A Markov-based vulnerability assessment for the design of on-board distributed systems in the concept phase

Habben Jansen, A. C.; Kana, A. A.; Hopman, J. J.

DOI
10.1016/j.oceaneng.2019.106448

Publication date
2019

Document Version
Accepted author manuscript

Published in
Ocean Engineering

Citation (APA)
Habben Jansen, A. C., Kana, A. A., & Hopman, J. J. (2019). A Markov-based vulnerability assessment for the design of on-board distributed systems in the concept phase. *Ocean Engineering*, *190*, Article 106448. https://doi.org/10.1016/j.oceaneng.2019.106448

A Markov-based vulnerability assessment for the design of on-board distributed systems in the concept phase

A.C. Habben Jansen*, A.A. Kana*, J.J. Hopman*

Department of Maritime & Transport Technology, Delft University of Technology,
Mekelweg 2, 2628 CD, Delft, The Netherlands

Abstract

Naval ships are designed to operate in a hostile environment. As such, vulnerability is an important aspect that needs to be assessed during the design. With the increased interest in electrification and automation on board naval ships, the vulnerability of distributed systems has become a major topic of interest. However, assessing this is not trivial, especially during the concept phase, where the level of detail is limited, but consequences of design decisions are large. Many existing vulnerability methods assess the vulnerability of pre-defined concepts, and focus on systems rather than capabilities. To address this, a new method for assessing the vulnerability of distributed systems in the concept phase has been developed. This method not only evaluates the vulnerability of a pre-defined concept, but also provides direction for finding other, potentially better solutions. This is done from a capabilities perspective. The method helps ship designers and naval staff in setting vulnerability requirements, developing new concepts, and identifying trade-offs in capabilities. The method uses a discrete Markov chain and the

---

*Corresponding author.
Email address: `A.C.HabbenJansen@tudelft.nl` (A.C. Habben Jansen)

Preprint submitted to Ocean Engineering *August 2, 2019*

eigenvalues of the associated transition matrix. A test case considering vulnerability of a notional Ocean-going Patrol Vessel (OPV) with two different powering concepts illustrates the method.

Keywords:
Naval ship vulnerability, Distributed systems, Concept ship design, Markov chain, Eigenvalues

1. Introduction

Naval ships are designed to operate in a hostile environment, which exposes them to an ever-present risk of getting hit by weapon deployment of an enemy. A hit may result in damage, such as failed structures, flooded compartments, impaired systems, or personal injuries. Consequently, the ship and its crew may no longer be able to perform the intended operations. In order to mitigate the risk of damage, survivability is a major design driver during the design of the ship, as explained by e.g. Ball and Calvano (1994). Various definitions of survivability exist. A commonly used definition provided by Said (1995), who defines survivability specifically for ships as "the capability of a ship and its shipboard systems to avoid and withstand a weapons effects environment without sustaining impairment of their ability to accomplish designated missions". Survivability consists of three major components: susceptibility, vulnerability, and recoverability. Susceptibility refers to the inability of a ship to avoid damage, while vulnerability refers to the inability to withstand damage. Recoverability is defined by Said (1995) as "the ability of a ship and its crew to prevent loss and restore mission essential functions given a hit by one or more threat weapons". These three major elements are usually considered with an external man-made hostile environment in mind. However, other circumstances can also impose a need to consider survivability, such as accidental fires, collisions, damage resulting from heavy seas, or cascading failures that result from increasingly complex system design. Examples of non-hostile environments that have resulted in damage include the collision of the KNM Helge Ingstad (BBC (2018)) and repeated power failures on board Type 45 Destroyers (Elgot (2016)).

Considering these three major elements, vulnerability is in particular governed by the design of the ship. As such, it is the primary focus of many research efforts. Susceptibility can also be addressed during the design, but it is observed that in some cases hits can not be avoided, even if susceptibility reduction measures have been taken (Schulte (1994)), (Reese et al. (1998)), (Duchateau et al. (2018)). Recoverability is mainly governed by active on board response, and is therefore addressed to a lesser extent in ship design research. However, some examples of dedicated recoverability research exist, such as the work of Piperakis and Andrews (2012) and Janssen et al. (2016). The present paper focusses on vulnerability.

Various ship design areas can contribute to reducing the vulnerability during the design of the ship. Reese et al. (1998) have identified structural integrity, seakeeping, floodable length, damage stability, and system separation as primary topics of interest. Most of the measures that can be taken with respect to these topics are aimed at obtaining an "intelligent layout", which is deemed the most effective protective measure by Brown (1991). Traditionally, vulnerability has mainly been addressed from a weapons effect perspective, with a focus on fire, blast, and fragmentation for above water hits, and damage stability for underwater hits. Such topics continue to be relevant for recent research (e.g. Boulougouris et al. (2017)). However, developments in the field of naval ship design impose a need for a stronger focus on the vulnerability from a systems perspective, as automation and electrification are design drivers of today's naval ships (Brefort et al. (2018)), (Dougal and Langland (2016)). This trend commenced in the 1980s and has since become more distinct as a result of growing electrical demands for existing and future sensors and weapon systems (Clayton et al. (2000); Doerry (2015)).

Doerry (2015) identifies several advantages of an Integrated Power System (IPS), where the ship's propulsion and the electrical system are combined in one power system. These advantages include an improved support of high-power mission systems, higher efficiencies of prime movers and propulsors, and more flexibility in the general arrangement. In order to enable an IPS, complex networks for distributing vital commodities such as electricity, fluids, air, and data are indispensable. The systems that provide those commodities are known as either distributed systems, a term used by e.g. Doerry (2006) or distribution systems, a term used by e.g. de Vos and Stapersma (2018). There is a slight and subtle difference between these terms. Distributed systems are systems that are distributed throughout the ship, where distribution systems are systems that distribute vital commodities. In practice, these systems usually cover both characteristics, and the terms can be regarded interchangeable. This also applies to the present paper, which uses the term distributed systems.

With the increasing interest in IPSs, the distributed systems networks become more complex and interdependent. This makes them more opaque and difficult to understand during the design. As a result, latent design errors may occur. These may result in cascading failures, which have a negative influence on the vulnerability (Brefort et al. (2018)). To identify and prevent such cascading failures, the vulnerability of distributed systems needs to be addressed in the early design stage (Goodrum et al. (2018)). Various terminologies exist for the early stage ship design. This paper uses the terminology of Andrews (2018), which refers to the early design stage as the concept phase. A further discussion on the concept phase is provided in Section 2.1. The concept phase is associated with several challenges and is often regarded as the most challenging in ship design, as discussed by Andrews (2018), van Oers (2011), and Gillespie (2012), among others. This is caused by several reasons, which include, but are not limited to the need for creativity in exploring and defining solutions, the large number of potential solutions, and the potential variability of the design requirements over time. In addition to that, the problem knowledge and level of detail are limited in the concept phase, while decisions made in this phase have a major influence on the committed costs (Duchateau (2016)). These challenges apply to all ship design areas, but are considered from a vulnerability perspective in the present paper. More specifically, three challenges for assessing vulnerability in the concept phase are identified:

- <u>Limited level of detail</u>: The level of detail of a vulnerability assessment in the concept phase needs to be limited enough to be used in a short time frame on a potential large number of concepts, but detailed enough to provide useful estimations of the vulnerability of these concepts.

- Generating vs. analysing concepts: In order to investigate whether a concept is likely to meet the requirements, a physically realisable model needs to be developed and tested. However, an assessment of a pre-defined concept usually provides results of which the applicability is limited to that specific concept. Assessing a pre-defined concept may therefore be of limited use for generating novel concepts, or design space exploration. Hence, a need for a more generalised method for vulnerability assessments arises.

- Systems vs. capabilities: Requirements for vulnerability usually are developed and formulated in terms of residual mission capability, in combination with a pre-defined damage or weapon impact. In other words, the vulnerability requirements are operationally oriented (Reese et al. (1998)). Yet, concept designs are usually defined in terms of compartments and systems. Though systems and capabilities are inextricably connected, the availability of systems is not necessarily a metric for the availability of residual capabilities. In addition to that, the required residual capabilities may be dependent on the impact level of a hit. This requires a vulnerability assessment from a capabilities perspective, in addition to a systems perspective.

Various tools and methods exist for assessing the vulnerability of naval ships. These are discussed in more detail in Section 2. It turns out that many existing vulnerability tools - including some that are aimed at the concept phase - require a significant level of detail, such as a general arrangement, a structural plan, or a systems design. Though some tools with a lower level of detail exist as well, none of them addresses both other challenges.

In order to address this gap, a new method for assessing vulnerability has been developed. The method uses a basic definition of a ship concept, which includes compartments, main systems, and their routings. The probability of availability for various levels of residual capabilities are calculated on the basis of a discrete Markov chain. Due to this mathematical set-up, it is not only possible to evaluate the vulnerability of a specific concept, but also to obtain guidance towards other, potentially better concepts. This is achieved by an evaluation of the eigenvalues of the transition matrix of the discrete Markov chain.

The remainder of this paper is organised as follows. First, a literature overview is provided in Section 2, which evaluates existing tools for assessing vulnerability of naval ships and other domains. Subsequently, the new method is explained in Section 3, including the mathematical set-up of the discrete Markov chain and the eigenvalues of the associated transition matrix. The application of the method is demonstrated with a test case in Section 4. Section 5 provides the results of the test case. Conclusions are drawn in Section 6. This section also provides recommendations for further research.

2. Literature overview

2.1. Design process in the concept phase

A commonly used approach for the design of a complex product or system is Systems Engineering (Kossiakoff et al. (2011)). This approach has also been adopted for naval ship design, and has previously been described as Total Ship Systems Engineering (TSSE). This covers all topics of ship design, and is not limited to vulnerability. The five stages in TSSE are requirement definition, requirement analysis, synthesis, verification, and validation (Brouwer (2008)). In systems engineering theory, defining the requirements is independent of the solution(s), i.e. the ship concept(s). However, the second concept phase challenge that has been identified in Section 1 reflects that the two are not strictly separated in the case of designing naval ships (and several other types structures). This has been discussed in more detail by Andrews (2011, 2018). As such, the concept phase of naval ship design benefits from an approach where design requirements and concepts are developed simultaneously, with the right level of detail at the right time.

To bring more structure into the concept phase of naval ship design, this phase can be subdivided into three design activities: concept exploration, concept studies, and concept design Andrews (2018). In concept exploration a wide, exploratory investigation of all possible options for layouts, capabilities, and technologies is executed. This is carried out at a limited level of detail. Based on the results of concept exploration, a limited number of alternatives (about 1-5) are investigated in more detail. During this stage, design drivers and the impact of design decisions on performance and cost are investigated in more detail. Subsequently, the concept design stage aims at providing sufficient information on capability and cost for ensuring that the further design process can be executed coherently. The end of this stage usually leads to commitment to a more substantial design and acquisition effort. Though these three activities are described as subsequent to each other, they may overlap in practice. The overall objective of the concept phase is to elucidate what is wanted and what is affordable.

A key feature of the concept phase is that the focus lies on decision making. Though this is inextricably connected with generating concepts, these concepts are not generated for detailed design and production, but for elucidating requirements. As such, the concept phase mostly benefits from generalisations rather than specific information on performance characteristics of individual concepts. This is not limited to vulnerability, but holds for ship design in general. From a vulnerability perspective, however, most methods carry out an analysis of a pre-defined concept, which relates to the specific perspective. Further research into the generalised perspective may contribute to developing methods that are more suitable assessing vulnerability in the concept phase, to feed into the concept phase of the ship as a whole.

2.2. Methods for assessing naval ship vulnerability

Various vulnerability assessment methods and tools exist. Some of these methods are aimed to be used in practical ship design by navies or shipyards, while others are developed from a more fundamental research perspective. Examples of the former type include the commercially developed tools RESIST (TNO (2018)), SURVIVE (Schofield (2009)) and SURMA (Surma Ltd. (2018)). These tools provide high fidelity assessments of a ship exposed to one or more hits. They include damage effects such as pressure, flooding, and fragmentation. The results of these tools comprise overviews of the damage stability, availability of critical systems, and structural integrity after one or more hits. The computations in these tools are based on detailed techniques. RESIST, for example, uses algorithms that hold an intermediate position between Finite Element Methods (FEM) and Computational Fluid Dynamics (CFD), in addition to analytical and empirical formulas. Because of this level of fidelity, detailed plans such as a general arrangement, a structural arrangement, and a systems design are needed as input for these tools. This makes them highly useful for detailed design stages. For the concept phase they are of limited use, due to the required level of detail. For the concept phase a simplified version of SURVIVE exists, known as SURVIVE Lite (Schofield (2009)). This version can be used for more generic layouts and a reduced level of subdivision. Another tool, called PREVENT Heywood and Lear (2006) applies a similar level of detail.

Methods for assessing vulnerability in the concept phase exist as well. Many of these have a more fundamental or scientific background. Piperakis (2013) has developed a method that is specifically aimed at the concept phase. It integrates susceptibility, vulnerability, and recoverability in an method for assessing overall survivability. The method is layout-based. It combines existing tools with a newly developed recoverability method. The method is suitable for assessing a relatively low number of alternatives, but at a relatively high level of detail. This fits well in the concept definition phase. A comparable level of detail is considered by Goodfriend and Brown (2017). They only consider vulnerability, with a specific focus on distributed systems. Their method uses a multi-objective genetic algorithm to explore the design space, with high effectiveness, low cost, and low risk as objectives. The method has an exploratory nature, though it still requires a level of detail that may be more suitable for later design stages.

An method with a lower level of detail has been developed by van Oers et al. (2012). Their method uses an genetic optimisation algorithm that generates routings of distributed systems, where low vulnerability is one of the objective functions, quantified by minimising the loss of capability. The method only considers variations on the shortest path. In a follow-up study, Duchateau et al. (2018) also consider routings that may be longer, but potentially less vulnerable. They also use a genetic optimisation algorithm. On a similar level of detail, the vulnerability of distributed systems is considered by Kim and Lee (2012). However, their aim is not to generate routings, but to evaluate the availability of critical systems after one or more hits in a probabilistic fashion. They investigate a binomial method, a Poisson method, a tree diagram, and a Markov chain. Their method can be used with a limited level of detail, but the mathematical set-up becomes complex when the number of redundant components is increased. Furthermore, their method is well suited for evaluating pre-defined concepts, but does not provide guidance towards other - potentially better - concepts.

In addition to genetic algorithms and probabilistic models, networks are used as well for vulnerability assessments of distributed systems in the concept phase. Goodrum et al. (2018) combine two networks, one describing the compartments of the ship, and one describing systems design, to compute an operability score for damaged compartments. All compartments are considered individually. The translation from their operability score to residual capabilities is not addressed. However, since their method is network-based, it is very robust and quick, allowing large numbers of layouts and damage scenarios to be considered. Networks are also applied by de Vos and Stapersma (2018). They specifically focus on the logical connections between the components of distributed systems, and do not consider physical compartments or routings. Similar to van Oers et al. (2012) and Duchateau et al.

(2018), they use a genetic optimisation algorithm. Trapp (2015) also uses networks to model the logical connections between components of distributed systems, optimising for network flow.

Brefort et al. (2018) have developed an architectural framework in order to structure all these topics. They define the design of distributed systems in terms of the physical, logical, and operational architecture, and their overlaps. The framework is not a tool in itself, but aids in describing and understanding the various aspects and relationships of the design of distributed systems. The method of Shields et al. (2016) has a similar background. While not a design tool in itself, it provides an estimation of the complexity of a design with respect to survivability. Doerry (2007) proposes survivability metrics that enable better definition of power system requirements, from the perspective of the operational needs of the ship. This is also not a tool in itself, but aims to enable a better understanding of the link between design requirements and operational needs.

2.3. Methods for assessing vulnerability in other fields

Vulnerability assessments are also carried out in other fields of study. They are especially relevant for applications with flows through infrastructures, analogous with the flow through distributed naval ship systems. The number of applications is extensive, but three examples are discussed here in more detail.

A typical example of a non-naval vulnerability assessment is is the design of land power grids. Liu et al. (2012) have defined an operational vulnerability index to investigate the possible benefits of decentralised power generation. In terms of this index, a good network with respect to vulnerability is one in which the long-distance large-capacity power transmission is minimal. A major difference with naval ship applications is that land power grids consider only one type of flow (electricity), while for naval ships interdependencies with other types of flow, e.g. chilled water, data, and fuels, need to be considered. Furthermore, the operational vulnerability index is not based on damage or loss of systems or compartments, but on the efficiency of the transmission.

Another non-naval ship design example of a vulnerability assessment is the work of El-Rashidy and Grant-Muller (2014), who have performed a vulnerability assessment on a highway network. This example also considers one type of flow (cars), but they have taken into account that the vulnerability depends on various operational and external factors, such as different threats or traffic speeds. This is done by defining vulnerability attributes that are calculated based on basic road traffic parameters, such as the number of lanes, the speed of the cars, and the congestion density.

A third example is the assessment of a health care facility. Arboleda et al. (2009) have developed a methodology for assessing the operational vulnerability of a health care facility during disaster events. In contrast to the two methods mentioned earlier, this method takes into account that the system, in this case a health care facility, is dependent on different types of flow, such as water, power, and the transportation of medical supplies. However, it takes into account only one default operational scenario.

2.4. Gap analysis

In Section 1 three challenges for assessing the vulnerability of distributed systems in the concept phase have been identified. The first one concerns the level of detail. Based of the review in Paragraph 2.2 it can be stated that many of the methods require a considerable level of detail, which makes them less suited for the concept phase. This also holds for some of the methods and tools that are specifically aimed at the concept phase. Though less detailed methods exist as well, none of them address both other challenges, concerning generating vs. analysing concepts, and systems vs. capabilities. It is not uncommon to assess vulnerability at the capability level by describing a capability as a hierarchy of systems. Yet, the higher the level of such hierarchies becomes, the more challenging it is to attribute their vulnerability to specific parts of the ship concept, especially in the generalised perspective that is needed during the concept phase. The examples of vulnerability assessments in different fields of study are not directly applicable, since they consider only one type of flow or one default operational scenario. Hence, a vulnerability method that specifically links capabilities to the layout of a concept, with the generalised perspective needed for the concept phase, is still lacking in literature. In order to address this, a new method is introduced in this paper.

During the concept phase, relevant, feasible, and affordable design requirements are set, and design drivers and trade-offs are identified. In order to match with these activities, the vulnerability method gives insight on how the design of distributed systems influences the vulnerability. This is realised by assessing concepts, deliberately at a low level of detail. Due to the mathematical set-up of the method, the results have a generalised nature, and are not limited to the concepts that are defined upfront. In other words, specific concepts are used to develop generalised knowledge. Therefore, these concepts are meant to be used for creating insights and requirements, and are not necessarily meant to be worked out in more detail during later design stages.

3. Method

In order to assess the vulnerability of distributed systems on board naval ships, there is a need to describe the availability of (parts of) these systems, and the probability that the availability changes after one or more hits. To enable this, a discrete time Markov chain has been selected as an appropriate mathematical technique that forms the basis of the method. A significant benefit of a discrete time Markov chain is that it is based on probability. In terms of vulnerability this means that all damage scenarios can inherently be addressed at once, which reduces the need for modelling individual damage scenarios. The probabilistic nature of this technique is deemed appropriate, as it gives an overall indication of the ability of a concept to withstand damage. This fits well into the concept phase, where the focus is on comparing alternatives rather than working out individual concepts. Furthermore, the probabilistic nature can represent the real-life uncertainty as to whether and where hits will occur. Nevertheless, modelling individual damage cases, such as worst-case scenarios, remains indispensable during later design stages.

The base elements of a discrete Markov chain are a state vector $\mathbf{s}$ and a transition matrix T. These are now discussed in more detail by means of a simple illustrative layout, which has been introduced previously in Habben Jansen et al. (2018b). This layout comprises 9 compartments, positioned as a 3×3 grid. The layout contains two systems. System A is located in

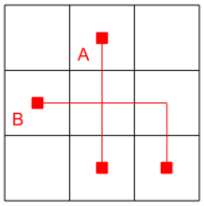


Figure 1: Graphical representation of the illustrative layout

3 compartments and system B is located in 4 compartments. The systems overlap in the central compartment. Figure 1 gives a graphical representation of this layout. It is assumed that both systems can individually be on or off. ‘On’ is defined as functioning, and ‘off’ is defined as not functioning due to a hit. Hence, the state vector becomes

$$\mathbf{s} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \end{bmatrix} \tag{1}$$

where

- s_1 is the probability for system A and B both being on
- s_2 is the probability for only system A being on
- s_3 is the probability for only system B being on
- s_4 is the probability for system A and B both being off

As a result of this definition, $\mathbf{s}$ is a stochastic vector, meaning its elements sum to 1. This is a requisite for a state vector of a discrete Markov chain. The transition matrix T describes the probability that $\mathbf{s}$ changes over time. For the set-up of this method, time is defined in number of hits, and is not related to a physical time scale. It is assumed that one hit occurs at each time step, disabling one of the compartments. The hit probability is uniform, regardless of the number or location of previous hits. Hence, compartments can be hit multiple times. If one or more systems are located in a compartment, they become unavailable. Repair of systems is not considered. Hence, T is dependent on the layout, and is row stochastic. With this information, the elements in T can be calculated. For the illustrative layout, the probability for s_1 to s_1, which is element $T_{1,1}$ is 3/9, as three of the nine compartments (the empty ones) can be hit without loosing system A or system B. The probability for s_1 to s_2, which is element $T_{1,2}$ is 3/9, as three compartments can be hit that result in loss of system B, while system A remains on. Similarly, elements $T_{1,3}$ and $T_{1,4}$ become 2/9 and 1/9, respectively. The same procedure can be followed for other elements in the transition matrix. For example, $T_{2,2}$ is the probability that, given system A is on and system B is off, the situation remains like that after a subsequent hit. This probability is 6/9, as six compartments can be hit that do not disable system A (the three empty compartments and all compartments of system B, except for the central compartment that is shared with system A). Following this procedure, T becomes as follows for the illustrative layout:

$$T = \begin{bmatrix} 3/9 & 3/9 & 2/9 & 1/9 \\ 0 & 6/9 & 0 & 3/9 \\ 0 & 0 & 5/9 & 4/9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

For the illustrative layout, the size of T is limited, and its elements were calculated by hand. However, the size of T quickly increases if more systems are considered. Hence, a script has been made for automatic computation of T for any layout. The input for the script is a $n_s \times n_c$ matrix, where n_s is the number of systems and n_c is the number of compartments. If system x is located in compartment y, element $\{x, y\}$ of the input matrix equals 1. Otherwise, it equals zero. Following the same procedure as for the manually derived T, the transition matrix is computed. By definition, its size is $2^{n_s} \times 2^{n_s}$. Subsequently, $\mathbf{s}$ and T can be used to calculate the probability for any state after any number of hits, using Equation 3:

$$\mathbf{s}(h) = \mathbf{s}(0) \cdot T^h \tag{3}$$

where h denotes the number of hits, and $\mathbf{s}(0)$ is the initial state vector. It is assumed that both systems are initially on, so the initial state vector becomes:

$$\mathbf{s}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

The probabilities of the four states can be plotted for an increasing number of hits, which is presented in Figure 2. As explained in more detail in Habben Jansen et al. (2018b), the fact that the two systems are located in one layout already makes them interdependent from a vulnerability perspective. This also holds in situations where there is no physical or logical overlap between the two systems. The results of Figure 2 are obtained by the matrix-vector multiplication of Equation 3. These results give information on what the shapes of the curves are, but not on why the curves are shaped that way. An explicit formulation of the curves can contribute to understanding the latter, and is one of the key contributions of this paper.

As the transition matrix is raised to higher powers of h, the explicit for-

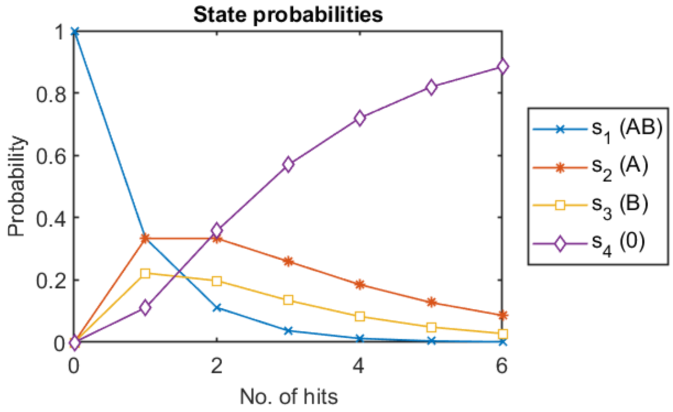


Figure 2: State probabilities for an increasing number of hits, associated with the illustrative layout

mulation can be obtained by applying matrix diagonalisation. By definition, the associated equation is:

$$T^h = PD^hP^{-1} \tag{5}$$

where D is a diagonal matrix with the eigenvalues of T on the diagonal, and P contains the respective eigenvectors. This holds if and only if all eigenvectors of T, i.e. all columns of P, are linearly independent (Lay (2006)). This paper does not contain a proof that this universally holds for the transition matrix of any layout in general. However, the authors are not aware of any layout, either conceptual or more advanced, where the columns of P are not linearly independent. This indicates - but does not proof - that linear independence occurs for any layout. The linear independence of the columns of P can be confirmed by computing the rank of P. In this study MATLAB is used for this, and for all other computations described in this paper. If the rank of P equals the number of columns in P, its columns are linearly independent. In addition to P, the diagonal matrix D needs to be constructed as well. This requires the eigenvalues of T. Since the vulnerability assessment only considers damage of systems, and not repairs, T is always an upper triangular matrix. Hence, the eigenvalues of T are the entries on its diagonal. As a result, the diagonal of D contains the same values as the diagonal of T. For the illustrative layout, this leads to:

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix} = \begin{bmatrix} 3/9 & 0 & 0 & 0 \\ 0 & 6/9 & 0 & 0 \\ 0 & 0 & 5/9 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The associated eigenvectors are the columns of P:

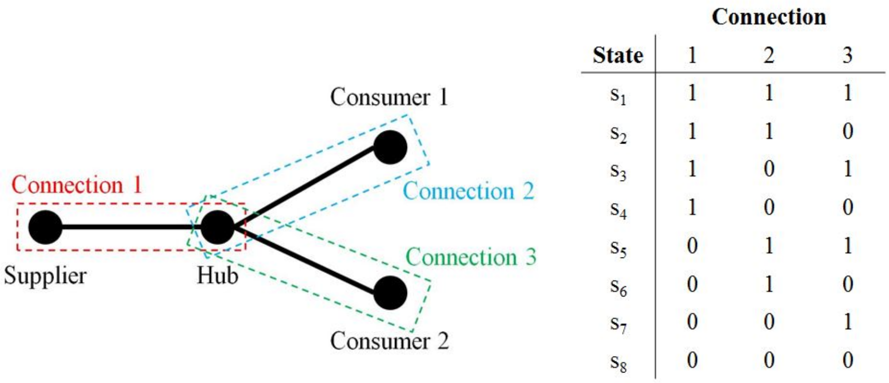
$$P = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

The rank of P is 4, so the columns of P are linearly independent. As such, matrix diagonalisation is indeed possible for this case, so the state probabilities can indeed be expressed explicitly. For the illustrative layout, this leads to:

$$
\begin{aligned}
Pr(s_1) &= \lambda_1^h &&= (3/9)^h && \text{(8a)}\\
Pr(s_2) &= -\lambda_1^h + \lambda_2^h &&= -(3/9)^h + (6/9)^h && \text{(8b)}\\
Pr(s_3) &= -\lambda_1^h + \lambda_3^h &&= -(3/9)^h + (5/9)^h && \text{(8c)}\\
Pr(s_4) &= \lambda_1^h - \lambda_2^h - \lambda_3^h + \lambda_4^h &&= (3/9)^h - (6/9)^h - (5/9)^h + 1 && \text{(8d)}
\end{aligned}
$$

These equations show that the state probabilities are only dependent on eigenvalues of T. A major advantage is that this holds for any layout with two systems, regardless of the number of compartments or the physical location of the systems in the compartments. Thus, a specific, pre-defined concept is used to generate generalized knowledge. This can be used to search for alternative solutions, and to evaluate the interdependencies between the states. Consider for example the situation where there is a desire to maximise the probability that both systems are on, i.e. to maximise $Pr(s_1)$. This probability is dependent on λ_1 only. This is element $(1, 1)$ of the transition matrix, i.e. the probability to remain in s_1 given that the previous state was s_1 already. In this example, this probability is 3/9, corresponding to the number of compartments where no systems are located. To increase the probability, this value needs to be increased. This implies that more compartments need to be empty, i.e. systems A and B need to be concentrated more, which is sensible from a physical perspective. However, this comes at a cost, as increasing λ_1 has a negative effect on the probabilities for s_2 and s_3. Hence, this leads to a high probability that both systems are on after one or more hits, but the probability that at least one of the systems is on, reduces. Though this result seems trivial for this illustrative layout, this method can be extended to larger, more complex concepts as well, enabling a better understanding of the trade-off between different levels of residual capabilities.

To scale up this method to layouts that can be used during the concept phase, several additional issues need to be addressed. These have previously been introduced in Habben Jansen et al. (2018a). Contrary to the systems in the illustrative layout, that contained only two components and a routing between them, distributed systems on board naval ships are part of multi-layered networks with one or more hub layers between suppliers and consumers (de Vos and Stapersma (2018)). In addition to that, these networks are often multiplex, resulting in interdependencies between the different commodities that flow through the network. For example, a chilled water unit is represented by a single node, while it is a consumer of electrical energy and a supplier of chilled water at the same time. As a result, it is not possible to simply state that a system is on or off. This depends on the availability of multiple components that may provide different types of commodities. Within the Markov chain this is addressed by describing the states as the availability of individual connections. These connections contain edges of the distributed systems network, including the start node and end node. Consider Figure 3 for an example, where it is assumed that Consumer 1 and Consumer 2 provide the same capability. The table at the right shows how the states for this example are defined. The capability is available if the network is in State 1, 2, or 3. If the network is in State 4, 5, 6, 7, or 8, the capability is not available. Note that this is independent of the transition matrix. Hence, the distributed systems network is subdivided in individual connections, and subsequently the Markov chain is calculated.



	Connection		
State	1	2	3
s_1	1	1	1
s_2	1	1	0
s_3	1	0	1
s_4	1	0	0
s_5	0	1	1
s_6	0	1	0
s_7	0	0	1
s_8	0	0	0

Figure 3: Definition of connections and states of the Markov chain of an example network, adapted from Habben Jansen et al. (2018a)

The interdependencies between connections (i.e. the fact that capabilities require a combination of certain edges) is accounted for after the Markov chain has been calculated. In the case of this example, the probability for having the capability available is found by adding the state probabilities of State 1, 2, and 3 together, as only they represent states where the capability is available.

The states, that describe the availability of individual connections, can be used to calculate the probability that certain levels of residual capability are available after one or more hits. These levels could for example be expressed as the ship's ability to perform the main functions 'fight', 'move' and 'float', where full residual capability includes all three functions, medium residual capability only contains 'move' and 'float', and minimal residual capability only includes 'float'. However, other definitions of residual capabilities can be applied as well. It is not possible to express the states directly as the availability of the capabilities, as multiple connections may contribute to a

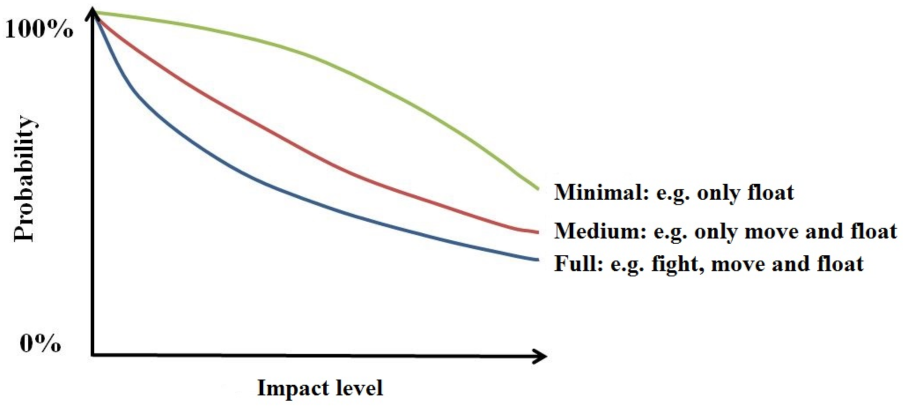


Figure 4: Qualitative example of the result provided by the capability-based vulnerability assessment. The probability for having full residual capability after a certain number of hits is smaller than the probability for having medium or minimal residual capability.

single capability. Likewise, a single connection may contribute to multiple capabilities. To obtain the probability that a certain level of residual capability is available, all states that contribute to that capability need to be added together, which provides a result that resembles the curves of Figure 4. As each state can be expressed as an explicit function of only the eigenvalues of T, this also holds for the explicit formulation of the capability curves of Figure 4. Hence, eigenvalues of T are the direct link between the ship concept (consisting of the compartments and the routed distributed systems network) and the shape of the curves, and can be used to study the interdependencies between the different levels of capability.

4. Test case

This section provides a test case to illustrate the application and contributions of the method. This is an extension of the test case that has previously been introduced in Habben Jansen et al. (2018a). It considers a notional Ocean-going Patrol Vessel (OPV) with an offensive weapon system, a defensive weapon system, and two propellers. Two powering concepts are considered:

1. Conventional: The propulsion system is mechanical, and is separated from the electrical distribution system, which powers both weapon systems. A forward and aft chilled water unit provide chilled water to the weapon systems.
2. IPS: Both the propulsion system and the weapon systems are powered by electrical power. The chilled water units for the weapon systems are located in the vicinity of the weapon systems.

The distributed systems networks and the physical location of the networks in the ship are presented in Figure 5 and Figure 6, respectively. Both concepts contain 12 connections. As a result, the number of states is $2^{12} = 4096$. The Markov chain has been set up according to the method described in Section 3, using the script for automatic generation of the transition matrix. The probability for each state has been calculated for up to 8 hits, meaning that up to 8 compartments are disabled. Subsequently, the states are combined to four levels of residual capability, as specified in Table 1. For each individual state it is checked whether it contributes one or more levels of residual capability. If so, the probability for that state is assigned to that residual capability. If not, the probability for that state is ignored. The result is a sum of contributing state probabilities for each level of residual capability, which is a quantitative version of the example result shown in Figure 4. For this test case, the result is presented in Figure 7. The horizontal axis

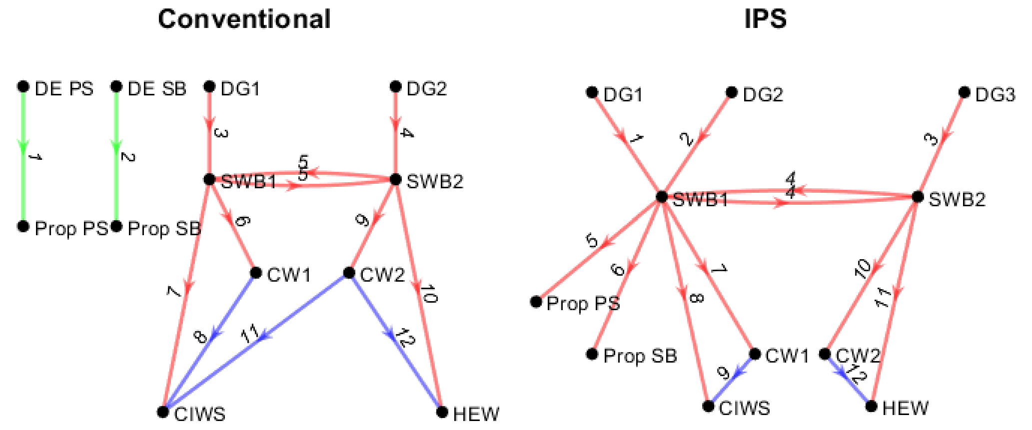


Figure 5: Distributed systems networks of the conventional and IPS concepts, adapted from Habben Jansen et al. (2018a). DE PS / SB = diesel engine port side / starboard, Prop = propeller, DG = diesel generator, SWB = switchboard, CW = chilled water plant, CIWS = close-in weapon system (defensive), HEW = high energy weapon (offensive). The edges are numbered 1-12, as denoted by the edge labels. Red edges denote electrical power connections, blue edges denote chilled water connections, and green power denotes mechanical energy connections. Between the SWBs there is only one connection, but it is directed both ways.

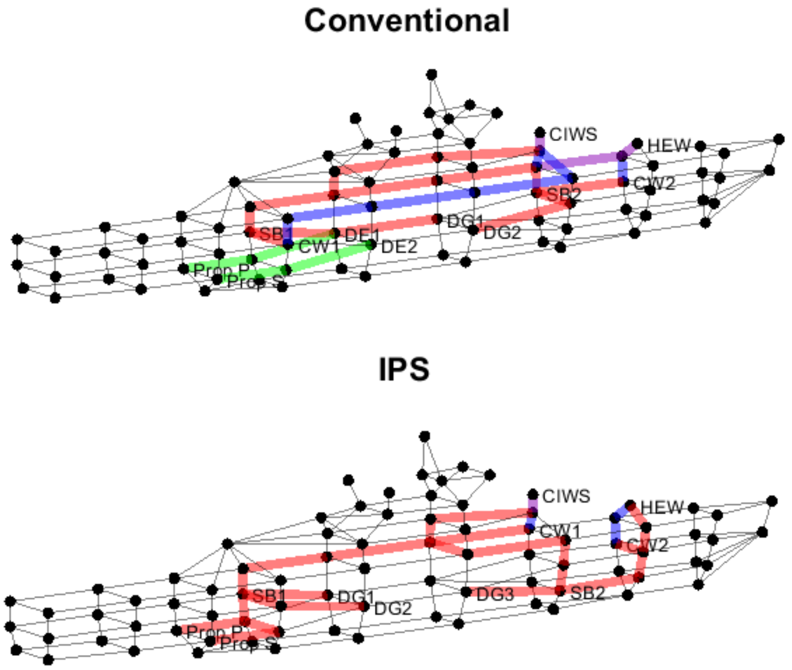


Figure 6: Physical location of the distributed systems networks in the ship, adapted from Habben Jansen et al. (2018a). A purple line denotes routings of both the electrical and chilled water network through the same compartments.

of this figure denotes the number of hits. The vertical axis is the probability that at least the level of required residual capability is met. For example, the probability to have at least minimal residual capability is higher for the conventional concept (solid purple line) than for the IPS concept (dashed purple line). The higher the number of hits, the bigger this difference becomes. This indicates that the conventional concept performs better than the IPS concept for this level of residual capability (i.e. at least propulsion at one side). For the level of considerable residual capability, i.e. at least full propulsion and the defensive weapon, the IPS concept performs better. This illustrates a trade-off in residual capabilities that needs to be made for these two particular concepts. It should be kept in mind, though, that the differences between the conventional concept and IPS concept are based on ship concepts with a limited level of detail. If one concept performs better than another concept at this stage, there is no definite guarantee that this also holds when both concepts are developed in more detail. However, the purpose of this assessment is not to select the best concept, but to identify the underlying rationale that leads to these levels of vulnerability.

The results, including the associated trade-off, have previously been interpreted in a qualitative fashion only. With the observation that the curves can be written as a function of the eigenvalues of the transition matrix, a formal mathematical description can be obtained, and is presented in this paper. This gives insight into why the curves are shaped in this particular fashion. The resulting knowledge can be used to search for alternative, potentially better concepts, and to identify and quantify interdependencies between the different levels of residual capability.

Table 1: Levels and specification of residual capability, adapted from Habben Jansen et al. (2018a)

Residual capability	**Description**
Full	Offensive and defensive weapons, two-shaft propulsion
Considerable	Defensive weapon, two-shaft propulsion
Moderate	Defensive weapon, one-shaft propulsion
Minimal	One-shaft propulsion

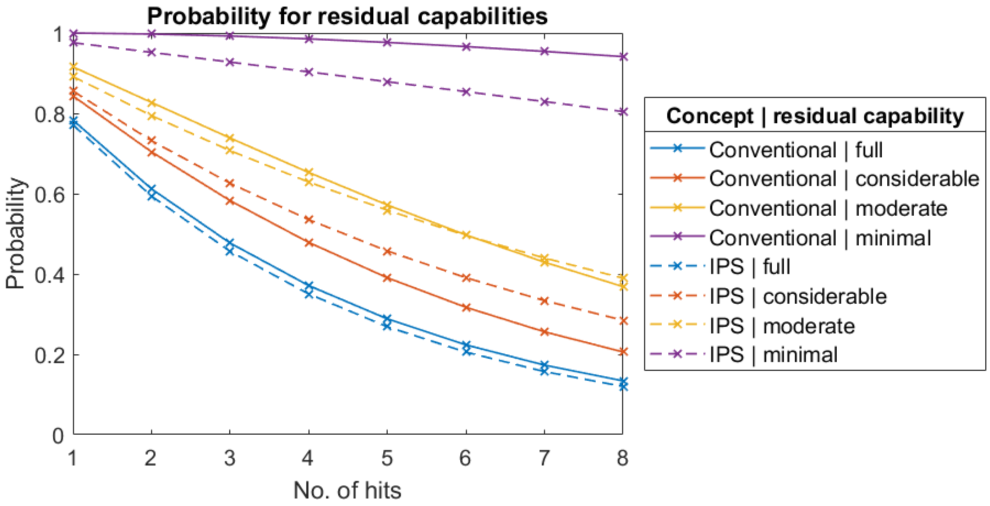


Figure 7: Probabilities for the four levels of residual capability for the conventional and IPS concept, adapted from Habben Jansen et al. (2018a)

5. Results

In order to formulate the probability curves for the states explicitly, matrix diagonalisation needs to be applied. This is only possible if the rank of P equals its size. In this case, this means that matrix diagonalisation is possible if the rank of P is 4096. Using MATLAB, this is found to be the case for both the conventional concept and the IPS concept. Hence, the curves of the different levels of residual capability can be expressed as:

$$Pr(c_i) = \sum_{k=1}^{n} f_k \cdot \lambda_k^h \tag{9}$$

where c_i denotes the i^{th} capability level (in this test case i runs from 1 to 4), λ_k denotes the k^{th} eigenvalue of the transition matrix (in this test case k runs from 1 to 4096, which is n, the number of states), h is the number of hits, and f_k is an integer factor that states how strong λ_k contributes to the curve, and which sign it has.

The expression of Equation 9 can be set up for each individual vulnerability curve. Table 2 gives the eigenvalues and factors for the curve that represents the probability for full residual capability of the conventional concept, as presented in Figure 7. The first column provides the state numbers of the eigenvalues that contribute to the curve (between 1 and 4096). The second column gives the actual value of these eigenvalues, as provided by the transition matrix. The third column states whether the eigenvalue corresponds positively or negatively to the curve, and how strong. The corresponding state definitions are included as well, where the numbers 1-12 relate to the individual connections in the concept, as defined in Figure 5. Several observations can be made:

Table 2: The eigenvalues, and their corresponding factors and state definitions, that contribute to the curve of full residual capability for the conventional concept. The numbers 1-12 in the state definitions relate to the individual connections in the concept, as defined in Figure 5

k	λ_k	f_k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	0.6747	2	1	1	1	1	1	1	1	1	1	1	1	1
3	0.6747	-2	1	1	1	1	1	1	1	1	1	1	0	1
81	0.7349	-2	1	1	1	1	1	0	1	0	1	1	1	1
129	0.6867	-1	1	1	1	1	0	1	1	1	1	1	1	1
131	0.6867	1	1	1	1	1	0	1	1	1	1	1	0	1
209	0.7470	1	1	1	1	1	0	0	1	0	1	1	1	1
257	0.6988	-1	1	1	1	0	1	1	1	1	1	1	1	1
259	0.6988	1	1	1	1	0	1	1	1	1	1	1	0	1
337	0.7590	1	1	1	1	0	1	0	1	0	1	1	1	1
513	0.6867	-1	1	1	0	1	1	1	1	1	1	1	1	1
515	0.6867	1	1	1	0	1	1	1	1	1	1	1	0	1
593	0.7470	1	1	1	0	1	1	0	1	0	1	1	1	1

- Though there is a total of 4096 states, only 12 states have eigenvalues that contribute to the probability for full residual capability of the conventional concept.

- Some eigenvalues have a positive contribution, while others have a negative contribution. Many of these eigenvalues occur in pairs that cancel each other out. For each pair, connection 11 is off in one of the corresponding states, while connections 7, 10, and 12 are on. This state is physically not possible, as connection 11 shares routings with these other connections through the same compartments. The meaning of the corresponding eigenvalues is strictly mathematical in this case. Nevertheless, they should not be ignored. If modifications to connection 11 are made in such way that it no longer routed together with other connections, the pair-wise cancellation may no longer be present.

- For every eigenvalue, the corresponding state includes the availability of at least connections 1, 2, 7, 9, 10, and 12. Relating this back to the distributed systems network of Figure 5, it turns out that these are all non-redundant connections.

- For some states, connections 6 and 8 are off. Their unavailability is related; individual availability of either connection 6 or connection 8 does not occur. For every state where connections 6 and 8 are off, there is a fellow state where connection 11 is off.

- For most states, either connection 3, 4, or 5 is off, but no combinations of these states.

These observations are the result of assessing this specific concept, but are not restricted to the physical routings of the concept. Therefore they provide valuable information that can be used to better understand and improve the existing concept. The following procedure is proposed for this:

1. From all curves representing the various levels of residual capability, select one to study in more detail. For this test case, the curve of full residual capability for the conventional concept is investigated.
2. Check how many eigenvalues contribute to the shape of that curve. This may be decisive for how the further assessment is carried out. In this case, 12 eigenvalues contribute to the curve. The number of contributing eigenvalues is no metric for the vulnerability - it can therefore not be stated that either more or less eigenvalues is 'better'. However, a smaller number of eigenvalues allows the designer to do a manually-oriented assessment, while for a larger number of contributing eigenvalues the assessment may require a more computational approach. This test case illustrates a manual approach. Assessment methods for larger numbers of eigenvalues are still subject of further research.
3. Check for repetitions or pairs in the contributing eigenvalues. For this test case, four pairs with connection 11 occur, leading to eigenvalues that cancel each other out. As such, only four eigenvalues contribute to the shape of the curve. They will be addressed in this test case.
4. Change the remaining connections in such way that the eigenvalues with positive factors increase and the eigenvalues with negative factors decrease. In order to increase an eigenvalue with a positive factor, the connections that are on in that state need to be reduced in size or

> concentrated. In order to decrease an eigenvalue with a negative factor, the connections that are off in that state need to be reduced in size or concentrated. From a mathematical point of view it could also be an option to increase the size of the routings of the other connections, but that would lead to increased vulnerability of those connections for the sake of a lower relative vulnerability of the other connections. As such, recommendations of size reduction and concentration of routings are preferred.

This can be applied to this test case. Consider the situation where there is a desire to increase the probability that there is full residual capability after one or more hits. The eigenvalues λ_{81}, λ_{209}, λ_{337} and λ_{593} have the strongest influence on this, as they are not cancelled out by other eigenvalues. More specifically, λ_{81} needs to be decreased, and λ_{209}, λ_{337} and λ_{593} need to be increased. For all these eigenvalues, connections 6 and 8 are off. In order to decrease λ_{81}, the routings of these connections should be made smaller or more concentrated. However, in order to increase λ_{209}, λ_{337} and λ_{593}, it is the other way around. This is a mathematical representation of conflicting requirements that result from interdependencies between the connections. However, a closer look at λ_{209}, λ_{337} and λ_{593} shows that either connection 3, 4 or 5 is off in their associated states. In order to increase these eigenvalues, the probability to remain in any of these states should be increased. This indicates that all routings related to connections other than 3, 4, 5, 6 or 8 need to be made smaller, or more concentrated. Connections 1 and 2 are related to propulsion, and are not easy to modify for the conventional concept. However, for connections 7, 9, 10, 11 and 12, two modifications are proposed:

1. Bring CW2 above SWB2, closer to the CIWS. This concentrates the routings of connections 9, 10 and 12, and reduces the routing length of connection 11.
2. Concentrate the routing from SWB1 to the CIWS (connection 7) with the routing between SWB1 and SWB2 (connection 5). This reduces the number of compartments solely occupied by the routing of connection 7.

The concept is adjusted accordingly, such as presented in Figure 8. The results of this adjustment are given in Figure 9. It can be observed that the adjustments have the desired effect, as the curve for full residual capability lies higher, indicating a higher probability for having this level of residual capability after one or more hits. At the same time the curves for moderate and considerable residual capability have increased as well. Hence, the positive effect of the modification goes beyond the level of residual capability that was originally considered. In this case this effect is positive. However, for other cases the residual capability of other levels may drop if the residual capability of the level that was originally considered is increased. This method elucidates and quantifies these interdependencies.

The same method can be applied to the IPS concept. Figure 7 shows that the probability for minimal residual capability, i.e. at least propulsion at one side, is significantly lower for the IPS concept (purple dashed curve) than for the conventional concept. Consider the situation where there is a desire to increase this probability. The eigenvalues and factors that determine the shape of this curve are presented in Table 3. For this case all factors are

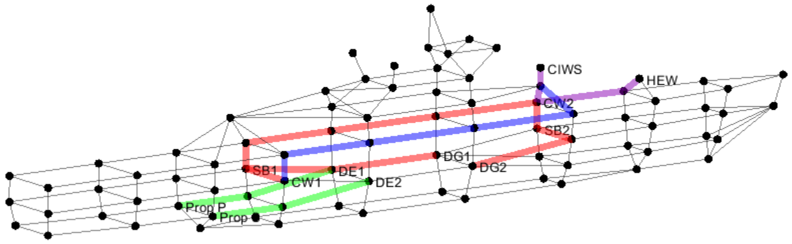


Figure 8: Physical location of the distributed systems networks in the adjusted conventional concept

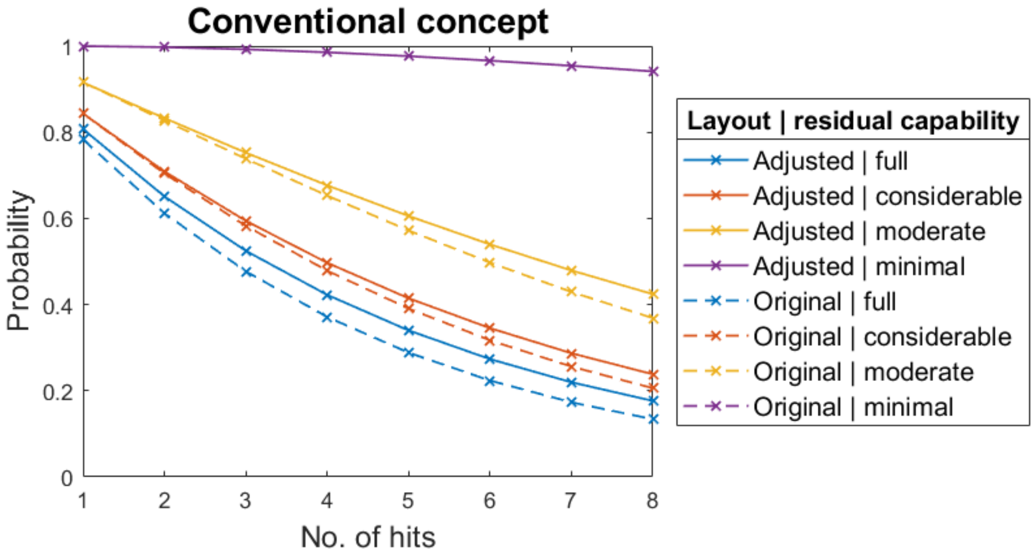


Figure 9: Probabilities for the four levels of residual capability for the adjusted conventional concept

either 1 or -1, so in order to determine the eigenvalues with the largest influence, the magnitude of the eigenvalues need to be considered. The largest eigenvalues, i.e. the eigenvalues with the largest influence on the curve, are λ_{1920}, λ_{1984} and λ_{2944}. The corresponding factor is 1 for all these eigenvalues, so they make a positive contribution to the curve. Hence, the probability for minimal residual capability increases when the magnitude of these eigenvalues increase. The state definitions associated with these eigenvalues show that connections 1, 2, 5, and 6 are on, while the other connections are off. In order to increase the probability for minimal residual capability, the number of compartments associated with these connections needs to be reduced, and concentration and/or separation of the associated system components and routings may be beneficial. To that end, SWB1 is relocated one compartment lower compared to the original IPS concept that was presented in Figure 6. As a result, the distance between SWB1 and both propellers reduces. In addition of that, only one compartment is a single point of failure, instead of two compartments for the previous situation. Since SWB1 has been relocated, the routings from DG1 and DG2 need to be adjusted as well. As a result, the routing from DG2 to SWB1 now has a partial overlap with the routing between SWB1 and the starboard propeller. However, this does not affect the power supply to the port side propeller, so the requirement to have a large probability for propulsion at one side at least is still met. Propulsion power can also be supplied via DG3 and SWB2, i.e. connections 3 and 4. However, the results of the eigenvalue assessment shows that these connections have a smaller influence on the shape of the curve. These connections are therefore left unchanged. The adjusted layout is presented in

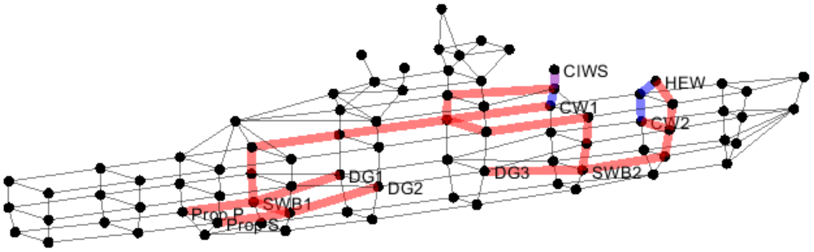


Figure 10: Physical location of the distributed systems networks in the adjusted IPS concept

Figure 10. The associated result is given in Figure 11. It can be seen that the probability for having at least minimal residual capability has increased significantly. Hence, the proposed solution has the desired effect. For the other levels of residual capability, the curves remain unchanged, so no trade-off needs to be made. This is because the capability that is considered, i.e. propulsion at one side at least, does not include components or routings of other systems.

6. Conclusions and recommendations

In this paper a method for assessing vulnerability of naval distributed ship systems is presented and illustrated. This method assesses the vulnerability in a quantitative fashion, from a capabilities perspective, in the concept phase. A major benefit of the method is that it does not only evaluate the vulnerability of an existing, pre-defined concept, but also provides direction for finding other, potentially better solutions. This is done from a capabilities perspective rather than from a systems perspective. The method accounts for the fact that the relation between individual connections and higher-level ca-

Table 3: The eigenvalues, and their corresponding factors and state definitions, that contribute to the curve of minimal residual capability for the IPS concept

k	λ_k	f_k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
64	0.8072	-1	1	1	1	1	1	1	0	0	0	0	0	0
128	0.8313	1	1	1	1	1	1	0	0	0	0	0	0	0
192	0.8193	1	1	1	1	1	0	1	0	0	0	0	0	0
832	0.9036	1	1	1	0	0	1	1	0	0	0	0	0	0
896	0.9277	-1	1	1	0	0	1	0	0	0	0	0	0	0
960	0.9157	-1	1	1	0	0	0	1	0	0	0	0	0	0
1088	0.8313	1	1	0	1	1	1	1	0	0	0	0	0	0
1152	0.8554	-1	1	0	1	1	1	0	0	0	0	0	0	0
1216	0.8434	-1	1	0	1	1	0	1	0	0	0	0	0	0
1856	0.9277	-1	1	0	0	0	1	1	0	0	0	0	0	0
1920	0.9518	1	1	0	0	0	1	0	0	0	0	0	0	0
1984	0.9398	1	1	0	0	0	0	1	0	0	0	0	0	0
2112	0.8193	1	0	1	1	1	1	1	0	0	0	0	0	0
2176	0.8434	-1	0	1	1	1	1	0	0	0	0	0	0	0
2240	0.8313	-1	0	1	1	1	0	1	0	0	0	0	0	0
2880	0.9157	-1	0	1	0	0	1	1	0	0	0	0	0	0
2944	0.9398	1	0	1	0	0	1	0	0	0	0	0	0	0
3008	0.9277	1	0	1	0	0	0	1	0	0	0	0	0	0
3136	0.8434	-1	0	0	1	1	1	1	0	0	0	0	0	0
3200	0.8675	1	0	0	1	1	1	0	0	0	0	0	0	0
3264	0.8554	1	0	0	1	1	0	1	0	0	0	0	0	0

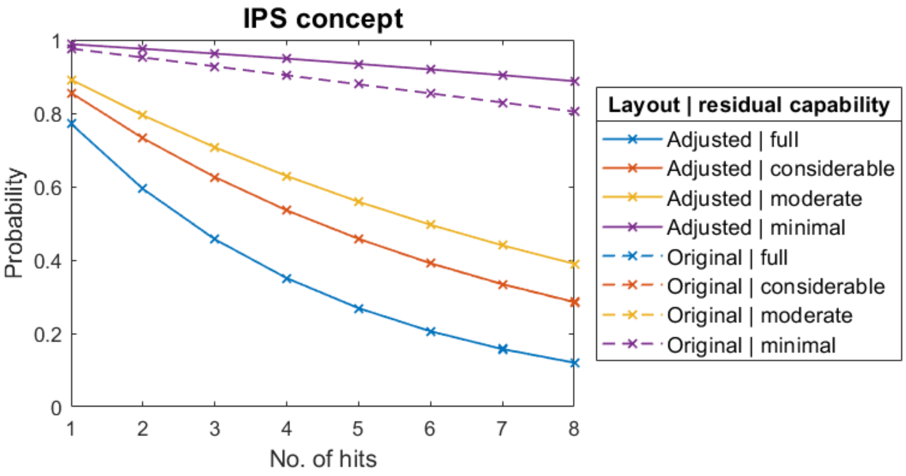


Figure 11: Probabilities for the four levels of residual capability for the adjusted IPS concept. The curves for considerable, moderate, and minimal residual capability are similar for the adjusted and the original concept.

pabilities is not necessarily one-to-one, and that potential trade-offs between various levels of capability may exist. An explicit mathematical formulation relates the availability of higher-level capabilities to specific connections in the distributed systems network that are decisive for this availability.

In addition to these general contributions, several specific conclusions can be drawn from the test case, where the vulnerability of a conventional powering concept and an IPS concept for a notional OPV has been assessed at various levels of required residual capability. It differs per level whether the conventional or IPS concept performs better. Though some of the differences are subtle, there is a major difference in the probability for minimal residual capability, i.e. having propulsion at one side at least. For this level of residual capability, the conventional concept performs significantly better than the IPS concept. This is because the number of compartments that is equipped with propulsion components and routings is larger for the IPS concept, making it more likely to get hit. Nevertheless, the method has successfully provided directions to modify the concept such that this improves, without compromising the performance for other levels of residual capability. The conventional concept has also been modified, with a goal to obtain a higher probability for full residual capability. This has indeed been achieved, also without compromising other levels of residual capability. It should be kept in mind that these results are based on an assessment with uniform hit probability. In earlier work of the authors it has been shown that other (user-defined) hit probability distributions can be applied as well (Habben Jansen et al., 2018b). These other types of distributions have not yet been applied for obtaining design recommendations, such as done in this paper. Opportunities arise for combining these two aspects, but the mathematical set-up and design implications are still subject of ongoing research. The same holds for scaling up the method to higher numbers of systems and routings. As discussed in Section 3, the size of the transition matrix increases exponentially with the number of connections that is considered. Currently this limits the size and complexity of the method to distributed systems comparable to the test case of this paper. Since this paper aimed to explain how design recommendations for reduced vulnerability can be obtained, rather than mimicking an actual design effort, this limited complexity is considered appropriate. However, opportunities for scaling the method and including more representative ship concepts are under consideration in ongoing work.

The vulnerability method presented in this paper considers system components and routings that can be either on or off. In other words, the method checks whether the power sources and sinks in the network are connected. However, in order to meet the various level of residual capabilities, there also needs to be sufficient effort and flow of the different commodities. Adding a network flow assessment to this method would increase the fidelity. Such an assessment can also evaluate in which damage cases an operational decision needs to be taken because there is a higher power demand than power supply. However, the design stage for which this method is meant should be taken into consideration while doing this, as the concept phase deliberately is associated with a low level of detail.

Acknowledgements

Funding for this research is provided by Ms. Kelly Cooper from the United States Office of Naval Research (ONR) under grant no. N00014-15-1-2752, and is gratefully acknowledged. Furthermore, the authors would like to thank the Defence Materiel Organisation of the Netherlands Ministry of Defence for their in-kind contribution to this research.

References

Andrews, D., 2011. Marine requirements elucidation and the nature of preliminary ship design. International Journal of Maritime Engineering 153.

Andrews, D., 2018. The Sophistication of Early Stage Design for Complex Vessels. International Journal of Maritime Engineering, Special Edition doi:10.3940/rina.ijme.2018.SE.472.

Arboleda, C., Abraham, D., Richard, J., Lubitz, R., 2009. Vulnerability assessment of health care facilities during disaster events. Journal of Infrastructure Systems 15, 149–161.

Ball, R., Calvano, C., 1994. Establishing the Fundamentals of a Surface Ship Survivability Design Discipline. Naval Engineers Journal 106, 71–74. doi:10.1111/j.1559-3584.1994.tb02798.x.

BBC, 2018. Helge Ingstad: Norway's warship collides with tanker in fjord. URL: `https://www.bbc.com/news/world-europe-46136564`.

Boulougouris, E., Winnie, S., Papanikolaou, A., 2017. Assessment of Survivability of Surface Combatants after Damage in the Sea Environment. Journal of Ship Production and Design 33, 156–165.

Brefort, D., Shields, C., Habben Jansen, A., Duchateau, E., Pawling, R., Droste, K., Jaspers, T., Sypniewski, M., Goodrum, C., Parsons, M., Yasin Kara, M., Roth, M., Singer, D., Andrews, D., Hopman, J., Brown, A., Kana, A., 2018. An Architectural Framework for Distributed Naval Ship Systems. Ocean Engineering 147, 375–385.

Brouwer, R., 2008. A Framework for Systems Engineering in Ship Design from a NATO Specialist Team Perspective, in: Engineering the Total Ship Symposium, American Society of Naval Engineers, Falls Church.

Brown, D., 1991. The future British surface fleet. Conway Maritime Press, London.

Clayton, D., Sudhoff, S., Grater, G., 2000. Electric ship drive and power system, in: Conference Record of the 2000 Twenty-fourth International Power Modulator Symposium, Norfolk. pp. 85–88. doi:10.1109/MODSYM.2000.896171.

Doerry, N., 2006. Zonal Ship Design. Naval Engineers Journal 118, 39–53. doi:10.1111/j.1559-3584.2006.tb00407.x.

Doerry, N., 2007. Designing Electrical Power Systems for Survivability and Quality of Service. Naval Engineers Journal , 25–34.

Doerry, N., 2015. Naval Power Systems: Integrated power systems for the continuity of the electrical power supply. IEEE Electrification Magazine , 12–21doi:10.1109/MELE.2015.2413434.

Dougal, R., Langland, D., 2016. Catching it early - Modeling and simulating distributed systems in early stage design. SNAME Marine Technology , 63–69.

Duchateau, E., 2016. Interactive evolutionary concept exploration in preliminary ship design. Ph.D. thesis. Delft University of Technology. Delft.

Duchateau, E., de Vos, P., van Leeuwen, S., 2018. Early stage routing of distributed ship service systems for vulnerability reduction, in: 13th International Marine Design Conference, Helsinki.

El-Rashidy, R., Grant-Muller, S., 2014. An assessment method for highway network vulnerability. Journal of Transport Geography 34, 34–43.

Elgot, J., 2016. British warships need multimillionpound refit to stop power failures. URL: `https://www.theguardian.com/uk-news/2016/jan/29/royal-navy-warships-multimillion`

Gillespie, J., 2012. A Network Science Approach to Understanding and Generating Ship Arrangements in Early-Stage Design. Ph.D. thesis. University of Michigan.

Goodfriend, D., Brown, A., 2017. Exploration of System Vulnerability in Naval Ship Concept Design. Journal of Ship Production and Design 33, 1–17.

Goodrum, C., Shields, C., Singer, D., 2018. Understanding cascading failures through a vulnerability analysis of interdependent ship-centric distributed systems using networks. Ocean Engineering 150, 36–47.

Habben Jansen, A., Duchateau, E., Kana, A., 2018a. Towards a novel design perspective for system vulnerability using a Markov chain, in: Proceedings of the 14th International Naval Engineering Conference, Glasgow.

Habben Jansen, A., Kana, A., Hopman, J., 2018b. An approach for an operational vulnerability assessment for naval ships using a Markov model, in: Proceedings of the 13th International Marine Design Conference, Helsinki.

Heywood, M., Lear, T., 2006. PREVENT A tool to reduce vulnerability early in the design, in: Proceedings of Warship 2006, London.

Janssen, J., Butler, J., Worthington, P., Geertsma, F., den Hartog, M., 2016. Autonomous, adaptive, aware: DINCS, in: Proceedings of the 13th International Naval Engineering Conference, Bristol.

Kim, K., Lee, J.H., 2012. Simplified vulnerability assessment procedure for a warship based on the vulnerable area approach. Journal of Mechanical Science and Technology 26, 2171–2181.

Kossiakoff, A., Sweet, W., Seymour, S., Biember, S., 2011. Systems Engineering Principles and Practice. 2nd ed., Wiley, Hoboken, New Jersey.

Lay, D., 2006. Linear Algebra and Its Applications. 3rd ed., Pearson Education, Boston.

Liu, C., Xu, Q., Chen, Z., Leth Bak, C., 2012. Vulnerability Evaluation of Power System Integrated with Large-scale Distributed Generation Based on Complex Network Theory, in: Proceedings of the 47th international Universities Power Engineering Conference (UPEC), Uxbridge.

van Oers, B., 2011. A packing approach for the early stage design of service vessels. Ph.D. thesis. Delft University of Technology.

van Oers, B., van Ingen, G., Stapersma, D., 2012. An integrated approach for the design of survivable ship services systems, in: Proceedings of the International Naval Engineering Conference (INEC), Edinburgh.

Piperakis, A., 2013. An Integrated Approach to Naval Ship Survivability in Preliminary Ship Design. Ph.D. thesis. University College London.

Piperakis, A., Andrews, D., 2012. A comprehensive approach to survivability assessment in naval ship design. International Journal of Maritime Engineering 156, 333–352. doi:10.3940/rina.ijme.2014.a4.307.

Reese, R., Calvano, C., Hopkins, T., 1998. Operationally Oriented Vulnerabiliity Requirements in the Ship Design Process. Naval Engineers Journal 110, 19–34. doi:10.1111/j.1559-3584.1998.tb02383.x.

Said, M., 1995. Theory and Practice of Total Ship Survivability for Ship Design. Naval Engineers Journal 107, 191–203. doi:10.1111/j.1559-3584.1995.tb03085.x.

Schofield, J., 2009. SURVIVE and SURVIVE Lite - survivability assessment from concept to operational support, in: Proceedings of the American Society of Naval Engineers Day, Maryland.

Schulte, J., 1994. An analysis of the historical effectiveness of anti-ship cruise missiles in littoral warfare. Ph.D. thesis. Naval Postgraduate School.

Shields, C., Sypniewski, M., Singer, D., 2016. Understanding the relationship between naval product complexity and on-board system survivability using network routing and design ensemble analysis, in: Proceedings of PRADS2016, Copenhagen.

Surma Ltd., 2018. SURMA survivability analysis. URL: `http://survivability.fi/surma/demo/`.

TNO, 2018. RESIST Lite. URL: `https://www.tno.nl/media/1644/resist_lite.pdf`.

Trapp, T., 2015. Shipboard Integrated Engineering Plant Survivable Network Optimization. Ph.D. thesis. Massachusetts Institute of Technology.

de Vos, P., Stapersma, D., 2018. Automatic topology generation for early design of on-board energy distribution systems. Ocean Engineering 170, 55–73. doi:10.1016/j.oceaneng.2018.09.023.