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BAYESIAN OMA OF OFFSHORE ROCK LIGHTHOUSES: SURPRISES WITH CLOSE MODES, SYMMETRY AND ALIGNMENT

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A set of seven rock lighthouses around the British Isles was studied by a combination of forced and ambient vibration tests executed with some extreme logistical constraints. Forced vibration testing of the circular section masonry towers combined with experimental modal analysis identified modes with alignment assumed the same as the shaker as well as some interesting effects of helideck retrofit, whereas operational modal analysis revealed the considerable degree of uncertainty in mode shape alignment. Hence Bayesian operational modal analysis was used to characterise the uncertainty and find the best representation of mode shape direction. While perfectly axisymmetric towers would show a single frequency omnidirectional mode, OMA reveals the split modes and allows un unbiased view of directionality. The variability and uncertainty of these mode shape directions are further revealed using Bayesian OMA.

Keywords: lighthouse, condition, assessment, system, identification, Bayesian, OMA

1. INTRODUCTION

Lighthouses are iconic structures that provide visual navigational aids to protect mariners and preserve vital maritime trade. Some of the most spectacular examples are the masonry towers constructed in the Victorian era (1837-1901) on rock outcrops (reefs) around the British Isles. A collaborative research project was established in 2016 to examine the performance of a set of these structures, evaluate the wave loads they experience and provide guidance for their future management. A significant component of this project has been evaluation of the lighthouse dynamic characteristics i.e. their modal properties, since these provide insight into the way the structures resist wave forces as well as the means to infer the distribution of extreme wave loads through extended monitoring of dynamic response. To this end an experimental programme of dynamic investigation was organised to provide sufficient information to calibrate finite element simulations and to infer wave loads [1] on a set of seven lighthouses.

This programme focused on a sequence of modal tests primarily relying on a 180 N electrodynamic shaker to produce input-output data for experimental modal analysis (EMA) using classical frequency domain methods (circle fit and GRFP). For five of the lighthouses topped by a steel helideck there were obviously two fundamental modes, rather similar to a tuned mass damper, with alternating phase angle of the lightweight steel helideck. Additionally, each of these modes was in fact a pair of modes occurring at very close frequencies, and EMA was unable adequately to identify these distinct modes in terms of likely slightly different frequencies and roughly orthogonal 'principal' directions.

As a complement to the shaker testing, the secondary ambient vibration exercise is described, the singular value spectra of acceleration responses presented and the Bayesian operational modal analysis (BAYOMA) process introduced, that allows identification and quantification of most probable values (MPV) and coefficients of variation (COVs) of natural frequency, damping ratio and mode shape.

Seven lighthouses were studied, but a sample of three of these, all with helidecks, is chosen for this paper. All three lighthouses masonry towers are 'concave elliptic frustums' following Smeaton's classic design on Eddystone Rocks (south of Plymouth), and have helidecks fixed to the crown of the masonry structure and straddling the lantern. Wolf Rock lighthouse was the first rock lighthouse to be equipped with a helideck, in 1973, to simplify, render safer and less costly access for lighthouse keepers and (when the lighthouses were automated) by maintenance crews. The three examples are operated by Trinity House, one of the three General Lighthouse Authorities of the British Isles.

Table 1: Lighthouse details

Lighthouse	Location	Built	Designer	Tower height	Modal test date
Les Hanois (TH)	Guernsey west coast 49°26′06.2″N 2°42′08.4″W	1860-1862	James Douglass	36 m	2/6/2016
Wolf Rock (TH)	15 km west of Land's End 49°56.72′N 5°48.50′W	1861-1869	James Walker	41 m	18/7/2016
Eddystone (TH)	21 km southwest of Plymouth 50°10′48″N 4°15′54″W	1879-1882	James Douglass	49 m	1010/2017- 11/10/2017

Design of British lighthouses evolved by trial and destruction. Smeaton's Tower on Eddystone Rocks (completed in 1759) was the classic design with multiple circular masonry courses comprising shaped and dovetailed granite blocks. Lighthouse construction technology evolved fast during the last half of the 19th century and Les Hanois Lighthouse was the first new lighthouse to employ dovetailed connections to blocks above. The construction method was used in all subsequent structures.

Detail at the foundation level is particularly important as it affects lighthouse axisymmetry. Fixing to the rocks for Wolf Rock [2] and Eddystone Lighthouses [3] was by metal bolts sunk into the foundation rock and 'fox-wedged' at each end. Such mechanical connections extend into lower courses and with the dovetailing and grouting, the lighthouses should act as monolithic granite towers continuous with the foundation rock. This was designed to prevent strong vibrations observed on earlier lighthouses, but which are still known to occur on Wolf Rock, due to its very exposed location. A major objective of the test programme was to identify the modal parameters through forced and ambient vibration measurements in order to evaluate their vulnerability to extreme breaking wave impact loads and to observe the response during long term monitoring of two lighthouses, one being Wolf Rock. Hence Wolf Rock modal parameters are particularly interesting.

2. RELEVANT RESULTS FROM EMA

The most important modes relating to loads from breaking wave impact are the fundamental modes that have lowest modal masses corresponding to mode shape scaling at the effective point of impact, which is usually about one third up the tower height. Modal mass normalised to unity at the crown of the masonry tower is a good proxy as mode shapes in the tower are similar between lighthouses.

Figure 1 shows the three lighthouses and their (two) fundamental mode shapes from EMA in one axis. Modal masses as described but the mode shapes are shown with unity overall maximum value.

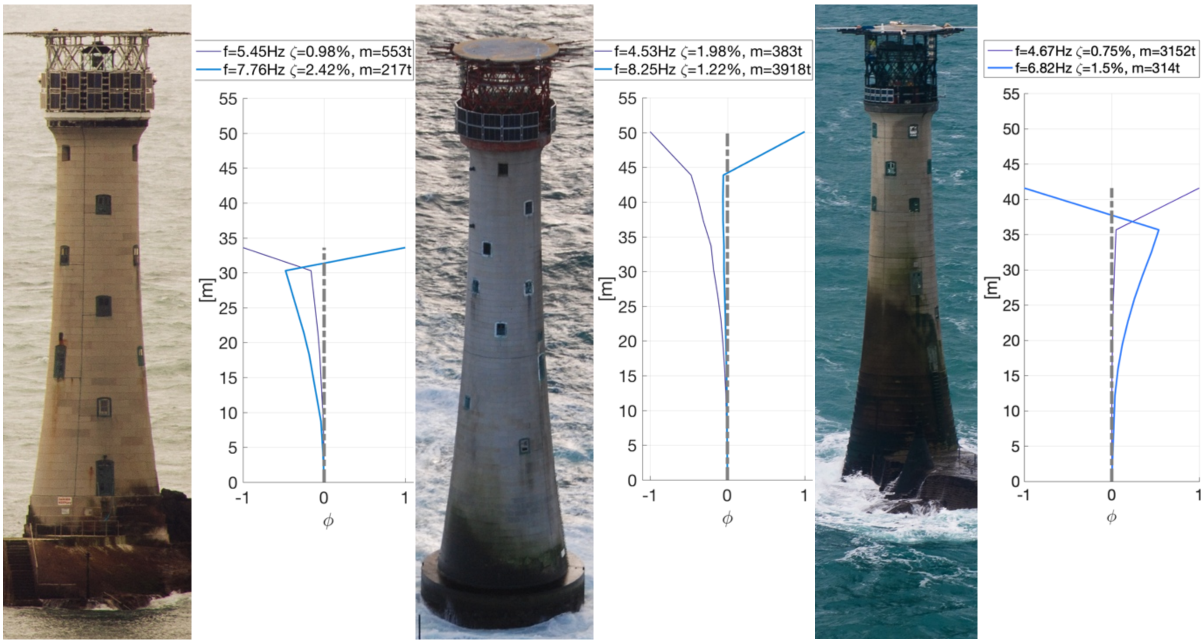


Figure 1 Rock lighthouses and their fundamental modes shapes from shaker testing. Left to right: Les Hanois, Eddystone and Wolf Rock. Photos by Emma Hudson, Trinity House and James Bassitt.

For Eddystone the second mode, and for Wolf Rock the first mode of the pair have very high modal mass due to the high valued helideck mode shape ordinate compared to the crown of the masonry tower. The modal parameters were extracted using the shaker aligned to a direction chosen for logistical convenience, usually to allow more accurate and consistent alignment of accelerometers at different levels within the lighthouse. Compared to buildings and bridges that have obvious cues to aid alignment, in lighthouses the steeply curved inner walls with irregularly spaced features such as windows, the various metallic fittings and electrical equipment, and the spiral staircases all conspire to confound accurate and consistent accelerometer alignment. Shaking in orthogonal direction generally identified almost identical modal parameters, the exception being Fastnet Lighthouse (not described here).

3. SINGULAR VALUE SPECTRA AND BAYESIAN

Power spectral densities (PSDs) of ambient response are the usual way to investigate modal properties. These are obtained as the diagonals of the cross-spectral density (CSD) matrix of all acceleration response channels obtained using the Welch procedure. The number of averages for fixed data length goes inversely with frequency line spacing and there is a trade-off between variance of spectral (line) variance error and bias in modal parameter estimates using some operational modal analysis (OMA) techniques. For stationary systems with stochastic stationary modal forces, the relationship of record length with uncertainty in modal parameter estimates has more recently been established [4], although it is well known that the longest possible duration recordings of stationary data should be used.

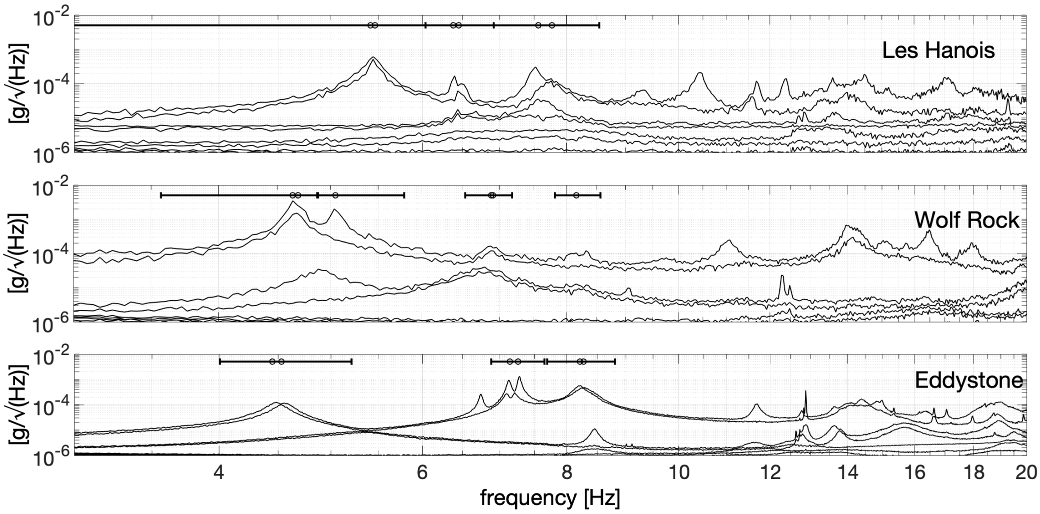
Due to logistical constraints (time, helicopter freight capacity) data for OMA (as well separately for EMA) comprised two setups to cover all levels in the lighthouse, there being 9 levels for Wolf Rock, 10 for Les Hanois and 11 for Eddystone, including helideck. Honeywell QA 750s were used, with sets of four (coloured) signal cables feeding drummed multicore signal cables from (Data Physics SignalCalc) analyser via power supply and connection boxes, with the acquisition set up in the service or battery room at the top of the tower. Cables were run externally up the helideck structure to the flat helipad level. Les Hanois and Wolf Rock used two setups of biaxial pairs, Eddystone used biaxial pair references and other accelerometers rotated 90° between setups, which was a more efficient approach.

The whole operation of cabling was time consuming despite preparations based on limited Trinity House photographs, so time on station for Wolf Rock and Les Hanois between inbound and outbound daylight helicopter flights reduced available acquisition time, even before bug-chasing. Modal tests were only possible during visits of maintenance crew who took priority for accommodation, but for the larger Eddystone Lighthouse an overnight stay was possible allowing more time to work and longer to acquire data. The OMA data set details for the three lighthouses are given in Table 2.

Table 2: Ambient vibration data sets used

Lighthouse	Set	Levels and directions	Duration (s)	Sample rate (Hz)
Les Hanois	1	1,2,5,6,9,10 x+y;	940	204.8
	2	3,4,7,8,9,10 x+y;	340	204.8
Wolf Rock 2017	1	1,2,5,6,8,9 x+y;	960	256
	2	3,4,7,6,8,9 x+y;	64	256
2019	3	1,5,7,8 x+y;	28800 (8 h)	256
Eddystone	1	1-6y, 7-11x+y;	35968 (10 h)	128

CSD matrices for response data in each set used default rectangular windowing and zero overlap over multiple averages, with PSDs prepared using CSD diagonals, for inspection on site to choose shaker bandwidth for EMA. PSD plots cannot identify identify number of modes in a band, so singular value decomposition (SVD) was applied to CSD matrices, (frequency) line by line. Near the resonance bands of modes the rank of the CSD matrix at any frequency line should equal the number of contributing modes in the band, but in practice practice the rank is usually identified by a sudden reduction of singular values to noise. Hence SVD of the ambient response CSD matrix indicates the number of modes clearly through traces of singular values (SVs) peaking in a frequency band. SVD plots for the the three lighthouses (using the longer recordings when available) are shown in Figure 2.



EMA:
5.45/5.45 Hz
7.76/7.77 Hz

EMA:
4.67/4.68 Hz
6.81/6.83 Hz

EMA:
4.53/4.53 Hz
8.25/8.30 Hz

Figure 2: SVD of averaged ambient response data spectra, with corresponding EMA values. Error bars indicate chosen frequency bands and initial estimates for BAYOMA identification.

Les Hanois SVD clearly shows two modes at 5.45 Hz and at least two modes around 7.76 Hz. A generator was running (at the standard 1500 revolutions/minute, with about 10% variation depending on load) and a subharmonic around 6.2 Hz was the suspected cause of PSD peaks, but presence of two SVD peaks suggests otherwise. For Wolf Rock, modes around 4.7 Hz and 6.8 Hz correspond to EMA estimates but there are also additional peaks e.g. at 5 Hz and 8 Hz not identified using EMA. Eddystone Lighthouse benefitted from much better understanding of the modal properties of the structural type, the largest set of (16) accelerometers including many biaxial pairs and an overnight stay that allowed for more comprehensive and long duration measurements. Hence the SVD shows clear mode pairs in bands around 4.3 Hz and 8.2 Hz, plus extra modes which remain mysterious.

As a possible complement to the EMA and specifically to provide some information on mode direction in the absence of forcing, OMA of the ambient response data was initially attempted using a conventional second generation technique i.e. NExT/ERA [5]. Compared to the clean shapes produced by EMA, OMA mode shapes were rather disappointing, with unclear alignment and non-smooth shapes, particularly for Wolf Rock. A significant problem with NExT/ERA is that results depend on operator heuristics i.e. preparation of the CSDs and choice of parameters such as Hankel matrix block size. Compared to EMA there is a high degree of judgement involved in deciding what is a convincing result and the process is complicated by small changes in environmental conditions between setups.

Bayesian operational modal analysis (BAYOMA) [6][7] yields the probability density function of modal properties using the FFT of ambient (vibration) time series, with preselected mode frequency estimates in frequency bands, both chosen based on singular value spectra. Using Bayes' theorem, given the data, the most probable value (MPV) of the set of modal parameters Θ is found by minimising a likelihood function (the 'negative log of the likelihood function' or NLLF) for the given data while the curvature (of NLLF) at the minimum point provides the covariance of Θ. In BAYOMA, the modes are assumed to be classically damped while the noise and modal force are assumed to be independent and to have constant PSDs within the frequency bands (e.g. those shown in Figure 2). BAYOMA identifies mode shapes (not operating deflection shapes), but these need not be orthogonal in the measured DOFs.

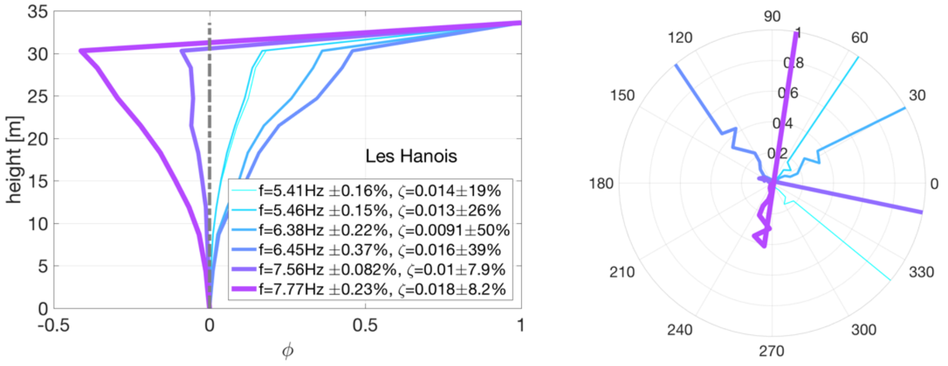
In the case of multiple data sets, mode shape pieces are glued using a global least square method [8], but for challenging cases where the signal to noise ratio is low in some sets, a multi-set Bayesian algorithm may be adopted, although presently it only works efficiently when the modes are well-separated. Neither requirement held for the lighthouse data: for Wolf Rock the very short second measurement set was rescued from the end of using a shaker measurement due to lack of time on station.

4. BAYOMA RESULTS

4.1. Mode shapes

Results are presented as mode shape plots for Les Hanois, Wolf Rock and Eddystone lighthouses in Figure 3, gluing sets (as described) to obtain the full mode shapes in elevation and plan. For the plan views the accelerometer x axis is at 0°, and for the elevation views the best fit vertical plane of each mode is identified and the mode is rotated through the plane angle with respect to the x axis to provide a common view. Mode shapes for Les Hanois and Eddystone appear smooth whereas those for Wolf Rock are, unexpectedly, not smooth, most likely due to the very short data duration of set 2.

Three closely spaced mode pairs are identified for Les Hanois. The first and last pair correspond to the EMA pairs but the second pair (at the previously mentioned generator subharmonic frequency) was not clear enough for identification in the FVT. All damping estimates are believable and the modes in each pair are approximately orthogonal within the pair but at different alignments for different pairs. Eddystone data are good quality and like Les Hanois there are three mode pairs, with the second unclear in and missed by EMA, and all three pairs are (self) orthogonal.



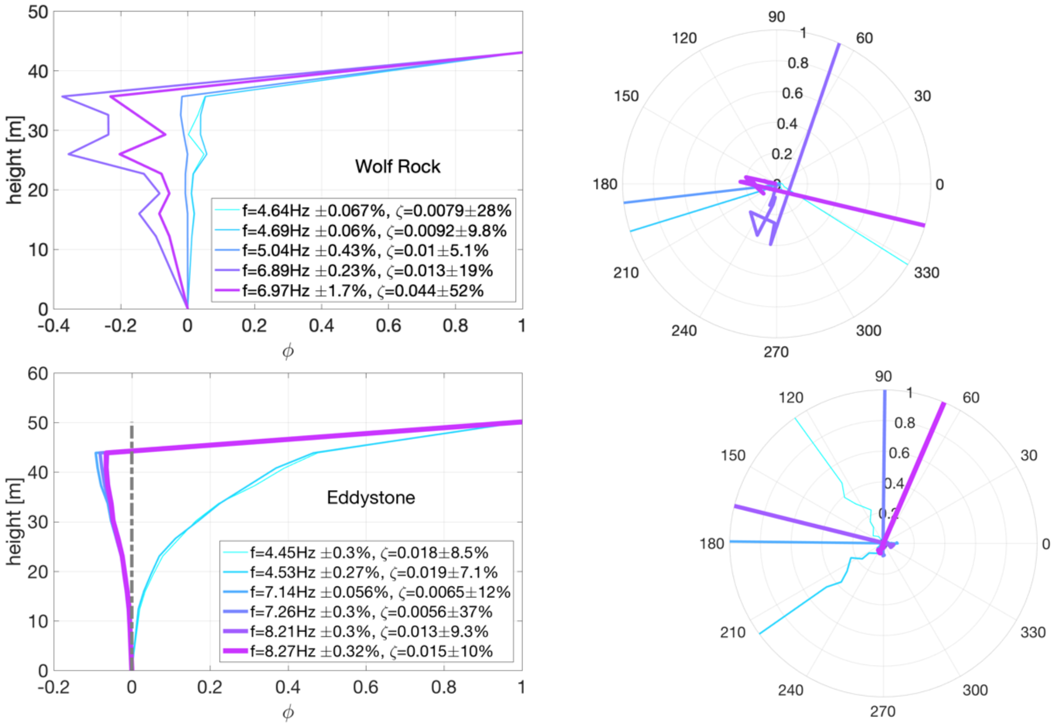


Figure 3: BAYOMA modes. Left elevation, rotated to the same plane; right in plan.

4.2. Mode directionality

Usually modal tests for identifying sway and torsional modes of buildings are planned with a specific orientation and aligned with e.g. core walls, rectangular columns or building envelope [9]. For buildings with square or rectangular section, modes are expected to align with the two symmetry axes. Even with clear symmetry this is not guaranteed, and directions need not align precisely with geometric symmetry axes, neither do the directions need to be at exact right angles at any level.

For structures like lighthouses with circular section there may be no expected alignment which begs questions of whether modes have predominant directions, and how they reflect variation of structural stiffness and mass. Few full-scale tests on supposedly axisymmetric structures have been reported, the closest analog to the studied lighthouse being industrial chimneys. Studies rarely report full-height mode shapes or their alignment, although some papers e.g. [9] do report very close modal frequencies.

The BAYOMA mode shape alignment variability for Wolf Rock is presented in Figure 4. The MPV of mode shapes are illustrated as vectors originating from the vertical axis through the lighthouse. Being vector-valued the dominant uncertainty of each mode shape is obtained from the eigenvector (maximum eigenvalue) of its posterior covariance matrix multiplied by the square root of the eigenvalue. This ±1σ uncertainty is shown as dashed lines that are roughly perpendicular to the vector tip with thicker lines for second mode of the pair and estimates from both sets are shown. Set 2 duration was approximately one third of set 1 duration and this is reflected in the larger variance for the first mode pair. While mode pairs are self-orthogonal, directions of the three mode pairs do not correlate.

The first (slightly lower frequency) mode of the pair seems to align with brickwork landing. FEM mode alignment is rotated a little anticlockwise for the first pair but is inconsistent with that for the second pair. The FEM mode frequencies are a good match with the experimental modes and a curious result is that the first mode frequency of the helideck alone is remarkably close to the frequency of the additional mode at ~5 Hz identified by both BAYOMA (Figure 3).

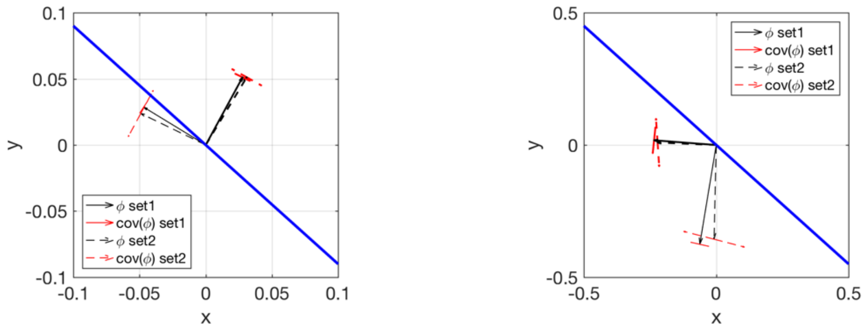


Figure 4: Wolf Rock first (left columns) and second (right columns) mode pairs at Level 8, normalised to unit at helideck. Blue lines are in direction of landing platform and entrance.

4.3. Mode stationarity

Non-stationarity of MPs has already been mentioned. Various factors such as wind speed, temperature and internal live loading can affect structure mass, stiffness and energy dissipation. Modal parameter variation and correlation with such factors is sometimes very clear [11][12] but it can sometimes be challenging to identify beneath the uncertainty in the modal parameter estimation. This is a particularly pertinent problem for the significant bodies of research concerned with diagnosing structural condition via MP changes [12] and inverse identification (of loading and/or structure) [13].

Following a maintenance visit to Wolf Rock in 2019 extended recordings of multiple accelerometer array are available. Estimates shown in Figure 5 are from 5-minute segments; for frequencies as error bars representing ±1σ identification uncertainty about MPVs and for mode shapes as circles with diameter proportional to uncertainty. Using the same frequency range among the plots highlights the differing frequency ‘closeness’ as well as the inherent MP variation. The segment length is a trade-off between controlling identification uncertainty (the longer the better) and modelling error arising from potential MP variation between segments (the shorter the better). The mode shape uncertainty and variation indicate a clear link with closeness of frequencies

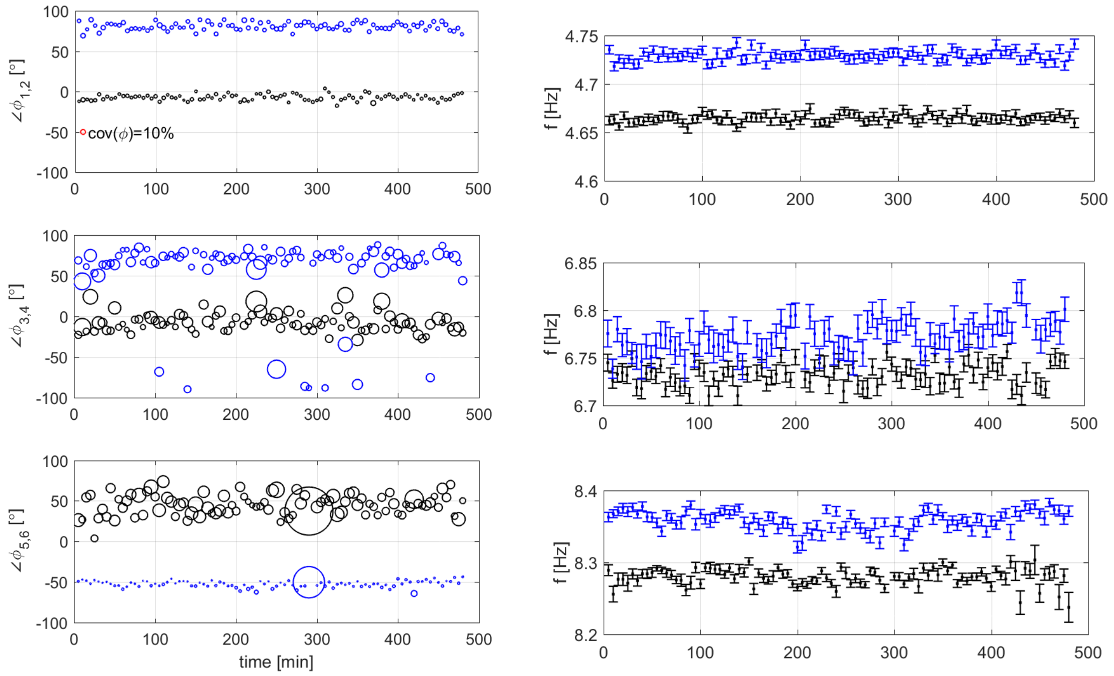


Figure 5: Mode pair mode shape (left) and frequency (right) variation for overnight recording at Wolf Rock Lighthouse.

5. CONCLUSIONS

The dynamic properties of this set of iconic rock lighthouses are important to study for the purpose of vulnerability assessment but the results are useful for studying the problem of distinguishing and aligning very close modes. Mysteries remain about the structural origins of some of the modes but the outcomes are very useful for further development of BAYOMA.

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