

## Intersection crossing of cooperative multi-vessel systems

Chen, Linying; Negenborn, Rudy; Hopman, Hans

**DOI**

[10.1016/j.ifacol.2018.07.062](https://doi.org/10.1016/j.ifacol.2018.07.062)

**Publication date**

2018

**Document Version**

Final published version

**Published in**

IFAC-PapersOnLine

**Citation (APA)**

Chen, L., Negenborn, R., & Hopman, H. (2018). Intersection crossing of cooperative multi-vessel systems. *IFAC-PapersOnLine*, 51(9), 379-385. <https://doi.org/10.1016/j.ifacol.2018.07.062>

**Important note**

To cite this publication, please use the final published version (if applicable).  
Please check the document version above.

**Copyright**

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

**Takedown policy**

Please contact us and provide details if you believe this document breaches copyrights.  
We will remove access to the work immediately and investigate your claim.

# Intersection Crossing of Cooperative Multi-vessel Systems

Linying Chen, Rudy R. Negenborn, Hans Hopman

*Department of Maritime and Transport Technology, Delft University of Technology, Delft, The Netherlands*

L.chen-2@tudelft.nl

## Abstract:

A Cooperative Multi-Vessel System (CMVS) is a system consisting of multiple coordinated vessels. Vessels utilize Vessel-2-Vessel and Vessel-2-Infrastructure communication to making decisions with negotiating and/or collaborating with each other for a common goal. Due to the geographic limitations of banks and navigation rules and regulations, in straight waterways, the cooperation of vessels usually results in train-like formations. This behavior is similar to the highway platooning of vehicles. A particular challenge arises when such platoons have to cross waterway intersections. At the intersections, the vessel trains need to interact with others. However, research on the interaction between vehicle platoons is still lacking.

This paper focuses on the cooperation of vessels at waterway intersections. We propose a framework for cooperative scheduling and control of CMVSs at intersections. The actions of the vessels are determined by solving two problems: Waterway Intersection Scheduling (WIS) and Vessel Train Formation (VTF). Firstly, the process of the vessels passing through an intersection is regarded as consumption of space and time. The WIS helps to find a conflict-free schedule for the vessels from different directions. By solving the WIS problem, each vessel's desired time of arrival can be determined. Then, the actions of vessels are determined using a distributed Model Predictive Control algorithm in the VTF problem. Agreement among the vessels is achieved via serial iterative negotiations. Simulation experiments are carried out to illustrate the effectiveness of the proposed framework. We compare the passing time of each vessel, and the total passing time in three scenarios: non-cooperative case, partially-cooperative case, and fully-cooperative case. With the proposed cooperative framework, vessels can have smoother trajectories. The total passing time and the passing time for each vessel also benefit from the cooperation. Besides, the proposed framework can be extended to the whole waterway network where other infrastructure (bridges and locks) exists.

© 2018, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

## Keywords:

Cooperative Multi-vessel System, Waterway Intersection Control, Vessel Train Formations, Distributed Model Predictive Control

## 1. INTRODUCTION

Optimizing the waterborne transport system requires not only automation of the individual vessels, but also coordination among vessels. In this paper, a system consisting of multiple coordinated autonomous vessels is referred to as Cooperative Multi-Vessel System (CMVS) (Chen et al., 2016). In such a system, vessels can negotiate and/or collaborate with each other for improving overall safety, efficiency, and/or environmental sustainability, via Vessel-to-Vessel and Vessel-to-Infrastructure communication.

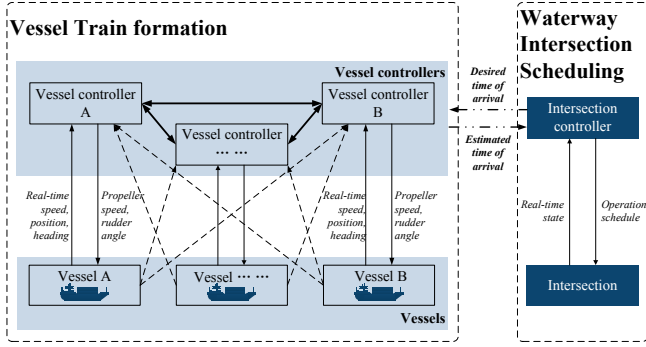
As typical cooperative behaviors, flocking of aerial vehicle and highway platooning of vehicles have been studied for decades (Olfati-Saber, 2006; Li et al., 2017). They can provide important references for the CMVS study. However, there are significant differences between the cooperation of vessels and flocking and platooning behaviors. Firstly, the configuration of a CMVS differs from typical flocking

behaviors. Due to the geographic limitations of banks, in straight waterways, a fleet of vessels usually has a train-like formation. Secondly, waterways have intersections where the vessel trains need to interact with others. Therefore, the cooperative behavior of vessels at the intersections needs to be emphasized. Whilst, research on the interaction between vehicle platoons is still lacking.

This paper focuses on the cooperation of vessels at waterway intersections. We propose a framework consisting of Waterway Intersection Scheduling (WIS) and Vessel Train Formation (VTF) for the cooperative control of CMVSs. WIS is used to find out the desired time of arrival of each vessel. Whilst, VTF is used for stable distance keeping in straight waterways, while helping the vessels arrive at a certain point at a desired time in sequence.

The remainder of this paper is organized as follows. In Section 2, the framework for cooperation at intersections is proposed. As parts of the framework, the WIS and VTF

Fig. 1. Framework for CMVSs at intersections



problems are stated. Subsequently, the formulation and algorithms for the two problems are presented in Section 3. In Section 4, simulation experiments are carried out to illustrate the potential of the proposed method. The results are compared with the non-cooperative cases and the partially-cooperative cases. Conclusions and future research directions are given in Section 5.

## 2. FRAMEWORK FOR CMVSs AT INTERSECTIONS

The framework of cooperative navigation of CMVSs is shown in Fig. 1. Through communication and cooperation, a vessel keeps a stable distance with respect to nearby vessels. Due to the limitation of banks, the vessels move to a train-like formation in straight waterways. When approaching an intersection, vessels report their estimate time of arrival to the intersection controller. Then, the controller makes a schedule, and tells those vessels the desired time of arrival at the intersection. After passing through the intersection, vessels sailing in the same waterways form new vessel trains for safe navigation.

Therefore, the cooperation of CMVSs at intersections is divided into two parts. The WIS is to create a conflict-free schedule on arrival time at the intersections, while the VTF is used to make the vessel arrive at its desired arrival time. The VTF is also responsible for the conflicts of vessels in the same CMVS.

### 2.1 Waterway Intersection Scheduling Problem

In this paper, we assume that a waterway can be represented by two lanes, and that all vessels sail along the starboard side. Thus, an intersection can be divided into four blocks. As shown in Fig. 2, a vessel passing through an intersection can be regarded as occupying some blocks for a certain period. Therefore, the WIS problem can be formulated as a Job Shop Scheduling Problem (JSSP), in which several jobs need to be processed by a number of machines in a given order.

A significant difference between WIS and typical JSSP is, however, that vessels usually do not stop at the intersection. Thus, for a vessel, there is no waiting time between blocks which it should visit. Therefore, the WIS problem is a variant of a JSSP. The aim is to minimize the makespan, i.e., the time when all vessels passing through the intersection, under the following conditions:

- Sequential constraint: a vessel passes through the blocks in a predetermined sequence;

Fig. 2. Occupation of crossing blocks

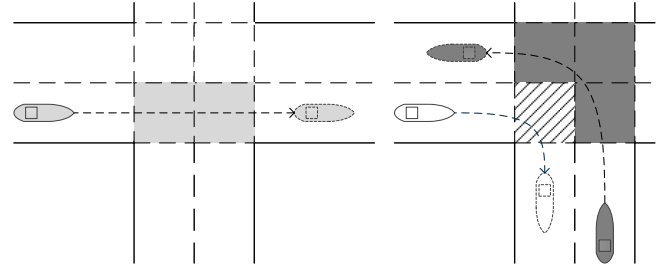
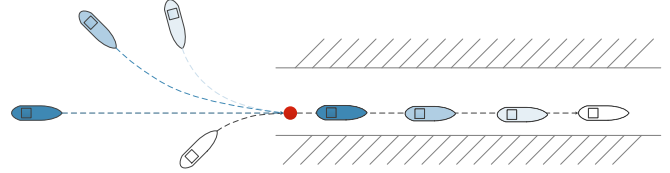


Fig. 3. Vessel train formation



- No-wait constraint: a vessel has to enter the next block immediately when it leaves a block;
- Disjunctive constraint: other vessels cannot enter a block until the one inside leaves the block.

### 2.2 Vessel Train Formation Problem

As mentioned, in straight waterways, the cooperation of vessels usually results in a train-liked formation, as shown in Fig. 3. VTF problem refers to making the vessels moving in formation. Because every vessel has a desired arrival time at a certain point, such as the first block it should visit at an intersection, the vessels can change their positions in the fleet. Thus, lateral operation, i.e., overtaking behavior, is allowed. Besides, vessels usually have predetermined paths. Thus, in VTF problem, vessels' behavior follows three rules:

- Path following: attempt to follow the predetermined paths;
- Aggregation: attempt to stay close to nearby vessels;
- Collision avoidance: avoid collisions with nearby vessels.

## 3. PROBLEM SOLVING

In this section, the WIS problem and the VTF problem are formulated and solved.

### 3.1 Modeling of CMVSs

A CMVS consists of  $N$  vessels. Let the vessels be regarded as mass points. The dynamics of vessel  $i$  are given by the following linear discrete-time model:

$$\begin{cases} p_i(k+1) = p_i(k) + q_i(k) \\ q_i(k+1) = q_i(k) + u_i(k) \end{cases} \quad (1)$$

where  $p_i, q_i, u_i \in \mathbb{R}$  denote the position, velocity and acceleration of vessels  $i$ , respectively.

Due to the limitations of sensors, vessels can only receive and broadcast information over a limited range. Thus, given an interaction range  $r_i > 0$ , a vessel only communicates and interacts with vessels in this range. Then, a

CMVS can be represented by a graph  $G = (\mathcal{V}, e)$  that consists of a set of vertices and edges. Vessels are the vertices  $\mathcal{V} = \{1, 2, \dots, n\}$ . The set of edges  $e$  represents the communication and interaction possibilities between the vessels:

$$e = \{(i, j) : i, j \in \mathcal{V}, \|p_i - p_j\| \leq \min(r_i, r_j), j \neq i\} \quad (2)$$

where,  $\|\cdot\|$  is the norm. In this paper, we use infinity norm to represent the distance between vessels. If an edge exists between vessel  $i$  and  $j$ , they are neighbors, i.e.,  $i \in N_j$ ,  $j \in N_i$ .

### 3.2 Mixed integer programming for WIS

According to the statement in Section 2.1, the WIS problem can be formulated as follows:

$$\text{minimize } T_{\max} \quad (3)$$

$$\text{subject to } \forall i \in \mathcal{V}, \forall j \in \mathcal{V}, i \neq j, \forall a \in \mathcal{C} :$$

$$T_{\max} \geq s_{ia} + t_{ia}, \quad (4)$$

$$s_{ia} \geq r_i, \quad (5)$$

$$\frac{d_{ia}}{v_{\max}} \leq t_{ia} \leq \frac{d_{ia}}{v_{\min}}, \quad (6)$$

$$s_{i(a+1)} = s_{ia} + t_{ia}, \quad (7)$$

$$s_{ja} \geq s_{ia} + t_{ia} \text{ OR } s_{ia} \geq s_{ja} + t_{ja}, \quad (8)$$

$$s_{ja} \geq s_{ia} + t_{i,\text{safe}} \text{ OR } s_{ia} \geq s_{ja} + t_{j,\text{safe}}. \quad (9)$$

where,  $\mathcal{V}$  is the set of vessels;  $\mathcal{C}$  is the set of crossing blocks. In Equation (3),  $T_{\max}$  is the makespan, i.e., the total time needed by all vessels to pass through the intersection. Therefore, it is larger than or equal to the passing time of each vessel at each block, i.e., the sum of the arrival time of vessel  $i$  at block  $a$  ( $s_{ia}$ ) and the time vessel  $i$  need to pass through block  $a$  ( $t_{ia}$ ) in (4). Equation (5) means for each vessel  $i$ , there is an earliest arrive time  $r_i$ ;  $t_{ia}$  is determined by (6), where  $d_{ia}$  is the length of the path that vessel  $i$  passes through block  $a$ , and  $v_{\max}$ ,  $v_{\min}$  are the maximum and minimal speed of vessel  $i$ . Equation (7) is for the sequential and no-wait constraint, and (8) is for the disjunctive constraint. Equation (9) represents that the interval between the arrival time of each vessel at the same block should be larger than a predefined safe time interval  $t_{i,\text{safe}}$ .  $t_{i,\text{safe}}$  is calculated by safety distance  $d_{\text{safe}}$  and the velocity of the vessel, i.e.,  $t_{i,\text{safe}} = \frac{d_{\text{safe}}}{d_{ia}/t_{ia}}$ .

Mixed Integer Programming (MIP) is one of the best known exact methods for solving JSSP (Ku and Beck, 2016). In the WIS problem, the scheduling is for the vessels arrive in a certain period. It is assumed to be a small-size problem, that an MIP can solve within a reasonable amount of time. Thus, our WIS problem can be formulated as an MIP problem with the constraints (8) and (9) replaced by the following constraints:

$$\begin{cases} s_{ia} + t_{ia} \leq s_{ja} + \kappa(1 - \chi_{ij,a}) \\ s_{ja} + t_{ja} \leq s_{ia} + \kappa\chi_{ij,a} \end{cases} \quad (10)$$

$$\begin{cases} s_{ia} + t_{i,\text{safe}} \leq s_{ja} + \kappa(1 - \chi_{ij,a}) \\ s_{ja} + t_{j,\text{safe}} \leq s_{ia} + \kappa\chi_{ij,a} \end{cases} \quad (11)$$

where  $\kappa$  is an arbitrarily large number,  $\kappa \gg \sum_{i=1}^n \sum_{a=1}^m t_{ia}$ , and  $\chi_{ij,a}$  is a binary variable,

$$\chi_{ij,a} = \begin{cases} 1, & \text{if vessel } i \text{ pass block } a \text{ before } j, \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

### 3.3 Serial iterative DMPC for VTF

MPC has been popular in practical applications since its very early days (Negenborn and Maestre, 2014). For waterborne transport, MPC has been applied to vessel path following (Zheng et al., 2016b), heading control (Li and Sun, 2012), and collision avoidance (Abdelaal et al., 2016). Besides, distributed MPC has been used for cooperative control of networked vehicles (Keviczky et al., 2008). Research indicates that MPC has many advantages for the control of large-scale networked systems (Negenborn and Maestre, 2014). Therefore, in this paper, we consider MPC as a suitable approach for the control of vessels in CMVSs.

According to the three rules stated in Section 2.2, the objective of a single vessel in a CMVS can be described as

$$J_i(u_i(k)) = \sum_{\tau=1}^{H_p} \sum_{j \in N_i} (\alpha \|p_i(k + \tau | k) - w_i(k + \tau)\|_{\infty} + \beta \|d_{ij|i}(k + \tau | k) + \delta_{ij}(k + \tau | k)\|_{\infty} + \gamma \|u_i(k + \tau - 1 | k)\|_{\infty}) \quad (13)$$

The three parts in the equation represent trajectory following, aggregation and control efforts, respectively:  $\alpha$ ,  $\beta$  and  $\gamma$  are the weights on trajectory following, aggregation and control efforts;  $H_p$  is the predict horizon;  $\tau$  is the  $\tau$ th time step in the prediction horizon;  $w_i(k)$  is the reference trajectory;  $d_{ij|i}(k + \tau | k)$  is the distance between vessel  $i$  and vessels  $j$  calculated by vessel  $i$  with the position of  $j$  that  $i$  received ( $p_{j|i}$ ) i.e.,  $d_{ij|i}(k + \tau | k) = \|p_i(k + \tau | k) - p_{j|i}(k + \tau | k)\|_{\infty}$ ;  $\delta_{ij}$  is introduced for aggregation ( $d_{ij|i}(k + \tau | k) \leq r$ ,  $r = \min(r_1, r_2, \dots, r_n)$ ),  $-r \leq \delta_{ij|i}(k + \tau | k) \leq r$ ;  $u_i(k)$  indicates control input over the prediction horizon.

To simplify the model, in this paper, at each time step  $k$ , vessels are assumed to be in the same CMVS over the prediction horizon. Then, the control problem for the cooperation in a CMVS can be expressed as

$$\text{minimize } \sum_{i=1}^n J_i(u_i(k)) \quad (14)$$

$$\text{subject to } \forall i \in N, j \in N_i, \forall k \in T, \forall \tau \in H_p, \quad (15)$$

$$u_{\min} \leq \|u_i(k + \tau | k)\|_2 \leq u_{\max}, \quad (16)$$

$$v_{\min} \leq \|q_i(k + \tau | k)\|_2 \leq v_{\max}, \quad (17)$$

$$d_{ij|i}(k + \tau | k) \geq d_{\text{safe}}, \quad (18)$$

$$-r \leq \delta_{ij|i}(k + \tau | k) \leq r. \quad (19)$$

According to Richards and How (2002), problem (14)-(19) can be transferred into a MIP problem. Based on the method proposed by Negenborn et al. (2008) and Zheng et al. (2016a), the VTF problem can then be decomposed into several local problems that each vessel can solve by itself using Alternative Direction Method of Multipliers (ADMM). The cooperation among those vessels can be reached with serial information exchange and iterations adopting the Serial iterative ADMM-based DMPC proposed by Chen et al. (2018) (see Algorithm 1).

**Algorithm 1.** Serial iterative ADMM-based DMPC

```

while  $k \leq T$  do
   $z^0(k) := u_i^0(k) := [u_i^{\text{end}}(k : \text{end} | k-1); \mathbf{0}]$ ;
   $\lambda_i^0(k) := \mathbf{0}$ ;  $\rho := \rho_{\text{ini}}$ ;
  for  $s = 1 : S$  do
     $jd_g := 0$ ;  $N_{\text{jump}} := 0$ 
    for  $i = 1 : N$  do
      // Subcontrollers solve the subproblem
       $u_i^s(k) := \arg \min_{u_i(k)} \left( J_i(u_i(k)) \right.$ 
         $\left. + \lambda_i^{s-1}(k)' (u_i(k) - z_i^{s-1}(k)) \right.$ 
         $\left. + \rho_i/2 - \|u_i(k) - z_i^{s-1}(k)\|_2^2 \right)$ ;
      if solution does not exit then
         $u_i^s(k) := u_i^{s-1}(k)$ ;  $N_{\text{jump}} := N_{\text{jump}} + 1$ ;
      // Update global variable and Lagrange multiplier
       $z_i^s(k) := u_i^s(k)/2 + z_i^{s-1}(k)/2 + \lambda_i^{s-1}(k)/\rho_i$ ;
       $\lambda_i^s(k) := \lambda_i^{s-1}(k) + \rho_i (u_i^s(k) - z_i^s(k))$ ;
       $ZX_i^s := \tilde{A}_i x_i(k) + \tilde{B}_i z_i(k)$ ;
      // Update primal and dual residual and tolerance
       $R_{\text{pri},i}^s := u_i^s(k) - z_i^s(k)$ ;
       $R_{\text{dual},i}^s := z_i^s(k) - z_i^{s-1}(k)$ ;
       $\varepsilon_{\text{pri},i}^s := \sqrt{N n_u} \varepsilon^{\text{abs}}$ 
         $+ \varepsilon^{\text{rel}} \max \{ \|u_i^s(k)\|_2, \|z_i^s(k)\|_2 \}$ ;
       $\varepsilon_{\text{dual},i}^s := \sqrt{N n_u} \varepsilon^{\text{abs}} + \varepsilon^{\text{rel}} \|\lambda_i(k)^s\|_2$ ;
      // Stopping check
      if  $\|R_{\text{pri},i}^s\|_2 \leq \varepsilon_{\text{pri},i}^s$  and  $\|R_{\text{dual},i}^s\|_2 \leq \varepsilon_{\text{dual},i}^s$ 
        then  $jd_g := jd_g + 1$ ;
      // Update the penalty parameter
      case  $\|R_{\text{pri},i}^s\|_2 > 10 \|R_{\text{dual},i}^s\|_2$   $\rho_i := 2\rho_i$ ;
      case  $\|R_{\text{dual},i}^s\|_2 > 10 \|R_{\text{pri},i}^s\|_2$   $\rho_i := \rho_i/2$ ;
      // Send  $ZX_i^s$ ,  $jd_g$  and  $N_{\text{jump}}$  to others
    if  $jd_g = N$  and  $N_{\text{jump}} = 0$  then break;
  // Update the states and move to next step

```

### 3.4 Baseline scenario

In the next section, the proposed framework is compared with a baseline scenario which simulates the vessel traffic without cooperation. The arrivals of the vessels at a certain place, such as a lock, a port or an intersection, are usually regard to be random. Poisson distribution is usually used to simulate the arrivals (Kuo et al., 2006). Therefore, in the baseline scenario, the arrivals of the vessels follow a Poisson distribution.

The collision avoidance actions in the baseline scenario are determined based on the the Artificial Potential Field (APF) method. The core of APF is to take known obstacles into consideration by building a representation of the environment by potential gradients. An attractive field is assigned to the target, whilst negative field represent obstacles and so the vessel is repelled at these locations. The APF method has been widely used for simulating ship navigation (Xue et al., 2011; Rong et al., 2015). In this

paper, we use the form proposed by Xue et al. (2011) to calculate the attractive force and repulsive force, as shown in (20)-(25).

$$\mathbf{F} = \mathbf{F}_{\text{att}} + \mathbf{F}_{\text{rep}} \quad (20)$$

$$\mathbf{F}_{\text{att}} = -\nabla \mathbf{U}_{\text{att}} \quad (21)$$

$$\mathbf{F}_{\text{rep}} = \sum_{i=1}^{n_{\text{obs}}} \mathbf{F}_{\text{rep},i} = \sum_{i=1}^{n_{\text{obs}}} -\nabla \mathbf{U}_{\text{rep},i} \quad (22)$$

$$\mathbf{U}_{\text{att}} = \zeta \|\mathbf{p}_{\text{goal}} - \mathbf{p}\|^m \quad (23)$$

$$\mathbf{U}_{\text{rep},i} = \begin{cases} 0 & \text{if } \|\mathbf{p}_{\text{obs},i} - \mathbf{p}\| > \rho_0 \\ \mu_{\text{rep},i} & \text{if } d_{\text{safe}} < \|\mathbf{p}_{\text{obs},i} - \mathbf{p}\| \leq \rho_0 \\ \infty & \text{if } \|\mathbf{p}_{\text{obs},i} - \mathbf{p}\| \leq d_{\text{safe}} \end{cases} \quad (24)$$

$$\mu_{\text{rep},i} = \frac{1}{2} \eta \left( \frac{1}{\|\mathbf{p}_{\text{obs},i} - \mathbf{p}\| - d_{\text{safe}}} - \frac{1}{\rho_0 - d_{\text{safe}}} \right)^2 \times \|\mathbf{p}_{\text{goal}} - \mathbf{p}\|^n \quad (25)$$

where  $\mathbf{F}$ ,  $\mathbf{F}_{\text{att}}$ , and  $\mathbf{F}_{\text{rep}}$  are the total force, attractive force and repulsive force, respectively;  $\mathbf{U}_{\text{att}}$  is the attractive potential due to the goal;  $\mathbf{U}_{\text{rep},i}$  is the repulsive potential due to the obstacle  $i$ ;  $\mathbf{p}$ ,  $\mathbf{p}_{\text{goal}}$  and  $\mathbf{p}_{\text{obs},i}$  are the position of own ship and obstacle  $i$ ;  $\zeta$ ,  $m$ ,  $\eta$ , and  $n$  are model parameters to adjust the strength of the attractive and repulsive potential;  $\rho_0$  is the influence range.

## 4. SIMULATION EXPERIMENTS

In this section, simulations of three scenarios are compared to show the potential of the proposed framework:

Scenario I: Non-cooperative case – the arrivals of the vessels follow a Poisson distribution, and the collision avoidance actions are determined by APF;

Scenario II: Partially-cooperative case – several vessel trains arrive at the intersection from different waterways, and the collision avoidance actions are determined by APF;

Scenario III: Fully-cooperative case – several vessel trains arrive at the intersection from different waterways, pass through the intersection with the proposed cooperative framework, and again form new vessel trains.

Scenario I is the baseline. Vessels in Scenario II use the same collision avoidance method as in Scenario I. Meanwhile, vessels in Scenario II and Scenario III have the same arrival pattern.

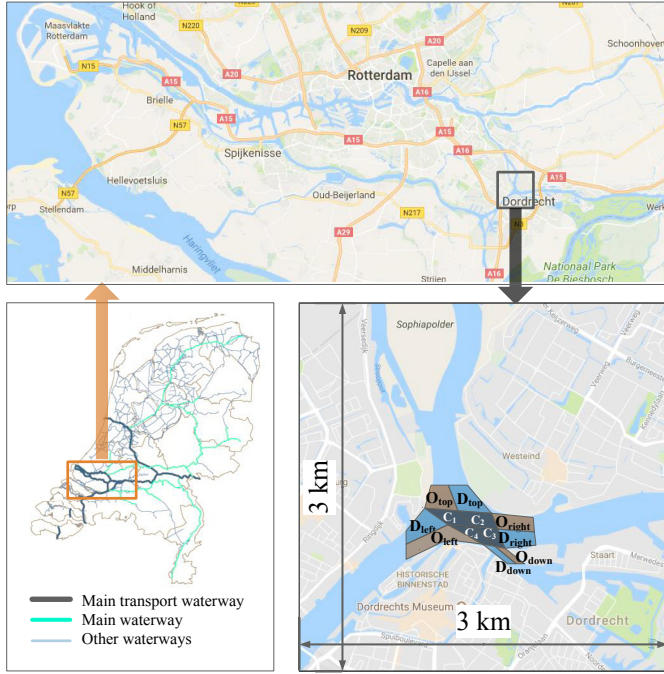
### 4.1 Simulation setup

As a connection of the four busiest inland waterways, a major intersection near the Port of Rotterdam is chosen as the simulation area (Fig. 4). The area is  $3\text{km} \times 3\text{km}$ , including straight waterways connecting to the intersection. The intersection area is divided into 4 blocks, i.e.,  $C_1 - C_4$ . In the results, the passing time of a vessel refers to the time it enters and leaves the blocks.

Ten vessels form three vessel trains, approaching the intersection from three different directions. The origins, destinations and the sequences of crossing blocks of the vessels are given in Table 1. CMVS 1 ( $V_0 - V_4$ ) starts from the left, and  $V_0$  starts earliest while  $V_4$  starts latest. CMVS 2 ( $V_5 - V_7$ ) starts from the right with the order



Fig. 4. Simulation area



Source: Statistics Netherlands (2009); Google map (2017)

Table 1. Setup for vessels in the Vessel trains

From	To	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>
O <sub>left</sub>	D <sub>top</sub> : V <sub>0</sub>	(C <sub>4</sub> , s <sub>04</sub> , p <sub>04</sub> )	(C <sub>1</sub> , s <sub>01</sub> , p <sub>01</sub> )	(C <sub>2</sub> , s <sub>02</sub> , p <sub>02</sub> )
	D <sub>right</sub> : V <sub>1</sub>	(C <sub>4</sub> , s <sub>14</sub> , p <sub>14</sub> )	(C <sub>3</sub> , s <sub>13</sub> , p <sub>13</sub> )	
	D <sub>right</sub> : V <sub>2</sub>	(C <sub>4</sub> , s <sub>24</sub> , p <sub>24</sub> )	(C <sub>3</sub> , s <sub>23</sub> , p <sub>23</sub> )	
	D <sub>right</sub> : V <sub>3</sub>	(C <sub>4</sub> , s <sub>34</sub> , p <sub>34</sub> )	(C <sub>3</sub> , s <sub>33</sub> , p <sub>33</sub> )	
	D <sub>down</sub> : V <sub>4</sub>	(C <sub>4</sub> , s <sub>44</sub> , p <sub>44</sub> )		
O <sub>right</sub>	D <sub>down</sub> : V <sub>5</sub>	(C <sub>2</sub> , s <sub>52</sub> , p <sub>52</sub> )	(C <sub>3</sub> , s <sub>53</sub> , p <sub>53</sub> )	(C <sub>4</sub> , s <sub>54</sub> , p <sub>54</sub> )
	D <sub>left</sub> : V <sub>6</sub>	(C <sub>2</sub> , s <sub>62</sub> , p <sub>62</sub> )	(C <sub>1</sub> , s <sub>61</sub> , p <sub>61</sub> )	
	D <sub>left</sub> : V <sub>7</sub>	(C <sub>2</sub> , s <sub>72</sub> , p <sub>72</sub> )	(C <sub>1</sub> , s <sub>71</sub> , p <sub>71</sub> )	
O <sub>top</sub>	D <sub>right</sub> : V <sub>8</sub>	(C <sub>1</sub> , s <sub>81</sub> , p <sub>18</sub> )	(C <sub>4</sub> , s <sub>84</sub> , p <sub>48</sub> )	(C <sub>3</sub> , s <sub>83</sub> , p <sub>83</sub> )
	D <sub>left</sub> : V <sub>9</sub>	(C <sub>1</sub> , s <sub>91</sub> , p <sub>91</sub> )		

V<sub>5</sub> → V<sub>6</sub> → V<sub>7</sub>. CMVS 3 (V<sub>8</sub> – V<sub>9</sub>) starts from the top, and V<sub>8</sub> starts earlier.

The arrivals of those vessels in Scenario I follows Poisson distributions. The average time intervals between the arrivals in the three directions are 35s, 40s, and 45s, respectively. In the Scenario II and III, the three vessel trains have been formed before the simulations start. The time interval is for each vessel train is the same, i.e., 25s.

In Scenario III, the vessels sailing towards the same direction form a new vessels train after crossing the intersection. That is, V<sub>1</sub>, V<sub>2</sub>, V<sub>3</sub>, and V<sub>8</sub> form a vessel train towards the right; V<sub>6</sub>, V<sub>7</sub> and V<sub>9</sub> form a vessel train navigating to the left; V<sub>4</sub> and V<sub>5</sub> form the vessel train going downwards; V<sub>0</sub> is the only one moving upwards.

Parameters used in the simulation are given in Table 2. Vessels in the simulations should follow predetermined reference paths. The reference paths are determined by the A\*BG algorithm we proposed in Chen et al. (2016).

Table 2. Parameters in the simulation

General	$v_{\max}$	$v_{\min}$	$d_{\text{safe}}$				$t_{\text{safe}}$	$\rho_0$	
	5m/s	0m/s	100m (vessels) 15m (bank)				20s	200m (vessels) 30m (bank)	
VTF	$u_{\max}$	$u_{\min}$	$M$	$H_{\text{p}}$	$\alpha$	$\beta$	$\gamma$	$\varepsilon^{\text{abs}}$	$\varepsilon^{\text{rel}}$
	1m/s <sup>2</sup>	0m/s <sup>2</sup>	16	10	10	1	1	10 <sup>-3</sup>	10 <sup>-3</sup>
APF	$m$		$n$		$\zeta$			$\eta$	
	2		2		10			20	

#### 4.2 Results and discussion

Figure 5 shows the trajectories of each vessel in the three scenarios. The vessels in the same original vessel train have the same color with different brightness. The vessels which form the same new vessel train after passing the intersection have the same line type (the same in Figure 7). As shown in Figure 5, in straight waterways, the trajectories in Scenario II are smoother than in the other two scenarios, because agreements have been already reached in each vessel train, and the vessels do not need to avoid collision with each other. In Scenario III, due to the desired time of arrival, overtaking occurs to change the formation of the vessel trains. Vessels in Scenario III have the smoothest trajectories when passing through the intersection owing to the conflict-free schedule.

The WIS result is shown in Figure 6. At the intersection, vessels cross certain blocks (C<sub>1</sub> – C<sub>4</sub>) in a given order without waiting between the blocks, and one block can serve one vessel one time. The arrival time intervals are at least 20s. The dashed line indicates the vessels that are initially in the same CMVS. Most vessels still stay together with different orders. However, some vessels in the same CMVS may have large arrival time intervals in order to make full use of the time and space, such as V<sub>5</sub> and V<sub>7</sub> in CMVS 2.

Figure 7 shows the changes of the vessel speeds in the three scenarios. The vessels in the same CMVS at the beginning are shown in the same subplot. The collision avoidance actions the vessels take in Scenario I are usually heading changes, which is not shown in the figure. In Scenario III, before crossing the intersection, vessels have to adjust their speed for arriving at the desired time. Consequently, frequent speed changes occur when the vessels are in straight waterways, and then, the speed becomes stable.

Table 3 gives the makespan and passing time of each vessel in the three scenarios. In Scenario II, because the vessels arrive with small time intervals, they need to decelerate to avoid collisions with others. Thus, the average passing time in Scenario II becomes longest. Scenario III has the shortest makespan and average passing time. From the perspective of a single vessel, most vessels benefit from the cooperation regarding shorter passing time.

#### 5. CONCLUSIONS AND FUTURE RESEARCH

Optimizing the waterborne transport system requires not only automation of the individual vessels, but also coordination among vessels. Due to the geographic limitations of banks and navigation rules and regulations, the cooperation of vessels has several significant difference compared with flocking of aerial vehicle and platooning of vehicles.

Fig. 5. Trajectories of the vessels in the three scenarios

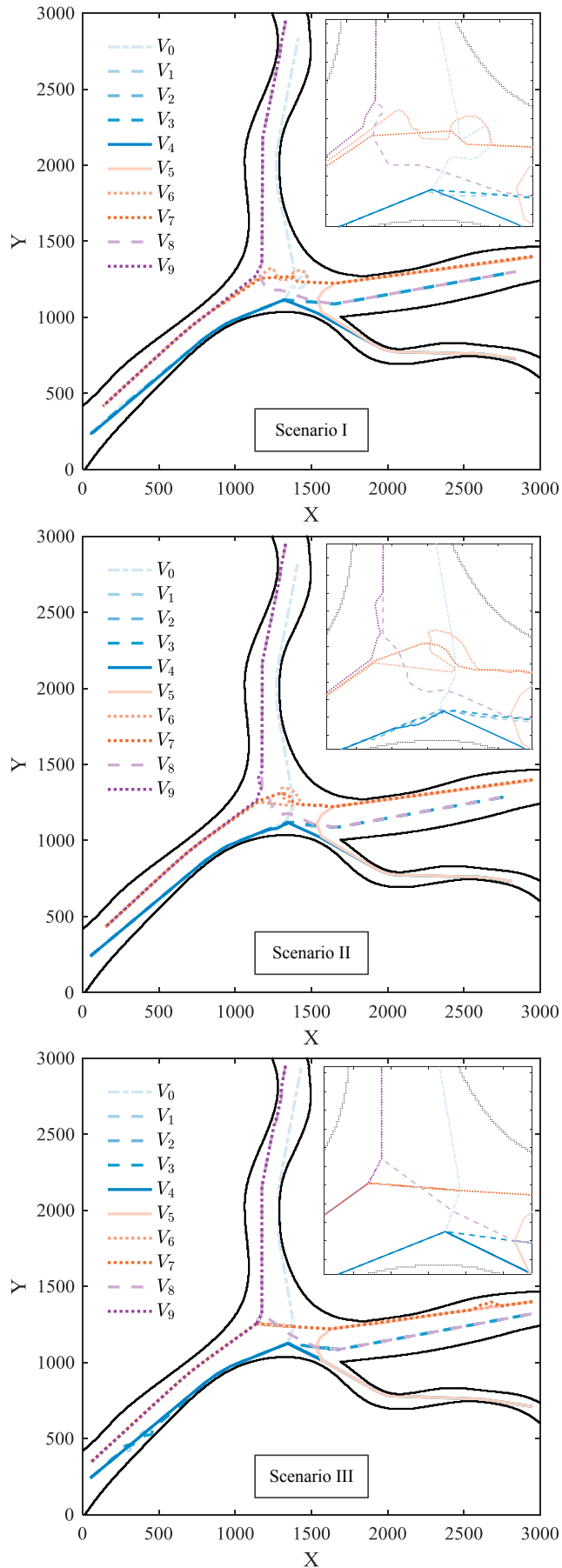


Fig. 6. WIS results

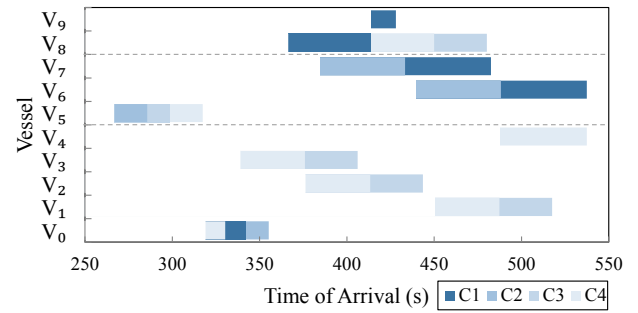


Fig. 7. Speed changes

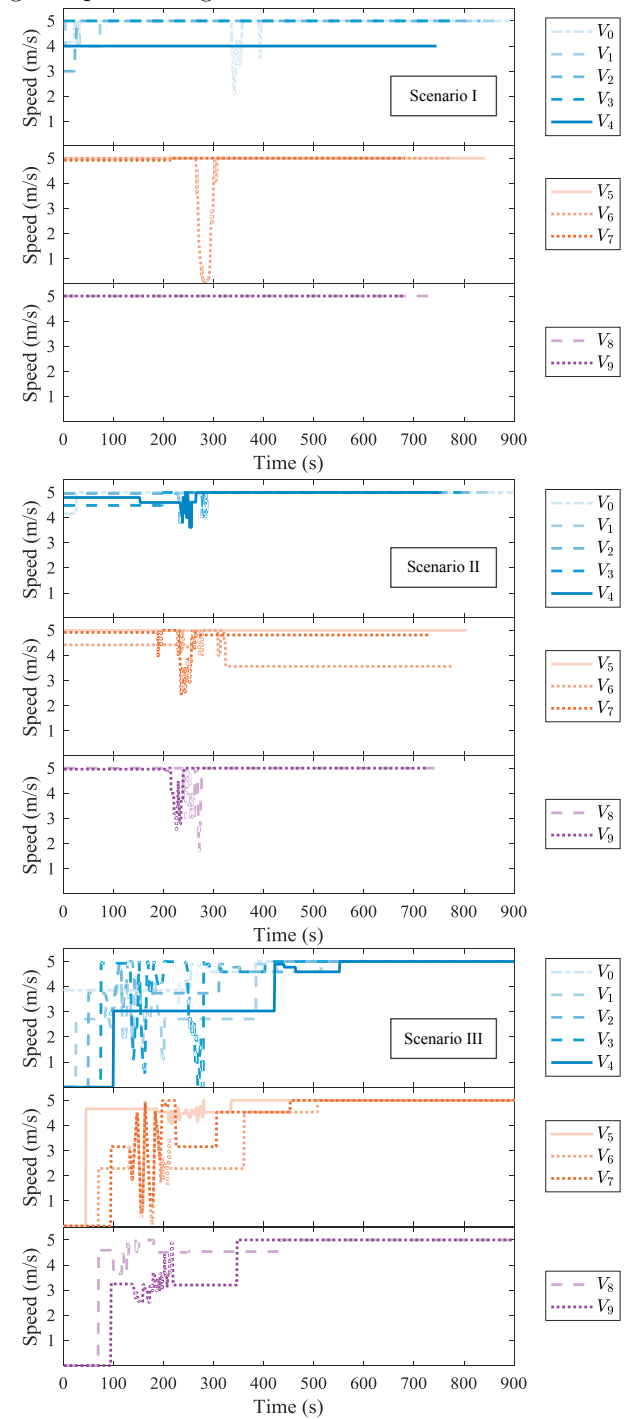


Table 3. Overall makespan and the passing time of each vessel (s)

Vessel	Scenario I	Scenario II	Scenario III
V <sub>0</sub>	77	31	36.56
V <sub>1</sub>	68	69	67.49
V <sub>2</sub>	68	67	67.49
V <sub>3</sub>	67	68	67.49
V <sub>4</sub>	59	49	50.09
V <sub>5</sub>	51	51	51.00
V <sub>6</sub>	199	240	98.25
V <sub>7</sub>	102	119	98.25
V <sub>8</sub>	125	128	114.03
V <sub>9</sub>	10	12	14.76
Average	82.6	83.4	66.54
Makespan	288	295	270.98

A fleet of vessels can have a train-like formation, and at the intersections, the vessel trains need to interact with one another.

This paper focuses on the cooperation of vessels at waterway intersections. We propose a framework consisting of the Waterway Intersection Scheduling and the Vessel Train formation for the control of CMVSs. The WIS addresses the inter-CMVS conflicts, while the VTF addresses the intra-CMVS conflicts. Simulations are carried out to show the potential of the proposed framework. Compared with the non-cooperative and partially cooperative scenarios, the fully cooperative scenario with our framework has the smoothest trajectories and shortest makespan.

In the next step, the WIS and VTF problems will be formulated in a closed-loop form: intersection schedulers also consider the feedback that the vessels send. Moreover, the proposed framework will be extended to the whole waterway network. Other infrastructures, such as bridges and locks, are also considered. Similarly, vessels passing through movable bridges and locks are also the occupation of space and time. When those infrastructures are networked, they are interdependent. For example, delays at one lock are significantly affected by operations at nearby locks. Therefore, the interdependence between the infrastructures should be considered in future research.

## ACKNOWLEDGEMENTS

This research is supported by the China Scholarship Council under Grant 201406950041.

## REFERENCES

- Abdelaal, M., Frnzle, M., and Hahn, A. (2016). NMPC-based trajectory tracking and collision avoidance of underactuated vessels with elliptical ship domain. *IFAC-PapersOnLine*, 49(23), 22 – 27.
- Chen, L., Negenborn, R.R., and Hopman, J.J. (2018). Distributed model predictive control for vessel train formation of cooperative multi-vessel systems. Technical report, Dept. of Maritime & Transport Technology, Delft University of Technology.
- Chen, L., Negenborn, R.R., and Lodewijks, G. (2016). Path planning for autonomous inland vessels using A\* BG. In A. Pias, M. Ruthmair, and S. Voß (eds.), *Proceedings of International Conference on Computational Logistics*, volume 9855 of *Lecture Notes in Computer Science*, 65–79. Springer International Publishing Switzerland.
- Google map (2017). <https://www.google.nl/maps/@51.8242567,4.6738837,14.25z>.
- Keviczky, T., Borrelli, F., Fregene, K., Godbole, D., and Balas, G.J. (2008). Decentralized receding horizon control and coordination of autonomous vehicle formations. *IEEE Transactions on Control Systems Technology*, 16(1), 19–33.
- Ku, W.Y. and Beck, J.C. (2016). Mixed integer programming models for job shop scheduling: A computational analysis. *Computers & Operations Research*, 73(Supplement C), 165 – 173.
- Kuo, T.C., Huang, W.C., Wu, S.C., and Cheng, P.L. (2006). A case study of inter-arrival time distributions of container ships. *Journal of Marine science and technology*, 14(3), 155–164.
- Li, S.E., Zheng, Y., Li, K., Wang, L.Y., and Zhang, H. (2017). Platoon control of connected vehicles from a networked control perspective: Literature review, component modeling, and controller synthesis. *IEEE Transactions on Vehicular Technology*, PP(99), 1–1.
- Li, Z. and Sun, J. (2012). Disturbance compensating model predictive control with application to ship heading control. *IEEE Transactions on Control Systems Technology*, 20(1), 257–265.
- Negenborn, R.R., De Schutter, B., and Hellendoorn, J. (2008). Multi-agent model predictive control for transportation networks: Serial versus parallel schemes. *Engineering Applications of Artificial Intelligence*, 21(3), 353 – 366.
- Negenborn, R.R. and Maestre, J.M. (2014). Distributed model predictive control: an overview and roadmap of future research opportunities. *Control Systems*, 34(4), 87–97.
- Olfati-Saber, R. (2006). Flocking for multi-agent dynamic systems: algorithms and theory. *IEEE Transactions on Automatic Control*, 51(3), 401–420. doi: 10.1109/TAC.2005.864190.
- Richards, A. and How, J.P. (2002). Aircraft trajectory planning with collision avoidance using mixed integer linear programming. In *Proceedings of the 2002 American Control Conference*, volume 3, 1936–1941.
- Rong, H., Teixeira, A., and Soares, C.G. (2015). Evaluation of near-collisions in the tagus river estuary using a marine traffic simulation model. *Zeszyty Naukowe / Akademia Morska w Szczecinie*, 43(115), 68–78.
- Statistics Netherlands (2009). Dutch inland waterways. <https://www.cbs.nl/en-gb/news/2009/48/dutch-inland-waterway-system-plays-important-part-in-goods-transport>.
- Xue, Y., Clelland, D., Lee, B.S., and Han, D. (2011). Automatic simulation of ship navigation. *Ocean Engineering*, 38(17), 2290 – 2305.
- Zheng, H., Negenborn, R.R., and Lodewijks, G. (2016a). Fast ADMM for distributed model predictive control of cooperative waterborne AGVs. *IEEE Transactions on Control Systems Technology*, PP(99), 1–8.
- Zheng, H., Negenborn, R.R., and Lodewijks, G. (2016b). Predictive path following with arrival time awareness for waterborne AGVs. *Transportation Research Part C: Emerging Technologies*, 70, 214–237.