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# A CFD approach for the flotation analysis of pipelines in liquefied sand

## Une approche CFD pour l'étude de la flottation des pipelines dans les sables liquéfiés

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**ABSTRACT:** Submarine buried pipelines may often interact with shallow layers of loose sand that are particularly prone to liquefaction. When liquefaction is triggered by environmental loading and/or mechanical vibrations, the soil tends to behave as a viscous solid-fluid mixture and the pipeline to undergo either flotation or sinking. While a few indications can be found in the literature on the triggering condition for flotation/sinking, no conclusive methods are available to estimate the displacement of the pipe when liquefaction cannot be avoided. This preliminary work shows that combining in numerical simulations fluid and soil mechanics may successfully serve such a goal. The proposed modelling approach is compared to the results of small-scale pipe flotation tests, with emphasis on existing knowledge gaps and indications for future research on the subject.

**RÉSUMÉ:** Les pipelines enfouis dans les fonds marins sont susceptibles d'interagir avec des couches peu profondes de sable lâche sujet à la liquéfaction. Lorsque la liquéfaction est causée par des charges environnementales et/ou des vibrations mécaniques, le sol tend à se comporter comme un mélange visqueux solide-fluide, et le pipeline peut alors soit flotter, soit s'engloutir. Si des données sont disponibles dans la littérature sur les conditions menant à la flottaison ou l'engloutissement du pipeline, aucune méthode conclusive n'existe pour estimer son déplacement lorsque la liquéfaction ne peut être évitée. Cette note de travail préliminaire montre que combiner simulations numériques de mécanique des fluides et de mécanique des sols peut permettre d'atteindre cet objectif. L'approche de modélisation proposée est comparée aux résultats d'essais de flottaison à petite échelle, mettant en évidence les verrous scientifiques et de futures lignes de recherche sur le sujet.

**Keywords:** Consolidation; finite element modelling; liquefaction; offshore engineering; pipes & pipelines

## 1 INTRODUCTION

Submarine pipelines are widely employed to transport hydrocarbons through the ocean from

wells to production and distribution plants. When directly laid on the seabed, pipelines interact with very shallow soil layers characterised by low effective stress levels, which orients major

construction costs towards stabilisation measures (Cheuk et al., 2008, Randolph et al., 2011). A valid option in this context is to lay pipelines in trenches, either left open or back-filled with rocks or sand. This kind of 'secondary stabilisation' is commonly required in regions with intense fishing and ship anchoring operations, or in cold environments for thermal insulation (Finch and Machin, 2001). Pipelines in trenches backfilled with coarse-grained materials are exposed to the detriments of soil liquefaction, as backfills are unavoidably very loose and remoulded. Liquefaction can be triggered by mechanical vibrations, wave action, earthquakes, tidal fluctuations, etc. (Sumer et al., 1999), and cause very large pipeline displacements after flotation or sinking. Since pipe routes may hardly avoid all areas prone to liquefaction, the understanding and prediction of pipeline flotation/sinking in liquefied sand is of utmost importance.

The present note is a preliminary work regarding a new CFD<sup>1</sup> approach to numerically simulate the post-liquefaction flotation of pipelines in presence of reconsolidation. The goal is to predict the extent and timing of pipe uplift when liquefaction cannot be prevented.

## 2 PREVIOUS EXPERIMENTAL STUDIES

### 2.1 Relevant literature

After the first pioneering studies in the United States (Pipeline Flotation Research Council, 1966), the hazard of pipeline flotation in liquefied sand received attention for North Sea offshore developments as well (Silvis, 1990, Sumer et al., 1999). To date, most research on the subject has focused on the application of Archimedes' principle to pipe-liquefied-sand systems (Damgaard et al., 2006). De Groot and Meijers (1992) first proposed that the unit weight of liquefied sands is typically around 18 kN/m<sup>3</sup>,

whereas Teh et al. (2006) suggested that relevant to pipeline flotation/sinking are critical state conditions. Different studies, almost contemporary, inferred from experiments that the liquefied sand density needed for pipe motion is generally not uniform within the soil domain, but rather (linearly) increasing along the depth (Sumer et al., 2006, Teh et al., 2006, Damgaard et al., 2006).

It should be noted, however, that none of the above studies could make final conclusions on estimating the distance travelled by floating or sinking pipelines. This seems especially relevant to decide on the need for mitigation measures when liquefaction cannot be avoided.

### 2.2 Reference small-scale experiments

The numerical developments presented herein are validated against the results of small-scale tests recently performed Deltares (Delft, Netherlands) and reported by Horsten (2016).

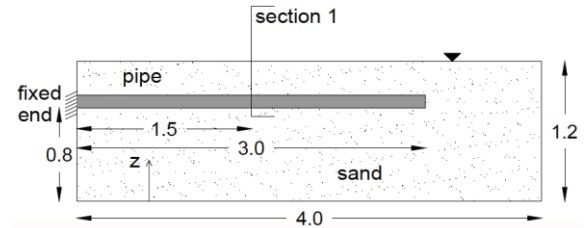


Figure 1. Experimental set-up as described in Horsten (2016) – dimensions in meters.

Table 1. Pipe geometrical/mechanical properties –  $L$  = length,  $t$  = thickness,  $D$  = diameter,  $I$  = section moment of inertia,  $\rho_{HDPE}$  = mass density,  $E_{HDPE}$  = Young's modulus

	<b>L</b>	<b>T</b>	<b>D</b>	<b>I</b>
	[m]	[mm]	[mm]	[m <sup>4</sup> ]
Pipe 1	3	17	110	$3.5 \cdot 10^{-6}$
Pipe 2	3	17	160	$1.6 \cdot 10^{-5}$
Pipe 3	3	17	200	$2.3 \cdot 10^{-5}$
		$\rho_{HDPE} = 950 \text{ kg/m}^3$		$E_{HDPE} = 1100 \text{ MPa}$

<sup>1</sup>Computational Fluid Dynamics

As illustrated in Figure 1, the experimental set-up featured a fixed-end pipe buried in a saturated sand layer. After sand liquefaction was triggered by hitting the soil container with a hammer, the motion of three different HDPE<sup>2</sup> pipes was recorded along with the evolution of the pore water pressure at different depths. Relevant geometrical/ mechanical properties of the three pipes are listed in Table 1.

### 3 CFD MODELLING OF PIPE FLOTATION

This section describes the numerical method used to simulate pipeline flotation in liquefied sand. The approach relies on the assumption of soil fully liquefied at the onset of pipeline flotation, which justifies resorting to CFD simulations and one-phase, non-Newtonian fluid modelling. A simplified, soil mechanics-based approach to include relevant reconsolidation effects is introduced and critically discussed.

#### 3.1 Governing equations and PFEM discretisation

The flow of liquefied soils and their interaction with structures has already been effectively reproduced via Computational Fluid Dynamics (CFD) simulations, for instance in relation to debris avalanches (Hwang et al., 2006, Boukpeti et al., 2012, Pastor et al., 2014). In this work, CFD simulations based on the so-called Particle Finite Element Method (PFEM) have been performed in the version developed by Cremonesi et al. (2010). The method employs a fully lagrangian description of free-surface fluid flow, intrinsically suitable for fluid-structure interaction problems involving large deformations. The liquefied soil mass is assumed to flow as a viscous incompressible fluid, with the conservation of linear momentum and mass (Navier-Stokes equations) fulfilled over the moving fluid volume  $\Omega_t$  and time interval  $(0, T)$ :

$$\begin{aligned} \rho \frac{D\mathbf{u}}{Dt} &= \nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{b} & \text{in } \Omega_t \times (0, T) \\ \nabla \cdot \mathbf{u} &= 0 & \text{in } \Omega_t \times (0, T) \end{aligned} \quad (1)$$

In Equation (1),  $D\mathbf{u}/Dt$  represents the material time derivative of the local velocity vector  $\mathbf{u}$ ,  $\boldsymbol{\sigma}$  the Cauchy stress tensor,  $\rho$  the mass density and  $\mathbf{b}$  the external body force vector. The above equations are discretised in space with linear interpolation functions for both independent field variables,  $\mathbf{u}$  and  $\boldsymbol{\sigma}$ ; time integration is performed through the implicit backward Euler algorithm and Newton-type iterations at each analysis step. The unavoidable mesh distortion associated with large deformations is handled through a remeshing procedure based on Delaunay tessellation (Cremonesi et al., 2010).

#### 3.1.1 Soil-pipe interaction

A plane-strain 2D version of the above method has been used to reduce the computational burden, with the pipe modelled as a circular rigid body. Soil-pipe interaction is reproduced via a staggered Dirichlet-Neumann scheme (Cremonesi et al., 2010): the velocity of the rigid body is applied to the fluid interface as a Dirichlet boundary condition, fluid stresses along the pipe boundary are integrated and fed as forces into the pipe rigid motion equations.

#### 3.2 Bingham modelling of liquefied sand

Assuming the liquefied soil to flow as a one-phase, incompressible fluid is coherent with the notion of 'total stress analysis': accordingly, solid and fluid phases are undistinguished and stresses are all meant as total.

The flow of liquefied sands is most often modelled under the assumption of Bingham rheological behaviour. The non-Newtonian Bingham idealisation applies to mixtures with high sediment concentration, and entails a linear rheological law beyond a material-specific 'yield

<sup>2</sup> HDPE: high-density polyethylene

stress' threshold (O'Brien and Julien, 1988). In the simplest case of pure shear flow, the Bingham relationship between shear stress and shear strain rate reads as:

$$\dot{\gamma} = \begin{cases} 0 & \text{for } \tau < \tau_y \\ (\tau - \tau_y)/\eta & \text{for } \tau \geq \tau_y \end{cases} \quad (2)$$

where  $\eta$  and  $\tau_y$  represent the viscosity and the yield stress of the liquefied soil. For 2D/3D problems, a multiaxial version of Equation (2) can be easily obtained (Cremonesi et al., 2017).

### 3.3 Re-consolidation effects

It is in general not possible to reproduce pipeline flotation by disregarding sand re-consolidation (Bonjean et al., 2008). That would imply resorting to two-phase, effective stress simulations, unfortunately at a level of complexity beyond the current state of the art. Further, soil constitutive models capturing the transition from solid-like to fluid-like hydro-mechanical behaviour would be needed, to be used in large deformation simulations through e.g. the PFEM (Monforte et al., 2017). Material models of the mention kind are currently under development (Redaelli et al., 2016), but not yet fully applicable to coupled hydro-mechanical problems.

The goal of this work is to propose a simpler analysis framework, based on one-phase CFD but enhanced with basic consolidation theory by the following simplifying assumptions:

- (1) the re-consolidating liquefied sand can be regarded as a Bingham fluid with rheological properties,  $\eta$  and  $\tau_y$ , evolving in time and space;
- (2) the physical driver of such a rheological evolution is the gradual dissipation of the excess pore pressure in the actual two-phase material;
- (3) pore pressure dissipation and fluid rheology can be linked by first solving at each time an uncoupled consolidation

problem, then feeding back its effect on  $\eta$  and  $\tau_y$  into transient CFD calculations.

Given the illustrative purpose of this note, a simplified version of the above approach has been implemented for a preliminary assessment of its predictive potential.

Despite the presence of the pipe, it is assumed that the main principles of 1D linear consolidation theory can be used to describe pore pressure dissipation in the soil. Under the assumption of constant total mean pressure  $p$ , variations in effective mean pressure  $p'$  would only occur in the real two-phase system at the expense of the pore pressure  $u$ , i.e.  $\Delta p = 0 \rightarrow \Delta u = -\Delta p'$ . The decrease in  $u$  is thus linked to a proportional increase of the yield threshold  $\tau_y$  at each soil element:

$$\Delta \tau_y = \frac{M}{\sqrt{3}} \Delta p' = -\frac{M}{\sqrt{3}} \Delta u \quad \text{in } \Omega_t \times (0, T) \quad (3)$$

In Equation (3)  $M = (6 \sin \phi') / (3 - \sin \phi')$  and  $\phi'$  soil critical state friction angle. The time evolution of  $\Delta u$  in  $\Omega_t$  should in general be derived from a separate pore pressure dissipation analysis, either analytical or numerical. An example relevant to this study is the following analytical solution of the 1D linear consolidation equation:

$$\begin{aligned} \Delta u(z, t) &= u(z, t) - u(z, t = 0) \\ &= \sum_{m=1}^{\infty} \frac{8\gamma'H}{(2m-1)^2\pi^2} e^{-\left(\frac{2m-1}{2}\right)^2 \frac{\pi^2}{H^2} c_v t} \cos\left(\frac{2m-1}{2} \frac{\pi}{H} z\right) \\ &\quad - \gamma'H \left(1 - \frac{z}{H}\right) \end{aligned} \quad (4)$$

associated with zero-pressure (top) and impermeable (bottom) boundary conditions, and linear initial distribution of the pore pressure  $u(z, t = 0)$  – consistent with a linear depth-distribution of the effective mean pressure right before the occurrence of liquefaction. In Equation (4),  $H$  is the thickness of the liquefied sand layer along the elevation coordinate  $z$  (Figure 1),  $\gamma'$  the effective

unit weight of the mixture, and  $c_v$  the 'equivalent' consolidation coefficient.

It seems also reasonable to believe that re-consolidating liquefied sands undergo variations in viscosity as well. In the lack of well-established theories, the empirical relation by Pierson and Costa (1987) is heuristically considered to link the viscosity of the liquefied mixture  $\eta$  [Pa·s] to its porosity ( $n$ , [%]):

$$\eta = 0.112 \exp[0.163(100 - n)] \quad (5)$$

The same consolidation theory underlying Equation (4) can also be used to estimate the evolving porosity (volumetric strain) in Equation (5) after setting a soil stiffness value suitable for low-confinement conditions.

## 4 NUMERICAL SIMULATIONS

### 4.1 Simulation set-up

2D plain-strain PFEM simulations have been performed by discretising the liquefied soil domain with linear triangular elements. Velocity no-slip boundary conditions have been set along all container walls, with constant pressure imposed at the top surface<sup>3</sup>. Figure 2 shows as an example the PFEM mesh adopted for pipe 1 in Table 1. In the lack of specific measurements, a uniform density of 1900 kg/m<sup>3</sup> has been set for the liquefied soil (Sumer et al., 2006, Teh et al., 2003).

The 3D effect of the fixed left-edge (Figure 1) has been considered in 2D simulations by introducing an elastic restoring force proportional to the pipe displacement in the vertical rigid body motion equation. Hereafter, relevant simulation parameters are first set with exclusive reference to pipe 1 ( $D = 100$  mm), then predictions of pipe 2-3 experiments ( $D = 160, 200$  mm) are presented (Table 1). Experimental and

numerical pipe displacements are compared for the mid-section 1 indicated in Figure 1.

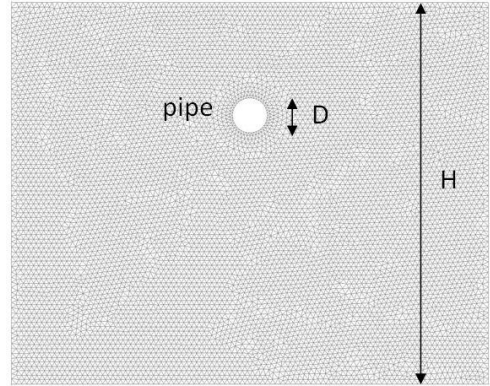


Figure 2. PFEM mesh for the simulation of pipe 1 flotation –  $D = 110$  mm, Horsten (2016)

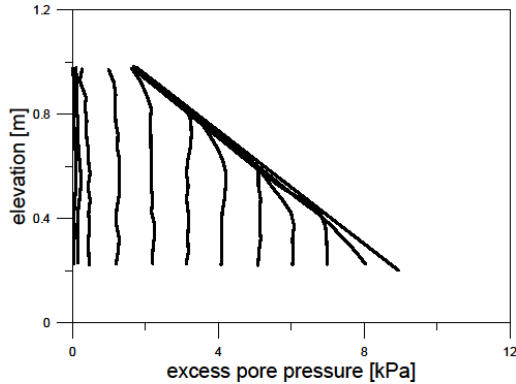
### 4.2 Model calibration

The proposed CFD model requires as main input data the mass density  $\rho$  and the Bingham parameters ( $\eta$  and  $\tau_y$ ) for the liquefied sand, along with their space/time evolution laws during re-consolidation. Figure 3a confirms for pipe 1 that significant pore pressure dissipation occurs while the pipe floats in the liquefied sand. Figure 3b displays the excess pore pressure decay recorded at  $z = 1$  m, right above the pipe and thus most relevant to its flotation. Unsurprisingly, the simple linear solution (5) ( $c_v = 0.03$  m<sup>2</sup>/s) cannot reproduce the experimental trend: this is due to the severe/sharp variations exhibited by the soil stiffness and permeability in the low-stress regime during the fluid-to-solid transition. An illustrative remedy to this shortcoming is for instance to manufacture an alternative analytical solution by combining (5) and the following assumption of piecewise-constant  $c_v$ :

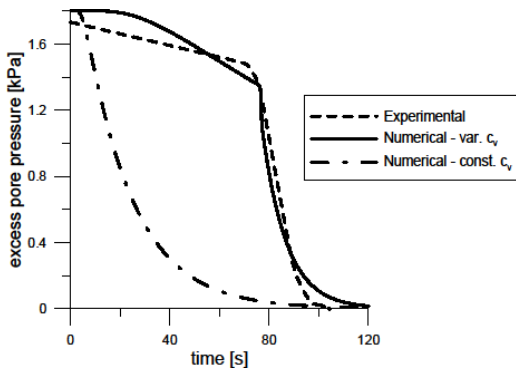
$$c_v = \begin{cases} c_{v1} = 0.005 \text{ m}^2/\text{s} & \text{for } t \leq 80\text{s} \\ c_{v2} = 0.06 \text{ m}^2/\text{s} & \text{for } t > 80\text{s} \end{cases} \quad (6)$$

<sup>3</sup> Since  $\tau_y$  is pressure-independent in Equation (2), adding free water above the liquefied soil has no effect on the simulation results.

Assumption (7) is a mere phenomenological attempt to simply capture the expected increase in  $c_v$  during re-consolidation (e.g. 'switch-time' of 80 s has indeed no theoretical background), and avoids the simulation of a non-linear consolidation problem in this preliminary work. Finally, Equation (3) provides the variation in space and time of  $\tau_y$ , after setting  $M(\phi = 30) = 1.2$  (indicative value) and  $\tau_y(z, t = 0) = 0$ .



(a) isochrones for  $0.2m \leq z \leq 1m$  and  $0s \leq t \leq 110s$  (10s time-spacing)



(b) measurement and analytical approximations at  $z=1m$

Figure 3. Excess pore pressures during pipe 1 test (Horsten, 2016)

As for viscosity, the effects on pipe flotation of two different settings are compared, namely constant  $\eta=500$  Pa·s, and  $\eta$  linearly increasing from 500 to 3000 Pa·s in 30 s. The latter setting is a crude approximation of the real physics, although consistent with Equations (4)-(5) and an

average Young's modulus for oedometer compression equal to 30 kPa – value consistent with the recommendations of Janbu (1963) and Muir Wood (2009).

Figure 4 overviews the impact of all above modelling assumptions:

- (1) the initial value of Bingham viscosity, is crucial to capture the initial/tangent flotation velocity. The value  $\eta=500$  Pa·s agrees, for instance, with what suggested by Huang et al. (2011);
- (2) the rate of pore pressure dissipation, and in turn of  $\tau_y$  regain, is most influential on the maximum flotation displacement. Keeping an average, constant  $c_v$  is inappropriate, the actual physics of soil re-consolidation and its time evolution need to be properly reproduced;
- (3) the viscosity of the liquefied soil is hardly constant, Figure 4 supports the need for increasing  $\eta$  during re-consolidation.

## 5 MODEL PREDICTIONS

The same simulation set-up with varying  $c_v$  and  $\mu$  calibrated for pipe 1 has been applied to predict the uplift measured for pipes 2 and 3 (Horsten, 2016). The experimental vs numerical comparisons in Figures 5-6 show that the same  $c_v$ - $\mu$  time-trends identified for  $D = 110$  mm are still suitable for  $D = 160, 200$  mm, notwithstanding the different pore pressure fields that are expected for pipes of different diameters.

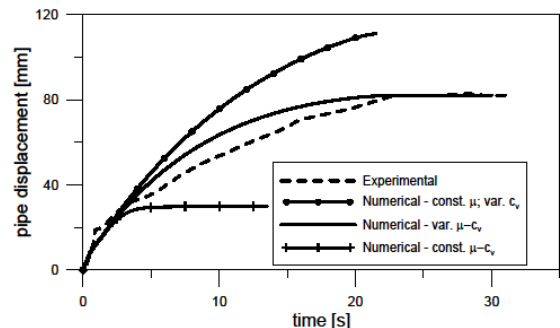


Figure 4. Experimental vs numerical flotation curves for pipe 1 test (Horsten, 2016)

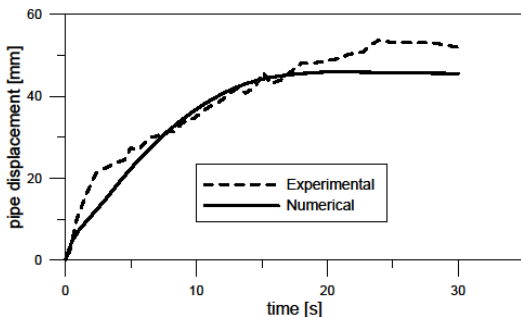


Figure 5. Model prediction of pipe 2 uplift (Horsten, 2016)

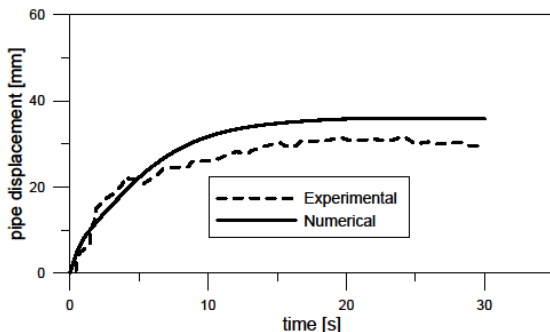


Figure 6. Model prediction of pipe 3 uplift (Horsten, 2016)

## 6 CONCLUDING REMARKS

This work explored the potential of CFD total stress analyses for describing the flotation of buried pipelines in liquefied sand. In particular, the chance of enhancing standard Bingham modelling with soil consolidation concepts was successfully investigated. As a first take on the subject, simplifications were introduced to capture globally the effects of sand re-consolidation (increase in strength and viscosity), and shown to be promising ingredients for the CFD simulation of pipe flotation. Future improvements to the proposed approach may come from: (i) a more rational modelling of re-consolidation effects, accounting for the actual, non-linear mechanics of the process; (ii) use of a 2D/3D numerical model for the uncoupled

prediction of pore pressure dissipation during flotation, accounting for the moving pipe boundary.

## 7 REFERENCES

- Bonjean, D., Erbrich, C. & Zhang, G. Pipeline Flotation Assessment in Liquefiable Soil. Offshore Technology Conference, 2008. Offshore Technology Conference.
- Boukpeti, N., White, D. & Randolph, M. 2012. Analytical modelling of the steady flow of a submarine slide and consequent loading on a pipeline. *Géotechnique*, 62, 137.
- Cheuk, C., White, D. & Bolton, M. D. 2008. Uplift mechanisms of pipes buried in sand. *Journal of geotechnical and geoenvironmental engineering*, 134, 154-163.
- Cremonesi, M., Ferri, F. & Perego, U. 2017. A basal slip model for Lagrangian finite element simulations of 3D landslides. *International Journal for Numerical and Analytical Methods in Geomechanics*, 41, 30-53.
- Cremonesi, M., Frangi, A. & Perego, U. 2010. A Lagrangian finite element approach for the analysis of fluid–structure interaction problems. *International Journal for Numerical Methods in Engineering*, 84, 610-630.
- Damgaard, J. S., Sumer, B. M., Teh, T., Palmer, A., Foray, P. & Osorio, D. 2006. Guidelines for pipeline on-bottom stability on liquefied noncohesive seabeds. *Journal of waterway, port, coastal, and ocean engineering*, 132, 300-309.
- De Groot, M. & Meijers, P. 1992. Liquefaction of trench fill around a pipeline in the seabed. *BOSS 92: Behaviour of offshore structures*, 1333-1344.
- Finch, M. & Machin, J. Meeting the challenges of deepwater cable and pipeline burial. Offshore Technology Conference, 2001. Offshore Technology Conference.
- Horsten, T. 2016. *Pipe uplift in liquefied sands*. Master, Delft University of Technology.
- Huang, Y., Zhang, W., Mao, W. & Jin, C. 2011. Flow analysis of liquefied soils based on smoothed particle hydrodynamics. *Natural hazards*, 59, 1547-1560.



- Hwang, J.-I., Kim, C.-Y., Chung, C.-K. & Kim, M.-M. 2006. Viscous fluid characteristics of liquefied soils and behavior of piles subjected to flow of liquefied soils. *Soil Dynamics and Earthquake Engineering*, 26, 313-323.
- Janbu, N. Soil compressibility as determined by odometer and triaxial tests. Proc. Europ. Conf. SMFE, 1963. 19-25.
- Monforte, L., Arroyo, M., Carbonell, J. M. & Gens, A. 2017. Numerical simulation of undrained insertion problems in geotechnical engineering with the Particle Finite Element Method (PFEM). *Computers and Geotechnics*, 82, 144-156.
- Muir Wood, D. 2009. *Soil mechanics: a one-dimensional introduction*, Cambridge University Press.
- O'Brien, J. S. & Julien, P. Y. 1988. Laboratory analysis of mudflow properties. *Journal of hydraulic engineering*, 114, 877-887.
- Pastor, M., Blanc, T., Haddad, B., Petrone, S., Morles, M. S., Dremptic, V., Issler, D., Crosta, G., Cascini, L. & Sorbino, G. 2014. Application of a SPH depth-integrated model to landslide run-out analysis. *Landslides*, 11, 793-812.
- Pierson, T. C. & Costa, J. E. 1987. A rheologic classification of subaerial sediment-water flows. *Debris flows/avalanches: process, recognition, and mitigation. Reviews in Engineering Geology*, 7, 1-12.
- Pipeline Flotation Research Council 1966. ASCE preliminary research on pipeline flotation. *J. Pipeline Div., Am. Soc. Civ. Eng.*, 92, 27-71.
- Randolph, M., Gourvenec, S., White, D. & Cassidy, M. 2011. *Offshore geotechnical engineering*, Spon Press New York.
- Redaelli, I., Di Prisco, C. & Vescovi, D. 2016. A visco-elasto-plastic model for granular materials under simple shear conditions. *International journal for numerical and analytical methods in geomechanics*, 40, 80-104.
- Silvis, F. Wave induced liquefaction of seabed below pipeline. Proc., Young Geotechnical Engineers' Conf, 1990.
- Sumer, B. M., Fredsøe, J., Christensen, S. & Lind, M. 1999. Sinking/flotation of pipelines and other objects in liquefied soil under waves. *Coastal Engineering*, 38, 53-90.
- Sumer, B. M., Hatipoglu, F., Fredsøe, J. & Hansen, N.-E. O. 2006. Critical flotation density of pipelines in soils liquefied by waves and density of liquefied soils. *Journal of waterway, port, coastal, and ocean engineering*, 132, 252-265.
- Teh, T., Palmer, A., Bolton, M. & Damgaard, J. 2006. Stability of submarine pipelines on liquefied seabeds. *Journal of waterway, port, coastal, and ocean engineering*, 132, 244-251.
- Teh, T., Palmer, A. & Damgaard, J. 2003. Experimental study of marine pipelines on unstable and liquefied seabed. *Coastal Engineering*, 50, 1-17.