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Successful transfer of algebraic skills from mathematics into physics in senior pre-university education

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Successful transfer of algebraic skills from mathematics into physics in senior pre-university education

Proefschrift

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"There is no end to education.
It is not that you read a book, pass an examination, and finish with education.
The whole of life, from the moment you are born to the moment you die, is a process of learning."

Jiddu Krishnamarti

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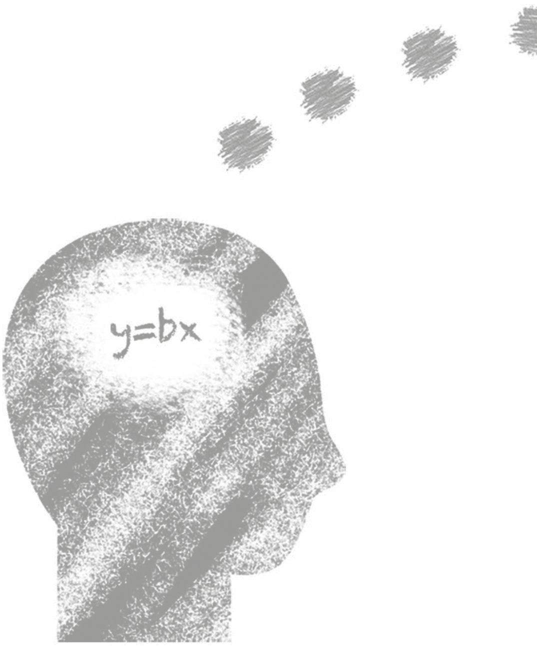
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Chapter 1

General Introduction

1.1 Background to The Study

Mathematics and natural sciences are intimately related (Atiyah, 1993; Dierdorp, Bakker, van Maanen, & Eijkelhof, 2014). Especially, the relation between mathematics and physics is the most intimate and oldest (Atiyah, Dijkgraaf, & Hitchin, 2010). Galileo paid attention to this relationship and said that the book of nature is written in the language of mathematics: *"…without which it is humanly impossible to understand a single word of it; without these one is wandering in a dark labyrinth"* (Drake, 1957, p. 237). Indeed, in the following centuries, scientists such as Newton with the universal law of gravitation, Maxwell explaining the behavior of electromagnetism, Einstein with his theory of special and general relativity, others such as Bohr, Heisenberg and Schrödinger in the development of modern quantum mechanics, and quite recently the search for a 'theory of everything' demonstrated the importance of mathematics to explain and understand physical phenomena.

Accordingly, the close relationship between mathematics and science subjects is also pivotal in both secondary and higher education. While mathematics offers students the tools by which quantitative relationships in science subjects can be represented, modelled, calculated and predicted, science subjects provide meaning to mathematics through rich and relevant contexts in which mathematics can be applied (Dierdorp, Bakker, van Maanen, & Eijkelhof, 2014). Despite this intimate relationship, however, students encounter difficulties with applying mathematics into science, in particular in physics, indicating a lack of transfer between these subjects (e.g., Redish & Kuo, 2014; Wong, 2018).

Even if students have a solid grasp of mathematics, their application in science subjects still can be poor. Remarkably, except for a couple of studies (e.g., Cui, 2006; Rebello et al., 2007) and projects such as SONaTe (Zegers et al., 2003) and SaLVO (2019), this phenomenon has *hardly* been studied, implying a knowledge gap in this area of research. In their pioneering work with pre-course tests of algebraic and trigonometric knowledge and skills taken by 200 students initiating a physics course, Hudson and McIntire (1977) have shown that students who were successful on mathematics tests, were poor on physics tests. Their solid grasp of mathematics was insufficient for transfer.

The lack of transfer above has consequences for science teachers, especially for physics teachers, leaving less time for their core business of teaching physics. This may be frustrating and time-consuming, overshadowing the science content that needs to be taught (SLO, 2019; Roorda, 2012). In addition, in a large number of countries, science curricula are overloaded, compelling science teachers to fit their program into a seriously reduced instruction time (e.g., Lyons, 2006). This can make inefficient transfer of mathematics in physics even more harmful. In the Netherlands, where this study is conducted, in the last few years physics problems requiring mathematics such as the application of mathematics, especially algebraic skills, has become much more important in senior pre-university physics education, especially in national final physics examinations (SLO, 2019). Thus, examining this transfer phenomenon is relevant from both an educational and a scientific point of view.

1.1.1 *Early Perspectives of Transfer*

Transfer of knowledge has been discussed in the area of learning and instruction for over 100 years (e.g., Larsen-Freeman, 2013; Leberman, et al., 2016; Lobato, 2003). Within this rich body of research, there are many approaches to investigate transfer. Below we will discuss some of these approaches for the early, the cognitive and the situated views of transfer.

Early perspectives on transfer were based on the mental abilities of a person whose intellectual performance was believed to rely on the basic mental functions 'attention, judgement and memory'. Training these functions was thought to enhance the ability to transfer knowledge to new situations (Karakok, 2009). This belief is based on the *general effects* view where students were required to take, for example, Latin and geometry courses. It was believed that these courses would make the students' minds think more logically, in a sense disciplining their minds and enhancing their abilities in other subjects. In 1903, Thorndike challenged the general effects view and proposed *the theory of identical elements*. He demonstrated that learners who performed well on the test of a specific content, did not enhance their learning in the new situation. Thorndike concluded that transfer could only happen if both the initial and targeted task shared identical elements. This transfer approach influenced many researchers (e.g., Bassok, 1990; Gick & Holyoak, 1983) in the 20th century presenting an initial learning task followed by a target task where, according to the researchers, both tasks shared similar features. However, researchers stated that beyond tasks with identical elements, Thorndike's theory had limited applications (Mestre, 2005). Indeed, Judd (1939) in his theory of deep structure claimed that learners might have different ideas on the sameness of the initial and the target task, rather than sameness according to the researchers (for a more comprehensive explanation see for example Tuomi-Gröhn & Engeström (2003)). According to Judd (1939), transfer was not the consequence of effortlessly and mindlessly rote memorization, but determined by the degree to which the learner was aware of underlying shared causal principles between two situations.

1.1.2 *Traditional Transfer*

While Judd (1939) represents the cognitive view (also referred to as traditional transfer) in which there is more emphasis on transfer from one situation to another situation, the situated view (also referred to as contemporary transfer) gives much attention to construction of sameness between situations as seen by the learner. In the last decades there has been a shift from traditional towards contemporary perspectives of transfer (e.g., Lobato, 2006).

Traditionally, transfer has been defined as "*the ability to apply knowledge learned in one context to new contexts*" or "*the ability to extend what has been learned in one context to new contexts*" (Mestre, 2005, p. 156). A feature of these definitions is the role of 'what has been learned/knowledge' that is applied to another situation (Roorda et al., 2014). In this view, the researcher examines whether a learner transfers knowledge from initial learning to the target (transfer) task. Such studies are conducted from the researchers' perspective who looks for improved performance from the initial learning to the transfer task. The research questions examine issues such as "*Can students successfully apply knowledge previously acquired in the learning task to the transfer task?*" and "*What conditions facilitate transfer*" (Karakok, 2009, p.27). To answer these questions, the researcher pre-defines the structural similarities (sameness) between both tasks. In addition, transfer is a static construct where students can either apply their knowledge from an initial task to a transfer task or cannot.

1.1.3 Contemporary Views of Transfer

Most of the studies carried out under the cognitive view reported failure of transfer from the initial task to the target task (e.g., Mestre, 2005; Lobato, 2006; Roorda, 2012). Many researchers claimed that this failure was due to the definition of transfer in which knowledge could be separated from the situations where it was learned instead of being an overall connected experience (e.g., Lobato & Siebert, 2002). Also, methodologically there were some major concerns about this perspective. The transfer tasks focused on the researchers' view who designed the initial and the transfer task in such a way that both tasks shared similar features (sameness). In addition, transfer was unidirectional: learners apply their knowledge in the target situation after they were exposed to the initial situation. However, several studies showed that transfer was not a static unidirectional but a dynamic bidirectional process (e.g., Marongelle, 2004; Zandieh, 2004). This implies that knowledge in an initial and transfer task mutually interact and that learners may also continue to develop knowledge even in the transfer tasks. Contemporary views of transfer involve aspects such as socio-cultural issues and available resources during initial learning that cognitive models neglect (e.g., Lobato, 2003; Ozimek, 2004). Transfer is viewed from the learners' point of view, and the researchers' job is to figure out what learners exactly transfer.

Some examples of these contemporary models are the *Actor-oriented Transfer* approach (Lobato, 2006; Karakok, 2009; Roorda, 2012) that will be explained in the next section, the *Affordances and Constraints Approach* (Greeno et al. 1993. & 1996), the *Boundary-crossing Approach* (Akkerman & Bakker, 2011; Tuomi-Gröhn & Engeström, 2003) and Recontextualization (e.g. Moore, 2012; Nowacek, 2011). The Affordances and Constraints approach investigates the degree to which participating in an activity influences the learners' ability (while the learner is aware of the affordances and constraints of the activity) to participate in a different activity in a new situation. The Boundary-crossing Approach is an alternative metaphor for transfer "*to capture the often more complex efforts by people who move not only forth but also back; boundary crossing is therefore bidirectional and dynamic. Moreover, it is oriented towards both the personal and the collective. The concept of boundary crossing thus draws attention to a wider range of relevant processes involved in integrating different types of knowledge to be learned and used in different contexts*" (Bakker & Akker, 2014, p. 224). Furthermore, Nowacek (2011) builds on genre theory and situates students as agents of integration within a theory of transfer as Recontextualization. She unpacks transfer as recontextualization with five principles such as "*multiple avenues of connection [exist] among contexts, including knowledge, ways of knowing, identities, and goals*"(p. 21), "*transfer can be both positive and negative and ... there is a powerful affective dimension of transfer*" (p. 25) and that "*meta-awareness is an important, but not a necessary, element of transfer*" (p. 30).

1.1.4 *The Actor Oriented Transfer Approach*

According to Lobato (2006), the actor-oriented transfer approach views transfer as the "*personal construction of similarities between activities where the 'actors,' i.e. learners, see situations as being similar*" (Lobato, 1996 & 2003). This implies that the main focus is the actor (learner) who sees the target situation (for example, a given task or problem or the experiences during teaching) similar to the initial learning situation (for example, an initial task or problem or the experiences during interviews). Within this view, the evidence for transfer is gathered by "*scrutinizing a given activity by any indication of influence from previous activities and by examining how people construe situations as similar*" (Lobato & Siebert, 2002, p.89). So, any indication of influence from the previous tasks on the target task is regarded to be evidence for actor-oriented transfer (Karakok, 2009). Indeed, the researcher does not decide or prioritize what learners should transfer. Instead, the researcher adopts a learner-centered view to figure out what learners transfer and examine how these are supported by the environment (Roorda et al., 2014).

Some studies conducted under the cognitive view concluded that students do not transfer knowledge from, for example, mathematics lessons to physics problems (Cui, 2006; Karakok, 2009; Roorda, 2012; Roorda et al., 2014). However, when data were analyzed from the actor-oriented transfer perspective, students did transfer. They constructed similarities between situations in mathematics lessons and physics problems. The findings of these studies inform the researcher about the students' learning process, rather than, based on the end results of learning, the observation that transfer happened or not. Within these studies the focus is on how students connect their previous experiences during teaching to new experiences during the interviews, where explicit or implicit similarities between both experiences were considered as evidence for transfer.

1.1.5 *Views of Transfer in This Study*

Later in this study students during interviews are asked to solve algebraic physics problems (target tasks) in regular physics textbooks for which solution algebraic skills are needed. We expect that these algebraic skills are learned in mathematics class from regular mathematics textbooks (previous learning situation). To determine whether transfer occurred or not (we quantized transfer), we adopted the traditional transfer approach by comparing students' solution sets to the physics tasks with our solution sets. This means that the degree to which transfer occurred, was determined by the researchers' perspective. To some extent, we also payed attention to the actor-oriented transfer approach. Other contemporary views were not adopted, because they were not concerned with algebraic problem-solving in upper secondary education. Indeed, the actor-oriented approach, and especially the study of Roorda (2014) fitted our research. Therefore, we followed the line of Roorda (2014) who operationalized the actor-oriented transfer "*as a search for students' personal constructions of relations between (1) learning from mathematics and physics classes and (2) interview tasks*" (p. 863). For instance, when students explicated that they learned a specific problem-solving approach from their mathematics textbook. In short, beyond the traditional approach to measure the degree of transfer, to some extent we adopted the actor-oriented transfer approach by paying attention to previous learning derived from what they said during the interviews. While earlier studies on actor-oriented transfer studied field notes taken in class to gain deeper insight into students' previous learning situation, we only payed attention to what they said about previous learning in mathematics and physics class, and, to some extent, how algebraic skills were presented in their mathematics textbooks.

1.1.1 Why Students Lack Transfer of Mathematics in Science Subjects

There are at least four possible causes for the lack of transfer. In this study we focused on compartmentalized thinking (Osborne, 2013), teachers' beliefs (Schoenfeld, 2014) about transfer, especially mismatches between naïve beliefs and those required for classroom practice, discrepancies between pedagogical approaches to *how* mathematics is learned in mathematics class and applied in science class (Alink et al., 2012), especially mathematics and science textbooks *and* mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001), especially symbol sense behavior (Drijvers et al., 2011). Below we will elaborate upon these issues.

Compartmentalized Thinking

As to compartmentalized thinking, students may see mathematics and science subjects as unrelated subjects (Nashon & Nielsen, 2007). Compartmentalized thinking can be very persistent: students who think they just started a completely new subject after they left the mathematics classroom and entered the physics classroom. This phenomenon is consolidated and intensified since in many countries those subjects are taught separately (SLO, 2019; Honey, Pearson & Schweingruber, 2014; 'TIMMS & PIRLS', 2019).

On the other hand, mathematics and science subjects are two different subjects. What we mean with 'unrelated subjects' is that students face difficulties because they do not understand the mathematics context in which the problems are embedded (Frykholm & Glasson, 2005). In this regard, Furner & Kumar (2007, p.186) state that "*The separate subject curriculum can be viewed as a jigsaw puzzle without any picture. If done properly, integration of math and science could bring together overlapping concepts and principles in a meaningful way and enrich the learning context. Learning situated in such enriched (macro) contexts often lead to meaningful learning experiences*". How this can be done properly, is explained in the sections below.

Teachers' Beliefs about Transfer

Transfer from mathematics to physics is indeed problematic, but very little is known about transfer of algebraic skills from mathematics to physics in senior pre-university education. Thus, we first need to examine this problem. Indeed, do teachers really acknowledge such a transfer problem involving algebraic skills, and what are their *beliefs* about aspects influencing transfer? These are relevant questions that can provide insight into this specific transfer problem involving algebraic skills. Therefore, a problem analysis involving interviews with mathematics and physics teachers in senior pre-university education is needed. The idea to ask teachers about their beliefs about transfer is no coincidence. Indeed, it is well-known that teachers' beliefs strongly influence their behavior (Borg, 2015; Mansour, 2009; Schoenfeld, 2014). Consequently, teachers' beliefs about transfer influence their behavior on how they deal with transfer issues in teaching practice. For instance, a physics teacher who *naïvely* believes that a lot of practice in mathematics class with algebraic skills will automatically improve transfer of these skills to physics class. They neglect insight (conceptual understanding) in the underlying mathematics in physics problems and may soon find themselves re-teaching basic mathematics and become frustrated. Thus, naïve beliefs can impede transfer of learning. In addition, in many countries, science curricula are overloaded, compelling science teachers to fit their program into a seriously reduced instruction time (e.g., Lyons, 2006), making inefficient transfer of mathematics in physics even more harmful.

According to researchers, beliefs can be organized into a belief system containing a set of mutually supporting beliefs (e.g., Lumpe et al., 2012; Pajares, 1992). This is illustrated in figure 1, where the upper rectangle describing 'View nature of mathematics' represents a belief system (Ernest, 1991) containing a basis for the teachers' espoused (mental) models of learning and teaching mathematics. These models are influenced by the constraints and opportunities of the powerful social context of teaching (the dashed rectangle in the middle) that is"*a result of a number of factors including the expectations of others, such as students, their parents, fellow teachers and superiors*" (Ernest, 1991, p. 290). Then, these espoused models are transformed into classroom practice (enacted models). In figure 1, the enacted models are depicted by the three subsequent rectangles about 'learning mathematics', 'teaching mathematics' and 'using mathematics texts'. In short, the upper two rows of rectangles are concerned with a teachers' belief system, and the other rectangles with teaching practice. The distinction between espoused and enacted is essential, since earlier studies have shown that there can be a great disparity between both models (Brown & McNamara, 2011; Lloyd, Veal, & Howell, 2016).

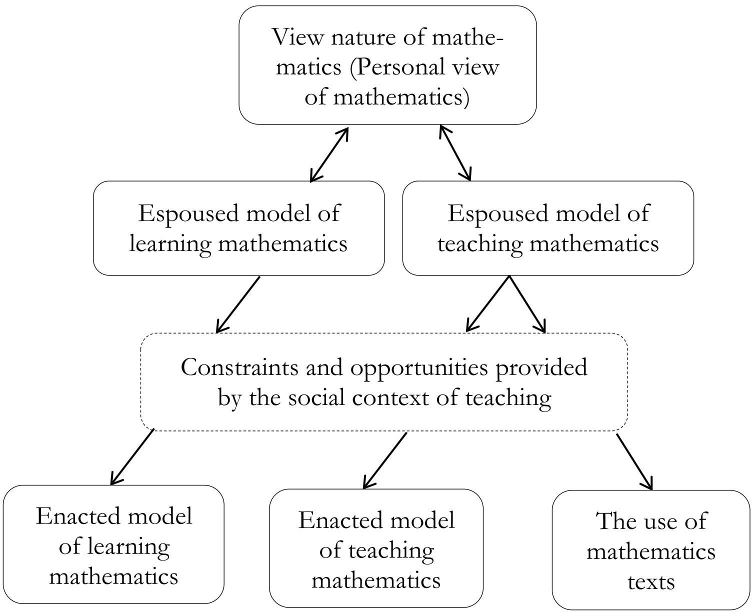


Figure 1. Scheme describing how a teachers' belief system is influenced by the social context of teaching. Adopted from Ernest (1991).

Now imagine the case of the physics teacher advocating automatic transfer. A change of such naïve beliefs (espoused models) into *desirable beliefs* (beliefs about transfer that after being transformed into behavior (enacted models) improves transfer of students), requires awareness of their belief systems, reflect on them and change naïve beliefs into desirable beliefs about transfer. For instance, a teacher who pays attention to basic algebraic skills and becomes aware of the importance of insightful learning for practicing algebraic skills in physics class.

We note that the model in figure 1 is greatly simplified. The relationships between the espoused and enacted models in relation with the social context are far more complex and far less mechanistic than in that model. The enacted models for example, are not separated from the social context of teaching, but embedded in it. And all espoused and enacted models are part of an interactive system. Moreover, "*pressures at any point, such as in classroom practices, will feed-back and may influence all the other components*" (Ernest, 1991, p. 291).

Unfortunately, transformations from naïve into desirable beliefs can be very tough to realize, since according to cognitive psychology changes in behavior are attributed to *the structure* of a belief system, and not to individual beliefs (Leatham, 2006; Misfeldt & Aguilar, 2016). The structure of a belief system is composed of several collections of beliefs with both varying centrality and psychological strength. The strongest beliefs are the hardest to change. They are localized in the center and strongly connected to many other central beliefs. The more peripheral weak beliefs are founded and derived upon the central beliefs. According to Singletary (2012), the central and peripheral beliefs can be considered as a group of concentric circles. The innermost circles with small radii contain the central beliefs and are gradually transforming into the outer circles with increasing radii containing weak peripheral beliefs.

Mismatches in Pedagogical Approaches

The discrepancies between pedagogical approaches between mathematics and science subjects may be related to the content of textbooks, since in many countries including the Netherlands, textbooks mediate between curricula (intended curriculum) and the actual teaching in classrooms (the implemented curriculum) (SLO, 2019; van Zanten & van den Heuvel-Panhuizen, 2014). Most Dutch teachers follow them closely and teach them accordingly to their students. Thus, to a very large extent textbooks shape teaching practice (e.g., TIMMS & PIRLS, 2019). With respect to distinct pedagogical approaches in mathematics and science curricula, this may influence teachers and students. It might be the case that, for example, while a mathematics teacher applies the equation triangle (mnemonic) without insight to solve for C in $A = B \cdot C$, a physics teacher solves for u in the analogous expression $F_{spring} = C \cdot u$ in physics class using algebraic skills with insight. Such mismatches can be quite confusing for students, thereby impeding transfer (Alink et al., 2012; Quinn, 2013; Roorda, 2012).

Lack of Symbol Sense Behavior

Successful learning of mathematics and hence the application of mathematics in physics, depends on mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001). Mathematical proficiency contains five intertwined strands. These are adaptive reasoning (capacity for logical thought, reflection, explanation, and justification), strategic competence (ability to formulate, represent, and solve mathematical problems), conceptual understanding (comprehension of mathematical concepts, operations, and relations), productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, one's own efficacy) and procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately). These strands are illustrated in figure 2.

In algebraic problem-solving such as the two examples above, especially the third and last strands are of major importance (Bokhove, 2011; Drijvers, 2011, 2015). Together, they shape algebraic expertise that refers to algebraic skills with particular attention to procedural fluency in relation to conceptual understanding. In figure 3 (Drijvers et al., 2011, p. 22), it can be seen that algebraic expertise extends from basic algebraic skills to symbol sense. While basic algebraic skills involves basic procedures, symbol sense refers to algebraic skills with emphasis on conceptual understanding and involves the ability to consider an algebraic expression carefully, to identify its relevant aspects and to choose a wise systematic problem-solving strategy based on these aspects. Symbol sense consists of "*an intuitive feel for when to call on symbols in the process of solving a problem, and conversely, when to abandon a symbolic treatment for better tools*" (Arcavi 1994, p. 25). On the concept level, basic algebraic skills involve procedural work with a local focus and algebraic reasoning. Symbol sense deals with strategic work with a global focus and attention to algebraic reasoning. Strategic work refers to a student who is in control of the work and tries to find a different systematic strategy when an approach appears to be inappropriate. Having a global focus is related to recognition of patterns in mathematical expressions or physics formulas. Algebraic reasoning is concerned with issues such as symmetry considerations. In this study we focus on the relationship between local and global, and procedural and strategic work during algebraic problem-solving in physics class.

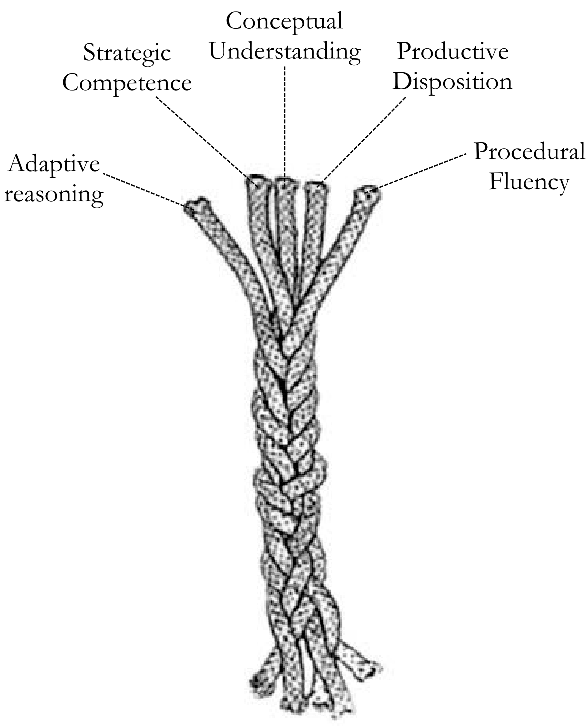


Figure 2. The five interwoven strands of mathematical proficiency. Adopted from Kilpatrick et al. (2001).

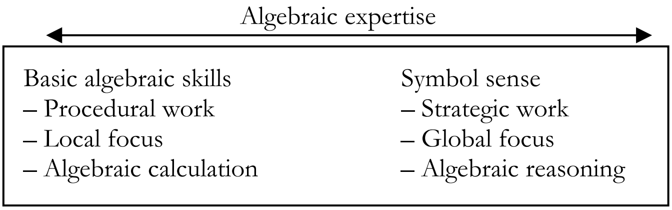


Figure 3. Algebraic expertise as a spectrum ranging from basic skills to symbol sense. From Drijvers et al. (2011, p. 22).

Through examples, Arcavi (1994) described eight symbol sense behaviors, showing the close relationship between basic algebraic skills and insight. With respect to algebraic problem-solving, behavior number six, i.e. flexible manipulation skills is of major importance and deals with being in control of the work and have the ability to flexibly manipulate expressions (both technical and with insight). Flexible manipulation skills contains the two interconnected components having a gestalt view on algebraic expressions and dealing in an appropriate way with their visual salience (Kirshner & Awtry, 2004; Wenger, 1987). While the former component includes "*the ability to consider an algebraic expression as a whole, to recognize its global characteristics, to 'read through' algebraic expressions and equations, and to foresee the effects of a manipulation strategy*" (Bokhove & Drijvers, 2010, p. 43), visual salience deals with visual cues of algebraic expressions. Moreover, visual salience consists of pattern salience that is related to sensitivity towards patterns in algebraic expressions, for example, cancelling out common factors *and* local salience relating to sensitivity towards local algebraic symbols, i.e. visual attractors such as fractions, square root signs, and exponents, for example, expanding brackets. In short, flexible manipulation skills plays a key role in algebraic problem-solving in mathematics and science class where mathematics is applied. Improving flexible manipulation skills and thus symbol sense behavior involves sufficient sensitivity towards local salience and pattern salience of algebraic expressions during algebraic problem-solving.

Furthermore, in this study we distinguish between 'systematic algebraic strategies' (or 'systematic algebraic approaches') and the application of 'ad hoc strategies' ('ad hoc approaches'). With 'systematic algebraic approaches' we refer to using a systematic, rule-based problem-solving approach where algebraic skills are used with insight, where 'rule' refers to the standard rules for multiplication and division of powers, such as $y^{\mathrm{a}} \cdot y^{\mathrm{b}} = y^{\mathrm{a+b}}$, that play the role of algebraic axioms in high school algebra. In short, in this study using systematic algebraic strategies are associated with applying algebraic skills systematically and correctly.

As a working definition of 'ad hoc approaches' we use mathematical strategies that are not based on standard algebraic rules with insight. They only work for a specific case, and may lead to fragmented knowledge, impeding generalization of algebra. In more sophisticated problems where insight is needed rather than ad hoc strategies, students may get stuck. Also, using them depends on the approval of an authority such as a teacher or a textbook. On the other hand, they can be useful as initial attempts to solve a problem (Roorda, 2012).

In this study, applying 'systematic algebraic strategies' becomes visible through the application of algebraic skills during procedures involving basic algebraic skills and having a gestalt view on algebraic expressions and dealing with their visual salience. Accordingly, the word *successful* in the title of this thesis refers to the application of systematic algebraic strategies during algebraic problem-solving in physics. Furthermore, the term *successful* (see title of this thesis) was operationalized by measuring both the extent to which students demonstrated symbol sense behavior and the degree to which they used basic algebraic skills correctly.

1.1.2 Coherent Mathematics and Science Education

A remedy for overcoming the aforementioned aspects underlying the lack of transfer including compartmentalized thinking, teachers' naïve beliefs, discrepancies between pedagogical approaches in mathematics and physics textbooks, and lack of symbol sense behavior may be coherent mathematics and science education (CMSE) (e.g., Berlin & White, 2012, 2014; Mooldijk & Sonneveld, 2010; National Academies Press, 2019; Ríordáin et al., 2016). Indeed, like in the interdisciplinary Science Technology Engineering and Mathematics education, or *STEM* in short (van Breukelen, 2017; National Science and Technology Council, 2013; SLO, 2019; 'TIMMS & PIRLS', 2019), mathematics lies at the heart of the CMSE approach. This approach aims at connection between mathematics and science education through alignment of various aspects such as notations, concept descriptions, pedagogical approaches and the organization of the learning process in order to establish a logical learning line across both subjects.

There is a close relationship between CMSE and transfer (e.g., Alink et al., 2012; Roorda, 2012), for CMSE is based on the traditional transfer paradigm in which mathematics (initial learning situation) is applied in other subjects (new learning situation) (Alink et al., 2012; Larsen-Freeman, 2013; Leberman et al., 2016).

The terms 'coherent' and 'alignment' above can have different meanings in different studies (Roorda, 2012). First of all, 'coherent' may be an essential part of the following constructs (1) 'coherent profiles', (2) 'coherent education' and (3) 'coherent knowledge' that share the term 'coherent'. According to van den Akker (2004), however, they may refer to different levels of the curriculum. The first refers to the curriculum (We note that there are 4 profiles in the Netherlands. Concerning this study, only two of the 4 profiles are relevant and contain a certain combination of mathematics and science subjects that will be explained in the following sections), the second to what has been implemented, and the third to what has been achieved. In this study we follow the line of Roorda (2012). When we refer to 'coherence between mathematics and science subjects', we refer to number (2). In other words, teachers or textbook publishers connecting both subjects in terms of aforementioned alignment through various aspects. When we use 'coherence' in relation with students, we refer to the achieved level, i.e. the extent to which students experience coherence across both subjects (e.g., Frykholm & Glasson, 2005; Furner & Kumar, 2007; Mooldijk & Sonneveld, 2010). This may become visible when students are aware of the intimate relationship between both subjects. In this regard, the tools provided in mathematics class may become a versatile, widely applicable machinery to tackle problems in science class. Conversely, their awareness of science as a meaningful context in which mathematics can be applied, can contribute to the transfer of mathematics in science subjects, especially in physics. Therefore, students who experience coherence across both subjects is of major importance for transfer. Furthermore, we assume a reciprocal relation between CMSE and transfer. When students experience coherence across mathematics and science subjects, transfer from mathematics to science subjects can be improved, and vice versa, improving transfer helps can help them to experience coherence between these subjects.

Beyond 'coherent' and 'alignment', another term that may need further explanation is 'integration'. With the latter we refer to any attempt to connect both subjects by means of the three levels above. Furthermore, the integration of both subjects might encompass teaching in which, for instance, mathematics is entirely used as a language and tool for science subjects, or science subjects that are entirely part of mathematics (Furner & Kumar, 2007). Teachers may have different beliefs on how they think about integration of both subjects. In short, 'integration' is used as a construct to denote the entire set of attempts to connect both subjects across the three levels identified by Van den Akker (2004).

1.2 Aim and Relevance of The Study

In this study we aim to improve upper secondary education students' transfer of algebraic skills from mathematics into physics. This goal is guided by the central research question "*How can the transfer of algebraic skills from mathematics into physics be improved for solving algebraic physics problems that occur in upper secondary education?"*. To answer this question, we conducted five studies. The first three were follow-up studies researching (1) teachers' beliefs, (2) teachers' core beliefs and (3) teachers' belief systems. Among other things, these three studies involved aforementioned compartmentalized thinking, naïve beliefs and mismatches in pedagogical approaches in mathematics and physics class. Also, actors such as teachers and textbooks that may play a role in CMSE were involved. During study (4), we examined students' symbol sense behavior during algebraic problem-solving in physics, and in study (5) the effectiveness of activation of prior mathematical knowledge during algebraic problem-solving in physics. In study (5) we also studied symbol sense behavior.

Even though there is research on transfer focusing on the application of mathematics to science (e.g., Karam, 2014; Potgieter, Harding, & Engelbrecht, 2008; Roorda, Goedhart, & Vos, 2014), our extensive literature research with web-search engines such as Google Scholar and ProQuest on scholarly articles revealed that the relationship between transfer of algebraic skills and the other issues above have not been studied before. We already explained why studying transfer in relation with teachers' beliefs was essential in this study. This also applies for the role of symbol sense behavior. Below, we will briefly discuss each of these studies.

The first study (1) may offer a continuum of beliefs about aspects influencing transfer. Sufficient insight into such beliefs may help reduce physics teachers focus on science content rather than spending extra time on re-teaching mathematics and become frustrated. They may also enhance students' transfer and help them to experience coherence between these subjects.

Study (2) is a follow-up study that aims at reducing large amounts of data into a small set of core beliefs that contain *constraints* including naïve beliefs that are harmful for transfer and *affordances* that improve transfer. In other words, such a set aimed to provide the essence of the large amounts of data about teachers' beliefs. Instead of focusing on large amounts of data, one may also focus on the set of major aspects that influence transfer. Conversely, the large amount of data from study (1) is the result of aforementioned problem analysis to examine whether teachers indeed acknowledged specific transfer problems involving algebraic skills to physics, and to gain insight into the various aspects that influence transfer.

To reduce data from study (1), we used pattern coding that in many textbooks is described in a general way (e.g., Saldaña, 2013). Contrary to such a general approach, we intended to use this second cycle coding technique in a more systematic and refined manner. We especially aimed to develop a specific approach to further reduce code trees including large amounts of data. This study also functions as 'a bridge' between the first and the third study. In short, beyond a study on teachers' core beliefs, our purpose was to develop a systematic and refined method to reduce the code tree containing large amounts of coded data, since this was not present in earlier studies. In addition, if we would have aimed at combining study (2) and study (1) into a single study, it was considered as too large by peer-reviewed international Journals on science education.

Study (3) investigated whether and to which extent there is the possibility to extract belief systems that might contain naïve and desirable beliefs from the set of core beliefs above. We have already seen how naïve beliefs (espoused models) are transformed in teaching practice (enacted models) that may be harmful for *how* students deal with the application of algebraic skills in physics problems. Therefore, it is important that such belief systems containing naïve beliefs are known. Then, for instance, well-informed mathematics and science teacher educators can use professional teaching programs (Guskey, 2002) to make mathematics and science teachers aware of their belief systems (espoused models), reflect on them and change naïve beliefs into desirable beliefs that enhance transfer after they are transformed into behavior, for instance, teaching practice involving pedagogical strategies that improve transfer (enacted models). Otherwise, because of the powerful socialization effect in school (see figure 1), teachers are often observed to stick to the same ineffective classroom practice (Brown & McNamara, 2011). In addition, contrary to Ernest (1991) who *theoretically* clustered (categorized) teachers into social groups based on their belief systems, we *empirically* examine the possibility to cluster teachers based on their belief systems about CMSE and transfer. With a cluster, we mean a group of teachers that have similar belief systems. We also research whether the belief system model from cognitive psychology including the strong central and weak peripheral beliefs (Leatham, 2006; Singletary, 2012; Misfeldt & Aguilar, 2016), and the belief system model of Ernest (1991) can be used together to provide further understanding about CMSE and transfer. Besides evaluation of both models, we also intend to investigate whether both models can be regarded as respectively *microscopic* and *macroscopic* lenses through which belief systems can be viewed that complement each other.
Furthermore, also combining study (2) and (3) would return a single study that would have been considered too large for publication in peer-reviewed international science Journals.

In study (4), symbol sense and especially symbol sense behavior is studied for the first time outside mathematics education. Besides deeper understanding of students' algebraic problem-solving abilities in physics, it may also contribute to the evaluation of this concept. Moreover, our extensive literature research also reveals that the mechanisms behind the lack of transfer from mathematics in science subjects are under researched. Students having a solid grasp of mathematics, but facing difficulties in applying this subject in physics, is even *highly* under researched. This study may provide insight into the underlying mechanisms of such students' problem-solving in physics in which symbol sense is involved.

Study (5) was based on insights from the previous four studies in which compartmentalized thinking, teachers' beliefs about transfer, mismatches between pedagogical approaches in mathematics and physics textbooks, *and* symbol sense behavior are viewed together. Those areas of research were not studied together before. We used the CMSE approach (e.g., Frykholm & Glasson, 2005; Furner & Kumar, 2007; Berlin and White, 2012, 2014) to bring together these areas and combat the lack of transfer. Besides providing new insights about transfer, especially the traditional transfer perspective in those areas of research, it can also contribute to the evaluation of both transfer framework. To some extent, this also applies for the actor-oriented transfer approach, since this view was only used to gain insight into what students said about previous learning during the interview. Furthermore, this is the first time that shift-problems (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013) are examined outside mathematics education to gain deeper understanding of students' mathematical problem-solving abilities, especially algebraic problem-solving in physics. Beyond providing information about the usability of shift-problems in physics education, the transformation of insights from the previous four studies into small interventions (among other insights we used activation of prior mathematical knowledge) on tasks in textbooks can provide practical and scientific knowledge about using algebraic skills with insight.

Furthermore, the studies above can offer insights that are of importance for the international mathematics and science audience. These involve curricula, textbooks, individual mathematics and physics teachers, collaboration between them, and mathematics and science teacher educators aiming at students' transfer of mathematics in physics and help them to experience coherence between these subjects.

1.3 Context of The Study

The researchers in this study were all affiliated with the mathematics and science teacher education program of Delft University of Technology that is located in the Netherlands. Thus, we approached Dutch secondary schools rather than those from another country. We discuss the Dutch context in relation to education, especially that of secondary education that consists of three lower years and three upper years. In the Netherlands upper secondary education is used for both senior general secondary education and senior pre-university education. In this study upper secondary education refers to senior pre-university education that starts in grade 10. For this and stylistic reasons we used the terms senior pre-university education and upper secondary education interchangeably to denote the same.

According to the OECD (2018), the Netherlands is considered as an advanced industrial nation where both mathematics and science education are high on the governmental agenda (Ministry of Education, Culture and Science, 2018). Internationally, Dutch students in upper secondary education score accordingly on mathematics and science assessments, including assessments on physics ('TIMMS & PIRLS', 2019).

Pre-university education in the Netherlands consists of three lower years (junior pre-university education) and three senior years. Senior pre-university education, also referred to as 'Tweede Fase', starts in grade 10 (when they are 15 or 16 years old) where students have to choose between the four profiles 'Culture and Society', 'Economics and Society'', 'Nature and Health' and 'Nature and Technic' (SLO, 2019). In this study we focus on the latter two profiles in which both science subjects are involved. As for physics students, the main difference between both 'Nature' profiles is that 'Nature and Health' students follow mathematics A, and the other students mathematics B. While mathematics A includes algebraic skills, there is a major focus on applied calculus, stochastics and statistics. Mathematics B focusses on analysis, geometry, algebra and algebraic skills, formulas and equations and mathematical thinking activities. We conclude that mathematics B puts much more emphasis on algebraic skills than mathematics A. Moreover, the latter subject is mainly appropriate for students who aim to follow economical or biomedical sciences after they finished secondary education, and mathematics B for students who aim to study STEM subjects.

The idea behind these profiles is threefold and includes providing students a broad general development, creating coherence between the school subjects in a profile, and offering students a more independent way of learning that fits better with the working method in higher education. This second purpose is also related to enhancement of students' transfer (Alink et al., 2012). However, until now several studies of the SLO (2019) have shown that the connection between subjects of the 'Nature' profiles has been realized to a very limited extent, i.e. mathematics has remained an unrelated subject.

1.3.1 Mathematics in Dutch Pre-university Education

Even though there is difference in emphasis on algebra in mathematics A and mathematics B, the algebra in these curricula is considered to be sufficient to tackle algebraic problems in physics class (SLO, 2019). Yet, some teachers think that mathematics B should be compulsory for physics students. Quantitative research is needed to generalize this belief for the Dutch context. Moreover, the content of these subjects is determined by aforementioned curricula that describe the general educational core goals and the more specific standards. These core goals and standards are tested in national final examinations.

The algebraic skills in both curricula mainly focus on algebraic activity involving, for example patterns of relationships between numbers, implicit or explicit generalizations, and mathematical operations with variables, formulas and expressions (Drijvers, 2011). While a formula refers to algebraic expressions with real measurable quantities such as pressure, an expression can be a formula or an abstract algebraic expression with abstract mathematical variables (placeholders).

The algebraic skills described in mathematics curricula refer to the entire set of mathematical activities above. Moreover, algebraic skills are divided into specific skills that is close to basic algebraic skills and general skills that are close to symbol sense behavior.

An essential part of algebraic skills that are described in the mathematics curricula and widely used in mathematics class are algebraic techniques (Drijvers, 2011). They are used to manipulate expressions. Some examples of algebraic techniques are 'substitution of variables' and 'division of both sides'. Therefore, algebraic techniques play a key role in this study.

Concerning connection between these curricula with the science curriculum, especially with physics, there is no explicit reference to alignment through, for example pedagogy of mathematical approaches. This also applies for the organization of the learning process in order to achieve a logical learning line across both subjects. In practice, however, certain mathematical concepts are used in physics class before they were introduced in mathematics class (Alink et al., 2012). As a consequence, the lack of alignment across these subjects may be confusing for students and impede both transfer and help them experiencing coherence between these subjects (e.g., Berlin & White, 2012, 2014; Mooldijk & Sonneveld, 2010; Ríordáin et al., 2016).

1.3.2 Physics in Dutch Pre-university Education

Students in the Netherlands start with physics in grade 8. In this year, there is a strong context-concept approach in physics textbooks (e.g., van Bemmel et al. 2013), the number of formulas describing physical quantities is negligible, and algebraic skills to manipulate formulas are rarely used. In grade 9, there is again a context-concept approach in textbooks where formulas are used more frequently (e.g., Alkemade et al., 2014), but still, the level of algebraic skills required is still low. In grade 10, immediately after the transition from LSE to upper secondary education the intended level of algebraic skills increases substantially (e.g., Ottink et al., 2014). Furthermore, contrary to upper secondary education, the curricula for grade 8 and 9 of LSE are not described in curricula and tested in national final central examinations, leaving textbook publishers much room for shaping their content. As a consequence, the discrepancies between content of different textbooks written for the same grade can be large. This does not apply to the curricula in upper secondary education which are tested in national final examinations.

The physics formulas used in upper secondary education are symbolic representations of usually proportionalities that consist of real, measurable quantities. For instance, whereas the kinetic energy $E_{\text{kin}} = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2$ is proportional to the quantities m and $(v_{\text{final}})^2$, the centripetal force $F_{\text{C}} = G \cdot \frac{m \cdot v^2}{r}$ is inversely proportional to r. These formulas in algebraic physics problems and the algebraic skills needed to solve them are described in the physics curriculum (SLO, 2019). In addition, these formulas can also be found in BINAS, a natural sciences information booklet that students use during regular physics tests and the final examination. Over the last few years, physics problems for which solution algebraic skills are needed have become more important in senior pre-university education. They contain algebraic curve straightening, dimensional analysis and derivation of formulas. Moreover, solving algebraic physics problems correctly requires mathematically correct procedures with sufficient algebraic expertise including basic algebraic skills and symbol sense behavior.

1.4 Research Questions and Methodologies

In figure 4 we illustrated the dissertation flow chart including the five *explorative* studies that we have conducted. The dashed arrows between the studies (1), (2) and (3) indicate that these are follow-up studies investigating teachers' beliefs. The bold arrow from the three follow-up studies *and* the bold arrow from study (4) pointing to study (5) indicate that insights from these four studies are used to design and conduct study (5). In the previous sections, we have explained the logic behind the arrows. Below, we will elaborate upon these studies by describing their research questions and their corresponding methodologies.

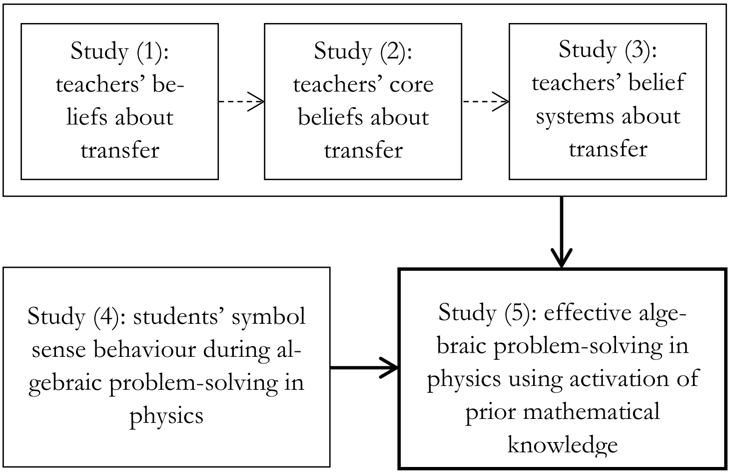


Figure 4. Dissertation flow chart of our five studies.

In the qualitative study (1) we examined the two sub questions (1a) "*How do mathematics and physics teachers characterize the transfer problem in the case?*", and (1b) "*What sort of beliefs do mathematics and physics teachers' beliefs have about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*". While question (1a) was asked to check whether teachers acknowledged this type of transfer problem, question (1b) aimed to gain insight into the various aspects that influence transfer. To answer these questions, we selected 10 mathematics and 10 physics teachers using convenience sampling (Bryman, 2015), i.e. they should be available and willing to participate in this study. These teachers were interviewed by means of a semi-structured questionnaire including a concrete case about a student transfer problem. The interviews were transcribed verbatim and analysed using open and axial coding to obtain a hierarchical code tree.

Study (2) examined the research questions (2a) "*How can a systematic, refined method be developed to reduce code trees of coded data into a single dataset?*" and (2b) "*What are the core beliefs of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*". This qualitative study was conducted immediately after study (1). To extract belief systems from the code tree containing s of coded data, we further developed pattern coding (e.g., Gibson & Brown, 2009; Saldaña, 2013). This second cycle coding technique was used to reduce coded data. In short, it grasps the essence of data and leaves out less important details. Different from Gibson and Brown (2009) and Saldaña (2013) offering general knowledge on how to further reduce coded data, we worked out pattern coding in detail.

In the quantitative study (3) we studied the research question (3) "*What are the belief systems of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*". For this purpose, we designed a webpage to conduct an online survey among 503 mathematics and physics teachers in the Netherlands. Next, we carefully converted the dataset including 16 core beliefs of study (2) into 16 claims. During this process we used the six functions of language of Jakobson to make sure that all claims were phrased clearly (Hébert, 2011). These claims were incorporated in an online multi-criteria assessment tool that we have developed, since no such tool was available. We asked the teachers to select their top five claims, and distribute 50 points over these claims, thereby identifying their belief system. The teachers were selected by means of self-selection sampling (Bryman, 2015). After they selected their top 5, we used the clustering technique agglomerative hierarchical clustering (AHC) (Everitt & Dunn, 2001) to categorize teachers based on their belief systems.

Study (4) concerned a qualitative study with a quantitative component in which we investigated the research question (4) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems?*". To gain insight into their symbol sense behavior during algebraic problem-solving in physics, we used convenience sampling to select 6 students who were available and willing to participate in this study; three of them from a regular school A, and three of them from a regular school B. Based on the Dutch ten-point grading system, these students had a sufficient mathematics grade and an insufficient physics grade (< 5.5). This grade criterion was to ensure that students' difficulties with algebraic physics problems were mainly because of insufficient *application* of basic algebraic skills in physics, and not related to a lack of basic mathematics.

Following Bokhove & Drijvers (2011), we designed tasks that should trigger students to solve algebraic physics problems and provide insight into their algebraic expertise including basic algebraic skills and symbol sense behavior. Next, we conducted task-based interviews among these students who were videotaped while problem-solving and thinking aloud. Both videotaped data and students' work were analysed using the seven consecutive phases proposed by Powell et al. (2003). For the operationalization of our research question we used a coding scheme. The coding process was based on analyzing videotaped episodes, the transcripts of the audio part of videotaped data, and the students' written solution set (students' work) to the tasks. Their solution set was compared to our systematic solution set, coded using our coding scheme and assigned to a score. This score was a measure for the extent to which students demonstrated symbol sense behavior and the degree to which they used basic algebraic skills correctly during algebraic problem-solving in physics class. Therefore, this score was also a measure for the degree to which students *successfully* transferred algebraic skills from mathematics to physics. We adopted the traditional view on transfer (Leberman et al., 2016), since we were interested in to the degree to which students used a systematic, rule-based problem-solving approach in which algebraic skills were used with insight. Hence, we compared the students' solution sets to our systematic solution set that contained the operationalized systematic algebraic strategies in terms symbol sense behaviour and basic algebraic skills above. Beyond the traditional approach to measure the degree of transfer, to some extent we adopted the actor-oriented transfer approach by paying attention to previous learning derived from what they said during the interviews. For instance, whether they explicated that they learned a specific problem-solving approach from their mathematics textbook (previous learning situation). So, the actor-oriented transfer approach was only used to gain insight into their previous learning situation, and not to measure the degree of transfer.

Furthermore, the design of our solution set was based on previous studies stating that algebraic skills should not prioritise basic skills or insight, but incorporate both basic algebraic skills and insight (e.g., Bokhove, 2011; Drijvers et al, 2011). Therefore, both basic algebraic skills and symbol sense behavior were explicitly worked out in the systematic solution set. We note that we obtained this insight also independently from the studies on teachers' beliefs.

For the qualitative study (5) containing a quantitative component we examined the research question (5) "How can activation of prior mathematical knowledge be used effectively to improve students' symbol sense behavior in upper secondary education when solving algebraic physics problems?". We divided this question in two sub questions: (5a) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems that occur in their physics textbooks?*" and (5b) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving the same algebraic physics problems that occur in their physics textbooks after activation of prior mathematical knowledge?*". To gain insight into their symbol sense behavior during algebraic problem-solving in physics, we again used convenience sampling to select 3 students who were available and willing to participate in this study. We selected students having a sufficient mathematics grade and an insufficient physics grade (< 5.5). Based on the iterative 3D-principle (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013) that will be explained in chapter six, we designed new tasks that contained different physics contexts than those in study (4). Again, these tasks should trigger students' algebraic problem-solving in physics and provide insight into their algebraic expertise. Next, in light of sub question (5a), students solved these tasks while being videotaped and thinking aloud (round 1). Two weeks later, we carried out small interventions by presenting the same problems as shift problems to them. Again, we asked students to solve these problems while being videotaped and thinking aloud (round 2).

For the design of shift-problems we used insights from the studies on teachers' beliefs about transfer and insights from study (4). From the studies on teachers' beliefs we especially used the importance of activation of prior mathematical knowledge. The latter provided algebraic hints at the start of these tasks to improve students' systematic problem-solving abilities, in particular symbol sense behavior. The other important insight that we used from the studies on teachers' beliefs was combined with insights from the study on symbol sense behavior. We addressed that algebraic skills, especially algebraic techniques should be applied in a similar way to how these were learned in their mathematics textbooks (Reichard et al., 2014). Other relevant insights from the studies (1) through (4) are explained in detail in later chapters.

Concerning data analysis, except for using different tasks and thus a different systematic solution set to assess students' solutions, our analysis was similar to that of study (4). After students' work of both rounds were analysed and assigned to a score, we examined the effectiveness of our intervention by checking to which extent their basic algebraic skills and symbol sense behavior were improved. Similar to study (4), this score was also a measure for the extent to which students *successfully* transferred algebraic skills from mathematics to physics. Since we compared students' work to our systematic solution set, transfer was viewed through the lens of the traditional perspective (Leberman et al., 2016). Again, the degree to which transfer occurred, was measured by determining the extent to which students demonstrated symbol sense behavior and the degree to which they used basic algebraic skills correctly during algebraic problem-solving in regular physics textbooks. As in the previous study (4), our second lens, i.e. the actor-oriented transfer approach did not influence this score, for it was only used to gain insight into what students said about previous learning.

Furthermore, to check reliability of our results, the analysis of each study was carried out independently by several researchers including the first and second authors, and cross-checked afterwards. Discrepancies between results were always discussed and if required, adjustments in those areas were made. This led to 100% agreement on the results among the researchers.

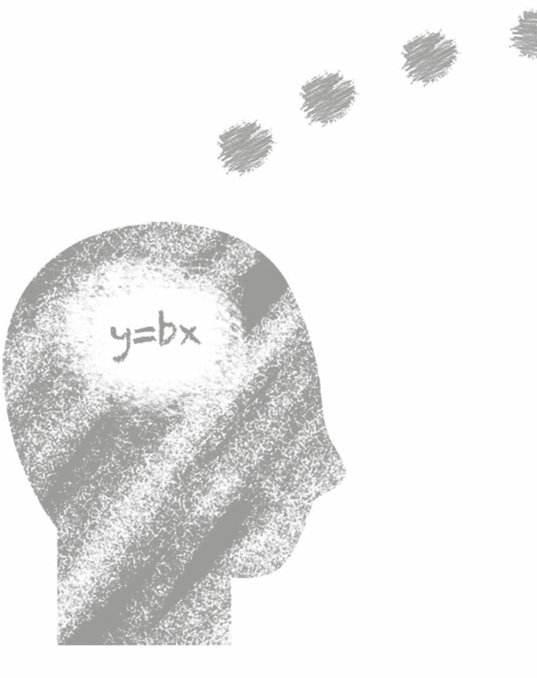
1.5 Dissertation Outline

This dissertation contains seven chapters. Except for study (5) in Chapter 6, Chapter 2 through 5 are adapted from international journals. For this reason, there can be some overlap in texts between chapters. These chapters with studies are depicted in table 1 above. As can be seen, in Chapter 2 we present study (1). Chapter 3 is concerned with study (2) and Chapter 4 deals with study (3). In Chapter 5 we present study (4). Finally, in Chapter 7 we present the general conclusion and discussion of this study.

Table 1. Dissertation outline including chapters and corresponding studies.

C	ST	Rationale	RQ	Participants		Data collection				
				S	T	I	L	OS	Q	TBI
1		General introduction								
2	(1)	Teachers' beliefs about improving transfer	1a, 1b		•	•	•		•	
3	(2)	Teachers' core beliefs about improving transfer	2				•			
4	(3)	Teachers' belief systems about improving transfer	3	•	•		•	•	•	
5	(4)	Students' symbol sense behavior during problem-solving in physics	4	•	•	•	•		•	•
6	(5)	Activation of prior-knowledge during problem-solving in physics	5a, 5b	•	•	•	•		•	•
7		General conclusion and discussion								

C = chapter; I = interview; L = literature; OS = online survey; Q = questionnaire; RQ = research question; S = student; ST = study; T = teacher; TBI = task-based interview.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Chapter 2

Teachers' Beliefs about Improving Transfer[1]

[1] This chapter has been published in adapted form as: Turşucu, S., Spandaw, J., Flipse, S., & de Vries M. J. (2017). Teachers' beliefs about improving transfer of algebraic skills from mathematics into physics in senior pre-university education. *International Journal of Science Education*, *39*(5), 587-604.

2.1 Introduction

Internationally, educational experts, teachers and policy makers have stressed the need to integrate - or at least to make connections - between mathematics education and science education (Berlin & White, 2012, 2014; Frykholm & Glasson, 2005; Furner & Kumar, 2007). As part of coherent mathematics and science education (CMSE), the United States' NCTM (2000) states that students should be able to transfer their knowledge of mathematics in different contexts outside mathematics. However, research has shown that students encounter mathematical difficulties in science subjects, implying a lack of transfer between these subjects (Cui, 2006; Karakok, 2009; Roorda, 2014).

Although researchers use different notions of transfer (e.g. Akkerman & Bakker, 2011;, 2006; Karakok, 2009; Roorda et al., 2014; Tuomi-Gröhn & Engeström, 2003), as described in the previous chapter, one can safely say assert that transfer from mathematics to physics is indeed problematic. However, very little is known about transfer of algebraic skills from mathematics to physics in senior pre-university education. Therefore, we first need to examine whether teachers really acknowledge such a transfer problem, and what their *beliefs* are about aspects that influence transfer in this area of research. These are major questions about this specific transfer problem. This legitimizes conducting a problem analysis involving interviews with mathematics and physics teachers in senior pre-university education. Asking teachers about their beliefs about transfer of algebraic skills from mathematics to physics in senior pre-university education is important, since it is well-known that teachers' beliefs strongly impact their behavior (e.g., Borg, 2015; Schoenfeld, 2014). Consequently, teachers' beliefs about transfer impact their behavior on how they deal with transfer issues in teaching practice. For instance, a physics teacher who *naïvely* thinks that a lot of practice in mathematics lessons with algebraic skills will automatically improve transfer of these skills to physics lessons. Since they neglect insight into the underlying mathematics in physics problems, they soon find themselves re-teaching basic mathematics, leaving less time for their core business of teaching physics. In short, naïve beliefs can impede transfer in physics class. In addition, in many countries, science curricula are overloaded, compelling science teachers to fit their program into a seriously reduced instruction time (e.g., Lyons, 2006), making inefficient transfer of mathematics in physics even more harmful. Furthermore, in recent years, physics problems for which solution mathematics such as the application of algebraic skills is needed, have become more important in upper secondary physics education (grade 10, 11 and 12 of pre-university education). Therefore, examining this transfer phenomenon is relevant from both educational and scientific point of view.

2.1.1 Research Aim and Research Questions

This paper aims to report the findings of a qualitative study on mathematics and physics teachers' beliefs about improving transfer of algebraic skills from mathematics into physics. Two research questions will be answered: (1a) *How do mathematics and physics teachers characterize the transfer problem in the case?*, and (1b) *What sort of beliefs do mathematics and physics teachers' beliefs have about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*

These teachers' beliefs can be organized into a belief system (Ernest, 1991), which contains a set of mutually supporting beliefs. In this study we investigate the individual beliefs, rather than belief systems. As a working definition of 'belief' we used quotes such as "*In my opinion..*", "*I believe..*"', "*I think..*" (Pajares, 1992).

2.2 Background

2.2.1 Coherent Mathematics and Science Education (CMSE)

Mathematics and science are closely connected (Atiyah, 1993). Mathematics provides the tools by which quantitative relationships in science subjects can be represented, modeled, calculated and predicted. Science offers meaning to mathematics by means of rich and relevant contexts in which mathematics can be applied (Dierdorp, Bakker, van Maanen, & Eijkelhof, 2014). Education that has the aim to foster this connection forms the basis of CMSE and is of vital importance for students (Berlin & White, 2012, 2014). Connecting these subjects is possible through alignment, such as using compatible notations, concept descriptions and pedagogy of mathematical methods. Sufficient attention for the connection between mathematics and physics may improve students' transfer of algebraic skills to physics and strengthen the extent to which students experience coherence across both subjects (e.g., Frykholm & Glasson, 2005; Furner & Kumar, 2007).

Another way to connect both subjects is through organization of the learning process in order to achieve a logical learning line across both subjects. In practice, unfortunately, it still happens that certain mathematical concepts are used in physics class *before* they were introduced in mathematics class (Alink, Asselt, & Braber, 2012).

The CMSE approach is based on traditional transfer of learning (Singley & Anderson, 1989): application of knowledge learned in a one situation (initial learning) to a new situation. Haskell (2001) states that this is universally accepted as the ultimate aim of teaching. Within this model the expert (teacher) determines whether transfer occurs or not. However, 'there is little agreement in the scholarly community about the nature of transfer, the extent to which it occurs, and the nature of its underlying mechanisms.' (Barnett & Ceci, 2002, p. 612). Hence, there has been a shift from traditional to alternative models, such as actor-oriented transfer. Within this framework the expert tries to understand the process in which the actor (student) constructs similarities between the initial learning situation and the new situation (Lobato, 2003). The extent to which transfer occurs moves from the experts' to the actors' point of view.

2.2.2 Belief Systems and Classroom Practice

Beliefs play a critical role in organizing knowledge and information and have a major impact on behavior (Ernest, 1991; Pajares, 1992). As stated earlier, beliefs can be organized into a belief system containing a set of mutually supporting beliefs (e.g., Lumpe et al., 2012; Pajares, 1992). Ernest's (1991) model describing the relation between a belief system and classroom practice is illustrated in figure 1 in Chapter 1. The upper rectangle describing 'View nature of mathematics' represents a belief system (Ernest, 1991) containing a basis for the teachers' espoused (mental) models of learning and teaching mathematics. These espoused and enacted models are influenced by the dashed rectangle in the middle including constraints and opportunities of the powerful social context of teaching that is "*a result of a number of factors including the expectations of others, such as students, their parents, fellow teachers and superiors*" (Ernest, 1991, p. 290). These espoused models are transformed into classroom practice (enacted models). In figure 1, the enacted models are displayed by the three rectangles 'learning mathematics', 'teaching mathematics' and 'using mathematics texts'. So, while the upper two rows of rectangles represent a teachers' belief system, the lowest two rectangles are concerned with classroom practice. The distinction between espoused and enacted is essential, since case studies have shown that there can be a great disparity between both models (Brown & McNamara, 2011; Lloyd, Veal, & Howell, 2016). In this study the espoused models in figure 1 may refer to, for instance teachers' naïve beliefs (Schoenfeld, 1985) about transfer which is harmful for transfer in teaching practice (enacted models). To change such naïve beliefs into desirable beliefs about transfer, teachers have to be aware of the relation between their beliefs in relation with classroom practice (Ernest, 1991), reflect on them and reconcile their espoused and enacted beliefs.

2.2.3 The Unifying Role of Mathematics

Naïve beliefs about transfer may be related to beliefs about the unifying role of mathematics, i.e. the fact that similar expressions and formulas used in different contexts outside mathematics can be reduced to the same abstract mathematics (Atiyah et al., 2010), beliefs about drilling of basic algebraic skills (Wu, 1999), such as adding fractions, substitution and completing the square (Drijvers, 2011), and beliefs about automatic transfer. The latter refers to. teachers who think that transfer of mathematics to science happens *automatically* as long as students practice a lot in mathematics class. However, such beliefs do not consider conceptual understanding (Kilpatrick et al., 2001) and could lead to routine based on 'tricks'. On the other hand, too much focus on conceptual understanding can impede basic skills. We conclude that both basic skills and conceptual understanding should be taught in an integrated approach (Drijvers, 2011). This may improve transfer.

Whereas some scientists view mathematics as the 'servant of the sciences', some mathematicians may consider mathematics to be 'the queen of the sciences' (Atiyah, 1993). They often perceive applied mathematics as inferior to pure mathematics. Some even refuse to discuss applications. Such beliefs may conceivably influence transfer. It goes without saying that these views are not typical of the whole mathematical community. In fact, one finds various ideas about the role of mathematics in science, the difficulties and the importance of teaching and learning transfer among mathematicians, even among those whose taste and interest are skewed toward the theoretical end of the mathematical spectrum.

2.2.4 Coherent Mathematics and Science Education and Transfer in The Classroom

CMSE depends on actors such as teachers, school organization, curriculum and policy makers (Schmidt, Wang, & McKnight, 2005). Since these actors interact with one another, their involvement makes CMSE and transfer a rather complex process. As Schmidt, McKnight, and Raizen (1997, p. 92) explain: "*each 'actor' pursues his or her own 'life' –his or her goals, visions, plans, processes, and efforts to satisfy those to whom he or she is accountable*".

In this study we restrict ourselves to teachers' beliefs about curricula, textbooks and teachers. Teachers follow the textbooks very closely and these textbooks are shaped by the curricula (SLO, 2019).

Furthermore, we assume a reciprocal relation between CMSE and transfer. When students experience coherence across mathematics and science subjects by means of *meaningful* contexts, transfer from mathematics to science subjects can be improved, and improving transfer can help them to experience coherence between mathematics and subjects.

2.2.5 Curricula and Textbooks in Dutch Secondary Education

Pre-university education in the Netherlands consists of three junior years and three senior years. In senior pre-university education students in physics class should choose between mathematics A and B. The latter puts much more emphasis on algebra than the former. The content of mathematics and physics subjects are mainly determined by the national final examinations at the end of secondary school, and specified in curricula (SLO, 2019).

Textbooks mediate between the core goals of education (the intended curriculum) and the actual teaching in classrooms (the implemented curriculum). Hence, textbooks are referred to as the potentially implemented curriculum (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). The limited description of the core goals in the curricula leave publishers room for different interpretations. Their textbooks are followed very closely by both teachers and students (SLO, 2019).

2.3 Methodology

In this section we will *first* explain how we collected our data. *Second*, the semi-structured questionnaire used in this study is presented. Finally, we will discuss the methods used to analyze our data.

2.3.1 Data Collection

Convenience sampling (Bryman, 2015) was used to gather data from ten Dutch mathematics and ten Dutch physics teachers. Each group of respondents consisted of eight male and two female teachers; they were qualified to teach in senior pre-university education and had at least five years of teaching experience. These numbers are in good agreement with the gender ratio in senior pre-university education in the Netherlands: about 15 percent of mathematics teachers, and 5 percent of physics teachers is female (Mullis, Martin, Kennedy, Trong, & Sainsbury, 2009). The respondents were interviewed by means of a questionnaire and gave consent to reporting. Each interview was conducted privately in an appropriate, silent place chosen by the teacher and took 30 – 45 minutes. Afterwards, it was transcribed *verbatim* for analysis. For the teachers' names we used pseudonyms.

2.3.2 Semi-structured Questionnaire

In order to investigate research questions (1a) and (1b), we used a semi-structured questionnaire (Bryman, 2015) that was based on one specific *case* about a transfer problem. The questions were based on this case. For the case and the questionnaire, see table 1.

Table 1. Semi-structured questionnaire, which was based on the transfer problem in the case.

Question number	Rationale
	Case: during a physics lesson a student does not recognize that the physics formula (formula in short) for displacement, $s = \frac{1}{2}at^2$, has a similar algebraic structure as the mathematical equation (equation in short), $y = bx^2$. This student is also unable to express *t* in terms of *s*. However, earlier that day during mathematics, the student managed to express *x* in terms of *y*, implying that besides a lack of recognition, the student is not able to apply algebraic skills from mathematics to physics successfully. Now we want the same student to *recognize* that in both situations a similar algebraic structure is used.
1	*Is this a familiar problem?*
2	*Do you consider it an important problem?*
3	*What may be the reason?*
4	*As a physics (mathematics) teacher, what would you do a about it?*
5	*What may the mathematics (physics) teacher do about it?*
6	*What does it mean for the formal physics (mathematics) curriculum?*
7	*What does it mean for the math- and physics textbooks?*
	We also want this student to be competent in the *application* of algebraic skills from mathematics into physics. In this case, to express *t* in terms of *s*: $t = \sqrt{\frac{2s}{a}}$.
8	*Do you consider it an important problem?*
9	*What may be the reason?*
10	*As a physics (mathematics) teacher, what would you do a about it?*
11	*What may the mathematics (physics)teacher do about it?*
12	*What does it mean for the formal physics(mathematics) curriculum?*
13	*What does it mean for the math- and physics textbooks?*
14	*To what extent do you follow textbooks during teaching?*
	In the above-mentioned case, it can be seen that mathematics and physics are closely related to one another. Teachers appear to have different ideas about their relation.
15	*How do you see the relation between math- and physics?*
16	*Do you have any cooperation with your mathematics colleagues?*
17	*How do you see the optimal cooperation with your mathematics colleagues?*
	Our pre-university physics education is permeated with algebraic problems from mathematics, such as the case above.
18	*How can the application of algebraic skills from mathematics to physics be improved for solving algebraic problems that occur in our pre-university physics education?*

2.3.3 Data Analysis

We used open coding (Bryman, 2012; Saldaña, 2013) to label each fragment of the transcripts, giving us a short description of teachers' beliefs about both research questions. This process resulted for each of the twenty interviews in a set of labels identifying teachers' beliefs.

Next, we used axial coding, consisting of two steps. In the first step labels with roughly the same content were grouped together, leading to a grouping of the labels. Each group of labels was summarized as a subtheme. A subtheme had to contain at least three different beliefs of at least three different teachers. Otherwise it was marked as an outlier. In the second step we grouped the 28 subthemes into 9 core themes (see Table 2). Thus, we obtained one common code tree for all twenty teachers. This tree is a hierarchical structure consisting of core themes as main branches. These cores themes then branch out into smaller branches, called subthemes. The next and finest level of the hierarchy, the leaves of the tree, are the underlying teachers' beliefs about CMSE and aspects influencing students' transfer of algebra to physics.

To enhance reliability of our results, the whole process of open and axial coding was independently carried out by an independent researcher. The two common code trees overlapped for approximately 80% on each of the three levels (labels, subthemes and core themes). The two researchers then discussed the remaining 20%. After some adjustments in these parts of the tree, this led to consensus among the two researchers about the common code tree. Finally, the whole process was (sample-wise) double checked by the second and third author.

2.4 Results

First, we present our results for research question (1a), then for research question (1b).

2.4.1 Research Question (1a): Characterizing The Case

Research question (1a) is related to the case and to questions 1, 2 and 8 of the questionnaire in Table 1. As to question 1, nine out of ten mathematics teachers, and eight out of ten physics teachers acknowledged the case. As to question 2, nine mathematics and nine physics teachers considered it an important problem. For question 8 we found that nine mathematics and nine physics teachers considered it important that students are competent at the transfer of algebraic skills from mathematics into physics.

Table 2. Teachers' beliefs about aspects influencing students' transfer and aspects about CMSE.

Core theme/ subtheme	Mathematics teachers	Physics teachers
1. Coherence	126	135
1.1 Alignment	2/1[a]	10/6
1.2 Collaboration and cooperation	85/10	75/10
1.3 Ideal collaboration and cooperation	39/10	50/10
2. Curriculum	65	86

2.1 Curriculum (general)	25/9	10/7
2.2 Mathematics curriculum	23/10	31/10
2.3 Physics curriculum	17/10	45/10
3. Education	7	26
3.1 Junior pre-university education	07/5	26/7
4. Pedagogy of algebra	82	72
4.1 Algebraic skills	40/10	26/7
4.2 Algebraic techniques	7/4	8/5
4.3 Practice (general)	21/9	30/9
4.4 Practice within mathematics	9/5	3/3
4.5 Practice within physics	5/3	5/3
5. Relation between scientific subjects	87	52
5.1 Mathematics and physics	27/10	15/10
5.2 Mathematics within physics	35/10	23/10
5.3 Physics within mathematics	25/10	14/10
6. School subjects	30	20
5.1 Mathematics	19/7	13/6
5.2 Physics	11/6	7/4
7. Teacher	193	112
6.1 Mathematics teacher	97/10	48/10
6.2 Physics teacher	96/10	64/10
8. The use of textbooks	143	139
8.1 Following textbooks	31/10	43/10
8.2 Mathematics textbook	66/10	31/10
8.3 Physics textbook	37/10	45/10
8.4 Textbook general	9/5	20/7
9. Transfer	144	89
9.1 Activating prior knowledge	8/5	10/4
9.2 Affordances (specific)	34/10	8/5
9.3 Constructing relations (general constraints)	27/10	23/9
9.4 Constructing relations (specific constraints)	75/10	48/10
9.5 Focus on students	1/1[b]	1/1[c]

Note. [a] This subtheme is considered as an outlier
[b] This subtheme is considered as an outlier
[c] This subtheme is considered as an outlier

2.4.2 *Research Question (1b): Common Code Tree*

We found a continuum of teachers' beliefs (approximately thirteen hundred beliefs) which can be organized in nine core themes and their twenty-eight subthemes. See Table 2. For example, the core theme 'School subjects' contains the subthemes 'Mathematics' and 'Physics'. The number '30' next to 'School subjects' is the total number of beliefs about this core theme uttered by the mathematics teachers. The numbers '11/6' next to the subtheme '6.2 Physics' in the same column mean that among these 30 beliefs 11 belonged to this subtheme and they were uttered by 6 teachers. We found three outliers: the subtheme 'Alignment' for mathematics teachers and the subtheme 'Focus on students' for both teacher groups.

2.5 Results Interpretation

In this section we will first interpret the results regarding research questions (1a) and (1b). Next, we will discuss three teacher groups' beliefs about improving transfer of algebraic skills to physics. Finally, we will discuss limitations and recommendations.

The quotations below are taken from the interviews. For stylistic reasons, we use the words 'believe' and 'think' interchangeably to describe teachers' beliefs. The word 'collaboration' refers to activities in which teachers work together, such as designing teaching materials. The word 'cooperation' refers to conversations without such activities.

Our analysis below shows some inconsistencies within the set of beliefs of many interviewees. Indeed, during the second half of the interview many teachers expressed opinions contradicting their own opinions during the first half of the interview. For example, most mathematics and physics teachers first expressed the opinion that extensive algebraic practice in math class alone should solve the transfer problem, but later the same teachers said that algebraic practice is also needed in physics class.

2.5.1 Research Question (1a): Insight into The Case

Most of the interviewed teachers acknowledged the case of Table 1. This justifies our interviews with both mathematics and physics teachers. It is remarkable that the mathematics teachers acknowledged the case, even though they did not encounter this problem in their own classroom. This may imply that mathematics teachers discuss this problem with physics teachers. Mathematics teachers think that the transfer problem occurs especially in the first year of senior pre-university education, rarely in the next years. This seems reasonable, since the level of algebraic skills needed in physics increases substantially in the transition from junior pre-university education to senior pre-university education.

Most of the physics teachers believed that the well-performing students in mathematics B do not encounter any transfer problems at all. This belief is supported by the fact that mathematics B puts a much stronger emphasis on algebra than mathematics A.

Regarding questions 2 and 8, most physics teachers believed that in recent years the transfer of mathematics to physics has become more important, because of the role of mathematics in the final central physics examinations. They think that the relation (they often used the word 'link') between mathematics and physics has to be emphasized more strongly. Most of the mathematics teachers, however used the word 'application'; they mention the importance of applying mathematics to another subject such as physics.

2.5.2 Research Question (1b): Core Themes

Below we discuss the subthemes for each core theme (see Table 2).

Core theme 1: coherence
The subtheme 'Alignment' was considered an outlier for mathematics teachers. Most of the physics and mathematics teachers mention the need to align the learning lines in mathematics and physics using the textbooks. This connection is of key importance: it may improve

students' transfer of mathematics into physics and also strengthen the extent to which they experience coherence between these subjects (e.g., Roorda, 2012; Quinn, 2013).

As for the subtheme 'Collaboration and cooperation' most of the physics teachers said they were willing to collaborate, but they strongly believed that mathematic teachers do not feel the need for collaboration. As one of them said: "*It is difficult to communicate with mathematics teachers*". Consequently, there is little interaction between mathematics and physics teachers. If there is any interaction at all, this consists of individual efforts on a small scale during informal meetings. Indeed, our data indicates the existence of *two* types of mathematics teachers. The first type, which represents the majority, does not feel the need to collaborate with physics teachers, supporting physics teachers' beliefs. They think that "*They* [physics teachers] *have a problem, and they have to find us*". The second type does collaborate with physics teachers. These mathematics teachers also feel the need to align the content of mathematics and physics subjects across time.

The next subtheme 'Ideal collaboration and cooperation' assumes the absence of constraints (see figure 1 of Chapter 1). Most of the mathematics teachers believe that more collaboration with their colleagues from physics would be desirable in this ideal situation. The difference between this ideal (espoused) beliefs of mathematics teachers and their lack of collaboration with physics teachers (enacted beliefs) can be caused by constraints (see figure 1 of Chapter 1). Indeed, they often mentioned huge workload as an impeding factor. The physics teachers believe that an ideal collaboration would result in alignment of notations, equations, formulas and algebraic techniques in both subjects.

Core theme 2: curriculum

Both mathematics and physics teachers use the words 'connection' and 'integration' interchangeably to indicate CMSE. Concerning the subtheme 'Curriculum (general)' most of the physics teachers believed that there is the need to integrate or at least make connections between the mathematics and physics curriculum. Although they did not explicate what this integration or connection should look like, they believed that these should be visible through the content standards, probably because they observe which algebraic skills their students lack in physics class. In contrast, most of the mathematics teachers believed that such integration or connection is not needed. Presumably, they were unaware of the type of mathematical skills that students lack during physics lessons.

Regarding the subtheme 'Mathematics curriculum' most of the mathematics and physics teachers believed that the content standards should include physics contexts in which algebraic skills are involved. For instance, manipulating formulas and solving for a variable. A small number of these teachers (including some math teachers!) state that they are unaware of the content of the mathematics curriculum. Indeed, most teachers rely on textbooks as a substitute for the curriculum. They trusted that these textbooks represent this curriculum accurately and follow these books very faithfully (SLO, 2019; Zanten & van den Heuvel – Panhuizen, 2014). Most of the mathematics teachers desired the incorporation – in the curriculum or in the texbooks; for many teachers that's the same thing – of a content standard about recognizing the algebraic structure of formulas and equations in physics.

For the subtheme 'Physics curriculum', most of the mathematics and physics teachers would like to see an emphasis on algebraic skills, for example manipulating formulas and solving for variables in the physics curriculum. Some mathematics teachers provided quite explicit suggestions about what is needed in the physics curriculum. This result is quite remarkable, since most mathematics teachers mentioned that they did not feel the need to integrate both curricula. Physics teachers also wish for a content standard about recognition of the algebraic structure of formulas and equations in physics. However, a small number of physics teachers seems to be satisfied with the actual physics curriculum: "*There is no need to add anything*".

Core theme 3: education

Most mathematics and physics teachers believed that in the last year of junior pre-university education there is a lack of emphasis on algebraic skills in mathematics lessons. Physics teachers mention that they observed this lack mainly in the first year of senior pre-university education. As mentioned above, this belief is shared by most mathematics teachers.

Core theme 4: pedagogy of algebra

Concerning the subtheme 'Algebraic skills' most of the physics teachers again mentioned that the lack of sufficient algebraic skills to tackle transfer problems mainly occured in the first year of senior pre-university education. This result is in agreement with the subsections 'Discussion on characterizing the case' and the core theme 'Education'. Mathematics teachers believed that more practice with algebraic skills will improve transfer.

The subtheme 'Algebraic techniques' concerns mathematical tools used to solve algebraic problems such as cross multiplication and cover-up method (Drijvers, 2011). Both mathematics and physics teachers think there is a mismatch between algebraic techniques learned in mathematics and physics. They think that more alignment between these algebraic techniques is needed.

As to the subtheme 'Practice (general)', both groups believed that the lack of practice with transfer problems analogous to the case impedes the transfer of algebraic skills to physics. They believed that more practice in both physics and mathematics is required to improve transfer. This is illustrated by a quote from a physics teacher, referring to both subjects: "*In physics class students should practice with formulas analogous to* $s = \frac{1}{2}at^2$ *and in math class with equations analogous to* $y = bx^2$. *This will help students to solve this transfer problem*". Similar statements were made by many other mathematics and physics teachers.

Regarding the subtheme 'Practice within mathematics' most of the mathematics and physics teachers believed that extensive practice in math class with algebraic skills is both necessary and sufficient. This is illustrated by the quote "*They need lots of practice during mathematics classrooms. Then, application into physics will happen automatically*". A small number of mathematics and physics teachers believed that in math classes more practice with transfer problems analogous to the case is needed.

As for 'Practice within physics', most mathematics and physics teachers said that in physics classes more practice with physics problems involving algebraic skills is needed. This contradicts their previous statement about the alleged sufficiency of practice in math class and automatic transfer. A small number of mathematics and physics teachers suggested activation of prior mathematical knowledge by starting with the mathematics problem in the case (see Table 1), followed by algebra problems in physics.

Summarizing, both teacher groups put a strong emphasis on practice with transfer problems similar to the case and stated that this would improve students' transfer. This belief may be regarded as naïve and can be associated with the idea of basic skills first (Wu, 1999). However, there is no single teacher who mentioned and related this matter to conceptual understanding in both activities. This result is important, because it might indicate that these teachers overlooked a serious risk: putting too much emphasis on basic skills could push conceptual understanding of the underlying mathematics to the background (Drijvers, 2011; Kilpatrick et al., 2001) and could impede transfer of algebra to science. Hence, teachers who develop common learning strategies aiming at transfer should take into account both basic skills and conceptual understanding. Note that this result may also partly explain the lack of transfer in earlier studies (Cui, 2006; Karakok, 2009; Karam, 2014; Roorda, 2012).

Core theme 5: relation between scientific subjects

Concerning the subtheme 'Mathematics and physics' most of the mathematics and physics teachers said that mathematics and physics are two inextricably intertwined subjects. Only a small number of mathematics and physics teachers mentioned that both subjects should be regarded as separate disciplines. Some mathematics teachers viewed mathematics as the 'Queen of all sciences' (Atiyah, 1993): mathematics should remain pure, because application of mathematics would degrade it.

As to the subtheme 'Physics within mathematics' most of the physics teachers viewed mathematics as the 'Servant of science'.

As to the role of 'Mathematics within physics', most teachers in both groups mentioned 'aid' and 'tool'. Some of the math teachers even mentioned 'mathematics serves physics' (cf. Atiyah, 1993).

Before we conducted the interviews with teachers, we hypothesized that teachers who view mathematics as the 'Queen of all Sciences' might not feel the need to bother about transfer. However, analysis of the data shows the opposite: these teachers *did* make suggestions about tackling transfer. This result seems to indicate that they were aware of the importance of applying mathematics in physics, even though they had purist views about mathematics.

Core theme 6: school subjects

Concerning the subtheme 'Mathematics' most of the physics teachers believed that physics should provide good contexts for math class. Examples from physics in math class makes mathematics more understandable and offer new insights to the students. This result matches with the view expressed in Alink, Asselt and Braber (2012) and Berlin and White (2012, 2014), who stated that science contexts offer meaning to mathematics in which it can be applied, and they contribute to help students experiencing coherence between these subjects. Unfortunately, the beliefs of the interviewed mathematics teachers were too diverse to draw general conclusion.

This diversity of mathematics teachers' beliefs also holds for the subtheme 'Physics'. Still, a small number of them argued that the mathematics used in physics class should be restricted to mathematics A, since some students in physics class do not study mathematics B (see the first paragraph of subsection 'Classroom Actors' for information about mathematics A and

B). Physics teachers share this belief. They added that mathematics A students encounter more difficulties with algebraic problems in physics class than mathematics B students. This belief may indicate that physics teachers were aware of the fact that some of their students have less training in algebraic skills because they do not study mathematics B, although none of the physics teachers explicated this. The lack of perceived urgency to cooperate with physics teachers is not typical for the community of mathematics teachers as a whole. Indeed, many teachers may appreciate the need to promote transfer.

Core theme 7: teacher

Concerning the subtheme 'Mathematics teachers', most of the mathematics and physics teachers agreed that math teachers should incorporate more physics context in their lessons. Furthermore, they should emphasize the close relationship between mathematics and physics. Most of the physics teachers mentioned that mathematics teachers should include exercises similar to the case. A small number of physics teachers mentioned that mathematics teachers should be competent in the mathematics content. They should use other variables than x and y. Another small number of physics teachers stated that mathematics teachers should be acquainted with the physics curriculum, but they did not specify to what extent. A small number of mathematics and physics teachers believed that mathematics teachers should stick to pure mathematics: "*Mathematics should avoid all forms of physics*". Unlike the physics teachers, the math teachers in this group viewed mathematics as the 'Queen of all sciences' (Atiyah, 1993). However, these math teachers made suggestions on how to improve transfer. Another small number of mathematics teachers mentioned practicing exercises similar to the case, and use other variables than just x and y.

Regarding the subtheme 'Physics teacher' most of the physics teachers referred to the desirability of teacher-centered practice of physics problems which involve algebra in physics class. They also believed that showing similarities between different equations and formulas is beneficial. They emphasized that students should practice with exercises similar or analogous to the case. Prior mathematical knowledge that is related to the mathematics involved in physics problems should be activated. Physics teachers should use x and y as well as the conventional quantities in physics. This can be regarded as an extension of activation of prior mathematical knowledge. A small number of physics teachers used their privately developed teaching materials to train students' mathematical skills: "*Students' performance on algebraic skills were bad. I became frustrated and developed my own teaching material*". This may be due to the absence of sufficient attention on algebraic skills in the current physics methods. Another small number of physics teachers mentioned the lack of time to focus on algebraic skills in physics problems. This shortage of time is often observed in schools in the Netherlands, and most probably related to physics teachers' busy daily routine (Alink, Asselt, & Braber, 2012). As opposed to physics teachers, most of the mathematics teachers put a stronger emphasis on activating prior mathematical knowledge. They believed that they should relate physical quantities to the variables x and y used in mathematics, write down a formula from physics next to the corresponding mathematical equation on the blackboard. They also mentioned practice with exercises similar to the case, and explanation of the close relationship between mathematics and physics. A small number of mathematics teachers stated that physics teachers explicating transfer problems in physics lessons to mathematics teachers may also help to reduce these transfer problems.

We conclude there are many beliefs about what mathematics and physics teachers should do to deal with the problem of transfer. According to Davison et al. (1995) these beliefs help students to, "…*explore the connections between mathematics and science and begin to see the relevancy of mathematics in the reality of science and vice versa*" (p. 228). Sufficient attention for these connections may help to improve transfer and enhance students experiencing coherence across both subjects (Frykholm & Glasson, 2005; Furner & Kumar, 2007; Berlin & White, 2012, 2014).

Core theme 8: the use of textbooks

As to the subtheme 'Following textbooks', all interviewees mentioned that they are highly textbook-driven. This is in line with earlier research (Stein & Smith, 2010; Van den Heuvel-Panhuizen & Wijers, 2005). In some countries countries this matter is not that extreme as in the Netherlands (e.g., TIMMS & PIRLS', 2019).

The beliefs belonging to the subtheme 'Mathematics textbook' indicate the existence of two main types of mathematics teachers. The first type claimed to be satisfied with the use of contexts in mathematics textbooks, for example "*There is enough context* [in the mathematics method]". The second type would like to see more context in mathematics textbooks. They also frequently stated the lack of sufficiently many formulas and physics exercises. Most of the physics teachers advocated the inclusion of more physics context in mathematics textbooks, such as formulas, exercises and algebraic skills needed to solve physics problems. A small number of physics teachers, however, disagreed on this point.

Concerning the subtheme 'Physics textbook' the majority of the interviewed physics teachers believed that physics textbooks need introduction paragraphs containing prior mathematical knowledge about the physics content that will be treated. They argued that some physics textbooks do this adequately, whereas others do not. Only two physics teachers were satisfied with the actual content of physics textbooks. Most of the mathematics teachers strongly believed that activating prior mathematical knowledge is of key importance in the approach of tackling transfer problems.

Regarding the subtheme 'Textbook (general)' the teachers use the words 'connection' and 'integration' interchangeably to indicate connection in terms of alignment between mathematics and physics textbooks. Most of the physics teachers would like to see this connection in textbook series. This should be made possible through exercises analogous to the case, equations and corresponding formulas that are treated together, or alignment of algebraic techniques in both textbooks. They also mentioned alignment of textbooks on a general level, without making any concrete suggestions. Mathematics teachers' beliefs are split: while one part claimed there is the need to connect both textbooks, the other part showed very little enthusiasm. Supporters of connection would like to see two separate textbooks, with the physics textbooks referring to mathematics textbooks, and vice versa. Data shows that teachers who advocated such connection do not speak about one single integrated textbook, but two separate textbooks making connections to one another through content.

Still, aiming at CMSE through connections between the content of mathematics and physics textbooks is a rather complex process, which depends on good collaboration between other actors than just textbook publishers, such as policy makers (Schmidt, Wang, & McKnight, 2005).

Core theme 9: transfer

Although mathematics and physics teachers' beliefs about the subtheme 'Activating prior knowledge' are very fragmented, they all mentioned the importance of activation of prior mathematical knowledge in physics class.

For the subtheme 'Constructing relations (affordances)' most of the mathematics and physics teachers believed that the transfer problem in the case may be overcome when students recognize similarities in the algebraic structures of equations and formulas. They also think that physics teachers have to relate quantities of formulas more often to the variables x and y from mathematics. This can be interpreted as activation of prior mathematical knowledge.

Regarding the subtheme 'Constructing relations (general constraints)' most of the mathematics and physics teachers claimed that the main reason for students facing difficulties with transfer problems analogous to the case is because students see both subjects as two entirely separate subjects: "*Students think that they have entered an entirely new subject when just having left math and entered the physics classroom*". This is close to literature where students view mathematics and science, in particular physics are seen as two unrelated subjects (Cui, 2006; Karakok, 2009; Roorda, 2012).

As to the subtheme 'Constructing relations (specific constraints)' most of the mathematics and physics teachers stated that the variable names x and y are often used in mathematics, but impede transfer. A small number of mathematics and physics teachers also claimed that this transfer is impeded when students rely too much on mathematical 'tricks', such as the equation triangle. These teachers seemed to be aware of the necessity of conceptual understanding (Kilpatrick et al., 2001). Furthermore, a small number of mathematics teachers believed that the absence of automation in solving transfer problems impedes transfer too. However, from data it is not clear what they exactly meant with automation.

2.5.3 Three Approaches to Transfer

Most of the interviewees belong to one of the following three groups. The first and largest group believed that the transfer problem should by intensive algebraic practice in math class. Then, they claimed, transfer of algebraic skills into physics happens automatically. The second and smallest group stated that the transfer problem should be solved by practicing algebraic physics problems in physics class. The third group lies between these opposite views. These teachers believed that the transfer problem can only be solved by comprehensive algebraic practice in both mathematics and physics class. For example, algebra problems in math class should use contexts and notations from physics, and physics teachers should activate prior mathematical knowledge. Both physics and mathematics teachers should emphasize the connections between their subjects.

The beliefs in the first group are linked to the unifying role of mathematics (Atiyah, 1993) and can be interpreted as prioritizing basic skills (Wu, 1999). The second groups' beliefs can be described as reinventing the same mathematical wheel in different physics contexts.

Presumably, the same wheels also have to be reinvented in other subjects using algebra such as chemistry and economics. Although this approach does not concern mathematics lessons, it still can be viewed as prioritizing basic skills, but in science contexts.

We concluded that the first and second groups ignored the development of conceptual understanding in their teaching (Kilpatrick et al., 2001), meaning that the understanding of the underlying mathematics could be pushed into the background (Drijvers, 2011). However, for optimal transfer conditions there must be a focus on both basic skills and conceptual understanding (Bransford et al., 2000; Kilpatrick et al., 2001; Roorda, 2014). Thus, teachers' beliefs prioritizing basic skills can be seen as naïve (Schoenfeld, 1985). Teachers who transform such beliefs (espoused model) into teaching practice (enacted model) may be confronted with a great disparity between beliefs and what they observe in the classroom (see figure 1 of Chapter 1), i.e. a lack of transfer. Teachers should be aware of the existence of such beliefs, reflect on these and reconcile with their classroom practice. Without reflectivity teachers are often observed to adopt similar practices in the classroom (Ernest, 1991; Pajares, 1992). As a consequence, this may impede students' transfer in physics class.

The third group's beliefs about transfer are most comprehensive, since they consider an integrated approach. This may contribute to students experiencing coherence across both subjects.

2.5.4 Limitations of This Study and Recommendations

For this study we interviewed 10 qualified mathematics and 10 qualified physics teachers from senior pre-university education from different schools in the Netherlands within a radius of approximately 50 kilometers. Each teaching group contained eight male and two female teachers, being in good agreement with the gender ratio in senior pre-university education in the Netherlands (Mullis et al., 2009). These teachers were selected using convenience sampling and had varying years of teaching experience, i.e. ranging from 5 to 40 years. Based on these characteristics, we conclude that our sample may be representative for Dutch teachers who teach at senior pre-university education

The content of mathematics and physics subjects in the Netherlands is determined by mainly the national final examination at the end of the secondary school (e.g., SLO, 2019; 'TIMMS & PIRLS', 2019), and described in curricula through both the general educational core goals and the more specific standards, shaping to a very large extent the content of textbooks and teachers who quite strictly follow these textbooks (SLO, 2019; Van den Heuvel-Panhuizen & Wijers, 2005). Thus, to a great extent, Dutch teachers' beliefs about transfer are influenced by textbooks. We do not expect much difference in the content of textbook series, implying that teachers' beliefs above would not differ significantly from each other.

Based on the results above we expect this study to be generalizable for mathematics and physics teachers teaching in senior pre-university education in the Netherlands, and also for those who teach in senior general secondary education. However, we do not expect that this holds for preparatory vocational secondary education, because the mathematical skills needed in physics are fundamentally different from those in senior general secondary education and senior pre-university education. This can lead to different teachers' beliefs about transfer. In many countries, however, the combination of such final examination shaping textbooks with textbook-driven teachers does not exist (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002), implying that our results are not generalizable to these countries.

In terms of diversity in teachers' beliefs we did not observe much change after a total of eight interviews including four mathematics and four physics teachers, indicating saturation for both teacher groups.

Three of our subthemes were removed, because these did not consist of at least three different beliefs mentioned by at least three different teachers. This criterion is slightly arbitrary. Data reduction is associated with grasping the essence and leaving out less important details. Outliers may contain important information about missing teachers' beliefs as the subtheme 'Focus on students' showed. This subtheme contained two teachers who mentioned that transfer should be examined through the eyes of the student. Teachers should pay attention to questions like "*How did the student construct transfer?*". They focused on students, rather than the traditional approach focusing on transfer from the experts' view (teacher). In short, they adopted a contemporary view such as the actor-oriented approach (Lobato, 2003). Therefore, in terms of improving students' transfer of algebraic skills into physics, we recommend to further investigate the focus on this matter.

Referring to figure 1 of Chapter 1, we recommend examining mathematics teachers who had purist beliefs (espoused beliefs), but nevertheless made suggestions about improving transfer. How do they deal with transfer problems in the classroom (enacted beliefs) if they have such purist beliefs?

2.6 Conclusion

Regarding research question (1a) "*How do mathematics and physics teachers characterize the transfer problem in the case?*" we found that nearly all mathematics and physics teachers acknowledged the case presented to them and considered it important that students are competent at the transfer of algebraic skills from mathematics into physics. They think that transfer problems occur especially in the first year of senior pre-university education.

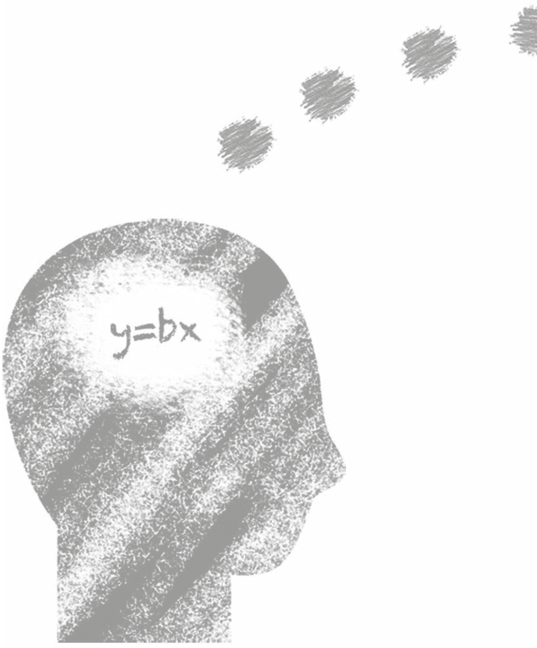
To answer research question (1b) "*What sort of beliefs do mathematics and physics teachers' have about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*" we used open and axial coding to analyze the interviews and found one common code tree for both teacher groups, including nine core themes: Coherence, Curriculum, Education, Pedagogy of algebra, Relation between scientific subjects, School subjects, Teacher, The use of textbooks and Transfer (see table 2). These core themes contained a continuum of teachers' beliefs about aspects influencing students' transfer above, including beliefs on how to improve this transfer, and aspects about CMSE, including aspects that may enhance students experiencing coherence between these subjects. When solved, these aspects may help reduce science teachers' frustrations, who spend extra time on repeating mathematics in science classes.

We have seen that most of the teachers believed that transfer does not happen because students see both subjects as separate disciplines. This is close to the idea that both subjects are viewed as unrelated to each other (Cui, 2006; Karakok, 2009; Roorda, 2012).

Contrary to physics teachers, most of the mathematics teachers mentioned that they do not feel the need to collaborate and cooperate with physics teachers. This may impede the development of common teaching strategies to tackle transfer problems. We noted that lack of perceived urgency to cooperate with physics teachers is not typical for the community of mathematics teachers as a whole. Indeed, many teachers may appreciate the need to promote transfer.

With regard to their views about improving transfer, most interviewees fit into one of the following groups. The first and largest group think that the transfer problem is solved by intensive practice in math class. The second and smallest group believes the opposite: the transfer problem should be tackled by algebraic problems in physics class. Finally, the intermediate group believes in comprehensive algebraic practice in both mathematics and physics class. Conceptual understanding is ignored by all teachers from the first two, extreme groups and by some teachers of the intermediate group. Furthermore, since we conducted qualitative research involving a small number of teachers, these results are not generalizable to the whole Dutch mathematics of physics community teaching in senior pre-university education, let alone to other countries.

Some of the teachers' beliefs can be organized into a belief system (Ernest, 1991), i.e. into a set of mutually supporting beliefs about transfer. Further research should investigate to which extent this is the case and which beliefs they contain. This is explained in study (2) and (3) of the next two Chapters.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Chapter 3

Teachers' Core Beliefs about Improving Transfer[2]

[2] This chapter has been published in adapted form as: Turşucu, S., Spandaw, J., Flipse, S., & de Vries, M. J. (2018). Teachers' core beliefs about improving transfer of algebraic skills from mathematics into physics in senior pre-university education. *Eurasia Journal of Mathematics Science and Technology Education*, *14*(10), em1596.

3.1 Introduction

Teachers have the experience that students encounter difficulties when applying mathematics in physics class (Ivanjek, Susac, Planinic, Andrasevic, & Milin-Sipus, 2016; Karam, 2014; Quinn, 2013). Such transfer problems can be intractable and concern students of all ages, including those in pre-university education (Awodun, Omotade & Adeniyi, 2013; Basson, 2002; Molefe, 2006; Roorda, 2012).

Hiele (1974) was among the first authors to explain scientifically why pre-university students faced problems with applying mathematics, especially algebraic skills to physics. He points out two main causes: mathematics and physics which are taught as two separate school disciplines, and the difference in pedagogical approaches between mathematics and physics teachers. For instance, for the lens formula in geometrical optics a typical physics teacher writes $b^{-1} + v^{-1} = f^{-1}$ $(b \neq 0, v \neq 0)$, while a typical mathematics teacher writes $(b - f)(v - f) = f^2$.

The lack of transfer may also be related to the mismatch between teachers' beliefs and teaching practice[3]. Indeed, beliefs have a major impact on teacher behavior (Ernest, 1991; Pajares, 1993). For instance, when physics teachers *naïvely* (Schoenfeld, 2014) think that extensive practice in mathematics lessons automatically lead to transfer in physics. Since they ignore mathematics in physics class, they may find themselves re-teaching basic mathematics. This may be both frustrating and time-consuming, overshadowing the physics content that needs to be taught (Roorda, Goedhart, & Vos, 2014; Turşucu, Spandaw, Flipse, & de Vries, 2017).

Although teachers' beliefs are relevant for transfer in relation with classroom practice *and* coherence across both subjects (Furner & Kumar, 2007; Berlin & White, 2012, 2014), they are not studied extensively. Therefore, this study is relevant from both educational and scientific point of view.

3.1.1 *Article Aim and Research Question (2)*

This study, i.e. study (2), is a follow-up study that aims at reducing large amounts of coded data (Table 1 in Chapter 1) into a small dataset of core beliefs that contain *constraints* including naïve beliefs that can be harmful for transfer, and *affordances* that can improve transfer. The word 'core' in core belief should not be confused with the stable and unchangeable beliefs (Haney & McArthur, 2002). In contrast to such stable and unchangeable beliefs, these core beliefs are the final product of data reduction of the common code tree including the continuum of teachers' beliefs about transfer of algebraic skills into physics, and coherence across both subjects from the previous qualitative exploratory study (Turşucu, Spandaw, Flipse, & de Vries, 2017). Indeed, rather than focusing on large amounts of data, one may also focus on a single dataset containing major aspects that influence transfer. Conversely, the large amounts of data from study (1) offers insight into the continuum of aspects influencing transfer.

[3] A detailed explanation of teachers' beliefs about transfer in relation with classroom practice can be found in chapter 1.

To reduce data from study (1), we used pattern coding that in many textbooks is described in a general way (e.g., Saldaña, 2013). Contrary to such a general approach, we intended to use this second cycle coding technique in a more systematic and refined manner. For this purpose, we needed to develop a specific approach to further reduce code trees containing large amounts of data. In other words, to further reduce our code tree and study teachers' core beliefs, we needed to develop a systematic and refined method to reduce code trees containing large amounts of coded data, since such an approach was not present in earlier studies. In addition, even if we would have aimed at combining study (2) and study (1) into a single study, it was considered as too large by peer-reviewed international Journals on science education. Furthermore, this study also functions as 'a bridge' between the first and the third study where we aim to extract belief systems about transfer. Indeed, contrary to the common code tree, these core beliefs provide data small enough to extract belief systems including an organized set of mutually supporting core beliefs about CMSE and transfer in one single data reduction step.

In this study we examine the following research questions: (2a) "*How can a systematic, refined method be developed to reduce code trees containing large amounts of data into a single dataset?*", and (2b) *"What are the core beliefs of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?"*.

3.2 Background

3.2.1 Coherent Mathematics and Science Education and Transfer

Mathematics and science are intertwined disciplines (Atiyah, Dijkgraaf, & Hitchin, 2010). Mathematics offers science a formal language in which quantitative relationships can be described, evaluated and predicted. On the other hand, science offers meaning to mathematics through contexts. Education aiming at enhancement of connection between both subjects lies at the heart of coherent mathematics and science education[4] (CMSE), and is of major importance to students (Alink, Asselt, & Braber, 2012; Berlin & White, 2012, 2014).

Connecting mathematics and science, especially physics is possible through alignment such as using compatible notations, concept descriptions, pedagogy of equivalent mathematical methods, and organization of the learning process across time (Alink, Asselt, & Braber, 2012). The latter implies that the required mathematics has already been explained *before* it is used in physics lessons (Roorda, 2012).

Traditionally, the application of mathematics (initial learning) (Bransford, Brown, & Cocking, 2000; Larsen-Freeman, 2013; Singley & Anderson, 1989) into physics (new learning situation) forms the foundation of CMSE. Educational researchers view this approach to transfer as one of the main goals of education (Haskell, 2001). In this framework, the existence of transfer is determined by the expert (teacher) in advance and measured by comparing the learners (student) test answers with that of the experts' correction scheme (Cui, 2006;

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[4] A detailed explanation of CMSE can be found in chapter 1.

Lobato, 2003; Rebello et al., 2007). However, "*little agreement in the scholarly community about the nature of transfer, the extent to which it occurs, and the nature of its underlying mechanisms*" (Barnett & Ceci, 2002, p. 612) led to a shift from traditional to alternative models, such as actor-oriented transfer. As to actor-oriented transfer, transfer is defined as the students' construction of similarities between the initial and new learning situation (Lobato, 2003). The expert tries to understand *how* they are constructed. So, even though the transfer task is designed by the expert (for instance, a teacher), the goal of actor-oriented transfer is "*to elicit activities by a learner, and to investigate if and how students construct similarities between this task* [transfer task] *and earlier activities*" (Roorda et al., 2014, p. 4). Therefore, transfer is observed from the students' point of view.

Furthermore, in this study, we assume a reciprocal relation between CMSE and transfer. When students experience coherence across mathematics and science subjects by means of *meaningful* contexts, transfer from mathematics to science subjects can be improved, and improving transfer can help them to experience coherence between mathematics and subjects.

3.2.2 The Three Classroom Actors

Dutch students start with senior pre-university education in grade 10 (when students are 1 15 or 16 years old) after they finished three years of junior pre-university education. In that year, they should choose between mathematics A and mathematics B. The former puts less attention on algebra than the latter. The content of these subjects is determined by curricula (SLO, 2019) and tested in national final examinations. These curricula shape the textbooks which are closely followed by the teachers and their students (SLO, 2019; van Zanten & van den Heuvel – Panhuizen, 2014). Hence, curricula, teachers and textbooks are the three main actors in Dutch pre-university education.

According to Turşucu et al. (2017), some of the Dutch teachers they interviewed, were not sufficiently aware of the content of curricula. For them textbooks were the curricula. Thus, their beliefs about CMSE and transfer were influenced by the content of textbooks. For instance, a physics teacher who discovers that the method of *how* the algebraic technique substitution (Drijvers, 2011) is explained in the physics textbook is different from that in the mathematics textbook. It might be the case that for some Dutch teachers textbooks are a substitute for curricula. The overloaded science curricula (SLO, 2019) may compel them to use their time very efficiently. As a result, they may only follow their books, rather than examine their curricula, influencing their beliefs about CMSE and transfer.

In this study, the actors students, teachers and textbooks are referred to as the three classroom actors, since they play a major role in classroom practice.

3.2.3 Teachers' Individual Beliefs about CMSE and Transfer

In the previous study we examined mathematics and physics teachers' beliefs about CMSE and transfer (Turşucu, Spandaw, Flipse, & de Vries, 2017), and not the organized teachers' belief systems (Ernest, 1991) containing a set of mutually supporting beliefs. We answered the two sub questions: (1a) "*How do mathematics and physics teachers characterize the transfer problem* *in the case?*" (see Table 1 in the previous Chapter), and (1b) "*What sort of beliefs do mathematics and physics teachers' beliefs have about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*".

In that study, we used convenience sampling to gather data from 10 mathematics and 10 physics teachers who were qualified to teach in senior pre-university education and had at least 5 years of teaching experience. The interviews were conducted by means of a semi-structured questionnaire that was based on one specific *case* containing a transfer problem from mathematics to physics (table 1 in the previous chapter). Afterwards, each interview was transcribed verbatim for analysis.

We used open coding (Bryman, 2012) to label each fragment of the transcripts, which provided a short description of teachers' individual beliefs regarding sub questions (1a) and (1b). For each of the twenty transcripts this led to a set of labels identifying teachers' beliefs. Then, we used axial coding including two steps. First, labels with the same content were put together, resulting in a grouping of the labels. Each group of labels was summarized as a subtheme and included at least three different beliefs uttered by at least three different teachers. If not, it was considered as an outlier. In the subsequent step, we organized 28 subthemes into 9 core themes (coherence, curriculum, education, pedagogy of algebra, relation between scientific subjects, school subjects, teacher, the use of textbooks and transfer), see table 2 in the previous chapter. Hence, we obtained one hierarchical structured common code tree for all 20 teachers, with the core themes as main branches. The latter branches out into subthemes, the smaller branches. The leaves of the tree are the last and finest level of the hierarchy and represent the underlying continuum of approximately 1.300 individual teachers' beliefs about aspects influencing students experiencing coherence across these subjects and aspects influencing transfer.

3.2.4 Findings in a Nutshell

In the previous study, almost all teachers acknowledged the case and considered it important that students are competent at applying algebraic skills in physics (Turşucu et al., 2017).

In line with literature (Karakok, 2009; Dierdorp, Bakker, van Maanen, & Eijkelhof, 2014; Nashon & Nielsen, 2007; Roorda, Goedhart, & Vos, 2014) most teachers believe that transfer did not occur, because of compartmentalized thinking in which students see math and science as unrelated disciplines.

Some mathematics teachers mentioned that they did not feel the need to collaborate with physics teachers. On the other hand, all physics teachers in our sample mentioned that they were willing to work together with mathematics teachers. Even though this can be harmful for transfer in teaching practice, we emphasized that such views on mathematics do not represent the mathematics community, for there can be various ideas about the role of mathematics in science, even mathematicians promoting transfer

Data indicated the existence of two extreme, opposite beliefs about how transfer may be realized. The first one is related to prioritizing basic skills (Wu, 1999) in mathematics class, and the second to reinventing the same mathematical wheel in different physics contexts. An intermediate group thinks that only an integrated approach can solve the transfer problem.

Furthermore, using Ernest's (1991) belief system model[5] (figure 1 in Chapter 1), we explained how teachers' naïve beliefs about transfer (espoused models) influenced by the social context of teaching were transformed into classroom practice that can be harmful for transfer (enacted models). In this respect, the upper rectangle describing 'View nature of mathematics' in figure 1 containing the rectangles about learning and teaching mathematics (espoused models) was considered a belief system with naïve beliefs about transfer and CMSE. The other two (lowest) rows were concerned with teaching practice. We emphasized that the distinction between the espoused and enacted models was necessary, since case studies have shown a great disparity between both models (Cooney, 1985). Through professional development programs (Guskey, 2002) teachers can be made aware of such beliefs, reflect on the mismatches between both models and reconcile them to enhance transfer.

3.2.6 From Individual Beliefs to Core Beliefs

The common code tree (table 2 in the previous chapter) contains about 1300 teachers' beliefs. When a teacher is asked to select, for instance, a top 5 from these beliefs to identify his/her belief system, this would be a very difficult task. In addition, our common code tree is the result of first order coding that according to literature can be further reduced into a much smaller dataset using pattern coding (e.g. Saldaña, 2013), i.e. a second cycle coding technique. For instance, we might go from 1300 beliefs to 20 core beliefs that grasp the essence of the code tree. Selecting a top 5 from such a small dataset is much easier for teachers than from a code tree containing large amounts of data. Therefore, we carried out data reduction on this code tree by using pattern coding. In fact, we further developed pattern coding. This process is described in the section below.

3.3 Methodology

Pattern coding grasps the essence of the common code tree and leaves out less important details. In this section we explain how we used this method in three consecutive data reduction steps D1, D2 and D3 (D = data reduction step).

Different from, for example Gibson & Brown (2009) and Saldaña (2013) who offer general directions and explanations on how to *further* reduce coded data, we worked out their method in detail to reduce the common code tree including the continuum of about 1300 beliefs. We think that our approach to pattern coding is elegant since we used refined and systematic data reduction steps (see figure 2, 3 and 4), and offers a generally applicable second cycle coding tool to further reduce data of (common) code trees containing large amounts of data.

[5] A detailed explanation of Figure 1 can be found in Chapter 1.

3.3.1 D1: Data Reduction Step 1

After the common code tree (table 2 in the previous chapter) was split into the code tree for mathematics and physics teachers, we followed D1 including two sub steps. *Firstly,* we reduced the collected individual beliefs of each subtheme to zero up to seven summarizing beliefs for each teaching group. These summarizing beliefs contain the essence of the individual beliefs for each subtheme. *Secondly*, we grouped the summarizing beliefs of different sub themes belonging to the same core theme. This led to summarizing beliefs for each teacher group and is shown in Figure 2. Herein, the collected beliefs refer to the collected individual teachers' beliefs of table 2.

Figure 2. Data reduction step 1: the forming of summarizing beliefs.

3.3.2 D2: Data Reduction Step 2

As shown in figure 3, in the second data reduction step we combined both previous datasets including summarizing beliefs into one single dataset. Each core theme of the mathematics group was compared to the same core theme of the physics group. Summarizing beliefs that had the same content were grouped to form main beliefs.

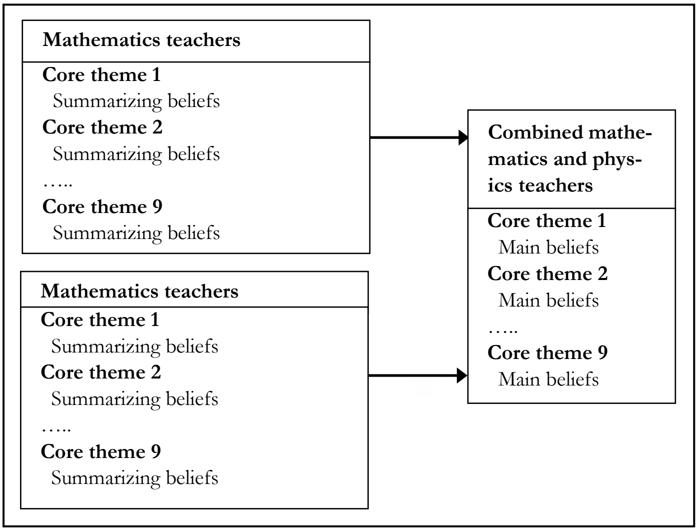


Figure 3. Data reduction step 2: the forming of main beliefs.

3.3.3 D3: Data Reduction Step 3

The last step of pattern coding is shown in figure 4 and concerned the reduction of core themes together with main beliefs into core beliefs. This process consisted of two smaller steps. Firstly, we replaced the nine core themes including main beliefs by three categories. The new categorization differed from the structure of corethemes, and described (1) the causes for the lack of transfer, (2) the effects of the lack of transfer, and (3) the potential solutions from different perspectives. In this regard, the main beliefs of each core theme were attached to one of these categories. *Secondly*, the set of differently categorized main beliefs were reduced into one single set of core beliefs. To this extent, main beliefs with the same content were removed. This led to the remaining beliefs, which are the core beliefs.

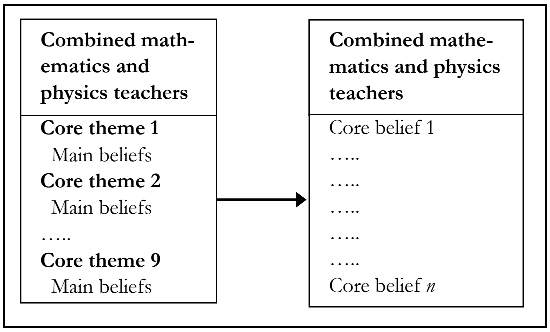


Figure 4. Data reduction step 3: the forming of core beliefs.

During the triangulation process, the steps D1, D2 and D3 were independently carried out by an independent researcher. After each round the first author and the independent researcher crosschecked the results. Each dataset was thoroughly discussed and led to 100% agreement among both researchers.

3.4 Results

The results of this study are described in the subsequent sub sections 'D1: Forming of Summarizing Beliefs', 'D2: Forming of Main Beliefs' and 'D3: Forming of Core Beliefs'. Note that the terms 'summarizing' in summarizing beliefs and 'main' in main beliefs to distinguish between the different phases of the data reduction steps during pattern coding, especially to emphasize that these are necessary steps to find one dataset containing core beliefs. In addition, the terms 'summarizing', 'main' and 'core' were not present in the method of Gibson & Brown (2009) and Saldaña (2013). As stated earlier, in the next study we aim to extract belief systems from this dataset. Contrary to the code tree of the previous study containing large amounts of data, one dataset of core beliefs makes it possible to distill belief systems in a single data reduction step.

3.4.1 D1: Forming of Summarizing Beliefs

The results of the first step of pattern coding (D1) are presented in table 3. The first column 'Core theme' represents the nine core themes that we found in the previous study (table 2 in the previous chapter). The second column 'Summarizing beliefs' including the bold numbers '76/64' refers to the total number of summarizing beliefs. The first number deals with mathematics teachers and the second with physics teachers. This also holds for the other numbers. For instance, the second core theme 'Curriculum' consists of five summarizing beliefs for mathematics and seven summarizing beliefs for physics teachers.

Table 3. Core themes including summarizing beliefs.

Core theme	Summarizing beliefs
	76/64
1. Coherence	11/12
2. Curriculum	5/7
3. Education	2/3
4. Pedagogy of algebra	11/10
5. Scientific subjects	9/6
6. School subjects	6/5
7. Teacher	11/11
8. The use of textbooks	12/8
9. Transfer	14/10

3.4.2 D2: Forming of Main Beliefs

The results of the second step of pattern coding (D2) are presented in table 4.

The second column 'Main beliefs' with bold number '31' combines mathematics and physics teachers' summarizing beliefs and refers to the total number of main beliefs.

Similarly, for example number '5' corresponds to the core theme 'Coherence' and includes five main beliefs.

Table 4. Core themes including main beliefs.

Core theme	Main beliefs
	31
1. Coherence	5
2. Curriculum	2
3. Education	1
4. Pedagogy of algebra	3
5. Scientific subjects	4
6. School subjects	2
7. Teacher	3
8. The use of textbooks	4
9. Transfer	7

3.4.3 D3: Forming of Core Beliefs

The result of the third step of pattern coding (D3) is presented in table 5 and contains the set of sixteen core beliefs. This list consists of beliefs about CMSE and beliefs influencing students' transfer of algebraic skills from mathematics into physics. This dataset was further organized into the five main categories 'Collaboration', 'Curricula', 'Students', 'Teachers' and 'Textbooks'. Each of these contains core beliefs which are related to each other. The first two core beliefs are part of 'Collaboration', core beliefs number '3' up to '6' of 'Curricula', number '7' up to '9' of 'Students', number '10' up to '14' of 'Teachers' and the last two core beliefs belong to the main category 'Textbooks'.

Table 5. Sixteen core beliefs.

Core belief number	List of core beliefs
1	Mathematics teachers often lack time for cooperation
2	There is a lack of collaboration between mathematics and physics teachers
3	Algebraic skills taught in mathematics A do not match sufficiently with physics
4	Mathematics contains less algebra
5	Mathematics should incorporate more physics contexts
6	The physics curriculum should contain manipulation of formulas
7	Transfer can be stimulated if students practice in different physics contexts
8	Transfer is being hindered because students regard mathematics and physics as separate subjects
9	Transfer often will occur spontaneously if students recognize the contexts
10	Both mathematics and physics teachers can stimulate transfer
11	There is no consensus whether mathematics and physics teachers should be able to teach basic mathematics that is needed for transfer
12	Transfer can be stimulated if mathematics and physics teachers agree on the used notations for formulas
13	Transfer can be stimulated if prior knowledge is activated in physics class
14	Transfer can be stimulated if students are taught to see connections between contexts
15	Mathematics and physics teachers stick to the lesson book
16	There is no consensus whether mathematics and physics textbooks should be adapted

3.5 Results Interpretation

In this section we will first interpret the core beliefs. Subsequently, the five main categories are discussed. We finalize with the limitations of this study and make some recommendations for further research.

3.5.1 Core Themes Versus Main Categories

In short, through pattern coding (Saldaña, 2013) the nine core themes in table 2 in the previous chapter were further reduced into the five main categories in table 5. This means that the main categories somehow are related to the nine core themes. Indeed, this was the case. The main category *'Collaboration'* corresponds to the core theme 'Coherence (core theme 1, or 1 in short)' in table 2 in Chapter 2. In an analogous manner, *'Curricula'* corresponds to 'Curriculum (2)', 'Education (3)' and 'School subjects (6)'; *'Students'* to 'Pedagogy of algebra (4)' and 'Transfer (9)'; *'Teachers'* to 'Relation between scientific subjects (5)' and 'Teacher (7)'; *'Textbooks'* to 'The use of textbooks (8)' (Turşucu et al., 2017). These correspondences may seem a bit rough, since some of the main categories slightly overlap with the same core theme. Nevertheless, we conclude that these matches are reasonable. For instance, core belief number seven of the main category *'Students'* corresponds to individual teachers' beliefs such as more practice with different physics problems improves transfer (subtheme 'Practice within physics' of the core theme 'Pedagogy of algebra' (4)). Core belief number '8' matches with, for example the individual teachers' belief that mathematics class ends when the student enters the physics class (subtheme 'Constructing relations (general constraints)' of 'Transfer' (9)).

3.5.2 Loss of Information

The main difference between table 2 in the previous chapter and table 5 concerns the information density: the continuum of circa 1300 individual teachers' beliefs were condensed into sixteen core beliefs. Therefore, some of the detailed information in table 2 in chapter 2 about CMSE and transfer of algebraic skills in physics were lost. For instance, core belief number 6 'The physics curriculum should contain manipulation of formulas' does not offer any information about the sort manipulation of formulas. In contrast, table 2 in chapter 2 includes detailed information such as practice with transfer problems analogous to the case. Another example is core belief number 10 'Both mathematics and physics teachers can stimulate transfer'. While the latter does not explicitly describe *how* to stimulate transfer, table 2 includes detailed information, for example physics teachers who should write down the mathematics equation $y = bx^2$ next to the corresponding physics formula $s = \frac{1}{2}at^2$ in physics class.

3.5.3 Main Category '1': Collaboration

The first two core beliefs refer to a lack of collaboration between mathematics and physics teachers. The majority of mathematics teachers (as opposed to physics teachers) said that they did not feel the need to collaborate with physics teachers (Turşucu, Spandaw, Flipse, & de Vries, 2017). They believe that physics teachers encounter problems and should contact them [mathematics teachers] for solutions to transfer problems (espoused models). Accordingly, there are no meetings between both departments (enacted models). See figure 1 of chapter 1 for both models. The only interactions between them concern individual efforts on a small scale during informal meetings. The lack of time and a huge workload among mathematics teachers were regarded as impeding factors, which supports the first core belief. The lack of perceived urgency to cooperate with physics teachers is not typical for the community of mathematics teachers as a whole. Indeed, many teachers may appreciate the need to promote transfer.

On the other hand, a small number of mathematics teachers already was working together with physics teachers. They emphasized the importance of aligning both subjects across time. In this way, certain mathematical concepts were not introduced in mathematics class before they were used in physics class (Alink et al., 2012; Roorda, 2012).

We concluded that for teaching practice aiming at CMSE (Berlin & White, 2012, 2014; Davison, Miller, & Metheny, 1995), and improving students' transfer of algebraic skills from mathematics in physics, a shift in mathematics teachers' espoused beliefs in figure 1 of Chapter 1 (Ernest, 1991) towards more openness to collaboration was needed (enacted beliefs). This involves the development of common pedagogical strategies such as alignment of notations, formulas and the application of algebraic skills in both subjects.

3.5.4 Main Category '2': Curricula

Core beliefs number '3' up to '6' refer to beliefs about the content of mathematics and physics subjects and thus belong to the main category 'Curricula'. Number '3' follows from the belief that the algebra involved in mathematics A is insufficient for that needed in physics class. Indeed, we had already seen this in the previous study (Turşucu, Spandaw, Flipse, & de Vries, 2017). Extensive quantitative research is required to determine whether this belief is widely shared among Dutch mathematics and physics teachers. Even if the latter would be the case, it does not imply that mathematics A is insufficient for physics. Moreover, both mathematics A and mathematics B curricula are designed in such a way that the algebra involved in these subjects should be sufficient for physics (SLO, 2019).

As to number '4', both teacher groups claimed that mathematics contains less algebra. So, besides mathematics A, mathematics B also seems to lack sufficient algebra for physics students. This result seemed to contradict the belief above that *only* mathematics A does not contain algebra. Nevertheless, from the previous study we know that most of the mathematics and physics teachers wish to see that the content standards of both subjects should include the application of algebraic skills (e.g., manipulating formulas and solving for a variable) in physics contexts. This desire is close to core belief number '5'. Furthermore, the mismatch between number '3' and '4' may be related to teachers' contradicting opinions between the first and the second half of the interviews.

We note that too much focus on physics contexts on algebra lessons may be harmful for transfer Bransford, Brown, and Cocking (2000). Curriculum designers aiming at CMSE and transfer need to take this issue into consideration. They should focus on an 'integrated' approach in which algebraic skills are mainly taught algebraically with some applications in physics context. Such an approach may be visible through explicit descriptions of different aspects of algebraic skills in the content standards, for example analogous explanation of an algebraic method in both curricula.

Both groups believed that the actual physics curriculum should contain descriptions about manipulation of formulas (number '6'). Nevertheless, the current physics curriculum already includes explicit descriptions about manipulation of formulas. This emphasizes our earlier findings on physics teachers who were unaware of the actual physics curriculum (Turşucu, Spandaw, Flipse, & de Vries, 2017). Although mathematics teachers are not directly involved in physics class, some of them shared this belief. For Dutch teachers, who usually quite strictly follow their textbooks (SLO, 2019) and faithfully think that textbooks reflect the intended curriculum (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002), textbooks *were* the curriculum. Therefore, we regard core belief number '6' as naïve (Schoenfeld, 2014).

We think that transfer can be improved when designers of mathematics curricula and those engaged in physics curricula put effort into collaboration aiming at pedagogy of equivalent mathematical methods regarding the application of algebraic skills. For instance, using algebraic techniques (Drijvers, 2012) in the same way. Design principles should also focus on organization of the learning process across time (Alink et al., 2012).

Furthermore, for two main reasons it is probably better that the alignment above does not lead to attempts to integrate both subjects. Firstly, mathematics has a serving role which is not restricted to physics, but also includes subjects such biology and chemistry. Secondly, it has an intrinsic unifying role: similar expressions and formulas used in different contexts outside mathematics can be reduced to the same abstract mathematics (Atiyah et al., 2010).

3.5.5 Main Category '3': Students

Core belief number '7' is related to basic algebraic skills first: thoroughgoing practice in physics class should automatically lead to transfer. However, earlier studies have shown that successful execution of basic skills in school mathematics also involves conceptual understanding (e.g., Kilpatrick, Swafford & Findell, 2001; Wu, 1999). Not considering the latter may lead to routine based on 'tricks', thereby impeding transfer (Drijvers, 2011; Roorda, 2012; Turşucu, Spandaw, Flipse, & de Vries, 2017). Thus, this belief is identified as naïve. The same argument holds for extensive practice in mathematic class with algebraic skills. To improve transfer, both basic skills and conceptual understanding should be taught in an integrated manner: "*Advocates of insightful learning are often accused of being soft on training. Rather than against training, my objection to drill is that it endangers retention of insight. There is, however a way of training - including memorisation - where every little step adds something to the treasure of insight: training integrated with insightful learning.*" (Freudenthal, 1991).

We think that practicing algebraic skills should happen in both classes with emphasis on conceptual understanding of the underlying mathematics behind these skills. Concrete indications are provided in table 1 in the previous chapter. In physics class students should practice with formulas analogous to $s = \frac{1}{2}at^2$, and in mathematics class with equations analogous to $y = bx^2$.

Core belief number '8' is in line with earlier literature on compartmentalized thinking in which students saw math and physics as unrelated subjects (Dierdorp et al., 2014; Osborne, 2013). The quote "*Math is for math class*" (Nashon & Nielsen, 2007, p. 97) summarizes our finding. This may be reinforced by the fact that in the Netherlands (and in many other countries) both subjects are taught as separate disciplines (Roorda, 2012). Reducing this mental wall could be possible through coherent mathematics and science education (CMSE) (Berlin & White, 2014), which aims at fostering connections between mathematics and science education. Indeed, similar to STEM education (e.g., van Breukelen, 2017; National Science and Technology Council, 2013; 'TIMMS & PIRLS', 2019), mathematics lies at the heart of the CMSE approach. Beyond CMSEs unifying role, it may also provide meaningful contexts to the other disciplines.

As to core belief number '9', recognition of the same algebraic structure in a mathematics equation and a physics formula does not necessarily lead to transfer, but could be an essential precondition for a strategy in which algebraic skills are applied successfully. Therefore, we regard core belief number '9' as naïve. As stated earlier, naïve beliefs (espoused models) are harmful for transfer, since they do not match with beliefs needed for classroom practice that can enhance transfer (enacted models) (Schoenfeld, 2014). Teacher educators who are well-informed about both models could use figure 1 of Chapter 1 to make teachers aware of their naïve beliefs and make them reflect on the mismatches between both models and reconcile them to improve transfer. Otherwise, because of the socialization effect of teaching, teachers are often observed to stick to the same ineffective teaching practice (Cooney, 1985).

3.5.6 Main Category '4': Teachers

Core belief numbers '10', '12', '13' and '14' confirm earlier research on the crucial role of teachers regarding transfer (Alink et al., 2012; Quinn, 2013). In addition to the aforementioned collaboration between both departments (mesoscopic level in school), individual efforts of mathematics and physics teachers (microscopic level in school) in respectively mathematics and physics classes could enhance students seeing connections between both subjects (number '14'). As to physics teachers, it is probably better that activation of prior mathematical knowledge in physics class is concerned with mathematics involved in physics problems (number '13'). For instance, physics teachers could write $y = b \cdot x^2$ next to $s = \frac{1}{2} \cdot a \cdot t^2$, and solving for x in $y = b \cdot x^2$ next to solving for t in $s = \frac{1}{2} \cdot a \cdot t^2$. Physics teachers could also develop their own teaching materials in which such examples are elucidated in detail. Similar arguments hold for mathematics teachers, for example mentioning that the equation of a parabola $y = b \cdot x^2$ has the same mathematical structure as the distance formula $s = \frac{1}{2} \cdot a \cdot t^2$in physics. Such interventions are small enough to be easily adopted by mathematics teachers, and provide context and meaning for the formal language of mathematics (Dierdorp et al., 2014).

The belief that transfer is stimulated when there is agreement on the used notations for formulas (number '12') is in line with earlier literature (e.g., Roorda, 2012; Quinn, 2013). A key example concerns that formulas used in physics class should be equivalent to those used in mathematics class.

Remarkably, only half of both math and physics teachers are united behind the statement that both teacher groups should be able to teach basic mathematics (number '11'). This belief seems to indicate that half of the mathematics and physics teachers think that students' transfer is independent of whether mathematics and physics teachers possess a solid basis of mathematics. This belief is quite astonishing. Indeed, if teachers have not mastered basic algebraic skills, then probably many of their students also lack these skills. This makes transfer very difficult to occur. It is very likely that teachers lacking basic mathematics, also lack sufficient basic algebraic skills. Therefore, core belief number '11' simply overlooks the fact that both mathematics and physics teachers should be sufficiently knowledgeable in explaining basic mathematics. Therefore, we regard core belief number '11' as naïve. We also think that this belief can be more harmful to transfer than the former naïve beliefs.

Furthermore, the belief above follows from core theme '5' (table 2 in the previous chapter). The relationship between mathematics and physics is extremely strong and goes back thousands of years (Atiyah, Dijkgraaf, & Hitchin, 2010); for example, Galileo mentioned that the book of nature is written in the language of mathematics. We think that even such historical facts brings responsibilities for physics teachers: they should be able to teach basic mathematics.

We conclude that there are many concrete things that individual mathematics and physics teachers could do to connect both subjects. In most cases these concern small interventions, feasible for teachers. Even mentioning that math and physics are not unrelated subjects, but closely related to each other may contribute to students' transfer and also enhance students experiencing coherence between these subjects.

3.5.7 Main Category '5': Textbooks

In line with earlier research (SLO, 2019; van den Heuvel – Panhuizen & Wijers, 2005; van Zanten & van den Heuvel – Panhuizen, 2014) core belief number '15' confirms that Dutch teachers are highly textbook-driven and teach these to their students. In short, to a large extent the content of mathematics and physics textbooks shape what students learn. But what if these textbooks contain mismatches on how algebraic skills are learned?

Algebraic techniques are part of the machinery of algebraic skills and pivotal in algebraic manipulation of formulas. Hence, the question "*To what extent do differences in pedagogical methods to how algebraic techniques are treated in mathematics and physics textbook series affect students solving algebraic physics problems where these techniques are needed?*" is worthwhile to investigate in a new study. It could give insight into the underlying mechanisms that affect students' application of these techniques in such physics problems and provide design principles about how pedagogical methods could be used in curricula and textbooks to improve transfer.

Although there is hardly any scientific research examining alignment in textbooks between these subjects, there have been talks between mathematics and physics textbook publishers on this matter (Alink et al., 2012). Textbook publishers mentioned that Dutch teachers follow textbooks faithfully and teach them to their students (confirming earlier studies above). They think that textbooks focusing on alignment could strengthen students experiencing coherence across these subjects. We believe that it is probably better that curriculum designers take this matter into account, since curricula determine the content of textbooks.

In practice, however, even if alignment should have been explicitly described in these curricula (ideal scenario), the idea to develop textbook series for this purpose remains difficult. Probably, the main reasons are twofold: mathematics and physics textbook publishers working separately, and the absence of learning lines aiming at coherence across both subjects (Alink et al, 2012).

Furthermore, among the respondents there was no consensus whether the content of mathematics and physics textbooks should be adapted (number '16'). This may imply that some mathematics and physics teachers believed that there is no need to adapt the content of math and physics textbooks. However, Alink et al. (2012) mention that during various consultations with teachers, the teachers frequently asked themselves why actual textbooks did not pay attention to alignment. Therefore, we regard core belief number '16' as naïve. This result also may implies that our 'what if' question above about potential pedagogical mismatches in these textbooks was legitimate.

We conclude that alignment of both subjects is crucial for CMSE and transfer (e.g., Konijnenberg, Paus, Pieters, Rijke & Sonneveld, 2015; Mooldijk & Sonneveld, 2010). This includes using compatible pedagogical strategies to teach algebraic methods, using compatible notations, compatible concept descriptions, and especially the organization of the learning process across time in which mathematics had already been explained *before* it was used in physics class. We think that textbook publishers should take these issues into account.

3.5.8 Limitations of This Study and Recommendations

The core beliefs obtained in this study were extracted from the common code tree (table 2 in the previous chapter). The latter resulted from open and axial coding (Bryman, 2012) of transcripts of twenty interviews with ten mathematics and ten physics teachers who were qualified to teach in senior pre-university education (Turşucu, Spandaw, Flipse, & de Vries, 2017). They were selected from different regular Dutch schools in urban, rural and sub-urban areas within a radius of ± 50 kilometres. Each teaching group consisted of eight male and two female teachers, being in good agreement with the gender ratio in Dutch senior pre-university education (Mullis et al., 2009). They had varying teaching experience ranging from five to forty years. Therefore, we think that our sample may be representative for Dutch teachers in senior pre-university education.

The common code tree in table 2 in the previous chapter contained teachers' beliefs that were saturated, because we did not see much change in the diversity of teachers' individual beliefs after a total of eight interviews in both teach groups. Since core beliefs follow from this common code tree, they can also be regarded as saturated.

Based on the sample properties together with saturation of core beliefs, the results of this study may be generalizable for the major part of mathematics and physics teachers teaching in senior pre-university education in the Netherlands. This also holds for teachers in general secondary education, but not for prepatory vocational secondary education. In PVSE, the mathematics needed in physics are different from those than in senior pre-university education and general pre-university education (SLO, 2019). This may lead to different beliefs.

In the Netherlands the content of subjects is mainly determined by the national final examination (e.g., SLO, 2019; 'TIMMS & PIRLS', 2019), shaping to a very large extent the content of textbooks and teachers who quite strictly follow these and teach to their students (SLO, 2019; van Zanten & van den Heuvel – Panhuizen, 2014). Consequently, textbooks influence teachers' beliefs about CMSE and transfer. In many countries however, such combination of final examinations and textbook-driven teachers does not exist (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Therefore, we do not expect that our results are generalizable to other countries outside the Netherlands. Although (to our knowledge) there are no other studies teachers' beliefs about transfer of mathematics in physics, the latter (non-generalizable) result does not trivialize the fact that teachers do observe students experiencing difficulties in physics class (e.g., Basson, 2002; Ivanjek et al., 2016; Karam, 2014; Quinn, 2013; Roorda, 2012). In this sense, our study shares the finding that teachers acknowledged students encountering algebraic difficulties in physics, and even mention the importance of being competent at it. Other studies in which algebraic skills play a more profound role confirm these findings (e.g., Awodun et al., 2013; Bolton & Ross, 1997; Hameed, Metwally, Al Shaya, & Abdo, 2015; Hudson & McIntire, 1977). For instance, in a study examining the mathematics performance among 120 senior pre-university physics students in physics class (four different schools in the North-West Province of South Africa), the results have shown a very poor level of application of algebraic skills (Molefe, 2006). These students tend to treat mathematics and physics as two unrelated subjects. Moreover, the individual studies above are in line with large scale international studies in which there is a clear decline in students' achievements in physics related to insufficient mathematical competence in a number of different countries (e.g., Mullis et al., 2016; Nilsen et al., 2013).

Within the triangulation process, some of the core beliefs have been removed because these were regarded as outliers. This was the case when less than three different teachers uttered a summarizing belief less than three times. To some degree this measure is arbitrary. But what if outliers contain important information about missing core beliefs such as the integration of the mathematics and physics curriculum or the textbooks. We recommend to further investigate this matter.

Furthermore, we recommend identifying teachers who stated that both mathematics and physics teachers are not required to be sufficiently knowledgeable to teach basic mathematics (number '11'). Among all naïve beliefs, this can be the most harmful for transfer. Conducting in depth-interviews (Bryman, 2015) with those teachers may provide insight into why they have such harmful beliefs for transfer. Similar to aforementioned cases, we recommend these teachers to take part in professional teaching programs, since basic mathematics lies at the heart of transfer. Teacher educators could pay attention on their espoused and enacted models (see figure 1 of Chapter 1) to change mathematics and physics teachers' naïve belief in one that may improve transfer in classroom practice.

3.6 Conclusion

To answer research question (2a) "*How can a systematic, refined method be developed to reduce code trees containing large amounts of data into a single dataset?*", we further developed the second cycle coding technique 'pattern coding' that is described by Gibson and Brown (2009) and Saldaña (2013). Whereas they briefly described general methods to further reduce coded data, we indeed developed a systematic and refined method. This process is described in the figures 2, 3 and 4 in the Methodology section. We think that our approach to pattern coding is generally applicable to further reduce large amounts of coded data.

Regarding research question (2b) "*What are the core beliefs of mathematics and physics teachers about students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*", we found one dataset containing sixteen core beliefs containing constraints and affordances influencing both transfer and students experiencing coherence across these subjects. In short, the common code tree in table 2 in the previous chapter including the continuum of teachers' beliefs was further reduced into sixteen core beliefs. These core beliefs provided new insight into the individual teachers' beliefs, not found in the previous study. They were grouped into five main categories, i.e. 'Collaboration' (number '1' and '2'), 'Curricula' (number '3' up to '6'), 'Students' (number '7', '8' and '9'), 'Teachers' (number '10', '11' and '12') and 'Textbooks' (number '15' and '16'). So, to enhance students experiencing coherence across these subjects, and solve the transfer problem in the *case*, one needs to focus on these categories.

Core belief numbers '6', '7', '9', and '16' concerned naïve beliefs (espoused model) which may stand in the way of both transfer and students experiencing coherence across these subjects (enacted model). Through professional development programs teachers with such beliefs could be made aware of their transfer impeding naïve beliefs, reflect on them and reconcile their espoused and enacted models (see figure 1 of Chapter 1). This may enhance students experiencing coherence across these subjects and transfer.

We have seen that 'Collaboration' between both departments was of major importance to tackle transfer problems, thereby confirming earlier studies.

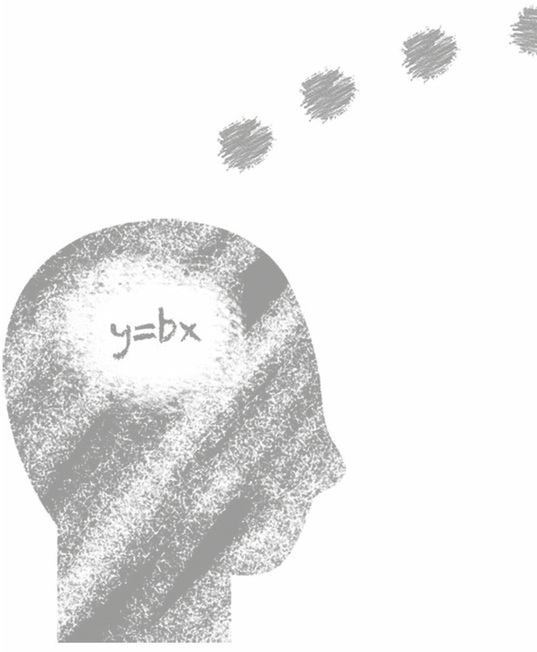
As to 'Curricula', teachers believed that both mathematics A and B lack sufficient algebra for physics. This result seems to contradict the belief that only mathematics A is insufficient for physics.

With respect to 'Students', teachers claimed that the lack of transfer is due to compartmentalized thinking in which students see mathematics and physics as unrelated subjects. Again, this finding confirms earlier research. To improve transfer, it is probably better that both basic skills and insightful learning are be taught in an integrated manner.

'Textbooks' should be designed in such a way that last-mentioned integration is taken into consideration. This may enhance both transfer and students experiencing coherence across these subjects.

Contrary to the common code tree, these core beliefs provided data small enough to extract belief systems including an organized set of mutually supporting core beliefs about CMSE and transfer in one single data reduction step. Quantitative research could investigate to which extent this is the case and which core beliefs these belief systems contain. This is explained in study (3) of the next Chapter.

Furthermore, based on the results above and the ease with which we further reduced coded data, we concluded that our approach to pattern coding was very useful.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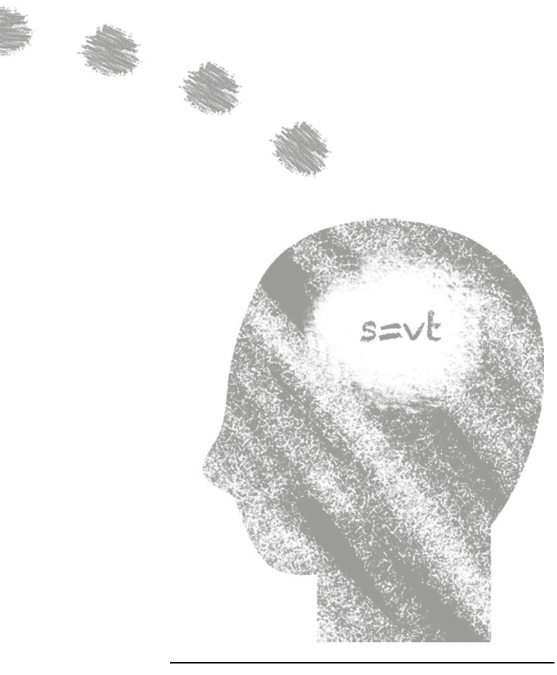
Chapter 4

Teachers' Belief Systems about Improving Transfer[6]



[6] This chapter has been published in adapted form as: Turşucu, S., Spandaw, J., Flipse, S., Jongbloed, G., & de Vries M. J. (2018). Teachers' belief systems about improving transfer of algebraic skills from mathematics into physics in senior pre-university education. *International Journal of Science Education*, *40*(12), 1-27.

4.1 Introduction

4.1.1 Complex Transfer Phenomenon

The application of mathematics in science is of major importance for students (Dierdorp, Bakker, van Maanen, & Eijkelhof, 2014; Quinn, 2013). However, research has shown that students encounter difficulties in science problems where mathematics is needed (Bagno, Berger & Eylon, 2008; Cui, 2006; NCTM, 2000; Karakok, 2009; Rebello et al., 2007; Roorda, 2014). For instance, Roorda (2012) mentions that his pre-university students who were taught differentiation in mathematics class, did not recognize in physics that the speed formula $v_t = v_0 + gt$ is the derivative of the displacement formula $x_t = x_0 + v_0 t + \frac{1}{2} g t^2$.

Even if students are successful in mathematics class, transfer to science class is not guaranteed (Karam, 2014). The study of Rebello et al. (2007) with students enrolled in an algebra-based physics course after they had taken a trigonometry course, shows that even though their mathematics knowledge was sufficient, they encountered difficulties with mathematics in physics which was related to their inappropriate application of mathematics to physics problems. This makes transfer a complex phenomenon which needs to be further researched.

4.1.2 Why Students are not Capable of Transferring Mathematics to Science Class

An important reason for the lack of transfer is related to compartmentalized thinking (Gellish et al., 2007) in which students view mathematics and science as two unrelated subjects. This phenomenon is reinforced since mathematics and science in secondary education and at college level are taught as separate school disciplines. Overcoming this mental wall may be possible by means of coherent mathematics and science education (CMSE) (Berlin & White, 2012, 2014). CMSE[7] aims at education that fosters connections between mathematics and science subjects.

The absence of transfer may also be related to the mismatch between teachers' beliefs[8]. Indeed, beliefs strongly affect teacher behavior (Borg, 2015). For instance, mathematics and physics teachers who may *naïvely* think that transfer occurs automatically when students are extensively trained in mathematics class in the application of basic algebraic skills. In practice, however, they spend time on re-teaching mathematics in physics class, precious time they would rather spend on teaching physics, leaving less time for their core business of teaching physics (Turşucu et al., 2017; Roorda, 2012). Thus, naïve beliefs are harmful for transfer. In addition, in many countries' science curricula are overloaded, compelling teachers to fit their curriculum into a seriously reduced instruction time (e.g., Lyons, 2006). This lack of time makes inefficient transfer of algebra even more harmful.

[7] A detailed explanation of CMSE can be found in chapter 1.

[8] A detailed explanation of teachers' beliefs about transfer in relation with classroom practice can be found in chapter 1.

4.1.3 *Article Aim and Research Question (3)*

There is a large body of research on transfer focusing on the effect of initial learning on new learning (Karam, 2014; Potgieter, Harding & Engelbrecht, 2008; Rebello et al., 2007; Roorda, Goedhart, & Vos, 2014). This study extends the latter, but is different in the sense that it examines *belief systems* containing an organized set of mutually supportive beliefs (Borg, 2015; Mansour, 2009) about improving transfer of algebraic skills from mathematics into physics. Thus, we do not investigate *individual beliefs*. We especially examine belief systems consisting of *desirable* (Leathem, 2006) and *naïve* beliefs (Schoenfeld, 2014) about CMSE and transfer. Our extended definition of desirable (originally, beliefs that science teacher educators want teachers to hold) refers to beliefs required for transfer enhancing teaching practice. This can be collaboration between the mathematics and physics department on using the same pedagogy in teaching mathematics (e.g., Alink, Asselt & Braber, 2012; Roorda, 2012; Quinn, 2013), for example the application of the algebraic methods.

The relationship between belief systems containing naïve beliefs and transfer is highly under researched. In fact, our extensive literature research yielded only our previous studies on both individual beliefs and core beliefs about transfer of *algebraic skills* from mathematics in physics (Turşucu, Spandaw, Flipse, & de Vries, 2017; Turşucu, Spandaw, Flipse, & de Vries, 2018a).

Better understanding of this relationship is relevant from both an educational and a scientific point of view. As to science education, it may provide relevant information for the mathematics and physics curricula, teachers, textbooks and science teacher educators aiming at improvement of transfer and students experiencing coherence across these subjects. Regarding the first, there is the possibility to connect the physics curriculum through content standards to the mathematics curriculum, for example, by means of compatible notations. Consequently, these standards can be used as design principles for physics textbooks. For instance, mathematics textbooks providing physics context, or introduction paragraphs in physics textbooks in which equations and corresponding physics formulas are treated together (activation of prior mathematical knowledge). With regard to teachers, we refer to what both mathematics and physics teachers can do in their classrooms individually, and also to collaboration between both departments. Science teacher educators can use professional teaching programs (Guskey, 2002) to make science teachers aware of their belief systems, reflect on them and change naïve beliefs such as automatic transfer above into desirable beliefs about transfer. Otherwise, because of the powerful socialization effect in school, teachers are often observed to stick to the same ineffective teaching (Brown & McNamara, 2011).

Concerning scientific relevance, this study could contribute to the evaluation of a *microscopic* (Misfeldt, Jankvist & Aguilar, 2016) and a *macroscopic* (Ernest, 1991) belief system model. The former model has its roots in cognitive psychology and deals with individual beliefs, and how they are related to each other in the structure of a belief system. The latter is concerned with how the social context of teaching (e.g., students, teachers and textbooks) influences a teachers' belief system. In the next section we will explain these models and their relation in detail. This explains why we examined *belief systems*, rather than *individual beliefs*.

This paper reports the findings of a quantitative exploratory study on teachers' belief systems containing possibly naïve beliefs about improving transfer of algebraic skills to physics. The research question (3) is "*What are the belief systems of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*".

4.2 Background

4.2.1 *Transfer as The Backbone of Coherent Mathematics and Science Education*

Whereas mathematics can be viewed as the formal language of physics (Kjeldsen & Lützen, 2015), physics provides context and meaning for mathematics (Dierdorp, Bakker, van Maanen, & Eijkelhof, 2014). Connection of both disciplines is possible through alignment, for example, similar concept descriptions, pedagogy of algebraic techniques and organization of the learning process. The latter concerns organization of the learning process to achieve a logical learning line across both subjects. In practice, however, certain mathematical concepts are taught in physics class *before* these were explained in math (Alink, Asselt & Braber, 2012). Fostering alignment between both subjects may foster both transfer of mathematics into physics and students' experiencing coherence across these subjects.

Transfer forms the backbone of the CMSE-approach and refers traditionally to the application of initial learning in a new learning situation (Lobato, Rhodehamel & Hohensee, 2012), and is regarded as one of the main goals of education (Mestre, 2006). The expert (teacher) determines whether transfer happens or not by checking the learners' (student) test answers to the expert's correction scheme, implying a binary outcome: transfer exists or not. A key example concerns assessment of students' solving for a variable in a formula, for example solving for the mass m in the formula for kinetic energy $E_k = \frac{1}{2} \cdot m \cdot v^2$. Each solution that does not match with $m = \sqrt{\frac{2 \cdot E_k}{m}}$ of the correction scheme is regarded incorrect, and therefore associated with the absence of transfer.

Furthermore, in this study, we assume a reciprocal relation between CMSE and transfer. When students experience coherence across mathematics and science subjects by means of *meaningful* contexts, transfer from mathematics to science subjects can be improved, and improving transfer can help them to experience coherence between mathematics and subjects.

4.2.2 *Microscopic Belief System Model*

Beliefs are instrumental in the definition of tasks and selection of cognitive tools, which are used to interpret, plan and make decisions concerning tasks (Borg, 2015). They play an essential role in organizing knowledge and information, i.e. they help individuals to define the world and themselves. Therefore, beliefs have a major impact on teacher behavior.

And what about changes in teacher behavior? They are attributed to the structure of a belief system, rather than to individual beliefs (Leatham, 2006; Misfeldt, Jankvist & Aguilar, 2016). Such structures contain certain collections of mutually supportive beliefs with both varying *centrality* and *psychological strength*. Beliefs in the center of the system are psychologically strong, and strongly connected to many other central beliefs, making them hard to change. Other beliefs, i.e. the more peripheral weak beliefs are founded and derived upon these central beliefs. The terms 'central' and 'peripheral' can be visualised as a group of concentric circles with varying radii (Singletary, 2012). While the inner-most circles represent the strong central beliefs, the outer circles concern the weak peripheral beliefs.

Both 'central' and 'peripheral' beliefs can be desirable or naïve. While the former improves transfer in physics lessons, the latter is harmful. It can occur that the central area of a teachers' belief system contains (1) a set of only desirable beliefs, (2) a set of only naïve beliefs, or a (3) 'mixed state' with a combination of desirable and naïve beliefs about transfer in the central area. Another picture is that of a 'mixed state' (4) distributed over the central-peripheral area. The first scenario is ideal, since all beliefs form a strong, coherent set of mutually supporting beliefs. Teachers with such beliefs are likely to improve students' transfer in the classroom. The second scenario contains only harmful beliefs. Changing such strong beliefs would be very difficult. This makes it a least ideal scenario. We expect that our extracted belief systems belong to scenario (4).

This picture with strong and weak beliefs resembles that of an atom having a structure with a nucleus holding together various particles into a stable system. Particles in the nucleus can be regarded as the connected central and psychologically strong beliefs. The other particles which are further from the nucleus and less tightly bound to the system, correspond to the peripheral weak beliefs. Hence, the belief system model above with varying centrality and psychological strength of beliefs can be regarded as a microscopic system.

4.2.3 Macroscopic Belief System Model

The upper rectangle in figure 1[9] of Chapter 1 represents a teachers' belief system (view) about the nature of mathematics (Ernest, 1991) containing the teachers' espoused (mental) models of teaching and learning mathematics. Influenced through constraints and affordances of the social context of teaching, these mental models are transformed (see downward arrows) into teaching practice (enacted models). In short, the upper two rows of rectangles are concerned with a teachers' belief system, and the other rectangles with teaching practice. The distinction between espoused and enacted is essential, since case studies have shown that there can be a great disparity between teacher's espoused and enacted models (e.g., Brown & McNamara, 2011; Lloyd, Veal, & Howell, 2016).

In this study, we investigate belief systems including desirable and naïve beliefs. Hence, the upper rectangle in figure 1 of Chapter 1 with underlying espoused models represents a teachers' belief system with desirable and naïve beliefs about improving transfer. Contrary to the microscopic model above, figure 1 of Chapter 1 can be regarded as macroscopic. We think that a macroscopic view is needed to understand and explain how, for example teachers and textbooks in the social context of teaching interact and influence transfer via espoused and enacted models. A microscopic view puts a lens on these espoused models, providing

[9] A detailed explanation of Figure 1 can be found in Chapter 1.

detailed understanding of how various central and peripheral beliefs of a belief system are related with each other.

What do both cognitive microscopic and macroscopic models concretely mean for classroom practice, for instance considering scenario 4? In case of the physics teacher strongly convinced about automatic transfer, he also might have weak a belief since, for example he is not that sure whether mathematics teachers providing physics context in mathematics class improves transfer (espoused models). In practice, however, he observes the opposite: students facing difficulties with mathematics in physics class, and the physics context contributing to enhancement of transfer (enacted models). Although there is a discrepancy between both models, this does not necessarily imply that the physics teacher will reflect on his espoused and enacted models and reconcile them to improve transfer. For that to happen, he needs to change his *strong* and *weak* beliefs rooted in respectively the *central* and *peripheral* area of his belief system. In contrast to his weak belief, his strong belief will resist heavily to change since it is connected to other neighbouring strong beliefs about transfer.

4.2.4 Curricula, Teachers and Textbooks in Secondary Education

Dutch students start with pre-university education consisting of two phases at the age of 12 (grade 7). The first phase involves three years of junior pre-university education junior pre-university education, and the second phase three years of senior pre-university education. Junior pre-university education offers a variety of subjects accessible for all students. Based on their interests and talents, students in senior pre-university education should choose between one of the four profiles 'culture and society', 'economics and society', 'science and health' or 'science and technology' preparing them for university education. Physics students of the last cluster should choose mathematics B, and those of 'science and health' mathematics A or mathematics B. Mathematics B puts more emphasis on algebra than mathematics A (Dutch Institute for Curriculum Development, 2019). The content of these subjects is determined by curricula (the intended curriculum) through core goals.

Textbooks (potential curriculum) mediate between these curricula and the actual teaching in classrooms (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002). The limited description of the core goals leaves publishers room for different interpretations in textbooks. Teachers strictly follow these textbooks and teach these to their students (SLO, 2019; van Zanten, M. & van den Heuvel - Panhuizen, 2014). Many teachers are unaware of curricula (e.g., Turşucu, Spandaw, Flipse, & de Vries, 2017): for them textbooks *are* the curriculum.

This study examines belief systems with naïve beliefs about transfer by focusing on the key players in senior pre-university education, i.e. curricula, teachers and textbooks (Alink, Asselt & Braber, 2012). Science teachers, for example, were not considered as key players in upper secondary education. Their role can be pivotal during science teacher education programmes leading to a teaching qualification. This may also be the case for the short professional educational programmes. But when these teachers finish their education and are 'in service', then their teaching is directly influenced by curricula, teachers and textbooks.

4.2.5 Ernest's Educational Model

In his theoretical-educational model Ernest (1991) combines the philosophy of mathematics, theories of intellectual and ethical development, and the sociological-historical theory into five different belief systems, each representing a social group: 'Industrial Trainers' ('IT' in short), 'Old Humanists' ('OH'), 'Progressive Educators' ('PrE'), 'Public Educators' ('PuE') and 'Technological Pragmatists' ('TP'). These groups have different views (beliefs) about twelve different themes such as 'View of mathematics', 'Theory of the child', 'Theory of society', 'Mathematical aims' and 'Theory of Teaching Mathematics'. Ernest's model can be regarded as a matrix consisting of five columns (each representing a social group) and 12 rows (each representing a theme) about the set of moral values, theory of society and philosophy of mathematics. The intersection of a column and a row corresponds to a social groups' belief. Below we will briefly describe these social groups.

The 'IT' perspective is authoritarian and basic skills-centered, and concerns those teaching the workforce of industry. As to CMSE and transfer, this perspective does not consider insightful learning, since for a well-balanced approach aiming at transfer both basic skills (Rittle-Johnson, Schneider & Star, 2015) and conceptual understanding should be taught in an integrated manner (Drijvers, 2011; Kilpatrick, Swafford & Findell, 2001). They view knowledge as dualistic (either knowledge may be right or wrong) and mathematics as absolute (unquestionable set of facts). The teacher demands rigid discipline of hard working, competitive students who attempt to acquire basic mathematical skills and numeracy, and social training in obedience. The calculator is restricted and there is no room for discussion and collaboration. Social issues and the interests of social groups have no place in mathematics (are neutral).

The pure mathematics centered 'OH' group refers to the elitist and cultured class. Knowledge is relativistic (its truth depends on the context), but math absolute. Because of their strong reliance on pure mathematics (especially logic), they are math-centered (not authority as 'IT'). For them mathematics is the 'Queen of the Sciences' (Colyvan, 2012). They refuse to discuss applications, let alone issues on transfer of mathematics in science subjects.

Furthermore, these teachers grant anyone to have his or her own opinion, and they encourage their students to climb up the hierarchy of knowledge as far as possible: they should understand and be appreciate advanced mathematics.

The perspective of 'PrE' is learner-centered. Their decisions are humanistic and based upon universal principles such as empathy and caring. Knowledge is considered relativistic and mathematics absolute. Teachers offer non-intrusive guidance in a secure learning environment involving exploration, discovery, play, discussion and collaboration. Students should gain confidence, creativity and self-expression through mathematics. Their openness to collaboration may enable them to develop common teaching strategies between both departments, since this may be a powerful manner to tackle transfer problems (Roorda, 2012).

The group 'PuE' aims at education based on democratic socialist principles and values. Their perspective is social justice-centered. Knowledge is regarded relativistic. Mathematics is viewed as the fallible product of human invention (uncertain and corrigible). Collaboration and discussion are important pedagogical components. Like 'PrE', their focus on collaboration may enable both departments working together on transfer problems. Through math, students should be empowered and liberated as critical and literate citizens in society.

As to 'TP', knowledge is multiplistic (it may be sometimes right, sometimes wrong) whereas mathematics is absolute. Learning involves acquisition of basic skills (Rittle-Johnson, Schneider & Star, 2015) and solving practical problems using mathematics and information technology. Even though they are more progressive than 'IT' (e.g., not anti-calculator and not computer), there is again emphasis on procedural fluency, but no attention for insightful learning. Basic and problem-solving skills prepare them for the demands of industry. Indeed, they represent the interest of the industry.

Following Ernest (1991), we categorized teachers into groups based on their belief systems. Unlike our clusters, however, Ernest's (1991) groups are theoretical and not based on empirical data. Finally, his social groups are 'package deals': the members of his groups are supposed to either embrace or reject complete sets of beliefs. His theoretical model does not admit belief systems mixing aspects of different groups. Neither does he allow for different *degrees* of belief. In practice, certain social groups may have ideas that overlap with other groups. We expect that this also holds for the participants in our study.

4.2.6 Study (2)

The dataset of table 5 in chapter 3 including sixteen core beliefs in study (2) is sufficiently reduced to extract belief systems with desirable and naïve beliefs in a last data reduction step. We will explain this in detail below.

4.3 Methodology

Firstly, we explain the transformation of sixteen core beliefs into sixteen claims. Secondly, the development of an online survey and how we ran it is explained. This survey included sixteen claims that were used in the short questionnaire. Thirdly, we will describe the data collection method. Fourthly, the extraction of belief systems is described.

4.3.1 Step 1: Forming of Claims

The sixteen core beliefs were converted into sixteen claims, which were clearly phrased using the six functions of language of Jakobson (Hébert, 2011; Waugh, 1980). Briefly, these six functions are (1) the 'referential function' that corresponds to the context and describes a situation, object or mental state, (2) the 'emotive function' that is concerned with the sender and his emotions, (3) the 'conative function' expressing how he receiver is engaged, (4) the 'poetic function' focusing on the message itself: is the message factual or poetic?, (5) the 'phatic function' referring to the language used for the sake of interaction and is therefore associated with the channel of communication, and (6) the 'metalingual function' relating to the language to discuss or describe itself. Also, the relation between research question (3) and the claims was emphasized. We combined both requirements. The claims were (1) phrased positively, (2) understandable without any explanation, (3) verbalized as factual as possible, and (4) used well-known language for both mathematics and physics teachers. This should lead to sixteen claims about students experiencing coherence across these subjects and transfer of algebraic skills from math in physics.

To warrant intersubjectivity, the whole procedure above was discussed by the first and third author and two independent researchers until 100 % consensus was reached on the formulation of claims among all researchers. Next, these claims were transformed into a short questionnaire. This process including the content of this questionnaire is explained in the following sub sections.

4.3.2 Step 2: Development of Online Survey and Running it

The online survey we developed for mathematics and physics teachers consisted of four parts, each corresponding to a separate webpage (Sue & Ritter, 2012). On the *first* webpage teachers were introduced to background information about the first author, together with the aims and purpose of the survey. On the *second* webpage respondents were asked to answer questions about their gender, profession and number of years of teaching experience. The sixteen claims were incorporated in the short questionnaire and used on the *third* webpage. Here, mathematics and physics teachers were asked to select their top 5. Because of two reasons the questionnaire was introduced through the transfer problem in the case (Turşucu, Spandaw, Flipse, & de Vries, 2017): to give the respondents an idea about what is meant with algebraic skills and since the claims were based on these skills. Webpage *four* deals with the multi-criteria assessment tool (Belton & Stewart, 2002) that we have developed, because there was no such digital tool available. This is illustrated in figure 2 below. To gain more insight into the relative weights between these five claims (top 5), teachers were asked to distribute 50 points over these claims.

The number '50' in the center of this tool has a dynamic display that changes with each distribution operation and returns the remaining points. In the pilot-study social science experts and teachers mentioned that this display helped them to concentrate on the distribution of their points over the claims, and gain more insight into the remaining points after each operation. This helped them achieving a well-balanced point distribution.

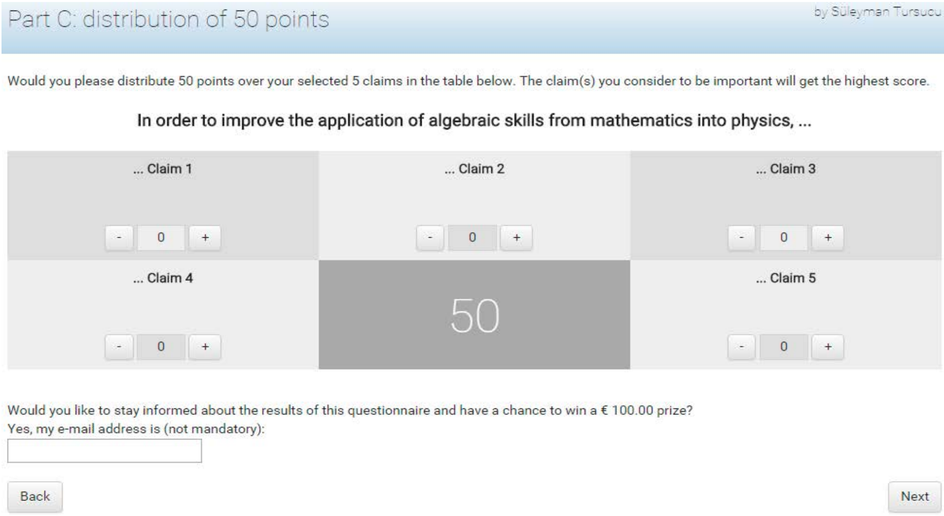


Figure 2. Multi criteria assessment tool in which teachers distribute 50 points over their selected 5 claims.

4.3.3 *Step 3: Data Collection Method*

We used self-selection sampling (Bryman, 2015) to gather data. About 300 schools in urban areas including most capitals of provinces and the bigger cities in the Netherlands were invited to participate in this study. We also invited about 100 sub-urban and 30 rural schools. The respondents consisted of mathematics and physics teachers from all levels of secondary education. To achieve a high response rate among these respondents we offered a € 100,00 reward.

4.3.4 *Step 4: Extraction of Teachers' Belief Systems*

The notion of 'belief system' was modelled as follows. For each of the 16 claims we have an empirical distribution of scores. We clustered the teachers such that all teachers in a certain cluster have similar scores on the 16 claims. A belief system is then the system of 16 empirical distributions for the corresponding cluster of teachers. So, to find belief systems we had to cluster the teachers. To this extent we used the clustering technique *agglomerative hierarchical clustering* (AHC) (Everitt & Dunn, 2001). After the belief systems were extracted, the first and second author triangulated on which of these were considered as useful for this study and which were not. They reached 100% consensus on the final set of belief systems.

4.4 Results

Firstly, we will present the sixteen claims translated from Dutch to English. *Secondly*, the teachers who participated in this study are shown. *Thirdly*, the total scores of experienced teachers are presented. *Fourthly* and *fifthly*, the belief systems of respectively the very and most experienced teachers are shown.

4.4.1 Sixteen Claims

The set of sixteen claims is depicted below in table 1. Each core belief in the previous study (Turşucu, Spandaw, Flipse, & de Vries, 2018a) was converted into the corresponding claim using the criteria above. The naïve claim numbers are 2, 3, 7, 9 and 16, and were already identified in the previous study on teachers' core beliefs (Turşucu et al., 2017).

Table 1. Set of sixteen claims about CMSE and improving transfer.

Claim number	List of claims
	To improve the application of algebraic skills from mathematics into physics…
1	… the collaboration between mathematics and physics teachers should have more priority.
2	… mathematics A should contain more algebraic skills than is the case now.
3	… mathematics should contain more algebra.
4	… mathematics teachers need more time to cooperate with physics teachers.
5	… the content of mathematics and physics textbooks should be adjusted.
6	… mathematics and physics teachers should be able to explain relevant basic knowledge about mathematics.
7	… mathematics and physics teachers should follow the content of their textbooks.
8	… students should recognize physics contexts.
9	… students should practice more algebraic skills during physics lessons.
10	… mathematics and physics teachers should use the same notations in formulas.
11	… prior mathematical knowledge should be activated during physics lessons.
12	… students should see relations between contexts of both mathematics and physics.
13	… mathematics and physics teachers should work together to improve the application of these algebraic skills.
14	… mathematics should incorporate more physics contexts.
15	… to a lesser extent students should see mathematics and physics as separate subjects.
16	… the physics curriculum should contain more manipulation of formulas than is the case now.

4.4.2 Participating Teachers in This Study

Our sample included 503 teachers from all levels of secondary education. After some preliminary analyses, we decided to focus on qualified senior pre-university teachers, since the transfer problems under consideration take mainly place in their grade 10 classes (Turşucu, Spandaw, Flipse, & de Vries, 2018a; Roorda, 2012). We restricted our attention to teachers with more than 5 years of teaching experience. This left us with 274 senior pre-university teachers, including 188 male and 86 female teachers. The results are listed below in table 2.

During the analysis in the next section we will refer to these 274 teachers as ‘experienced teachers’. The 97 teachers (61 male and 36 female) with more than 10 years of teaching experience will be called ‘very experienced teachers’, and the 118 teachers (89 male and 29 female) with more than 20 years of teaching experience ‘most experienced teachers’.

Table 2. Distribution of years of teaching experience over number of participating mathematics and physics teachers.

Teaching experience (years)	6 – 10	11 – 20	20 –	Total
Mathematics	31	59	77	167
Physics	28	38	41	107
Total	59	97	118	274

4.4.3 Total Scores of Experienced Teachers

A first impression of the opinions of these 274 experienced teachers can be obtained from the relative scores on the 16 claims. These are given in figure 3 (normalized to a total of 1) for the three subgroups of 59 experienced teachers (white bar), the 97 very experienced (grey bar), and the 118 most experienced teachers (black bar). For the sake of clarity, next to figure (3), we displayed table 1. Remarkably, not a single teacher had chosen claim number 7 in his or her top 5! Claim number 11 has the highest total score.

To gain more insight into the differences of the three experience levels with respect to the distribution of the scores, we plotted a boxplot for each of the 16 claims (figure 4), again for the whole group of 274 qualified teachers with more than 5 years of teaching experience.

We see that most claims have zero median and most scores have quite a few outliers, i.e. scores larger than the third quartile plus 1.5 times the interquartile range (Cohen, Cohen, West, & Aiken, 2013). Correlations between the 16 claims were low. Even when we restrict to the subsample of 97 very experienced teachers or 118 most experienced teachers, all correlations were small. Their squares (explained variance) were smaller than 0.10. Small correlations make principal component analysis (PCA) and factor analysis ‘pointless’ (Everitt & Hothorn, 2011, p. 157). This finding was confirmed by the PCA scree-plots. These lacked a clear ‘elbow’, which implies that there is no natural choice for choosing the number of dimensions.

Table 1. Set of sixteen claims about CMSE and improving transfer.

Claim number and corresponding claim
To improve the application of algebraic skills from mathematics into physics…
1. … the collaboration between mathematics and physics teachers should have more priority.
2. … mathematics A should contain more algebraic skills than is the case now.
3. … mathematics should contain more algebra.
4. … mathematics teachers need more time to cooperate with physics teachers.
5. … the content of mathematics and physics textbooks should be adjusted.
6. … mathematics and physics teachers should be able to explain relevant basic knowledge about mathematics.
7. … mathematics and physics teachers should follow the content of their textbooks.
8. … students should recognize physics contexts.
9. … students should practice more algebraic skills during physics lessons.
10. … mathematics and physics teachers should use the same notations in formulas.
11. … prior mathematical knowledge should be activated during physics lessons.
12. … students should see relations between contexts of both mathematics and physics.
13. … mathematics and physics teachers should work together to improve the application of these algebraic skills.
14. … mathematics should incorporate more physics contexts.
15. … to a lesser extent students should see mathematics and physics as separate subjects.
16. … the physics curriculum should contain more manipulation of formulas than is the case now.

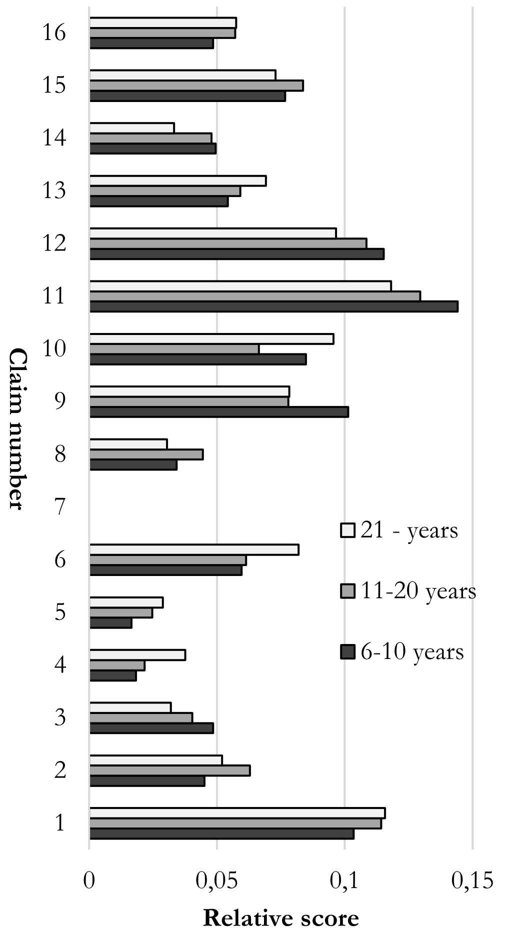


Figure 3. Relative total scores of the 274 experienced teachers over the sixteen claims.

AHC produces a tree, also called dendrogram. The method starts with the leaves of the tree, i.e. the teachers in the case at hand. It joins the individuals whose scores are closest with respect to a specified measure. Next, the distances between the clusters thus obtained are compared and the closest clusters are joined to agglomerated clusters. In the next step, one gets next level agglomerated clusters. And so on, until all teachers are teachers are joined in one supercluster at the root (see e.g., Figure 5). The distance between clusters which are merged in clusters of the next level is called the 'height' (Figure 5). Two choices should be

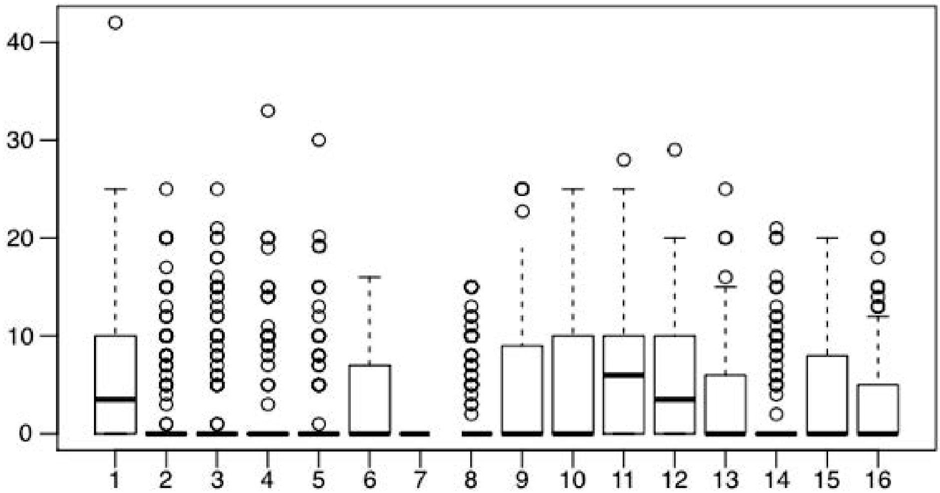


Figure 4. Boxplot of the total scores of 274 teachers showing medians, outliers and quartile scores.

made to produce a tree: a distance function on the set of 16-dimensional scores and a 'linkage method' used to define the distance between clusters. We took complete linkage as recommended by Everitt & Hothorn (2011). This means that the distance between two clusters X and Y is defined as the maximal distance between points x in X and y in Y.

We used the statistical programming language R to produce the trees using the command 'hclust'. Dichotomized scores (taking 1 for positive scores and 0 for zero scores) in combination with Manhattan distance and complete linkage produced the trees with the clearest clustering. For example, in Figure 5 we see that the 97 very experienced teachers with 11 – 20 years of teaching experience form six clusters. Note that the first splitting near the root occurs at the same height ($h = 8$) in the tree. If we use non-dichotomized scores, we get trees whose splitting heights are not so nicely and evenly distributed as in Figure 5 below. Different choices of the height, which are quite arbitrary, would lead to different number of clusters.

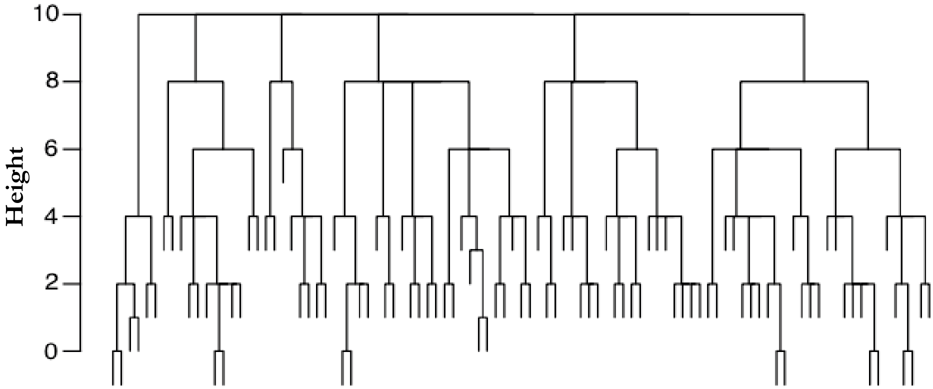


Figure 5. Dendogram for the 97 very experienced teachers using dichotomized scores.

4.4.4 Belief Systems of Very Experienced Teachers

The tree for the 97 qualified senior pre-university education mathematics and physics teachers with more than 10 years of teaching experience is shown in Figure 6. At the most fundamental level, this tree clusters these teachers into 6 clusters, which we will call BS1(VE) to BS6(VE) ('BS' for 'belief system' and 'VE' for 'very experienced'). The 'height' is the Manhattan distance between the clusters. The 6 clusters BS1(VE) to BS6(VE) merge at height 10, which means that the distance between these clusters equals 10. Their sizes are 6, 12, 8, 24, 20 and 27. They contain 3, 9, 5, 18, 9 and 15 mathematics teachers and 3, 3, 3, 6, 11 and 12 physics teachers, respectively. We focused on the three largest clusters BS4(VE), BS5(VE) and BS6(VE), covering 74% of the 97 teachers, since the other three clusters are rather small.

A first impression of these belief systems can be obtained from figure 6, which shows the mean (non-dichotomized) scores of these three clusters on the 16 claims. The total of the 16 means equals 50 for each cluster. The error bars correspond to one standard deviation.

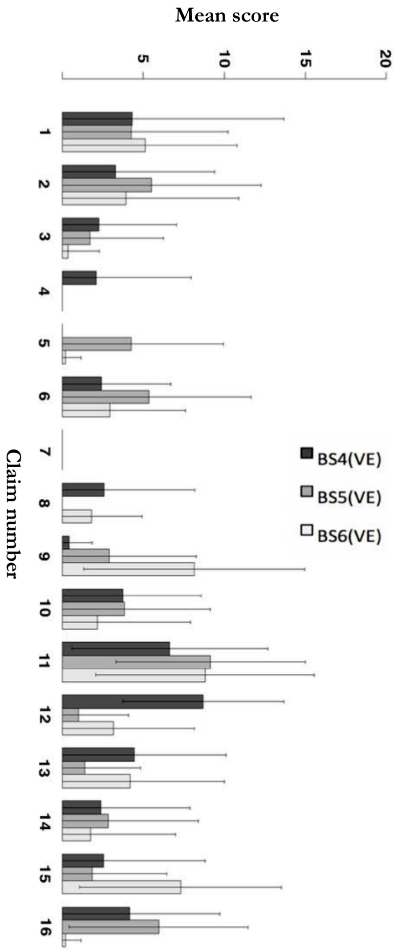


Figure 6. Mean scores and errors for the three largest clusters BS4(VE), BS5(VE) and BS6(VE) very experienced teachers.

Table 1. Set of sixteen claims about CMSE and improving transfer.

Claim number and corresponding claim
To improve the application of algebraic skills from mathematics into physics…
1. … the collaboration between mathematics and physics teachers should have more priority.
2. … mathematics A should contain more algebraic skills than is the case now.
3. … mathematics should contain more algebra.
4. … mathematics teachers need more time to cooperate with physics teachers.
5. … the content of mathematics and physics textbooks should be adjusted.
6. … mathematics and physics teachers should be able to explain relevant basic knowledge about mathematics.
7. … mathematics and physics teachers should follow the content of their textbooks.
8. … students should recognize physics contexts.
9. … students should practice more algebraic skills during physics lessons.
10. … mathematics and physics teachers should use the same notations in formulas.
11. … prior mathematical knowledge should be activated during physics lessons.
12. … students should see relations between contexts of both mathematics and physics.
13. … mathematics and physics teachers should work together to improve the application of these algebraic skills.
14. … mathematics should incorporate more physics contexts.
15. … to a lesser extent student should see mathematics and physics as separate subjects.
16. … the physics curriculum should contain more manipulation of formulas than is the case now.

4.4.5 *Belief Systems of Most Experienced Teachers*

We performed the same analysis for the group of 118 most experienced qualified pre-senior university teachers, i.e. teachers with more than 20 years of teaching experience. They are different individuals than those in the clusters in the analysis of the 97 very experienced teachers. The tree is given in Figure 7 below. We again found 6 belief systems, which we denoted as BS1(ME) ('ME' for 'most experienced') up to BS6(ME). The belief systems contain 9, 26, 9, 15, 27 and 32 teachers, respectively. We ignored the small belief systems BS1(ME), BS3(ME) and BS4(ME). Belief system 2 contains 20 math teachers and 6 physics teachers. For BS5(ME) these numbers are 17 and 10, and for BS6(ME) these numbers are

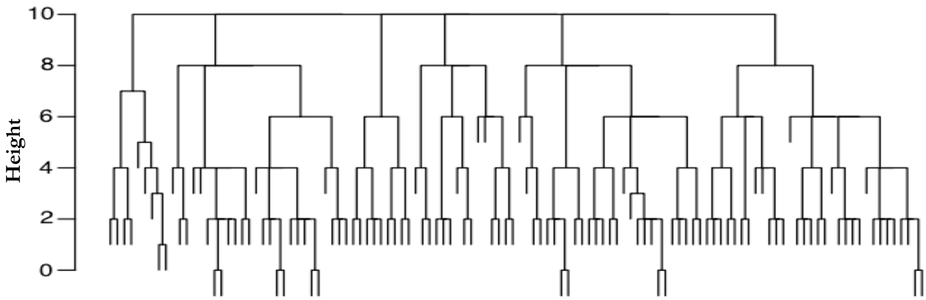


Figure 7. Dendogram for the 118 most experienced teachers using dichotomized scores.

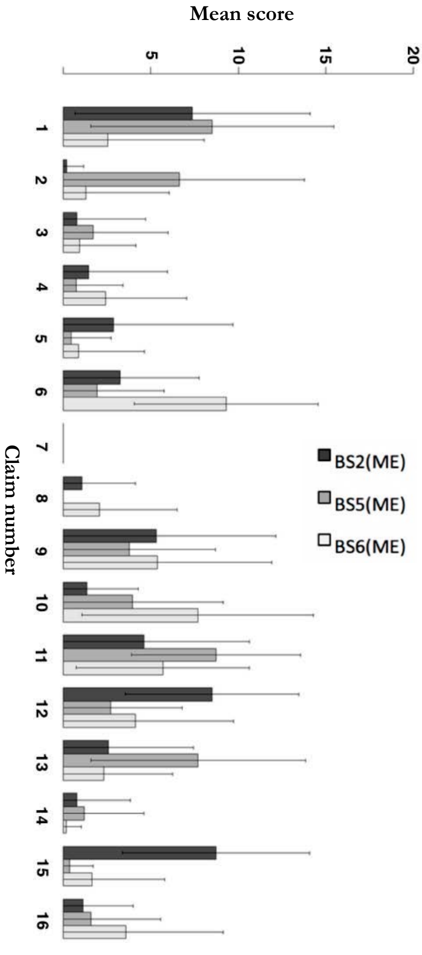


Figure 8. Mean scores and errors for the three largest clusters BS2, BS5 and BS6 of most experienced teachers.

Table 1. Set of sixteen claims about CMSE and improving transfer.

Claim number and corresponding claim
To improve the application of algebraic skills from mathematics into physics…
1. … the collaboration between mathematics and physics teachers should have more priority.
2. … mathematics A should contain more algebraic skills than is the case now.
3. … mathematics should contain more algebra.
4. … mathematics teachers need more time to cooperate with physics teachers.
5. … the content of mathematics and physics textbooks should be adjusted.
6. … mathematics and physics teachers should be able to explain relevant basic knowledge about mathematics.
7. … mathematics and physics teachers should follow the content of their textbooks.
8. … students should recognize physics contexts.
9. … students should practice more algebraic skills during physics lessons.
10. … mathematics and physics teachers should use the same notations in formulas.
11. … prior mathematical knowledge should be activated during physics lessons.
12. … students should see relations between contexts of both mathematics and physics.
13. … mathematics and physics teachers should work together to improve the application of these algebraic skills.
14. … mathematics should incorporate more physics contexts.
15. … to a lesser extent students should see mathematics and physics as separate subjects.
16. … the physics curriculum should contain more manipulation of formulas than is the case now.

23 and 9, respectively. The three belief systems BS2(ME), BS5(ME) and BS6(ME) cover 72% of the 118 most experienced teachers.

Figure 8 shows the mean scores on the 16 claims for the three largest belief systems BS2(ME), BS5(ME) and BS6(ME). Again, the total of the 16 means equals 50 for each belief system. As in figure 7, the error bars again correspond to 1 standard deviation. We immediately see some remarkable features, for example BS2(ME) gives a high score to claim 15, BS5(ME) puts much weight on claims 2 (naïve belief) and 13, whereas BS6(ME) is distinguished by its high score on claim 6.

4.5 Results Interpretation

In this and later sections the words *claim* and *belief* are used interchangeably to denote one of the sixteen claims in table 1. We focused on belief systems including beliefs with the highest scores and neglected those with the lowest scores.

4.5.1 Very Experienced Teachers: BS4(VE), BS5(VE) and BS6(VE)

The highest scores in BS4(VE) belong to claims 12, 11, 13, 1 and 16 (naïve belief). The focus is on collaboration between mathematics and physics teachers and making links between math and physics class. Prior mathematical knowledge should be activated in physics class and students should see (or rather: be helped to see) the relations between contexts used in math and physics class. Mathematics and physics should work together to improve transfer of algebraic skills to physics. More generally, collaboration between these teachers should have more priority. The lowest scores in BS4(VE) belong to claims 7, 5 and 9, which center around the textbooks. Teachers with this belief system believed that following or adjusting textbooks is not important to improve transfer, nor do they believed that algebraic skills should be practiced more in physics class.

The highest scores in BS5(VE) belong to claims 11, 16 (naïve belief), 2 (naïve belief) and 6. The focus is on the teacher and on the curriculum. The physics teachers should activate prior mathematical knowledge in their classes and they should be able to explain basic mathematics. The physics curriculum should pay more attention to manipulation of formulas and the curriculum for mathematics A should give more weight to algebraic skills than now is the case. The lowest scores in BS5(VE) correspond to claims 7, 4, 8 and 12 which centre around students. Teachers believed that students seeing relations between contexts used in both math and physics class is not important to improve transfer. Nor they believed that more collaboration of mathematics and physics teachers is the key to improvement of transfer.

Finally, the highest scores in BS6(VE) were given to claims 11, 9 (naïve belief) and 15. These centre around the student and the teacher. Their mathematical pre-knowledge should be activated, they should practice algebraic skills in physics class more than they do now, and they should learn to see the connections between mathematics and physics. The lowest scores in BS6(VE) belong to claims 7, 4, 16, 5 and 3. There is a focus on the textbook and the curriculum. These teachers believed that the content of textbooks should not be followed, nor do they believe that its content should be changed.

4.5.2 *Comparing BS4(VE), BS5(VE) and BS6(VE)*

The most remarkable differences among the strong claims between BS4(VE) and BS5(VE) are the scores on claims 12, 13 and 6. BS4(VE) rates claim 12 nine times more important and claim 13 three times more important than does BS5(VE). We conclude that collaboration between mathematics and physics teachers and connecting contexts used in physics and math class are far more important to BS4(VE) than to BS5(VE). Conversely, claim 6 is rated twice as high by BS5(VE) compared to BS4(VE). Hence, physics teachers explaining basic mathematics is deemed more important in BS5(VE) than in BS4(VE).

The strong claim of BS4(VE) with the highest ratio BS4(VE) : BS6(VE) is number 16 (naïve belief) (BS4(VE) = 22 × BS6(VE)). The strong claim of BS6(VE) with the highest ratio BS6(VE) : BS4(VE) is number 9 (naïve belief) (BS6(VE) = 20 × BS4(VE)). We conclude that BS4(VE) teachers believed much stronger than their BS5(VE) colleagues that more algebra in the physics curriculum is beneficial for the transfer of algebra to physics. BS6(VE) teachers, on the other hand, believe stronger in algebraic practice in physics class.

Finally, we compare BS5(VE) to BS6(VE). The most distinguishing strong claims are number 16 (naïve belief) (BS5(VE) = 32 × BS6(VE)), number 15 (BS6(VE) = 4 × BS5(VE)), and 9 (naïve belief) (BS6(VE) = 3 × BS5(VE)). Teachers belonging to BS5(VE) believed more strongly than BS5(VE) teachers that adjustments of the physics curriculum are important to improve transfer. Conversely, BS6(VE) teachers have a stronger focus on student related claims: these students should learn to see the connections between mathematics and physics and they should practice algebra in physics class.

Table 3 below summarizes this analysis. The items "11, 9 (naïve belief), 15" in the last column of the first row are the numbers of the strongest beliefs in BS6(VE). Similarly, "7, 5, 9 (naïve belief)" in the third row of the first column are the numbers of the weakest beliefs in BS4(VE). Finally, "6 (2)" in the BS4(VE)-row of the BS5(VE)-column means that among the strong beliefs of BS5(VE), claim 6 had the highest ratio BS5(VE) : BS4(VE), namely BS5(VE) = 2 × BS4(VE). The entries "12 (9), 13 (3)" in the BS5(VE)-row of BS4(VE)-column means that among the strong claims of BS4(VE), number 12 and 13 had the highest ratio BS4(VE) : BS5(VE), namely 9 and 3, respectively.

Table 3. Comparison of strong beliefs in BS4(VE), BS5(VE) and BS6(VE).

	strong	12, 11, 13, 1, 16	11, 16, 2, 6	11, 9, 15
weak	row < column	BS4(VE)	BS5(VE)	BS6(VE)
7 5 9	BS4(VE)		6 (2)	9 (naïve belief) (20)
7 4 8 12	BS5(VE)	12 (9) 13 (3)		15 (4) 9 (naïve belief) (3)
7 4 16 5 3	BS6(VE)	16 (naïve belief) (22)	16 (naïve belief) (32)	

4.5.3 Most Experienced Teachers: BS2(ME), BS5(ME) and BS6(ME)

The claims with the highest scores in BS2(ME) are claim number 15, 12, and 1. The focus is on students and on collaboration. Teachers believed that to improve transfer, students should learn to see the relations between physics and mathematics, and teachers should work together. The claims with the lowest scores in BS2(ME) are claim number 7, 2, 14 and 3. These centre around curricula. Teachers in this belief system believed that changing curricula are not the most important steps to improve transfer.

The claims with the highest scores in BS5(ME) are claim number 11, 1, 13, and 2 (naïve belief). The focus is on collaboration. Teachers in this believe system stated that mathematics and physics teachers should collaborate to improve the algebraic skills of their students, physics teachers should activate mathematical pre-knowledge in their classes, whereas mathematics A classes should pay more attention to algebra. The claims with the lowest scores in BS5(ME) are claim number 7, 8, 15 and 5. These centre around students and on the textbooks. Teachers in this belief system do not attach much importance to their students seeing connections between mathematics and physics. Neither they believed that teachers should follow the content of textbooks or make adjustments in textbooks.

The claims with the highest scores in BS6(ME) are claim number 6, 10, 11 and 9 (naïve belief). The focus is on teachers. To improve transfer, they should both be able to explain basic mathematics and use the same notations. Physics teachers should activate mathematical pre-knowledge and students practice more algebraic skills in physics class. The claims with the lowest scores in BS6(ME) are claim number 7, 14 and 5. Teachers in this belief system believed that following and changing textbooks will not help greatly to improve transfer.

4.5.4 Comparing BS2(ME), BS5(ME) and BS6(ME)

The strong claims distinguishing most between BS2(ME) and BS5(ME), i.e. the strong claims for BS2(ME) with the highest ratio BS2(ME): BS5(ME) and the strong claims for BS5(ME) with the highest ration BS5(ME): BS2(ME), are number 15 (BS2(ME) = 24 × BS5(ME)) and number 2 (naïve belief) (BS5(ME) = 34 × BS2(ME)). Connecting mathematics and physics is far more important for BS2(ME) than for BS5(ME), whereas more algebra in the mathematics A curriculum is far more important to BS5(ME).

The strong claims distinguishing best between BS2(ME) and BS6(ME) are number 15 (BS2(ME) = 5 × BS6(ME)), number 1 (BS2(ME) = 3 × BS6(ME)), and number 10 (BS6(ME) = 6 × BS2(ME)). BS2(ME) focuses more on students, who should learn to see the connections between mathematics and physics. BS6(ME) focuses on the teachers, who should use compatible notations.

Finally, we compare BS5(ME) to BS6(ME). The most distinguishing strong claims are number 2 (naïve belief) (BS5(ME) = 5 × BS6(ME)), number 1 (BS5(ME) = 3 × BS6(ME)), number 13 (BS5(ME) = 3 × BS6(ME)), number 6 (BS6(ME) = 5 × BS5(ME)), and numbers 16 (naïve belief) (BS6(ME) = 2 × BS5(ME)). Teachers in BS5(ME) stated that collaboration between mathematics and physics teachers and adapting the mathematics A curriculum are more important for improving transfer than their colleagues in BS6(ME) believed. Teachers adhering to BS6(ME), on the other hand, attached more weight than BS5(ME) teachers do to adapting the physics curriculum and to physics teachers explaining basic mathematics. In short, BS5(ME) teachers focused on math class and math teachers, whereas BS6(ME) teachers focused on physics class and physics teachers. Table 4 above summarizes this analysis.

Table 4. Comparison of strong beliefs in BS2(ME), BS5(ME) and BS6(ME).

	strong	15, 12, 1	11, 1, 13, 2	6, 10, 11, 9
weak	row < column	BS2(ME)	BS5(ME)	BS6(ME)
7 2 1 4 3	BS2(ME)		2 (naïve belief) (34)	10 (6)
7 8 1 5 5	BS5(ME)	15 (24)		6 (5) 16 (2)
7 1 4 5	BS6(ME)	15 (5) 1 (3)	2 (5) 1 (3) 13 (3)	

4.5.5 Naïve Beliefs: Very and Most Experienced Teachers

Based on the analysis above, we present 5 belief systems, each consisting of desirable and naïve beliefs which are depicted and explicated in table 5 below. BS2(ME) is not included, since it has no naïve beliefs. The strength of beliefs in increasing order is also illustrated. This information is obtained from the figures 6 and 8, and will be used in the last section. It can be seen that the number of naïve beliefs in a belief system may differ. Except for BS5(VE), the other clusters contain one naïve belief. Remarkably, claim numbers 6 and 16 are absent for the most experienced teachers. On the other hand, the very and most experienced teachers share claim numbers 2 and 9.

Table 5. The 5 belief systems we have found include desirable and naïve (in bold) beliefs. The last column concerns the increasing strength of beliefs in a belief system.

Belief System	Naive beliefs	Desirable beliefs	Increasing strength of beliefs
BS4(VE)	**16**	1, 11, 12, 13	**16**, 1, 13, 11, 12
BS5(VE)	**2, 16**	6, 11	6, **2**, **16**, 11
BS6(VE)	**9**	11, 15	15, **9**, 11
BS5(ME)	**2**	1, 11, 13	**2**, 13, 1, 13
BS6(ME)	**9**	6, 10, 11	**9**, 11, 10, 6

Also remarkable is that claim number 16' of BS4(VE), number '2' of BS5(ME) and number '9' of BS6(ME) are both naïve and the weakest beliefs. The naïve beliefs number '2' and '16' of BS5(VE), and number '9' of BS6(VE) are also weak. Overall, we can say that the naïve beliefs in the last column are the weakest beliefs, i.e. have least strength.

4.5.6 Comparing Very and Most Experienced Teachers

BS4(VE) seems to correspond to BS5(ME), both attaching much weight to claims 1, 11 and 13. To improve transfer, they focused on collaboration. Hence, we refer to both clusters as the 'Collaboration-oriented group' ('COG'). BS5(VE) matches with BS6(ME), both believed strongly in claims 6 and 11. Since they focused on teachers, we called them the 'Teacher-oriented group' ('TOG'). BS6(VE) and BS2(ME) we called the 'Student-oriented group' ('SOG') because they share a strong belief in claim 15 with a clear focus on students.

4.6 Discussion and Conclusion

With regard to research question (3) *"What are the belief systems of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?"* we conducted an online survey among 503 mathematics and physics secondary school teachers. The survey included the sixteen claims in table 1 about improving transfer. Teachers were asked to select their top 5. Data were analysed by means of agglomerative hierarchical clustering (AHC). We focused on teachers with more than 10 years of teaching experience in senior pre-university education. This led to the large clusters BS2(ME), BS5(ME) and BS6(ME) for the 118 most experienced teachers (ME), and the large clusters BS4(VE), BS5(VE) and BS6(VE) for the 97 very experienced teachers (VE). Except for BS2(ME), the other clusters consisted of both desirable and naïve beliefs. Despite these naïve beliefs, overall each of these clusters contain an organized set of mutually supporting beliefs about transfer. Since such clusters with coherent beliefs are interpreted as belief systems (Misfeldt, Jankvist & Aguilar, 2016; Singletary, 2012), we have empirical evidence for the existence of belief systems. To a certain extent, this also justifies Ernest's (1991) theoretical idea to cluster teachers based on their belief systems.

4.6.1 BS2(ME), Claim Numbers 7 and 11

Following the line of Leathem (2006) and Singletary (2012), the peripheral area of BS2(ME) enabling weak beliefs may be empty, i.e. may not contain *naïve* beliefs about transfer. In addition, the central part includes desirable beliefs that can be transfer enhancing. Consequently, teachers transforming such belief systems (espoused models) into teaching practice (enacted models) (Ernest, 1991) are more likely to foster transfer than teachers having belief systems with naïve beliefs. Indeed, with regard to improvement of transfer, there is a match between the espoused and enacted models. Remarkably, this was also the only belief system lacking the desirable claim (number 11) with the highest total score among all other clusters. On the other hand, claim number 7 was absent in BS2(ME). Like all other clusters they seem to be aware of this naïve belief, but *naively* seem to be unaware of the importance of claim number 11. We recommend to further examine this apparent contradiction, for example, by investigating why teachers in this cluster neglected this claim by means of a qualitative in depth-interview. Furthermore, the top scores of claim number 11 in the remaining clusters is not that surprising. Indeed, the importance of activation of prior knowledge is a well-known issue in the context of learning and instruction in relation to better students' achievements (e.g., Hailikari, Katajavuori, & Lindblom-Ylanne, 2008).

Claim number 11 has implications for science education. Regarding curriculum materials, policy makers should take this matter into account as a key design principle in which both subjects are connected to each other. These curricula are transformed into textbooks which are closely followed by Dutch teachers who teach them to their students (SLO, 2019; van Zanten, M. & van den Heuvel - Panhuizen, M., 2014). This strongly suggests that physics textbooks need introduction paragraphs containing prior mathematical knowledge about the physics subject that will be treated. For instance, when the task is to solve for m in the aforementioned kinetic energy formula $E_k = \frac{1}{2} \cdot m \cdot v^2$, the textbook may first refer to the mathematics equation $y = b \cdot x^2$ that students have already seen in mathematics class, also explicating that both expressions are analogous with respect to their algebraic structure. After solving for the mathematics equation $x = \sqrt{\frac{2 \cdot y}{b}}$, the textbook solves for the physics formula $m = \sqrt{\frac{2 \cdot E_k}{m}}$. Beyond textbooks, also physics teachers play a role in connecting mathematics to physics. Even mentioning that formulas in physics are rooted in mathematics class or writing mathematics and physics expressions next to each other may contribute to transfer (Alink, Asselt & Braber, 2012; Turşucu, Spandaw, Flipse, & de Vries, 2017). Furthermore, the mathematics teacher can reinforce this transfer process by referring to the physics class.

In case of science teacher educators, the implication is twofold. Through both 'professional teaching programs' (Guskey, 2002) and 'science teacher education programmes leading to a teaching qualification', they should make 'in service' and 'pre-service' science teachers aware of their belief systems, reflect on them and change naïve beliefs into desirable beliefs about transfer. Otherwise, their naïve beliefs will probably lead to teaching practice which can be harmful for CMSE and transfer.

Although activation of prior knowledge is considered important, it requires sufficient organization of the learning process. Mathematical concepts should be taught *before* they are explained in physics class. In case of physics teachers above, they may examine students' mathematics textbooks or have conversation with mathematics teachers to align both subjects. Regarding textbooks, these should be connected through content developed by the same publisher. Otherwise, i.e. when two different publishers are involved, alignment is made very difficult since each of them pursue different aims (Alink, Asselt & Braber, 2012).

The connections above are pivotal in overcoming compartmentalized thinking (Gellish et al., 2007) and could strengthen coherence between mathematics and science education (Berlin & White, 2012, 2014).

The finding that not a single teacher had chosen claim number 7 above, shows that they feel that following the content of textbooks does not contribute sufficiently to improving transfer. Further research, for example, textbook analysis could provide insight into the extent to which mathematics and physics textbooks take this matter into account. We also recommend conducting qualitative interviews with teachers, since both studies complement each other.

4.6.2 Other Relevant Desirable Beliefs

Beyond claim number 11, other desirable beliefs with high scores are 1, 6, 10 and 13 (see table 5). Remarkably, claim numbers 1, 10 and 13 all concern collaboration between mathematics and physics teachers. Contrary to the previous qualitative study in which the major part of mathematics teachers mentioned that they did not feel the need to work together with physics teachers (We emphasize that one cannot generalize from our small sample to the complete population of Dutch mathematics teachers) (Turşucu et al., 2017), this quantitative study revealed that many mathematics teachers think that collaboration between both departments should be given more priority (number 1). This is good news, since sufficient collaboration is essential to enhance students perceiving coherence across mathematics and physics subjects and transfer (Berlin & White, 2012, 2014; Quinn, 2013).

On the individual level, we *even* think that explaining relevant basic mathematical knowledge (number 6) should be a pre-requisite in science teacher education programmes leading to a teaching qualification. Furthermore, both on the collaboration and the individual level the mathematics and physics subjects should be connected through alignment of equations, formulas, notations (number 10), better application of algebraic skills (number 13) and the same pedagogy of teaching algebraic skills such as the application of algebraic techniques to manipulate expressions in both subjects. Such alignment aspects also apply to curricula and textbooks used in senior pre-university education (Alink, Asselt & Braber, 2012). In addition, science teacher educators should use these aspects in teaching materials for science teacher education programmes. Here, we did not mention mathematics teacher educators, because basic mathematical knowledge is already incorporated in the senior pre-university education mathematics curriculum (SLO, 2019), and thus part of the senior pre-university education teacher education programme. Furthermore, without a solid basis in school mathematics, especially school algebra, working on students' transfer of algebraic skills to physics is hardly possible. Probably, this makes individual mathematics and physics teachers pivotal in the classroom. Still, there should be collaboration above between both teacher groups. We conclude that both individual and collaborative efforts are like two sides of the same coin.

4.6.3 Belief Systems with Naïve Beliefs

Based on earlier studies (Leathem, 2006; Misfeldt, Jankvist & Aguilar, 2016; Singletary, 2012), the weakest beliefs in a belief system are located in the peripheral area. Since we had theoretically grounded naïve beliefs (e.g., Alink, Asselt & Braber, 2012; Turşucu, Spandaw, Flipse, & de Vries, 2018a; Roorda, 2012; Quinn, 2013) (second column of table 5), one would expect that the empirically determined weakest beliefs (last column of table 5) are these already identified naïve beliefs. Indeed, there is a match between them, implying that we verified a theoretical construct with empirical data. We conclude that the naïve beliefs in table 5 correspond to the weak beliefs rooted in the peripheral area of the belief systems we have found. On the other hand, the desirable beliefs in the last column have the strongest psychological strength and are strongly connected to other central beliefs.

To change the empirically naïve beliefs into desirable beliefs, clusters in the peripheral area in which they are stored, should be disturbed. This is possible, since naïve beliefs are the weakest beliefs. The good news is that most of the teachers' strong beliefs in these clusters are already desirable; they can be transfer enhancing. Otherwise, i.e. when the naïve beliefs would have been very strong (aforementioned second scenario), it would be very difficult to change them.

To make such changes one can use the microscopic belief system model above and the macroscopic model of Ernest (1991) together. Through professional development programs (Guskey, 2002), science teacher educators who are well-informed about these models could make teachers having naïve beliefs *explicitly* aware of their belief systems. They can use figure 1 of Chapter 1 with espoused and enacted models to explain how for example, teachers, students and textbooks influence a teachers' belief system. Thereafter, teachers may reflect on them to improve transfer. Thus, teachers belonging to BS5(VE) need extra attention, since they have not one, but two naïve beliefs in their belief system.

The remarkable difference in the number of belief systems with naïve beliefs, and the number of naïve beliefs between the most experienced and very experienced teachers in table 5, may be explained by the extent of their awareness of the harmful nature of naïve beliefs (espoused models) for teaching practice (enacted models). This implies that the more experienced teachers are, the less their number of clusters containing naïve beliefs, and the less their number of naïve beliefs become.

Furthermore, we have seen that the distinction between the macroscopic and microscopic model of belief systems turned out to be useful. The first model containing espoused and enacted models (Ernest, 1991) was used to explain and understand how the social context of teaching influences a teachers' belief system, and the second one which is a detailed cognitive description of the espoused models to understand how the weak naïve beliefs and the strong desirable beliefs in a belief system are related to each other. We recommend future studies on this matter to use both models together.

4.6.4 Ernest's Social Groups and Our Groups

As expected, the construction of an educational matrix model analogous to that of Ernest (1991) was not possible, because our obtained belief systems were *not* pairwise disjoint, i.e. some beliefs occur in several belief systems. For example, claim number 11 appeared in several belief systems, whereas Ernest's five social groups (belief systems) are pairwise disjoint. Our model of the notion of 'belief system', being a system of 16 score distributions, is more detailed than Ernest's black-or-white approach in which a belief system either contains or excludes a given claim. Furthermore, Ernest's model is theoretical, whereas our model was based on empirical data. Indeed, we can ask to what extent Ernest's 'technological pragmatist' really exists.

Ernest's model and this study concern different issues, but have mathematics education in common. As to 'Industrial Trainers' ('IT') and the 'Teacher-oriented group' ('TOG'), both of them share the emphasis on teachers to respectively teach mathematics (authoritarian) and improve transfer. Whereas for 'IT' math teachers should drill their students basic mathematics according to 'back to the basics' (Ernest, 1991, p. 129), for 'TOG' they should be able to explain basic mathematics and students should activate prior mathematical knowledge (espoused models). Contrary to 'IT', the 'TOG' perspective is more likely to enhance transfer (enacted models), since there is a match between their espoused and enacted models. To improve transfer, 'IT' should reconcile both models by considering insightful learning (Kilpatrick, Swafford, & Findell, 2001) with their naïve belief on basic skills. The latter also holds for the industry-centred 'Technological Pragmatists' ('TP').

The 'Old Humanists' ('OH') have no direct commonalities with the clusters that we have obtained. Nevertheless, there is one issue that needs to be mentioned. In the previous *qualitative* study (Turşucu, Spandaw, Flipse, & de Vries, 2017) we have seen that like 'OH' some math teachers (small number *N*) view math as the 'Queen of the Sciences' (Colyvan, 2012) and refuse to discuss applications in mathematics class. This *quantitative* study should also contain teachers belonging to 'OH', since our sample is much larger than that of the previous study. They might have participated in this study because of external motivations, for example, winning the € 100,00 reward. Such teachers cannot have coherent clusters, since they are not interested in applications. Accordingly, their data might have been 'lost' as noise.

The social group 'Progressive Educators' ('PrE') and 'Student-oriented group' ('SOG') are both student-centred. The latter group thinks that this will improve transfer, whereas 'PrE' that this will enhance students' confidence, creativity and self-expression. Like 'PrE', the decisions of 'SOG' might be based upon universal principles such as empathy and caring for children. In addition, 'SOG' might also be as progressive as 'PrE' towards collaboration. Unfortunately, our data do not show these claims.

Furthermore, like 'OH' the group, Public Educators' ('PuE') do not have direct commonalities with 'COG', 'SOG' or 'TOG'.

Finally, it is worthwhile to mention that neither the belief systems identified in this study, nor those described by Ernest (1991) focused on teaching practice in which both basic algebraic skills and conceptual understanding are taught in an integrated manner (Drijvers, 2011; Rittle-Johnson, Schneider & Star, 2015). For students perceiving coherence across both subjects and transfer to occur, both concepts should be treated together.

4.6.5 Limitations of This Study

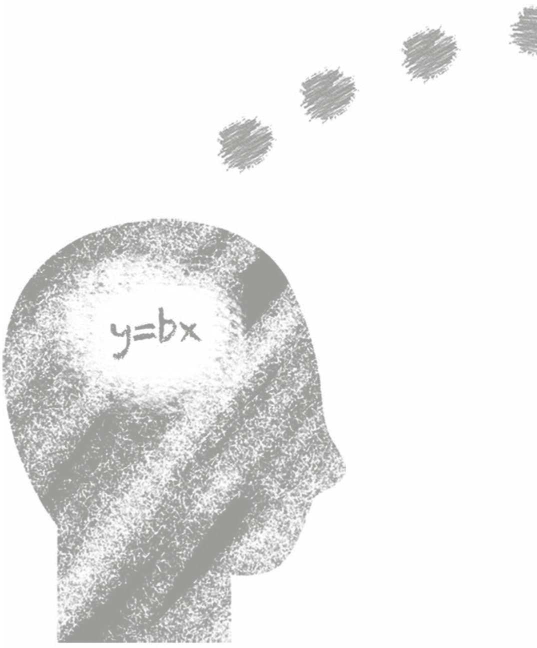
Our findings are based on 118 qualified VE teachers, and 97 ME teachers in senior pre-university education. This group of 215 teachers consists of 136 mathematics and 79 physics teachers. The Netherlands has 2903 qualified mathematics and 1330 qualified physics teachers in senior pre-university education ('Dutch Ministry of Education', 2017), also giving a ratio of roughly 2: 1. Furthermore, the 118 VE teachers had a gender-ratio (male-female) of 1:1 for mathematics, and 5:1 for physics teachers. For the 97 ME teachers this was 2:1 for mathematics and 13:1 for physics teachers. Unfortunately, there were no data available on the gender-ratio of qualified ME and VE mathematics and physics teachers in Dutch senior pre-university education, so we cannot judge how well our sample represents the national situation. The vast majority in our sample is familiar with both senior pre-university education and senior general secondary education (SGSE). Furthermore, the algebraic skills needed in physics class in both types of education are quite similar. Therefore, in case of representability of our sample above, we expect our results to be generalizable for both senior pre-university education and senior general pre-university education in the Netherlands. However, this does not hold for preparatory vocational secondary education, since the algebraic skills needed in senior pre-university education and senior general pre-university education are fundamentally different from those in preparatory vocational secondary education (SLO, 2019). This would lead to different belief systems (claims) about CMSE and transfer. At this point we recommend conducting a separate study to investigate the national gender-ratios of both mathematics and physics teachers in senior pre-university education in relation with the number of years of teaching experience. This could provide information about the national representativeness and generalizability of this study.

The content of subjects in senior pre-university education in the Netherlands is determined through curricula that contain both the general educational core goals and the more specific standards, which are tested in national final examinations. To a very large extent, these curricula shape the content of textbooks and also teachers who faithfully follow and teach these to their students (SLO, 2019; van Zanten, M. & van den Heuvel - Panhuizen, M., 2014). Consequently, they affect teachers' beliefs (claims) about CMSE and transfer. Such curricula do not exist in many other countries (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002). Thus, we do not expect that our results are generalizable to other countries outside the Netherlands – even if this study was generalizable for Dutch senior pre-university education.

Among three classes of clustering techniques, i.e. AHC, optimization methods and mixture models, we used the first option and discarded the third option, because it pre-supposes knowledge of latent variables. A standard optimization method is the *k*-means method. It has several disadvantages. First, it imposes a spherical structure on the data, i.e. it assumes that the data points are grouped in more or less spherical clusters in 16-dimensional space. We have no a priori reason to expect this to be true. Second, one has to choose the number *k* of clusters in advance. Third, the algorithm starts with random cluster centers, which resulted in rather different clusters in each run of the *k*-means algorithm. Therefore, we used agglomerative hierarchical clustering, since it does not have these disadvantages. Furthermore, this method also turned out to be objective, leading to nicely distributed clustering of belief systems including very clear splitting heights.

During the analysis of the belief systems, we focused on the three largest clusters for both teacher groups, and neglected 28% of the most experienced teachers and 26% of the very experienced teachers. Thus, we based our results on respectively the 72% and 74% of the extracted clusters. But what if these small clusters contained important information about, for example, naïve beliefs (claims)? Such beliefs may impede transfer. Therefore, we recommend to further investigate this matter.

Reliability of the statistical analysis.
Figures 6 and 8 show the mean scores and standard deviations of the large clusters for the very experienced and most experienced teachers, respectively. Some error bars are quite large compared to the means. This large dispersion within a belief system is usually due to a few outliers in the cluster. Our description of the main characteristics of the two triples of belief systems and the differences between these belief systems remains valid even taking this dispersion into account.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Chapter 5

Search for Symbol Sense Behavior[10]

[10] This chapter has been published in adapted form as: Turşucu, S., Spandaw, & de Vries, M. J. (2018). Search for Symbol Sense Behaviour: Students in Secondary Education Solving Algebraic Physics Problems. *Research in Science Education*, *48*(5), 1-27.

5.1 Introduction

Mathematics plays a major role in science education (Karam, 2014; Roorda, Goedhart & Vos, 2014). However, research has shown that students face difficulties when applying mathematics in science subjects (e.g., Redish & Kuo, 2014; Quinn, 2013). Such transfer[11] problems can be persistent and concern students of all ages.

Even if students' knowledge of mathematics is sufficient, its *application* in science subjects is not guaranteed. Except for a couple of studies (e.g., Cui, 2006; Rebello et al., 2007), this phenomenon has hardly been researched. The pioneering study of Hudson & McIntire (1977) with pre-course tests of algebraic- and trigonometric knowledge and skills taken by 200 students initiating a physics course, has shown that a solid grasp of mathematics is not sufficient to guarantee the application of these in physics: although the student has a solid grasp of mathematics, the performance in physics can be poor.

Several researchers have stated that a major reason for the lack of transfer above is related to compartmentalized thinking (e.g., Turşucu et al., 2018c; Nashon & Nielsen, 2007; Quinn, 2013) in which students see mathematics and science as two unrelated subjects. In many countries, teaching these subjects separately consolidates and intensifies this phenomenon (e.g., SLO, 2019; 'The National Academies Press', 2018; 'TIMMS & PIRLS', 2019).

Berlin & White (2010, 2012, 2014) suggest that a remedy for compartmentalized thinking may be coherent mathematics and science education (CMSE) that is of major importance for students. The idea behind CMSE[12] is fostering *connection* between mathematics and science education through, for example, alignment of notations. In addition, improvement of *mathematical proficiency* (Kilpatrick, Swafford, & Findell, 2001) including the five interwoven strands adaptive reasoning, conceptual understanding, procedural fluency, productive disposition and strategic competence may also help improve the application of mathematics in science subjects. In algebra education, especially crucial are the second and third strands. Together, these strands form algebraic expertise, referring to algebraic skills with particular emphasis on procedural fluency in relation to conceptual understanding (Andrá et al., 2015; Drijvers, 2011; Arcavi, 1994). The algebraic skills involving conceptual understanding are called symbol sense, relating to the ability to first consider an algebraic expression carefully, to find its relevant aspects, and to choose a *wise* systematic problem-solving strategy based on these aspects. Symbol sense contains "*an intuitive feel for when to call on symbols in the process of solving a problem, and conversely, when to abandon a symbolic treatment for better tools*" (Arcavi, 1994, p. 25). By means of examples, Arcavi described eight behaviors. These examples demonstrated the intimate relationship between procedural skills and conceptual understanding as if they were two sides of the same coin of algebraic expertise.

Flexible manipulation skills are regarded as a key behavior of symbol sense and deal with the ability to flexibly manipulate expressions (both technical and with insight) and being in control of the work. Flexible manipulation skills consist of two important, intertwined characteristics, which are having a gestalt view on algebraic expressions, and handling in a suitable way with their visual salience (Kirshner & Awtry, 2004). The former concept includes "*the*

[11] A detailed explanation of the controversial transfer phenomenon can be found in chapter 1.

[12] A detailed explanation of CMSE can be found in chapter 1.

ability to consider an algebraic expression as a whole, to recognize its global characteristics, to'read through' algebraic expressions and equations, and to foresee the effects of a manipulation strategy" (Bokhove & Drijvers, 2010; p. 43). The latter deals with visual cues of algebraic expressions. As part of visual salience, Wenger (1987) distinguished between pattern salience that is related to sensitivity towards patterns in algebraic expressions, and local salience relating to sensitivity towards local algebraic symbols, i.e. visual attractors such as fractions, square root signs and exponents. Hence, flexible manipulation skills play and therefore algebraic expertise including basic algebraic skills and symbol sense play a major role in the transfer of mathematics to science subjects, in particular to physics.

Furthermore, in this study, we assume a reciprocal relation between CMSE and transfer. When students experience coherence across mathematics and science subjects by means of *meaningful* contexts, transfer from mathematics to science subjects can be improved, and improving transfer can help them to experience coherence between mathematics and subjects.

5.1.1 Research Aim and Research Question (4)

This study aims to report the findings of a qualitative study with a quantitative component on symbol sense behavior of students in upper secondary education. The research question (4) is "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems?*".

According to teachers in upper secondary education, among students encountering difficulties with applying mathematics in algebraic physics problems, grade 10 students face the biggest problems (Turşucu et al., 2017). Therefore, we selected grade 10 students to gain *deeper* insight into their algebraic problem-solving abilities, especially their basic algebraic skills and their symbol sense behavior.

For the operationalization of symbol sense behavior, we followed the line of Bokhove & Drijvers (2010) in the sense that we examined students' basic algebraic skills and sensitivity towards local salience and pattern salience. Different from them, we investigated algebraic physics problems. We did not focus on the meaning or nature of physical concepts, because the emphasis is on algebraic skills learned in mathematics class and applied to physics problems.

In this study, our working definition of *successful* (see title of this thesis) refers to using systematic algebraic strategies during algebraic problem-solving in physics. For the operationalization of symbol sense behavior, we followed the line of Bokhove & Drijvers (2010) in the sense that we examined students' basic algebraic skills and having a gestalt view on algebraic expressions and dealing with their visual salience. We operationalized systematic algebraic strategies by measuring the extent to which students demonstrated symbol sense behavior and the degree to which they applied basic algebraic skills correctly during algebraic problem-solving in physics.

For stylistic reasons we will use the concepts procedural skills and procedural fluency (Kilpatrick, Swafford, & Findell, 2001) interchangeably to refer to the same basic algebraic skills. This also applies to conceptual understanding and insightful learning. Furthermore, our sample contained one male and five female students. Therefore, we used 'she' to refer to students participating in this study.

Furthermore, we expected that the algebraic skills that students applied during problem-solving (target tasks) in physics class were learned in mathematics class from regular mathematics textbooks (previous learning situation). We determined the extent to which transfer occurred by adopting the traditional transfer approach (e.g. Mestre, 2015). Indeed, we were interested into the extent to which students used a systematic, rule-based problem-solving approach in which algebraic skills were used with insight. Therefore, we compared the students' solution sets to our systematic solution set that contained the operationalized systematic algebraic strategies above. Hence, the degree to which transfer occurred, is determined by the researchers' perspective. Beyond the traditional view, to some extent, we adopted the actor-oriented transfer approach. Other contemporary views were not adopted, because they were not concerned with algebraic problem-solving in upper secondary education. Indeed, the actor-oriented approach, and especially the study of Roorda (2014) fitted our research. Therefore, we followed the line of Roorda (2014) who operationalized the actor-oriented transfer "*as a search for students' personal constructions of relations between (1) learning from mathematics and physics classes and (2) interview tasks*" (p. 863). For instance, when students explicated that they learned a specific problem-solving approach from their mathematics textbook. In short, beyond the traditional approach to measure the degree of transfer, to some extent we adopted the actor-oriented transfer approach by paying attention to previous learning derived from what they said during the interviews. While earlier studies on actor-oriented transfer studied field notes taken in class to gain deeper insight into students' previous learning situation, we only payed attention to what they said about previous learning in mathematics and physics class, and, to some extent, how algebraic skills were presented in their mathematics textbooks.

5.1.2 Relevance of This Study

Our extensive literature research with various web-search engines such as Google Scholar and ProQuest on scholarly articles reveals the absence of studies investigating symbol sense behavior in algebraic physics problems. Since we examine physics rather than mathematics, investigation of symbol sense behavior may add to the evaluation of this concept. Our literature study also reveals that the mechanisms behind the lack of successful application of mathematics in science subjects are under researched. In addition, students having a solid grasp of mathematics, but facing difficulties in applying this subject in physics, is *highly* under researched. This study may offer insights in how students apply algebraic skills from mathematics in physics and provide insight into these underlying mechanisms that can be used by curriculum developers, mathematics and physics teachers, mathematics and science teacher educators and textbook publishers aiming to improve the application of mathematics in physics, and strengthen students experiencing coherence across these subjects (e.g., Alink, van Asselt, & den Braber, 2012; Berlin & White, 2012, 2014).

As to curriculum developers aiming at CMSE, our study may provide design principles that connects the physics curriculum to that of mathematics. This may be a content standard dealing with the same pedagogical approach to using algebraic skills (Turşucu et al., 2018b). Whether students are in the mathematics or in the physics classroom, they may not be confused about different ways of how algebraic skills are applied and impede compartmentalized thinking (Nashon & Nielsen, 2007; Quinn, 2013). Physics textbooks may contain, for example, introduction paragraphs where physics formulas ($h = \frac{1}{2} \cdot g \cdot t^2$) are treated together with corresponding mathematical expressions ($y = b \cdot x^2$) that students learned in mathematics class. On the individual level, mathematics teachers may provide context to algebra by examining analogous physics problems. On the collaboration level it may be possible to develop common problem-solving strategies where algebraic skills are used with insight into both subjects. Through professional teaching programmes, mathematics and science teacher educators can make mathematics and physics teachers aware (Girvan, Conneely, & Tangney, 2016) of the underlying mechanisms above and discuss remedies.

These issues above are of major importance for many countries aiming at enhancement of the application of algebraic skills from mathematics in physics class, especially improving symbol sense behavior in algebraic physics problems.

5.2 Background

5.2.1 Mathematics in Dutch Upper Secondary Education

The researchers in this study were all affiliated with the academic science teacher education program in Delft in the Netherlands. Hence, we approached Dutch secondary schools rather than those from another country. In this regard, it may worthwhile to discuss shortly the Dutch context in relation to education, especially that of secondary education (SE) that contains three lower years, i.e. lower secondary education and three upper years, i.e. upper secondary education.

According to the 'OECD' (2018), the Netherlands is regarded as an advanced industrial nation where both mathematics and science education are high on the governmental agenda ('Ministry of Education, Culture and Science', 2018). Internationally, Dutch students in upper secondary education score accordingly on mathematics and science assessments, including assessments on physics ('TIMMS & PIRLS', 2019).

In the first year of upper secondary education, grade 10 students who follow physics have to choose between mathematics A and mathematics B. The latter puts more emphasis on algebra than the former. The content of these subjects is described in curricula (SLO, 2019). These curricula contain both the general educational core goals and the more specific standards, which are tested in national final examinations.

Because of the difference in emphasis on algebra in both mathematics subjects, some teachers stated (Turşucu et al., 2017) that mathematics B should be compulsory for physics students. For this belief to be generalizable for the Dutch context, quantitative research is needed.

The algebraic skills in both curricula are mainly associated with algebraic activity (SLO, 2019). Although it is hard to characterize the latter, it involves activities such as implicit or explicit generalizations, patterns of relationships between numbers, and mathematical operations with variables, formulas and expressions (Drijvers, 2011). As a working definition of the concept of formula we used algebraic expressions with real measurable quantities (e.g., speed). An expression can be a formula involving physical quantities or an abstract algebraic expression with abstract mathematical variables (placeholders).

Mathematics curricula refer to *algebraic skills* to cover the entire set of mathematical activities above. These skills are divided into *specific* skills and *general* skills. The first concept deals with knowledge about algebra and manipulation skills, and hence is close to basic algebraic skills (Kieran, 2013; Wu, 1993). General skills contain developing systematic problem–solving strategies and showing insight into the structure of expressions. We conclude that algebraic expertise is not mentioned in these curricula, but the description of using algebraic skills with insight is identical to that of symbol sense in earlier studies (e.g., Andrá et al., 2015; Arcavi, 2005).

Algebraic techniques are part of algebraic skills and used to manipulate expressions (Drijvers, 2011). Hence, they play a key role in this study. Some well–known techniques are 'substitution' used to replace single variables in expressions, and 'multiplication of both sides' where the left and right side of the *equals sign* is multiplied by the same variable. In the next section we will discuss these techniques.

The application of algebra in mathematics A is mainly related to contexts from everyday life. To some extent, this also holds for mathematics B that contains many more abstract problems requiring algebraic proof than mathematics A does.

Regarding the connection between these curricula with the physics curriculum, there is no explicit reference to alignment through compatible notations, concept descriptions and pedagogy of mathematical approaches. Even though it is of major importance for students, also reference to the organization of the learning process in order to achieve a logical learning line across both subjects is absent. As a result, certain mathematical concepts are used in physics class before they were introduced in mathematics class (Turşucu et al., 2018c). Furthermore, the algebra used in mathematics curricula is considered to be sufficient to tackle algebraic problems in physics class (SLO, 2019).

The connection above is of major importance in a very large number of countries, especially in secondary education (e.g., Alink, van Asselt, & den Braber, 2012; Berlin & White, 2012, 2014). A lack of alignment across these subjects, may be confusing for students and impede both their application of algebraic skills in physics and CMSE. For instance, for the lens formula in geometrical optics a mathematics teacher may write $(O-f)(i-f)=f^2$, while a physics teacher writes $O^{-1}+i^{-1}=f^{-1}$ $(O\neq 0, i\neq 0)$ (Turşucu et al., 2018b). Such mismatches may also hold for concept descriptions and the pedagogy of mathematical approaches above. Therefore, curriculum developers should explicate the importance of connection across both subjects.

5.2.2 Algebra in Physics

Dutch students in secondary school start with physics in their second year of LSE (grade 8). Quantities are introduced through a strong context-concept approach (e.g., Bemmel et al., 2013). The number of formulas describing physical quantities in this year is negligible, let alone using algebraic skills to manipulate formulas. In the next year, formulas are used more frequently (e.g., Alkemade et al., 2014), but the algebraic skills level needed to manipulate them is low. This changes in grade 10, immediately after the transition from lower secondary education to upper secondary education where the intended level of algebraic skills increases substantially (e.g., Ottink et al., 2014). Indeed, as mentioned above, among students in upper secondary education, grade 10 students face the biggest difficulties with applying mathematics in algebraic physics problems (Turşucu et al., 2017).

Most of the physics formulas are symbolic representations of proportionalities containing real, measurable quantities expressed in various symbols. For instance, whereas the potential energy $E_{\text{pot.}} = m \cdot g \cdot h$ and the period of a spring-mass system $T = 2 \cdot \pi \cdot \sqrt{\frac{m}{C}}$ are proportional to h and $\sqrt{m}$ respectively, the attractive gravitational force $F_{\text{G}} = G \cdot \frac{m \cdot M}{r^2}$ is inversely proportional to r^2. The formulas in algebraic physics problems and the algebraic skills needed to solve them are described in the Dutch physics curriculum (SLO, 2019). These formulas can also be found in BINAS, a natural sciences information booklet that students use during regular physics tests and the final examination.

In recent years, algebraic physics problems including algebraic curve straightening, for example, the curve related to Boyle's law $P \cdot V = c$, derivation of formulas such as deriving the escape velocity $v_{\text{escape}} = \sqrt{\frac{2 \cdot G \cdot M_{\text{sun}}}{r}}$ from our solar system, and dimensional analysis, for instance showing that the quantity acceleration due to gravity g in the free fall formula $h = \frac{1}{2} \cdot g \cdot t^2$ has the unit $\frac{m}{s^2}$, have become more important in upper secondary education. Solving them correctly, i.e. using mathematically correct procedures requires sufficient algebraic expertise with basic algebraic skills and demonstration of symbol sense behavior.

5.2.3 Mathematics and Physics Teaching

In many countries (Stein & Smith, 2010) including the Netherlands (SLO, 2019; Turşucu et al., 2017), textbooks mediate between both the core goals and standards of education (the intended curriculum) and the actual teaching in classrooms (the implemented curriculum). They are very closely followed by teachers, who teach their students from these books. Therefore, to a very large extent, textbooks shape classroom practice. As to algebra education, physics students apply mathematics and especially algebraic skills that they have learned in mathematics textbooks to solve algebraic physics problems in physics lessons.

Since explicit reference to connection in Dutch mathematics and physics curricula is absent, there is also no alignment between mathematics and physics textbooks. This can impede students' successful application of mathematics in physics (e.g., Alink, van Asselt, & den Braber, 2012; Berlin & White, 2012, 2014). Hence, the connection between these subjects mainly depends on individual efforts. For instance, physics teachers designing teaching materials that aim to align both subjects through content. Such individual attempts are also of major importance for countries without curricula containing both the general educational core goals and the more specific standards, which are tested in national final examinations.

5.2.4 *Algebraic Expertise in Detail*

Similar to Arcavi (1994), Drijvers (2011) views algebraic expertise as a spectrum extending from basic algebraic skills (Kieran, 2013; Wu, 1993) to symbol sense involving conceptual understanding. This is illustrated in figure 3 of Chapter 1. Whereas basic algebraic skills deal with procedural work with a local focus and algebraic reasoning, symbol sense concerns strategic work with a global focus and emphasis on algebraic reasoning. In our case, strategic work refers to a physics student who is in control of the work and seeks for a different systematic approach when a strategy appears to be insufficient. Having a global focus is related to recognition of patterns in physics formulas and equations where these formulas are involved. Algebraic reasoning deals, for example, with extreme cases and symmetry considerations. Analogous to Bokhove & Drijvers (2010), we focus on the relationship between local and global, and procedural and strategic work.

Concerning the relationship between basic skills (Kieran, 2013; Wu, 1999) and conceptual understanding (Schoenfeld, 2016), the last decades have been an arena for a long-standing debate called '*Math Wars*' related to how students best acquire algebraic expertise: by first practicing standard procedures or focusing on insightful learning? This pedagogical war led to conflicting ideas about designing curricula, content of textbooks and effective teaching practice. In recent years, this discussion shifted towards the belief that both procedural skills and conceptual learning should be taught in an integrated manner (Rittle-Johnson, Schneider & Star, 2015). To improve algebraic expertise, one needs to view their relationship as bidirectional and continuous: "*understanding of concepts makes basic skills understandable, and basic skills can reinforce conceptual understanding*" (Bokhove & Drijvers, 2010, p. 43). This view on algebraic expertise will also be our point of departure.

As for teaching practice, such an integrated approach may involve ideas for teaching algebraic skills in both mathematics and physics class. For instance, to show that g in the formula $h = \frac{1}{2} \cdot g \cdot t^2$ has the unit $\frac{m}{s^2}$, one may first isolate g. Solving for g requires insight into algebraic techniques. Physics textbooks may need introductory paragraphs summarizing prior mathematical knowledge that students learned in their mathematics textbooks. This idea is based on the importance of activation of pre-knowledge in the context of learning and instruction in relation to better students' achievements (e.g., Hailikari, Katajavuori, & Lindblom-Ylanne, 2008; Turşucu et al., 2018c). For instance, solving the analogous mathematical expression $y = b \cdot x^2$ for x gives $x = \sqrt{\frac{y}{b}}$. Next, g in $h = \frac{1}{2} \cdot g \cdot t^2$ is solved. Substitution of units in $g = \frac{2 \cdot h}{t^2}$ is regarded as basic algebraic skills. This integrated approach can be extended to mathematics, where physics formulas may be written next to mathematical expressions.

5.2.5 *Systematic Algebraic Strategies Versus Ad Hoc Strategies*

In this study we distinguish between 'systematic algebraic strategies', i.e. using algebraic skills with insight as described in the curriculum (SLO, 2019), and the application of ad hoc strategies. For stylistic reasons we use 'ad hoc strategies' and 'ad hoc approach' interchangeably to denote the same. This also holds for 'systematic algebraic strategies' and 'systematic algebraic approaches'. As a working definition of 'systematic algebraic strategies' we use a systematic, rule-based problem-solving approach in which algebraic skills are used with insight, where 'rule' refers to the standard rules for multiplication and division of powers, such as $x^{\mathrm{a}} \cdot x^{\mathrm{b}} = x^{\mathrm{a+b}}$, which play the role of algebraic axioms in high school algebra. Therefore, using systematic algebraic approaches are associated with applying algebraic skills systematically and correctly. With 'ad hoc strategies' we refer to mathematical strategies that are *not* based on standard algebraic rules with insight, and only work for a specific case that may lead to fragmented knowledge, impeding generalization of algebra. Especially, in more sophisticated problems for which insight is needed rather than ad hoc strategies, students may get stuck. In addition, applying them depends on the approval of an authority. For instance, a student may be skilful in using the formula triangle of the form $a = b \cdot c$, but gets stuck when solving for b in $a = b \cdot c \cdot d$. She only succeeds after a teacher, i.e. authority introduces a new ad hoc approach. While such strategies may not always yield correct solutions, they can be useful as initial attempts to solve a problem (Roorda, 2012). Thus, ad hoc strategies may be harmful for students' application of algebraic skills in physics, mainly because of the lack of insight into algebraic skills.

Concrete examples of ad hoc strategies are the application of mnemonics such as the formula triangles above, substitution of numbers for variables to verify whether an operation will result in a valid outcome or guessing a solution for a problem and then working backwards.

In this study, using algebraic skills with insight becomes visible through the application of algebraic techniques during procedures involving both basic algebraic skills and symbol sense behaviour, i.e. having a gestalt view on algebraic expressions and dealing with their visual salient aspects local salience and pattern salience. Our working definition of *successful* in the title of this dissertation refers to the application of systematic algebraic strategies during algebraic problem-solving in physics class, which is operationalized by measuring the extent to which students demonstrate symbol sense behavior and the degree to which students apply basic algebraic skills properly. We assume that students learned these algebraic skills in mathematics class from their teachers who strictly follow their mathematics textbooks (SLO, 2019). Our operationalization of both basic algebraic skills and demonstration of symbol sense behaviour is explained in the next section below.

5.3 Methodology

5.3.1 Selection Criteria for Participants

To gain insight into students' symbol sense behavior during algebraic problem-solving in physics, we needed two different groups, each containing three anonymized grade 10 physics students from a regular school. To this extent, we used convenience sampling (Bryman, 2015) to find two physics teachers who together with their students were *available* and *willing* to participate in this study. Next, we used the 'Interview Protocol Physics Teachers' in the third subsection of the appendix to conduct an interview with them and select appropriate students. To ensure appropriate length and clearness, this protocol was redesigned several times and tested on different teachers and social scientists during the pilot-phase prior to this study. The physics teachers used 'Magister', a student monitoring system for secondary education ('accounts.magister.net', 2019) to select appropriate students based on their mathematics and physics grades. Based on the Dutch ten-point grading system, these students had a sufficient mathematics grade and an insufficient physics grade, i.e. < 5.5. This grade criterion indicates that students' difficulties with algebraic physics problems were mainly because of insufficient application of algebraic skills in physics, and not related to a lack of basic mathematics. To ensure that it was legitimate to compare the students' individual results and that of both groups, the students should use the same mathematics and physics textbook *and* have a similar knowledge domain at the start of these interviews. These selection criteria were so strong, that we only found two schools satisfying these criteria. For Group (I) of school (I) this yielded Aron (the only male student in this study), Beth and Chloe with mathematics A, *and* for Group (II) of school (II) Diana, Eve and Fiona with mathematics B of Group (II). They all used the physics textbook *SysNat* (Ottink et al., 2014a) and the same mathematics textbook series Getal & Ruimte: Reichard et al. (2014a) for Group (I) and Reichard et al. (2014b) for Group (II). The details are presented in table 1. The physics grades range from 5.0 up to 5.4 and for mathematics from 6.4 up to 7.9.

Table 1. Mathematics and physics grades of the respondents.

	Aron	Beth	Chloe	Diana	Eve	Fiona
Grade mathematics A	7.0	7.0	6.4	–	–	–
Grade mathematics B	–	–	–	6.6	7.1	7.9
Grade physics	5.0	5.4	5.3	5.2	5.4	5.3

5.3.2 Design of The Tasks

The tasks were designed so that they should trigger students solving algebraic physics problems and provide insight into their algebraic expertise with basic algebraic skills and symbol sense behavior. Based on these design principles, we first selected four exercises from the physics textbook SysNat (Ottink et al., 2014a; 2014b) that we called 'Ideal gas' (Task 1), 'Falling stone' (Task 2), 'Uniform circular motion' (Task 3) and 'Spring-mass system' (Task 4). Only 'Task 3' was identical to that of the physics textbook. The other exercises were adjusted into algebraic physics problems described in symbolic representations (Goldin, 2000). Next, these tasks were solved by grade 10 students who did not participate in this study. This pilot-phase provided us information about the appropriateness of these problems, such as *clearness*, *length* and that these problems were doable by students. After analyzing their work, some of the tasks were slightly adjusted to meet our design principles. This resulted in 'The Tasks' described in the appendix. The 'Systematic Solution Set to The Tasks' (solution set in short) are also presented in the appendix. For stylistic reasons we abbreviated procedures involving basic algebraic skills as 'BAS', local salience as 'LS' and pattern salience as 'PS'. Furthermore, we focused on rewriting formulas and solving kinematic energy relations.

5.3.3 Design of Task-based Interviews (TBIs)

The TBIs have their origin in clinical interviews that were used by Piaget (1954) to gain deeper understanding of students' cognitive development. Such clinical studies are regarded as qualitative research. Conducting TBIs to gain insight into students' problem-solving *behaviors* follow from gaining deeper understanding of students' cognitive development. Therefore, TBIs are part of qualitative research (e.g., Bokhove & Drijvers, 2010; Maher & Sigley, 2014). This implies that our study can also be considered as a qualitative study. The quantitative component of this study is related to the operationalization of *successful* in the title of this dissertation, which was operationalized by measuring both the extent to which students demonstrated symbol sense behavior and the degree to which students apply basic algebraic skills properly. Our interviews were carefully designed so that students had only interaction with the tasks and the interviewer. Therefore, the TBIs were conducted by two independent researchers (one per group) in an appropriate, quiet place. The TBIs took approximately 40 minutes and were based on a structured protocol consisting of *two* parts that was designed in such a way that the instructions were *clear* and students could easily work with it. Based on the feedback of the non-participating students above, some parts of this protocol were reorganized and rewritten. These adjustments led to 'Interview Protocol for Students' in the fourth subsection of the appendix. In the *first* part students were asked questions about their background, the textbooks they used and their mathematics and physics grades were double checked. In the *second* part they solved 'The Tasks' while thinking aloud (Charters, 2003). The interviews were videotaped. Next, the audio part was transcribed *verbatim*, for which the students gave consent. The interviewer only interrupted when a procedure or reasoning was not clear enough or she remained silent for one minute. We used stimulated recall techniques (Geiger, Muir & Lamb, 2016) to get as much information as possible on the students' solutions. If necessary, we provided small hints.

5.3.4 Data Analysis: Phase 1 – Phase 4

Data analysis of videotaped data comprises seven consecutive phases (Powell, Francisco & Maher, 2003), not requiring a rigid order. Regarding this study, in 'phase 1' (viewing attentively the video data) we acquired a first and general understanding of how the respondents solved the algebraic physics problems. Since their behavior was video recorded, we could easily identify the first interesting and relevant observations such as the application of ad hoc strategies. 'Phase 2' (describing the video data) was less important, because the relevant information in videotaped data was captured in detail by the interview transcripts. This process is described in 'phase 4'. In 'Phase 3' (identifying critical events) we identified critical events, i.e. students' application of algebraic techniques, ad hoc approaches and other relevant steps during problem solving in which a mathematical explanation or argument was involved. These events are further described in 'phase 5'. In 'phase 4' (transcribing) the audio part of videotaped data were transcribed *verbatim*.

5.3.5 Data Analysis: Phase 5

In 'phase 5' (coding) we operationalized research question (4) through the coding scheme (spread sheet) in table 2. The coding process was based on analyzing videotaped episodes (to some extent), the transcripts of the audio part of videotaped data, and the students' written solution set to the tasks. Their solution set was compared to our systematic solution set in Appendix A and coded afterwards using table 2. The dots in the cells indicate that they are empty and should be filled in. This process is explained in the next subsection. Thus, table 2 is complete.

Prior to the TBIs, we identified both the required algebraic technique (Drijvers, 2011) and the needed basic algebraic skills or symbol sense type in the systematic set. Later, this was compared to students' written solution set and assigned to scores to gain insight into their symbol sense behavior (Bokhove, 2011). For instance, to solve sub task a) of 'Task 1 Ideal gas' in Appendix A systematically, procedure 1 requires the application of 'multiplication of both sides' of $\frac{P \cdot V}{T} = C$ by T which yields $P \cdot V = C \cdot T$, since $T \cdot \frac{P \cdot V}{T} = T^{1-1} \cdot P \cdot V = P \cdot V$. This implies that this procedure requires students' sensitivity towards the exponent -1 in T^{-1}, and is associated with the symbol sense type local salience.

Table 2. Coding scheme to analyze students' symbol sense behavior.

Time	Subtask	Algebraic Techniques	Ad hoc strategies	Trigger
…	1a	…	…	…
…	1b	…	…	…
…	2a	…	…	…
…	2b	…	…	…
…	3a	…	…	…
…	3b	…	…	…
…	4	…	…	…

Students may use seemingly different approaches than manipulating exponents above. For instance, cancelling out variables $\cancel{m} \cdot g \cdot h = \frac{1}{2} \cdot \cancel{m} \cdot (v_{final})^2$ that is mathematically equivalent to working with exponents. Such procedures are also correct and do not affect students' symbol sense behavior. This also holds for interchanging procedures.

How was the coding scheme used?

The first column 'Time' in Table 2 refers to the interview timeline in which a critical event was observed.

The column 'Subtask' refers to the subtasks in Appendix A. Except for the columns 'Time', 'Subtask' and 'Ad hoc strategies', the columns 'Algebraic Technique' and 'Trigger' contained drop-down boxes with each having different options. Based on the used algebraic technique during a procedure, one of the options 'multiplication of both sides', 'division of both sides', 'substitution', 'taking the square root of both sides', 'squaring both sides' and 'subtraction from both sides' of the third column 'Algebraic Technique' was selected (coded) in the spreadsheet. If instead of the latter, she applied ad hoc strategies, the details were thoroughly described in the column 'Ad hoc strategies'.

The last column 'Trigger' contains the options 'positive', 'negative' or 'missed opportunity'. A procedure was coded 'positive' or 'negative' when she chose the right procedure, thereby respectively executing correctly with score '1', and incorrectly with score '0,5'. 'Missed opportunity' refers to the third scenario when she overlooked a required procedure (for instance, when the student made no attempt to solve the task), or used ad hoc approaches. This led to SSB (%), the symbol sense behavior percentage per subtask. Other scores are OSSB (%), the overall symbol sense behavior percentage for the whole set of subtasks, and OBAS (%), the overall basic algebraic skills percentage for the whole set of subtasks. These two scores are calculated by $\frac{sum\ of\ all\ sub\ task\ scores}{25} \cdot 100\ \%$ and $\frac{sum\ of\ all\ sub\ task\ scores\ requiring\ BAS\ only}{5} \cdot 100\ \%$ respectively. As can be seen in the solution set of appendix A, the number '25' in OSSB (%) is the sum of twenty procedures involving local salience and pattern salience, and five involving basic algebraic skills. Performing each procedure flawlessly yields the maximum score of 25.

Furthermore, a student was regarded procedurally fluent if OBAS (%) ≥ 90.0 % (4,5 out of 5 points). We regarded OSSB (%) to be sufficient when OSSB (%) ≥ 80.0 % (20 out of 25 points). Based on these criteria, *successful* in the title of this dissertation refers to the application of systematic algebraic strategies with OBAS (%) ≥ 90.0 % (the criterion for applying basic algebraic skills successfully) and OSSB (%) ≥ 80.0 % (the criterion for *successful* demonstration of symbol sense behaviour).

5.3.6 Data Analysis: Phase 6 and 7

In 'phase 6' (constructing storyline) we identified the ad hoc strategies that were used, and then determined the students' SSB (%) per subtask followed by OBAS (%) and OSSB (%). This can be seen the first three subsections of the next section.

As to 'phase 7' (composing narrative), the transcripts, students' written solution set, and the findings from 'phase 6' were further integrated, leading to a narrative containing common findings. These are presented in the last two subsections of the next section.

To enhance reliability of our results (Bryman, 2015), the two independent researchers crosschecked their results. Next, this was double checked by the first author, who found an overlap of approximately 95%. After discussing the remaining 5%, some adjustments were made which led to 100% agreement among them.

5.4 Results

To show variability and chose maximal variation in the selection of cases, we only present the results of Diana, Eve and Chloe (Although each students' work was analyzed in detail).

5.4.1 Diana

Diana has a 6.6 for mathematics B and a 5.2 for physics. She used the permutation strategy to solve task 1a and b, 3a and task 4. With the tasks 3a and 4 she faced serious difficulties, and for task 2a and 3a she needed hints, but she did not use ad hoc approaches. Consequently, her interview lasted long, i.e. circa 60 minutes.

Her symbol sense behavior characteristics per subtask can be found in table 3. The second (1a) up to the eighth column (4) of the first row, each represent a subtask. The second row 'Missed opportunity' shows the sum of how many procedures per subtask she made no attempt to solve a task or used ad hoc strategies. For subtask 2b, this number is one. Furthermore, three procedures were correct (positive score: 3) and one was not (negative score: 0.5).

Table 3. Symbol sense behavior characteristics of Diana per subtask.

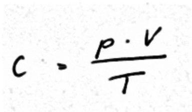
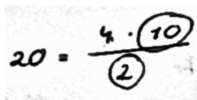
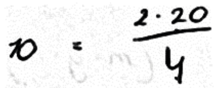
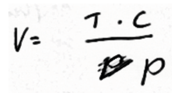
	1a	1b	2a	2b	3a	3b	4
Missed opportunity	2	2	1	1	1	3	1
Negative score	-	-	0.5	0.5	1	0.5	0
Positive score	-	-	2	3	0	1	3
Subtask max. score	2	2	4	5	3	5	4
SSB (%)	0	0	62.5	70.0	33.3	30.0	75.0

We calculated her SSB (%) per subtask. For subtask 2b this becomes $\frac{0,5+3}{5} \cdot 100\% = 70.0\ \%$. Her total number of 'Missed opportunity' corresponds to 11 (six ad hoc strategies and five overlooked procedures) including one basic algebraic skills procedure, eight local salience and two pattern salience procedures. This corresponds to a large 44.0 % of the perfect score. Her lowest SSB (%) concerns the subtasks 1a and 1b (0 % each) and her highest subtask 4 (75.0 %). The latter corresponds to the well-performed first three procedures. For the last procedure, she substituted numbers for variables and got stuck. Diana's OBAS (%) is $\frac{3.5}{5} \cdot 100\ \% = 70.0\ \%$. Hence, she lacks a solid domain of basic algebraic skills. Her OSSB (%) is $\frac{11.5}{25} \cdot 100\ \% = 46.0\ \%$. This score was mainly due to the application of ad hoc approaches, implying the absence of sensitivity towards systematic procedures with local salience and pattern salience. Instead of showing insight into the application of algebraic techniques, there was a focus on the permutation strategy. This is illustrated in table 4. So, based on our working definition of *successful* in the title of this thesis, Diana was unsuccessful in both applying basic algebraic skills (OBAS (%) = 70.0 % $\leq$ 90.0 %) and demonstration symbol sense behavior (OSSB (%) = 46.0 % $\leq$ 80.0 %).

Based on trial–and–error, she permutated numbers to discover the arrangement of valid outcomes for subtask 1a. Although this strategy provided right outcomes for subtask 1a, 1b, 3a and 4, she made mistakes when the substituted number of variables became larger. Especially, when numbers are identical, but represent different variables. For instance, for subtask 3a she substituted numbers for $G \cdot \frac{m \cdot M}{r^2} = \frac{m \cdot v^2}{r}$, but lost her overview and failed in making the next step and needed a hint. This process of using an ad hoc approach followed by failure and a hint also applies for subtask 4.

During the evaluation of her work, Diana indicated that the permutation strategy was "*actually a residual technique acquired in mathematics class in Grade 8*". This may imply that they learned ad hoc strategies, rather than the mathematics teacher putting emphasis on insight into *why* and *how* algebraic skills are used systematically.

Table 4. Diana's permutation strategy to solve subtask 1a.

Step 1. The variables in are  substituted into	
Step 2. As a strategy to solve for variable $\boldsymbol{V}$, Diana iteratively permutes (trial and error) the numbers while checking that the result of the division remains valid.	
Step 3: Finally, she substitutes the corresponding variables back, which remarkably leads to the correct solution:	

The corresponding fragment to subtask 1a:
"*What I can do is... $C = 8$, and then P is 8, $V = 2$ and $T = 16$. So, if P should be 8 then I should interchange C and P. So, the formula would be... oh no! I must find V. I should interchange 2 and 8. So, wait. If I write $8 = \frac{8 \cdot 2}{16}$ and if I want V, then V in this case is 2. No, that's not going to work. If I interchange C and P, then I get 64 divided by 16 and that is 2. Uhmm, I'm going to take other numbers. $C = 4$, P becomes... uhmm... together they should be 8, so I get 4 again. Uhmm, 20... $C = 20$, $C = \frac{P \cdot V}{T}$ =... divided by... what to do next? 2 or no, $\frac{4 \cdot 10}{2}$, then $V = 10$. If I'll get 10, then the formula must look like.. hmm... $\frac{2 \cdot 20}{4}$... yes. So, then I get $\frac{2 \cdot 20}{4}$ and that is right. So $10 = V$, $2 = T$, times... $20 = C$ divided by 4 is P. Let me check... uhmm... is this okay? Oh no! Yes... yes!*"

5.4.2 Eve

Eve is a female student with a 7.1 for mathematics B and a 5.4 for physics. As to her TBI, she often used a combination of algebraic techniques with insight and the permutation strategy for task 1a and 1b and the numbering strategy for task 2a, 3a and 4. The numbering strategy is different than the former strategy and aims at simplification of formulas or validation of solutions with insight. She frequently switched from systematic algebraic strategies to ad hoc strategies, and only used a hint for 2b. Her interview took 67 minutes.

Eve's symbol sense behavior characteristics are shown in table 5. Her 'Missed opportunity' corresponds to seven including one basic algebraic skills procedure, five local salience and one pattern salience procedure (six ad hoc approach and one overlooked procedure).

Table 5. Symbol sense behavior characteristics of Eve per sub task.

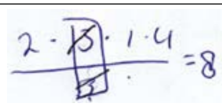
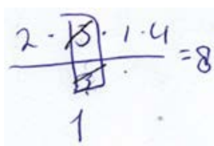
	1a	1b	2a	2b	3a	3b	4
Missed opportunity	1	2	1	-	1	1	1
Negative score	-	-	-	1	0.5	-	0.5
Positive score	1	-	3	3	1	4	2
Sub task max. score	2	2	4	5	3	5	4
SSB (%)	50.0	0	75.0	80.0	50.0	80.0	62.5

This corresponds to 28.0 % of the maximum OSSB (%), which is low compared to Diana. Her highest SSB (%) is both for subtask 2b (80.0 %) and subtask 3b (80.0 %), and her lowest subtask 1b (0%) where she used the numbering strategy to validate her answer. As Diana, this lower score was mainly related to using ad hoc strategies. Contrary to other subtasks, the algebraic techniques involved in 2b and 3b were used in a right manner and with insight.

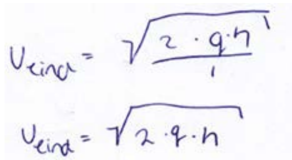
Both her OBAS (%) of 70.0 % (3.5 points) and her OSSB (%) with 64.0 % (16 points) are insufficient, but her OSSB (%) is higher than that of Diana. Without the application of ad hoc strategies, especially in task 1 and 2a, both her OBAS (%) and OSSB (%) would have been sufficient. Eve's numbering strategy for subtask 2a is depicted in table 6. Even if this approach led to the correct solution, she did not understand why it was legitimate to cancel out the masses *m* in the square root sign. This is related to a lack of insight into algebraic skills. So, like Diana, Eve was unsuccessful in both applying basic algebraic skills (OBAS (%) = 70.0 % ≤ 90.0 %) and demonstration of symbol sense behavior (OSSB (%) = 64.0 % ≤ 80.0 %).

After her TBI, Eve mentioned that both the numbering and the permutation strategy were learned in mathematics class in grade 8. This confirms Diana's statement on the permutation strategy, since they are classmates since grade 8. Nevertheless, Diana did not use the numbering strategy.

Table 6. Eve's numbering strategy to solve subtask 2a.

Step 1: the variables in are substituted in		
Step 2: verification that $\frac{3}{3}$ is equal to	 1	

Step 3: the corresponding variables are substituted back, with the two variables m being cancelled out:



The corresponding fragment to the steps 1, 2 and 3:
"*I think that one of both m's should be cancelled out. Yes,* $\frac{m}{m}$ *is* 1. *So, maybe I should take away the upper m and then... no. I must put the number* 1 *in front of* m*, but then I'll get the same number. So, let me see... let us say that we will get* $\frac{2\cdot3\cdot1\cdot4}{3}$ *if you take away the* 3. *And then this is* [$\frac{3}{3}$] *equal to* 1. *And, normally, this should... should...* 24 *divided by* 3. *This should give* 8. *If I take away* $\frac{3}{3}$*... and then there, then... maybe I should put there a* 1*? Okay. So, both m's cancel out. So, this gives* $2\cdot g\ \frac{2\cdot g\cdot h}{1}$. *And that* [1] *you could also leave out. So, finally I get* v_{final} *is* $2\cdot g\cdot h$."

5.4.3 Chloe

Chloe is a female student with a 6.4 for mathematics A and a 5.3 for physics. She worked much faster than Diana and Eve, and *only* used the cross-multiplication strategy to solve task 1a, 1b and 3a. Chloe seems to have automated this approach that she performed smoothly for task 1. Hence, her TBI lasted 37 minutes.

Chloe's symbol sense behavior characteristics are displayed in table 7. Her 'Missed opportunity' is thirteen and contains one basic algebraic skills procedure, eight local salience and nine pattern salience procedures (five ad hoc approaches and eight overlooked procedures). This equivalent to 52.0 % of the maximum OSSB (%), implying a lack of symbol sense behavior. Her highest SSB (%) is for subtask 2a (75.0 %) where she lost points for the last two procedures. During the third procedure she did not multiply the value 0.5 with 2. Her lowest SSB (%) is for task 1 (0 %). Chloe's insufficient OBAS (%) of 70.0 % (3.5 points) is identical to that of Diana and Eve. Her OSSB (%) of 38.0 % (9.5 points) is lower than them. In summary, like the other students, Chloe was unsuccessful in both applying basic algebraic skills (OBAS (%) = 70.0 % ≤ 90.0 %) and demonstration of symbol sense behavior (OSSB (%) = 38.0 % ≤ 80 %).

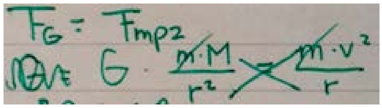
Table 7. Symbol sense behavior characteristics of Chloe per subtask.

	1a	1b	2a	2b	3a	3b	4
Missed opportunity	2	2	-	3	1	4	1
Negative score	-	-	1	1	-	-	0.5
Positive score	-	-	2	-	2	1	2
Subtask max. score	2	2	4	5	3	5	4
SSB (%)	0	0	75.0	20.0	66.7	20.0	62.5

In table 8 we show Chloe's cross-multiplication strategy in subtask 3a. Although Chloe intended, she forgot to cross multiply and at the same time cancelled out the masses *m*. Probably, she lost her overview.

For subtask 2b, she cancelled out the masses in $m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot (v_{final})^2 + F_{res} \cdot h$. This procedure is forbidden and implies that the previous cancellation in subtask 3a was not based on understanding, but on routine based on ad hoc approaches. Indeed, during the evaluation Chloe mentioned that she used ad hoc strategies, but did not understand why these were mathematically incorrect. For instance, for subtask 1 she writes $\frac{P \cdot V}{T} = \frac{C}{1}$ and then cross-multiplied, yielding $P \cdot V = C \cdot T$. It turned out that she learned this approach from the mathematics A textbook. Furthermore, as for task 3b, Chloe mentioned that she lost her overview and got stuck because of the large number of variables.

Table 8. Chloe's cross-multiplication strategy and cancellation of m's to solve subtask 3a.

Step 1: she first uses the cross-multiplication strategy and then immediately cancels out the masses during the same procedure.	
Step 2: although she forgets to perform cross-multiplication, she writes the correct result of the previous procedure. In the same procedure, she cancels out the radiuses.	
The corresponding fragment to the steps 1 and 2: "*I think I'll first use the cross-multiplication strategy. So, this multiplied by that, and this multiplied by that. This is much easier to do, since there are no fractions involved. Now I should cancel out the masses. Then we get* $G \cdot \frac{M}{r^2} = \frac{v^2}{r}$. *And now I can cancel out the radiuses.*"	

5.4.4 OBAS (%) and OSSB (%)

In the previous section OBAS (%) was incorporated into OSSB (%). To gain more insight into the relation between them, we placed OBAS (%) next to OSSB (%), see figure 2. Both Beth (A) with mathematics A and Fiona (B) with mathematics B had insufficient OSSB (%) and OBAS (%). Aron (A)'s OSSB (%) was sufficient (80.0 %), but his OBAS (%) was not. Except for Aron, all students lacked both sufficient basic algebraic skills and insight. So, he was the only student who demonstrated symbol sense behavior successfully. This also implies that in terms of tasks requiring symbol sense behavior, he was the only student who successfully transferred algebraic skills from mathematics into physics.

We also calculated the ratio of $\frac{\text{OBAS (\%)}}{\text{OSSB (\%)}}$ per student. Except for Diana and Chloe, this number for the other students is roughly 1. For them, their OBAS (%) might be used as a reasonable predictor for their OSSB (%).

On the individual level, Aron has both the highest OSSB (%) (80,0 %) and OBAS (%) (76.0 %). The lowest OSSB (%) and OBAS (%) are respectively for Chloe (38.0 %) and Fiona (50.0 %). Furthermore, Chloe, Diana and Eve have the same OBAS (%) (70.0 %), but a different OSSB (%).

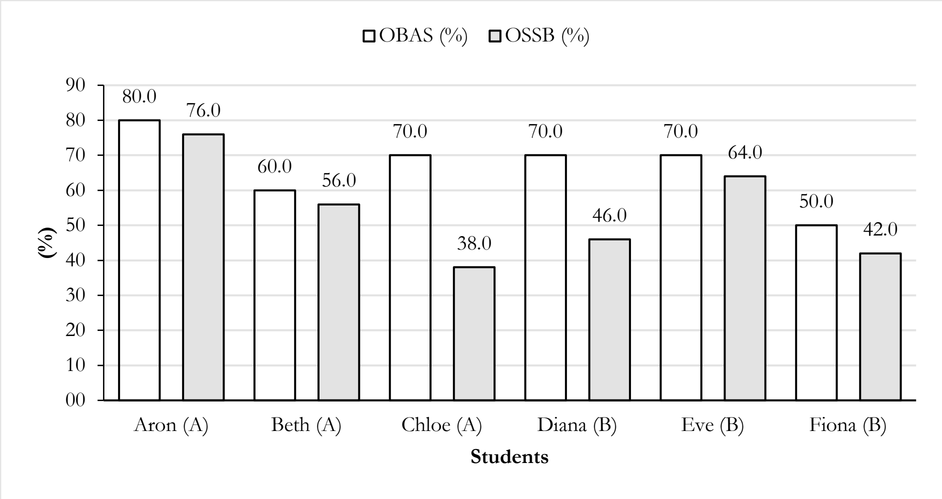


Figure 2. OBAS (%) and OSSB (%) per student

We also calculated ⟨OBAS (%)⟩, i.e. the average of OBAS (%) for each mathematics group. This also applies for ⟨OSSB (%)⟩, i.e. the average of OSSB (%). For group (I) we found ⟨OBAS (%)⟩ = 70.0 % and ⟨OSSB (%)⟩ ≈ 56.7 %, and for group (II) ⟨OBAS (%)⟩ = 63.3 % and ⟨OSSB (%)⟩ = 50.7 %. Concerning ⟨OSSB (%)⟩, there is a difference of 6.7 % between these groups. For ⟨OBAS (%)⟩ this is 6.0 %. Although group I performed slightly better than II, these differences are reasonably small, and can be neglected. Furthermore, we found ⟨OBAS (%)⟩ = 70.0 % and ⟨SSB (%)⟩ ≈ 55.3 % for the average OSSB (%) of all students. These values converge to the findings above: none of the students have both sufficient procedural skills and symbol sense behavior.

5.4.5 ⟨SSB (%)⟩ per Subtask

Below in figure 3 we displayed ⟨SSB (%)⟩, i.e. the average SSB (%) per subtask among all students.

Because of the criterion OSSB (%) ≥ 80.0 %, ⟨SSB (%)⟩ ≥ 80.0 % was regarded sufficient. None of the tasks met this criterion, which confirms students' insufficient OSSB (%). Students' average OBAS (%) was not incorporated in figure 3, since subtask 1a, 1b and 4 did not contain basic algebraic skills procedures.

We may say that there are two regimes of scores. Sub task 1a, 1b and 3b belong to the very low (ranging from 33.3% up to 41.7%), and subtasks 2a, 2b, 3a and 4 to the less high scores (ranging from 52.8 % up to 68.8 %). For subtask 1a (41.7 %) and 1b (33.3 %), Chloe, Diana, Eve and Fiona used ad hoc approaches, which strongly impeded ⟨OSSB (%)⟩ of both subtasks.

Regarding subtask 2a, most students faced difficulties with procedure 2. They were required to think globally and demonstrate sensitivity towards pattern salience by dividing both sides by *m*. They performed well on the other procedures. Especially, the last procedure for which they achieved the maximum score. This contributed to a higher ⟨SSB (%)⟩, i.e. 68.8 %.

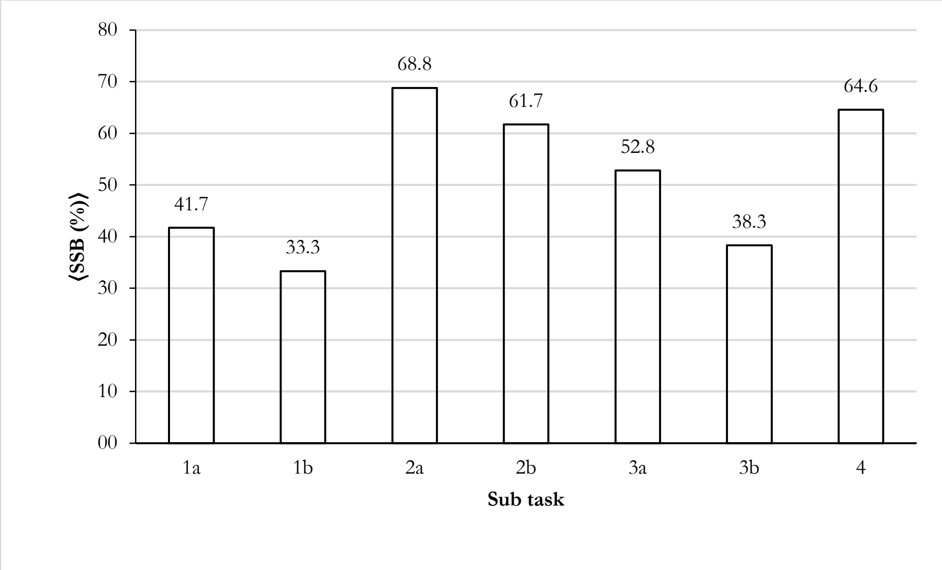


Figure 3. ⟨SSB (%)⟩ per subtask

With respect to subtask 2b, most students lost points in the first and fourth procedure. Only one student performed flawlessly on the first procedure involving basic algebraic skills. For the fourth procedure students encountered problems to cancel out the $\frac{1}{2}$ in $\frac{m \cdot g \cdot h - F_{\text{res.}} \cdot h}{m} = \frac{1}{2} \cdot (v_{\text{final}})^2$. After multiplying both sides with 2, they got rid of the $\frac{1}{2}$ in the term $\frac{1}{2} \cdot (v_{\text{final}})^2$. Still, they struggled with the meaning of 2 in the expression $2 \cdot \frac{m \cdot g \cdot h - F_{\text{res.}} \cdot h}{m}$. They often only multiplied $m \cdot g \cdot h$ by 2, thereby ignoring $\frac{-F_{\text{res.}} \cdot h}{m}$.

As to 3b, most of the students faced problems with procedures 3, 4 and 5. Chloe, Aron, Beth and Fiona explicitly mentioned that they lost their overview and were discouraged because of the large number of variables (the largest among all subtasks) in $G \cdot \frac{M}{r} = \left(\frac{2 \cdot \pi \cdot r}{T}\right)^2$. The same applies, to some extent, for 3a. Here, most students used ad hoc strategies to solve it, explaining the insufficient ⟨SSB (%)⟩ = 52.8 % above.

Concerning task 4, students built upon insights acquired in the preceding subtasks, partially driven by hints. With ⟨OSSB⟩ = 64.6 %, these insights resulted in a similar high score as 2a.

5.5 Discussion and Conclusion

The purpose of this study was to measure the extent to which upper secondary students demonstrate symbol sense behavior when solving algebraic physics problems. The main difference with previous studies (e.g., Bokhove, 2011; Drijvers, 2015) is that these tasks contain expressions with variables relating to real, measurable physical quantities, and other studies to abstract mathematical variables without meaning in real life.

With regard to operationalization of symbol sense behavior, we followed the line of Bokhove & Drijvers (2010) in the sense that we focused on students' sensitivity towards gestalt view and visual salient aspects of algebraic expressions. While they used a digital environment to assess students' work, we deployed traditional pen-and-paper settings involved in other studies (Arcavi, 1994, 2005; Wenger, 1987). Moreover, aforementioned studies have a predominantly qualitative character, whereas our coding scheme in combination with the systematic solution set including clearly worked out systematic procedures, provided us the opportunity to investigate symbol sense behavior qualitatively with a quantitative component. This component should not be confused with quantitative research to generalize results from a larger sample population (Bryman, 2015). Instead, beyond qualitative explanations, it provided us quantitative insight into whether students successfully applied systematic algebraic approaches that became visible through the application of basic algebraic skills and symbol sense behaviour.

We expect that our systematic algebraic approaches can also be used in other science subjects., for example, in algebraic manipulations involving the Hardy-Weinberg equation in biology $p^2 + 2 \cdot p \cdot q + q^2 = 1$ or the Nernst equation in chemistry $E = E^0 + \frac{R \cdot T}{n \cdot F} \ln \left[\frac{[\text{Ox}]}{[\text{Red}]} \right]$. Furthermore, this study was based on a theoretical model (Powell, Francisco & Maher, 2003) with detailed consecutive steps to analyze videotaped data, not present in earlier studies.

We adopted a similar strategy as Bokhove (2011) by selecting tasks should trigger students solving algebraic physics problems and provide insight into their procedural skills and symbol sense behavior. Indeed, this was the case, contributing to the internal validity of this study. In addition, the way we investigated basic algebraic skills (Kieran, 2013; Wu, 1999) and symbol sense behavior were very helpful in analyzing both aspects.

Regarding the research question (4) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems?*" we observed that students lacked both sufficient symbol sense behavior and a solid grasp of basic algebraic skills, mainly due to the time-consuming ad hoc strategies and overlooked procedures. Ad hoc approaches only worked for basic expressions containing fewer variables. In problems with more variables, students got stuck and were unable to explain why operations with ad hoc strategies led to problems. On the subtask level some students showed symbol sense behavior. Overall, students were unsuccessful in the transfer of algebraic skills that students learned in mathematics class to solve algebraic physics problems in physics class.

To measure transfer, we adopted the traditional transfer view (e.g. Mestre, 2015) and compared students' solution sets to our systematic solution set. This perspective of transfer offered us sufficient insight into the degree to which students both applied basic algebraic skills correctly and demonstrated symbol sense behaviour. This perspective was very useful in this area of research. Beyond this approach, to some extent we adopted the actor-oriented transfer perspective by examining what students said during the interviews. This provided us information about their previous learning situation. For example, students mentioned that they learned some ad hoc strategies from their teachers. Since Dutch teachers are highly textbook-driven, this may provide information about the textbooks they use. Since such strategies are harmful for transfer, this is important information for teachers and textbook publishers. We conclude that the traditional approach was very useful to measure the degree of transfer, and the other approach was useful to gain insight into previous learning.

Our findings are in line with earlier studies, stating that using ad hoc strategies leads to fragmented knowledge, impedes generalization of algebra and can be harmful for conceptual understanding. Instead of such strategies, students should learn systematic algebraic problem-solving strategies as in the solution set (see Appendix). This involves a rule-based problem-solving approach in which algebraic skills are used with insight, where the term 'rule' plays the role of algebraic axioms in high school algebra. It is probably better that students have a gestalt view on algebraic expressions and deal with their visual salience. This solution set should be considered and implemented by curriculum developers, textbooks and textbook publishers, teachers and mathematics and science teacher educators aiming at successful application of mathematics in physics and strengthening students experiencing coherence across mathematics and science subjects, especially physics. Regarding the first, we recommend physics curricula adopting content standards that refer to the importance of using algebraic skills to solve problems with insight as described in the mathematics curriculum (SLO, 2019).

Concerning activation of prior mathematical knowledge, our research has shown that it is probably better to use the same pedagogy of algebraic skills, especially algebraic techniques as in mathematics curricula. The latter should emphasize the importance of science context, especially algebraic physics problems analogous to mathematics problems. For the science audience, regardless of whether these are curricula containing both the general educational core goals and the more specific standards, which are tested in national final examinations, and shaping the content of textbooks, we recommend physics textbooks to avoid the time-consuming ad hoc strategies such as the permutation strategy. With respect to algebraic problem-solving, mathematics textbooks should include systematic procedures with insight similar to that in the solution set in the Appendix, thereby paying attention to differentiation of algebraic techniques during procedures, for example, 'substitution' and 'multiplication of both sides' of the equals sign. This may contribute to conceptual understanding of algebraic skills. Physics problems should be included to provide context for corresponding mathematics problems. We recommend physics textbooks adopting paragraphs where physics problems are introduced through corresponding mathematics problems that students have learned in mathematics class. Again, we emphasize the importance of activation of pre-knowledge (e.g., Hailikari, Katajavuori, & Lindblom-Ylanne, 2008) and using identical problem-solving pedagogies to that in mathematics textbooks. We note that such emphasis requires sufficient organization of the learning process. Otherwise, certain mathematical concepts may be taught in physics class *before* they are explained in mathematics class (Alink et al., 2012; Turşucu et al., 2018c). These design principles may have major implications for textbook publishers. In many countries they are bound to one discipline, since each of them pursues different aims. Our study indicates that it is probably better that mathematics and physics publishers work together to develop textbook series in which these principles are incorporated.

With regard to mathematics and physics teachers, it is pivotal that they are able to explain basic mathematics. This should be a pre-requisite for pre-service teachers following science teacher education programs leading to a teaching qualification. Probably, this is the most important matter in improving the application of mathematics in physics. Furthermore, the issues such as activation of prior-knowledge and using identical pedagogies in systematic problem-solving with insight, also apply for individual mathematics and physics teachers. In addition, even *mentioning* that physics formulas are rooted in mathematics class, writing mathematics and physics expressions next to each other, or relating physical quantities to the variables x and y used in mathematics can impede compartmentalized thinking (Turşucu et al., 2018b). Similar issues hold for mathematics teachers, for example, mentioning that algebraic skills are used in science classes, especially in physics.

In many countries including the Netherlands (Turşucu et al., 2018b), physics teachers' curricula are overloaded (e.g., Lyons, 2006). As a consequence, teachers can lack time for sufficient collaboration with other departments. We recommend mathematics and physics teachers to systematically reserve some fixed amount of time in their school timetables. This may compel teachers to stick to their schedules. In addition, informal meetings may also offer a solution. Overall, we think that such collaborative efforts should focus on alignment of both subjects that is feasible to adopt in teaching practice.

Through professional teaching programmes, science teacher educators can make teachers aware (Girvan, Conneely, & Tangney, 2016) of the mechanisms underlying students' difficulties when applying mathematics in science subjects, especially in physics. This also applies for providing solutions to combat these difficulties. Similarly, we recommend mathematics teacher educators to address these issues to their audience consisting of teachers and future teachers. Overall, both teacher educators should emphasize the importance of using systematic algebraic strategies, rather than ad hoc strategies lacking insight.

Our research has shown that the application of ad hoc strategies may help students to solve basic algebraic problems. However, there are risks for the longer term. Students can become dependent on an authority, i.e. a teacher or a textbook that tells them what is mathematically correct and what is not. In addition, mathematics can become a collection of incoherent and misunderstood strategies for them. Students often do not know the boundaries of such strategies, i.e. where they apply and where not. Especially, in new and more sophisticated situations students encounter difficulties. We conclude that ad hoc approaches can be harmful for the application of algebraic skills with insight. Furthermore, with a rule-based problem-solving approach that becomes visible through the application of algebraic skills during procedures involving basic algebraic skills and symbol sense behavior, students are flexible and able to handle such new and more sophisticated problems. These issues above can be of major importance for many countries aiming at enhancement of the application of algebraic skills from mathematics in physics class, especially improving symbol sense behavior in algebraic physics problems.

Observing symbol sense behavior is not a straightforward affair (Bokhove & Drijvers, 2010), for it is not easy to recognize whether students rely on procedural skills or demonstrate insight into expressions. Indeed, both concepts are intertwined, which is illustrated in figure 3 of Chapter 1 (Drijvers, 2011). Nevertheless, we succeeded quite easily using the numerical criteria OSSB (%) $\geq$ 80.0 % and SSB (%) $\geq$ 80.0 %. Similarly, we used the criterion OBAS (%) $\geq$ 90.0 % for the observation of procedural fluency (basic algebraic skills). Even though this might look as if we decoupled basic algebraic skills and the symbol sense items local salience and pattern salience of OSSB (%), this is not the case. Investigation of OBAS (%) was helpful in gaining insight into the extent to which students used basic algebraic skills correctly. In addition, OBAS (%) was already incorporated in OSSB (%), indicating the intertwinement above.

The comparable performance of both mathematics groups confirms that the algebra involved in both subjects is sufficient to tackle algebraic physics problems (SLO, 2019).

Our findings support earlier studies in which students encounter difficulties applying mathematics in physics (e.g., Turşucu al., 2017; Roorda, Vos & Goedhart, 2014). This contributes to the relevance of this study.

5.5.1 Limitations of This Study and Further Recommendations

Even though we aimed at a 50 % –50 % gender-ratio (Bryman, 2015), our sample consisted of 1 male and 5 female students. This was due to strong selection principles needed to safeguard the quality of this study. Firstly, the respondents should be selected from two regular schools having a sufficient mathematics and an insufficient physics grade ($<$ 5.5). They were required to follow the same mathematics and physics textbook series (Reichard, 2014a; Ottink et al., 2014) and have a similar knowledge domain in these subjects at the moment of interviews. Finally, they should be willing to participate in this study.

On the other hand, based on our aforementioned extended literature study, there are no indications that a sample with an equal number of male and female students would have generated fundamentally different results. Instead, they may be similar. Indeed, students' performance in terms of OBAS (%) and OSSB (%) is mainly related to a combination of grades for both subjects and a similar knowledge domain, rather than on gender. Thus, it is very likely that male and female students with similar grades will show similar performance. As a result, the composition of our sample should not be seen as a limiting factor. Despite this, it is worthwhile to elaborate on this matter, since this may add to the internal validity (Bryman, 2015) of this study.

As stated earlier, the grade-criterion above was to ensure that students' difficulties with algebraic physics problems were mainly because of insufficient application of algebraic skills in physics, and not related to a lack of basic mathematics (Kieran, 2013; Wu, 1999). In addition, the other criteria were essential to make sure that it was legitimate to compare the results of Group-I and Group-II. This also applies for comparing individual students in both groups. We note that students' poor physics grades can also be the result of the absence of a variety of aspects, for example, having a positive attitude towards physics or understanding physical concepts.

Since this study was based on a very small sample, the findings cannot be extended to the whole Dutch population. Neither is it representative for the Dutch context. The individual differences among mathematic A and mathematics B students' grades, how they apply algebraic skills and their symbol sense behavior characteristics are too fundamental.

When stuck, students got hints to help them proceed. For instance, Aron got stuck in procedure 3 of subtask 2a and asked: "*Is this okay?*". He was given the hint "*Look carefully at* $m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2$ *and the next step*". He continued his work with correct procedures. Thus, we think that these hints did not influence their scores.

As to design principles, we recommend incorporation of two aspects in the interview protocol. It should explicate that students should reserve time reading the questions carefully, since they immediately started solving the tasks. Another problem was that half of the students worked fast and did not check their solutions after they finished their problems. These contained sloppy mistakes which may have been overcome if they had carefully re-examined their work. They would have benefited from a guideline emphasizing re-examination of their 'finished' work, which is supported by earlier studies (Hattie & Timperley, 2007; Shute, 2008), and may add to their meta-cognitive skills.

The criteria OBAS (%) ≥ 90.0 % and OSSB (%) ≥ 80.0 % were not chosen arbitrarily, but the result of consensus among all authors after a series of discussions. Firstly, physics students should have a solid foundation of basic algebraic skills, irrespective of having mathematics A or mathematics B. This sheds light on the relatively high number of OBAS (%). However, meeting this criterion does not imply a similar high OSSB (%). Indeed, beyond basic skills, symbol sense behavior depends on conceptual understanding (Drijvers, 2011). Hence, we have chosen OSSB (%) ≥ 80.0 % instead of, for example, 90.0 %.

Our findings have consequences for teaching practice. Should there be focus on procedural skills, or emphasis on insightful learning? We have seen that students experienced difficulties because their basic procedures were not automated, the problem required unusual reasoning, or the automated procedures were insufficient to tackle unusual problems. Thus, both procedural skills and insight should be taught in an integrated manner, which is in line previous studies (Arcavi, 2005; Bokhove, 2011; Drijvers, 2015).

Two mathematics A students learned the cross-multiplication strategy from their textbook *Getal en Ruimte* (Reich et al, 2014a). This textbook series has the largest market share in the Netherlands, influencing large number of highly textbook-driven teachers who teach them to their students (SLO, 2019; van Zanten & van den Heuvel-Panhuizen, 2014). In short, textbooks determine *how* and *what* students learn. As to the *harmful* ad hoc strategies, we strongly recommend conducting a textbook analysis of *Getal en Ruimte* in which different types of such strategies are mapped and analyzed. We also recommend communicating the findings to the publisher.

Furthermore, all respondents were involved in grade 10. Their lack of basic algebraic skills and symbol sense behavior might indicate insufficient attention to algebraic skills in grade 9. This may also be examined through textbook analysis.

In the next study (5) in Chapter 6, we used insights from this study and the other three studies to carry out interventions in physics textbooks to improve transfer.

5.6 Appendix

5.6.1 The Tasks

Task 1 ideal gas

The formula for an enclosed ideal gas is given by $\frac{P \cdot V}{T} = C$. Here, P is the pressure of the gas in Pa, V its the volume in m^3 and T its temperature in K.

a) Solve for V in $\frac{P \cdot V}{T} = C$.

b) Solve for T in $\frac{P \cdot V}{T} = C$.

Task 2 falling stone

The potential energy of a stone is given by the formula $E_{pot.} = m \cdot g \cdot h$. Here, $E_{\text{pot.}}$ is the potential energy of the stone in J, m its mass in kg, g its acceleration due to gravity in $\frac{m}{s^2}$ and h its height in $meter$. When the stone is dropped from a height h through a medium without air resistance the potential energy is only converted into kinetic energy. At the moment the stone hits the ground this energy is $E_{\text{kin.}} = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2$. Here, $E_{\text{kin.}}$ is the kinetic energy of the stone in J, m its mass in kg and v_{final} its final speed in $m \cdot s^{-1}$.

a) Solve for v_{final} in $E_{\text{pot.}} = E_{\text{kin}}$.

Now we add air resistance to the medium. As a consequence, when the falling stone hits the ground its potential energy is converted into kinetic energy and heat. For the heat we can write $Q = F_{\text{res.}} \cdot s$. Here, Q is the falling stone's produced heat in J, $F_{\text{res.}}$ the average air resistance in N, and h the height in m. At the moment the stone hits the ground we can write $E_{\text{pot.}} = E_{\text{kin.}} + Q$.

b) Solve for F_{wr} in $E_{\text{pot.}} = E_{\text{kin.}} + Q$.

Task 3 uniform circular motion

Imagine that the earth is orbiting around the Sun in uniform circular motion. In order to the earth remain in orbit the attractive gravitational force F_{G} between the earth and the Sun must equal the centripetal force F_{C} between these objects. Hence, we can write $G \cdot \frac{m \cdot M}{r^2} = \frac{m \cdot v^2}{r}$. Here, G is the universal constant of gravitation in $N \cdot \text{m}^2 \cdot \text{kg}^{-2}$, m the earth's mass in kg, M the Sun's mass in kg, v the earth's constant speed around the Sun in $m \cdot s^{-1}$ and r is the distance between the centers of the masses in m.

a) Solve for v^2 in $F_{\text{G}} = F_{\text{C}}$.

The earth's speed around the Sun is given by $v = \frac{2 \cdot \pi \cdot r}{T}$. Here, r is the distance between the mass centres in m, T the time it takes to complete one orbit around the Sun in s.

b) Derive the formula $\frac{r3}{T^2} = \frac{G \cdot M}{4\pi^2}$ by using $G \cdot \frac{m \cdot M}{r^2} = \frac{m \cdot v^2}{r}$ and $T = \frac{2 \cdot \pi \cdot r}{v}$.

Task 4 spring-mass system

The period of a spring-mass system is given by $T = 2 \cdot \pi \cdot \sqrt{\frac{m}{C}}$. Here, T is the period in seconds m the mass of the attached object in kg and C the spring constant in $N \cdot m^{-1}$. Solve for m in $T = 2 \cdot \pi \cdot \sqrt{\frac{m}{C}}$.

5.6.2 Systematic Solution Set to The Tasks

For stylistic reasons we abbreviated procedures involving basic algebraic skills as 'BAS', local salience as 'LS' and pattern salience as 'PS'.

Task 1 ideal gas

a) Procedure 1 (LS): multiplication of both sides of $\frac{P \cdot V}{T} = C$ with T. Result: $P \cdot V = C \cdot T$; procedure 2 (LS): dividing both sides of $P \cdot V = C \cdot T$ through P. Result: $= \frac{C \cdot T}{P}$.

b) Procedure 1 (LS): multiplication of both sides of $\frac{P \cdot V}{T} = C$ with T. Result: $P \cdot V = C \cdot T$; procedure 2 (LS): dividing both sides of $P \cdot V = C \cdot T$ through C. Result: $\frac{P \cdot V}{C} = T$.

Task 2 falling stone

a) Procedure 1 (BAS): substitution of $E_{pot.} = m \cdot g \cdot h$ and $E_{kin.} = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2$ in $E_{pot.} = E_{kin.}$. Result: $m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2$; procedure 2 (PS): division of both sides of $E_{pot.} = E_{kin.}$ through m. Result: $g \cdot h = \frac{1}{2} \cdot (v_{\text{final}})^2$; procedure 3 (LS): multiplication of both sides of $g \cdot h = \frac{1}{2} \cdot (v_{\text{final}})^2$ with 2). Result: $2 \cdot g \cdot h = (v_{\text{final}})^2$; procedure 4 (LS): taking the square root of $2 \cdot g \cdot h = (v_{\text{final}})^2$on both sides. Result: $\sqrt{2 \cdot g \cdot h} = v_{\text{final}}$.

b) Procedure 1 (BAS): substitution of $E_{\text{pot.}} = m \cdot g \cdot h$, $E_{\text{kin.}} = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2$ and $Q = F_{\text{res.}} \cdot h$ in $E_{\text{pot.}} = E_{\text{kin.}} + Q$. Result: $m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2 + F_{\text{res.}} \cdot h$; procedure 2 (PS): substraction of $F_{\text{res.}} \cdot h$ from both sides of $m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2 + F_{\text{res.}} \cdot h$. Result: $m \cdot g \cdot h - F_{\text{res.}} \cdot h = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2$; procedure 3 (LS): division of both sides of $m \cdot g \cdot h - F_{\text{res.}} \cdot h = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2$ through m. Result: $\frac{m \cdot g \cdot h - F_{\text{res.}} \cdot h}{m} = \frac{1}{2} \cdot (v_{\text{final}})^2$; procedure 4 (LS): multiplication of both sides of $\frac{m \cdot g \cdot h - F_{\text{res.}} \cdot h}{m} = \frac{1}{2} \cdot (v_{\text{final}})^2$with 2. Result: $2 \cdot \frac{m \cdot g \cdot h - F_{\text{res.}} \cdot h}{m} = (v_{\text{final}})^2$; procedure 5 (LS): taking the square root of $2 \cdot \frac{m \cdot g \cdot h - F_{\text{res.}} \cdot h}{m} = (v_{\text{final}})^2$on both sides. Result: $\sqrt{2 \cdot \frac{m \cdot g \cdot h - F_{\text{res.}} \cdot h}{m}} = v_{\text{final}}$.

Taks 3 uniform circular motion

a) Procedure 1 (BAS): substitution of $F_G = G \cdot \frac{m \cdot M}{r^2}$ and $F_G = \frac{m \cdot v^2}{r}$ in $F_G = F_C$. Result: $G \cdot \frac{m \cdot M}{r^2} = \frac{m \cdot v^2}{r}$; procedure 2 (PS): dividing both sides of $G \cdot \frac{m \cdot M}{r^2} = \frac{m \cdot v^2}{r}$ through m. Result: $G \cdot \frac{M}{r^2} = \frac{v^2}{r}$; procedure 3 (PS): multiplication of both sides of $G \cdot \frac{m \cdot M}{r^2} = \frac{m \cdot v^2}{r}$ with r. Result: $G \cdot \frac{M}{r} = v^2$.

b) Procedure 1 (BAS): substitution of $v = \frac{2 \cdot \pi \cdot r}{T}$ in $G \cdot \frac{M}{r} = v^2$. Result: $G \cdot \frac{M}{r} = \left(\frac{2 \cdot \pi \cdot r}{T}\right)^2$; procedure 2 (BAS): execution of previous procedure. Result: $G \cdot \frac{M}{r} = \frac{4 \cdot \pi^2 \cdot r^2}{T^2}$; procedure 3 (LS): division of both sides of $G \cdot \frac{M}{r} = \frac{4 \cdot \pi^2 \cdot r^2}{T^2}$ through 4. Result: $G \cdot \frac{M}{4 \cdot r} = \frac{\pi^2 \cdot r^2}{T^2}$; procedure 4 (LS): division of both sides of $G \cdot \frac{M}{4 \cdot r} = \frac{\pi^2 \cdot r^2}{T^2}$ through π^2. Result: $G \cdot \frac{M}{4 \cdot \pi^2 \cdot r} = \frac{r^2}{T^2}$; procedure 5 (PS): multiplication of both sides of $G \cdot \frac{M}{4 \cdot \pi^2 \cdot r} = \frac{r^2}{T^2}$ with r. Result: $r \cdot G \cdot \frac{M}{4 \cdot \pi^2 \cdot r} = r \cdot \frac{r^2}{T^2} \rightarrow G \cdot \frac{M}{4 \cdot \pi^2} = \frac{r^3}{T^2}$.

Taks 4 spring-mass system

Procedure 1 (LS): division of both sides of $T = 2 \cdot \pi \cdot \sqrt{\frac{m}{C}}$ through 2. Result: $\frac{T}{2} = \frac{2 \cdot \pi \cdot \sqrt{\frac{m}{C}}}{2} \rightarrow \frac{T}{2} = \pi \cdot \sqrt{\frac{m}{C}}$; procedure 2 (LS): division of both sides of $\frac{T}{2} =\cdot \pi \cdot \sqrt{\frac{m}{C}}$ through π. Result: $\frac{T}{2 \cdot \pi} = \sqrt{\frac{m}{C}}$. procedure 3 (LS): squaring both sides of $\frac{T}{2 \cdot \pi} = \sqrt{\frac{m}{C}}$. Result: $\frac{T^2}{4 \cdot \pi^2} = \frac{m}{C}$; procedure 4 (LS): multiplication of both sides of $\frac{T^2}{4 \cdot \pi^2} = \frac{m}{C}$ with C. Result: $C \cdot \frac{T^2}{4 \cdot \pi^2} = C \cdot \frac{m}{C} \rightarrow \frac{C \cdot T^2}{4 \cdot \pi^2} = m$.

5.6.3 Interview Protocol Physics Teachers

Introduction
This interview will take approximately 15 minutes and is part of the PhD-study of the interviewer, Süleyman Turşucu. By means of a questionnaire, we will ask you to respond to questions about the background of some of your grade 10 physics students, and the mathematics and physics textbooks they use.

Purpose of this interview
We aim to select three grade 10 physics students having a sufficient mathematics grade and an insufficient physics grade (< 5.5). Later, during an interview these students will be asked to solve algebraic physics problems while being videotaped and thinking aloud (Charters,

2003). We hope that these interviews will provide insight into how the application of algebraic skills from mathematics in physics may be improved.

Interview approach
Would you please read aloud the questions of the *questionnaire* below one by one, and provide answers? This conversation can be audiotaped so that it can be listened back. Furthermore, the students' names will be anonymized. Would you consent to your students participating in this study?

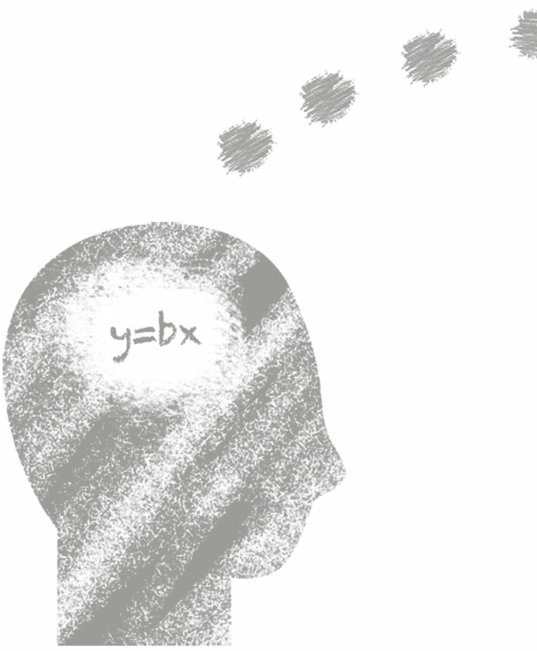
Questionnaire
1. Would you please provide us the names of your grade 10 physics students who have a sufficient mathematics grade and an insufficient physics grade (< 5,5)?
2. Would you please provide us insight into the attitude of these students towards learning?
3. Would you please tell us which physics textbook these students use, and why this textbook was chosen?
4. Would you please tell us which mathematics textbook these students use?

5.6.4 Interview Protocol Physics Students

Introduction
This interview contains two parts. In the *first* part we ask you general questions about your background, and in the *second* part we ask you to solve four tasks including algebraic physics problems while thinking aloud and being videotaped.

First part: general questions
1. Would you please tell me why you chose mathematics and science subjects?
2. Would you please tell me your opinion about mathematics and physics?
3. Would you please provide me the grades for mathematics and physics?

Second part: solving algebraic physics problems
Would you please solve these four tasks 'Task 1: ideal gas', 'Task 2: falling stone', 'Task 3: uniform circular motion' and 'Task 4: spring-mass system' below while thinking aloud. Please write down as many intermediate steps as possible. I will only interrupt you when a procedure or reasoning is not clear enough, or it remains silent for about one minute. The information that you provided in the first and second part of this interview will only be used for my PhD-research. Your name will be anonymized.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Chapter 6

Effectiveness of Using Shift-problems[13]

[13] This chapter is submitted for publication in *Eurasia Journal of Mathematics Science and Technology Education*.

6.1 Introduction

Mathematics has a pivotal role in science education, for it offers the tools by which quantitative relationships can be calculated, modelled, represented and predicted (Dierdorp, Bakker, van Maanen & Eijkelhof, 2014). On the other hand, science provides rich, relevant and meaningful context for mathematics. Despite this close relationship, students in a large number of countries struggle with applying mathematics in science subjects, implying insufficient transfer[14] between these subjects (Jonas et al., 2017; NCTM, 2013; 'TIMMS & PIRLS', 2019; Wong, 2018).

Transfer of mathematics to science subjects, especially physics is not *even* guaranteed when students have a solid grasp of mathematics. One of the very rare studies conducted by Rebello et al. (2007) shows that students solid grasp of mathematics enrolled in an algebra-based physics course faced problems with mathematics in physics. They concluded that the lack of transfer was not due to students' mathematics knowledge, but inappropriate application of algebra to physics problems.

According to Nashon and Nielsen (2007), a major reason for the lack of transfer is related to *compartmentalized thinking* in which students see mathematics and science as two unrelated subjects. This phenomenon is reinforced since in many countries both subjects are taught separately (SLO, 2019; Honey, Pearson & Schweingruber, 2014; 'TIMMS & PIRLS', 2019). In this respect, Furner & Kumar (2007, p.186) pointed out that "*The separate subject curriculum can be viewed as a jigsaw puzzle without any picture. If done properly, integration of math and science could bring together overlapping concepts and principles in a meaningful way and enrich the learning context. Learning situated in such enriched (macro) contexts often lead to meaningful learning experiences*". In addition, there can be a mismatch in pedagogical approaches between mathematics and science teachers (Turşucu et al., 2018c), especially in teaching mathematical methods such as algebraic techniques (Drijvers et al., 2011).

A remedy for both compartmentalized thinking and the mismatch above may be coherent mathematics and science education (CMSE) that is of major importance for students (Berlin & White, 2010, 2012, 2014). The idea behind CMSE is fostering *connection* between mathematics and science education through, for example, alignment of notations. As part of CMSE, the discrepancy in pedagogical approaches may require alignment of teaching approaches, and also improvement of *mathematical proficiency* (Kilpatrick, Swafford, & Findell, 2001) that contains the five interwoven strands adaptive reasoning, conceptual understanding, procedural fluency, productive disposition and strategic competence. Concerning algebra education, especially important are the second and third strands. Together, these strands form algebraic expertise (Arcavi, 1994; Andrá et al., 2015) that includes algebraic skills with emphasis on procedural fluency in relation to conceptual understanding. Symbol sense is the part of algebraic skills involving conceptual understanding, relating to "an intuitive feel for when to call on symbols in the process of solving a problem, and conversely, when to abandon a symbolic treatment for better tools" (Arcavi, 1994, p. 25). Moreover, symbol sense contains the ability to choose a *wise* systematic problem-solving strategy based on relevant aspects of an algebraic expression.

[14] A detailed explanation of the controversial transfer phenomenon can be found in the previous chapter.

Through examples, Arcavi identified eight behaviors of symbol sense that show the intertwinement between conceptual understanding and procedural skills. A key behavior of symbol sense is flexible manipulation skills. While being in control of the work, flexible manipulation skills includes flexible manipulation of expressions (both technical and with insight). Two interconnected characteristics of flexible manipulation skills are having a gestalt view on algebraic expressions and dealing in a suitable manner with their visual salience. Gestalt view contains "*the ability to consider an algebraic expression as a whole, to recognize its global characteristics, to'read through' algebraic expressions and equations, and to foresee the effects of a manipulation strategy*" (Bokhove & Drijvers, 2010, p. 43). Visual salience deals with local salience, i.e. sensitivity towards visual cues in local symbols such as exponents, fractions and square root signs, and pattern salience relating to sensitivity towards global patterns in algebraic expressions. This implies that flexible manipulation skills and thus algebraic expertise including basic algebraic skills and symbol sense play a major role in the transfer of mathematics to science subjects, in particular to physics. Even though symbol sense is pivotal in relation to transfer, our extensive literature study with various web search engines including Google Scholar and ProQuest revealed that their relationship has hardly been studied, especially the role of activation of prior mathematical knowledge to improve transfer to physics. In fact, there is no single study examining such relationship with emphasis on activation of pre-knowledge. This makes it worthwhile to investigate this matter. In the following sections we will discuss the aim, present the research question and the relevance of this study.

6.1.1 Research Aim and Research Question (5)

In this study we researched the effectiveness of activation of students' prior mathematical knowledge to solve physics problems in regular physics textbooks (Ottink et al., 2014) for which solution algebraic skills were needed. Students solved these problems during interviews. Such problems were presented as *shift problems* (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013) which use instructional models to carry out small interventions on tasks. In many studies the gap between advice offered in educational research and established classroom practices is too large. Only very few teachers can incorporate such large changes in their teaching (Wiliam, Leahy, 2012). As a consequence, such studies often have very limited impact on teaching practice. Shift problems, on the other hand, are small interventions that can be easily implemented by students and teachers. In this study, the instructional model activates prior mathematical knowledge by providing hints at the start of these tasks to improve students' systematic problem-solving abilities, especially symbol sense behavior. Regarding the pedagogy of algebra, we used algebraic skills, especially algebraic techniques similar to how these were learned in their mathematics textbooks (Reichard et al., 2014).

These problem-solving activities during interviews were guided by the research question "*How can activation of prior mathematical knowledge be used effectively to improve students' symbol sense behavior in senior pre-university education when solving algebraic physics problems?*". This question was divided in two sub questions: (5a) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems that occur in their physics textbooks?*", and (5b) "*To what extent do students in senior pre-university education demonstrate symbol sense behavior* *when solving the same algebraic physics problems that occur in their physics textbooks after activation of prior mathematical knowledge?*'.

Earlier studies on algebraic problem-solving in physics have shown (Turşucu et al., 2017; 2018b; 2018c) that among students in senior pre-university education grade 10 students encounter the biggest transfer problems. Hence, we selected grade 10 physics students to gain deeper understanding in their algebraic problem-solving skills, especially their basic algebraic skills and symbol sense behavior, and whether and to which extent we could improve their problem-solving skills.

Similar to the previous study, our working definition of *successful* in the title of this thesis refers to the application of systematic algebraic strategies during algebraic problem-solving in physics. Similar to the previous study, we operationalized such strategies by measuring both the extent to which students demonstrated symbol sense behavior and the degree to which they applied basic algebraic skills properly during interviews. Symbol sense behaviour became visible through the application of algebraic techniques during procedures involving basic algebraic skills and having a gestalt view on algebraic expressions and dealing with their visual salience.

For stylistic reasons we used the concepts conceptual understanding and insightful learning (Kilpatrick, Swafford, & Findell, 2001) interchangeably to refer to conceptual understanding. This also holds for procedural fluency, procedural skills and basic algebraic skills, and 'her' and 'the student', since we had more female than male participants.

In this study we do not focus on the meaning or nature of physical concepts. The emphasis is on algebraic skills learned in mathematics class and applied to physics problems.

Furthermore, as in the previous study we expected that the algebraic skills that students used during problem-solving (target tasks) in regular physics textbooks were learned in mathematics class from regular mathematics textbooks (previous learning situation). To determine the degree to which transfer occurred, we adopted the traditional transfer approach (e.g. Mestre, 2015) by comparing students' solution sets to our systematic solution set. This is done by the operationalization of systematic algebraic strategies above. Therefore, the extent to which transfer occurred, was determined by the researchers' perspective. To some extent, we also adopted the actor-oriented transfer approach. In this regard, we followed the same approach as in the previous study (4) and operationalized the actor-oriented transfer approach "*as a search for students' personal constructions of relations between (1) learning from mathematics and physics classes and (2) interview tasks*" (Roorda et al., 2014; p. 863).

6.1.2 Educational and Scientific Relevance

Our literature study also reveals that this is the first time that shift-problems are used outside mathematics education. Therefore, this study may contribute to the evaluation of shift-problems. In addition, the case of students having a solid grasp of mathematics, but struggle with the application of mathematics in physics is highly under examined. This study may provide deeper understanding in why such students struggle with physics problems, thereby providing insight into the underlying mechanisms of *how* they apply mathematics in physics, and also offer insight into *how* activation of prior mathematical knowledge may be used for the benefit of the international mathematics and science audience, especially curriculum developers, mathematics and physics teachers, mathematics and science teacher educators and textbook publishers who aim to enhance transfer between both subjects and strengthen students experiencing coherence across these subjects. Concerning the first group, it may provide design principles to connect the mathematics curricula to that of physics through core goals and content standards dealing with, for example, the same pedagogy of using algebraic skills. Mathematics and physics textbooks may follow identical problem-solving approaches which are followed by individual teachers in mathematics and physics class. For students, this is an appropriate transfer scenario (e.g., Alink, van Asselt, & den Braber, 2012; Berlin & White, 2012, 2014). Regarding collaboration, both departments may develop strategies in which algebraic skills are used with emphasis on conceptual understanding. In case of mathematics and science teacher educators, they may use the findings of this study in professional teaching programmes to make teachers aware (Turşucu et al., 2018c; Girvan, Conneely, & Tangney, 2016) of aspects that influence transfer, i.e. aspects that impede and enhance transfer. Together, the issues above are likely to improve transfer of algebraic skills in physics, especially basic algebraic skills and symbol sense behavior in algebraic physics problems.

6.2 Background

6.2.1 Coherent Mathematics and Science Education and Transfer

As mentioned above, a remedy for the transfer problem is coherent mathematics and science education (CMSE in short). Similar to Science Technology Engineering and Mathematics education, or *STEM* education in short (van Breukelen, 2017; National Science and Technology Council, 2013; SLO, 2019; 'TIMMS & PIRLS', 2019), mathematics lies at the heart of the CMSE approach. The CMSE approach aims to connect subjects (in this regard mathematics and science subjects) through the alignment of various aspects such as notations, concept descriptions, pedagogical approaches and the organization of the learning process in order to establish a logical learning line across both subjects. The latter requires that certain mathematical concepts are already taught in mathematics class before these were used in science class.

The CMSE approach is based on the traditional transfer perspective in which mathematics (initial learning situation) is applied in other subjects (new learning situation) (Alink et al., 2012; Larsen-Freeman, 2013; Leberman et al., 2016). Therefore, there is a very close relationship between CMSE and transfer.

The terms 'alignment' and 'coherent' can have different meanings in different studies (Roorda, 2012). First of all, 'coherent' may be used in (1) 'coherent profiles', (2) 'coherent education' and (3) 'coherent knowledge'. Even though all of them share the word 'coherent', they may refer to different levels of the curriculum (Van den Akker, 2004). The first is concerned with the curriculum that is tested in Dutch national final examinations. 'Coherent education' refers to what has been implemented, and 'coherent knowledge' to what has been achieved. In this study we follow the line of Roorda (2012). When we refer to 'coherence between mathematics and science subjects', we refer to teachers or textbook publishers connecting both subjects in terms of aforementioned alignment through various aspects (number (2)). 'Coherence' in relation with students denotes the achieved level in number (3). In other words, the extent to which students experience coherence across both subjects (e.g., Frykholm & Glasson, 2005; Furner & Kumar, 2007; Mooldijk & Sonneveld, 2010). For this to happen, students should be aware of the intimate relationship between both subjects. In that case, the tools offered in mathematics class may become a versatile, widely applicable machinery to solve problems in science class. Conversely, their awareness of science as a *meaningful* context in which mathematics can be used, may improve the transfer of mathematics in science subjects, especially in physics. Thus, students experiencing coherence across both subjects is of major importance for transfer.

In this study, we assume a reciprocal relation between CMSE and transfer. When students experience coherence across mathematics and science subjects by means of *meaningful* contexts, transfer from mathematics to science subjects can be improved, and improving transfer can help them to experience coherence between mathematics and subjects.

In many countries, CMSE in teaching practice depends on various players such as the curriculum and science teacher educators (Schmidt, Wang, & McKnight, 2005; Turşucu et al., 2018c). Concerning curricula (intended curriculum), they describe the general core goals of education and the specific standards which are tested in national final examinations. Textbooks mediate between these curricula and the actual teaching in classrooms (implemented curriculum) which are closely followed by teachers and their students (SLO, 2019). Thus, for classroom practice aiming at CMSE, the key players in the Netherlands are curricula, textbooks and teachers (van Zanten, M. & van den Heuvel - Panhuizen, M., 2014). For researchers, the textbooks' central role in teaching practice makes it worthwhile to design shift-problems in which instructional models replace a small part of regular textbooks (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013). In this study we used this pragmatic approach to design instructional models by means of hints about using algebraic techniques in a similar way to that in mathematics textbooks. By providing hints we aim to activate prior mathematical knowledge and improve both students' procedural fluency and symbol sense behavior during algebraic problem solving in physics and strengthen.

Furthermore, in this study, we assume a reciprocal relation between CMSE and transfer. When students experience coherence across mathematics and science subjects by means of *meaningful* contexts, transfer from mathematics to science subjects can be improved, and improving transfer can help them to experience coherence between mathematics and subjects.

6.2.2 Context of The Study

All authors in this study were affiliated with the mathematics and science teacher education program of Delft University of Technology in the Netherlands. Therefore, rather than students from countries abroad, they were selected from Dutch secondary schools. For this reason, we shortly draw the Dutch context in relation to education, especially that of pre-university education. The latter lasts six years and consists of three lower years, i.e. lower pre-university education and three upper years, i.e. senior pre-university education (from 12 to 18 years of age).

The Netherlands is regarded as an advanced industrial country (OECD, 2018) where both mathematics and science education at secondary and college level play an important role in the Dutch political agenda (Ministry of Education, Culture and Science, 2018), implying that government officials and policy makers are paying serious attention to both subjects and their relation. Internationally, Dutch lower and senior pre-university education students score accordingly in assessments of these disciplines, including that of physics ('TIMMS & PIRLS', 2019).

6.2.3 Algebra in Senior Pre-university Mathematics Education

The algebraic skills in Dutch mathematics curricula mainly deal with algebraic activity, for example, algebraic manipulation of formulas and expressions, and patterns of relationships between variables. While a formula refers to an expression containing real measurable quantities such as temperature, an expression can be an abstract algebraic expression with abstract mathematical variables (placeholders), or a formula with physical quantities (Drijvers et al., 2011).

According to mathematics curricula, the whole set of mathematical activities above are part of algebraic skills. The latter is divided into *general* skills including developing systematic problem–solving strategies with insight, and *specific* skills dealing with algebraic knowledge and manipulation skills. Thus, specific skills in these curricula are identical to basic algebraic skills. Symbol sense is not explicitly mentioned, but is very close to general skills. The description of general and specific skills together is identical to that of algebraic expertise.

In this study we focus on the application of algebraic techniques which are a key element of algebraic skills, especially in algebraic manipulation of expressions. To solve basic algebraic problems correctly, these techniques should be automated. In this sense, they can be seen as part of the basic algebraic skills machinery. Some well-known algebraic techniques are 'multiplication of both sides' and 'substitution'. The former multiplies both sides of the equation by the same variable, and the latter replaces single variables in expressions. The spectrum of these techniques is presented and thoroughly described in the next section.

Furthermore, since explicit reference to connection in Dutch mathematics and physics curricula is absent, the alignment through issues such as equations, formulas and pedagogy of using algebraic techniques are absent in mathematics and physics class. Unfortunately, this also applies for the organization of the learning process in order to reach an appropriate learning line of concepts between mathematics and physics (Alink, Asselt, & Braber, 2012).

6.2.4 Algebra in Senior Pre-university Physics Education

Physics education in the Netherlands starts in grade 8 (age 13 or 14 years) and corresponds to the second year of lower pre-university education. Since in the next year students decide whether to choose physics or not, grade 8 is regarded as crucial for potential physics students. Physics is introduced through a strong context-concept approach (Turşucu et al., 2018c) where students start with basic quantities such as force and length. The number of formulas is negligible, so students hardly apply algebraic skills to manipulate formulas. This changes slightly in the next year, but the number of formulas is still very limited, and the required level of algebraic skills is low. After the transition to senior pre-university education in grade 10, the level of algebraic skills changes significantly (Turşucu et al., 2017).

Physics formulas are algebraic representations of proportionalities, consisting of two or more variables representing real, measurable quantities. Most formulas express proportionality. They involve only products, quotients and powers, but no sums or differences (Drijvers et al., 2011). For instance, the formula for kinetic energy $E_{\text{kin.}} = \frac{1}{2} \cdot m \cdot (v_{\text{final}})^2$ is proportional to $(v_{\text{final}})^2$ and that of an enclosed ideal gas $\frac{P \cdot V}{T} = C$ is inversely proportional to T. Although proportionalities are of major importance in physics, these are insufficiently addressed in mathematics. This may have consequences for students perceiving coherence across mathematics and physics subjects and transfer.

The formulas are described in the physics curriculum ("Netherlands institute for curriculum development", 2019). This also holds for the algebraic skills needed to solve these formulas. The latter can also be found in BINAS, the natural sciences information booklet. Students use it in tests and the national final physics examination.

According to the physics curriculum, students should be able to derive formulas, for example, the derivation of Keplers' law $G \cdot \frac{M}{4 \cdot \pi^2} = \frac{r^3}{T^2}$, straighten algebraic curves such as the period of a pendulum $T = C \cdot \sqrt{l}$ and conduct dimensional analysis, for example, proving that $[r]$ in the attractive gravitational force $F_G = G \cdot \frac{m \cdot M}{r^2}$ has the unit m. Over the last few years, such algebraic physics problems have become more important in the upper secondary education physics program. Moreover, these problems require mathematically correct solutions where algebraic expertise include procedural fluency and having a gestalt view on algebraic expressions and dealing with their visual salience by means of local salience and pattern salience of algebraic expressions.

6.2.5 Closer Look at Algebraic Expertise

The spectrum of algebraic expertise extends from basic algebraic skills to symbol sense and is shown in figure 3 in chapter 1 (Arcavi, 1994; Turşucu et al., 2018c; Drijvers et al., 2011). While symbol sense deals with strategic work with a global focus and emphasis on algebraic reasoning, basic algebraic skills are concerned with procedural work with a local focus and algebraic calculation. Algebraic reasoning deals, among other techniques, with extreme cases and symmetry considerations.

Demonstration of a global focus deals with recognizing patterns in formulas. In this study, strategic work refers to a physics student being in control of her work during algebraic problem solving. The student seeks for a different systematic approach when a strategy is insufficient. Our focus is on the intertwinement between local and global, *and* procedural and strategic work.

A key issue in algebraic problem-solving is how dealing with basic algebraic skills and insight into teaching practice should look like. Should teachers put emphasis on basic skills or insight, or think about some kind of hybrid state? Indeed, these questions are very relevant (Schoenfeld, 2016) and lie at the heart of *'Math Wars'*, i.e. a long-standing debate about the best way to acquire algebraic expertise. This discussion resulted in clashing ideas about curricula, textbooks and classroom practice. During the past few years there has been a shift towards teaching basic skills and insightful learning through an integrated approach (Turşucu et al., 2018c; Rittle-Johnson, Schneider & Star, 2015): "*Without insight, there is no skill, and without skill, there is no insight*" (Drijvers, 2011, p.141), indicating that their relationship is bidirectional and continuous. We adopt this view on algebraic expertise. For classroom practice, such view on algebraic expertise may have consequences for teaching algebraic skills in both subjects. Mathematics teachers introducing the equation $y = b \cdot x^2$ may provide context and meaning by referring to the analogous formula for kinetic energy in physics $E_{\text{kin.}} = \frac{1}{2} \cdot m \cdot (v)^2$. Physics teachers may do the opposite which can be considered as activation of prior mathematical knowledge. Earlier studies on learning and instruction have shown that activation of pre-knowledge may contribute to better students' achievements (e.g., Hailikari, Katajavuori, & Lindblom-Ylanne, 2008; Roorda, 2012). When solving for v in $E_{\text{kin.}} = \frac{1}{2} \cdot m \cdot v^2$, physics teachers should use the same pedagogical approach learned in mathematics class, i.e. when solving for x in $y = b \cdot x^2$. While students should be skillful in basic operations such as isolation of $x^2$and v^2, they should also understand and be able to explain the mathematics behind these operations.

6.2.6 Systematic Algebraic Approach vs. Ad Hoc Approach

This study distinguishes between using 'ad hoc strategies' and applying 'systematic algebraic strategies' that includes applying algebraic skills with insight as described in the mathematics curriculum (SLO, 2019).

For stylistic reasons we use systematic algebraic strategies and systematic algebraic approach interchangeably to denote the same. This also holds for 'ad hoc strategies' and 'ad hoc approach'. With systematic algebraic approach we refer to a "*rule-based problem-solving approach in which algebraic skills are used with insight, where 'rule' refers to the standard rules for multiplication and division of powers, such as $x^a \cdot x^b = x^{a+b}$, which play the role of algebraic axioms in high school algebra*" (Turşucu et al., 2018c, p. 5). With ad hoc strategies we refer to "*mathematical strategies that are not based on standard algebraic rules with insight, and only work for a specific case that may lead to fragmented knowledge, impeding generalization of algebra*". Even though such strategies may be useful as initial steps or even lead to correct solutions (Roorda, 2012), in general students may get stuck in more sophisticated problems requiring insight. Moreover, students may become dependent on an authority such as a teacher telling them whether an approach is algebraically correct or not. So, ad hoc approaches may be harmful for students' transfer of mathematics in physics, mainly because students lack insight into algebraic skills. Other examples of ad hoc strategies are guessing a solution for a problem and then working backwards or the application of mnemonics such as the formula triangles.

In this study, applying algebraic skills with insight systematically becomes visible using algebraic techniques during procedures involving basic algebraic skills and those involving symbol sense behaviour. The latter implies having a gestalt view on algebraic expressions and handling in a suitable way with their local and pattern salience. In short, using systematic algebraic strategies are associated with applying algebraic skills systematically and correctly. In addition, as in the previous section, our working definition of the word *successful* in the title of this dissertation refers to using systematic algebraic strategies during algebraic problem-solving in physics textbooks. Such strategies are operationalized by measuring the extent to which students demonstrate symbol sense behavior and the degree to which they use basic algebraic skills correctly. We assume that students learned these algebraic skills in mathematics class from their teachers who strictly follow their mathematics textbooks (SLO, 2019).

6.3 Methodology

6.3.1 Selection Criteria for Students

In this explorative qualitative study with a quantitative component we aimed to gain deeper understanding in students' symbol sense behavior, and to investigate whether shift-problems designed to activate prior mathematical knowledge can improve students' problem-solving in physics for which solution algebraic skills are needed.

We used convenience sampling (Bryman, 2015) to find one physics teacher together with three students who were *available* and *willing* to participate in this study. Next, we used the 'Physics Teachers' Interview Protocol' that is presented in the first section of the appendix to interview the teacher and select appropriate physics students. During the pilot-phase prior to this study, this protocol was tested on different teachers and social scientists and redesigned several times until the authors agreed on appropriate length and clearness. Selection of students were based on a sufficient mathematics grade and an insufficient physics grade for which the physics teacher used 'Magister', a secondary education student monitoring system ('accounts.magister.net', 2018). According to the Dutch ten-point grading system, a student' grade is insufficient if her grade is less than 5.5. This grade criterion indicates that students' difficulties with algebraic physics problems were mainly because of insufficient application of algebraic skills in physics, and not related to a lack of basic mathematical skills.

We also took into account that the students should have a similar knowledge domain at the start of these interviews. Otherwise, it would not have been legitimate to compare their individual performances. Because of these strong selection criteria, we only found one school satisfying them that yielded the anonymized grade 10 physics students Clare, Mary and Sam (the only male student in this study). These students all followed mathematics and used the mathematics textbook *Getal en Ruimte* (Reichard et al., 2014) and the physics textbook *Natuurkunde Overal* (Sonneveld et al., 2014). Furthermore, as can be seen in table 1, their mathematics grade is ranging from 6.8 up to 7.9 and their physics grade from 4.8 up to 5.9. Although Sam's 5.9 did not perfectly meet our requirement, we selected this student. In the last section we elaborate on this matter and legitimize our choice.

Table 1. Students' mathematics and physics grades

	Clare	Mary	Sam
Grade physics	5.0	4.8	5.9
Grade mathematics	6.9	6.8	7.9

6.3.2 Design of Shift-problems

As mentioned above, the students were asked to solve the tasks about algebraic physics problems in two rounds: first, as they appeared in the physics textbook (Ottink et al., 2014), and after two weeks in the second round as *shift problems* (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013). The design of shift problems was based on the iterative 3D-principle in which the interaction between, in subsequent order, the teacher (D1), the resources (textbooks) (D2) and the students (D3) is described. D1 starts with a learning objective of the teacher, i.e. in this study the improvement of symbol sense behavior, especially the transfer of algebraic skills including algebraic techniques from mathematics to physics for solving algebraic physics problems. Therefore, the tasks should trigger students problem-solving and provide insight into their algebraic expertise with basic algebraic skills and symbol sense behavior. Based on these criteria, in D2 we selected the algebraic physics problems *'Task 1: Specific heat capacity', 'Task 2: Thermal resistance' and 'Task 3: Charged particles'* from the physics textbook *SysNat* (Ottink et al., 2014). These tasks are described in symbolic representations (Goldin, 2000) and focus on algebraic manipulations. During a pilot-phase these problems were solved by *other* grade 10 students who did not participate in this study. This offered us information about clearness, length and usability, i.e. that these tasks were doable by students. Afterwards, some of the tasks were adjusted considering these issues. This resulted in 'The Tasks' containing the three algebraic physics problems above which are presented in the second subsection of the appendix. Next, we imagined a hypothetical learning trajectory (Arthur Bakker, 2004) by predicting how students will react when solving these problems. For instance, we expected students to apply the ad hoc strategies. In D3, they were videotaped while solving these tasks thinking aloud (Charters, 2003).

As to D1' where the apostrophe refers to the second cycle of the 3D-principle, students' work of D3 was analyzed to add refinements to the tasks. We also used insights from the study in which we examined symbol sense behavior (Turşucu et al., 2018c). Pivotal was that students should avoid the application of transfer impeding ad hoc approaches. Instead, these refinements should focus and trigger the application of systematic algebraic strategies with insight. Next, we examined how algebraic techniques are used in the mathematics textbook *Getal en Ruimte* (Reichard et al., 2014) to provide similar pedagogical approaches to students that they learned in their mathematics textbook. In D2' these principles were used to design algebraic hints for the tasks aiming at activation of prior mathematical knowledge that we refer to as 'Activation Hint'. We also designed a 'Strategic hint' providing information about how to start a task. The 'Activation hint' together with the 'Strategic hint' we called *systematic hints*. The tasks as they appeared in the physics textbook including these systematic hints form shift-problems.

Immediately before starting with shift-problems, students were introduced by an exemplary task explaining how systematic hints are used. Both the introductory task and shift problems are presented in the fourth subsection 'Shift-problems' of the appendix. The systematic hints are summarized below.

Concerning 'Strategic hints' we demonstrated through a systematic algebraic approach how to go from $y = ab$ to $a = \frac{y}{b}$ in task 1, and from $y = \frac{a}{\frac{c}{d}}$ to $y = \frac{ad}{c}$ in task 2a; that if

$y = ab$ and $z = ab$, then $y = z$ for task 2b, and how to go from $y = \frac{a}{b}$ to $a = yb$ for task 3. Regarding 'Activation hints' we asked first to solve for c, then to apply the [ ]-operator that is used to determine the units of quantities. So, the [ ]-operator is applied to c, i.e. replacing c by $[c]$ and the other quantities by their units for task 1; analogously application of the [ ]-operator to R_{therm} in task 2a; to rewrite R_{therm} to $1/R_{therm}$ for task 2b, and to first to solve for f and then to apply the [ ]-operator to f. We note that the [ ]-operator refers to the unit-operator that is used for dimensional analysis. Moreover, Dutch physics textbooks use this type of notation.

In D3', two weeks after D3, again while being videotaped students solved these shift-problems after being introduced to these tasks by an example explaining how systematic hints are applied. Their work in both D3 and D3' was analyzed with regard to the extent to which they demonstrated symbol sense behavior and the degree to which they used basic algebraic skills correctly. To distinguish between D3 and D3' we refer to D3 as the 'first round' and to D3' as the 'second round'. In short, during the first round the tasks were presented as they appeared in their physics textbook without guidance. Next, in the second round we offered a 'Strategic hint' for how to start with the task, and an 'Activation hint' to activate prior mathematical knowledge in the sense that algebraic techniques were applied in a similar way as in their mathematics textbook.

Furthermore, the students were only interrupted if they remained silent for a while or when a procedure or a reasoning was not clear enough. Stimulated recall techniques were used (Geiger, Muir & Lamb, 2016) to get as much information as possible on students' work. If needed, students were given small hints.

Design of students' task-based interviews (TBIs)

The algebraic problem-solving activities in this study took place during interviews conducted by an independent researcher. Such interviews guided by a protocol are called task-based interviews (TBIs) (Maher & Sigley, 2014). In this study, the TBIs were designed in such a way that the students only had interaction with the tasks and the interviewer. Therefore, they were conducted by the independent researcher in an appropriate, quiet place. Each of these TBIs lasted approximately half an hour. The interview protocol consisted of two parts. During the design process in the pilot phase, we took into account that the instructions were clear and that the non-participating students above could easily work with it. Based on their feedback, some parts of this protocol were adjusted, resulting in the 'Interview Protocol Physics Students' that is displayed in the fifth subsection of the appendix. In the *first* part of our protocol students were asked questions about their background, the textbooks they used, and their mathematics and physics grades were double checked. In the *second* part they were guided by the protocol to solve 'The Tasks'. We note that the same protocol was also used to solve shift-problems. The interviewer only interrupted when the student remained silent for one minute or a procedure or reasoning was not clear enough. Stimulated recall techniques (Geiger, Muir & Lamb, 2016) were used to get as much information as possible on the students' work. For those who got stuck and could not continue or asked for help, we provided small hints.

Historically, TBIs have their origin in clinical interviews that were used to gain deeper insight into students' cognitive development (Piaget, 1954). Such clinical interviews are regarded as qualitative studies (Powell, Francisco & Maher, 2003; Turşucu et al., 2018c). Hence, conducting TBIs to gain deeper understanding of students' symbol sense behavior during algebraic problem-solving can also be considered as qualitative research. Furthermore, the quantitative component of our qualitative study follows from the quantization of basic algebraic skills and symbol sense behavior which is used to compare students' individual performance and will be discussed in the following subsections.

6.3.3 Data Analysis

Analysis of videotaped data

We used the seven phases of Powell, Francisco & Maher (2003) to analyze videotaped data that do not require a rigid order. In 'phase 1' we obtained a first and general understanding of how students solved the tasks. Next, interesting and relevant findings were identified. The description of the videotaped data in 'Phase 2' was less important, since detailed information about the audio part of videotaped data was already transcribed verbatim in 'Phase 4', for which the students gave consent. In 'Phase 3' we identified critical events such as ad hoc strategies, the application of algebraic techniques and other relevant information during problem-solving requiring mathematical motivation. These are described in 'phase 5' where we operationalized our research question using the coding scheme (spread sheet) in table 2.

The coding process was based on the students' written solution set to both 'The Tasks' and 'Shift-problems', the transcripts of the audio part of videotaped data and to some extent on the analysis of videotaped episodes. Students' written work of round one and two were compared to the 'Systematic Solution Set' (solution set in short) in the third subsection of the Appendix and coded using table 2. This means that students' work was assigned to scores to gain insight into their algebraic problem-solving skills for each round and to determine to which extent shift-problems improved the extent to which they demonstrated symbol sense behavior and the degree to which they used basic algebraic skills correctly. For instance, to solve 'Task 1: specific heat capacity' in the appendix systematically and with insight, the first procedure requires division of both sides of $Q = m \cdot [c] \cdot \Delta T$ by m, yielding $\frac{Q}{m} = [c] \cdot \Delta T$. Since $\frac{m \cdot [c] \cdot \Delta T}{m}$ is equivalent to $[c] \cdot \Delta T \cdot m \cdot m^{-1}$, this procedure may require sensitivity towards the exponent -1 in m^{-1} and is associated with the symbol sense type local salience. Students may use other strategies, for example, cancellation of the masses in $\frac{\cancel{m} \cdot [c] \cdot \Delta T}{\cancel{m}}$. Such correct mathematical procedures do not affect scores, neither does interchanging procedures.

Table 2. Coding scheme to analyze students' symbol sense behavior

Time	Task	Algebraic techniques	Ad hoc strategies	Trigger
…	1	…	…	…
…	2	…	…	…
…	3	…	…	…

The coding process

The observation of critical events during coding was reported in the first column 'Time' of table 2. The entries of 'Task' represent the tasks in both the first and the second round. Contrary to the columns 'Time', 'Task' and 'Ad hoc strategies', the remaining columns consist of drop-down boxes, each having different options. Concerning 'Algebraic Techniques', students' algebraic technique during a procedure anchored the selection (coding) of one of the options 'multiplication of both sides', 'division of both sides', 'substitution' and 'inverting both sides' in the spreadsheet. If the student used ad hoc approaches instead of systematic algebraic strategies, the details were described in 'Ad hoc strategies'. The column 'Trigger' involved the options 'positive', 'negative' or 'missed opportunity'. For appropriate procedures, this was coded either 'positive' with score '1' indicating a flawless procedure, or an incorrect 'negative' procedure with score '0.5'. The student who used ad hoc strategies or overlooked a required procedure, corresponds to the third scenario 'Missed opportunity' with score '0'. As a result, these scores led to SSB (%) per task. In addition, the overall symbol sense behavior percentage for all tasks, i.e. $\text{OSSB}\ (\%) = \frac{\text{sum of all sub task scores}}{11} \cdot 100\ \%$, and $\text{OBAS}\ (\%) = \frac{\text{sum of all sub task scores requiring BAS only}}{4} \cdot 100\ \%$ the overall basic algebraic skills percentage for all tasks were calculated. Each of the numbers '11' in OSSB (%) and '4' in OBAS (%) are the sum of respectively eleven procedures involving local salience, pattern salience, and basic algebraic skills, *and* 'five' basic algebraic skills procedures. This implies that a flawless performance in both cases yields the maximum score of 11 for OSSB (%) and the maximum score of 5 for OBAS (%).

Furthermore, a student was considered procedurally fluent when OBAS (%) $\geq$ 75 % (3 out of 4 points). OSSB (%) was sufficient if OSSB (%) $\geq$ 72.7 % (8 out of 11 points).

Based on these criteria, *successful* application of systematic algebraic strategies and thus of algebraic skills corresponds to OBAS (%) $\geq$ 75 % (the criterion for applying basic algebraic skills successfully) and OSSB (%) $\geq$ 72.7 % (the criterion for successful demonstration of symbol sense behaviour).

6.3.4 Towards Common Findings

Regarding 'phase 6' (constructing storyline), in subsequent order we identified the ad hoc strategies in both rounds, students' SSB (%) per task, OBAS (%) and finally OSSB (%). Next, during 'phase 7' (composing narrative), the findings from 'phase 6', students' written solution set and the transcripts were merged into common findings.

With respect to reliability (Bryman, 2015), the first and second author independently crosschecked their results. They found an overlap of approximately 90 %. After discussing the remaining 10 %, some adjustments were made which led to 100% agreement among them.

6.4 Results

In this section we used the term 'hint' to denote hints provided in the first and second round by the independent researcher to help students proceed when they got stuck. So, these hints were not part of the systematic hints. Furthermore, we especially examined tasks for which students maximized their score to 100 % in the second round using systematic hints. In case of two or more of such maximum scores, we only discussed the task that provided the most relevant information.

6.4.1 Clare

Clare is a female student with a 5.0 for mathematics and a 6.9 for physics. In round one, she faced serious difficulties with task 2b and 3 where she used the ad hoc approach swapping strategy. She only used one hint for task 1. In round two, Clare did not use any hints.

Below in table 3 we show her symbol sense behavior characteristics for both rounds. The second (1) up to the fifth column (3) of the first row each represent a task in round one; the sixth (1') up to the ninth column (3') of the first row each represent a task in round two. To distinguish between both rounds, in round two we used apostrophes next to the tasknumbers. 'Missed opportunity', the first column of the second row demonstrates the sum of how many procedures per task she used ad hoc strategies (and) or made no attempt to solve the task.

Table 3. Symbol sense behavior characteristics of Clare per task in both rounds

	1	2a	2b	3	1'	2a'	2b'	3'
Missed opportunity	2	-	2	3	2	-	1	-
Negative score	-	0.5	-	-	-	-	-	-
Positive score	1	1	-	1	1	2	1	4
Task max. score	3	2	2	4	3	2	2	4
SSB (%)	33.3	75.0	0	25.0	33.3	100.0	50.0	100.0

The entry 'Task max. score', i.e. the fifth row of the first column gives the maximum score of a task. For instance, regarding task 2a in round one, Clare's 'Missed opportunity' equals 0; she has one correct procedure with positive score '1' and one that is incorrect with negative score '0.5'. Overall, her total number of 'Missed opportunity' for round one is seven and consists of five overlooked procedures (four local salience and one pattern salience procedure). In the second round this number is reduced to three including two local salience and one pattern salience procedure. For many overlooked procedures she performed algebraic manipulation 'by head' without writing them on paper. This makes it very difficult to decipher the underlying mechanisms of such procedures. Despite this, we calculated the percentage of her total number of 'Missed opportunity' in round one corresponding to 63.6 % of the perfect score. For round two, this number was 27.3%, implying that Clare significantly reduced the number of overlooked procedures in round two without using ad hoc approaches. Her lowest SSB (%) is 0 % for task 2b and 25.0% for taks 3 in round one, and 33.3% for taks 1' in round two. Her highest SSB (%) corresponds to 75.0% for task 2a in round one, and to 100% for both tasks 2a' and 3' in round two. For the latter two tasks, Clare used flawless procedures involving basic algebraic skills and local salience procedures. Her OBAS (%) improved from $\frac{3.5}{4} \cdot 100\ \% = 87.5\ \%$ in the first round to 100.0 % in the second round. Thus, her OBAS (%) is sufficient. Conerning her OSSB (%) over both rounds, this improved from $\frac{3.5}{11} \cdot 100\ \% = 31.8\ \%$ to $72.7\ \%$. The latter implies that Clare's symbol sense behavior is sufficient. Indeed, she nicely adopted the provided systematic hints for both tasks 2a' and 3', and demonstrated sensitivity towards local salience and pattern salience of expressions. Even though she struggled with task 2b that only contains pattern salience procedures, her score slightly increased from 0 % to 50 %.

In the first round Clare was successful in applying basic algebraic skills (OBAS (%) = 87.5 % ≥ 75.0 %) but unsuccessful in demonstration of symbol sense behaviour (OSSB (%) = 31.8 % ≤ 72.7 %). In the second round, she was successful in both aspects of algebraic skills (OBAS (%) = 100.0 % and OSSB (%) = 72.7 % ≥ 72.7 %). Therefore, she successfully transferred algebraic skills that she has learned in mathematics class to physics class.

For task 3, Clare maximized her SSB (%) and went from 25% to 100% in the second round. This is depicted in table 4 that gives insight into how she applied the swapping strategy in the first round. In line 2 Clare swapped the whole term $\frac{Q \cdot q}{r^2}$ in the denominator of $f = \frac{F_C}{\frac{Q \cdot q}{r^2}}$ into $f = \frac{F_C \cdot r^2}{Q \cdot q}$, but did not understand why this step was mathematically legitimate. Even though her solution for this problem yields the correct answer, Clare's procedure lacked a rule-based problem-solving approach in which algebraic skills were used with insight. Remarkably, she used the same approach for task 2b where she mentioned: "*Bottom one goes to top, middle one remains*". It turned out that her mathematics teacher taught her how to swap such terms. In the second round, again after swapping $\frac{Q \cdot q}{r^2}$ in $f = \frac{F_C}{\frac{Q \cdot q}{r^2}}$ she writes $f = \frac{F_C \cdot r^2}{Q \cdot q}$. Next, Clare paused for longer than one minute after which the interviewer interrupted. Then, as can be seen in line 4', she decided to multiply both sides with r^2. She explained that both sides of the equals sign should be divided by $Q \cdot q$ yielding $f = \frac{F_C \cdot r^2}{Q \cdot q}$. Clare noticed that she got the same answer as in the begin of round 2, but that the swapping approach was much easier to use: "*I also divided here by swapping this term and that was much easier for me. I did not really used these [systematic] hints. With these hints, it takes much more time to solve this question*". Nevertheless, she correctly adopted the systematic hints of task 3.

Overall, by adopting the systematic hints in the second round in an appropriate way, Clare converted her lower scores for the high number of 'Missed opportunity' into positive scores. The [ ]-operator was applied flawlessly in all questions. She mentioned that the 'Activation hints' of task 2a and 3 were very useful. To a less extent this also holds for task 1. Despite this, she mentioned that 'her way' of problem-solving including the swapping strategy and algebraic manipulation by memory was easier and quicker to perform. However, Clare did not understand that the systematic hints she applied in the second round helped her to prevent making errors and finishing the problems successfully. This may change when Clare becomes aware of the benefits of such hints, for example, when algebraic manipulations in the next years of senior pre-university education become more difficult. Thus, it is worthwhile to show and make her aware of the importance and effectiveness of using systematic hints.

Table 4. Clare's solutions for task 3 in both rounds

Round 1: SSB (%) = 25.0 %	Round 2: SSB (%) = 100.0 %
$F_c = f \cdot \frac{Q \cdot q}{r^2}$ $f = \frac{F_c}{\left(\frac{Q \cdot q}{r^2}\right)}$ Line 2 $f = \frac{F_c \cdot r^2}{Q \cdot q}$ $f = \frac{[N] \cdot [m^2]}{[C] \cdot [C]}$ $f = \frac{N \cdot m^2}{C^2}$ $f = N \cdot m^2 \cdot C^{-2}$	$F_c = f \cdot \frac{Q \cdot q}{r^2}$ $f = \frac{F_c}{\frac{Q \cdot q}{r^2}} = \frac{F_c \cdot r^2}{Q \cdot q}$ $F_c = f \cdot \frac{Q \cdot q}{r^2}$ $F_c \cdot r^2 = \frac{f}{1} \cdot \frac{Q \cdot q}{r^2}$ Line 4' $F_c \cdot r^2 = f \cdot Q \cdot q$ $f = \frac{F_c \cdot r^2}{Q \cdot q}$ $[f] = \frac{N \cdot m^2}{C^2} = N \cdot m^2 \cdot C^{-2}$

6.4.2 Mary

Mary is a female student with a 4.8 for mathematics and a 6.8 for physics. In round one, she struggled with task 1 and 3 for which she used the numbering strategy. She only used a hint for task 2a. In round two, Mary did not ask for hints.

Her symbol sense behavior characteristics for both rounds are displayed in table 5. Mary's total number of 'Missed opportunity' in the first round was seven including two overlooked pattern salience procedures and five local salience procedures for which she used the numbering strategy. In round two, she reduced this total number to three, by applying the numbering strategy for local salience procedures three times. Because she reduced the number of ad hoc approaches and even did not overlook procedures in round two, the percentage of her total number of 'Missed opportunity' dropped from 63.6 % for the first round to 27.3 % for the second. These scores are quite similar to that of Clare. Mary's lowest SSB (%) is 0 % for task 2b and 3 in round one (identical to Clare), and highest for task 2a. While her score for task 3 remained the same in round 2, task 2b and 3 increased to the maximum score where she adopted the systematic hints appropriately. For task 3 and 3' she used the numbering strategy for identical procedures, remarkably avoiding all systematic hints. Mary's OBAS (%) was already 100.0 % in the first round, and did not change in the second. Therefore, she has a perfect grasp of basic algebraic skills. Her OSSB (%) improved from 36.4 % to 72.7 % in the second round. Mary's similar scores for the tasks in round two yielded the same OSSB (%) as Clare. Based on this result, Mary has sufficient symbol sense behavior. Her OSSB (%) would have even been higher if she avoided the application of ad hoc strategies in task 3'. In summary, in the first round Mary was successful in applying basic algebraic skills (OBAS (%) = 100.0 %) but unsuccessful in demonstration of symbol sense behaviour (OSSB (%) = 36.4 % ≤ 72.7 %). In the second round, she was successful in both aspects of algebraic skills (OBAS (%) = 100.0 % ≥ 75.0 % and OSSB (%) = 72.7 % ≥ 72.7 %). Therefore, she successfully transferred algebraic skills that she has learned in mathematics class to physics class.

Table 5. Symbol sense behavior characteristics of Mary per task in both rounds

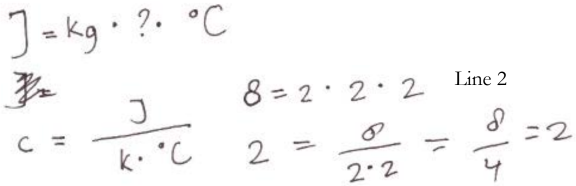
	1	2a	2b	3	1'	2a'	2b'	3'
Missed opportunity	2	-	2	3	-	-	-	3
Negative score	-	-	-	-	-	-	-	-
Positive score	1	2	-	1	3	2	2	1
Task max. Score	3	2	2	4	3	2	2	4
SSB (%)	33.3	100.0	0	25.0	100.0	100.0	100.0	25.0

In table 6 below, we depicted Mary's solutions for task 1 where she increased her SSB (%) from 33.3 % to the perfect score. In round one she wrote the correct solution that was based on the numbering strategy. As can be seen in line 2, Mary first substituted the units of the expression $J = kg \cdot ? \cdot °C$ into numbers resulting in $8 = 2 \cdot 2 \cdot 2$. Since the task is to solve for c and $c = 2$, she solved for 2 in the numerical expression yielding $2 = \frac{8}{2 \cdot 2}$. Then, she re-substituted units for numbers giving her the correct expression $c = \frac{J}{kg \cdot °C}$: "*Yes, if you for example take* 8*, that is* 2 *times* 2 *times* 2 *and* $2 = 8$ *divided by* 2 *times* 2*, because that is* 8 *divided by* 4*, and* 8 *divided by* 4 *is* 2*. So, yes, I think this is the solution* ".

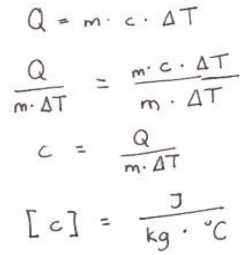
In the second round, she started with the problem *after* carefully reading the systematic hints and divides both sides of $Q = m \cdot c \cdot \Delta T$ by $m \cdot \Delta T$ (line 2'), and correctly wrote the solution: "*First, I'm going solve for* c*. Then, I'm going to divide through* m *and* ΔT*. This gives* c *is* Q *divided by* m *multiplied by* ΔT*, and then I'm going to put* c *in brackets. And then writing all units. I really liked these [systematic] hints*". It can be seen that she applied these hints in a well-structured manner. We conclude that although the solution in round one was correct, a rule-based problem-solving approach in which algebraic skills are used with insight as in the second round was absent.

Table 6. Mary's solutions for task 1 in both rounds

Round 1: SSB (%) = 33.3 %	Round 2: SSB (%) = 100.0 %



Line 2



Line 2'

In general, like Clare, Mary's application of systematic hints transformed her lower scores for the high number of 'Missed opportunity' of round one into positive scores in round two. The [ ]-operator was used correctly in all problems. Because of these hints, Mary almost did not use the numbering strategy that was frequently used in the first round. She was quite diligent in reading the systematic hints carefully and applying them in a much more structured manner. Contrary to Clare, she regularly went back to systematic hints of previous problems to use these for other problems. Thus, she seemed to be aware of the benefits of the systematic hints. Consequently, the 'Activation hint' of task 2a was applied on task 2b and 3. She found the 'Activation hints' of task 1 and 2a very useful. This also applies for the 'Strategic hint' of task 3. When triggered just a few more times by such hints, Mary may soon make the wholehearted transition from ad hoc strategies to systematic algebraic problem-solving with insight.

6.4.3 Sam

Sam is the only male student in this study. His grade for mathematics is a 7.9 and for physics a 5.9. In round one, he faced some difficulties with questions 1 and 3, and only used a hint for task 3. Sam did not use hints in round two. Furthermore, for both rounds he did not use ad hoc approaches.

His symbol sense behavior characteristics for both rounds are illustrated in table 7 below. Concerning both rounds, Sam has only one 'Missed opportunity' in the first where he overlooked a local salience procedure. Consequently, the percentage of his total number of 'Missed opportunity' is low, i.e. 9.1 % of the perfect score and even 0 % in round two.

Table 7. Symbol sense behavior characteristics of Sam per task in both rounds

	1	2a	2b	3	1'	2a'	2b'	3'
Missed opportunity	1	-	-	-	-	-	-	3
Negative score	-	-	-	1.5	-	-	-	-
Positive score	2	2	2	1	3	2	2	4
Task max. Score	3	2	2	4	3	2	2	4
SSB (%)	66.7	100.0	100.0	62.5	100.0	100.0	100.0	100.0

These percentages are in contrast to those of the other students. Indeed, they used ad hoc strategies and overlooked procedures for many times. Sam did not use ad doc strategies. Together with one overlooked procedure above, this explains the discrepancy between them. Besides his lowest SSB (%) of 62.5 % in round one for task 3, he had the maximum score for taks 2a and 2b. These numbers are also in contrast with Clare and Mary who both had zero scores in round one. In round two, Sam even maximized task 1 and 3 by gaining the perfect score. Like Mary, his OBAS (%) was already 100.0 % in the first round, and did not change in the second. Since his symbol sense behavior characteristics were much better than the other students, he had also the highest OSSB (%) that improved from 77.3 % in the first to the perfect score in the second round. So, Sam is the only student whose basic algebraic skills and symbol sense behavior were already sufficient in the first round. He is also the only student with a perfect score for all tasks in the second round, implying the he increased his scores maximally. So, based on our working definition of *successful* in the title of this thesis, Sam successfully applied algebraic skills (OBAS (%) = 100.0 %) and demonstrated symbol sense behaviour (OSSB (%) = 77.3 % ≥ 72.7 %) in the first round. In the second round, beyond his perfect score for OBAS (%), he even gained the perfect score for OSSB (%). Therefore, he successfully transferred algebraic skills that he has learned in mathematics class to physics class.

In table 8 below, we displayed Sam's work for task 3 where he improved his SSB (%) from 62.5 % to a flawless score. In the first round he correctly divided both sides of $F_C = f \cdot \frac{Q \cdot q}{r^2}$ through $\frac{Q \cdot q}{r^2}$ yielding a term with double divisions $f = \frac{F_C}{\frac{Q \cdot q}{r^2}}$ in line 3. Therafter, he intended to stop working on this task, but then proceeded when the interviewer asks him how to get rid of the double divisions (hint). After substituting the quantities in this formula, he tried to simplify this expression. Still, he was not able to get rid of those divisions. Sam suspects that he might cross out m^2 in line 5. On the left side in line 6 he multiplies both the numerator and the denominator of this expression with $\frac{m^2}{m^2}$. On the the right side of the same line, Sam divided away m^2, but mentioned not being sure whether this procedure was correct: "*I don't know whether this is allowed. If I did it right. May be I can cross-out some terms. I don't know whether it is allowed*". In fact, crossing-outs these terms was a guess, without any mathematical understanding of this procedure. In the next round, he correctly divided away both sides of the initial formula through $\frac{Q \cdot q}{r^2}$. Subsequently, he multiplied the new expression by $\frac{r^2}{r^2}$ and finds $f = \frac{F_C \cdot r^2}{Q \cdot q}$. After applying the [ ]-operator and substituting units in the formula he found the correct solution. Interestingly, Sam applied the 'Activation hint' of problem 2a: "*I didn't really use this [systematic] hint. I used that of a previous one [problem 2a]. Although I understand the [systematic] hint, I didn't use it for this task* ".

Overall, except for the divisions above, Sam did not encounter serious transfer problems. Contrary to Clare and Mary, he did not use ad hoc strategies a single time. Remarkably, he already used both the []-operator and systematic algebraic strategies in the first round. He found the 'Activation hint' of task 2a very useful and made repeated statements that the algebraic skills underlying the 'Activation hints' were already taught in mathematics class not too long ago. Indeed, in the second round he used those strategies effectively and maximized his performance with a perfect OSSB (%). Sam is on the right track; he just needs to practice and develop his already existing tendency to use ad hoc approaches.

Table 8. Sam's solutions for task 3 in both rounds

Round 1: SSB (%) = 62.5 %	Round 2: SSB (%) = 100.0 %
$F_c = f \cdot \frac{Q \cdot q}{r^2}$ $\frac{F_c}{\left(\frac{Q \cdot q}{r^2}\right)} = f$ Line 2 $\frac{[F_c]}{\frac{[Q]\cdot[q]}{[r^2]}} = [f]$ $\frac{N}{\left(\frac{C \cdot C}{m^2}\right)} = [f]$ $\frac{N}{\left(\frac{C^2}{m^2}\right)} = [f]$ Line 5 $\frac{N \cdot m^2}{C^2} = [f]$ $\frac{N \cdot \frac{m^2}{m^2}}{\frac{C^2}{m^2} \cdot \frac{m^2}{m^2}} = [f]$ $\frac{N \frac{m^2}{m^2}}{C^2 \cdot m^2} = [f]$	$\frac{F_c}{\left(\frac{Q \cdot q}{r^2}\right)} = \frac{f \cdot \frac{Q \cdot q}{r^2}}{\left(\frac{Q \cdot q}{r^2}\right)}$ $\frac{F_c}{\frac{Q \cdot q}{r^2}} \cdot \frac{r^2}{r^2} = f$ $\frac{F_c \cdot r^2}{Q \cdot q} = f$ $[f] = \frac{[F_c]\cdot[r^2]}{[Q]\cdot[q]} = \frac{N \cdot m^2}{C \cdot C} = \frac{N \cdot m^2}{C^2}$

6.4.4 *OSSB (%) in Both Rounds*

To gain more insight into the effectiveness of our intervention, in figure 2 we displayed OSSB (%) of the first round next to that of the second. To distinguish between both rounds, like in table 3, 5 and 7 where we used apostrophes for the tasks in the second round, here we used OSSB (%)'. This is illustrated in figure 2. Since OBAS (%) and OBAS (%)' were already incorporated in OSSB (%) and OSSB (%)' respectively in the previous subsection, we did not depict them.

For all students there is a major increase in OSSB (%) from the first to the second round. While Sam was the only student having both sufficient procedural fluency (OBAS (%) $\geq$ 75 %) and symbol sense behavior (OSSB (%) $\geq$ 72.7 %) already in round one, the other students demonstrated that in the second. We also calculated the ratio $\frac{\text{OSSB (\%)}'}{\text{OSSB (\%)}}$ that gives the relative increase of students' OSSB (%) over both rounds and therefore may be seen as the effectiveness of our intervention by means of shift-problems. For Clare we have $\frac{\text{OSSB (\%)}'}{\text{OSSB (\%)}} = \frac{72.7}{31.8}$ = 2.3 for Mary $\frac{72.7}{36.4}$ = 2.0 and for Sam $\frac{100.0}{77.3}$ = 1.3. These numbers confirm our earlier findings: the systematic hints provided in the second round led to a major increase in students' symbol sense behavior.

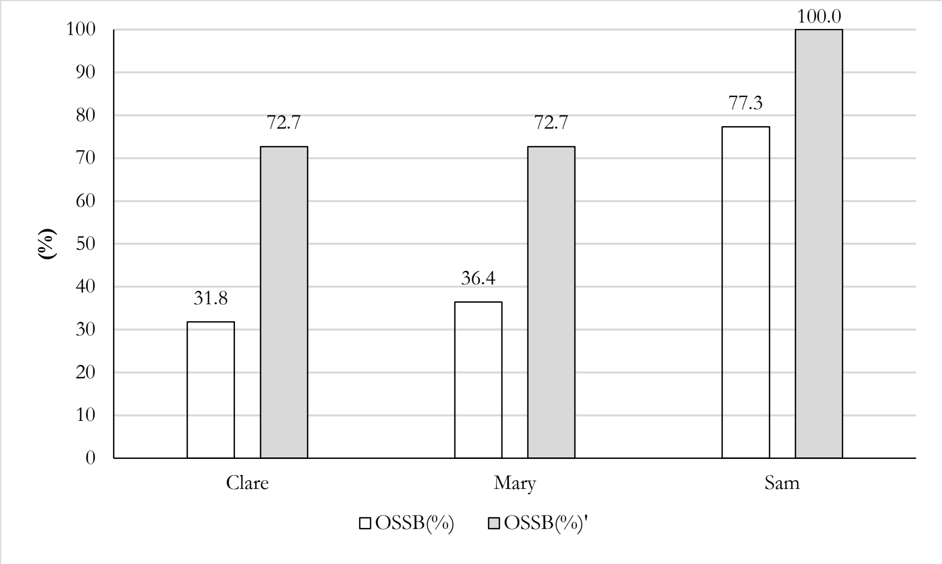


Figure 2. OSSB (%) of the first round next to OSSB (%)’ of the second

We also calculated ⟨OSSB (%)⟩ and ⟨OSSB (%)⟩’ where the symbols ⟨ ⟩ refer to the average. Therefore, ⟨OSSB (%)⟩ is the average of OSSB (%) among all students, and ⟨OSSB (%)⟩’ the average OSSB (%)’ among all students. This number increased from 48.5 % in the first to 81.8 % in the second round. So, while the average OSSB (%) of all students was insufficient in round one, this changed to sufficient in the second, again indicating the effectiveness of how we implemented shift-problems.

6.4.5 *Average SSB (%) per Task in Both Rounds*

In figure 3 we displayed ⟨SSB (%)⟩, i.e. the average SSB (%) per task among all students in round one, and analogously ⟨SSB (%)⟩’ for round two. Round one may be characterized by two regimes: the very low scores for task 1 (44 %), task 2b (33.3 %) and task 3 (37.5 %), *and* the very high score for task 2a (91.7 %). Furthermore, because of the criterion OSSB (%) ≥ 72.7 %, ⟨SSB (%)⟩ ≥ 72.7 % was considered sufficient. Only task 2a met this criterion. Indeed, contrary to task 2a, the other tasks contained a combination of many ad hoc approaches and overlooked procedures, strongly impeding students’ OSSB (%). The next round may be characterized by the high regime including task 1 (77.8 %), task 2b (83.3 %) and task 3 (75.0 %), and the highest regime including task 2a (100.0 %). In the next round, all these tasks met the criterion above, with task 2a even reaching the perfect score. This finding confirms what we have seen before: in round two students adopted systematic hints and applied them quite successfully.

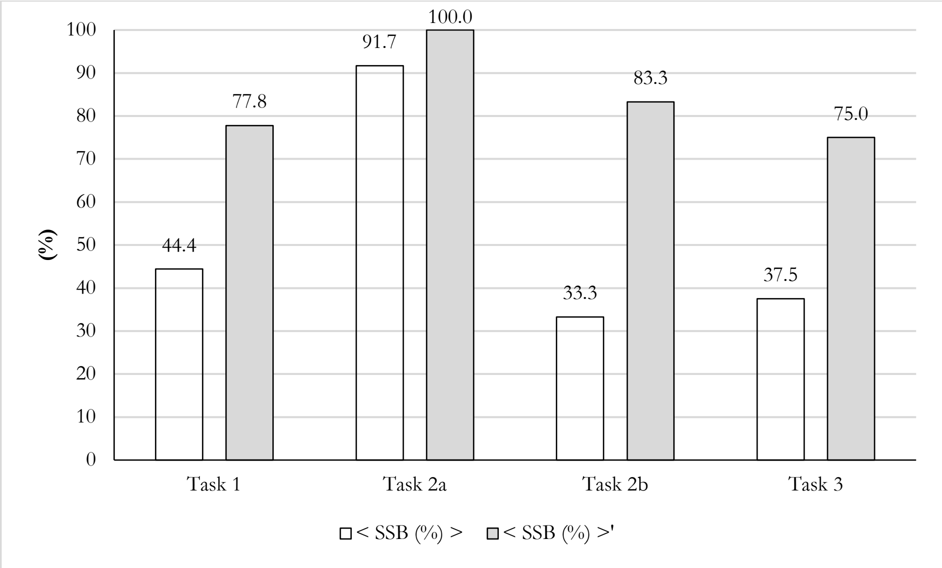


Figure 3. ⟨SSB (%)⟩ per task among all students next to ⟨SSB (%)⟩’ of the second

On the individual task level, for task 1 in the first round students mainly missed points because they did not divide both sides of $Q = m \cdot c \cdot \Delta T$ through m and ΔT. In addition, Mary used the numbering strategy a few times. The sufficient ⟨SSB (%)⟩ in the next round was mainly due Mary and Sam who flawlessly applied the systematic hints.

As to task 2a, students' solid domain of basic algebraic skills resulted in excellent performances for both basic algebraic skills procedures. The already sufficient ⟨SSB (%)⟩ in round one reached the perfect score in the second. The perfect scores in both rounds make it very difficult to argue to which extent the systematic hints were useful.

Task 2b consisted of two pattern salience procedures. Prior to the interviews, we expected this task to be the most difficult problem. Indeed, this was the case for round one where except for Sam the other students struggled seriously. Starting with $R_{\text{thermal}} = \frac{d}{\lambda \cdot A}$, they did not see the inverse relationship $\frac{1}{R_{\text{thermal}}} = \frac{\lambda \cdot A}{d}$. They also did not see that $P = \frac{\lambda \cdot A \cdot \Delta T}{d}$ in the second procedure can be written as $P = \frac{\lambda \cdot A}{d} \cdot \Delta T$. So, although the first procedure would be correct, they would perform incorrectly on the second. Thereafter, ⟨SSB (%)⟩ improved, where Mary and Sam used systematic hints that were mainly adopted from other questions such as 2a. Unfortunately, Clare did not understand the purpose of them. In short, we may say that the systematic hints provided for task 2b were not helpful for students. It might be the case that these systematic hints should be adjusted into more appropriate ones.

Task 3 is the question containing the largest number of variables. Hence, compared to other questions, it is also the task requiring the largest number of procedures. We observed that students struggled with dividing away the fraction $\frac{Q \cdot q}{r^2}$ in $F_C = f \cdot \frac{Q \cdot q}{r^2}$. Their work involving ad hoc strategies and overlooked procedures, strongly impeded ⟨SSB (%)⟩ for this task. It may be the case that especially Mary and Sam loosed their overview, got stuck because of the large number of variables and failed in making the final step; Mary, since she substituted numbers for $F_C = f \cdot \frac{Q \cdot q}{r^2}$ to understand the arrangement of the valid outcome for f, and Sam since he could not get rid of the doubled division term. Clare overlooked procedures because she immediately swapped the fraction $\frac{Q \cdot q}{r^2}$ into the denominator without understanding the mathematical legitimacy of this step, i.e. $f = \frac{F_C}{(\frac{Q \cdot q}{r^2})}$. During the second round, all students used systematic hints to solve this task. Remarkably, instead of the 'Activation hint' of this question, students used that of task 2a and multiplied both sides of $F_C = f \cdot \frac{Q \cdot q}{r^2}$ with $\frac{r^2}{r^2}$. Mary, however, lost points since she also used the numbering strategy.

6.5 Discussion and Conclusion

The aim of this study was to examine the possibility to improve students' symbol sense behavior through activation of prior mathematical knowledge during algebraic problem-solving in physics. To this extent we developed shift-problems. The main difference with earlier studies investigating shift-problems (Palha, Dekker, & Gravemeijer, 2015; Palha, Dekker, Gravemeijer, van Hout-Wolters, 2013) is that they were for the first time designed outside mathematics education, i.e. in physics education. In addition, except for the study examining symbol sense behavior in physics problems (Turşucu et al., 2018c), this is the second time that this concept was studied outside mathematics education. Also, our tasks consisted of expressions with variables representing real, measurable quantities in physics. Other studies used abstract variables in mathematics without meaning in real life.

Regarding operationalization of symbol sense behavior, we followed the method that we have developed to research students' basic algebraic skills and sensitivity towards gestalt view and visual salience by means of local salience and pattern salience of algebraic expressions (Turşucu et al., 2018c). To assess students' work, we used traditional pen-and-paper settings present in previous studies (Arcavi 1994, 2005; Wenger 1987) and not a digital environment such as Bokhove & Drijvers (2010). Unlike aforementioned studies that mainly have a qualitative character, we assessed students' work both qualitative and quantitatively, i.e. students' performance on procedures concerned with basic algebraic skills and symbol sense behavior were both quantized. To this extent, we used the coding scheme of table 2 together with the systematic solution set including clearly worked out procedures in the appendix. Furthermore, we analysed videotaped data in detail following the seven consecutive steps in the theoretical model of Powell et al. (2003). Except for the study of (Turşucu et al., 2018c), such detailed analysis is not present in earlier studies.

As to the applicability of our systematic algebraic problem-solving approach with insight involving activation of mathematical knowledge, we expect this approach to be applicable to other other science subjects. (and even other disciplines). For instance, solving for a variable in both Poiseuille's law $Q = \frac{\pi \cdot P \cdot r^4}{8 \cdot \gamma \cdot l}$ in biology and the equilibrium equation $K_{eq} = \frac{[C]^c \cdot [D]^d}{[A]^a \cdot [D]^d}$ in chemistry.

Similar to Bokhove & Drijvers (2010), the tasks were carefully selected to trigger students' problem-solving and to offer deeper understanding of students' performance including basic algebraic skills and symbol sense behavior. Indeed, this was the case in both rounds contributing to the internal validity of this study.

Concerning the sub question (5a) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems that occur in their physics textbooks?*", two of the three students' symbol sense behavior was insufficient in the first round. This was mainly due to overlooked procedures and the application of ad hoc strategies lacking a rule-based problem-solving approach in which algebraic skills are used with insight. Regarding sub question (5b) "*To what extent do students in senior pre-university education demonstrate symbol sense behavior when solving the same algebraic physics problems that occur in their physics textbooks after activation of prior mathematical knowledge?*", students demonstrated sufficient symbol sense behavior. This implies that students adopted the systematic hints that we provided appropriately and increased their symbol sense behavior in the second round. Beyond symbol sense behaviour, they also improved their application of basic algebraic skills. They strongly benefited from our intervention including activation of mathematical knowledge where we offered rule-based hints with insight, especially algebraic skills that were treated in a similar way to how they were learned in their mathematics textbooks (Reichard et al., 2014). Therefore, our research question "*How can activation of prior mathematical knowledge be used effectively to improve students' symbol sense behavior in senior pre-university education when solving algebraic physics problems?*" can be answered that using algebraic skills in the same way as in mathematics textbooks to activate prior mathematical knowledge was quite effective. In short, we have shown that *successful* transfer of algebraic skills from mathematics to physics (see title of this thesis) is possible when both subjects use the same pedagogy in teaching algebraic skills.

To determine to which degree transfer occurred, we adopted the traditional transfer approach (e.g. Mestre, 2015) by comparing students' solution sets to our systematic solution set. This view of transfer provided us sufficient insight into the extent to which students used basic algebraic skills correctly and to which extent they demonstrated symbol sense behaviour. Therefore, this perspective was very useful in this area of research. In addition, to some extent, we also adopted the actor-oriented transfer approach by paying attention to previous learning derived from what students said during the interviews. For instance, students mentioned that they learned some ad hoc strategies from their teachers. Since Dutch teachers are highly textbook-driven, this may provide information about the textbooks they use in relation with transfer. In short, the traditional approach was very useful to measure the degree of transfer, and the actor-oriented transfer approach to gain insight into their previous learning.

We have seen that using ad hoc strategies may help students to solve basic algebraic problems, but there are risks for the longer term, especially in more sophisticated problems requiring insight. Applying them depends on the approval of an authority such as a teacher or a textbook explaining them what is mathematically correct and what is not. As a consequence, mathematics can become a set of incoherent strategies lacking conceptual understanding. Since students generally do not know the boundaries of ad hoc approaches, they also do not know where they apply and where not. Therefore, ad hoc strategies are harmful for the application of algebraic skills with insight, confirming earlier studies (e.g., Turşucu et al., 2018c; Drijvers et al., 2011; Roorda, 2012). Instead of such strategies, students should use *systematic algebraic strategies* involving a rule-based problem-solving approach in which algebraic skills, especially algebraic techniques are used with insight as in the systematic solution set in the appendix. We recommend that these findings should be considered and implemented by the mathematics and science audience, especially curriculum developers, mathematics and physics teachers, mathematics and science teacher educators and textbook publishers aiming to foster transfer between both subjects and strengthen students experiencing coherence across these subjects.

As to the first group, we suggest using content standards that emphasize the importance of using algebraic skills with insight as described in the mathematics curriculum (SLO, 2019). Prior mathematical knowledge should be activated through the same pedagogy of applying algebraic skills, especially algebraic techniques that occur in mathematics curricula. Content standards of the latter should pay attention to the importance of algebraic physics problems analogous to mathematics problems.

Moreover, independent of whether those curricula determine the textbook content, it is probably better that mathematics and physics textbook publishers avoid ad hoc strategies such as the numbering strategy. Instead, mathematics textbooks should pay attention to systematic algebraic strategies where algebraic skills, especially different forms of algebraic techniques are emphasized, for example, 'inverting both sides of the equation. In addition, we suggest adopting science context such as physics problems. In terms of CMSE, it is probably better that physics problems in physics textbooks are introduced through paragraphs containing analogous mathematics problems that students have learned in mathematics class. Systematic hints can be taught together with systematic procedures as in the solution set. Even though CMSE is important, it requires sufficient organization of the learning process in order to achieve a logical learning line across both subjects. In practice, unfortunately, it still happens that certain mathematical concepts are used in physics class before they were introduced in mathematics class (Alink et al., 2012; Turşucu et al., 2017, 2018c; Roorda, 2012).

On the individual level, physics teachers explaining relevant basic mathematics should be a pre-requisite for pre-service teachers involved in science teacher education programmes leading to a teaching qualification –this does not apply for mathematics teachers, because basic mathematical knowledge is considered to be part of mathematics teacher education programmes. Besides the application of systematic algebraic strategies, we recommend mathematics teachers to refer to the importance of mathematics in physics. For instance, mentioning that mathematics is applied in science subjects such as physics, writing mathematics expressions next to physics formulas can improve transfer and strengthen students experiencing coherence across these subjects (Turşucu et al., 2018a). Physics teachers can, for example, emphasize that algebraic skills learned in mathematics class are applied in physics class and also use the same pedagogy of systematic algebraic approaches as mathematics teachers. Such issues require teachers from both departments to communicate with each other. However, internationally teachers' curricula are overloaded (e.g., Lyons, 2006). Thus, such collaborative efforts between teachers should be feasible to adopt in teaching practice. In this respect, we suggest individual efforts on a small scale during informal meetings.

Regarding mathematics and science teacher educators, we recommend that both of them make 'in service' and 'preservice' teachers aware of the harmful ad hoc strategies and their underlying mechanisms. Moreover, there is the possibility to develop partly integrated teaching materials emphasizing the danger of such strategies for students and propose common identical pedagogical approaches to combat ad hoc strategies through systematic algebraic approaches. Furthermore, the application of algebraic skills with insight that becomes visible during basic algebraic skills and symbol sense behavior procedures is discussed in mathematics education. We suggest science teacher educators to explain the importance of this concept for problem-solving in physics in relation to activation of prior mathematical knowledge to teachers and future teachers.

The concepts of basic algebraic skills and symbol sense behavior are intertwined (see figure 1). Hence, it is not easy to recognize on which concept students rely (Bokhove & Drijvers, 2010). Despite this, we succeeded in recognizing them in both rounds quite easily using the numerical criteria OSSB (%) ≥ 72.7 % and SSB (%) ≥ 72.7 %. In the same way with the OBAS (%) ≥ 75 % criterion we were able to observe and measure basic algebraic skills procedures. Moreover, investigation of OBAS (%) and OSSB (%) separately might indicate that we decoupled both concepts. In fact, we studied OBAS (%) to gain deeper understanding into the degree to which students had a solid grasp of basic algebraic skills. On the other hand, OBAS (%) was already part of OSSB (%), keeping their intertwinement intact.

As for students who faced problems applying mathematics in physics, the findings above, especially in the first round resonate with earlier studies (e.g., Turşucu, 2018c; Dierdorp et al., 2014; Rebello et al., 2007; Roorda et al., 2014; Wong, 2018. This also applies for previous studies on learning and instruction emphasizing the importance of activation of pre-knowledge for better students' achievements (e.g., Hailikari, Katajavuori, & Lindblom-Ylanne, 2008; Roorda, 2012). Indeed, our intervention with shift-problems activating prior-mathematical knowledge improved students' performance.

6.5.1 *Limitations and Further Recommendations*

For this study, we aimed at interviewing three students. Over two rounds of problem-solving, we expected that this would provide us sufficient data, which was indeed the case. Therefore, we ignored a gender ratio of 50%–50%. Our extended literature research reveals that there are no indications that a sample with an uneven gender ratio would yield different results. It is very likely that students' OBAS (%) and OSSB (%) in both rounds are related to a similar knowledge domain at the start of these interviews and their mathematics and physics grades. Regardless of sexes, such sample may have generated comparable results. Therefore, we do not view the composition of our sample as a limiting factor.

The selection criteria in this study were so strict that we only found one school meeting these requirements. Still, we *exactly* found three students, i.e. the number of students we aimed for. Among them, only one student whose physics grade was 5.9 (instead of < 5.5) did not perfectly meet these criteria. We selected this student because his (Sam) physics grade was in the 'danger zone'. Contrary to the other female students whose OSSB (%) scores were insufficient in the first round, Sam's OSSB (%) performance was sufficient in both rounds.

This may lead to the following question: "*Would these scores have been insufficient if his physics grade was insufficient?*". This may or may not be the case, but this question is too difficult to answer. For instance, he might have repeated his class and improved his problem-solving skills. On the other hand, his mathematics grade of 7.9 that is approximately 1.0 higher than the other students, may also not explain Sam's performance. Indeed, in the study of Turşucu et al. (2018c) that only assessed symbol sense behavior in algebraic physics problems, students with similar mathematics grades performed quite poorly on the transfer tasks leading to both insufficient OBAS (%) and OSSB (%) scores.

Except for additional information about the introductory task and the systematic hints provided in the second round, students were asked to solve the same questions in both rounds. Could this have caused undue bias? We do not think that the student's memory of the first round caused undue bias leading to a 'distorted picture' of students' OBAS (%) and OSSB (%) performance in the second round. First of all, round two took place two weeks after the first. Secondly, the combination of introductory task and the systematics hints shaped a different context than the same problems in the first round. Finally, there are no indications that point in this direction. The students seem not to be aware that shift-problems involved the same questions as the first round.

With regard to representativity of our sample, these students are not representative for the Dutch context. For instance, their problem-solving skills and symbol sense behavior characteristics are too different. In addition, the findings following from this study cannot be extended for the Dutch population, neither to other countries outside the Netherlands. Indeed, our sample size is simply too small. Yet, concerning pedagogical approaches to algebraic problem-solving in grade 10 mathematics and physics textbooks, there may be similarities between the Netherlands and other countries with national final examinations which are described in curricula through the general educational core goals and the more specific standards, shaping textbook driven mathematics and physics teachers who teach those textbooks to their students.

In this study, the students were provided *systematic hints* in the second round to improve their symbol sense behavior. Other *hints* were offered when students got stuck to help them proceed. We expect that these hints did not influence our results. For instance, during task 3 of the first round (see line 5 of table 8), Sam got stuck and mentioned "*I do not know whether I can further simplify this term, but at least until I can*". The interviewer responded: "*Can you simplify it one step further? To avoid having two divisions in that term?*". Thereafter, even he performed incorrectly, Sam continued his work.

Some of the tasks were not read carefully by students, especially the systematic hints. In addition, for most of the tasks the students worked too fast resulting in sloppy mistakes that may have been overcome if they had carefully re-examined their work. Hence, we suggest adjusting the interview protocol and incorporate two design principles: it should explicate that students should read the questions very carefully and re-examine their work after they finished their tasks. Earlier studies state that such adjustments may add to students' metacognitive skills (Hattie & Timperley, 2007; Shute, 2008). This in turn can improve students' transfer.

In this research, a student was considered procedurally fluent when her OBAS (%) ≥ 75 %. For symbol sense behavior we used OSSB (%) ≥ 72.7 %. These criteria were not chosen arbitrarily, but the result of consensus among all authors after a series of discussions. The norm for a solid grasp of basic algebraic skills was agreed on 3 out of 4 points. Symbol sense behavior extends basic algebraic skills, since it also requires insight into algebraic skills. Thus, we took 8 out of 11 points, i.e. a minimum OSSB (%) score of 72.7 %.

The findings above offer insight into the relation between procedural fluency and symbol sense behavior, and how these concepts should be treated in teaching practice. Our findings show the effectiveness of using systematic algebraic strategies involving a rule-based approach where basic skills are used with insight to combat problems that require unusual reasoning. The discussion is not about focussing on basic skills or insight. Instead, both basic skills and symbol sense behavior should be taught in an integrated manner. This confirms aforementioned statement "*Without insight, there is no skill, and without skill, there is no insight*" (Drijvers 2011, p. 141).

Two of the three students used the harmful ad hoc strategies. One of them mentioned that she learned the swapping approach from her mathematics teacher which might indicate that her teacher learned this strategy from her mathematics textbook (Reichard et al. 2014a). This method has the largest market share in the Netherlands, influencing a large number of highly textbook-driven teachers teaching them to their students (SLO, 2019; van Zanten and van den Heuvel- Panhuizen 2014). Therefore, we suggest further research investigating to which extent such strategies are involved in this type of textbook series. We strongly recommend researchers sharing their findings with the textbook publisher. If needed, we suggest making publishers aware by pointing out the risks for teachers and students such strategies entail. This may also be a point of attention for grade 9. Grade 10 students' lack of insight into the application of algebraic skills in round one may be due to insufficient emphasis on a rule-based problem-solving approach on conceptual understanding in grade 9. Therefore, we recommend examining grade 9 textbooks to elaborate on this matter.

6.6 Appendix

6.6.1 Physics Teachers' Interview Protocol

Introduction

We are grateful that you participated in this interview that will take about 20 minutes and is part of the PhD-study of the interviewer, Süleyman Turşucu. We have some questions about the background of your grade 10 physics students, and their mathematics and physics textbooks.

Interview aim

The purpose of this interview is to select three grade 10 physics students who have a sufficient mathematics grade (≥ 5,5) and an insufficient physics grade. Later, during interviews they will solve algebraic physics problems while being videotaped and thinking aloud (Charters, 2003). These interviews may offer deeper understanding about how students' application of algebraic skills from mathematics in physics can be improved.

Interview strategy
Could you please answer the questions of the part '*Questions about background of students*' below? We note that the students' names will be anonymized. Would you consent to your students who are engaged in this study?

Questions about background of students
1. Could you please offer us the grade 10 students' names having a sufficient mathematics grade (≥ 5,5) and an insufficient physics grade?
2. Could you please offer us insight into their attitude towards learning?
3. Could you please tell us which physics textbook they use, and why this textbook was chosen?
4. Could you please tell us which mathematics textbook they use?

6.6.2 The Tasks

Task 1 specific heat capacity
When an object is heated, its temperature increases. The formula for this phenomenon is given by: $Q = m \cdot c \cdot \Delta T$. Here, Q is the thermal energy in J, m is the mass in kg, c is the specific heat capacity and ΔT is the temperature change in K.

Derive the unit of the specific heat capacity. First, solve for c.

Task 2 thermal resistance
The thermal resistance $R_{\text{thermal}} = \frac{d}{\lambda \cdot A}$ is a measure of the thermal conductivity of an object. Here, d is the thickness of the material in m, A is the area of the material in m^2 and λ is the thermal conductivity in $W \cdot m^{-1} \cdot K^{-1}$.

a) Derive the unit of R_{thermal}.

The heat flow is given by $P = \frac{\lambda \cdot A \cdot \Delta T}{d}$. It can be shown that the relation between P and R_{thermal} is $P = \frac{\Delta T}{R_{\text{thermal}}}$.

b) Show that this is the case.

Task 3 charged particles
The attractive force between two charged particles is given by $F = f \cdot \frac{q_1 \cdot q_2}{r^2}$. Here, *f* is a constant, q_1 and q_2 are the magnitudes of both charged particles in C and r the distance between the centers of the particles in m.

Derive the unit of the constant f. First, solve for f.

6.6.3 Systematic Solution Set

Task 1 specific heat capacity

Procedure 1 (LS): division of both sides of $Q = m \cdot [c] \cdot \Delta T$ with m; result: $\frac{Q}{m} = [c] \cdot \Delta T$. Procedure 2 (LS): division of both sides of $\frac{Q}{m} = [c] \cdot \Delta T$ with ΔT; result: $[c] = \frac{Q}{m \cdot \Delta T}$. Procedure 3 (BAS): substitution. Each quantity in the formula is replaced with its corresponding unit, except for $[c]$; result: $[c] = \frac{J}{kg \cdot K}$. The solution $[c] = J \cdot kg^{-1} \cdot K^{-1}$ was also regarded as correct.

Task 2 thermal resistance

a) Procedure 1 (BAS): substitution. Each quantity in the formula is replaced with its corresponding unit, except for $[R_{\text{thermal}}]$; result: $[R_{\text{thermal}}] = \frac{m}{W \cdot m^{-1} \cdot K^{-1} \cdot m^2}$. Procedure 2 (BAS): $[R_{\text{thermal}}] = m^{+2-2} \cdot W^{-1} \cdot K$; result: $[R_{\text{thermal}}] = W^{-1} \cdot K$.

b) Procedure 1 (PS): inverting both sides of $R_{\text{thermal}} = \frac{d}{\lambda \cdot A}$; result: $\frac{1}{R_{\text{thermal}}} = \frac{\lambda \cdot A}{d}$.
Procedure 2 (PS): substitution. Replace $\frac{\lambda \cdot A}{d}$ in $P = \frac{\lambda \cdot A \cdot \Delta T}{d}$ by $\frac{1}{R_{\text{thermal}}}$; result: $P = \frac{\Delta T}{R_{\text{thermal}}}$.

Task 3 charged particles

Procedure 1 (LS): multiplication of both sides of $F_C = \frac{f \cdot Q \cdot q}{r^2}$ with r^2; result: $r^2 \cdot F_C = f \cdot Q \cdot q$. Procedure 2 (LS): division of both sides with q; result: $\frac{r^2 \cdot F_C}{q} = f \cdot Q$. Procedure 3 (LS): division of both sides with Q; result: $\frac{r^2 \cdot F_C}{Q \cdot q} = f$. Procedure 4 (BAS): substitution. Each quantity in the formula is replaced with its corresponding unit, except for $[f]$; result: $[f] = \frac{m^2 \cdot N}{C^2}$. The solution $[c] = m^2 \cdot N \cdot C^{-2}$ was also regarded as correct.

6.6.4 Shift Problems

Exemplary task

The force of gravity at the earth's surface on an object is given by $F_g = m \cdot g$. Here, m is the mass in kg and g its acceleration due to gravity in $\frac{m}{s^2}$.

a) Solve for m

'Activation hint'. In mathematics class you have learned that

> when $y = \boldsymbol{a} \cdot b$ and you want to solve for $\boldsymbol{a} = \cdots$, then the b on the right side should be 'taken away'. This can be done by dividing both sides of the equals sign through b. So, $\frac{y}{b} = \frac{\boldsymbol{a} \cdot b}{b}$ and because $\frac{b}{b} = 1$, this gives $\boldsymbol{a} = \frac{y}{b}$.

Application of the 'Activation hint' yields $F_g = m \cdot g$, $\frac{F_g}{g} = \frac{m \cdot g}{g}$, $m = \frac{F_g}{g}$.

b) Find the unit of F_g

'Strategic hint'. First use the unit-operator [ ] and replace F_g by $[F_g]$, and the other quantities by their units.

Application of the 'Strategic hint' yields $[F_g] = \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}$.

Now you have seen how the 'Activation hint' and the 'Strategic hint' are applied, I ask you to apply them in the tasks below. Please follow the hints in subsequent order.

Task 1 specific heat capacity (see subsection 'The Tasks')

Derive the unit of the specific heat capacity. First, solve for c.

(1). 'Strategic hint 1': first solve for c.

(2). 'Activation hint': in mathematics class you have learned that…

> …when $y = \boldsymbol{a} \cdot b$ and you want to solve for $\boldsymbol{a} = \cdots$, then the b on the right side should be 'taken away'. This can be done by dividing both sides of the equals sign through b. So, $\frac{y}{b} = \frac{\boldsymbol{a} \cdot b}{b}$ and because $\frac{b}{b} = 1$, this gives $\boldsymbol{a} = \frac{y}{b}$.

(3). 'Strategic hint 2': use the unit-operator [ ].

Task 2 thermal resistance (see subsection 'The Tasks')

a) Derive the unit of R_{thermal}.

(1). 'Strategic hint'. First use the unit-operator [ ].

(2). 'Activation hint'. In mathematics class you have learned that…

> …when $y = \frac{a}{\frac{c}{d}}$, this can be simplified into a simpler expression by multiplying both sides of the equals sign by $\frac{d}{d} = 1$ giving $y = \frac{a}{\frac{c}{d}} \cdot \frac{d}{d}$. Now, the denominator becomes $\frac{\boldsymbol{c}}{\boldsymbol{d}} \cdot d$. Since, $\frac{d}{d} = 1$, the denominator equals $\boldsymbol{c}$. Finally, we write $y = \frac{a\boldsymbol{d}}{\boldsymbol{c}}$.

The heat flow is given by $P = \frac{\lambda \cdot A \cdot \Delta T}{d}$. It can be shown that the relation between P and R_{thermal} is $P = \frac{\Delta T}{R_{\mathrm{thermal}}}$.

b) Show that this is the case.

(1). 'Strategic hint'. Rewrite R_{therm} into $1/R_{therm}$.

(2). 'Activation hint'. In mathematics class you have learned that…

> …when $y = ab$ and $z = ab$ then the following relation apply: $y = ab = z$. This means that $y = z$.

Task 3 charged particles

Derive the unit of the constant f. First, solve for f.

(1). 'Strategic hint'. First solve for f.

(2). 'Activation hint'. In mathematics class you have learned that…

…when $y = \frac{a}{b}$ and you want to solve for $a = \cdots$, then the $\frac{1}{b}$ on the right side should be 'taken away'. This can be done by multiplying both sides of the equals sign by b. So, $y.b = \frac{a}{b} \cdot b$ and because $\frac{b}{b} = 1$, this gives $a = yb$.

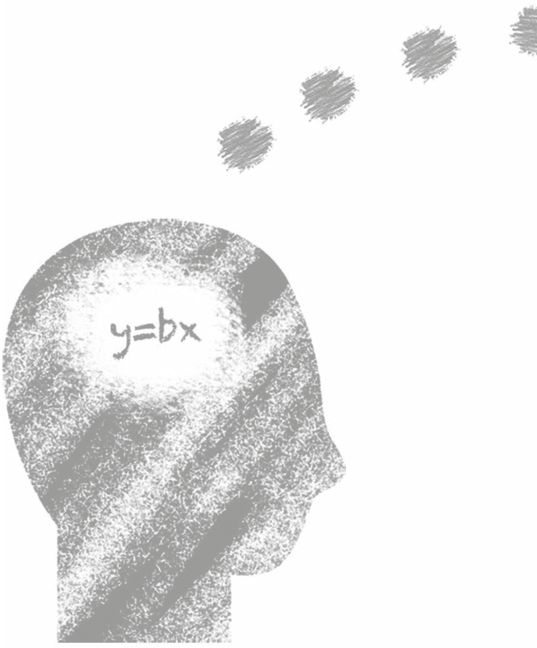
(3). 'Strategic hint'. Use the unit-operator [ ].

6.6.5 Physics Students' Interview Protocol

Introduction
We are grateful that you participated in this interview that consists of two parts. In the *first* section we ask you some questions about your background. The *second* section is concerned with algebraic problem-solving in physics where we ask you to solve four tasks while thinking aloud and being videotaped.

First section: questions about your background
1. Could you please tell me why you chose mathematics and science subjects?
2. Could you please tell me your ideas about mathematics and physics?
3. Could you please offer me your grades for mathematics and physics?

Second section: algebraic problem-solving in physics
Could you please solve the three tasks 'Task 1: *specific heat capacity*', 'Task 2: *thermal resistance*' and 'Task 3: charged particles' below while thinking aloud. Could you please write down as many intermediate steps as possible. You will only be interrupted if it remains silent for approximately one minute, or a procedure or reasoning is not clear enough. Your name will be anonymized, and all information that you offered during this interview will only be used for a PhD-research.


y=bx

Chapter 7

General Conclusion and Discussion

7.1 Introduction

In this dissertation we investigated the central research question *"How can the transfer of algebraic skills from mathematics into physics be improved for solving algebraic physics problems that occur in upper secondary education?"*. To answer this question, we conducted five studies. The research questions of these studies are presented below.

In the first study we carried out a problem analysis, since transfer from mathematics to physics was problematic, but very little was known about transfer of transfer of algebraic skills from mathematics to physics in senior pre-university education. Therefore, we examined the two research questions (1a) "*How do mathematics and physics teachers characterise the transfer problem in the case?*", and (1b) "*What sort of beliefs do mathematics and physics teachers' beliefs have about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education (SPE)?*". Question (1a) was asked to check whether teachers acknowledged this type of transfer problem, and question (1b) aimed to gain insight into the various aspects that influence transfer. We interviewed teachers by means of a semi-structured questionnaire including a concrete case about a student transfer problem. The interviews were transcribed verbatim and analyzed using open and axial coding to obtain a hierarchical code tree containing large amounts of data, i.e. teachers' beliefs.

The second study was a follow-up study that aimed to reduce data from study (1) into a small set of core beliefs that contains *constraints* including naïve beliefs that can be harmful for transfer and *affordances* that can improve transfer, and other aspects that can influence CMSE. In this respect, we examined the research questions (2a) "*How can a systematic, refined method be developed to reduce code trees containing large amounts of data into a single dataset?*" and (2b) "*What are the core beliefs of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*". We used pattern coding that in many textbooks is described in a general way (e.g., Saldaña, 2013). Contrary to such a general approach, we intended to use this second cycle coding technique in a more systematic and refined manner. We especially aimed to develop a specific approach to further reduce code trees including large amounts of data. This study also functioned as 'a bridge' between the first and the third study. In short, beyond a study on teachers' core beliefs, we aimed to develop a systematic and refined method to reduce the code tree containing large amounts of coded data, since this was not present in earlier studies. In addition, even if we would have aimed at combining study (2) and study (1) into a single study, it was considered as too large by peer-reviewed international Journals on science education, and therefore considered as unpublishable.

For study (3) we investigated the research question (3) "*What are the belief systems of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*". Such belief systems can be organized into a set of mutually supporting of beliefs. These belief systems were extracted from the small set of core beliefs. Some of these core beliefs may contain the harmful naïve beliefs for teaching practice. This makes examining belief systems in this area of research relevant for classroom practice.

Study (4) examined the research question "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems?*". In this respect, we aimed to gain insight into the underlying mechanisms of students' problem-solving in physics class in which symbol sense behaviour was involved. Improving symbol sense behavior of students, can enhance transfer of algebraic skills from mathematics to physics class. To determine to which degree transfer occurred, we adopted the traditional perspective of transfer. To some extent, the actor-oriented transfer approach was used to gain insight into what students said about their previous learning during the interviews.

In the fifth and last study we used insights from the previous four studies in which compartmentalized thinking, teachers' beliefs about transfer, mismatches between pedagogical approaches in mathematics and physics textbooks, *and* symbol sense behavior are viewed together to combate the lack of transfer. We examined the research question (5) "*How can activation of prior mathematical knowledge be used effectively to improve students' symbol sense behavior in upper secondary education when solving algebraic physics problems?".* This question was divided in two sub questions: (5a) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems that occur in their physics textbooks?*", and (5b) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving the same algebraic physics problems that occur in their physics textbooks after activation of prior mathematical knowledge?*". Similar to study (4), we adopted the traditional perspective of transfer to determine the extent to which transfer occurred. To some extent, we also adopted the actor-oriented transfer approach, again to gain insight into what students said about previous learning during the interviews.

The main results and conclusions of these studies are presented below. In the sub section 'General Conclusion' we bring these results and conclusions together to answer the central research question. Thereafter, in the sub section 'General Discussion' we will first evaluate the models involved in this study and discuss the theoretical contributions to educational research. Next, we will present the limitations of this study followed by recommendations for future research. Finally, we will discuss the implications for educational practice, especially for the mathematics and science audience.

7.2 Main Results and Conclusions of Study (1)

To answer the two research questions of the first qualitative study, we interviewed 10 mathematics and 10 physics teachers from regular Dutch schools who were qualified to teach in senior pre-university education and had at least five years of teaching experience. They were selected using convenience sampling, i.e. they were both available and willing to participate in this study. The interviews were conducted by means of a semi-structured questionnaire including a concrete *case* about a students' transfer problem from mathematics to physics for which solution algebraic skills were needed. We conducted a literature study to design this questionnaire. The questions dealt with actors such as mathematics and physics teachers, collaboration between them, mathematics and physics curricula, and mathematics and physics textbooks. The interviews were transcribed verbatim for analysis, for which the teachers gave consent.

We used open coding (Bryman, 2015) to label each fragment of the transcripts, which provided a short description of teachers' individual beliefs. For each of the twenty transcripts this led to a set of labels identifying teachers' beliefs. Next, we used axial coding to organize these beliefs. This resulted in two identical code trees for both teacher groups. Thus, we obtained one common code tree that is depicted in table 1 below. This hierarchical structure

Table 1. Teachers' beliefs about aspects influencing students' transfer and aspects about CMSE.

Core theme/ subtheme	Mathematics teachers	Physics teachers
1. Coherence	126	135
1.1 Alignment	2/1[a]	10/6
1.2 Collaboration and cooperation	85/10	75/10
1.3 Ideal collaboration and cooperation	39/10	50/10
2. Curriculum	65	86
2.1 Curriculum (general)	25/9	10/7
2.2 Mathematics curriculum	23/10	31/10
2.3 Physics curriculum	17/10	45/10
3. Education	7	26
3.1 Junior pre-university education	7/5	26/7
4. Pedagogy of algebra	82	72
4.1 Algebraic skills	40/10	26/7
4.2 Algebraic techniques	7/4	8/5
4.3 Practice (general)	21/9	30/9
4.4 Practice within mathematics	9/5	3/3
4.5 Practice within physics	5/3	5/3
5. Relation between scientific subjects	87	52
5.1 Mathematics and physics	27/10	15/10
5.2 Mathematics within physics	35/10	23/10
5.3 Physics within mathematics	25/10	14/10
6. School subjects	30	20
5.1 Mathematics	19/7	13/6
5.2 Physics	11/6	7/4
7. Teacher	193	112
6.1 Mathematics teacher	97/10	48/10
6.2 Physics teacher	96/10	64/10
8. The use of textbooks	143	139
8.1 Following textbooks	31/10	43/10
8.2 Mathematics textbook	66/10	31/10
8.3 Physics textbook	37/10	45/10
8.4 Textbook general	9/5	20/7
9. Transfer	144	89
9.1 Activating prior knowledge	8/5	10/4
9.2 Affordances (specific)	34/10	8/5
9.3 Constructing relations (general constraints)	27/10	23/9
9.4 Constructing relations (specific constraints)	75/10	48/10
9.5 Focus on students	1/1[b]	1/1[c]

Note. [a] This subtheme is considered as an outlier
[b] This subtheme is considered as an outlier
[c] This subtheme is considered as an outlier

contained 9 core themes (main branches of the tree) and 28 subthemes (smaller branches). These core themes are 'coherence' (core theme 1), 'curriculum', 'education', 'pedagogy of algebra', 'relation between scientific subjects', 'school subjects', 'teacher', 'the use of textbooks' and 'transfer'. The leaves of the tree are the last and finest level of the hierarchy and represent the underlying continuum of approximately 1.300 individual teachers' beliefs that we have found.

Regarding research question (1a) we found that nearly all mathematics and physics teachers acknowledged the case presented to them and considered it important that students are competent at the transfer of algebraic skills from mathematics into physics. They believed that such problems mainly occur in grade 10.

Concerning research question (1b), the continuum of beliefs contained aspects influencing transfer of algebraic skills from mathematics into physics, including beliefs that impede and improve transfer, and aspects that may impede and enhance students experiencing coherence across these subjects. These aspects also contained naïve beliefs (espoused models) that stand in the way of transfer after they are transformed into classroom practice (enacted models) (see figure 1 in chapter 1). To change naïve beliefs about transfer, teachers have to be aware of the relation between these beliefs and their classroom practice, reflect about them and reconcile their espoused and enacted beliefs. Furthermore, when implemented appropriately, these aspects may help reduce science teachers' frustrations who spend extra time on repeating mathematics and improve students' transfer.

The majority of teachers believed that the lack of transfer is due to students who see both mathematics and physics as two unrelated subjects. This is emphasized by quotes such as "*two entirely separated subjects*" and "*two separate worlds*".

Contrary to physics teachers, most of the mathematics teachers mentioned that they do not feel the need to collaborate and cooperate with physics teachers. This may impede the development of collaboration between both departments, especially common pedagogical strategies to tackle transfer problems. It goes without saying that these views are not typical of the whole mathematical community. In fact, one finds various ideas about the role of mathematics in science, the difficulties and the importance of teaching and learning transfer among mathematicians, even among those whose taste and interest are skewed toward the theoretical end of the mathematical spectrum.

With regard to the teachers' views about improving transfer, we identified three groups. The first group stated that the transfer problem in the case should be solved by intensive algebraic practice in mathematics class. Then, transfer of algebraic skills into physics happens automatically. The second group believed that the transfer problem should be solved by practising algebraic physics problems in physics class. Both opposite groups focus on basic skills, but did not pay attention to conceptual understanding. Thus, their beliefs are considered naïve in relation with transfer. The third group lies between these opposite views. They believed that transfer can only be solved by comprehensive algebraic practice in both mathematics and physics class, for example, algebra problems in mathematics class could use contexts and notations from physics, and physics teachers could activate prior mathematical knowledge. Both teacher groups should emphasize the connections between both subjects. Still, not all teachers in this group payed sufficient attention to insightful learning.

Some of the teachers' beliefs could be organized into a belief system (Ernest, 1991), i.e. into a set of mutually supporting beliefs about transfer and CMSE. Further research investigated to which extent this is the case and which beliefs they contain. This is presented in study (2) and study (3) below.

7.3 Main Results and Conclusions of Study (2)

This qualitative follow-up study aimed at reducing the code tree including approximately 1300 beliefs into a set of core beliefs. This was a crucial step, since the large amounts of data that hampered us to extract belief systems in one data reduction step. Thus, study (2) also functioned as 'a bridge' between studies (1) and (3). We used the second cycle coding technique pattern coding that grasps the essence of coded data and leaves out less important details. Different from, for example, Gibson and Brown (2009) and Saldaña (2013) offering general knowledge on how to reduce coded data, we worked out pattern coding in detail to further reduce the common code tree (see table 1 in the previous sub section). The latter included the subsequent steps 'D1: forming of summarizing beliefs', 'D2: forming of main beliefs' and 'D3: forming of core beliefs' (see figure 2 through four in chapter 3). Our systematic approach provided a generally applicable second cycle coding tool to further reduce data of code trees containing large amounts of data.

As can be seen in table 2 below, we found 16 core beliefs about constraints and affordances influencing both students experiencing coherence across these subjects and students' transfer of algebraic skills into physics. These core beliefs were organized into the five main categories Collaboration (number 1 and 2), Curricula (number 3 through 6), Students (number 7, 8 and 9), Teachers (number 10 through 14) and Textbooks (number 15 and 16). Thus, the nine core themes in table 1 in the previous sub section were condensed into the five categories of table 2. We found which of the five main categories corresponded to the nine core themes.

Table 2. The set of 16 core beliefs.

Core belief number	List of core beliefs
1	Mathematics teachers often lack time for cooperation
2	There is a lack of collaboration between mathematics and physics teachers
3	Algebraic skills taught in mathematics A do not match sufficiently with physics
4	Mathematics contains less algebra
5	Mathematics should incorporate more physics contexts
6	The physics curriculum should contain manipulation of formulas
7	Transfer can be stimulated if students practice in different physics contexts
8	Transfer is being hindered because students regard mathematics and physics as separate subjects
9	Transfer often will occur spontaneously if students recognize the contexts
10	Both mathematics and physics teachers can stimulate transfer
11	There is no consensus whether mathematics and physics teachers should be able to teach basic mathematics that is needed for transfer
12	Transfer can be stimulated if mathematics and physics teachers agree on the used notations for formulas
13	Transfer can be stimulated if prior knowledge is activated in physics class

14	Transfer can be stimulated if students are taught to see connections between contexts
15	Mathematics and physics teachers stick to the lesson book
16	There is no consensus whether mathematics and physics textbooks should be adapted

Based on our literature study, some of the core beliefs were identified as the harmful *naïve* beliefs (number 6, 7, 9, 11 and 16), and the remaining ones as the transfer enhancing *desirable* beliefs. Indeed, naïve beliefs (espoused models) may stand in the way of transfer, i.e. after they are transformed into teaching practice (enacted models). Number 6 was considered naïve since the current physics curriculum already includes explicit descriptions about manipulation of formulas; concerning number 7, thoroughgoing practice in physics class may not necessarily improve transfer. Indeed, earlier studies have shown that besides basic skills in school mathematics one needs to focus on conceptual understanding; regarding number 9, recognition of the same algebraic structure in a mathematics equation and a physics formula does not necessarily lead to transfer; as to number 11, if teachers have not mastered basic algebraic skills, then probably many of their students also lack these skills, making transfer hardly possible; with respect to number 16, there is a lack of alignment between actual mathematics and physics textbooks. This may impede transfer. We concluded that through professional development programs aiming at transfer and coherence across both subjects, teachers should be made aware of their naïve beliefs, reflect on them and reconcile their espoused and enacted models (see figure 1 in chapter 1).

Furthermore, the dataset of 16 core beliefs was sufficiently reduced to extract belief systems in a last data reduction step. The latter is explained in study (3) below.

7.4 Main Results and Conclusions of Study (3)

To answer research question (3), we designed a digital environment to conduct an online survey among 503 Dutch mathematics and physics teachers from all levels of secondary education who were selected by means of self-selection sampling. The 16 core beliefs were converted into 16 claims (see table 3) and incorporated in an online multi-criteria assessment tool. Teachers were asked to select a top 5, and distribute 50 points over these claims, thereby identifying their belief system. We analysed the correlations between those 16 claims and found small correlations between them. Their squares (explained variance) were smaller than 0.10, making principal component analysis and factor analysis 'pointless'. Therefore, we used the clustering technique agglomerative hierarchical clustering to cluster, i.e. categorize teachers based on their belief systems that may contain the harmful *naïve* and the transfer enhancing *desirable* beliefs. After focusing on teachers with more than 10 years of teaching experience, we found three large clusters for those with more than 10 years of teaching experience who were called 'very experienced teachers'. We also found three large clusters for teachers with more than 20 years of teaching experience who were called 'most experienced teachers'. Except for *one* belief system belonging to the most experienced teachers, the other 5 clusters contained both desirable and naïve beliefs including claim number 2, 6, 9 and 16. These clusters turned out to be rather coherent (organized) sets of beliefs, and thus were interpreted as belief systems.

While Ernest (1991) *theoretically* categorized social groups ('Industrial Trainers', 'Old Humanists', 'Progressive Educators', 'Public Educators' and 'Technological Pragmatists') based on their belief systems about a variety of subjects, we *empirically* clustered teachers based on their belief systems about CMSE and transfer. This justified our idea to cluster certain groups based on their belief systems.

Contrary to Ernest's five social groups (belief systems) which are pairwise disjoint, our obtained belief systems were not. Indeed, some beliefs such as number 11 occur in several belief systems. Thus, the construction of an educational matrix model analogous to that of Ernest was not possible.

Table 3. Set of sixteen claims about CMSE and improving transfer.

Claim number	Claim
	To improve the application of algebraic skills from mathematics into physics…
1	… the collaboration between mathematics and physics teachers should have more priority.
2	… mathematics A should contain more algebraic skills than is the case now.
3	… mathematics should contain more algebra.
4	… mathematics teachers need more time to cooperate with physics teachers.
5	… the content of mathematics and physics textbooks should be adjusted.
6	… mathematics and physics teachers should be able to explain relevant basic knowledge about mathematics.
7	… mathematics and physics teachers should follow the content of their textbooks.
8	… students should recognize physics contexts.
9	… students should practice more algebraic skills during physics lessons.
10	… mathematics and physics teachers should use the same notations in formulas.
11	… prior mathematical knowledge should be activated during physics lessons.
12	… students should see relations between contexts of both mathematics and physics.
13	… mathematics and physics teachers should work together to improve the application of these algebraic skills.
14	… mathematics should incorporate more physics contexts.
15	… to a lesser extent students should see mathematics and physics as separate subjects.
16	… the physics curriculum should contain more manipulation of formulas than is the case now.

We found that the naïve beliefs in each of our clusters concerned the *weak* peripheral beliefs in the structure of a belief system. So, contrary to the *strong* central beliefs containing the desirable transfer enhancing beliefs, they are easy to change. We concluded that this could be done through professional development programmes in which well-informed science teacher educators make 'in service' and 'pre service' teachers having naïve beliefs (espoused beliefs) aware of their belief systems, reflect on them, and change these into desirable transfer enhancing beliefs for teaching practice (enacted models). Otherwise, because of the powerful socialization effect in school, teachers are often observed to stick to the same ineffective classroom practice.

We found similarities between the clusters of the 'most experienced teachers' and the clusters of the 'very experienced teachers'. This yielded three clusters that we called the 'collaboration-oriented group' attaching much weight to claims 1, 11 and 3; the 'teacher-oriented group' who strongly believe in the claims 6 and 11, *and* the 'Student oriented group' attaching much weight to claim 15.

Our idea to call Ernest's belief system model 'macroscopic model', and the belief system from cognitive psychology containing a structure with varying centrality and psychological strength of beliefs 'microscopic system', turned out to be useful. While the first model with espoused and enacted models was used to explain and understand how the social context of teaching influences a teachers' belief system, the second model provides a detailed cognitive description of the espoused models to understand how the weak naïve beliefs and the strong desirable beliefs in a belief system are related to each other.

From the set of 16 claims number 11 had the highest total score, and remarkably not a single teacher chose claim number 7. We concluded that according to the vast majority of teachers, activation of prior mathematical knowledge during physics is most likely to enhance transfer; the opposite applies for following textbooks.

Since our belief systems and the social groups of Ernest have mathematics education in common, we found commonalities between them. Both 'Industrial Trainers' and 'Technological Pragmatists' share with the 'Teacher-oriented group' the emphasis on teachers to respectively teach mathematics and improve transfer. 'Progressive Educators' and the 'Student-oriented group' are both student-centred. To improve transfer, one should pay attention to students. The social groups 'Old Humanists' and 'Public Educators' have no commonalities with our clusters. Furthermore, neither the belief systems identified in our study, nor Ernest's social groups focus on teaching practice in which both basic algebraic skills and conceptual understanding are taught in an integrated manner. For teaching practice aiming at CMSE and transfer, probably it is better to treat both concepts together. The next two studies, i.e. study (4) and (5) are concerned with students' symbol sense behavior in relation with improving the application of algebraic skills during algebraic problem-solving in physics class.

7.5 Main Results and Conclusions of Study (4)

For study (4) we used convenience sampling to select 3 mathematics A students from a regular school (I), and 3 mathematics B students from a regular school (II). These students had a sufficient mathematics grade and an insufficient physics grade, i.e. less than 5.5. This grade criterion was to ensure that students' difficulties with algebraic physics problems were mainly because of insufficient application of algebraic skills in physics, and not related to a lack of basic mathematics. We used algebraic physics problems (tasks) from the physics textbook SysNat. Following Bokhove & Drijvers (2011), these tasks were described in symbolic representations, should trigger students' algebraic problem-solving and offer insight into their algebraic expertise including basic algebraic skills and symbol sense behavior.

Students' algebraic expertise became visible through the application of algebraic techniques during procedures involving basic algebraic skills and having a gestalt view on algebraic expressions and dealing with their visual salient aspects local salience and pattern salience in expressions. We conducted task-based interviews with these students who were videotaped while problem-solving and thinking aloud. Both videotaped data and students' work were analyzed using the seven consecutive phases of Powell et al. (2003). This offered us deeper understanding of their algebraic problem-solving abilities in physics, especially in the underlying mechanisms of transfer.

An essential process during data analysis was the operationalization of research question (4). For this purpose, we developed a coding scheme. The coding process was based on analyzing videotaped episodes (to some extent), the transcripts of the audio part of videotaped data, and the students' work (written solution set to the tasks). Their solution set was compared to our systematic solution set, coded and assigned to a score using our coding scheme. Moreover, the design of the solution set was based on insights from earlier studies stating that algebraic skills should be used in an integrated manner by focusing on both basic algebraic skills and insight (e.g., Bokhove, 2011; Bokhove & Drijvers, 2010; Drijvers et al., 2011).

These scores included 'the symbol sense behavior percentage for each sub task' (SSB (%) per subtask) that gives insight into students' performance involving symbol sense procedures (algebraic procedures requiring insight) per sub task. So, SSB (%) does not refer to procedures requiring basic algebraic skills. We also measured 'the overall symbol sense behavior percentage for all sub tasks together' (OSSB (%) for the whole set of subtasks). This provided insight into students' performance involving all symbol sense procedures together. In addition, we measured the overall basic algebraic skills percentage for all sub tasks together (OBAS (%) for the whole set of subtasks). The latter was defined as algebraic procedures requiring only basic skills, not insight. Furthermore, a student was regarded procedurally fluent if OBAS (%) $\geq$ 90.0% (the extent to which a student applied basic algebraic skills correctly). OSSB (%) was considered sufficient when OSSB (%) $\geq$ 80.0% (the degree to which a student demonstrated symbol sense behavior). Based on these criteria, a student was considered successful in the transfer of algebraic skills from mathematics (see title of this thesis) into physics class when both criteria were met. Furthermore, to determine the degree to which transfer occurred, we adopted a traditional perspective of transfer. Beyond this approach, to some extent, we also adopted the actor-oriented transfer view to gain insight into what students said about previous learning during the interviews. We concluded both perspectives were very useful in this area of research.

Concerning research question (4), we found that students lacked both sufficient symbol sense behavior and a solid grasp of basic algebraic skills. We therefore concluded that students overall were *unsuccessful* in the transfer of algebraic skills that students learned in mathematics class to solve algebraic physics problems in physics class. This was mainly due to overlooked procedures and the application of ad hoc strategies including the cross-multiplication, the numbering, and the permutation strategy. The latter two strategies substitute numbers for variables. While the permutation strategy randomly checks several permutations to guess which one is correct, the numbering strategy substitutes numbers to check algebraic manipulations. Ad hoc approaches only worked for basic formulas containing few variables. In problems with more variables, students lost their overview and got stuck. Thus, we recommended students to avoid the application of ad hoc approaches. Instead, they should learn systematic algebraic problem-solving strategies as in the solution set. This involves a rule-based problem-solving approach in which algebraic skills are used with insight, where the term rule plays the role of algebraic axioms in high school algebra. Furthermore, the application of ad hoc strategies did not come as a surprise for us, since they were surfaced independently in the first three studies where teachers called them 'tricks'. The latter was viewed as something that could impede transfer when students relied too much on them. Furthermore, our results indicated that insufficient focus on conceptual understanding of algebra in some mathematics textbooks, could lead to reliance on poorly understood ad hoc strategies.

Regarding effective classroom practice, we concluded that algebraic skills should not focus on either basic algebraic skills or symbol sense behavior. Instead, both aspects should be taught in an integrated manner. As mentioned above, this is in line with the studies on teachers' beliefs. In the next study (5), we used insights from this study (4) and the previous three studies to carry out interventions in physics textbooks to improve transfer and students experiencing coherence across these subjects.

7.6 Main Results and Conclusions of Study (5)

Study (5) is a qualitative study with a quantitative component and based on the insights from previous studies. From study (3) on teachers' belief systems we used claim numbers 10 (mathematics and physics teachers should use the same notations in formulas), 11 (prior mathematical knowledge should be activated during physics lessons), 12 (students should see relations between contexts of both mathematics and physics), 13 (mathematics and physics teachers should work together to improve the application of algebraic skills) and 15 (to some extent students should see mathematics and physics as separate subjects). We note that claim number 11 was the most popular claim among 503 secondary education teachers, and in line with earlier studies emphasizing the importance of activation of prior knowledge in the context of learning and instruction yielding better students' achievements (e.g., Hailikari, Katajavuori, & Lindblom-Ylanne, 2008). Therefore, we selected claim number 11 as a major design principle. The other claims (10, 12, 13 and 15) aim at connection between mathematics and physics subjects, and thus also aim at improvement of students experiencing coherence across these subjects. Furthermore, we also adopted the insight that not a single teacher among 503 respondents had claim number 7 (mathematics and physics teachers should follow the content of their textbooks) in their top 5. We interpreted this as a need for partly adjusting algebraic problems in physics textbooks.

From the fourth study we used the insight to improve students' symbol sense behavior, in particular a systematic, rule-based problem-solving approach in which algebraic skills are used with insight, where rule refers to the standard rules for the division and multiplication of powers ($z^a z^b = z^{a+b}$), which play the role of algebraic axioms in secondary algebra education. Concretely, this approach has been clearly worked out in the systematic solution sets including basic algebraic skills and symbol sense procedures. Hence, we completely avoided any form of ad hoc strategies.

From the first three studies on teachers' beliefs about transfer and from study (4), we used the common insight that a well-balanced approach aiming at transfer of algebraic skills from mathematics into physics should focus on both basic algebraic skills (Rittle-Johnson, Schneider, & Star, 2015) and conceptual understanding, i.e. symbol sense behavior.

Finally, those insights from the studies on teachers' beliefs and the fourth study led to the following two major design principles: 'activation of prior mathematical knowledge' and 'using the same pedagogy of how algebraic skills are learned in mathematics textbooks'. Beyond our insight into claim number 7 above, the idea to include textbooks is related to our intention to explicitly carry out an intervention in physics textbooks. Indeed, according to the studies on teachers' beliefs textbooks are closely followed by Dutch teachers who teach them to their students (SLO, 2019; van Zanten & van den Heuvel-Panhuizen, 2014).

Next, these principles were used to design shift-problems by means of the iterative 3D-principle (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013). Shift-problems are concerned with small interventions in textbooks that are easily adopted by students and teachers. Based on the 3D-principle, we used convenience sampling to select three grade 10 students who had a sufficient mathematics grade and an insufficient physics grade, i.e. less than 5.5 according to the Dutch ten-point grading system. This grade criterion was to ensure that students' difficulties with algebraic physics problems were mainly because of insufficient application of algebraic skills in physics, and not related to a lack of basic mathematics. The algebraic physics problems were selected from the physics textbook SysNat (Ottink et al., 2014) (see chapter six for details) and included 'Task 1: specific heat capacity', 'Task 2: thermal resistance' and 'Task 3: charged particles. These tasks were described in symbolic representations (Goldin, 2000), focused on algebraic manipulations and should trigger students' problem-solving, thereby providing insight into their algebraic expertise. In students' work, algebraic expertise becomes visible by using algebraic techniques during procedures involving basic algebraic skills and having a gestalt view on algebraic expressions and handling in a suitable way with their visual salient aspects local salience and pattern salience.

To answer research question (5), we conducted two rounds of task-based interviews. The first round was conducted to answer sub question (5a), and the second round to answer sub question (5b). During these interviews students were asked to solve the tasks above while being videotaped and thinking aloud: in the *first round* as these tasks appeared in their physics textbook without guidance (Ottink et al., 2014), and after two weeks in the *second round* as shift problems (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013). Our shift-problems contained an instructional model to activate prior mathematical knowledge by providing hints at the start of these tasks. We offered a 'Strategic hint' for how to start with the task, and an 'Activation hint' to activate prior mathematical knowledge in the sense that algebraic techniques were applied in a similar manner to how these were learned in the students' mathematics textbooks (Reichard et al., 2014). Referring to study (3), the 'Activation hint' provided activation of prior mathematical knowledge (claim number 11). Our goal was to improve students' systematic problem-solving abilities, especially symbol sense behavior.

Both videotaped data and students' work were analysed in the same way as in study (4) using the seven consecutive phases of Powell et al. (2003). Again, this provided us deeper understanding of students' algebraic problem-solving abilities in physics, especially in the underlying mechanisms.

In this study (5), a student was regarded procedurally fluent if OBAS (%) $\geq$ 75.0% (the extent to which a student applied basic algebraic skills correctly). OSSB (%) was considered sufficient when OSSB (%) $\geq$ 72.7 % (the degree in which a student demonstrated symbol sense behavior). After students' work of both rounds were assigned to scores, we examined the effectiveness of our intervention by checking to which extent students' basic algebraic skills and their symbol sense behavior were improved. Based on these criteria, a student was considered successful in the transfer of algebraic skills from mathematics (see title of this thesis) into physics class when both criteria were met. Furthermore, to determine the extent to which transfer occurred, we adopted a traditional view on transfer. To some extent, we also adopted the actor-oriented transfer perspective and gained insight into what students mentioned about previous learning during the interviews. We concluded both perspectives were very useful in this area of research.

Concerning sub question (5a), we found that contrary to basic algebraic skills, the symbol sense behavior of two of the three students were insufficient in the first round. Indeed, while the average OBAS (%) of the three students was 95.8 %, the average OSSB (%) was 48.5 %. This insufficient number was mainly due to overlooked procedures and the application of ad hoc strategies lacking a rule-based problem-solving approach in which algebraic skillss are used with insight. The ad hoc strategies involved the numbering strategy and the swapping strategy.

The first strategy substitutes numbers for variables to check algebraic manipulations. The swapping strategy not present in study (4), first divides a single variable that is on one side of the equals sign by the expression that is on the other side of the equals sign. This expression is a fraction. Then, the student multiplies the single variable with the inverse of this expression. Again, as we have shown in the previous study, ad hoc strategies were already mentioned in the first two studies on teachers' beliefs hampering transfer when students rely too much on them.

Regarding sub question (5b), each student demonstrated sufficient symbol sense behavior. Their average of OSSB (%) was 81.8 % and their average of OBAS (%) even 100 %. The students adopted the systematic hints appropriately and increased their symbol sense behavior in the second round. We have seen that some activation hints were valued higher than others and accordingly used more often. Overall, the students benefited from shift-problems containing activation of mathematical knowledge where we offered rule-based hints with insight, especially algebraic techniques. These techniques were treated in a similar way to how they were learned in their mathematics textbooks.

Based on the results above, our answer to research question (5) is using algebraic skills in the same way as in mathematics textbooks to activate prior mathematical knowledge was quite effective and improved both students' basic algebraic skills and symbol sense behavior in upper secondary education when solving algebraic physics problems. We concluded that *successful* transfer of algebraic skills from mathematics to physics (see title of this thesis) is possible when both subjects use the same pedagogy in teaching algebraic skills.

We profited from the insights from the studies on teachers' beliefs about improving transfer of algebraic skills from mathematics into physics, and insights from study (4). These insights were used in an appropriate way to design shift-problems.

Similar to study (4), we observed that ad hoc approaches only worked for basic formulas containing few variables. In problems with more variables, students lost their overview and got stuck. Therefore, students should avoid the application of ad hoc approaches and apply systematic algebraic strategies as in the solution set. The latter involves a rule-based problem-solving approach in which algebraic skills are used with insight, where the term rule plays the role of algebraic axioms in high school algebra.

Since there are commonalities between study (4) and this study (5), some of the results indicated the same. This especially applies for the lack of emphasis on conceptual understanding of algebra that may be the case in some mathematics textbooks. This can lead to reliance on poorly understood ad hoc approaches. Instead, those textbooks should pay attention to effective teaching practice of algebraic skills and follow an integrated approach where there is attention to both basic algebraic skills and symbol sense.

7.7 General Conclusion

Now the main results and conclusions of the five studies above are presented, we can answer the central research question "*How can the transfer of algebraic skills from mathematics into physics be improved for solving algebraic physics problems that occur in upper secondary education?*". This is possible through the design of shift-problems containing an instructional model that provides systematic algebraic hints at the start of algebraic physics problems. These algebraic hints consist of a 'Strategic hint' for how to start with the task, and an 'Activation hint' to activate prior mathematical knowledge in a similar way to how algebraic skills are applied in mathematics textbooks. With algebraic skills we refer to systematic algebraic strategies involving a rule-based problem-solving approach in which algebraic techniques are used with insight. The term 'rule' refers to the standard rules for multiplication and division of powers, such as $y^a y^b = y^{a+b}$, that play the role of algebraic axioms in high school algebra. The systematic solution sets that we used in study (5) to assess students' work can be considered as a concrete example of such rule-based systematic algebraic approaches with insight. This also applies for the systematic solution set in study (4). Moreover, for each systematic solution set we clearly worked out algebraic expertise. The latter concept became visible through the application of algebraic techniques during procedures involving basic algebraic skills and symbol sense behavior, i.e. having a gestalt view on algebraic expressions and dealing in a suitable way with their visual salience (local salience and pattern salience).

Applying ad hoc strategies can be a major threat for transfer. Indeed, we have seen that ad hoc approaches are not based on standard algebraic rules with insight, but only work for a specific case. Especially, in more sophisticated problems with more variables students faced serious difficulties. Therefore, students should avoid the application of ad hoc strategies and prioritize the application of systematic algebraic strategies. This in turn can improve the transfer of algebraic skills from mathematics into physics and coherence across both subjects.

Furthermore, since we have shown that students improved their algebraic problem-solving abilities and thus transfer in study (5), we conclude that both the insights from the studies on teachers' beliefs about transfer and insights from study (4) were implemented appropriately in our intervention.

7.8 General Discussion

7.8.1 Evaluation of Models and Theoretical Contributions

In this sub section we will evaluate the various models that are involved in this study and discuss the theoretical contributions to educational research. We start with the three follow-up studies on teachers' beliefs about transfer. Finally, we discuss the last two studies on symbol sense behavior.

First three studies

Even though there is research on transfer of mathematics into science subjects, very little is known about transfer of algebraic skills from mathematics to physics in senior pre-university education. We were interested in whether teachers really acknowledged a transfer problem involving algebraic skills to physics class, and what were their *beliefs* about improving transfer in this area of research? Such relevant questions legitimized our problem analysis that we carried out in study (1).

We found that teachers acknowledged that students encounter difficulties with applying mathematics in science subjects, especially algebraic skills in physics. They believed that students should be competent in applying algebraic skills in physics class. This is in line with earlier studies on transfer (e.g., Cui, 2006; Jonas et al., 2017; Wong, 2018). Moreover, we found that teachers adopted the traditional transfer paradigm, rather than for instance, the contemporary actor-oriented transfer approach. They viewed transfer as the application of initial learning (mathematics class) in a new learning situation (physics class), again being in line with earlier studies on transfer (e.g., Larsen-Freeman, 2013; Leberman, et al., 2016; Wong, 2018). In addition, they stated that the main reason for the lack of transfer between both subjects is compartmentalised thinking, i.e. they believed that students see mathematics and science subjects as two separate subjects. They also stated that there is a lack of coherence between both subjects. These findings are close to earlier studies stating that students see mathematics and science subjects as unrelated subjects (e.g., Berlin & White, 2012, 2014; Claxton, 1991; Gellish et al., 2007; Quinn, 2013; Roorda, 2012; Osborne, 2013). Especially, the first study provided large amounts of data about aspects influencing transfer and CMSE, not present in earlier studies.

For teachers, probably the biggest remedy against the lack of transfer was activation of prior mathematical knowledge in physics class. This result is not that surprising, since the importance of pre-knowledge is already a well-known issue in the context of learning and instruction in relation to better students' achievements (e.g., Hailikari, Katajavuori, & Lindblom-Ylanne, 2008).

We found relevant insights about the unifying role of mathematics (Atiyah, 1993) in relation with pedagogical strategies to improve transfer. These new insights were not present in earlier studies on transfer of mathematics into science subjects.

The first group believed that the transfer problem should be solved by intensive algebraic practice in mathematics class. Then, transfer of algebraic skills into physics happens automatically. The second group stated that the transfer problem should be solved by practice with algebraic physics problems in physics class. We concluded that these extreme, opposite views both lacked conceptual understanding, resonating with earlier studies that emphasize that for transfer both basic algebraic skills and conceptual understanding should be taught in an integrated approach (Drijvers, 2011; Rittle-Johnson, Schneider, & Star, 2015). The third group lies between these extreme, opposite views. They believed that the remedy for the transfer problem is comprehensive algebraic practice in both mathematics and physics class. Even though there is no literature about this matter, we expect that this finding also holds for STEM-education where the first three letters are abbreviations for 'Science, Technology and Engineering' and indicate the close relationship between mathematics and these subjects. Indeed, mathematics has a unifying role within each of these subjects.

In the third study on teachers' belief systems (clusters), as expected, we have shown that teachers cannot be clustered analogous to Ernest's (1991) matrix model about belief systems (social groups) that are pair-wise disjoint. Indeed, first of all, whereas his social groups are theoretical, our belief systems were based on empirical data. Moreover, contrary to our clusters, his belief systems are 'package deals' where the members of his social groups are supposed to either embrace or reject complete sets of beliefs without admitting belief systems mixing individual beliefs of different groups. His social groups do not allow for different degrees of belief. For instance, we have seen that claim number '11' occured in five different belief systems. Therefore, our belief system model is much more detailed than Ernest's black-or-white approach in which a cluster either contains or excludes a given claim. In fact, we can ask whether, for instance, 'Progressive Educators' (Ernest, 1991) really exist.

Even though Ernest's (1991) theoretical social groups ('Industrial Trainers', 'Old Humanists', 'Progressive Educators', 'Public Educators' and 'Technological Pragmatists') and our final belief systems ('Collaboration-oriented group', 'Teacher oriented group' and the 'Student-oriented group') investigated different issues, both have mathematics education in common. In this regard, except for the 'Old Humanists' and 'Public Educators', we found commonalities between Ernest's (1991) and our groups. The social groups 'Industrial Trainers', 'Teacher-oriented group' and 'Technological Pragmatists' share the belief that transfer can be improved through education that puts emphasis to the role of the teachers. Likewise, together with 'Progressive Educators' the 'Student-oriented group' shared the belief that transfer can be improved by focusing on students, rather than teachers. We concluded that both our belief systems and those described by Ernest (1991) did not focus on classroom practice involving an integrated approach focusing on both basic algebraic skills and conceptual understanding. For CMSE and transfer, both concepts should be learned together (Drijvers, 2011; Rittle-Johnson, Schneider, & Star, 2015; Roorda, 2012).

The terms peripheral weak and central strong beliefs within the structure of a belief system are concepts used in cognitive psychology (Green, 1971; Misfeldt, Jankvist, & Aguilar, 2016). According to our extensive literature study, this was the first time in science education that the theoretically constructed weak naïve beliefs were indeed *weak*, and the strong desirable beliefs were indeed *strong*. We concluded that we verified a qualitative theoretical construct with quantitative empirical data. Moreover, the idea to call our clusters belief systems was legitimate. Each cluster turned out to be a rather coherent set of beliefs. Thus, these clusters were interpreted as belief systems, to a certain extent justifying Ernest's (1991) idea to cluster teachers based on their belief systems.

Furthermore, our idea to refer to the belief system from cognitive psychology as 'microscopic belief system model' and to that of Ernest (1991) as 'macroscopic belief system model' was useful. The first model offered a detailed cognitive description (Green, 1971; Misfeldt, Jankvist, & Aguilar, 2016) of the espoused models to understand how the weak (in this study naïve) beliefs and the strong (in this study desirable) beliefs in a belief system are related to each other. The second model with espoused and enacted models can be used to explain and understand how the social context of teaching through constraints and affordances influences a teachers' belief system. In short, both belief system models complemented each other. They were regarded as two sides of the same coin. Therefore, we proposed to use both models together and refer to them as the 'micro-macro belief system model'.

Concerning methodology, we used existing tools in an innovative way. For the second study on beliefs, we further developed pattern coding (second cycle coding technique) that can now be used to reduce code trees containing large amounts of data. Contrary to conventional approaches (e.g., Gibson and Brown, 2009; Saldaña, 2013) offering general knowledge on how to *further* reduce coded data, we worked out pattern coding in detail. This yielded a systematic and refined approach, not present in conventional approaches. For the third study, the innovation component above was twofold: agglomerative hierarchical clustering (AHC) was for the first time used in science education research to extract belief systems about transfer. For this purpose, we defined the notion of a belief system to be a system of 16 score distributions, which was also new in science education research. Furthermore, different from conventional multi-criteria assessment (mca) tools designed and used to assess different aspects of the sustainability of multiple options and to establish priorities of competing goals and objectives (e.g., Cinelli et al., 2014; Ding, 2008; Soebarto & Williamson, 2001), we used it to extract belief systems. In fact, we designed our mca-tool because there was no such tool available. Therefore, the way in which we designed and used our mca tool was renewing in multiple ways.

Study four and five

In both studies we measured the degree to which students showed both basic algebraic skills and symbol sense behavior during algebraic problem-solving in physics. While our tasks contained expressions with variables relating to real, measurable physical quantities, other studies consisted of abstract mathematical variables without meaning in real life. The latter was the main difference between our and previous studies on symbol sense behavior (e.g., Bokhove 2011; Drijvers 2015).

Concerning the operationalization of symbol sense behavior, in both studies we followed the line of Bokhove & Drijvers (2010) in the sense that we focused on having a gestalt view on algebraic expressions *and* dealing with their visual salience. Moreover, since symbol sense behavior was used for the first time outside mathematics education, especially phyiscs education and the way we operationalised this concept worked out well to gain insight into students' problem-solving abilities, we concluded that it is well-suited to use symbol sense behavior in this area of research. In addition, while Bokhove & Drijvers (2010) used a digital mathematical environment (DME) to assess students' work, our research involved traditional pen-and-paper settings (Arcavi, 1994, 2005; Wenger, 1987). Contrary to aforementioned studies having a predominantly qualitative character, our coding schemes together with the systematic solution sets consisting of clearly worked out systematic procedures, offered us the opportunity to examine basic algebraic skills and symbol sense behavior qualitatively with a quantitative component. This implies that we were also able to measure students' basic algebraic skills and symbol sense behavior quantitatively. In this regard, we used the term *successful* (see title of this dissertation) referring to the application of systematic algebraic strategies during algebraic problem-solving in physics class, which was operationalized by measuring the extent to which students demonstrated symbol sense behavior (OSSB (%)) and the degree to which students applied basic algebraic skills (OBAS (%)) properly.

Our approach to data analysis was based on a theoretical model of Powell, Francisco and

Maher (2003) who provided detailed consecutive steps to analyse videotaped data. We noted that such approach to data analysis was not present in earlier studies in this area of research.

Following Bokhove and Drijvers (2010), in both studies we selected tasks that should trigger students solving algebraic physics problems and provide insight into their procedural skills and symbol sense behavior.

According to Bokhove (2011), observing symbol sense behavior was not a straightforward affair. Indeed, since the concepts of basic algebraic skills and symbol sense behavior are intertwined, it could be difficult to recognize whether students rely on basic skills or show insight into algebraic expressions. Nevertheless, in both studies we succeeded easily using the aforementioned numerical criteria for OSSB (%) and SSB (%). Similarly, the OBAS (%) criterion was used to observe basic algebraic skills. This combination of qualitative research (analysing videotaped data) with a quantitative component is not present in earlier studies on symbol sense behavior.

Concerning study (4), the comparable performance of both mathematics groups showed that the algebra involved in both mathematics subjects was sufficient to tackle algebraic physics problems (SLO, 2019). This contrasted with teachers stating that physics students should choose mathematics B instead of mathematics A (Turşucu et al., 2017).

Except for our intervention in study (5), in both studies we observed that students lacked sufficient basic algebraic skills and symbol sense behavior. This was mainly because they often applied ad hoc strategies. These findings corroborated earlier studies, stating that using ad hoc strategies leads to fragmented knowledge, impedes generalization of algebra, and can be harmful for conceptual understanding (e.g., Drijvers et al., 2011; Roorda, 2012). Above all, these findings were in line with earlier studies in which students struggle with applying mathematics in physics (e.g., Jonas et al., 2017; Redish & Kuo, 2014; Wong, 2018), indicating a lack of transfer between both subjects.

Since we used both the coding scheme and the systematic solution set in both studies to quantify basic algebraic skills and symbol sense behavior, transfer was determined by the researchers' perspective, rather than by the students' construction of similarities between the initial and new learning situation (Lobato, 2003). Beyond this approach, to some extent we adopted the actor-oriented transfer view by investigating what students mentioned during the interviews. This offered us relevant information about their previous learning situation, for example, students mentioned that they learned certain ad hoc strategies from their teachers. We concluded that both transfer approaches were very useful in this area of research, contributing constructively to the evaluation of both transfer models. Moreover, we adopted the traditional transfer paradigm in a way similar to previous studies (e.g., Cui, 2006; Lobato, 2002; Rebello et al., 2007; Roorda et al., 2014). Our findings above extended the very few studies about students having a solid grasp of mathematics, but faced difficulties in applying mathematics in physics, in ways not present in earlier studies (e.g., Hudson & McIntire, 1977; Rebello et al., 2007). This also applied for the underlying mechanisms of transfer of learning.

As to shift-problems (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013), like symbol sense behavior, this is the first time that this concept is used outside mathematics education, especially in science education. Contrary to conventional educational research, shift-problems (Palha, 2013) aim to reduce the large gap between advice offered in educational research and what is established in classroom practice. Only very few teachers can incorporate such large changes in their teaching (Wiliam & Leahy, 2012). As a consequence, such studies often have very limited impact on teaching practice. Shift problems, on the other hand, are small interventions that are easily implemented by students and teachers. Indeed, in this study we have shown that this was the case. The instructional model of our intervention was based on insights from the studies on teachers' beliefs and symbol sense behavior. In this regard, we combined compartmentalized thinking, symbol sense behavior, teachers' beliefs and mismatches between pedagogy of different algebraic approaches to improve transfer of algebraic skills from mathematics to physics class. Beyond the fact that such combination of different areas of research was not present in previous studies on transfer (e.g., Jonas et al., 2017; Redish & Kuo, 2014; Wong, 2018), the way we implemented activation of prior mathematical knowledge in shift-problems turned in relation with those aspects turned out to be very effect transfer. Furthermore, the finding that students improved their problem-solving abilities implies that we also profited from the insights from the studies on teachers' beliefs about improving transfer from mathematics into physics for which solution algebraic skills were needed (for instance, activation of prior mathematical knowledge), and insights from study (4) on symbol sense behavior. These insights were used in an appropriate way to design shift-problems. In short, this study extended the very few studies on shift-problem lessons and shows that shift-problems are well-suited to use in science education, thereby making educational research feasible in teaching practice.

7.8.2 Limitations of The Study

In this sub section we will discuss the limitations of this study. As in the previous sub section, we will first present the three follow-up studies about transfer that is followed by the last two studies.

First three studies

The content of mathematics and science subjects in upper secondary education in the Netherlands are described in curricula that describe the general educational core goals and the more specific standards. These core goals and standards are tested in national final examinations. To a very large extent this shapes the content of textbooks and teachers who quite strictly follow them (SLO, 2019; van Zanten & van den Heuvel-Panhuizen, 2014). Thus, teachers' beliefs about transfer are greatly influenced by the content of textbooks. Since we do not expect much difference in the content of textbook series, teachers' beliefs would not differ significantly from each other. On the other hand, teachers' beliefs in the first study were saturated. Indeed, we did not see much change in the diversity of teachers' individual beliefs after eight interviews in both teacher groups. Since study (2) followed from the previous study, the core beliefs were regarded as saturated too. Therefore, the combination of national final examinations described through the general educational core goals and the more specific standards in curricula, and saturated beliefs made the results of both studies generalizable for mathematics and physics teachers teaching in Dutch upper secondary education. This also applied for senior general secondary education teachers. We did not expect that this holds for preparatory vocational secondary education, because the algebraic skills needed in physics were simply too different from those in upper and senior general secondary education. This could lead to different teachers' beliefs about transfer. In many countries, however, such combination of national examinations and curricula shaping textbooks above with textbook-driven teachers does not exist (e.g., The National Academies Press, 2018; 'TIMMS & PIRLS', 2019). Hence, our results in the first two studies were not generalizable to these countries.

Even though the results above are not generalizable to science teachers, especially physics teachers, we *did* observe students struggling with mathematics in physics class (e.g., Jonas et al., 2017; NCTM, 2013; Redish, 2017; Wong, 2018). So, our study corroborated earlier findings that students face problems with mathematics in science class, and that they should be competent at it.

Our sample in the quantitative follow-up study (3) on teachers' belief systems contained 118 very experienced and 97 most experienced teachers. They were all qualified to teach in upper secondary education. From these 215 teachers, 136 teachers taught mathematics and 79 physics. These numbers were in good agreement with the 2903 qualified mathematics and 1330 qualified physics teachers in upper secondary education in the Netherlands (Ministry of Education, Culture and Science, 2018), also giving a ratio of roughly 2:1. Moreover, our sample with 118 very experienced teachers had a gender-ratio of 1:1 for mathematics, and 5:1 for physics teachers. For the 97 most experienced teachers these numbers were 2:1 for mathematics and 13:1 for physics teachers. Unfortunately, there were no data available on the gender-ratio of qualified most experienced and very experienced mathematics and physics teachers in upper secondary education. This implied that we cannot judge how well our sample represented the national Dutch situation.

Like the results of the first two studies on teachers' beliefs, the results in study (3) on teachers' belief systems were largely shaped by the combination of national examinations and curricula that shape the content of textbooks. We therefore expected our results to be generalizable for both senior pre-university education and senior general secondary education in the Netherlands. This did not apply for preparatory vocational secondary education. This would lead to different belief systems (claims) about CMSE and transfer. Because of the same arguments as in the first two studies, we did not expect our results to be generalizable to other countries outside the Netherlands.

Among the three methods of clustering techniques optimization methods, mixture models and agglomerative hierarchical clustering, we used the third option and discarded the second option, because it presupposed knowledge of latent variables. A standard optimization method is the *k*-means method. Unfortunately, it had several disadvantages. Firstly, it imposed a spherical structure on the data, i.e. it assumed that the data points are clustered in more or less spherical clusters in 16-dimensional space. We had no a priori reason to expect this to be true. Secondly, one had to choose the number *k* of clusters in advance, and thirdly, the algorithm starts with random cluster centres resulting in rather different clusters in each run of the *k*-means algorithm. We therefore used agglomerative hierarchical clustering that had not these disadvantages. Since this method led to nicely distributed clustering of belief systems with very clear splitting heights, it was also objective.

Concerning reliability of the statistical analysis, some error bars were quite large compared to the mean scores and standard deviations of the large clusters for the most and very experienced teachers, respectively. This large dispersion in a belief system is usually due to a few outliers in the cluster. Even if we considered this dispersion, our description of the main characteristics of the two triples of belief systems and the differences between them remained valid.

Furthermore, to enhance reliability of our results, the analysis of each study was carried out independently by several researchers including the first and second authors and cross-checked afterwards. Discrepancies between results were always discussed and if required, adjustments in those areas were made. This led to 100% agreement on the results among the researchers.

Study four and five

The strong selection criteria in study (4) and (5) were required to safeguard the quality of these studies. Firstly, the students should be selected from regular schools and have a sufficient mathematics and an insufficient physics grade, i.e. less than 5.5 according to the Dutch ten-point grading system. They should follow the same mathematics (Reichard et al., 2014a; 2014b) and physics textbook series (Ottink et al. 2014; Sonneveld et al., 2014) and have a similar knowledge domain in these subjects at the moment of interviews. Finally, because we used convenience sampling, they should be *available* and *willing* to participate in this study. This resulted in a sample containing 1 male and 5 female students for study (4), and a sample of 1 male and 2 female students for study (5). The gender ratio might only be an issue for study (4). The strong selection criteria above hampered us to meet a 50% : 50% gender ratio. Fortunately, our extended literature study revealed that there were no indications that such a sample would have generated different results. Instead, they may be similar. Indeed, students' performance in terms of procedural fluency and symbol sense behavior is mainly related to a combination of grades for both subjects and a similar knowledge domain, rather than on gender. It is very likely that male and female students with similar grades will show similar performance. So, the composition of our sample was not be seen as a limiting factor. Still, it is worthwhile to elaborate on this matter, since this contributes to the internal validity (Bryman, 2015) of both studies.

The grade criteria for mathematics and physics subjects were imposed to ensure that students' difficulties with algebraic physics problems were mainly due to insufficient application of algebraic skills in physics, and not related to a lack of basic mathematics. The other criteria were imposed to legitimatize to compare the results of individual students. In addition, the same argument applied to compare both groups in study (4). A point of attention is that students' poor physics grades could also be the result of the absence of a variety of aspects, for example, understanding physical concepts or having a positive attitude towards physics.

In both studies we had very small samples, implying that the findings cannot be extended to the whole Dutch population, neither to other countries outside the Netherlands. For study (5) one may speculate how the results of a large-scale research may differ from our study. For instance, if the designed shift-problems in round two were implemented in 50 regular schools in the Netherlands. We stated that some of them may be close to our results, and others may differ more than what we have obtained. Still, we expected that students would improve the extent to which they applied basic algebraic skills correctly and demonstrated symbol sense behaviour in the second round. This implies that our intervention was independent of who conducted the research. Indeed, using insights from the first three studies on teachers' beliefs we focused on activation of prior mathematical knowledge in these tasks, in the same way to how algebraic skills are applied in mathematics textbooks. We believed that this would strengthen students experiencing coherence across mathematics and physics and improve transfer.

Regarding pedagogical approaches to algebraic problem-solving in upper secondary education, there can be similarities between our context and countries outside the Netherlands. Specifically, in countries with national final examinations in combination with curricula describing the general educational core goals and the more specific standards, shaping the content of textbooks and teachers and students who follow them faithfully. Furthermore, the students in both studies are not representative for the national Dutch situation. The individual differences among the students' grades and their OBAS (%) and OSSB (%) characteristics were simply too different.

For study (4) we had the criteria OBAS (%) ≥ 90.0% and OSSB (%) ≥ 80.0%. These numbers were 75 % and 72.7 % respectively in study (5). None of these values were chosen arbitrarily. In fact, they were the result of consensus among all authors after a series of discussions. First of all, the students should have a solid foundation of basic algebraic skills, explaining why OBAS (%) was slightly higher than OSSB (%). Indeed, symbol sense behavior extends basic algebraic skills and also depends on insight. Thus, OBAS (%) and OSSB (%) had not the same values.

The hints provided in both studies aimed at helping students to proceed when they got stuck. These hints did not cause any undue bias, neither did they influence students' procedural fluency and demonstration of symbol sense behaviour. This is illustrated in examples in the transcripts of videotaped data. Therefore, the findings of both studies were considered reliable. Furthermore, in study (5), except for the exemplary task and the systematic hints offered in the second round, students were asked to solve the same problems in round one. The second round was conducted two weeks after round one. Thus, we believed that this has not caused any undue bias, neither influenced students' performance. Also, the combination of the exemplary task and the systematics hints shaped a different context than the same problems in the first round. In addition, based on conversations with students there were no indications that pointed in this direction. They seemed not to be aware that shift-problems of the second round contained the same tasks as in the first round.

7.8.3 Further Research

Based on our analysis of the studies (1) through (5), there are four future research related topics. These are 'Further research on teachers' beliefs', 'Quality control', 'Research design' and 'Textbook issues'. Below we elaborate on these topics.

Further research on teachers' beliefs

In study (1), referring to figure 1 (Ernest, 1991) in chapter 1, some mathematics teachers had purist beliefs. Even though they perceived applied mathematics inferior to pure mathematics and refused to discuss applications (espoused beliefs), they made suggestions about improving transfer. We recommend identifying these mathematics teachers. How do they deal with transfer problems in the classroom (enacted beliefs) if they have such purist beliefs?

Although we expected that there were three groups concerning the unifying role of mathematics in relation with transfer to other subjects than physics, for instance STEM-education, we suggest future studies to investigate this matter. We also suggest future studies examining to which extent our findings apply to other countries. This may lead to a general framework explaining how the unifying role of mathematics is related to teachers' beliefs about enhancing transfer of mathematics in other subjects.

For the follow-up study (2), we recommend identifying mathematics teachers who believed that both mathematics and physics teachers are not required to be sufficiently knowledgeable to teach basic mathematics (espoused beliefs). Why do these teachers have such naïve beliefs about something that is of major importance for transfer? To answer this and the question above, we suggest conducting in depth-interviews. Concerning mathematics teachers with purist beliefs, this may provide deeper understanding about the relation between such beliefs (espoused models) and teaching practice (enacted models), and for the teachers with naïve beliefs, this may offer insight into why these teachers were unaware of their naïve beliefs about transfer. For both type of teachers, future research should focus on the relationship between espoused and enacted models and make them aware of their harmful beliefs for transfer.

In study (3) we found six large clusters (belief systems). Only one cluster consisted of the desirable transfer enhancing beliefs. The other clusters contained both naïve and desirable beliefs. Fortunately, most of the naïve beliefs within our clusters were weak. What if they were strong? Since naïve beliefs are harmful for transfer in relation to teaching practice, one would like to see them change into desirable beliefs. However, strong beliefs will resist heavily to change because they are connected to other neighboring strong beliefs about transfer. In case of carrying out interventions by means of shift-problems as in study (5), one needs teachers having belief systems with desirable beliefs. Indeed, contrary to teachers having naïve beliefs, we assume that teachers with desirable beliefs will participate more easily in such shift-problem studies. We therefore recommend future studies to investigate the question "*How can strong naïve beliefs be changed into desirable beliefs within the structure of a belief system?*". This may provide solutions making it easier for teachers having belief systems with naïve beliefs to participate more easily in research investigating shift-problems.

The result that not a single teacher among 503 respondents (!) had chosen claim number 7 in their top 5 implies that they believe that following the content of textbooks does not contribute sufficiently to improve transfer. For instance, because they might have believed that there are pedagogical mismatches between mathematics and physics textbooks. In fact, they can have various reasons underlying their beliefs. We therefore suggest conducting qualitative interviews with some of these mathematics and physics teachers to gain insight into why they did not choose claim number 7 in their top 5.

Data reduction is concerned with grasping the essence and leaving out less important details. But what if these details may have contained relevant information as for the core themes in study (1), core beliefs in study (2) and belief systems in study (3)? In this regard, we removed three outliers in the first study. These outliers were the missing subthemes that did not contain at least three different beliefs uttered by at least three different teachers. This criterion may be slightly arbitrary, since outliers may contain relevant information about missing teachers' beliefs such as the subtheme 'Focus on students'. Indeed, only two teachers mentioned the focus on students, thereby approaching transfer traditionally (Lobato, 2003). These two teachers seemed to adopt an alternative approach in which they tried to understand transfer as constructed by the student, i.e. from the students' point of view, and not from a teachers' perspective. Therefore, it is worthwhile to identify these teachers and examine how they exactly viewed transfer.

In the second study a core belief was considered an outlier when a summarizing belief was mentioned less than three times by less than three different teachers. Also, this criterion may be slightly arbitrary. Some of these missing core beliefs were related to the integration of mathematics and physics subjects through the curriculum or textbooks. We suggest studying this matter, since integration can be relevant for transfer. In the third study we focused on the three largest clusters for both teacher groups and neglected 28% of the most experienced teachers and 26% of the very experienced teachers. This implies that we based our findings on respectively the 72% and 74% of the extracted belief systems. It is very likely that some of these clusters contained naïve beliefs that can be harmful for students experiencing coherence across these subjects and transfer. We therefore recommend to further research this matter.

Quality control

Regarding study (3), our sample had a gender ratio (male-female) of 1:1 for mathematics, and 5:1 for qualified most experienced teachers. For the very experienced teachers this was 2:1 for mathematics and 13:1 for physics teachers. Unfortunately, according to the Dutch Ministry of Education, Culture and Science (2019), there were no data available on the gender-ratio of qualified most experienced and very experienced mathematics and physics teachers in Dutch upper secondary education. Therefore, we could not judge how well our sample represented the national situation. At this point, we recommend further studies to investigate the national gender-ratios of both mathematics and physics teachers in SPE in relation with the number of years of teaching experience. This should lead to statements about national representability and generalizability of this study.

Research design

Four issues related to research design may need further attention. Two of them were concerned with design principles and the other two with participants. For future studies conducting similar research on problem-solving as study (4) and (5), we recommend incorporating two important aspects in the interview protocol. The latter should explicate that students should take their time to read the tasks carefully, because they immediately started solving the problems. In addition, almost half of the students worked fast, and after finishing the tasks they did not check them. Hence, their work contained sloppy mistakes that may have been overcome when they had carefully re-examined their solutions. According to earlier research (Hattie & Timperley, 2007; Shute, 2008) the students would have benefited from a guideline explicating to reexamine their finished work. This may also enhance their metacognitive skills. Further research could examine to which extent students benefit from such a guideline during algebraic problem-solving in physics. The other issue deals with the design of systematic hints in study (5). Even though students adopted systematic hints, some of them were higher valued than others. We recommend further research to investigate what aspects some hints make more useful than others, yielding a complete set to draw from.

The students in study (5) worked alone on these problems. These students solved the tasks *individually*. It would be interesting to conduct similar studies with shift-problems during *collaborative* work in regular classrooms (Palha et al., 2013). How could such learning environments improve students' learning?

Furthermore, it is worthwhile to examine the research questions of study (5) in a large-scale study, for instance when $N = 50$ (schools) as mentioned in the previous section. Even though we expect that students will improve their symbol sense behavior in the second round, and thus transfer, there are no data available that confirm this expectation. We recommend future studies to examine this matter.

Textbook issues

A key actor in this study was the textbook, for it is very closely followed by both teachers and students (SLO, 2019; van Zanten & van den Heuvel-Panhuizen, 2014). This does also apply for many other countries with national final examinations like the Netherlands. In this regard, the studies on teachers' beliefs about improving transfer and the last two studies provided relevant information about both junior and senior pre-university education and put forward questions to be investigated in future studies.

Concerning aforementioned absence of claim number 7, we also suggest conducting an extensive literature study investigating important questions as "*What are the possibilities to connect both textbooks in terms of, for instance equations, formulas, same pedagogical approaches, and organization of the learning process?*" And above all, since mathematics and physics textbook publishers pursue different aims, "*What are the possibilities that mathematics and physics textbook publishers work together?*".

The lack of connection between both subjects became more evident in the following studies (4) and (5). We have seen that instead of emphasis on rule-based systematic algebraic strategies, some students learned ad hoc strategies such as the cross-multiplication approach from their mathematics textbook *Getal en Ruimte* (Reichard et al., 2014a). Since this textbook method has the largest market share in the Netherlands, it influences large number of teachers and students (SLO, 2019; van Zanten and van den Heuvel-Panhuizen, 2014). We therefore recommend further research conducting a textbook analysis of *Getal en Ruimte* in which different types of ad hoc strategies are mapped and analyzed. The findings should be communicated to the publisher. In addition, the lack of insight of grade 10 students' algebraic skills in round one was may be due to insufficient emphasis on a rule-based problem-solving approach with insight in grade 9. We recommend future studies examining grade 9 textbooks to elaborate on this matter.

Furthermore, based on our findings from the studies (1) through (5), it is very likely that there are mismatches between how algebraic skills are learned in mathematics textbooks and how these are applied in physics textbooks. This can impede students experiencing coherence across both subjects and transfer. We therefore suggest a comparative textbook analysis in the mathematics textbook series *Getal en Ruimte* and the physics textbook series *SysNat* that also has the largest market share in the Netherlands. These mismatches should be mapped and analyzed. Again, the findings should be communicated to the publisher.

7.8.4 Implications for Educational Practice

The implications in this section are in line with the coherent mathematics and science education (CMSE) approach that aims at connection between mathematics and science education, especially physics education. We have organized this section around the topics 'Curricula', 'Textbook Publishers', 'Teachers', and 'Teacher Educators'. We also discuss the implications of this study for 'STEM-education'. Below we will elaborate on these topics in detail.

Curricula

Referring to CMSE, the actual mathematics curricula for upper secondary education only mention that they are designed and tested as part of an integrated examination program (SLO, 2019) that involves mathematics and science subjects. Moreover, there is no explicit reference to alignment through, for example, compatible notations, the same pedagogy of using algebraic skills, and the organization of the learning process in order to achieve a logical learning line across both subjects. This also holds for the physics curriculum. In short, there is a lack of connection between the mathematics and physics curricula in Dutch senior pre-university education.

With connection we do not refer to fully integrated mathematics-physics curricula serving only both subjects. Indeed, mathematics has also a role in other (science) subjects, and thus to some extent has to avoid context. However, the situation is different for physics, since mathematics is most frequently used in physics. For instance, algebraic skills, calculus, geometry and trigonometry are widely used in physics. We therefore recommend making explicit reference to connect both curricula through the core goals and content standards. A key principle is the pedagogy of using identical mathematical approaches, especially the pedagogy of applying algebraic skills. In this respect, both curricula should explicate the importance of using systematic algebraic strategies as in the solution set of the appendices of study (4) and (5). With systematic algebraic approaches, we refer to a rule-based problem-solving approach in which algebraic skills are used with insight, where rule refers to the standard rules for multiplication and division of powers, such as $y^a \cdot y^b = y^{a+b}$, that play the role of algebraic axioms in high school algebra (Drijvers et al., 2011). In addition, also the application of identical equations and formulas, concept descriptions, compatible notations, and the organization of the learning process should be explicated. Furthermore, through core goals we recommend both curricula explicating that using ad hoc strategies can be harmful for applying algebraic skills with insight, and impede students experiencing coherence across these subjects and transfer of algebraic skills from mathematics into physics class. Indeed, in addition to the latter two studies, especially in our study on teachers' beliefs these issues were emphasized by both mathematics and physics teachers and considered as of major importance.

On the individual level, the physics curriculum should emphasize the importance of prior mathematical knowledge through the same pedagogy of applying algebraic skills, in particular algebraic techniques that occur in mathematics curricula. The content standards of these mathematics curricula should emphasize the importance of algebraic physics problems analogous to mathematics problems.

Textbooks

Internationally, independent of whether mathematics textbooks are determined by national final examinations at the end of secondary school, which are described in curricula through the general educational core goals and the more specific standards, we recommend paying attention to systematic algebraic approaches during problem-solving in mathematics class. In addition, to provide context, mathematics problems should adopt corresponding physics problems. Concretely, we refer to worked out systematic procedures with insight similar to that in the solution set in the last two studies, thereby placing emphasis on the differences between algebraic techniques during these procedures. For instance, division and inverting of both sides of the equals sign. Our study shows that it is probably better to pay attention and distinguish between procedures involving basic algebraic skills and symbol sense behavior during algebraic problem-solving in physics class.

Concerning physics textbooks, we recommend using paragraphs in which physics problems are introduced through corresponding mathematics problems that students have learned in their mathematics textbooks. We emphasize that activation of pre-knowledge in the context of learning and instruction is important in relation to better students' achievements (e.g., Hailikari et al. 2008). Moreover, non-routine physics problems or problems requiring unusual thinking should be presented as shift-problems containing systematic hints with strategic and algebraic hints that guide students during problem-solving. Again, these hints should use identical problem-solving pedagogies to that in mathematics textbooks.

We strongly recommend both mathematics and physics textbooks to avoid ad hoc strategies, especially mathematics textbooks where the cross-multiplication approach is widely used (We have checked this in, for example, *Getal en Ruimte* (Reich et al, 2014a). Furthermore, the application of aforementioned identical equations and formulas, concept descriptions, compatible notations should be explicated. This also holds for the organization of the learning process, since certain mathematical concepts are taught in physics class before they are explained in mathematics class (SLO, 2019; Alink et al., 2012).

The design principles above have important implications for textbook publishers. Indeed, in many countries, they are bound to one discipline ('TIMMS & PIRLS', 2019), since each of them pursues different aims. We therefore recommend mathematics and physics publishers to work together and develop textbook series in which these principles are incorporated.

Teachers

Activation of prior-knowledge, using identical pedagogies in systematic problem-solving with insight and other implications above, also apply for physics teachers. Even mentioning that physics formulas are rooted in mathematics class, writing mathematics and physics expressions next to each other, or relating physical quantities to the variables x and y used in mathematics to some extent can impede compartmentalized thinking, and thus improve transfer and students' demonstration of coherence across mathematics and science subjects, in particular physics (Quinn, 2013; Turşucu et al., 2018c). Similar issues hold for mathematics teachers, for example, mentioning that algebraic skills are used in physics.

These issues may have consequences for effective teaching practice. Should teachers pay attention to basic algebraic skills, or focus on conceptual understanding? We have seen that students experienced difficulties because, for example, their basic procedures were not automated or the automated procedures were insufficient to tackle unusual problems. We therefore recommend teachers to teach both procedural skills and insightful learning in an integrated manner.

Such implications may require teachers from both mathematics and physics departments to communicate with each other. However, in the Netherlands and also internationally, curricula are overloaded (e.g., Lyons, 2006). As a result, most teachers lack time to collaborate with each other (SLO, 2019). Thus, such collaborative efforts should focus on alignment of both subjects that is feasible to adopt in classroom practice. For instance, mathematics and physics teachers may systematically reserve some fixed amount of time in their school timetables, compelling them to stick to their schedules. In addition, informal meetings may also provide a solution.

Teacher educators

In the first three studies we have seen that many mathematics and physics teachers have naïve beliefs about transfer that can be harmful for teaching practice. A widely shared belief is automatic transfer (e.g. Turşucu et al., 2017). We have seen that some teachers claimed that extensive practice with algebraic skills in mathematics class automatically leads to the transfer of these skills in physics class. Another naïve belief was that mathematics and physics teachers are not necessarily required to be sufficiently knowledgeable to explain basic mathematics. Such and similar beliefs (espoused models) can be harmful for transfer, i.e. they can impede transfer when these are transformed into classroom practice (enacted models). To change naïve beliefs into *desirable* transfer enhancing beliefs, well-informed mathematics and science teacher educators in professional development programs (Girvan et al., 2016; Guskey, 2002) can make teachers aware of the relationship[15] between these beliefs and classroom practice (Ernest, 1991), reflect about them and reconcile their espoused and enacted models. Furthermore, since probably the most important matter in relation with transfer is that teachers should be sufficiently knowledgeable to explain basic mathematics, we recommend this to be a pre-requisite for pre-service teachers following science teacher education programs leading to a teaching qualification. This does not hold for mathematics teachers, because basic mathematics is considered to be part of mathematics teacher education programmes.

For the other two studies some mathematics teachers used ad hoc strategies (espoused models) that can also be harmful for transfer in classroom practice (enacted models). Again, we recommend teacher educators to reconcile teachers' espoused and enacted models above, and to pay attention to systematic algebraic strategies.

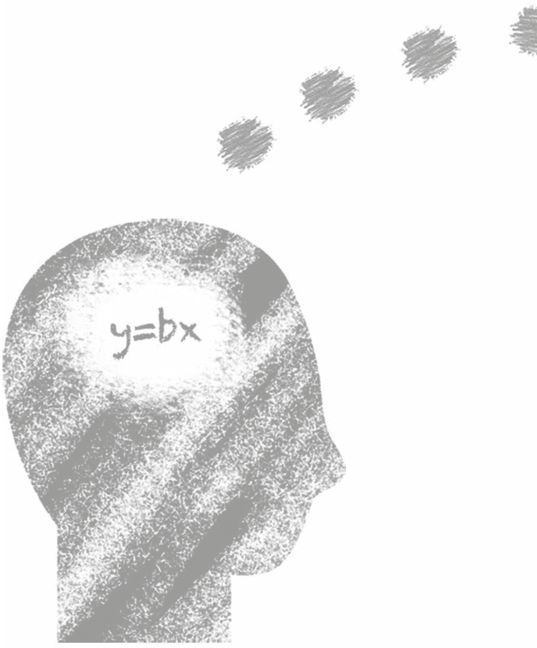
STEM education

In a large number of countries including the Netherlands, national governments aim for interdisciplinary Science, Technology, Engineering, and Mathematics Education (van Breukelen, 2017; National Science and Technology Council, 2013; SLO, 2019; 'TIMMS &

[15] A detailed explanation of teachers' beliefs about transfer in relation with classroom practice can be found in chapter 1.

PIRLS', 2019). At the heart of STEM education lies mathematics. This idea is based on the traditional transfer paradigm in which mathematics (initial learning situation) is applied in other subjects (new learning situation) (Larsen-Freeman, 2013; Leberman et al., 2016). Indeed, mathematics has a unifying role (Atiyah, Dijkgraaf, & Hitchin, 2010) for it can be applied in each of these subjects. Conversely, analogous mathematical forms in each of these subjects can be reduced to the same abstract mathematics. Because of this unifying role of mathematics, we recommend upper secondary education teachers involved in STEM education to connect their subject to mathematics. These teachers should use the same pedagogy of applying mathematics as in mathematics textbooks. Again, this includes a rule-based problem-solving approach in which mathematics is used with insight.

According to our extensive literature study, connecting mathematics with these STEM subjects through standard rules that play the role of algebraic axioms in high school algebra (Drijvers et al., 2011) has not studied before. There are no concrete examples of *what* such connections may look like in teaching practice. Therefore, instead of large collaborative efforts, it is probably better that STEM teachers aim at small collaborative efforts that are feasible for them. Especially, for mathematics teachers this demands a tremendous effort to answer STEM teachers needs.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Chapter 8

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Summary

Chapter 1: introduction

Science teachers have the experience that students in both secondary and higher education face difficulties with applying mathematics (initial learning situation) into science class (new learning situation), indicating a lack of transfer between these subjects (e.g., Redish & Kuo, 2014; Roorda, 2012; Wong, 2018). As a consequence, science teachers may be forced to re-teach basic mathematics. This may be frustrating and time-consuming, overshadowing the science content that needs to be taught. In addition, in a large number of countries, science curricula are overloaded, compelling science teachers to fit their program into a seriously reduced instruction time (e.g., Lyons, 2006), making inefficient transfer of mathematics in physics even more harmful.

Although such transfer problems are of major importance for both students and teachers, they are not studied extensively, especially the transfer from mathematics into physics. Except for a couple of studies (e.g., Hudson & McIntire, 1977; Rebello et al., 2007), the case of students having a solid grasp of mathematics but lack sufficient application in science is even *highly* under researched. Furthermore, in recent years, physics problems requiring mathematics such as the application of algebraic skills, have become more important in upper secondary physics education (grades 10, 11 and 12 of pre-university education). Therefore, examining this transfer phenomenon is relevant from both educational and scientific point of view.

In this both qualitative and quantitative explorative study, we aimed at improving the transfer of mathematics in physics. We especially focused on the application of algebraic skills to solve physics problems that occur in senior pre-university education. These problems contained physics formulas described in symbolic representations. For instance, the derivation of Kepler's third law using $G \cdot \frac{m \cdot M}{r^2} = \frac{m \cdot v^2}{r}$ and $T = \frac{2 \cdot \pi \cdot r}{v}$, or solving for m in the period of a spring-mass system $T = 2 \cdot \pi \cdot \sqrt{\frac{m}{C}}$. This goal was guided by the central research question "*How can the transfer of algebraic skills from mathematics into physics be improved for solving algebraic physics problems that occur in upper secondary education?*". To answer this question, we conducted five subsequent *explorative* studies using qualitative and quantitative methods. The first three were follow-up studies investigating (1) teachers' beliefs[16], (2) teachers' core beliefs and (3) teachers' belief systems. In study (4), we researched students' symbol sense behavior (SSB) during algebraic problem-solving in physics, and in study (5) the effectiveness of activation of prior mathematical knowledge during algebraic problem-solving in physics.

[16] In this study we carried out a problem analysis, since very little was known about transfer of algebraic skills from mathematics into physics in upper secondary education. A detailed explanation of this problem analysis can be found in chapter 1.

Chapter 2: study (1)

For this qualitative study we examined the two research questions (1a) "*How do* mathematics *and physics teachers characterise the transfer problem in the case?*", and (1b) "*What sort of beliefs do mathematics and physics teachers' beliefs have about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education (SPE)?*". To answer these research questions, we used convenience sampling to select 10 mathematics and 10 physics teachers from regular Dutch schools within a radius of approximately 50 km. These teachers were qualified to teach in senior pre-university education and had at least five years of teaching experience. They were interviewed by means of a semi-structured questionnaire including a concrete case about a students' transfer problem from mathematics to physics for which solution algebraic skills were needed. The questions dealt with aspects such as mathematics and physics teachers, collaboration between them, mathematics and physics curricula, and mathematics and physics textbooks. The interviews were transcribed verbatim for analysis, for which the teachers gave consent.

We used open coding (Bryman, 2015) to label each fragment of the transcripts, which provided a short description of teachers' individual beliefs regarding research questions (1a) and (1b). For each of the twenty transcripts this led to a set of labels identifying teachers' beliefs. Next, we used axial coding including two steps. First, labels with the same content were put together, resulting in a grouping of the labels. Each group of labels was summarized as a subtheme and included at least three different beliefs uttered by at least three different teachers. If not, it was considered as an outlier. In the subsequent step, we organized 28 subthemes into 9 core themes (coherence, curriculum, education, pedagogy of algebra, relation between scientific subjects, school subjects, teacher, the use of textbooks and transfer). Hence, we obtained one hierarchical structured common code tree for all 20 teachers, with the core themes as main branches. The latter branches out into subthemes, the smaller branches. The leaves of the tree are the last and finest level of the hierarchy and represent the underlying continuum of approximately 1.300 individual teachers' beliefs that we have found.

Regarding research question (1a) "*How do mathematics and physics teachers characterise the transfer problem in the case?*", we found that nearly all mathematics and physics teachers acknowledged the case presented to them and considered it important that students were competent at the transfer of algebraic skills from mathematics into physics. They believed that transfer problems occur especially in the first year of upper secondary education.

Concerning research question (1b), the continuum of beliefs underlying the 9 core themes above, contained aspects influencing students' transfer above, including beliefs on how to improve this transfer, and aspects about coherent mathematics and science education (CMSE; this approach aims at connecting mathematics to science subjects) including aspects that may enhance students experiencing coherence across these subjects. When implemented appropriately in classroom practice, these aspects may help reduce science teachers' frustrations, who spend extra time on repeating mathematics in science classes.

Contrary to physics teachers, most of the mathematics teachers mentioned that they did not feel the need to collaborate and cooperate with physics teachers. This can impede the development of common teaching strategies to tackle transfer problems. We noted that the lack of perceived urgency to cooperate with physics teachers is not typical for the whole mathematical community. Indeed, many teachers may appreciate the need to promote transfer.

With regard to their views about improving transfer, most interviewees fit into one of the following groups. The first and largest group believed that the transfer problem is solved by intensive practice in math class. The second and smallest group stated the opposite: the transfer problem should be tackled by solving algebraic problems in physics class. Finally, the intermediate group believed in comprehensive algebraic practice in both mathematics and physics class. Conceptual understanding was ignored by all teachers from the first two, extreme groups and by some teachers of the intermediate group. Some of the teachers' beliefs could be organized into a belief system (Ernest, 1991), i.e. into a set of mutually supporting beliefs about transfer. Further research should investigate to which extent this is the case and which beliefs they contain. This is explained in study (2) and study (3) below.

Chapter 3: study (2)

This qualitative follow-up study aimed[17] to extract teachers' belief systems. However, the common code tree including large amounts of data (about 1300 beliefs) hampered us to extract belief systems in one data reduction step. Therefore, we first aimed to further develop the second cycle coding technique pattern coding (Gibson en Brown, 2009; Saldaña, 2015) to reduce and grasp the essence, i.e. the core beliefs of the common code tree. Thus, we examined the following research questions (2a) "*How can pattern coding be further developed to obtain a systematic, refined method that can reduce code trees consisting of large amounts of data into a small dataset?*" and (2b) "*What are the core beliefs of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education?*". To answer question (2a), we worked out the methods of Saldaña (2015) in detail to reduce the common code tree. We used the subsequent steps 'D1: forming of summarizing beliefs', 'D2: forming of main beliefs' and 'D3: forming of core beliefs'. Contrary to the methods above that offer general knowledge on how to *further* reduce coded data, we think that our approach to pattern coding is elegant since we used refined and systematic data reduction steps providing a generally applicable second cycle coding tool to further reduce data of code trees containing large amounts of data.

Concerning research question (2b), we found 16 core beliefs about constraints and affordances influencing both students experiencing coherence across these subjects and transfer of algebraic skills into physics class. These core beliefs could be organized into what we named main categories: Collaboration, Curricula, Students, Teachers and Textbooks. This means that the nine core themes of study (1) were condensed into these five categories. In the teachers' eyes these are the main issues to focus on to improve transfer. We also found which main category corresponded to these nine core themes. Based on earlier literature on transfer, some of the core beliefs were identified as the harmful *naïve* beliefs, and others as the transfer enhancing *desirable* beliefs (espoused models) (Schoenfeld, 2014). When naïve beliefs are transformed into teaching practice, they may stand in the way of transfer (enacted

[17] A detailed explanation of why this study was necessary can be found in chapter 1.

models). Through professional development programmes (Guskey, 2002) aiming at transfer both espoused and enacted models should be reconciled.

Furthermore, the dataset of 16 core beliefs was sufficiently reduced to extract belief systems with desirable and naïve beliefs in a last data reduction step. This is explained in the study (3) below.

Chapter 4: study (3)

For this quantitative study we examined the research question (3) *"What are the belief systems of mathematics and physics teachers about improving students' transfer of algebraic skills from mathematics into physics for solving algebraic problems that occur in senior pre-university education (SPE)?"*. To answer this question, we needed to conduct an online survey with many mathematics and physics teachers in the Netherlands. For this purpose, we designed a webpage including an online multi-criteria assessment tool that we have developed, since no such tool was available. This tool contained the 16 core beliefs of study (2) that were transformed into 16 claims using the six functions of language of Jakobson to make sure that all claims were phrased clearly (Hébert, 2011; Waugh, 1980). Teachers were asked to select their top 5 claims, and distribute 50 points over them, thereby identifying their belief system. We used self-selection sampling (Bryman, 2015) to select 503 mathematics and physics teachers who had varying years of teaching experience. After they selected their top 5, we analyzed the correlations between those 16 claims and found small correlations between them. Their squares (explained variance) were smaller than 0.10. Such small correlations make principal component analysis (PCA) and factor analysis 'pointless' (Everitt & Hothorn, 2011). Therefore, we used the clustering technique agglomerative hierarchical clustering (AHC) (Everitt & Dunn, 2001) to analyze and gain insight into teachers' belief systems that might contain naïve and transfer enhancing beliefs (desirable beliefs).

Following Ernest (1991), we categorized teachers into groups based on their belief systems. We found 3 large clusters for the most experienced teachers (more than 10 years of teaching experience), and three large clusters for the very experienced teachers (more than 20 years of teaching experience). Five of them contained both naïve and desirable beliefs. These clusters turned out to be rather coherent sets of beliefs, and thus were regarded as belief systems. This *empirically* justifies Ernest's (1991) *theoretical* idea to cluster teachers based on their belief systems. Moreover, his social groups are 'package deals': the members of his groups are supposed to either embrace or reject complete sets of beliefs. He does not admit belief systems mixing aspects of different groups. Neither does he allow for different degrees of belief. In practice, certain clusters may have ideas that overlap with other groups. Indeed, this was the case for our belief systems. For instance, claim number 11 (activation of prior knowledge) appeared in all of our belief systems.

The distinction between the macroscopic and microscopic model of belief systems turned out to be useful. The first model containing espoused and enacted models (Ernest, 1991) was used to explain and understand how the social context of teaching influences a teachers' belief system, and the second one which is a detailed cognitive description of the espoused models to understand how the weak naïve beliefs and the strong desirable beliefs in a belief system are related to each other. We recommend future studies on this matter to use both models together.

We found relations between our groups and those of Ernest. Since naïve beliefs turned out to be weak in each cluster, science teacher educators can help science teachers to change their harmful naïve beliefs, into desirable transfer enhancing beliefs. Otherwise, because of the powerful socialization effect in school, teachers are often observed to stick to the same ineffective classroom practice (Brown & McNamara, 2011). Furthermore, we discussed some implications of our results for science teacher educators, curricula, teachers and textbooks.

The next two studies, i.e. study (4) and (5) are concerned with students' symbol sense behavior.

Chapter 5: study (4)

This qualitative study with a quantitative component studied the research question (4) *"To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems?"*. To gain insight into students' symbol sense behavior, we used convenience sampling to select 6 students who were available and willing to participate in this study. This yielded 3 mathematics A students from a regular school A, and 3 mathematics B students from a regular school B. Based on the Dutch ten-point grading system, they had a sufficient mathematics grade and an insufficient physics grade, i.e. less than 5.5. This grade criterion was to ensure that students' difficulties with algebraic physics problems were mainly because of insufficient application of algebraic skills in physics, and not related to a lack of basic mathematics. We designed tasks that should trigger students solving algebraic physics problems and provide insight into their algebraic skills with basic algebraic skills and symbol sense behavior. Next, we conducted task-based interviews among these students who were videotaped while problem-solving and thinking aloud. Both videotaped data and students' work were analyzed using the seven consecutive phases of Powell et al. (2003). An essential process during data analysis was the operationalization of research question (4). For this purpose, we used a coding scheme. During the coding process, we quantified both basic algebraic skills and symbol sense behaviour by comparing their solution set to the systematic solution set of our coding scheme. This implies that we adopted the traditional transfer paradigm (transfer determined from the researchers' perspective) to measure to which extent transfer occurred. To some extent, we also followed the actor-oriented transfer approach to gain insight into their previous learning. Together with the videotaped episodes (to some extent) and the transcripts of the audio part of videotaped data, this provided us deeper understanding of students' algebraic problem-solving abilities in physics, especially in the underlying mechanisms.

Our data confirmed that students did indeed struggle to apply algebra to physics, mainly because they lacked both sufficient basic algebraic skills and symbol sense behavior. We concluded that students were unsuccessful in the transfer of algebraic skills that students learned in mathematics class to solve algebraic physics problems in physics class.

They used ad hoc strategies instead of correct, systematic rule-based procedures involving insight. These ad hoc strategies included the cross-multiplication, the numbering, and the permutation strategy. They worked only for basic formulas containing few variables. In problems with more variables, students got stuck. The latter two strategies substitute numbers for variables. The permutation strategy randomly checks several permutations to guess which one is correct. The numbering strategy substitutes numbers to check algebraic manipulations.

Our results indicated insufficient focus on conceptual understanding of algebra in some mathematics textbooks, leading to reliance on poorly understood ad hoc strategies. Effective teaching of algebraic skills should not focus on either basic algebraic skills or on symbol sense behavior. Instead, both aspects should be taught in an integrated manner.

Our operationalization of symbol sense behavior turned out to be very useful for analysis. In contrast to other qualitative studies, we were able to measure symbol sense behavior quantitatively. In addition, this was also the case for measuring basic algebraic skills. This operationalization method should also be applicable to other science subjects.

Furthermore, we discussed some implications of our results for curricula, teachers, science teacher educators, and textbook publishers aiming at successful application of mathematics in physics, especially algebraic skills.

In the next study (5), we used insights from study (4) and the other three studies to carry out interventions in physics textbooks to improve transfer.

Chapter 6: study (5)

This qualitative study with a quantitative component is based on insights from the previous four studies and guided by research question (5) "*How can activation of prior mathematical knowledge be used effectively to improve students' symbol sense behavior in upper secondary education when solving algebraic physics problems?"*. This question was divided in the two sub questions (5a) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving algebraic physics problems that occur in their physics textbooks?*", and (5b) "*To what extent do students in upper secondary education demonstrate symbol sense behavior when solving the same algebraic physics problems that occur in their physics textbooks using activation of prior mathematical knowledge?*". To gain insight into their symbol sense behaviour during algebraic problem-solving in physics, we again used convenience sampling to select 3 students who were available and willing to participate in this study. For the same reason as in study (4), we selected students having a sufficient mathematics grade and an insufficient physics grade, i.e. less than 5.5. Based on the iterative 3D-principle (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013) that will be explained in later sections, we designed *new* tasks that contained different physics contexts than those in study (4). Again, these tasks should trigger students' algebraic problem-solving in physics and provide insight into their students' basic algebraic skills and symbol sense behavior. In light of sub question (5a), students solved these tasks while being videotaped and thinking aloud. This activity we called round 1. Two weeks later, we carried out small interventions by presenting the same problems as *shift problems*. The latter used prior mathematical knowledge by providing hints at the start of these tasks to improve students' systematic problem-solving abilities, especially symbol sense behavior. Here we used algebraic skills, especially algebraic techniques in a similar way to how these were learned in their mathematics textbooks (Reichard et al., 2014). This activity we called round 2. So, sub question (5a) is related to round 1, and sub question (5b) to round 2. Furthermore, except for the fact that we used different tasks and thus a different systematic solution set to check students' solutions, the analysis of data was carried out in the same way as in study (4). This also applied for the analysis of data in both rounds were we again quantified both basic algebraic skills and symbol sense behavior by comparing their solution set to the systematic solution set of our coding scheme and adopting both the traditional actor-oriented transfer approach. By comparing the extent to which students used basic algebraic skills correctly and demonstrated symbol sense behavior in both rounds, we determined the effectiveness of our intervention.

Contrary to the average OBAS (%) (overall basic algebraic skills percentage) among all students in round one, the average OSSB (%) (overall symbol sense behavior percentage) of 48.5 % among all students was insufficient. This again confirmed that students did indeed struggle to apply algebra to physics. After the intervention in round two, this number increased to 81.8 %, indicating sufficient symbol sense behavior among these students. In short, we have shown that *successful* transfer of algebraic skills from mathematics to physics is possible when both subjects use the same pedagogy in teaching algebraic skills. This also implied that the way we implemented activation of prior mathematical knowledge in shift-problems turned out to be very effective.

Chapter 7: general discussion and conclusion

In this section we first presented the main results and conclusions of all five studies. Next, these results and conclusions were put together to answer the central research question. Thereafter, we evaluated the models involved in this study and discussed the theoretical contributions to educational research. Then, we presented the limitations of this study followed by recommendations for future research. Finally, we discussed the implications of this study for educational practice, especially for the mathematics and science audience.

Samenvatting

Hoofdstuk 1: inleiding

Bèta-docenten hebben de ervaring dat leerlingen en studenten in respectievelijk het voortgezet- en hoger onderwijs moeite hebben met het toepassen van wiskunde in de bètavakken (Vakken zoals biologie, natuur- en scheikunde. Denk ook aan Elektrotechniek, Werktuigbouwkunde en dat soort vakken). Dit wijst op een gebrek aan transfer tussen deze vakken (e.g., Redish & Kuo 2014; Roorda, 2012; Wong, 2018). Het gevolg is dat docenten opnieuw basiswiskunde moeten onderwijzen. Dit kan frustrerend en tijdrovend zijn, en er blijft weinig tijd over voor de inhoud van de bètavakken. Bovendien geldt dat in veel landen dat de curricula van bètavakken overladen zijn, waardoor docenten gedwongen worden om hun programma te behandelen in een nog kortere instructietijd (e.g., Lyons, 2006). Hiermee wordt het gebrek aan transfer van wiskunde naar de bètavakken nog beperkter.

Ondanks het feit dat bovengenoemde transferproblemen van groot belang zijn voor het onderwijs, zijn deze onvoldoende bestudeerd. Dit geldt met name voor de transfer van wiskunde naar natuurkunde. Er is een zeer beperkt aantal studies (e.g., Hudson & McIntire, 1977; Rebello et al., 2007) over leerlingen die voldoende basiswiskunde beheersen, maar een gebrek aan transfer naar natuurkunde laten zien. Hiernaast zijn de laatste jaren natuurkundeopgaven die voor het oplossen ervan een sterk beroep doen op wiskunde, zoals de toepassing van algebraïsche vaardigheden, belangrijker geworden in ons bovenbouw vwo-natuurkundeonderwijs. Het onderzoek naar transfer is daarom zowel vanuit onderwijskundig- als wetenschappelijk oogpunt relevant.

Deze exploratieve studie die uit kwalitatief- en kwantitatief onderzoek bestaat, heeft ten doel om de transfer van wiskunde naar de natuurkunde te verbeteren. Onze focus lag op de toepassing van algebraïsche vaardigheden waarmee natuurkundeopgaven worden opgelost. Deze opgaven bevatten natuurkundeformules en zijn beschreven in symbolische representaties. Twee voorbeelden hiervan zijn de afleiding van de derde wet van Kepler met $G \cdot \frac{m \cdot M}{r^2} = \frac{m \cdot v^2}{r}$ en $T = \frac{2 \cdot \pi \cdot r}{v}$ en het vrijmaken van m in de formule voor trillingstijd $T = 2 \cdot \pi \cdot \sqrt{\frac{m}{C}}$.

De centrale onderzoeksvraag die hoort bij deze studie is "*Hoe kan de transfer van algebraïsche vaardigheden uit de wiskunde naar de natuurkunde worden verbeterd voor het oplossen van algebraïsche natuurkundeopgaven die zich voordoen in bovenbouw vwo?*". Om deze vraag te beantwoorden, hebben we vijf deelstudies uitgevoerd en gebruik gemaakt van kwalitatieve- en kwantitatieve onderzoeksmethoden. De eerste drie deelstudies betreffen opeenvolgende onderzoeken naar (1) de opvattingen van docenten[18], (2) kernopvattingen van docenten en (3) opvattingssytemen

[18] In deze studie hebben we een probleemanalyse uitgevoerd, omdat er zeer weinig bekend was over de transfer van algebraïsche vaardigheden uit de wiskunde naar natuurkunde in bovenbouw vwo-onderwijs. Een gedetailleerde uitleg hierover is te vinden in hoofdstuk 1.

van docenten over het bevorderen van transfer van algebraïsche vaardigheden uit de wiskunde naar natuurkunde. In deelstudie (4) hebben we het symbol sense gedrag van leerlingen onderzocht bij het oplossen van algebraïsche natuurkundeopgaven, en in deelstudie (5) de effectiviteit van het activeren van wiskundevoorkennis tijdens het oplossen van algebraïsche natuurkundeopgaven.

Hoofdstuk 2: deelstudie (1)

Voor deze kwalitatieve deelstudie hebben we de twee onderzoeksvragen (1a) "*Hoe karakteriseren natuur- en wiskundedocenten het transferprobleem in de casus?*", en (1b) "*Welke soort opvattingen hebben natuur- en wiskundedocenten over het verbeteren van de transfer van algebraïsche vaardigheden uit de wiskunde naar natuurkunde voor het oplossen van algebraïsche natuurkundeopgaven die zich voordoen in bovenbouw vwo?*" onderzocht. Om deze onderzoeksvragen te kunnen beantwoorden, hebben we een gelegenheidssteekproef gebruikt om 10 natuur- en 10 wiskundedocenten uit reguliere scholen geselecteerd. Deze scholen zijn representatief voor de Nederlandse context. De leraren waren bevoegd om les te geven in bovenbouw vwo en hadden minstens vijf jaar onderwijservaring. Zij zijn geïnterviewd door middel van een semi-gestructureerde vragenlijst met daarin een concrete casus over een transferprobleem uit de wiskunde naar natuurkunde. Voor de oplossing van dit transferprobleem moest de leerling algebraïsche vaardigheden gebruiken. De vragenlijst bestaat uit aspecten als natuur- en wiskundedocenten, samenwerking tussen beide docentengroepen, natuur- en wiskundecurricula, en wiskunde- en natuurkundeboeken. De interviews zijn vervolgens verbatim getranscribeerd voor analyse, waarvoor de leraren toestemming hebben gegeven.

Voor de analyse van de transcripties (uitgeschreven teksten van de interviews) hebben we eerst *open coderen* gebruikt (Bryman, 2015). Elk fragment van een transcript is gelabeled, wat een korte beschrijving gaf van de docentenopvattingen over de onderzoeksvragen (1a) en (1b). Voor elk van de twintig transcripties heeft dit geleid tot een reeks labels waarmee de opvattingen van docenten is vastgelegd. Hierna hebben we *axiaal coderen* gebruikt die uit twee stappen bestaat. Eerst zijn labels met dezelfde inhoud bij elkaar geplaatst, resulterend in een groepering van de labels. Elke groep labels werd samengevat als subthema en bestond uit ten minste drie verschillende opvattingen geuit door ten minste drie verschillende docenten. Zo niet, dan werd het beschouwd als een uitschieter. In de volgende stap organiseerden we de 28 subthema's in 9 kernthema's: samenhang, curriculum, onderwijs, pedagogiek van algebra, relatie tussen wetenschappelijke vakken, schoolvakken, docent, het gebruik van schoolboeken en transfer. Op deze manier hebben we voor alle 20 docenten een hiërarchische, gemeenschappelijke codeboom verkregen, met de kernthema's als hoofdtakken. Deze kernthema's vertakken zich in subthema's, de kleinere takken. De bladeren van de boom zijn het laagste en meest verfijnde niveau van de codeboom en representeren een continuüm van ongeveer 1.300 docentenopvattingen die wij hebben gevonden.

Met betrekking tot onderzoeksvraag (1a), zagen we dat vrijwel alle natuur- en wiskunde docenten de aan hen voorgelegde casus herkenden. Zij gaven aan het belangrijk te vinden dat leerlingen bekwaam zijn in de transfer van algebraïsche vaardigheden uit de wiskunde naar natuurkunde. Zij denken dat dit soort transferproblemen zich met name in vwo4 voordoen.

Wat betreft onderzoeksvraag (1b), bovengenoemd continuüm van docentenopvattingen bevat allerlei aspecten die van invloed zijn op transfer. Deze aspecten gaan over hoe transfer verbeterd kan worden, over samenhangend bètaonderwijs (dit concept heeft ten doel de samenhang tussen wiskunde en de sciencevakken te vergroten), waaronder aspecten die de mate van samenhang zoals die wordt ervaren door leerlingen kunnen vergroten. Wanneer deze aspecten op de juiste manier worden geïmplementeerd in het klaslokaal, kunnen zij de eerdergenoemde zorgen van docenten wegnemen.

In tegenstelling tot natuurkundedocenten voelen de meeste wiskundedocenten niet de behoefte om samen te werken met natuurkundedocenten. Dit kan de ontwikkeling van gemeenschappelijke onderwijsstrategieën om transferproblemen aan te pakken in de weg staan. Dergelijke opvattingen over wiskunde vertegenwoordigen niet de wiskundegemeenschap, want er kunnen veel wiskundigen zijn die voorstander zijn van het bevorderen van transfer en samenhangend bètaonderwijs.

Wat betreft docentenopvattingen over het verbeteren van transfer passen de meeste geinterviewde docenten in een van de volgende groepen. De eerste en grootste groep geloofde dat het transferprobleem uit de casus kan worden opgelost door veel te oefenen in wiskundelessen. De tweede en kleinste groep dacht het tegenovergestelde: het transferprobleem moet worden aangepakt door algebraïsche problemen te oefenen bij natuurkunde. De tussenliggende groep geloofde in een omvattender algebraïsche aanpak in zowel het natuur- als wiskundelokaal. Opvallend is dat inzicht werd genegeerd door leraren uit alle groepen.

Sommige opvattingen van leraren konden worden georganiseerd als opvattingssysteem (Ernest, 1991), i.e. een set coherente (samenhangend geheel) opvattingen over transfer. Verder onderzoek zou zich moeten richten op in hoeverre opvattingssystemen bestaan en uit welke opvattingen zij bestaan. Dit wordt uitgelegd in de deelstudies (2) en -(3) hieronder.

Hoofdstuk 3: deelstudie (2)

Deze kwalitatieve vervolgstudie op deelstudie (1) was bedoeld[19] om de opvattingssystemen van leraren uit de codeboom in kaart te brengen. Echter, vanwege de enorme data van ongeveer 1300 opvattingen was het niet mogelijk om dit in een enkele stap te doen. Hiertoe hebben we 'patroon coderen' (patterncoding), i.e. een tweede orde codeertechniek verder ontwikkeld (e.g., Gibson en Brown, 2009; Saldaña, 2015). Deze techniek reduceert de codeboom tot een set kernopvattingen die de essentie (kern) van de codeboom weergeeft. Onze onderzoeksvragen waren (2a) "*Hoe kan 'patroon coderen' verder worden ontwikkeld om te komen tot een systematische, verfijnde methode waarmee codebomen bestaande uit grote hoeveelheden data gereduceerd kunnen worden tot een beperkte dataset?*" en (2b) "*Wat zijn de kernopvattingen van natuur- en wiskundedocenten over het verbeteren van de transfer van algebraïsche vaardigheden uit de wiskunde naar natuurkunde voor het oplossen van algebraïsche natuurkundeopgaven die zich voordoen in bovenbouw vwo?*". Om

[19] Een gedetailleerde uitleg over waarom dit onderzoek nodig was, is te vinden in hoofdstuk 1.

vraag (2a) te beantwoorden, hebben we de methode van Gibson en Brown (2009) en Saldaña (2015) via een stappenplan in detail uitgewerkt. Hiermee waren wij in staat om de codeboom te reduceren. Het stappenplan bestond uit 'D1: vormen van samenvattende opvattingen', 'D2: vormen van hoofdopvattingen' en 'D3: vormen van kernopvattingen'. In tegenstelling tot bovenstaande methoden (e.g., Gibson & Brown, 2009; Saldaña, 2015) die vrij algemeen uitleggen hoe datareductie kan worden toegepast op reeds gecodeerde data, is onze benadering van 'pattern coding' elegant te noemen. Zij bestaat immers uit een geraffineerd en systematisch stappenplan. Onze manier van tweede orde datareductie is algemeen toepasbaar voor het reduceren van grote hoeveelheden gecodeerde data.

Wat onderzoeksvraag (2b) betreft, onze tweede orde datareductie resulteerde in een set van 16 kernopvattingen bestaande uit opvattingen over beperkende- en bevorderende factoren die van invloed kunnen zijn op zowel de transfer van algebraïsche vaardigheden uit de wiskunde naar natuurkunde, als op hoe leerlingen samenhang tussen wiskunde en de sciencevakken ervaren. Deze kernopvattingen zijn verder georganiseerd in wat we de 'kern categorieën' hebben genoemd. Dit zijn curricula, docenten, lesmateriaal, samenwerking en leerlingen. Dit betekent dat de negen kernthema's uit deelstudie (1) als het ware zijn gecondenseerd in deze vijf kern categorieën. Volgens de geïnterviewde docenten uit deelstudie (1) zijn deze vijf categorieën de belangrijkste zaken waarmee transfer kan worden verbeterd.
Tevens is uitgezocht welke kernthema's uit deelstudie (1) bij welke kern categorieën horen. Op basis van literatuur over transfer zijn sommige kernopvattingen geïdentificeerd als de transfer belemmerende *naïeve* opvattingen, en andere kernopvattingen als de transfer bevorderende *wenselijke* opvattingen (mentale modellen) (Ernest, 1991; Schoenfeld, 2014).
Naïeve docentenopvattingen kunnen transfer in de weg staan, omdat docenten in het klaslokaal hiernaar handelen. De opvattingen worden dus omgezet naar de onderwijspraktijk (praktische modellen). Door middel van professionele ontwikkelingsprogramma's (Guskey, 2002) die gericht zijn op transfer, kunnen de verschillen tussen de mentale- en de praktische modellen van docenten worden overbrugd en beide modellen op elkaar worden afgestemd.

Via een laatste datareductiestap kunnen nu uit de dataset met 16 kernopvattingen de opvattingssytemen van docenten worden verkregen. Dit wordt uitgelegd in deelstudie (3) hieronder.

Hoofdstuk 4: deelstudie (3)

Voor deze kwantitatieve deelstudie onderzochten we de onderzoeksvraag (3) *"Wat zijn de opvattingssystemen van natuur- en wiskundedocenten over het verbeteren van de transfer van algebraïsche vaardigheden uit de wiskunde naar natuurkunde voor het oplossen van algebraïsche natuurkundeopgaven die zich voordoen in bovenbouw vwo?"*. Om deze vraag te beantwoorden, hebben we een online enquête afgenomen onder 503 Nederlandse natuur- en wiskundedocenten. Zij zijn geselecteerd via zelfselectie. In dit kader is een webpagina ontworpen met daarin een online 'multi-criteria assesment tool' die we ook zelf hebben ontworpen, omdat deze tool simpelweg niet beschikbaar was. Deze 'multi-criteria assesment tool' bevatte de 16 kernopvattingen uit deelstudie (2) die waren omgezet in 16 claims. Hiervoor hebben we de zes functies van de taal van Jakobson gebruikt. Deze functies hadden ten doel dat alle claims zo duidelijk mogelijk werden geformuleerd (Hébert, 2011; Waugh, 1980). Aan docenten met verschillende jaren leservaring was gevraagd om uit de 16 claims een top 5 te selecteren die in hun ogen het meeste bijdragen aan transfer, en hierover 50 punten te verdelen. Hiermee werd hun opvattingssytemen vastgelegd.

Tijdens de analyse van data vonden we kleine correlaties tussen de 16 claims. Hun kwadraten (explained variance) waren kleiner dan 0,10. Dit soort kleine correlaties maakt PCA en factoranalyse 'onbeduidend' (Everitt & Hothorn, 2011). Daarom hebben we de clusteringstechniek agglomerative hierarchical clustering (AHC) (Everitt & Dunn, 2001) gebruikt. Deze techniek geeft ons inzicht in de opvattingssystemen van leraren met daarin naïeve- en wenselijke opvattingen en het gewicht dat zij toekennen aan de opvattingen in hun top 5.

Evenals Ernest (1991) hebben we leraren ingedeeld in groepen (clusteren) op basis van hun opvattingssystemen. We vonden 3 grote clusters voor de meest ervaren docenten (meer dan 10 jaar onderwijservaring) en drie grote clusters voor de zeer ervaren docenten (meer dan 20 jaar onderwijservaring). Behalve één opvattingssysteem, bevatten de overige vijf opvattingssystemen naïeve- en wenselijke opvattingen. Deze clusters bleken allemaal redelijk samenhangende sets van opvattingen te bevatten en werden daarom beschouwd als opvattingssystemen. Dit empirische resultaat rechtvaardigt het theoretische idee van Ernest (1991) om leraren te clusteren op basis van hun opvattingssystemen.

De sociale groepen van Ernest (1991) bestaan uitsluitend uit individuen die complete sets van opvattingen geheel omarmen of verwerpen. Met andere woorden, zijn opvattingssystemen laten niet toe dat dezelfde opvattingen kunnen voorkomen onder verschillende groepen. Hij kent ook geen verschillende gewichten toe aan opvattingen. In de praktijk geldt dat bepaalde clusters opvattingen hebben die overlappen met andere groepen. Inderdaad, dit is ook wat wij hebben gevonden. Zo kwam claim nummer 11 (activeren van voorkennis) voor bij alle groepen.

Ons besluit om onderscheid te maken tussen het macroscopische- en het microscopische model van opvattingssystemen was nuttig. Het eerste model bestaande uit mentale- en praktische modellen (Ernest, 1991) werd gebruikt om uit te leggen en te begrijpen hoe de sociale context van het onderwijs de opvattingssystemen van leraren kan beïnvloeden. Het andere model geeft een gedetailleerde cognitieve beschrijving van de mentale modellen. Hiermee kan worden begrepen hoe de in deze deelstudie zwak naïeve- en de sterk wenselijke opvattingen in een opvattingssysteem aan elkaar zijn gerelateerd. Omdat beide modellen elkaar aanvullen, bevelen we toekomstige studies aan om die samen te gebruiken. Er geldt dat de naïeve opvattingen die wij hadden gevonden zwak waren in elke cluster. Dit kan het mogelijk maken dat lerarenopleiders sciencedocenten helpen om hun transfer beperkende naïeve opvattingen te veranderen in wenselijke transfer bevorderende opvattingen. In het andere geval, kan het sterke effect van socialisatie op school ertoe leiden dat leraren vasthouden aan dezelfde minder effectieve manieren van lesgeven (Brown & McNamara, 2011).

Verder zijn op basis van bovengenoemde resultaten de onderwijskundige implicaties voor lerarenopleiders, curricula, leraren en studieboeken besproken. De volgende twee deelstudies hieronder, i.e. deelstudies (4) en -(5) betreffen onderzoek naar symbol sense gedrag bij leerlingen.

Hoofdstuk 5: deelstudie (4)

In dit kwalitatieve onderzoek met een kwantitatief onderdeel hebben we de volgende onderzoeksvraag (4) bestudeerd "*In hoeverre laten bovenbouw vwo-leerlingen symbol sense gedrag zien bij het oplossen van algebraïsche natuurkundeopgaven?*". Om deze vraag te beantwoorden, hebben we 6 leerlingen geselecteerd die beschikbaar en bereid waren om deel te nemen aan dit onderzoek. Dit leverde 3 wiskunde A-leerlingen op van een reguliere school (I), en 3 wiskunde B-leerlingen van een reguliere school (II).

Deze leerlingen stonden een voldoende voor wiskunde en een onvoldoende voor natuurkunde (< 5,5). Dit criterium was ervoor om te zorgen dat de problemen van leerlingen met algebraïsche problemen bij natuurkunde voornamelijk te wijten waren aan het toepassen van algebraïsche vaardigheden bij natuurkunde, en niet vanwege een gebrek aan voldoende basiswiskunde. De opgaven die we hebben ontworpen, moesten leerlingen triggeren om algebraïsche natuurkundeopgaven op te lossen en tevens inzicht verschaffen in hun algebraïsche expertise bestaande uit algebraïsche basisvaardigheden en symbol sense gedrag. De leerlingen hebben deze opgaven hardop denkend opgelost tijdens interviews die zijn opgenomen op video. De data zijn geanalyseerd met behulp van de zeven opeenvolgende fasen van Powell et al. (2003). Een essentieel onderdeel tijdens data-analyse was de operationalisering van de onderzoeksvraag waarvoor we een coderingsschema hebben gebruikt.

Tijdens het coderingsproces zijn de oplossingen van leerlingen vergeleken met onze systematische oplossingen. Hierna hebben we een score toegekend aan hun algebraïsche basisvaardigheden en symbol sense gedrag. Dit impliceert dat wij de mate van transfer hebben bepaald door gebruik te maken van het traditionele transfer paradigma (transfer wordt bepaald door de onderzoeker). In beperktere mate hebben we ook gebruik gemaakt van het actor-georiënteerde transfer paradigma. Dit gaf ons inzicht in hoe kennis was verworven.
De video-opnamen in combinatie met het werk van leerlingen en de transcripties van het audiodeel van videopnamen gaf ons dieper inzicht in de onderliggende mechanismen van hun algebraïsche vaardigheden tijdens het oplossen van algebraïsche problemen bij natuurkunde.

De resultaten bevestigden dat leerlingen inderdaad moeite hebben om wiskunde toe te passsen bij natuurkunde, met name vanwege beperkte algebraïsche basisvaardigheden en een gebrek aan symbol sense gedrag. Onze conclusie was dat leerlingen niet succesvol waren in het toepassen (transfer) van algebraïsche vaardigheden (geleerd in het wiskundelokaal) bij natuurkundeopgaven (in het natuurkundelokaal). In plaats van correcte, systematische algebraïsche procedures gebaseerd op standaard middelbare school wiskunderegels met inzicht gebruikten zij de ad-hoc strategieën cijferen, kruislinks vermenigvuldigen en permuteren. Bij cijferen en permuteren worden variabelen vervangen door getallen. De permutatiestrategie (*zonder* rekenkundig inzicht) controleert willekeurig meerdere permutaties om te raden welke juist is, en bij cijferen (*met* rekenkundig inzicht) worden getallen vervangen door variabelen om algebraïsche manipulaties te controleren. Deze strategieën werken alleen voor niet al te complexe formules met weinig variabelen. Bij opgaven met formules bestaande uit meerdere variabelen liepen leerlingen vast.

Onze resultaten wezen op onvoldoende focus op conceptueel begrijpen van algebra in sommige wiskundemethoden, wat leidt tot afhankelijkheid van onvoldoende begrepen ad hoc strategieën. Effectief lesgeven van algebraïsche vaardigheden moet niet gericht zijn op algebraïsche basisvaardigheden of symbol sense gedrag alleen. Beide concepten moeten op een geïntegreerde manier worden aangeleerd.

Onze operationalisering van symbol sense gedrag bleek erg nuttig voor analyse. In tegenstelling tot eerdere kwalitatieve studies, waren we in staat om symbol sense gedrag kwantitatief te maken. Dit gold ook voor algebraïsche basisvaardigheden. Deze manier van operationaliseren zou ook bij andere bètavakken kunnen worden gebruikt.

Verder hebben we enkele implicaties van onze resultaten besproken voor curricula, leraren, lerarenopleiders en tekstboekuitgevers die zich ten doel stellen om het toepassen van algebraïsche vaardigheden uit de wiskunde bij natuurkunde te bevorderen.

De inzichten die zijn verkregen uit deelstudie (4) en de andere drie deelstudies worden in de onderstaande deelstudie (5) gebruikt om interventies te plegen in natuurkundelesmateriaal om transfer te verbeteren.

Hoofdstuk 6: deelstudie (5)

Deze kwalitatieve deelstudie met een kwantitatief onderdeel is gebaseerd op inzichten uit de voorgaande vier deelstudies en bestudeert de onderzoeksvraag "*Hoe kan het activeren van wiskundevoorkennis effectief worden gebruikt om het symbol sense gedrag van bovenbouw vwo-leerlingen te verbeteren bij het oplossen van algebraïsche natuurkundeopgaven?* ". Deze vraag was verdeeld in twee deelvragen, namelijk (5a) "*In welke mate laten bovenbouw vwo-leerlingen symbol sense gedrag zien bij het oplossen van algebraïsche natuurkundeopgaven zoals die voorkomen in natuurkundeboeken?*", en (5b) "*In welke mate laten bovenbouw vwo-leerlingen symbol sense gedrag zien bij het oplossen van dezelfde eerder genoemde algebraïsche natuurkundeopgaven in natuurkundeboeken nadat wiskundevoorkennis is geactiveerd?*". Om meer inzicht te verkrijgen in symbol sense gedrag hebben we 3 leerlingen geselecteerd die beschikbaar en bereid waren om deel te nemen aan deze deelstudie. Deze leerlingen stonden vanwege dezelfde reden als bij de vorige deelstudie een voldoende voor wiskunde en een onvoldoende voor natuurkunde, i.e. kleiner dan een 5,5. Gebaseerd op het iteratieve 3D-principe (Palha, Dekker, Gravemeijer, & van Hout-Wolters, 2013), hebben we nieuwe opgaven ontworpen. Deze opgaven betroffen andere natuurkunde-contexten dan die in deelstudie (4). Evenals bij de vorige deelstudie moesten de opgaven de leerlingen triggeren om algebraïsche natuurkundeopgaven op te lossen en tevens inzicht te verschaffen in hun algebraïsche basisvaardigheden en symbol sense gedrag.

Om deelvraag (5a) te beantwoorden, hebben leerlingen in ronde 1 deze opgaven hardop denkend opgelost terwijl ze werden gefilmd. Twee weken later hebben we aan dezelfde opgaven kleine ingrepen gepleegd en deze opgaven in ronde 2 aangeboden aan dezelfde leerlingen als 'shift-problems'. Hierbij werd meteen bij het begin van deze opgaven via hints de wiskundevoorkennis van leerlingen geactiveerd. Het idee hierachter was dat leerlingen via deze interventie hun systematisch probleemoplossend vermogen, en in het bijzonder, hun symbol sense gedrag zouden verbeteren. De algebraïsche vaardigheden en hiermee samenhangende algebraïsche technieken zijn op een zeer vergelijkbare manier aangeboden als in hun wiskundeboeken (Reichard et al., 2014). Daarom heeft deelvraag (5a) betrekking op ronde 1, en deelvraag (5b) op ronde 2. Vanwege andere opgaven dan in deelstudie (4), waren de systematische oplossingen ook anders. De analyse van data werd op dezelfde manier uitgevoerd als in deelstudie (4). Voor de eerste- en de tweede ronde werd het werk van leerlingen geanalyseerd en gekoppeld aan een score die inzicht geeft in hun basis algebraïsche vaardigheden en hun symbol sense gedrag. Door deze scores in beide ronden te vergelijken, is de effectiviteit van onze interventie bepaald. Tijdens deze analyse hebben we wederom de mate van transfer hebben bepaald door gebruik te maken van het traditionele transfer paradigma. Ook was er weer gebruik gemaakt van het actor-georiënteerde transfer paradigma.

In tegenstelling tot het gemiddelde van algebraïsche basisvaardigheden onder alle leerlingen in de eerste ronde, was het gemiddelde symbol sense gedrag van 48,5% onder alle leerlingen onvoldoende. Dit bevestigde opnieuw dat leerlingen moeite hadden om algebra uit de wiskunde toe te passen bij natuurkunde.

Na de interventie in de tweede ronde steeg het percentage van 48,5% naar 81,8%. Dit wijst op voldoende symbol sense gedrag bij deze leerlingen. We hebben hiermee laten zien dat succesvolle transfer van algebraïsche vaardigheden uit de wiskunde naar natuurkunde mogelijk is wanneer er bij wiskunde en natuurkunde dezelfde didactische methoden voor het onderwijzen van algebraïsche vaardigheden worden gebruikt. Dit impliceert ook dat de manier waarop we hun wiskundevoorkennis hebben geactiveerd via 'shift-problems' zeer effectief is gebleken.

Hoofdstuk 7: algemene discussie en conclusie

In dit laatste hoofdstuk zijn eerst de belangrijkste resultaten en conclusies van alle vijf deelstudies gepresenteerd. Hierna zijn deze resultaten en conclusies samengebracht om de centrale onderzoeksvraag te beantwoorden. Daarna zijn de verschillende modellen uit ons onderzoek geëvalueerd en de theoretische bijdragen aan het onderzoeksveld besproken. Aansluitend zijn ook de beperkingen van de deelstudies besproken. Voor de actoren in het bèta- en wiskundeonderwijs bestaande uit curricula, lesmaterialen, leraren en lerarenopleiders hebben we een aantal aanbevelingen voor toekomstig onderzoek gedaan. Als laatste zijn de implicaties van deze deelstudies voor de onderwijspraktijk besproken.

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Curriculum vitae

Süleyman Turşucu was born on August the 31st of 1979 in his beloved Leerdam, the Netherlands. After finishing his secondary education in The Hague, he obtained his Master of Science degree in physics and Master of Education degree in physics education in respectively Leiden University and Utrecht University. During his teacher education programme he became interested in teaching mathematics and science subjects, especially teaching physics to secondary education students. Thereafter, he started his professional teaching journey in 2008 as a mathematics teacher. From 2009 until now he has taught physics in various secondary schools including preparatory vocational secondary education (VMBO), general secondary education (HAVO) and pre-university education (VWO) in different areas of the Netherlands. In 2014 he started his PhD research project entitled "*Successful transfer of algebraic skills from mathematics in physics*" under supervision of prof. dr. Marc de Vries and in collaboration with dr. Jeroen Spandaw in the group 'Science Education & Communication' (SEC) at Delft University of Technology in the Netherlands. This project was funded by the Netherlands Organization for Scientific Research (NWO). Currently, he works as a secondary education physics teacher at Wolfert Bilingual (Wolfert van Borselen TTO) in Rotterdam, and as a researcher and teacher at the Radboud University Nijmegen in the Netherlands. Furthermore, recently, in addition to his interest in transfer of learning and coherent mathematics and science education, he became interested in 'misinformation in science'.