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Adaptive state-feedback synchronization with distributed input: the cyclic case

Simone Baldi * **Muhammad Ridho Rosa** * **Paolo Frasca** **

* *Delft Center for Systems and Control, Delft University of Technology, The Netherlands*
** *Univ. Grenoble Alpes, CNRS, Inria, Grenoble INP, GIPSA-Lab, Grenoble, France and University of Twente, Department of Applied Mathematics, The Netherlands (e-mails: {s.baldi, MuhammadRidhoRosa}@tudelft.nl, paolo.frasca@gipsa-lab.fr)*

Abstract: Using a setting in which the input is communicated among neighbors (without exchanging any distributed observer variables), the problem of synchronizing an *acyclic* network of linear uncertain agents has been formulated recently as a distributed model reference adaptive control (MRAC) where each agent tries to converge to the model defined by its neighbors. In this work we show how to parametrize the distributed MRAC in *cyclic* and *undirected* graphs.

Keywords: Adaptive synchronization, heterogeneous uncertain agents, adaptive control.

1. INTRODUCTION

In recent years, cooperative control of multiagent systems has received increasing attention, due to its impact in formation flying, smart energy, traffic, and other areas (Ren et al. (2007); Bullo et al. (2012)). An important problem in cooperative control is to achieve in a distributed way (i.e. using local information) a common behavior for the entire network: this is the so-called synchronization problem (Dorfler and Bullo (2014); Turci et al. (2014); Gibson (2016); Casadei and Astolfi (2017)).
Synchronization has been studied for: uncertain but homogeneous agents (Li and Ding (2015); Ding and Li (2016)), or heterogeneous agents with limited uncertainty (Seyboth et al. (2016, 2015); Li et al. (2014); Mei et al. (2016)). It results that synchronization for agents that are concurrently heterogeneous and uncertain is still a major problem. Recently, to handle heterogeneity and uncertainty, it has been proposed to formulate the synchronization problem as a special *model reference adaptive control* (MRAC) in which each agents tries to converge to the model defined by its neighbors (Baldi and Frasca (2018); Harfouch et al. (2017a)). This formulation is based on 'feedback matching gains' (used to match each agent to the reference model, or leader) and 'coupling matching gains' (used to match each agent with its neighbors). Adaptive laws for both feedback and coupling gains are derived via Lyapunov analysis.
The distributed MRAC exploits a communication setting in which the input is communicated among neighbors (*distributed input* approach). This is alternative to another popular approach to synchronization, the *distributed observer* approach (Cai et al. (2017); Lu and Liu (2017); Baldi (2018)) where, in place of the input, an observation of the leader state is communicated among neighbors. Both the distributed input and the distributed observer approaches include a feedforward action and need to communicate auxiliary variables to the neighbors (inputs and observations, respectively) to reconstruct the reference signal. Despite these similarities, the distributed observer scheme can handle cyclic graphs and undirected graphs, while the distributed input with MRAC can be applied only to acyclic directed graphs. For this reason it finds main application in platooning, where no cyclic communication occurs Harfouch et al. (2017b).
Even if the distributed observer approach can be used in a larger number of cases, the distributed input approach is relevant because the dimension of the input vector is typically smaller than the dimension of the leader state vector, and thus communication with distributed inputs is less cumbersome. While hierarchical architectures have been proposed to remove cycles in the distributed input approach (Wang et al. (2016)), the open question that motivates this work is: is it possible to design MRAC algorithms based on distributed input with the ability to handle cyclic and undirected graphs? This work gives a positive answer by showing that the same MRAC parametrization derived for the acyclic graph case can be extended to cyclic and undirected graphs.
Notation: The transpose of a matrix or of a vector is indicated with X' and x' respectively. A directed graph (digraph) is indicated with the pair $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is a finite nonempty set of nodes, and $\mathcal{E} \in \mathcal{N} \times \mathcal{N}$ is a set of ordered pair of nodes, called edges. The adjacency matrix $\mathcal{A} = [a_{ij}]$ of an unweighted digraph is defined as $a_{ii} = 0$ and $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, where $i \neq j$.

2. THE ACYCLIC CASE

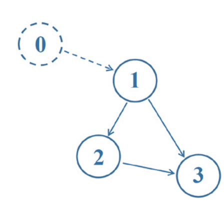

Fig. 1. A simple acyclic network

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To recall the synchronization results in the acyclic case, let us consider the network in Fig. 1. Three agents, denoted with indices 1, 2 and 3, have uncertain dynamics

$$\begin{aligned}\dot{x}_1 &= A_1 x_1 + b_1 u_1\\ \dot{x}_2 &= A_2 x_2 + b_2 u_2\\ \dot{x}_3 &= A_3 x_3 + b_3 u_3\end{aligned} \tag{1}$$

where $x_1, x_2, x_3 \in \mathbb{R}^n$ is the state, $u_1, u_2, u_3 \in \mathbb{R}$ is the input, and A_1, A_2, A_3 and b_1, b_2, b_3 are *unknown* matrices of appropriate dimensions, with possibly $A_1 \neq A_2 \neq A_3$ and $b_1 \neq b_2 \neq b_3$. Time index will be omitted whenever obvious. Consider the reference model

$$\dot{x}_m = A_m x_m + b_m r \tag{2}$$

where $x_m \in \mathbb{R}^n$ is the state of the reference model, $r \in \mathbb{R}$ is its reference input, and A_m and b_m are *known* matrices of appropriate dimensions, with A_m being Hurwitz so as to have bounded state trajectories x_m.

The synchronization task is achieved when $x_1, x_2, x_3 \to x_m$ for $t \to \infty$. Being the system matrices in (1) unknown, the synchronization task has to be achieved in an adaptive fashion. In order to have a well-posed adaptive control problem, the following assumptions should be verified.

Assumption 1. [Feedback matching conditions] There exist vectors k_1^*, k_2^*, k_3^* and scalars l_1^*, l_2^*, l_3^* such that

$$\begin{aligned}A_m &= A_1 + b_1 k_1^{*\prime}, \quad b_m = b_1 l_1^*\\ A_m &= A_2 + b_2 k_2^{*\prime}, \quad b_m = b_2 l_2^*\\ A_m &= A_3 + b_3 k_3^{*\prime}, \quad b_m = b_3 l_3^*.\end{aligned} \tag{3}$$

Assumption 2. The signs of the input vector fields, i.e. the signs of l_1^*, l_2^*, l_3^*, are known.

Assumptions 1 and 2 are classical conditions mutuated from MRAC (Tao (2003); Ioannou and Sun (2012)). We deal with the single-input case, although extension to the multi-input is possible following the related multivariable adaptive control theory. A consequence of Assumption 1 is the existence of coupling gains between neighboring agents.

Proposition 1. [Coupling matching conditions] There exist vectors k_{21}^*, k_{31}^*, k_{32}^* and scalars l_{21}^*, l_{31}^*, l_{32}^* such that

$$\begin{aligned}A_1 &= A_2 + b_2 k_{21}^{*\prime}, \quad b_1 = b_2 l_{21}^*\\ A_1 &= A_3 + b_3 k_{31}^{*\prime}, \quad b_1 = b_3 l_{31}^*\\ A_2 &= A_3 + b_3 k_{32}^{*\prime}, \quad b_2 = b_3 l_{32}^*.\end{aligned} \tag{4}$$

Proof. To derive (4), we find from (3)

$$b_1 = b_2 \frac{l_2^*}{l_1^*}, \quad A_1 - A_2 = b_2 k_2^{*\prime} - b_1 k_1^{*\prime} \tag{5}$$

which gives $k_{21}^{*\prime} = k_2^{*\prime} - \frac{l_2^*}{l_1^*} k_1^{*\prime}$ and $l_{21}^* = \frac{l_2^*}{l_1^*}$. Similar calculations hold for k_{31}^*, k_{32}^*, l_{31}^*, l_{32}^*. □

The synchronization of agent 1 to the reference model is the well-known model reference adaptive control (Tao, 2003, Chap. 4): it amounts to the controller

$$u_1(t) = k_1'(t) x_1(t) + l_1(t) r(t) \tag{6}$$

with k_1, l_1 the estimates of k_1^*, l_1^*, and to the adaptive laws

$$\begin{aligned}\dot{k}_1' &= -sgn(l_1^*)\gamma_k b_m' P e_1 x_1'\\ \dot{l}_1 &= -sgn(l_1^*)\gamma_l b_m' P e_1 r\end{aligned} \tag{7}$$

where $e_1 = x_1 - x_m$, the scalars $\gamma_k, \gamma_l > 0$ are adaptive gains, and P is a positive definite matrix satisfying

$$P A_m + A_m' P = -Q, \qquad Q > 0. \tag{8}$$

Proving synchronization exploits the Lyapunov function

$$V_1(e_1, \tilde{k}_1, \tilde{l}_1) = e_1' P e_1 + tr\left(\frac{\tilde{k}_i' \tilde{k}_i}{\gamma_k |l_i^*|}\right) + \frac{\tilde{l}_i^2}{\gamma_l |l_i^*|} \tag{9}$$

and the error dynamics

$$\dot{e}_1 = A_m e_{21} + b_1(\tilde{k}_1' x_1 + \tilde{l}_1 r). \tag{10}$$

The details are well known, cf. (Tao, 2003, Chap. 4).

The synchronization of agent 2 to agent 1 should avoid the use of r. This is possible via the controller

$$u_2(t) = k_{21}'(t) x_1(t) + k_2'(t)(x_2(t) - x_1(t)) + l_{21}(t) u_1(t) \tag{11}$$

and the adaptive laws

$$\begin{aligned}\dot{k}_{21}' &= -sgn(l_2^*)\gamma_k b_m' P (x_2 - x_1) x_1'\\ \dot{k}_2' &= -sgn(l_2^*)\gamma_k b_m' P (x_2 - x_1)(x_2 - x_1)'\\ \dot{l}_{21} &= -sgn(l_2^*)\gamma_l b_m' P (x_2 - x_1) u_1\end{aligned} \tag{12}$$

where k_{21}, k_2, l_{21} are the estimates of k_{21}^*, k_2^*, l_{21}^* respectively. The scalar l_2^* does not need to be estimated, only its sign is needed. The adaptation law in (12) is derived via the dynamics of the error $e_{21} = x_2 - x_1$

$$\dot{e}_{21} = A_m e_{21} + b_2(\tilde{k}_{21}' x_1 + \tilde{k}_2' e_{21} + \tilde{l}_{21} u_1) \tag{13}$$

with $\tilde{k}_{21} = k_{21} - k_{21}^*$, $\tilde{k}_2 = k_2 - k_2^*$, $\tilde{l}_{21} = l_{21} - l_{21}^*$. By taking the Lyapunov function

$$V_{21} = e_{21}' P e_{21} + tr\left(\frac{\tilde{k}_{21}' \tilde{k}_{21}}{\gamma_k |l_2^*|}\right) + tr\left(\frac{\tilde{k}_2' \tilde{k}_2}{\gamma_k |l_2^*|}\right) + \frac{\tilde{l}_{21}^2}{\gamma_l |l_2^*|} \tag{14}$$

we can calculate the time derivative of (14) along (13)

$$\begin{aligned}\dot{V}_{21} = &-e_{21}' Q e_{21} + 2(sgn(l_2^*) b_m' P e_{21} x_1' + \gamma_k^{-1} \dot{\tilde{k}}_{21}') \frac{\tilde{k}_{21}'}{|l_2^*|}\\ &+ 2(sgn(l_2^*) b_m' P e_{21} e_{21}' + \gamma_k^{-1} \dot{\tilde{k}}_2') \frac{\tilde{k}_2}{|l_2^*|}\\ &+ 2(sgn(l_2^*) b_m' P e_{21} u_1 + \gamma_l^{-1} \dot{\tilde{l}}_{21}) \frac{\tilde{l}_{21}'}{|l_2^*|}\\ = &-e_{21}' Q e_{21}\end{aligned} \tag{15}$$

where we have used (12). Using standard Lyapunov arguments and Barbalat's lemma we can show $\dot{V}_{21} \to 0$ as $t \to \infty$ and hence $e_{21} \to 0$.

To synchronize agent 3 to agents 1 and 2, let us derive the dynamics of the error $e_{31} = x_3 - x_1$ and $e_{32} = x_3 - x_2$

$$\begin{aligned}\dot{e}_{31} &= A_m e_{31} + b_3(u_3 - k_{31}^{*\prime} x_1 - k_3^{*\prime} e_{31} - l_{31}^* u_1)\\ \dot{e}_{32} &= A_m e_{32} + b_3(u_3 - k_{32}^{*\prime} x_2 - k_3^{*\prime} e_{32} - l_{32}^* u_2)\end{aligned} \tag{16}$$

which leads us to the controller

$$\begin{aligned}u_3(t) = &k_{31}'(t)\frac{x_1(t)}{2} + k_{32}'(t)\frac{x_2(t)}{2} + k_3'(t)\frac{e_{31}(t) + e_{32}(t)}{2}\\ &+ l_{31}(t)\frac{u_1(t)}{2} + l_{32}(t)\frac{u_2(t)}{2}\end{aligned} \tag{17}$$

and the adaptive laws

$$\begin{aligned}\dot{k}'_{31} &= -sgn(l^*_3)\gamma_k b'_m P(e_{31}+e_{32})x'_1\\ \dot{k}'_{32} &= -sgn(l^*_3)\gamma_k b'_m P(e_{31}+e_{32})x'_2\\ \dot{k}'_{3} &= -sgn(l^*_3)\gamma_k b'_m P(e_{31}+e_{32})(e_{31}+e_{32})'\\ \dot{l}_{31} &= -sgn(l^*_3)\gamma_l b'_m P(e_{31}+e_{32})u_1\\ \dot{l}_{32} &= -sgn(l^*_3)\gamma_l b'_m P(e_{31}+e_{32})u_2 \end{aligned} \tag{18}$$

where k_{31}, k_{32}, k_3, l_{31}, l_{32} are the estimates of k^*_{31}, k^*_{32}, k^*_3, l^*_{31}, l^*_{32} respectively. We derive the adaptation law in (18) via the dynamics of the error $e_{321} = e_{31} + e_{32}$ and the Lyapunov function

$$V_{321} = e'_{321}Pe_{321} + tr\left(\frac{\tilde{k}'_{31}\tilde{k}_{31}}{\gamma_k|l^*_3|}\right) + tr\left(\frac{\tilde{k}'_{32}\tilde{k}_{32}}{\gamma_k|l^*_3|}\right) + tr\left(\frac{\tilde{k}'_{3}\tilde{k}_{3}}{\gamma_k|l^*_3|}\right) + \frac{\tilde{l}^2_{31}}{\gamma_l|l^*_3|} + \frac{\tilde{l}^2_{32}}{\gamma_l|l^*_3|}. \tag{19}$$

It is possible to verify $\dot{V}_{321} = -e'_{321}Qe_{321}$ and $e_{321} \to 0$ using similar Lyapunov arguments as before. This shows that the state of agent 3 converges to the average of the states of agents 1 and 2. Since the states of agents 1 and 2 converge to the state x_m of the reference model, then the state of agent 3 will converge to x_m as well (cf. Rosa (2018) for the full details). Overall synchronization can be proved via the Lyapunov function $V_1 + V_{21} + V_{321}$.

Remark 1. The adaptation laws (12) *and* (18) *reminds the setting of systems stabilizing each other through adaptation (Narendra and Harshangi (2014)), with the peculiar difference that the directed path to the leader makes our adaptation always stable. On the other hand, (Narendra and Harshangi (2015)) discusses instability due to the absence of such leader.*

2.1 General acyclic case

Extending from Fig. 1, let us consider a set of N agents

$$\dot{x}_i = A_i x_i + b_i u_i, \quad i \in \{1,\dots,N\} \tag{20}$$

where agent 1 is the one that can access the reference r in (2). Assumptions 1 and 2 are generalized to

$$A_m = A_i + b_i k^{*\prime}_i,\ b_m = b_i l^*_i \tag{21}$$

with known signs of l^*_i.

Remark 2. Similarly to Proposition 1, one can verify the existence, for every pair of agents (i,j)*, of a constant vector* k^*_{ji} *and a scalar* l^*_{ji} *such that*

$$A_i = A_j + b_j k^{*\prime}_{ji}, \qquad b_j = b_2 l^*_{ji}. \tag{22}$$

For convenience of notation, let us rewrite (2) as

$$\dot{x}_0 = A_0 + x_0 + b_0 u_0 \tag{23}$$

with $x_0 = x_m$, $A_m = A_0 + b_0 k^{*\prime}_0$, $b_0 l^*_{10} = b_m$, $u_0 = k^{*\prime}_0 x_0 + l^*_0 r$, where k^*_0, l^*_0 are known and do not need to be estimated. This gives the controller (equivalent to (6))

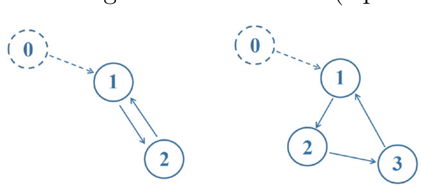

Fig. 2. Simple undirected (left) and cyclic (right) networks

$$u_1(t) = k'_{10}(t)x_1(t) + k'_1(t)(x_1(t) - x_0(t)) + l_{10}(t)u_1(t). \tag{24}$$

Under the following assumption a synchronization result holds.

Assumption 3. The directed communication graph is acyclic. In addition, the graph contains a directed spanning tree with the leader as the root node.

Theorem 1. Under Assumptions 1-3, consider the linear systems (20), with reference model (23), controllers

$$u_j(t) = \frac{\sum_{i=0}^N a_{ij}k'_{ji}(t)x_i(t)}{\sum_{i=0}^N a_{ij}} + k'_j(t)\frac{\sum_{i=0}^N a_{ij}(x_j(t)-x_i(t))}{\sum_{i=0}^N a_{ij}} + \frac{\sum_{i=0}^N a_{ij}l_{ji}(t)u_i(t)}{\sum_{i=0}^N a_{ij}} \tag{25}$$

with the index $i = 0$ used for the reference model (i.e. $a_{j0} \neq 0$ only for the root node), and update laws

$$\begin{aligned}\dot{k}'_{ji} &= -sgn(l^*_j)\gamma_k b'_m P\left[\sum_{i=0}^N a_{ij}e_{ji}\right]x'_i\\ \dot{k}'_{j} &= -sgn(l^*_j)\gamma_k b'_m P\left[\sum_{i=0}^N a_{ij}e_{ji}\right]\left[\sum_{i=0}^N a_{ij}e_{ji}\right]'\\ \dot{l}_{ji} &= -sgn(l^*_j)\gamma_l b'_m P\left[\sum_{i=0}^N a_{ij}e_{ji}\right]u_i \end{aligned} \tag{26}$$

where $e_{ji} = x_j - x_i$, and k_{ji}, k_i, l_{ji} are the estimates of k^*_{ji}, k^*_i, l^*_{ji} respectively. Then, all closed-loop signals are bounded and $e_i = x_i - x_m$, $e_{ji} = x_j - x_i$, with i,j such that $a_{ij} \neq 0$, converge asymptotically to zero.

Proof 1. The proof uses the Lyapunov function

$$V = \sum_{j=1}^N\left[\sum_{i=0}^N a_{ij}e_{ji}\right]' P\left[\sum_{i=0}^N a_{ij}e_{ji}\right] + \sum_{j=1}^N tr\left[\frac{\tilde{k}'_j\tilde{k}_j}{\gamma_k|l^*_j|}\right] + \sum_{j=1}^N\sum_{i=0}^N a_{ij}tr\left[\frac{\tilde{k}'_{ji}\tilde{k}_{ji}}{\gamma_k|l^*_j|}\right] + \sum_{j=1}^N\sum_{i=0}^N a_{ij}\frac{\tilde{l}^2_{ji}}{\gamma_l|l^*_j|} \tag{27}$$

Stability tools are similar as before and left to the reader. □

3. THE CYCLIC AND UNDIRECTED CASE

To understand the effect of cycles and undirected links, let us consider the undirected network of Fig. 2 (left). Let us assume we can calculate the inputs using the same method of Theorem 1. The ideal control actions are

$$\begin{aligned}2u_1 &= k^{*\prime}_1 x_1 + l^*_1 r + k^{*\prime}_1(x_1 - x_2) + k^{*\prime}_{12}x_2 + l^*_{12}u_2\\ u_2 &= k^{*\prime}_2(x_2 - x_1) + k^{*\prime}_{21}x_1 + l^*_{21}u_1.\end{aligned} \tag{28}$$

In order to unequivocally determine u_1 and u_2, the following equation should have a unique solution

$$\begin{bmatrix} 2 & -l^*_{12}\\ -l^*_{21} & 1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} = \begin{bmatrix}k^{*\prime}_1(2x_1 - x_2) + l^*_1 r + k^{*\prime}_{12}x_2\\ k^{*\prime}_2(x_2 - x_1) + k^{*\prime}_{21}x_1\end{bmatrix}.$$

From Proposition 1 we have $l^*_{12}l^*_{21} = 1$, so the determinant of the square matrix above is $2 - l^*_{12}l^*_{21} = 1$, and the ideal inputs u_1 and u_2 are well defined. In addition, the synchronization error dynamics with the ideal gains is

$$\begin{aligned} \dot{e}_1 + \dot{e}_{12} &= 2A_1x_1 + b_1k_1^{*\prime}x_1 + b_1l_1^*r - A_mx_m - b_mr \\ &\quad -b_1k_1^{*\prime}x_2 + k_{12}^{*\prime}x_2 + l_{12}^*u_2 - A_2x_2 - b_2u_2 \\ \dot{e}_{21} &= A_2x_2 + b_2k_2^{*\prime}(x_2 - x_1) + b_2k_{21}^{*\prime}x_1 + b_2l_{21}^*u_1 \\ &\quad -A_1x_1 - b_1u_1 \end{aligned} \quad (29)$$

which leads to

$$\dot{e}_1 + \dot{e}_{12} = A_m(e_1 + e_{12}), \quad \dot{e}_{21} = A_me_{21}. \quad (30)$$

Let us now consider the directed cyclic network of Fig. 2 (right) and calculate the inputs using the method of Theorem 1

$$\begin{aligned} 2u_1 &= k_1^{*\prime}x_1 + l_1^*r + k_1^{*\prime}(x_1 - x_3) + k_{13}^{*\prime}x_3 + l_{13}^*u_3 \\ u_2 &= k_2^{*\prime}(x_2 - x_1) + k_{21}^{*\prime}x_1 + l_{21}^*u_1 \\ u_3 &= k_3^{*\prime}(x_3 - x_2) + k_{32}^{*\prime}x_2 + l_{32}^*u_2. \end{aligned} \quad (31)$$

In order to unequivocally determine u_1, u_2 and u_3, the following equation should have a unique solution

$$\begin{bmatrix} 2 & 0 & -l_{13}^* \\ -l_{21}^* & 1 & 0 \\ 0 & -l_{32}^* & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 2k_1^{*\prime}x_1 + l_1^*r - (k_1^* - k_{13}^*)'x_3 \\ k_2^{*\prime}(x_2 - x_1) + k_{21}^{*\prime}x_1 \\ k_3^{*\prime}(x_3 - x_2) + k_{32}^{*\prime}x_2 \end{bmatrix}.$$

The determinant of the square matrix above is $2 - l_{13}^*l_{21}^*l_{32}^* = 1$, so even in this case the ideal inputs u_1, u_2 and u_3 are well defined. In addition, using similar calculations as in the previous case, it is possible to show that

$$\begin{aligned} \dot{e}_1 + \dot{e}_{13} &= A_m(e_1 + e_{13}) \\ \dot{e}_{21} &= A_me_{21}, \qquad \dot{e}_{32} = A_me_{32}. \end{aligned} \quad (32)$$

Moving beyond the analysis with ideal input, we have that in the presence of parametric uncertainties, the following result, which extends the parametrization in (Baldi and Frasca (2018)) to general graphs, holds.

Theorem 2. Under Assumptions 1 and 2, for any pair of agents (j, i), the dynamics of the synchronization error

$$\sum_{i=0}^{N} a_{ij}\dot{e}_{ji} = A_m \sum_{i=0}^{N} a_{ij}e_{ji} + b_j \cdot \quad (33)$$
$$\left[\sum_{i=0}^{N} a_{ij}\tilde{k}_{ji}'x_i + \tilde{k}_j' \sum_{i=0}^{N} a_{ij}e_{ji} + \sum_{i=0}^{N} a_{ij}\tilde{l}_{ji}u_i\right]$$

holds independently of the network connection.

Proof 2. The parametrization (33) turns out to be independent on the network connection thanks to the fact that all error dynamics can be homogenized to the reference model (A_m, b_m) via appropriate control gains. Therefore, the dynamics can be summed (cf. (29) or (32)) even in the presence of cycles and undirected links. □

Given the parametrization (33), one might be tempted to say that the algorithm in Theorem 1 can be used straightforwardly with the Lyapunov function (27). However, some attention must be paid when doing this: in fact, the actual input u_j may be not well defined for all time instants on general graphs. To explain this point, let us collect all inputs in (25) on the left-hand side, leading to $U\,[u_1 \;\cdots\; u_N]^T = [\beta_1 \;\cdots\; \beta_N]^T$ for an appropriate square matrix U depending on the estimates l_{ji}. On directed acyclic graphs, U can be made upper triangular, with positive weights in its main diagonal, thus U is always invertible. On general graphs, the invertibility of U depends on l_{ji} [1]. Despite this difficult analytic aspect, the simulations in the next section show that the algorithm in Theorem 1 can handle networks beyond Assumption 3, and U turns out to be invertible at all time instants.

Remark 3. The expression after (31) reveals that the agent are 'fictitiously' inverting the matrix U by only using neighbors' information. How such inversion is robust to delays is an open problem worth of future investigation.

4. SIMULATIONS

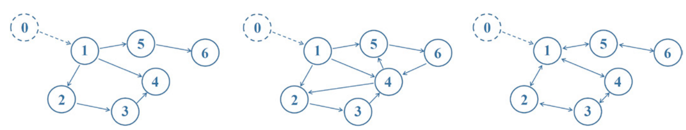

Fig. 3. Acyclic, cyclic and undirected networks

The simulations are carried out on a directed acyclic network, on a directed cyclic network, and on an undirected network, as shown in Fig. 3. Node 1 acts as the leader node and the reference model is indicated as agent 0. The agents are second-order linear agents in the canonical form

$$\dot{x}_i = \begin{bmatrix} 0 & 1 \\ a_{1_i} & a_{2_i} \end{bmatrix} x_i + \begin{bmatrix} 0 \\ b_{1_i} \end{bmatrix} u_i \quad (34)$$

with coefficients and initial conditions as in Table 1. The matrices are given in terms of $\dot{x}_0 = A_mx_0 + b_mr$ for the reference model and $\dot{x}_i = A_ix_i + b_iu_i$, $i \in \{1, \ldots, N\}$ for the other agents. The other design parameters are taken as: $\gamma_k = 3$, $\gamma_l = 0.3$, $Q = diag(1, 3)$, and all estimated gains are initialized to 0. Also note that $sgn(l_i^*) = 1$, $\forall i$. The simulations are carried out for a sinusoidal reference r of frequency 0.2 rad/s and amplitude 1. For the acyclic network, the resulting synchronization is shown in Figs. 4 and 7. All states converge asymptotically to the state of the reference model.

Table 1. Coefficients and initial conditions of agents

	a_1	a_2	b_1	x_0
agent #0	-0.5	-1	1	$[1 \;\; -1]'$
agent #1	-1	2	1	$[1 \;\; 1]'$
agent #2	-0.75	2.5	0.5	$[-1 \;\; -1]'$
agent #3	-1.25	2	1.25	$[-1 \;\; 0]'$
agent #4	-0.5	1	0.75	$[0 \; 1]'$
agent #5	-0.75	1	1.5	$[1 \;\; 0]'$
agent #6	-1.5	2.5	1	$[-1 \;\; 1]'$

In the directed cyclic graph, two cycles are present (2-3-4 and 4-5-6). Using the same parameters as in the previous simulations, the resulting synchronization is shown in Figs. 5 and 8. It can be seen that synchronization is slightly faster, at the price of a larger magnitude of the input. Finally, for the undirected graph the synchronization is shown in Figs. 6 and 9). It is observed that having bidirectional connections does not necessarily help synchronization: synchronization is slower than in the previous cases.

[1] A companion paper (Baldi et al. (2018)) shows that appropriate parameter projection can guarantee invertibility of U.

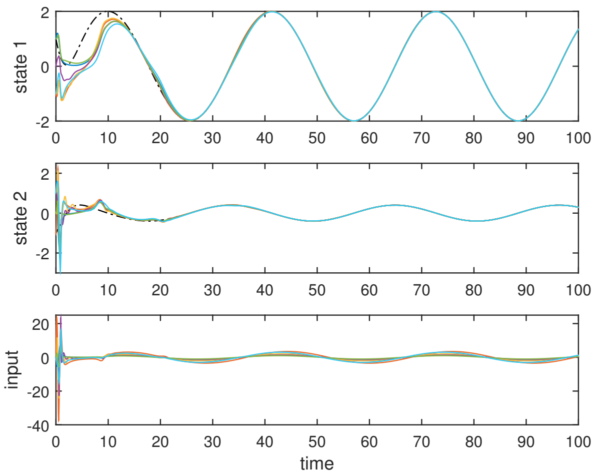

Fig. 4. Acyclic network: state/input for reference model (dashed) and agents (solid)

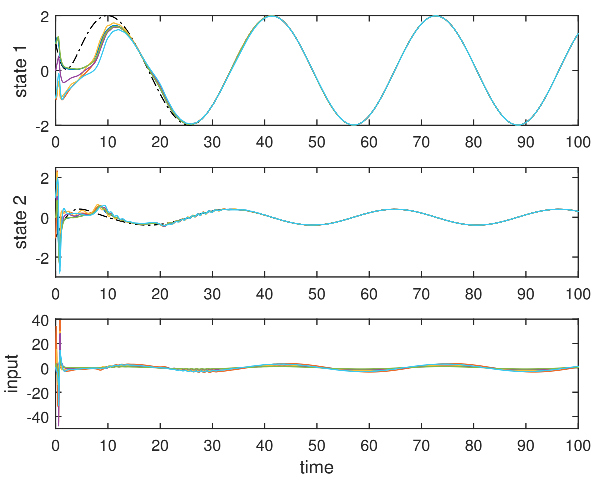

Fig. 5. Cyclic network: state/input for reference model (dashed) and agents (solid)

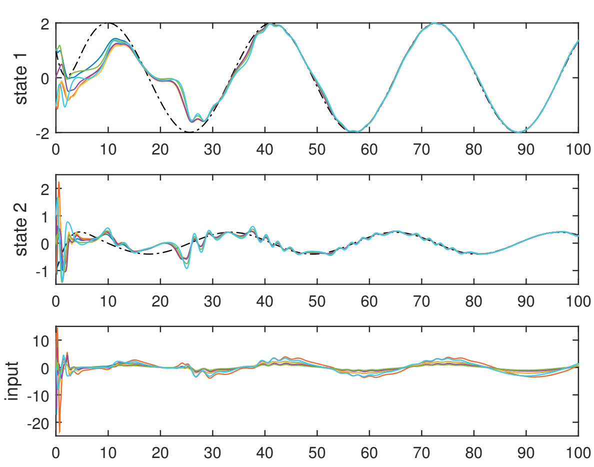

Fig. 6. Undirected network: state/input for reference model (dashed) and agents (solid)

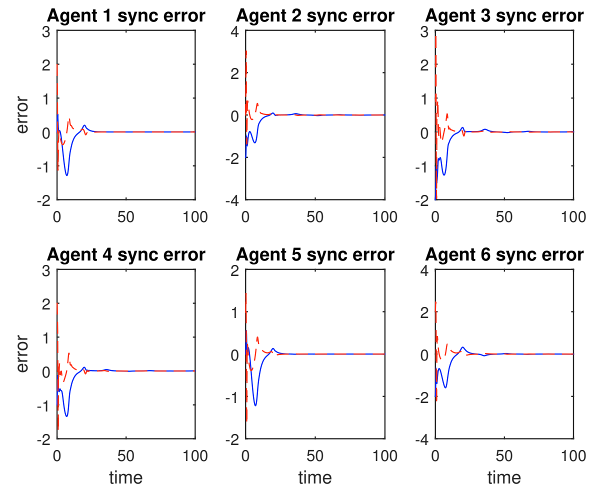

Fig. 7. Acyclic network: state synchronization errors for all agents

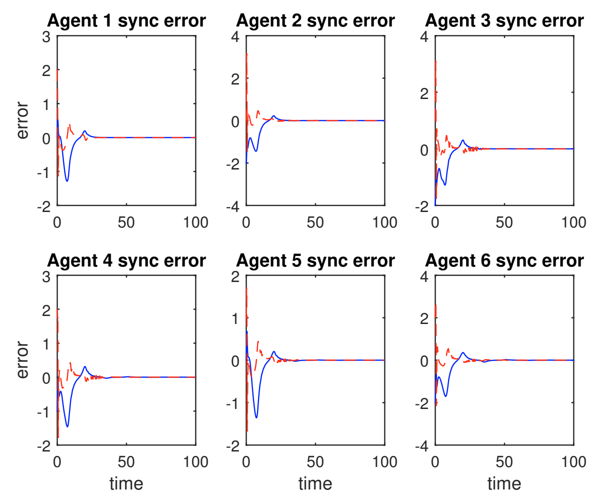

Fig. 8. Cyclic network: state synchronization errors for all agents

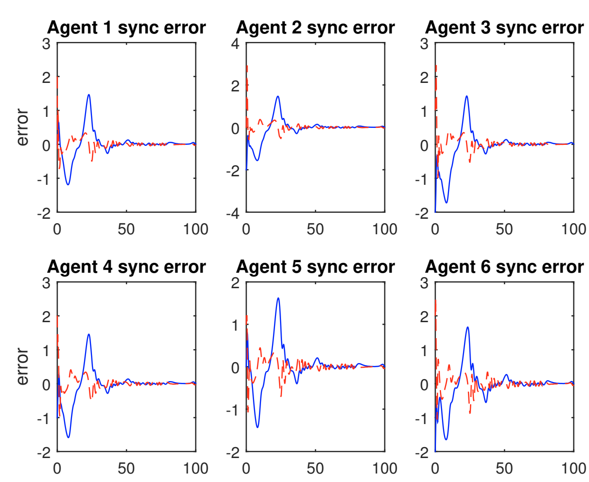

Fig. 9. Undirected network: state synchronization errors for all agents

5. CONCLUSIONS

We studied synchronization of uncertain agents via a MRAC formulation with distributed input. We showed that the parametrization derived for the acyclic case (Baldi and Frasca (2018)) can be extended to more general graphs. Despite a suitable Lyapunov function exists and ideal inputs (with ideal gains) might be well defined, it is difficult to prove that the actual inputs (with estimated gains) are well defined for all time instant. Simulations suggest so. Future work will include considering unmatched uncertainties (Lymperopoulos and Ioannou (2016)) and switching topologies by using adaptive switching strategies (Sang and Tao (2012); Yuan et al. (2017)).

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