is currently more suitable to conform to intrinsically curved surfaces, as it allows biaxial stretching and compression. A drawback of both approaches is that a full coverage of the target surface cannot be achieved, as the stretching is enabled through significant "opening up" of the material. Nonetheless, both methods are expected to receive considerable attention in future research, not only in the field of stretchable electronics but also as a pathway toward mechanical metamaterials. For example, the fractal kirigami patterns are very similar to earlier work on rotation-based auxetic mechanical metamaterials [130,131].

Discussion and conclusions

We reviewed current origami and kirigami techniques that could be used to approximate or conform to intrinsically curved surfaces. Starting off with some concepts from differential geo try, we highlighted the inherent difficulty of transforming sheets into intrinsically curved surfaces. Moreover, we expla the geometry of origami, which involves isometric deformat of developable surfaces and therefore retains the intrinsic flat of the starting material. While scientific research into orig and kirigami is still in its infancy, we could nonetheless ide several promising techniques for the transformation of flat sh into curved geometries.

Approximations of intrinsically curved surfaces

The origami and kirigami techniques that we have revie could essentially approximate intrinsically curved surface two different ways. One approach is to use origami and kiri to transform ordinary flat sheets into "metasheets" with sig cantly altered properties, which are then deformed into the de geometry. In a second approach, the prescribed fold and cut terns directly correspond to the final 3D shape and no additi deformation is required after folding. The kirigami-engineered ticity techniques [39,40,113,125,128,129] are examples of the approach, while the techniques with tucking molecules [59,8 and curved-crease couplets [97,98,101] are examples of the sec Some techniques could be classified in both categories. For ex ple, origami tessellations could be employed to texture shee

that they may be deformed into an intrinsically curved geometry [28,68,72,106], but (generalized) tessellations have also been calculated to fit a target surface once folded [73–77]. The lattice kirigami technique may be considered an example of the second approach, as the target shape is programmed into the flat sheet using appropriate cuts and folds. However, a pluripotent version of lattice kirigami has been also proposed [122], in which the same kirigami pattern could be folded to fit multiple curved surfaces.

Due to the developability constraint, no origami or kirigami technique can *exactly* fit a flat sheet onto an intrinsically curved smooth surface. Such surfaces could be approximated in a "global" sense, but locally the folded sheets remain intrinsically flat. Some techniques, such as lattice kirigami [62,122,123] or the origamizing technique [59,88] do imbue the sheets with Gaussian curvature, but this curvature remains concentrated in single points surrounded by developable patches, i.e. "non-Euclidean vertices" [10]. Even when a sheet of paper is crumpled, the majority of the paper remains developable and non-zero Gaussian curvature only arises at specific points due to local stretching of the paper [55,132]. However, owing to the different underlying mechanisms, some techniques will result in a "smoother" approximation of the target surface than others. This is an important factor to consider in applications where the surface topography plays a role, e.g. in fluid flow over an object. Surfaces approximated using origami tessellations or periodic pleating exhibit a textured surface topography. For the periodic pleating technique, this texture is in the form of sharp, parallel ridges. In the case of origami tessellations, the texture is determined by the specific unit cell geometry, with the square waterbomb or Ron Resch patterns resulting in a smoother surface than the Miura-ori pattern, for example. The axisymmetric geometries created by the curved-crease couplets could result in a relatively smooth surface due to the bent faces, although the frustration between the curved and straight creases might entail additional creases that disturb this smoothness. The smoothest approximation of the target surface is likely obtained with the origamizing technique, as the calculated crease pattern (almost) exactly folds into a polygon mesh of this surface. Naturally, a finer mesh results in a smoother representation, yet also entails a more complex folding process. The lattice kirigami technique results in a stepped surface approximation, consisting of many small units that simulate the convex and concave curvatures of the sheet. Interestingly, both the origamizing and lattice kirigami techniques bear strong similarities with computer graphics techniques used to represent 3D objects, either using polygonization of the surface or by means of a "voxel" (volume pixel) representation. Finally, the fractal kirigami technique allows conforming initially flat sheets, made from relatively rigid materials, to surfaces of non-zero Gaussian curvature through non-uniform opening of the cut pattern [40]. However, it was also mentioned that this opening of the perforations inhibits a full coverage of the target surface, which might be a drawback in certain applications.

Practical considerations

There are several practical challenges that need to be overcome in order to accelerate the adoption of origami and kirigami as a shape-shifting technique for development of advanced materi First and foremost is the challenge of the folding itself, which labor-intensive manual process in traditional origami. Althou all the macroscale paper models that we have presented in t review could be folded by hand, such manual folding becor increasingly complex at much smaller or much larger sca and for more advanced materials [14,133]. As a consequen self-folding (i.e. "hands-free" [11]) techniques are required wide range of such techniques have been developed dur recent years, in particular aimed at smaller length scales [9– 13–16,105,134–142]. While these techniques, for example, dif in terms of materials used, speed, actuation method, and suita length scales, the underlying principle is typically the same: self-folding behavior is programmed into the flat starting mat als, most often in the form of "active hinges" which are trigge by an external stimulus to activate folding. In many cases, sti ulus-responsive polymeric materials have been employed for t purpose. Examples are hydrogels that swell or de-swell upor change in aqueous environment [11,141,142], or shape mem polymers (SMP) that shrink when heated above the glass tran tion temperature [9,105,137–140]. Especially the use of th mallyresponsive SMP for self-folding has attracted the inter of many researchers due to its simplicity and versatility in ter of actuation method [138], for example via uniform oven he ing [122,137], localized joule heating [9], or localized heat by light or microwave absorption [138,139]. The shape mem effect is not limited to polymers but is also present in cert metallic alloys (giving rise to shape memory alloys (SMA)), m ing these materials also suitable for thermally activated s folding origami [134]. In addition to these more common te niques, many other actuation methods for self-folding have be developed such as the folding of rigid panels driven by surf tension [143–145] or cell traction forces [136] to create nar and microscale origami, and mechanically driven origami/k gami [146,147] approaches in which folding is achieved throu controlled buckling at specified locations. The reader interes in more detailed information on self-folding techniques a the associated materials is referred to other excellent reviews these topics [11,14,16,133,148,149].

Despite many recent developments, self-folding remain challenging task, in part, because of the need for sequential fo ing and control over the direction of folding and fold ang [9,11,150]. While many of these challenges have been address in recent years [9,15,138,150–153], self-folding origami demo strations have often been restricted to single folds or basic po hedral shapes [16,138–140,144], while demonstrations of m complicated patterns such as origami tessellations are not so common [137,152]. Given these inherent complexities, i not surprising that some of the reviewed origami and kiriga techniques are better suited for self-folding than others. Co pared to straight creases, self-folding of curved-crease origam expected to be more challenging despite some recent demonst tions [153]. The facet bending, which is inherent in this type non-rigid origami, would necessitate larger hinge actuati forces than for purely rigid origami with straight creases. Mo over, the possibility of arriving at a "locked" state during fold may further complicate the automated folding of curved-cre origami [93]. Comparing the tucking molecules origa

technique and the lattice kirigami technique, it has been argued that the latter is more applicable to self-folding [62,122,123]. The tucking molecules approach requires excessive material to be tucked away behind the outer surface, which is a cumbersome process involving many small (crimp) folds and high folding angles. This is in sharp contrast with the lattice kirigami technique, where no excessive material needs to be tucked away and a simple, repetitive folding pattern is used [122]. Indeed, self-folding of basic lattice kirigami units (millimeter and centimeter scale) has been recently demonstrated using localized heating [153] or controlled compressive buckling [146,154], but more complex 3D geometries have not yet been reported. Regarding the techniques that employ the "metasheet" approach, such as the origami tessellations and the kirigami-engineered elasticity, one could argue that these are less suited for self-folding as the sheets need to be actively deformed into the desired shape. In order for these sheets to self-fold into the target geometry, local control over the fold angles or gap opening would be required. Preliminary results show that the standard kirigami cut pattern, consisting of parallel straight cuts, could be actuated using thermally activated local shrinkage [125], but more investigations are needed for metasheets to automatically fit curved surfaces through remotely actuated opening or closing of folds and cuts.

In addition to the self-folding process, another practical consideration is related to locking the origami and kirigami structures in their curved folded geometries. This has, for example, been achieved by annealing titanium-rich origami structures at high temperatures [155]. Another approach is using sequential self-folding to include self-locking mechanisms [149,156]. Alternatively, the locking mechanisms might be inherent to the used origami or kirigami technique. For example, the origamizing technique with tucking molecules uses crimp folds to keep the tucks closed and maintain the desired shape on the outside surface [59]. On the contrary, the standard lattice kirigami cannot benefit from such excess material to lock the folds, although recent generalizations of lattice kirigami enable this to a certain extent by retaining some material for use as "fastening tabs" [123].

In addition to aspects such as self-folding and locking, another prominent challenge is related to the medium to which origami and kirigami are applied. Origami and kirigami are often considered to be scale-independent processes on zero-thickness surfaces. The thin paper sheets that have traditionally been used in these art forms are not too far from zero-thickness surfaces [157]. However, in engineering and scientific applications, the thickness of the flat starting materials cannot be ignored, especially not for applications where the ratio of the sheet thickness to other sheet dimensions is substantial. An important consequence of finite sheet thickness is that appropriate hinge design is required to enable obstruction-free folding and flat-foldability [158]. In recent years, several hinge design approaches to account for finite material thickness have been proposed for rigid-foldable origami [158–160]. In addition to hindering flat-foldability, the material thickness also affects the fold regions themselves, i.e. when the materials are actually folded and the folds are not replaced by hinges. Origami design methods typically assume perfectly sharp folds applied to the (zero-thickness) sheets, meaning that all folding is concentrated a a single line of infinitesimal width (such a sharp fold is co ered to be "G^0 continuous") [133,161]. While such idea sharp folds could be approximated to some extent in a thin p sheet, a fold in a finite thickness sheet will never be perf sharp but will rather be defined by some bent region with a tain radius of curvature, especially when thicker materia materials that cannot withstand high bending strains are [133,161]. Peraza et al., inspired by Tachi's origami approach, recently developed an origami design method b on "smooth" folds [161], which are bent surface regions of f width. Such "smooth" folds form the connection between rigid origami faces and are characterized by higher order geo ric continuity than the sharp folds. Smooth folds are not relevant for folding of thicker materials but also relevan self-folding techniques based on "active hinges" [161]. Swe or shrinking at these hinge locations also results in finite reg of bending rather than perfectly sharp folds [162]. This dis tion between folds in idealized origami (zero-thickness sh and folds in origami on real materials (non-zero-thick sheet) essentially revolves around the subtle difference betw bending and folding [11,157]. Lauff et al. described ben (i.e. smooth fold) as "distributed curvature" while foldin "localized curvature". However, Liu et al. correctly stated an overlap between bending and folding exists as it is diff to draw a clear boundary between localized and distributed vature [11]. Note that in this context, the term "curvat refers to single or extrinsic curvature. Bending and fol both result in zero Gaussian (intrinsic) curvature, see Fig [157].

In light of the techniques reviewed in this paper, origami finite material thickness is expected to be particularly chall ing in the tucking molecules approach, due to many small (which are considered to be sharp folds in the origamizer ware) and the requirement for flatfolding to tuck away ex material. As mentioned before, improvements to the stan tucking molecules approach have been proposed to make technique more apt to real applications with finite thick materials [81,89,161]. The approach based on the curved-cr couplets is limited by the fact that very high folding angles (to π) are required, which could be difficult to realize with thi materials. The techniques based on origami tessellations, centric pleating, and lattice kirigami are expected to be b suited for origami with finite thickness sheets as they generally characterized by simple fold patterns and already been (self-) folded from materials other than p [122,152,153,158,162]. Finally, the kirigami-engineered ela ity techniques do not require folding, hence they do not s from challenges with tight folds or flat-foldability. Neverthe the sheet thickness does influence the load at which out-of-p buckling of the struts occurs (see Fig. 12a), thereby making it an important design parameter [39,125].

Besides the thickness, another aspect that is often neglect "idealized" origami is the mechanical properties of the mate which may also hinder practical application of the origami kirigami techniques [162]. For example, permanently foldi finite thickness sheet entails a complex stress state invol plasticity and some degrees of stretching, aspects which

strongly linked to the mechanical properties of the sheet [55,104]. Furthermore, non-rigid origami also involves facet bending [90], which is strongly tied to the bending stiffness of the sheets that are used. As for the kirigami-engineered elasticity techniques, it has been already mentioned that these techniques are characterized by high stress concentrations, both for the out-of-plane buckling and in-plane rotation approaches [39,40,125,128]. Consequently, implementation of these techniques to real materials will require certain levels of understanding regarding the local material behavior under these high stress states [128]. Some techniques might therefore be more suitable than others for a given application depending on the chosen material.

As is clear from the preceding discussion, scientific and engineering origami/kirigami are not purely "scale-independent" processes that could be treated solely from a geometrical perspective. For example, self-folding techniques that are suitable for micro-scale origami are not necessarily suitable for architectural-scale origami (e.g. surface tension or cell traction forces). As a final remark, we note that traditional paper seems to remain an excellent medium for origami and kirigami, considering its balance of relative thickness, bending and tearing resistance, and the ability to withstand relatively sharp creases. However, this does not necessarily mean that other materials (on different scales) are less well suited as these materials might behave very similar to paper when used for origami or kirigami [113].

Outlook

Approximating intrinsically curved surfaces using origami and kirigami is relevant for many applications that can benefit from the specific advantages offered by these techniques: the ability to obtain complex geometries from (nearly) non-stretchable flat sheets and the ability to apply this on virtually any length scale. As such, the folding-and-cutting paradigm could enable the development of flexible electronics [38,40], shape-morphing materials [122], nano- and microscale devices [14,163], architectural structures [84,159], or any other complex geometry involving intrinsic curvature. The applications of origami and kirigami are not limited to static designs, but could also be of a more dynamic nature. In fact, certain fold patterns involving facet bending or curved creases may form energetic barriers between different folding states, giving rise to bi-stability and fast snapping motions that could be leveraged for switchable or tunable devices [103,164].

Interesting opportunities for origami and kirigami may be also found in the rapidly expanding field of biomedical engineering [36], with such examples as patterned micro-containers for controlled drug delivery [35], origami stent grafts [34], or self-folding tetherless micro-grippers [165]. A particularly interesting bio-application is the folding of 3D tissue scaffolds from flat sheets enriched with cell-regulating surface topographies. Self-folding of patterned scaffolds with simple geometries has already been demonstrated [166], but more complex curved geometries would be needed in order to better stimulate and guide tissue regeneration [167]. For example, it has been hypothesized that promising bone-mimicking scaffolds could be based on triply periodic minimal surfaces (TPMS) [168,169]. These are area-minimizing 3D surfaces with zero mean curvature (H) everywhere, correspon ing to negative (or zero) Gaussian curvature everywh ($H = \frac{1}{2}(\kappa_1 + \kappa_2) = 0$ and thus $\kappa_1 = -\kappa_2$) [42]. These intrinsica curved minimal surfaces are ubiquitous in biological syste [41,170–172] and are nature's best attempt at dealing with frustration of embedding constant negatively curved surfaces Euclidean 3-space (Hilbert's theorem) [173,174]. Current TP scaffolds are created with additive manufacturing [4,169], me ing that the surface topographies needed to enhance tis regeneration cannot be included. However, the origami and igami techniques we have reviewed here might enable tra forming patterned flat sheets into intrinsically curved scaffo through appropriate cutting and folding.

In conclusion, we have reviewed recent work on origami a kirigami to identify the techniques that enable shape shifting flat sheets into complex geometries. By introducing aspects fr differential geometry, in particular the Gaussian curvature, have illustrated the fundamental difference between flat she and intrinsically curved surfaces, which can explain g wrapping of spheres to wavy edges in plant leaves. While plane distortions could imbue the flat sheets with intrinsic cur ture, we have shown that origami and kirigami offer alternat approaches to *approximate* curved surfaces with (almost) stretching of the underlying material. Despite originating fr centuries-old art forms, the techniques we have reviewed h are promising for many applications across a broad range length scales. It could therefore be expected that the relativ recent interest in "scientific" origami and kirigami will only k on growing in the near future.

Acknowledgments

This research has received funding from the Europe Research Council under the ERC grant agreement n° [67757

References

[1] A.A. Zadpoor, J. Malda, Ann. Biomed. Eng. 45 (1) (2017) 1–11.
[2] M. Kadic et al., Appl. Phys. Lett. 100 (19) (2012) 191901.
[3] R. Hedayati, A.M. Leeflang, A.A. Zadpoor, Appl. Phys. Lett. 110 (9) (20 091905.
[4] F.S.L. Bobbert et al., Acta Biomater. 53 (2017) 572–584.
[5] E.B. Duoss et al., Adv. Funct. Mater. 24 (31) (2014) 4905–4913.
[6] T. Jiang, Z. Guo, W. Liu, J. Mater. Chem. A 3 (5) (2015) 1811–1827.
[7] O.Y. Loh, H.D. Espinosa, Nat. Nanotech. 7 (5) (2012) 283–295.
[8] S. Dobbenga, L.E. Fratila-Apachitei, A.A. Zadpoor, Acta Biomater. 46 (2016 14.
[9] S.M. Felton et al., Soft Matter 9 (32) (2013) 7688–7694.
[10] C.D. Santangelo, Annu. Rev. Conden. Map. 8 (1) (2016).
[11] Y. Liu, J. Genzer, M.D. Dickey, Prog. Polym. Sci. 52 (2016) 79–106.
[12] R. Guseinov, E. Miguel, B. Bickel, ACM Trans. Graph. 36 (4) (2017) 1–12.
[13] A.M. Hubbard et al., Soft Matter 13 (12) (2017) 2299–2308.
[14] J. Rogers et al., MRS Bull. 41 (02) (2016) 123–129.
[15] N. Bassik, G.M. Stern, D.H. Gracias, Appl. Phys. Lett. 95 (9) (2009) 91901.
[16] T.G. Leong, A.M. Zarafshar, D.H. Gracias, Small 6 (7) (2010) 792–806.
[17] M.Z. Li et al., J. Mater. Process. Technol. 129 (2002) 333–338.
[18] T. Gutowski et al., Compos. Manuf. 2 (1991) 147–152.
[19] Y. Klein, E. Efrati, E. Sharon, Science 315 (5815) (2007) 1116–1120.
[20] A.S. Gladman et al., Nat. Mater. 15 (4) (2016) 413–418.
[21] J. Kim et al., Science 335 (6073) (2012) 1201–1205.
[22] E. Sharon, M. Marder, H.L. Swinney, Am. Sci. 92 (2004) 254–261.
[23] E. Sharon et al., Nature 419 (2002) 579.
[24] H. Liang, L. Mahadevan, Proc. Natl. Acad. Sci. USA 108 (14) (2011) 5516–5
[25] K. Miura, Inst. Space Astronaut. Sci. Rep. 618 (1985) 1–9.

[26] Guest, S.D., Pellegrino, S., Inextensional wrapping of flat membranes, in: Motro, R., Wester, T. (eds.), First Int. Sem. on Struct. Morph., Montpellier, 1992, pp 203–215.
[27] S.A. Zirbel et al., J. Mech. Des. 135 (11) (2013) 111005.
[28] M. Schenk, S.D. Guest, Proc. Natl. Acad. Sci. USA 110 (9) (2013) 3276–3281.
[29] E.T. Filipov, T. Tachi, G.H. Paulino, Proc. Natl. Acad. Sci. USA 112 (40) (2015) 12321–12326.
[30] J.L. Silverberg et al., Science 345 (6197) (2014) 647–650.
[31] J.T. Overvelde et al., Nature 541 (7637) (2017) 347–352.
[32] M. Eidini, G.H. Paulino, Sci. Adv. 1 (2015) 8.
[33] S. Felton et al., Science 345 (6197) (2014) 644–646.
[34] K. Kuribayashi et al., Mater. Sci. Eng. A 419 (1–2) (2006) 131–137.
[35] C.L. Randall et al., Adv. Drug. Deliv. Rev. 59 (15) (2007) 1547–1561.
[36] C.L. Randall, E. Gultepe, D.H. Gracias, Trends Biotechnol. 30 (3) (2012) 138–146.
[37] R.J. Lang, Phys. World 20 (2) (2007) 30.
[38] Z. Song et al., Sci. Rep. 5 (2015) 10988.
[39] T.C. Shyu et al., Nat. Mater. 14 (8) (2015) 785–789.
[40] Y. Cho et al., Proc. Natl. Acad. Sci. USA 111 (49) (2014) 17390–17395.
[41] S. Hyde et al., The Language of Shape: The Role of Curvature in Condensed Matter: Physics, Chemistry and Biology, Elsevier Science, Amsterdam, The Netherlands, 1996.
[42] D. Hilbert, S. Cohn-Vossen, Geometry and The Imagination, Chelsea Publishing Company, New York, USA, 1990.
[43] J.R. Weeks, The Shape of Space, CRC Press, Boca Raton, FL, USA, 2001.
[44] C.R. Calladine, Theory of Shell Structures, Cambridge University Press, Cambridge, UK, 1983.
[45] V.A. Toponogov, V. Rovenski, Differential Geometry of Curves and Surfaces: A Concise Guide, Birkhäuser Boston, New York, NY, USA, 2005.
[46] V. Rovenski, Modeling of Curves and Surfaces with MATLAB, Springer Science & Business Media, New York, NY, USA, 2010.
[47] M. Spivak, A Comprehensive Introduction to Differential Geometry, vol. II, Publish or Perish, Inc., Houston, Texas, USA, 1990.
[48] B. O'Neill, Elementary Differential Geometry, Academic Press (Elsevier), Burlington, MA, USA, 2006.
[49] E. Sharon, E. Efrati, Soft Matter 6 (22) (2010) 5693.
[50] M. Marder, R.D. Deegan, E. Sharon, Phys. Today 60 (2) (2007) 33–38.
[51] R.D. Kamien, Science 315 (2007) 1083–1084.
[52] A.N. Pressley, Elementary Differential Geometry, Springer Science & Business Media, Dordrecht, The Netherlands, 2010.
[53] E. Efrati et al., Phys. D 235 (1–2) (2007) 29–32.
[54] H. Aharoni, E. Sharon, R. Kupferman, Phys. Rev. Lett. 113 (25) (2014) 257801.
[55] T.A. Witten, Rev. Mod. Phys. 79 (2) (2007) 643–675.
[56] E. Efrati, E. Sharon, R. Kupferman, J. Mech. Phys. Solids 57 (4) (2009) 762–775.
[57] E. Demaine, M. Demaine, Recent results in computational origami, in: T. Hull (Ed.), Origami 3: Proceedings of the 3rd International Meeting of Origami Science, Math, and Education (OSME 2001), A K Peters/CRC Press, Boca Raton, FL, USA, 2002, pp. 3–16.
[58] E.D. Demaine, M.L. Demaine, J.S.B. Mitchell, Comput. Geom. 16 (1) (2000) 3–21.
[59] T. Tachi, IEEE Trans. Vis. Comput. Graphics 16 (2) (2010) 298–311.
[60] Z. Abel et al., J. Comput. Geom. 7 (2016) 171–184.
[61] T.A. Evans et al., R. Soc. Open Sci. 2 (9) (2015) 150067.
[62] T. Castle et al., Phys. Rev. Lett. 113 (24) (2014) 245502.
[63] Miura, K., A note on intrinsic geometry of origami, in: Huzita, H., (ed.), First Int. Meeting of Origami Science and Technology, Ferrara, Italy, 1989, pp 239–249.
[64] D.A. Huffman, IEEE Trans. Comput. C-25 (10) (1976) 1010–1019.
[65] T. Hull, Project origami: activities for exploring mathematics, second ed., CRC Press, Boca Raton, FL, USA, 2012.
[66] R.C. Alperin, B. Hayes, R.J. Lang, Math. Intell. 34 (2) (2012) 38–49.
[67] E. Gjerde, Origami Tessellations Awe-Inspiring Geometric Designs, A K Peters/CRC Press, Boca Raton, FL, USA, 2008.
[68] K.A. Seffen, Philos. Trans. A Math. Phys. Eng. Sci. 370 (1965) (2012) 2010–2026.
[69] A.A. Evans, J.L. Silverberg, C.D. Santangelo, Phys. Rev. E Stat. Nonlin. Soft Matter Phys. 92 (1) (2015) 013205.
[70] L. Mahadevan, S. Rica, Science 307 (2005).
[71] C. Lv et al., Sci. Rep. 4 (2014) 5979.
[72] M. Schenk, S.D. Guest, Origami folding: A structural engineering approach, in: P. Wang-Iverson et al. (Eds.), Origami 5: Fifth International Meeting of Origami Science, Mathematics, and Education (5OSME), A K Peters/CRC Press, Boca Raton, FL, 2011, pp. 291–304.
[73] T. Tachi, J. Int. Assoc. Shell Spatial Struct. 50 (3) (2009) 173–179.
[74] J.M. Gattas, W. Wu, Z. You, J. Mech. Des. 135 (11) (2013) 1110111–1110
[75] P. Sareh, S.D. Guest, Int. J. Space Struct. 30 (2) (2015) 141–152.
[76] F. Wang, H. Gong, X. Chen, C.Q. Chen, Sci. Rep. 6 (2016) 33312.
[77] L.H. Dudte, E. Vouga, T. Tachi, L. Mahadevan, Nat. Mater. 15 (5) (2016) 588.
[78] Y. Chen et al., Proc. R. Soc. A 472 (2190) (2016) 20150846.
[79] R.D. Resch, The Topological Design of Sculptural and Architectural Sys Nat. Comp. Conf. and Exp., ACM, New York, 1973, pp. 643–650.
[80] Resch, R.D., Christiansen, H., The design and analysis of kinematic folded systems, in: IASS Symp. on Folded Plates and Prism. Struct., Vienna, 19
[81] T. Tachi, J. Mech. Des. 135 (2013) 111006.
[82] T. Tachi, Rigid folding of periodic origami tessellations, in: K. Miura (Eds.), Origami 6: Mathematics, American Mathematical Society, Provi RI, USA, 2016.
[83] H. Nassar, A. Lebee, L. Monasse, Proc. R. Soc. A 473 (2197) (2017) 2016
[84] T. Tachi, J. Geom. Graph. 14 (2) (2010) 203–215.
[85] X. Zhou, H. Wang, Z. You, Proc. R. Soc. A 471 (2181) (2015) 20150407.
[86] K. Song et al., Proc. R. Soc. A 473 (2200) (2017) 20170016.
[87] Lang, R.J., A computational algorithm for origami design, in: Twelfth Symp. on Comput. Geom., Philadelphia, 1996.
[88] Tachi, T., 3D origami design based on tucking molecules, in: Lang, R. J., Origami 4, Proceedings of 4OSME: 4th International Conference on Origa Science, Mathematics and Education, A K Peters/CRC Press, Boca Rato USA, 2009, pp 259–272.
[89] Demaine, E., Tachi, T., Origamizer: a practical algorithm for folding polyhedron, in: Aronov, B., and Katz, M. J., (eds.), 33rd Int. Symp. on Co Geom. (SoCG 2017), Schloss Dagstuhl–Leibniz-Zentrum fuer Infor Dagstuhl, Germany, 2017.
[90] E.D. Demaine et al., Symm. Cult. Sci. 26 (2) (2015) 145–161.
[91] J.P. Duncan, J.L. Duncan, Proc. R. Soc. A 383 (1784) (1982) 191–205.
[92] Geretschläger, R., Folding curves, in: Lang, R. J., (ed.), Origami 4, Procee of 4OSME: 4th International Conference on Origami in Science, Mathem and Education, A K Peters/CRC Press, Boca Raton, FL, USA, (2009), pp 151
[93] M. Kilian, A. Monszpart, N.J. Mitra, ACM Trans. Graph. 36 (3) (2017) 1-
[94] D. Fuchs, S. Tabachnikov, Am. Math. Month. 106 (1999) 27–35.
[95] M. Kilian et al., ACM Trans. Graph. 27 (2008) 75.
[96] A. Vergauwen, L.D. Laet, N.D. Temmerman, Comput. Aided Des. 83 (201 63.
[97] Leong, C. C., Simulation of nonzero gaussian curvature in origami by c crease couplets, in: Lang, R. J., (ed.), Origami 4, Proceedings of 4OSM International Conference on Origami in Science, Mathematics and Educ A.K. Peters/CRC Press, Boca Raton, FL, USA, 2009, pp 151–163.
[98] J. Mitani, Comput. Aided Des. Appl. 6 (1) (2009) 69–79.
[99] Lang, R. J., Origami Flanged Pots, http://demonstrations.wolfram OrigamiFlangedPots/, in: Wolfram Demonstrations Project, vol. 2017.
[100] R.J. Lang, Origami Design Secrets: Mathematical Methods for an Ancier second ed., A K Peters/CRC Press, Boca Raton, FL, USA, 2011.
[101] J. Mitani, A design method for axisymmetric curved origami with trian prism protrusions, in: P. Wang-Iverson et al. (Eds.), Origami 5: International Meeting of Origami Science, Mathematics, and Educ (5OSME), A K Peters/CRC Press, Boca Raton, FL, 2011, pp. 437–447.
[102] E.D. Demaine et al., Characterization of curved creases and rulings desig analysis of lens tessellations, in: K. Miura et al. (Eds.), Origami 6: Mathem American Mathematical Society, Providence, RI, USA, 2016, pp. 209–23
[103] N.P. Bende et al., Proc. Natl. Acad. Sci. USA 112 (36) (2015) 11175–1118
[104] E.D. Demaine, M.L. Demaine, V. Hart, G.N. Price, T. Tachi, Graphs Comb (3) (2011) 377–397.
[105] T. van Manen, S. Janbaz, A.A. Zadpoor, Mater. Horiz. (2017), https://do 10.1039/C7MH00269F.
[106] M. Schenk, Folded Shell Structures (Ph.D. thesis), University of Camb 2011.
[107] Tachi, T., Simulation of rigid origami, in: Lang, R. J., (ed.), Origa Proceedings of 4OSME: 4th International Conference on Origami in Sc Mathematics and Education, A K Peters/CRC Press, Boca Raton, FL, USA,
[108] Z.Y. Wei et al., Phys. Rev. Lett. 110 (21) (2013) 215501.
[109] P.O. Mouthuy et al., Nat. Commun. 3 (2012) 1290.
[110] M.A. Dias et al., Phys. Rev. Lett. 109 (11) (2012) 114301.
[111] M.A. Dias, C.D. Santangelo, EPL (Europhys. Lett.) 100 (5) (2012) 54005.
[112] Demaine, E. D., Demaine, M. L., Lubiw, A., Polyhedral sculptures hyperbolic paraboloids, in: 2nd Ann. Conf. of BRIDGES: Mathem Connections in Art, Music, and Science, Kansas, USA, 1999.
[113] M.K. Blees et al., Nature 524 (7564) (2015) 204–207.

[114] A. Lamoureux et al., Nat. Commun. 6 (2015) 8092.
[115] R.M. Neville, F. Scarpa, A. Pirrera, Sci. Rep. 6 (2016) 31067.
[116] K. Saito, F. Agnese, F. Scarpa, J. Intell. Mater. Syst. Struct. 22 (9) (2011) 935–944.
[117] S. Del Broccolo, S. Laurenzi, F. Scarpa, Compos. Struct. 176 (2017) 433–441.
[118] W.T. Irvine, V. Vitelli, P.M. Chaikin, Nature 468 (7326) (2010) 947–951.
[119] J.F. Sadoc, N. Rivier, J. Charvolin, Acta Crystallogr. Sect. A: Found. Crystallogr. 68 (4) (2012) 470–483.
[120] Martin, A.G., A basketmaker's approach to structural morphology, in: Int. Assoc. Shell Spatial Struct. (IASS) Symp., Amsterdam, The Netherlands, 2015.
[121] J. Charvolin, J.-F. Sadoc, Biophys. Rev. Lett. 06 (01n02) (2011) 13–27.
[122] D.M. Sussman et al., Proc. Natl. Acad. Sci. USA 112 (24) (2015) 7449–7453.
[123] T. Castle et al., Sci. Adv. 2 (9) (2016) e1601258.
[124] F. Wang et al., J. Appl. Mech. 84 (6) (2017) 061007.
[125] Y. Tang et al., Adv. Mater. 29 (10) (2016) 1604262.
[126] A. Rafsanjani, K. Bertoldi, Phys. Rev. Lett. 118 (8) (2017) 084301.
[127] S. Yang, I.-S. Choi, R.D. Kamien, MRS Bull. 41 (02) (2016) 130–138.
[128] Y. Tang et al., Adv. Mater. 27 (44) (2015) 7181–7190.
[129] Y. Tang, J. Yin, Extr. Mech. Lett. 12 (2017) 77–85.
[130] R. Gatt et al., Sci. Rep. 5 (2015) 8395.
[131] H.M.A. Kolken, A.A. Zadpoor, RSC Adv. 7 (9) (2017) 5111–5129.
[132] M. Ben Amar, Y. Pomeau, Proc. R. Soc. A 453 (1959) (1997) 729–755.
[133] E.A. Peraza-Hernandez et al., Smart Mater. Struct. 23 (9) (2014) 094001.
[134] E. Hawkes et al., Proc. Natl. Acad. Sci. USA 107 (28) (2010) 12441–12445.
[135] S. Janbaz, R. Hedayati, A.A. Zadpoor, Mater. Horiz. 3 (6) (2016) 536–547.
[136] K. Kuribayashi-Shigetomi, H. Onoe, S. Takeuchi, PLoS One 7 (12) (2012) e51085.
[137] M.T. Tolley et al., Smart Mater. Struct. 23 (9) (2014) 094006.
[138] Y. Liu et al., Soft Matter 8 (6) (2012) 1764–1769.
[139] D. Davis et al., RSC Adv. 5 (108) (2015) 89254–89261.
[140] D. Davis et al., J. Mech. Robot. 8 (3) (2016) 031014.
[141] J. Guan et al., J. Phys. Chem. B 109 (49) (2005) 23134–23137.
[142] G. Stoychev, N. Puretskiy, L. Ionov, Soft Matter 7 (7) (2011) 3277–3279.
[143] B. Gimi et al., Biomed. Microdevices 7 (4) (2005) 341–345.
[144] A. Azam et al., Biomed. Microdevices 13 (1) (2011) 51–58.
[145] D.H. Gracias et al., Adv. Mater. 14 (3) (2002) 235.
[146] Z. Yan et al., Adv. Funct. Mater. 26 (16) (2016) 2629–2639.
[147] Y. Zhang et al., Proc. Natl. Acad. Sci. USA 112 (38) (2015) 11757–11764.
[148] Y. Zhang et al., Nat. Rev. Mater. 2 (4) (2017) 17019.
[149] T. Van Manen, S. Janbaz, A.A. Zadpoor, Mater. Today 21 (2) (2018) 144–1
[150] T. Tachi, T.C. Hull, J. Mech. Robot. 9 (2) (2017). 021008-021008-021009.
[151] Y. Liu, B. Shaw, M.D. Dickey, J. Genzer, Sci. Adv. 3 (2017) 3.
[152] J.H. Na et al., Adv. Mater. 27 (1) (2015) 79–85.
[153] Q. Zhang et al., Extr. Mech. Lett. 11 (2017) 111–120.
[154] Y. Shi et al., Extr. Mech. Lett. 11 (2017) 105–110.
[155] B.Y. Ahn, D. Shoji, C.J. Hansen, et al., Adv. Mater. 22 (20) (2010) 2251–2
[156] Y. Mao et al., Sci. Rep. 5 (2015) 13616.
[157] Lauff, C., et al., Differentiating bending from folding in origami enginee using active materials, in: Proceedings of the ASME 2014 International De Engineering Technical Conferences & Computers and Informa Engineering Conference, 2014, p V05BT08A040.
[158] Y. Chen, R. Peng, Z. You, Science 349 (6246) (2015) 396–400.
[159] T. Tachi, Rigid-foldable thick origami, in: P. Wang-Iverson et al. (Eds.), Orig 5: Fifth International Meeting of Origami Science, Mathematics, Education (5OSME), A K Peters/CRC Press, Boca Raton, FL, 2011, pp. 253–
[160] R.J. Lang et al., J. Mech. Robot. 9 (2) (2017) 021013.
[161] E.A. Peraza Hernandez, D.J. Hartl, D.C. Lagoudas, Proc. R. Soc. A 473 (2 (2017) 20160716.
[162] N. An, M. Li, J. Zhou, Smart Mater. Struct. 25 (11) (2016) 11LT02.
[163] L. Xu, T.C. Shyu, N.A. Kotov, ACS Nano 11 (2017) 7587–7599.
[164] J.L. Silverberg, J.H. Na, A.A. Evans, et al., Nat. Mater. 14 (4) (2015) 389–3
[165] T.G. Leong et al., Proc. Natl. Acad. Sci. USA 106 (3) (2009) 703–708.
[166] M. Jamal et al., Biomaterials 31 (7) (2010) 1683–1690.
[167] A.A. Zadpoor, Biomater. Sci. 3 (2) (2015) 231–245.
[168] S.C. Kapfer et al., Biomaterials 32 (29) (2011) 6875–6882.
[169] S.B. Blanquer et al., Biofabrication 9 (2) (2017) 025001.
[170] K. Michielsen, D.G. Stavenga, J. R. Soc. Interface 5 (18) (2008) 85–94.
[171] J.W. Galusha et al., Phys. Rev. E Stat. Nonlin. Soft Matter Phys. 77 (5 P (2008) 050904.
[172] H. Jinnai et al., Adv. Mater. 14 (22) (2002) 1615–1618.
[173] M.E. Evans, G.E. Schröder-Turk, Asia Pacific Math. Newslett. 5 (2) (2015) 30.
[174] S.T. Hyde, G.E. Schröder-Turk, Interface Focus 2 (2012) 529–538.