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# An adaptive approach to longitudinal platooning with heterogeneous vehicle saturations

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**Abstract:** Adaptive CACC strategies have been recently proposed to stabilize a platoon with non-identical and uncertain vehicle dynamics (heterogeneous platoon). This work proposes a method to augment such strategies with a mechanism coping with saturation constraints (i.e. engine constraints). In fact, in a platoon of heterogeneous vehicles, engine constraints might lead to loss of cohesiveness. The proposed mechanism is based on making the reference dynamics (i.e. the dynamics to which the platoon should homogenize) ‘not too demanding’, by applying a properly designed saturation action. Such saturation action will allow all vehicles in the platoon not to hit their engine bounds. Cohesiveness will then be achieved at the price of losing some performance, which is in line with the state of art studies on this topic. Simulations on a platoon of 5 vehicles are conducted to validate the theoretical analysis.

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*Keywords:* Cooperative adaptive cruise control, engine constraints, heterogeneous platoon, model reference adaptive control.

## 1. INTRODUCTION

Cooperative Adaptive Cruise Control (CACC), a.k.a. platooning, is a way of grouping individual vehicles into platoons with a defined inter-vehicle spacing policy by using inter-vehicle communication in addition to on-board sensors Günther et al. (2016); Flores and Milanés (2018). Originally, platooning was studied in the ideal setting that all vehicles have the same dynamics (homogeneous platoon) Ploeg et al. (2014); Hafez et al. (2015). However, it was soon recognized that an effective CACC strategy should be able to cope with a substantial level of heterogeneity, either in the vehicle dynamics or in the environment. Platooning under various forms of heterogeneous constraints have been thus the object of many studies. Popular constraints are probably networked-induced constraints coming from wireless communication Acciani et al. (2018); Harfouch et al. (2018); Ploeg et al. (2015); Lei et al. (2011); Santini et al. (2015); Montanaro et al. (2014). Additionally, the performance of a platoon of nonidentical (heterogeneous) vehicles can be severely limited by saturating engines. A pioneering work considering the fundamental limitations and tradeoffs in the control of vehicular platoons was probably Warnick and Rodriguez (1994), which applies a systematic design procedure for addressing multiple saturating nonlinearities in platoons of vehicles. With similar intentions, Jovanovic et al. (2004) studied the fundamental limitations of the platooning problem, also with particular emphasis on saturation. Both works come to similar conclusions: the key idea in the design methodology is to modify an existing platoon controller with a supervisory logic that acts only when necessary to prevent saturation. The common result of these studies is that instability effects due to saturation can be systematically eliminated only at the price of losing performance.

Recently, the problem of engine saturation is emerging in platooning applications. This is mainly due to the fact that, because platooning has to be implemented over heterogeneous vehicles, some vehicles may struggle in maintaining a cohesive platoon. One can think about e.g. a family car trying to maintain cohesiveness in a platoon composed of many sport cars. Motivated by settings such as this one, a fruitful line of research has been conducted. Performance analysis of a team of unmanned (single integrator) vehicles that are subject to actuator faults is investigated in Semsar-Kazerouni and Khorasani (2010). Consensus control for homogeneous platoons with velocity constraints was the subject of Zegers et al. (2018). A low-gain control algorithm is designed in Gao et al. (2017) to accommodate the requirement of the input saturation. In Guo et al. (2018) a neural network-based distributed adaptive approach combined with sliding mode technique is proposed for vehicle-following platoons in the presence of input saturation. Of these works, only the last one addresses how to cope with uncertainties in vehicle dynamics, which require the controller to adapt their control action.

Recently, a CACC strategy was proposed that overcomes the homogeneity assumption and that is able to adapt its action and achieve string stability even with uncertain heterogeneous platoons with unknown engine performance losses Harfouch et al. (2017). The main idea of Harfouch et al. (2017) was that CACC can be formulated as a model reference control problem, where the leading vehicle plays the role of some reference dynamics that all other vehicle should try to match. This led, in further works by some of the authors, to a set of distributed matching conditions Baldi and Frasca (2018) that define the gains that each controller should have in order to match the

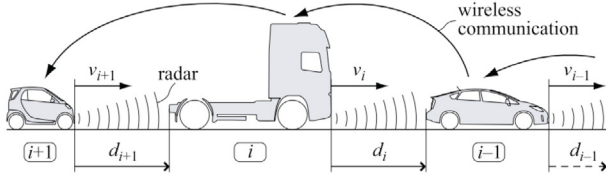


Fig. 1. CACC-equipped heterogeneous vehicle platoon (Ploeg et al. (2014))

reference dynamics. When the vehicle dynamics are uncertain, such matching gains can be learned via appropriate adaptive laws Baldi et al. (2018). Despite the effectiveness of this distributed model reference adaptation framework, the problem of input saturation remains not addressed. This work is meant to enhance the distributed adaptive framework in such direction. The main idea of this work is the following: inside platoons with input constraints, all vehicles should adapt to a reference model that is not ‘too demanding’. The reference model is made not ‘too demanding’ by appropriately saturating its control action. Therefore, the new perspective of this work is that saturation (on the reference vehicle) can have a positive effect on the cohesiveness of the platoon. This is clearly in line with the studies Warnick and Rodriguez (1994); Jovanovic et al. (2004), i.e. saturation can be systematically eliminated only at the price of losing performance.

The paper is organized as follows. In Section 2, the structure of a CACC-equipped platoon is presented. The proposed adaptive law to stabilize the platoon in the saturated scenario is studied in Section 3. Simulation results for the proposed strategy are presented in Section 4 along with some concluding remarks in Section 5.

## 2. CACC SYSTEM STRUCTURE

Consider a heterogeneous platoon with  $M$  vehicles. Fig. 1 shows the platoon where  $v_i$  and  $d_i$  represent the velocity (m/s) of vehicle  $i$ , and the distance (m) between vehicle  $i$  and its preceding vehicle  $i - 1$ , respectively. Furthermore, each vehicle in the platoon can only communicate with its preceding vehicle via wireless communication. The main goal of every vehicle in the platoon, except the leading vehicle, is to maintain a desired distance  $d_{r,i}$  between itself and its preceding vehicle. Consistently with most CACC literature, we will consider a one-vehicle look-ahead topology Ploeg et al. (2014). Extension to multiple-vehicle look-ahead topologies is in principle possible using the tools of Baldi and Frasca (2018).

A constant time headway (CTH) spacing policy is adopted to regulate the spacing between the vehicles, implemented by defining  $d_{r,i}$  as:

$$d_{r,i}(t) = r_i + hv_i(t), \quad i \in S_M$$

where  $r_i$  is the standstill distance (m),  $h$  the time headway (s), and  $S_M = \{i \in \mathbb{N} \mid 1 \leq i \leq M\}$  with  $i = 0$  reserved for the platoon’s leader (leading vehicle). It is now possible to define the spacing error (m) of the  $i^{th}$  vehicle

$$e_i(t) = d_i(t) - d_{r,i}(t) \quad (1)$$

$$= (q_{i-1}(t) - q_i(t) - L_i) - (r_i + hv_i(t)) \quad (2)$$

with  $q_i$  and  $L_i$  representing vehicle  $i$ ’s rear-bumper position (m) and length (m), respectively.

The control objective is to regulate  $e_i$  to zero for all  $i \in S_M$ . The following model, derived by Ploeg et al. (2014), is used to represent the vehicles in the platoon

$$\begin{pmatrix} \dot{d}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} v_{i-1} - v_i \\ a_i \\ -\frac{1}{\tau_i}a_i + \frac{1}{\tau_i}u_i \end{pmatrix}, \quad i \in S_M \quad (3)$$

with  $a_i$  and  $u_i$  representing the acceleration (m/s<sup>2</sup>) and external input (m/s<sup>2</sup>) of the  $i^{th}$  vehicle, and  $\tau_i$  (s) representing the vehicle’s driveline time constant. For the time being, let us focus on the unsaturated case, while the saturated case will be covered in the next section.

Substituting (1) in (3) we obtain the state space system

$$\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & -h \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} v_{i-1} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{pmatrix} u_i. \quad (4)$$

At this point, we define the leading vehicle’s model as

$$\begin{pmatrix} \dot{e}_0 \\ \dot{v}_0 \\ \dot{a}_0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_0} \end{pmatrix} \begin{pmatrix} e_0 \\ v_0 \\ a_0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_0} \end{pmatrix} u_0. \quad (5)$$

The leading vehicle’s model does not necessarily represent an actual vehicle, but rather it represents some desired dynamics to which all vehicles in the platoon should homogenize. Standard approaches to platooning had assumed all vehicles are already homogeneous, i.e. with the same dynamics  $\tau_0$  Ploeg et al. (2014); Hafez et al. (2015). Removing the homogeneous assumption implies considering that  $\forall i \in S_M$ ,  $\tau_i$  can be represented as the sum of

$$\tau_i = \tau_0 + \Delta\tau_i \quad (6)$$

where  $\Delta\tau_i$  is a perturbation of vehicle  $i$ ’s driveline dynamics from  $\tau_0$ . Two approaches can be used to address  $\Delta\tau_i$ : the first one is that  $\Delta\tau_i$  is perfectly known, leading to a robust control approach; the second one is that  $\Delta\tau_i$  is an unknown parameter, leading to an adaptive control approach. The main idea behind Harfouch et al. (2017) is that all vehicles can homogenize to (5) in an adaptive way.

Consequently, the model of a vehicle in a heterogeneous platoon is obtained using (6) in the third equation of (4)

$$\dot{a}_i = -\frac{1}{\tau_0}a_i + \frac{1}{\tau_0}[u_i + \Omega_i^* \phi_i], \quad (7)$$

where  $\Omega_i^* = -\frac{\Delta\tau_i}{\tau_i}$  is an unknown ideal constant scalar parameter, and  $\phi_i = (u_i - a_i)$  is the known scalar regressor. Using (7) in (4), we can define the vehicle model as the uncertain LTI of the following form

$$\begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & -h \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_0} \end{pmatrix} \begin{pmatrix} e_i \\ v_i \\ a_i \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} v_{i-1} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_0} \end{pmatrix} [u_i + \Omega_i^* \phi_i], \quad \forall i \in S_M. \quad (8)$$

## 3. ENGINE-CONSTRAINED CONTROL

Under the baseline conditions of identical vehicles ( $\Omega_i^* = \tau_0$ ,  $\forall i \in S_M$ ), Ploeg et al. (2014) derived the following CACC control

$$h\dot{u}_{i,bl} = -u_{i,bl} + \xi_{i,bl}, \quad \forall i \in \{0\} \cup S_M \quad (9)$$

$$\xi_{i,bl} = \begin{cases} K_p e_i + K_d \dot{e}_i + u_{i-1,bl}, & \forall i \in S_M \\ u_r, & i = 0, \end{cases} \quad (10)$$

where  $\xi_{i,bl}$  is an auxiliary input  $u_r$  is the platoon input representing the desired acceleration of the leading vehicle, and  $u_{i-1,bl}$  is received over the wireless communication between vehicle  $i$  and  $i-1$ .

Therefore, we can now design reference dynamics (to whose behaviour (4) and (5) should converge) as an "ideal" homogeneous platoon with  $\Omega_i^* = 0$  and  $u_i = u_{i,bl}, \forall i \in S_M$ . Substituting (9) in (8) and extending the state vector with  $u_{i,bl}$  we obtain the following reference model dynamics

$$\begin{aligned} \begin{pmatrix} \dot{e}_{i,m} \\ \dot{v}_{i,m} \\ \dot{a}_{i,m} \\ \dot{u}_{i,bl} \end{pmatrix} &= \underbrace{\begin{pmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ \frac{K_p}{h} & -\frac{K_d}{h} & -K_d & -\frac{1}{h} \end{pmatrix}}_{A_m} \underbrace{\begin{pmatrix} e_{i,m} \\ v_{i,m} \\ a_{i,m} \\ u_{i,bl} \end{pmatrix}}_{x_{i,m}} \\ &+ \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_d}{h} & \frac{1}{h} \end{pmatrix}}_{B_w} \underbrace{\begin{pmatrix} v_{i-1} \\ u_{i-1,bl} \end{pmatrix}}_{w_i}, \forall i \in S_M \end{aligned} \quad (11)$$

where  $x_{i,m}$  and  $w_i$  are vehicle  $i$ 's reference state vector and exogenous input vector, respectively. Consequently, (11) is of the following form

$$\dot{x}_{i,m} = A_m x_{i,m} + B_w w_i, \forall i \in S_M. \quad (12)$$

Furthermore, the leading vehicle model becomes

$$\begin{pmatrix} \dot{e}_0 \\ \dot{v}_0 \\ \dot{a}_0 \\ \dot{u}_0 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ 0 & 0 & 0 & -\frac{1}{h} \end{pmatrix}}_{A_r} \underbrace{\begin{pmatrix} e_0 \\ v_0 \\ a_0 \\ u_0 \end{pmatrix}}_{x_0} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{h} \end{pmatrix}}_{B_r} u_r. \quad (13)$$

The first question is how to modify (9) in the presence of uncertain perturbations as in (6): this question will be answered in Sect. 3.1. The second question is how to modify (9) and the (13) in the presence of saturation constraints: this question will be answered in Sect. 3.2.

### 3.1 Model reference dynamics

The dynamics (12) can be used as a reference model for the uncertain platoon's dynamics described by (5) and (8). With this scope in mind, we can augment the baseline controller (9) with an adaptive term

$$u_i = u_{i,bl} + u_{i,ad} \quad (14)$$

where  $u_{i,bl}$  is the baseline controller defined in (9) and  $u_{i,ad}$  the adaptive augmentation controller (to be constructed).

Replacing (14) into (5) and (8), and augmenting the state vector with  $u_{i,bl}$  results in

$$\begin{aligned} \begin{pmatrix} \dot{e}_i \\ \dot{v}_i \\ \dot{a}_i \\ \dot{u}_{i,bl} \end{pmatrix} &= \begin{pmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ \frac{K_p}{h} & -\frac{K_d}{h} & -K_d & -\frac{1}{h} \end{pmatrix} \underbrace{\begin{pmatrix} e_i \\ v_i \\ a_i \\ u_{i,bl} \end{pmatrix}}_{x_i} \\ &+ \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_d}{h} & \frac{1}{h} \end{pmatrix} \begin{pmatrix} v_{i-1} \\ u_{i-1} \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_0} \\ 0 \end{pmatrix}}_{B_u} [u_{i,ad} + \Omega_i^* \phi_i], \forall i \in S_M \end{aligned} \quad (15)$$

which can be written in the following form:

$$\dot{x}_i = A_m x_i + B_w w_i + B_u [u_{i,ad} + \Omega_i^* \phi_i]. \quad (16)$$

With the leading vehicle's model as in (13), the adaptive augmentation controller can be designed to compensate for the unknown term  $\Omega_i^* \phi_i$

$$u_{i,ad} = -\hat{\Omega}_i \phi_i \quad (17)$$

where  $\hat{\Omega}_i$  is the estimate of  $\Omega_i^*$ . Replacing (17) in (16) gives

$$\dot{x}_i = A_m x_i + B_w w_i - B_u \tilde{\Omega}_i^T \phi_i \quad (18)$$

where  $\tilde{\Omega}_i = \hat{\Omega}_i - \Omega_i^*$  is the parameter estimation's error vector. Defining the state tracking error as

$$\tilde{x}_i = x_i - x_{i,m} \quad (19)$$

we obtain the following dynamics

$$\dot{\tilde{x}}_i = A_m \tilde{x}_i + B_u \tilde{\Omega}_i^T \phi_i \quad (20)$$

*Remark 1.* Using the model reference adaptive tools in Harfouch et al. (2017), each vehicle can implement an adaptive law to drive  $\tilde{x}_i$  to zero, thus converging to the behavior of the nominal vehicle (the dynamics of the nominal vehicle represent the reference model). It is important to notice that each vehicle can calculate  $\tilde{x}_i$  by implementing a copy of the nominal vehicle. In the following, we want to show how such reference model can be modified in order to handle saturation constraints.

### 3.2 Saturated case

Let us design a stable reference model as the model of a nominal vehicle with appropriately designed saturation: in other words, we assume that each vehicle implements a copy of the reference model according to the following lines.

First, let us define  $\xi_{i,m} = K_p e_i + K_d \dot{e}_i + u_{i-1,m}$ . Then

$$h \dot{u}_{i,m} = \begin{cases} 0 & \text{if } u_{i,m} = u_{min,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} \geq 0 \\ -u_{i,m} + \xi_{i,m} & \text{if } u_{min,m} < u_{i,m} < u_{max,m} \\ & \text{or } u_{i,m} = u_{max,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} < 0 \\ & \text{or } u_{i,m} = u_{min,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} > 0 \\ 0 & \text{if } u_{i,m} = u_{min,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} \leq 0 \end{cases} \quad (21)$$

where  $u_{min,m}$  and  $u_{max,m}$  are the saturation levels to be designed. Such saturation levels guarantee that the reference model is not too demanding, in the sense that the vehicles will not hit their saturation bounds. It has to be noticed that (27) will provide an anti-windup action: in fact,  $\dot{u}_{i,m} = 0$  whenever the saturation bounds are hit. That is,  $u_{i,m}$  will stay at the saturation level, and will immediately exit the saturation whenever  $u_{i,m} = u_{max,m}$  and  $-u_{i,m} + \xi_{i,m} < 0$ , or  $u_{i,m} = u_{min,m}$  and  $-u_{i,m} + \xi_{i,m} > 0$ .

When saturation is hit, find  $\gamma$  such that  $-\gamma u_{i,m} + K_p e_i + K_d \dot{e}_i + u_{i-1,m} = 0$ . This leads to the saturated dynamics

$$\begin{pmatrix} \dot{e}_{i,m} \\ \dot{v}_{i,m} \\ \dot{a}_{i,m} \\ \dot{u}_{i,bl} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{\tau_0} & \frac{1}{\tau_0} \\ \frac{K_p}{h} & -\frac{K_d}{h} & -K_d & -\frac{\gamma}{h} \end{pmatrix}}_{A_m^\gamma} \underbrace{\begin{pmatrix} e_{i,m} \\ v_{i,m} \\ a_{i,m} \\ u_{i,bl} \end{pmatrix}}_{x_{i,m}} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_d}{h} & \frac{1}{h} \end{pmatrix}}_{B_w} \underbrace{\begin{pmatrix} v_{i-1} \\ u_{i-1,bl} \end{pmatrix}}_{w_i}, \forall i \in S_M \quad (22)$$

Let us now to design  $u_{min,m}$  and  $u_{max,m}$ . We can prove that  $u_{ad,i} \in [\bar{\Omega}(u_{i,min} - u_{i,max}), \bar{\Omega}(u_{i,max} - u_{i,min})]$ , where  $\bar{\Omega} = \max(|\Omega_{i,min}|, |\Omega_{i,max}|)$ , with  $\Omega_{i,min}$  and  $\Omega_{i,max}$  the minimum and maximum bounds on  $-\Delta\tau_i/\tau_i$ , and  $u_{i,min}$  and  $u_{i,max}$  the actual saturation levels of vehicle  $i$ . We used the fact that  $\phi_i = sat(u_i) - a_i$  belongs to  $[u_{i,min} - u_{i,max}, u_{i,max} - u_{i,min}]$  by exploiting the properties of a first order system with input  $sat(u_i)$  and output  $a_i$ . After establishing these bounds, we can say

$$\begin{aligned} u_{min,m} + \bar{\Omega}(u_{i,min} - u_{i,max}) &< u_i \\ &< u_{max,m} + \bar{\Omega}(u_{i,min} - u_{i,max}) \end{aligned} \quad (23)$$

where the result in Harfouch et al. (2017) that  $u_{i,bl}$  will converge to  $u_{i,m}$  has been used. From (23), one can design  $u_{min,m}$  and  $u_{max,m}$

$$u_{min,m} \geq u_{i,min} - \bar{\Omega}(u_{i,min} - u_{i,max}) \quad (24)$$

$$u_{max,m} \leq u_{i,max} + \bar{\Omega}(u_{i,max} - u_{i,min}) \quad (25)$$

*Remark 2.* The bounds (24) and (25) are such that the saturation bounds of the vehicles will not be hit. This implies that the nominal vehicle cannot express its full potentialities, which is in line with the studies Warnick and Rodriguez (1994); Jovanovic et al. (2004), i.e. saturation can be systematically eliminated only at the price of losing performance. Note that, in order to find  $\bar{\Omega}$  one must find some find bounds to the uncertainty  $-\Delta\tau_i/\tau_i$ : the more the heterogeneity of the vehicle, the more conservative the bounds. If the platoon would be completely homogeneous, (24) and (25) would become  $u_{min,m} \geq u_{i,min}$  and  $u_{max,m} \leq u_{i,max}$ , i.e. the saturation bounds of the reference model could be selected to be the same as the saturation of the vehicles.

The dynamics of the vehicle with saturation become

$$\dot{x}_i = A_m^\gamma x_i + B_w w_i + B_u [sat(u_{i,ad}) + \Omega^* \phi_i] \quad (26)$$

and

$$h\dot{u}_{i,bl} = \begin{cases} -\gamma u_{i,bl} + \xi_{i,bl} & \text{if } u_{i,m} = u_{min,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} \geq 0 \\ -u_{i,bl} + \xi_{i,bl} & \text{if } u_{min,m} < u_{i,m} < u_{max,m} \\ & \text{or } u_{i,m} = u_{max,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} < 0 \\ & \text{or } u_{i,m} = u_{min,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} > 0 \\ -\gamma u_{i,bl} + \xi_{i,bl} & \text{if } u_{i,m} = u_{min,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} \leq 0 \end{cases} \quad (27)$$

The last equation implies that  $u_{i,bl}$  follows a similar law as  $u_{i,m}$ : furthermore, when  $u_{i,bl} \rightarrow u_{i,m}$  the two inputs will saturate synchronously. We obtain the dynamics

$$\dot{x}_i = \begin{cases} A_m^\gamma \tilde{x}_i + B_u \tilde{\Omega}_i \phi_i & \text{if } u_{i,m} = u_{min,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} \geq 0 \\ A_m \tilde{x}_i + B_u \tilde{\Omega}_i \phi_i & \text{if } u_{min,m} < u_{i,m} < u_{max,m} \\ & \text{or } u_{i,m} = u_{max,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} < 0 \\ & \text{or } u_{i,m} = u_{min,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} > 0 \\ A_m^\gamma \tilde{x}_i + B_u \tilde{\Omega}_i \phi_i & \text{if } u_{i,m} = u_{min,m} \text{ and} \\ & -u_{i,m} + \xi_{i,m} \leq 0 \end{cases} \quad (28)$$

and the adaptive law (17) and

$$\dot{\tilde{\Omega}}_i = \Gamma_\Omega \phi_i \tilde{x}_i P_m B_u \quad (29)$$

with  $P_m$  a common symmetric positive-definite matrix satisfying

$$A_m^T P_m + P_m A_m < -Q_m \quad (30)$$

$$A_m^{\gamma T} P_m + P_m A_m^\gamma < -Q_m \quad (31)$$

with  $Q_m = Q_m^T > 0$  a design matrix. Stability cannot be studied here due to space limitations (we aim to address this point in an extended version of the work). Let us show the effectiveness of the approach via simulations.

*Remark 3.* From (30) and (31) it can be seen that stability relies on a common Lyapunov function between  $A_m$  and  $A_m^\gamma$  (i.e. between the unsaturated and saturated dynamics). A common Lyapunov function allows arbitrary switching among such dynamics, but also implies that  $A_m^\gamma$  (which can be eventually time-varying) should be close enough to  $A_m$  for  $P_m$  to exist. This implies that the unsaturated input in (27) should not be too far from the saturation bound. Using similar ideas as in Harfouch et al. (2017), one might look for multiple Lyapunov functions for the different regimes, resulting in average dwell time constraints when switching from the saturated to the unsaturated dynamics.

*Remark 4.* The bounds in (23) are necessarily conservative for two reasons: they are based on the worst-case uncertainty for  $\Omega_i$ ; they are based in the worst-case excursion for  $\phi_i = sat(u_i) - a_i$ . To decrease conservativeness, an efficiency factor can be added to (23). In simulations, we verified that an efficiency factor of  $2 \sim 3$  reduces conservativeness while still respecting all saturation bounds.

#### 4. ILLUSTRATIVE EXAMPLE

Consider a platoon of 5+1 vehicles, with the first one being the platoon leader, vehicle 0. Table 1 presents the platoon's characteristics. Table 1 also shows the true values of the

Table 1. Platoon parameters,  $M=5$ ,  $h=0.7s$

$i$	0	1	2	3	4	5
$\tau_i(s)$	0.6	0.5	0.7	0.45	0.7	0.8
$u_{min,i}$	-0.83	-1.5	-2.5	-1.0	-2.0	-2.5
$u_{max,i}$	0.83	1.5	2.5	1.0	2.0	2.5
$\Omega_i^*$		0.2	-0.143	0.333	-0.143	-0.25

constant parametric uncertainties  $\Omega_i^*$ ,  $\forall i \in S_M$ , which are

unknown to the designer. However, we assume to know the upper and lower bound of  $\Omega_i^*$ , that can be used to design  $u_{min,m}$  and  $u_{max,m}$ . Specifically, we have  $\Omega = 0.333$  and the worst case saturation bounds are  $u_{min,m} = -1 + 0.333 * 2 = -0.333$  and  $u_{max,m} = 1 - 0.333 * 2 = 0.333$ . After including an efficiency factor of 2.5 as explained in Remark 4, we obtain the bounds  $-0.83$  and  $0.83$ . The reference model (12) for the adaptive laws is characterized by  $K_p = 0.2$  and  $K_d = 0.7$ . The adaptive input (29) is designed using (30) with  $Q_m = 5I$  and  $\Gamma_\Omega = 80$ .

We run simulations in which the leading vehicle has a hard acceleration phase (with stop-and-go phase), followed by a deceleration phase, cf. Fig. 3. This is supposed to test how the platoon can keep cohesiveness during such acceleration and deceleration. Three scenarios are considered:

- No saturation with standard control;
- Saturation with standard control;
- Saturation with proposed control.

Fig. 2 shown the velocity response in case no saturation is present and the standard adaptive control of Harfouch et al. (2017) is adopted. It can be seen that all vehicles properly follow the velocity of the leader, which implies that platoon cohesiveness is attained.

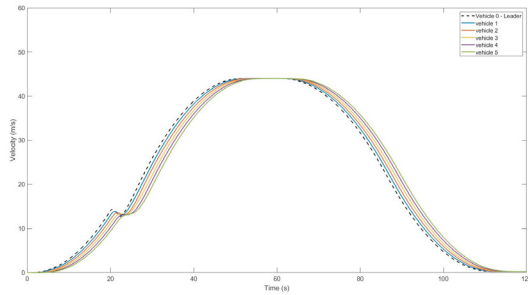


Fig. 2. No saturation with standard control: velocity response.

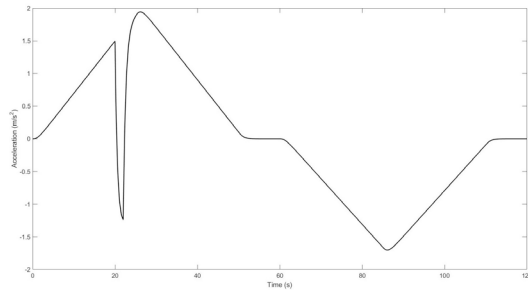


Fig. 3. No saturation with standard control: unconstrained leader acceleration.

In the simulation of Fig. 4, we add saturation, but we still keep the same control action. It is evident that vehicle 3 (which has very harsh saturation bounds, cf. Table 2) is incapable of following the follower speed, which implies that the platoon is not cohesive anymore. Vehicles 4 and 5 will clearly follow vehicle 3 which lost cohesiveness. The triangular shape of the velocity of vehicle 3 is the typical shape arising from the so-called wind-up phenomenon, highlighted in Fig. 5. Note that, even though vehicle 3 brakes at around time 68 seconds, its braking possibilities are also constrained: therefore, vehicle 3 will eventually

collide at around 80 seconds with vehicle 2, as it can be seen from the distance plot in Fig. 5.

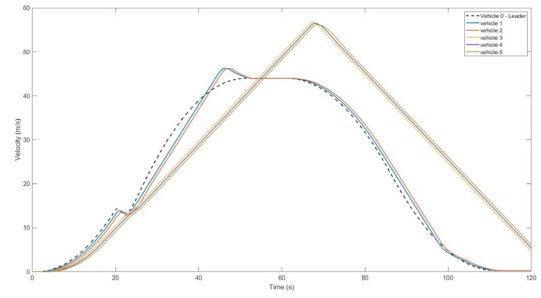


Fig. 4. Saturation with standard control: velocity response.

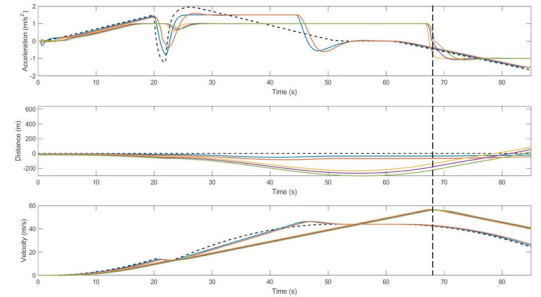


Fig. 5. Saturation with standard control: wind up and loss of cohesiveness

Finally, in the simulation of Fig. 6, we apply the proposed control action. It can be seen that this time all vehicles will maintain cohesiveness. Because of the saturation limits, cohesiveness is achieved at the price of reducing performance (the leading vehicle reaches a maximum speed of 30 m/s instead of 44 m/s): this is due to the fact that the reference model will apply saturation in order to result not too demanding for vehicles that might lose cohesiveness. This can be clearly seen from Fig. 7 where, as compared to Fig. 3 the high acceleration and deceleration peaks are chopped thanks to the saturation applied to the leading vehicle. The constrained inputs, within the expected limits for all vehicles, can be seen in Fig. 8.

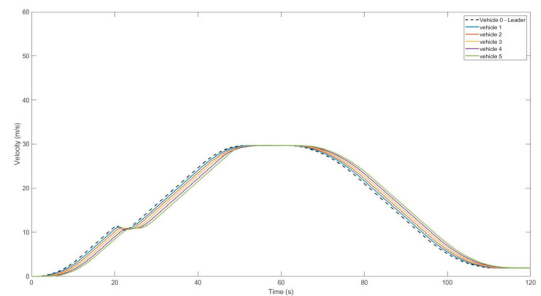


Fig. 6. Saturation with proposed control: velocity response.

## 5. CONCLUSIONS

In this work we have augmented adaptive CACC strategies with a mechanism to cope with saturation constraints. The mechanism is based on making the reference dynamics ‘not too demanding’, by applying a properly designed saturation. Such saturation will allow all vehicles in the platoon

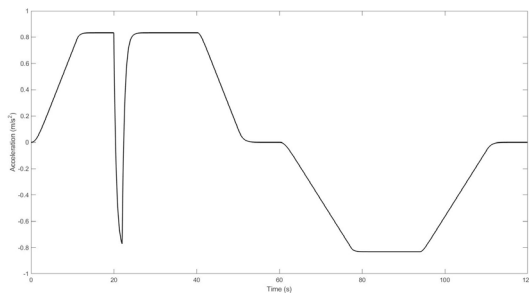


Fig. 7. Saturation with proposed control: constrained leader acceleration.

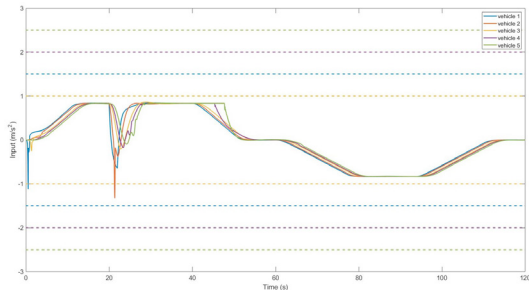


Fig. 8. Saturation with standard control: input response

not to hit their engine bounds, at the price of losing some performance. The results of this strategy are in line with the studies Warnick and Rodriguez (1994); Jovanovic et al. (2004), i.e. saturation can be systematically eliminated only at the price of losing performance.

In this work we have assumed the saturation bounds to be known. It would be relevant to study the case in which the saturation bounds can be learned on line, and thus  $u_{i,min}$  and  $u_{max,m}$  can be selected in an adaptive way. Furthermore,  $\tau_0$  is also assumed to be known. It would be relevant to learn in an adaptive way the best  $\tau_0$  that might lead to the best performance of the platoon.

## REFERENCES

- Acciani, F., Frasca, P., Stoorvogel, A., Semsar-Kazerooni, E., and Heijenk, G. (2018). Cooperative adaptive cruise control over unreliable networks: an observer-based approach to increase robustness to packet loss. In *2018 European Control Conference (ECC)*, 1399–1404.
- Baldi, S., Yuan, S., and Frasca, P. (2018). Output synchronization of unknown heterogeneous agents via distributed model reference adaptation. *IEEE Transactions on Control of Network Systems*. doi: 10.1109/TCNS.2018.2844260.
- Baldi, S. and Frasca, P. (2018). Adaptive synchronization of unknown heterogeneous agents: An adaptive virtual model reference approach. *Journal of the Franklin Institute*. doi: https://doi.org/10.1016/j.jfranklin.2018.01.022.
- Flores, C. and Milanés, V. (2018). Fractional-order-based acc/cacc algorithm for improving string stability. *Transportation Research Part C: Emerging Technologies*, 95, 381 – 393.
- Gao, W., Rios-Gutierrez, F., Tong, W., and Chen, L. (2017). Cooperative adaptive cruise control of connected and autonomous vehicles subject to input saturation. In *2017 IEEE 8th Annual Ubiquitous Computing, Electronics and Mobile Communication Conference (UEMCON)*, 418–423.
- Günther, H.J., Kleinau, S., Trauer, O., and Wolf, L. (2016). Platooning at traffic lights. *IEEE Intelligent Vehicles Symposium (IV)*, 1047–1053.
- Guo, X., Wang, J., Liao, F., and Teo, R.S.H. (2018). Cnn-based distributed adaptive control for vehicle-following platoon with input saturation. *IEEE Transactions on Intelligent Transportation Systems*, 19(10), 3121–3132.
- Hafez, M., Ariffin, M., Rahman, M.A., and Zamzuri, H. (2015). Effect of leader information broadcasted throughout vehicle platoon in a constant spacing policy. *IEEE International Symposium on Robotics and Intelligent Sensors*, 132–137.
- Harfouch, Y.A., Yuan, S., and Baldi, S. (2018). An adaptive switched control approach to heterogeneous platooning with intervehicle communication losses. *IEEE Transactions on Control of Network Systems*, 5(3), 1434–1444.
- Harfouch, Y.A., Yuan, S., and Baldi, S. (2017). An adaptive approach to cooperative longitudinal platooning of heterogeneous vehicles with communication losses. *20th IFAC World Congress*, 1352 – 1357.
- Jovanovic, M.R., Fowler, J.M., Bamieh, B., and D’Andrea, R. (2004). On avoiding saturation in the control of vehicular platoons. In *Proceedings of the 2004 American Control Conference*, volume 3, 2257–2262 vol.3.
- Lei, C., van Eenennaam, E.M., Wolterink, W.K., Karagiannis, G., Heijenk, G., and Ploeg, J. (2011). Impact of packet loss on CACC string stability performance. *IEEE 11th International Conference on ITS Telecommunications*, 381–386.
- Montanaro, U., M. Tufo, G.F., di Bernardo, M., Salvi, A., and Santini, S. (2014). Extended cooperative adaptive cruise control. *IEEE Intelligent Vehicles Symposium (IV)*, 605–610.
- Ploeg, J., Semsar-Kazerooni, E., Lijster, G., van de Wouw, N., and Nijmeijer, H. (2015). Graceful degradation of cooperative adaptive cruise control. *IEEE Transactions on Intelligent Transportation Systems*, 16(1), 488–497.
- Ploeg, J., van de Wouw, N., and Nijmeijer, H. (2014).  $l_p$  string stability of cascaded systems: Application to vehicle platooning. *IEEE Transactions on Control Systems Technology*, 22, 786–793.
- Santini, S., Salvi, A., Valente, A.S., Pescape, A., Segatayz, M., and Cignoz, R.L. (2015). A consensus-based approach for platooning with inter-vehicular communications. *IEEE Conference on Computer Communications*, 1158–1166.
- Semsar-Kazerooni, E. and Khorasani, K. (2010). Team consensus for a network of unmanned vehicles in presence of actuator faults. *IEEE Transactions on Control Systems Technology*, 18(5), 1155–1161.
- Warnick, S.C. and Rodriguez, A.A. (1994). Longitudinal control of a platoon of vehicles with multiple saturating nonlinearities. In *Proceedings of 1994 American Control Conference - ACC '94*, volume 1, 403–407 vol.1.
- Zegers, J.C., Semsar-Kazerooni, E., Ploeg, J., van de Wouw, N., and Nijmeijer, H. (2018). Consensus control for vehicular platooning with velocity constraints. *IEEE Transactions on Control Systems Technology*, 26(5), 1592–1605.