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Modelling growth principles of metropolitan public transport networks

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ABSTRACT

The development of metropolitan public transport networks often involves choosing between investing in extending radial lines or constructing ring connections. While the former enlarges network coverage the latter enhances network connectivity and reduces the need to perform detours. Moreover, investments might be better directed at increasing the capacity of already existing infrastructure. In this study we address the following question: how do transport networks in metropolitan areas evolve over time and how can we effectively model this growth as function of demand and cost function? The goal of this study is to determine the fundamental relations between population distribution, modal costs on the prevailing network structure and its evolution. The approach taken in this study offers a theoretical contribution to the field of transport network growth by combining principles from several research streams: transport geography, economics of network growth and network science. We propose an iterative investment model network analysis framework. The results of the network growth experiments manifest an overall trend in network growth with an early phase of expansion of the network, followed by a period of intensification manifested in capacity increments and finally adding some links that contribute to its densification. Furthermore, our findings suggest that bus networks include more ring-radial connections than Light Rail Train and Metro networks which are more concentrated on radial connections.

1. Introduction

The evolution of transport networks is determined by an interaction between changes in technology, society and the economy. A variety of network structures can be observed among transport networks worldwide (Zhang et al., 2015). Notwithstanding, little is known on the underlying principles that yield the prevailing network structures under different circumstances. Metropolitan public transport networks are currently undergoing exceptionally significant expansions. Development decisions often involve choosing between investing in extending radial lines. These decisions reflect a choice between enlarging the network coverage, or constructing ring connections to enhance network connectivity thereby reducing the need to perform detours. The following question thus arises: how do transport networks in metropolitan areas evolve over time and how can we effectively model this growth as function of demand and cost function? This question is addressed in this study by means of an iterative network growth model which allows investigating the fundamental relations between population distribution, modal costs and the prevailing network structure.

The modelling of transportation network growth and its history has been thoroughly reviewed by Xie and Levinson (2009). In their review they show progress within the field of modelling and analysing growth of transportation networks. According to Xie and Levinson (2009) the research studies into growing networks have followed five main streams: Transport geography, Optimisation and network design, Empirical models for network growth, Economics of network growth and Network science. Each of these streams covers a certain aspect of growing networks from a certain scientific background. In the years after this review by Xie and Levinson (2009) there have been new studies related to principles behind network growth. Louf et al. (2013) studied the emergence of hierarchical structures in cost driven growth models for spatial models. Growing a tree shaped network based on costs within a random distributed node space resulted in specific cost ratio conditions for hierarchical (hub and spoke) structures to appear. These studies are only completed for tree shaped random node networks. Consequently, they do not provide insight in networks where either hierarchy is not a factor or where the points are not randomly distributed over the space, like the metropolitan networks studied in this paper. Schultz et al. (2014) propose a growth model that describes how spatially embedded infrastructure networks might emerge. Their model consists of an initialisation phase, where the network grows in a tree shape based on cost minimisation, and then a growth phase where a trade-off exists between link redundancy and cost optimisation. The recent work of Saidi et al. (2017) mention the severe lack of models for

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ring-radial structures. The effects of transport modality and demand functions has only been researched in comparative scenarios or in research that assumes a fixed network not for growing (evolving) networks. Empirical findings concerning network evolution are scarce. Cats (2017) performed a longitudinal topological analysis of the evolution of the multi-modal Stockholm network and found that network development shifted from peripheral attachment to preferential attachment.

The process in which networks evolve over time is an important aspect in the growing interest in network science. Notwithstanding, according to Dupuy (2013), graph theory-based studies often result in a static representation of the network, hindering the analysis of network evolution. Furthermore, Ducruet and Beauguitte (2014) concluded that research concerning the evolution and dynamics of networks using network science concepts and methods has remained surprisingly unexplored as most studies adopt a static approach. We directly fill this gap in this study.

The efficiency of radial urban transport networks (mostly metro lines or urban rail networks) as well as their design have significant consequences for the prevailing network structure. Laporte et al. (1997) uses a radial urban network model with distinct core and an outer circular area to measure the effectiveness of various network topological shapes. Wang and Yang (2010) discuss the benefits of a ring-radial structure from six Chinese cities showing that networks with "ring plus radials" can decrease the mean topological distance and better reflect operation efficiency of the whole network. Vuchic (2002) classifies transit lines and discusses the advantages and disadvantages of radial and ring lines. Radial lines address the heaviest demands from suburban zones towards the core area or Central Business District where diametrical lines connect two (balanced) suburban zones passing this core/CBD zone. Ring lines are then connecting the radial lines creating opportunity for transfers and integrating the network by allowing shortcuts and robustness. Vuchic mentions the important functions of ring lines: Improving connectivity among radial lines, making trips between radial lines shorter and serving the busy areas that usually develop in a ring around the core/CBD area.

The design of radio-centric networks has been studied by Vaughan (1986) and Chen et al. (2015) who found the optimal spacing of radial and ring lines. Similarly, research by Tirachini et al. (2010) and Badia et al. (2014) use polar coordinate systems for optimising radial transit corridors. Saidi et al. (2016, 2017) developed a model that takes various cost effects into account and computes generalised transit passenger cost for a given network. However, the model relies on a detailed and static evaluation of the network. The model can thus only be used to compare a limited number of scenarios on the generalised passenger costs and does not provide any insight into network growth patterns.

In designing the backbone public transport network, development decisions often involve choosing between alternative high-capacity modalities such as Bus Rapid Transit (BRT), Light Rail Transit (LRT) and Metro. Bruun (2005) compared operating costs for BRT and LRT and found the conditions in which LRT dominates BRT. This usually stems from the difference in marginal costs for adding/removing capacity in peak hours. Tirachini et al. (2010) developed a framework to compare alternatives for a radio-centric urban network with radial lines from the borders to a central business district attempting to minimise total costs, similar to Boyd et al. (1978). Their results provide insight into the characteristics of each of the modes competing in the radial network structure. Badia et al. (2014) study the formation of an ideal fully radial network based on the modal technology used and assuming a uniform demand. Those results provide insight into the mechanics and relationships between cost functions and physical characteristics of a mode (such as stop spacing) on the network topology.

The goal of this study is to determine the influence that various demand distributions and operational cost functions have on the evolution and emerging topology of a monocentric urban transport network evaluated using network indicators. To the best of our knowledge, none of the previous studies has analysed the effects of transport modality and demand distribution on network growth patterns and the resulting network structure. To this end, an investment model network analysis framework is developed and applied to an idealized radio-centric network considered as a simplification of metropolitan urban public transport networks that exist in real life. The main contributions pertain to the examination of prevailing network structures for alternative modal supply properties and the relation between the underlying growth process and the evolution of key topological characteristics. As such, the contribution of this paper is theoretical rather than empirical. In relation to the abovementioned research streams identified by Xie and Levinson (2009), our research offers a combination of principles from three of the five streams: transport geography, economics of network growth and network science. The first two pertain to the principles used for network growth in relation to accessibility and cost-benefit analysis while the latter is used in analysing the prevailing and evolution of network properties.

The remaining of the paper is structured as follows. In the following section we detail the iterative network growth model developed in this study including the processes of generating and selecting candidate investments (Section 2). Thereafter we describe the experimental set-up employed in this study (Section 3) followed by the presentation of modelling results including the network evolutionary path as well as the final network structures (Section 4). We conclude with a discussion of the key findings and suggesting directions for future research (Section 5).

2. Methodology

2.1. Modelling framework

Fig. 1 depicts the modelling process of network growth decisions. The process starts with an initial network state and an origin-destination matrix which describes the latent travel demand. The core of the model consists of the following successive modules: (i) Input Parameters; (ii) Initialisation of the model by performing Network Generation, Trip Generation and Trip Distribution; (iii) Investment Candidate Generation; (iv) Investment Candidate Evaluation, and; (v) Scoring and Building. These steps generate a choice-set of investment alternatives, evaluate the consequences of each alternative investment and then execute the selected investment. These modules are detailed in the following sub-sections. Network growth terminates when none of the alternative investments yields a positive evaluation in the Cost Benefit Analysis decision as shown in the figure below.

2.2. Model input parameters

Users are able to input a predefined set of parameters. The use of these values will be explained in later paragraphs where respective parameters are used. Table 1 below lists all input parameters related to the grid, mode and demand properties along with a short description.

2.3. Initialisation

The initialisation for the network consists of several consecutive steps. First the grid input parameters are used to generate the underlying network. Thereafter the Trip Generation and Trip Distribution are performed to ensure all investments can be evaluated.

2.3.1. Network and trip generation

Let us consider a metropolitan urban area which can be abstracted in the form of an idealized radio-polar form. The study area can then be represented using an undirected polar-grid graph $G(N, E)$ where the set of nodes, N, corresponds to centroids of travel demand areas which may also serve as potential locations for (interchange) stations and the set of links, $E \subseteq N \times N$, representing direct connections between stations.

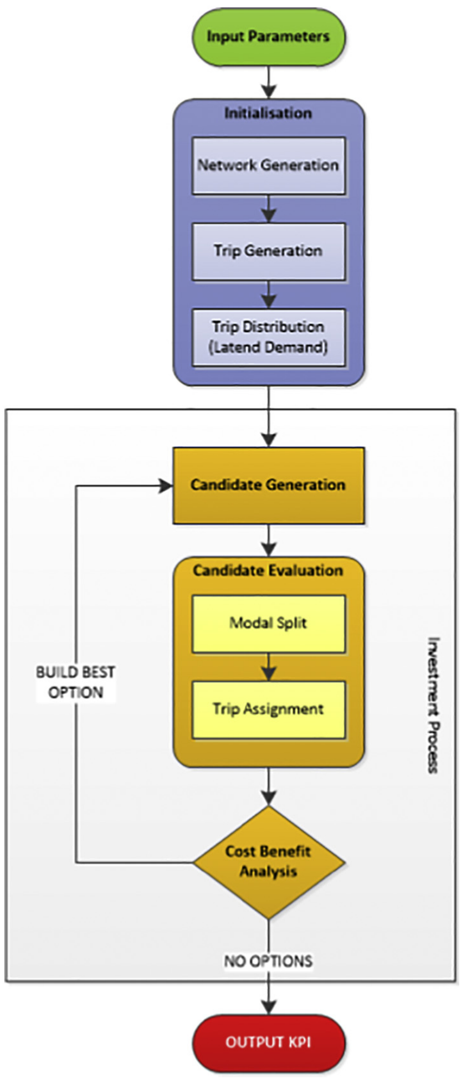

Fig. 1. Iterative network growth model workflow.

Table 1
List of model parameters.

Grid geometry parameters	
r_{max}	Maximum radius of the Urban Area
θ	Angle between radians
ψ	Number of ring lines
Mode parameters	
v^{mode}	Mode-specific operational speed per mode
β^{mode}	Mode-specific cost per km
κ^{mode}	Mode-specific capacity step
$\kappa^{max,\ mode}$	Mode-specific maximum capacity
Demand parameters	
χ	Total travel demand in the agglomeration area
ω	Value of Time

Networks are built within a pre-defined limited area containing a finite number of nodes. The area boundaries are based on the maximum radius r_{max} from the centre of the idealized monocentric agglomeration to its outer edges.

The symmetric polar-grid area is defined using the abovementioned set of geometrical parameters. This set of parameters defines the location of the nodes $i \in N$ in the plane defined by ρ and φ so that the position in the plane is defined by the distance and angle $r_i = [0, r_{max}]\ \forall\, n$ and $\varphi_i = [0, 2\pi]\ \forall\, n$, respectively. Furthermore, each of the nodes is assigned with a travel demand x_i that is generated from and attracted to this node based on the total number of trips in the agglomeration area, χ, and drawn from the distribution function and its respective parameters.

2.3.2. Trip distribution

Trip distribution is performed using a doubly constraint gravity model:

$$x_{ij} = k_i k_j x_i x_j g(c_{ij}) \tag{1}$$

Subject to:

$$x_i = \sum_{j \in N} x_{ij} \tag{2}$$

$$x_j = \sum_{i \in N} x_{ij} \tag{3}$$

$$k_i = \frac{1}{\sum_{j \in N} k_j x_j g(c_{ij})} \tag{4}$$

$$k_j = \frac{1}{\sum_{i \in N} k_i x_i g(c_{ij})} \tag{5}$$

where x_{ij} is the travel demand from origin $i \in N$ to destination node $j \in N$. Constraints (2) and (3) ensure that the total demand originating or destined from or to a certain node equals the total travel demand assigned to the respective node. k_i and k_j represent in constraints (4) and (5) the balancing factors in the gravity model where $g(c_{ij})$ specifies a deterrence function. c_{ij} is the travel cost when travelling between i and j, reflecting the impedance associated with making this trip. Many functions have been suggested for describing the impedance relation g and we here adopt the one proposed by Ortúzar and Willumsen (2011):

$$g(c_{ij}) = c_{ij}^{0.5} e^{-0.25 c_{ij}} \tag{6}$$

We approximate the travel cost by the network travel distance when using the polar distance. The generated Origin-Destination matrix constitutes thus the number of trips performed in a fully connected graph. Hence, it reflects thus the latent demand or in other words the maximum potential travel demand that can be realised in case all nodes are connected and all possible links have been constructed.

2.4. Candidate generation

Each iteration of the Investment Process (see Fig. 1) involves the generation of a set of candidate investments from which in the evaluation at most a single investment is selected. Options for investments are either expanding the network by adding a new connection or an enhancement of the network by means of increasing the capacity of an already existing link. Each of the network investment candidates are subject to some constraints. For the expansion candidates the assumption is that a transport network generated has to be a connected planar graph. Hence, the public transport network consists of a single connected component and that lines cannot cross each other. Barabasi and Albert (1999) showed that the benefits of connecting an additional node to an existing network given equal conditions is preferred over connecting any two other nodes that are detached from the original network. Arguably, providing an additional node access to all other connected nodes is likely to attract a larger share of the latent demand than connecting a single OD pair. Given the polar grid assumed earlier an additional limitation is in place: Only links for which either $r_i = r_j$ or $\varphi_i = \varphi_j$, hence only radial links (constant angle from origin) and ring links (constant radius from origin) are allowed. Because the graph is planar, the length of the elements is restricted to either only span a ring segment $(r_i \cdot \theta)$ or to only span a radial segment (r_{max}/ψ).

The paragraphs below provide detail in the process of generation a set of candidate investments.

2.4.1. Describing evolving network states with augmented adjacency matrices

In order to obtain the network expansion, in each candidate investment option the network state is defined using a variant of the adjacency matrix, A^q, in which the elements indicate the number of investments that have been made between two nodes, and q denotes the network development iteration. Similarly to a "conventional" adjacency matrix, 0 (zero) denotes the absence of a direct connection between nodes; whereas a non-zero value corresponds to the existence of a link. Unlike a conventional adjacency matrix, elements can take values *larger than one* (>1), since several investments related to the same link are possible, corresponding to capacity increments. Put simply, the augmented adjacency matrices A^q contain numbers $a_{ij}{}^q \geq 0$, indicating how many investments a particular link has undergone. Additionally, in the adjacency matrix $\widehat{A}$ elements are zero if and only if no link exists in the fully connected and strict planar graph G. We use $\widehat{A}$ below to describe investment options. Hence, no loops and multiple links connecting the same pair of nodes are allowed.

2.4.2. Description of investment candidates

With A^q and $\widehat{A}$, we can now describe investment candidates. Let E^q denote the investment candidates considered in iteration q. Element $e_{ij}{}^q \in E^q$ is one if the link has not been constructed yet and zero if the edge has been constructed. Therefore, elements $e_{ij}{}^q$ that are non-zero correspond to links that can be considered for investment. The matrix is determined by taking all potential connections and subtracting the current network adjacency matrix, i.e. $E^q = \widehat{A} - A^q$ with $e_{ij}{}^q = 0$ if $\sum_{i\in N} a_{ij}^q + \sum_{j\in N} a_{ij}^q = 0$. The latter constraint ensures that the network consists of a single connected component, i.e. there are no subnetworks which are unconnected. Note that we assume all investments are bidirectional. This implies that in the computations only the upper triangular matrix elements have to be considered—reducing the number of calculations required.

In addition to adding new connections, a fixed capacity increase can be considered for each existing link. To this end, a maximum capacity per link can be set as an input parameter. This means that the potential investment candidates may consist of all currently build links E^q as long as their capacity has not yet reached $\kappa^{max,\ mode}$.

2.5. Candidate evaluation

In order to evaluate the impacts of a candidate investment using a Cost Benefit Analysis later on, the benefits of the candidates have to be first assessed. This is performed in two steps: (i) the new modal split is calculated to find out how many public transport trips will be performed given the new network state, and; (ii) a non-capacitated assignment is performed to assess network saturation and the associated travel costs. These two steps are detailed in the following sections.

2.5.1. Modal split

The latent demand resulting from the trip generation and distribution steps are divided into public transport and non-public transport demand matrices. The share of demand that will travel using public transport is calculated for each candidate network investment. It is determined using a Binary Logit model for choosing between public transport and alternative modes as follows:

$$p_{ij}^{PT} = \frac{e^{u_{ij}^{PT}}}{e^{u_{ij}^{PT}} + e^{u_{ij}^{ALT}}}\ \forall\ i, j \in N \tag{7}$$

$$p_{ij}^{ALT} = \frac{e^{u_{ij}^{ALT}}}{e^{u_{ij}^{PT}} + e^{u_{ij}^{ALT}}}\ \forall\ i, j \in N \tag{8}$$

where p is the probability of choosing a certain mode {PT, ALT} and u is the respective utility, both calculated per OD pair. We assume that all users travel by the alternative modes (e.g. car, bike) if no public transport connection is available. Once available, a non-zero share will switch to public transport.

The disutility for PT and Alternative mode are computed as a function of the impedance imposed by travel time. Travel times are determined by the mode-specific speed parameters, $\upsilon^{PT}$and υ^{ALT}, and the distance between two nodes, c_{ij}:

$$u_{ij}^{PT} = -\frac{c_{ij}}{\upsilon^{PT}} \quad \forall\ i, j \in N \tag{9}$$

$$u_{ij}^{ALT} = -\frac{c_{ij}}{\upsilon^{ALT}} \quad \forall\ i, j \in N \tag{10}$$

2.5.2. Assignment

An all-or-nothing (AON) assignment is applied, assuming that all travellers take their respective shortest path to reach their destination. This ignores taste variations in route choice, as well as capacity effects that may in practice cause people to choose paths other than the shortest one. This is arguably reasonable under the assumption that the network is not overly saturated and that all services offer comparable conditions (in terms of frequency, comfort and price). Paths passenger flows are obtained by multiplying the modal split with the latent demand matrix. Links passenger flows are then obtained by superimposing all OD pair flows on the respective shortest paths and their summation per link.

2.6. Cost benefit analysis

Each candidate investment is evaluated by weighting the costs against the benefits as assessed by a planning authority. Both the benefits and costs are discounted for using the investment time-horizon of 30 years, set by the EC guidelines (Sartori, 2015). We propose the following scoring function (which translates both costs and benefits into monetary terms) to assess each candidate investment in the set E^q:

$$z(e) = \frac{\omega \bullet [f(A^{\hat{q}}) - f(A^q)]}{\sum_{mode\in M} \delta_e^{mode} \beta^{mode} c_e}\ \forall\ e \in E^q \tag{11}$$

The nominator depicts the total benefits stemming from the investment corresponding to travel time savings and the denominator is the cost associated with the investment under consideration. The benefits are the product of changes in total travel costs and the value of time ω. The network-wide travel impedance, f, for a given network state matrix, A^q, is calculated as follows:

$$f(A^q) = \sum_{i\in N} \sum_{j\in N\setminus i} [p_{ij}^{PT,A^q} \bullet x_{ij} \bullet c_{ij}^{PT,A^q} + (1 - p_{ij}^{PT,A^q}) \bullet x_{ij} \bullet c_{ij}^{ALT,\widehat{A}}] \tag{12}$$

In which $f(A^q)$ is the sum of the total travel impedance by public transport and by the alternative mode, each of which consists of the product of the respective demand flow and travel impedance for the respective network. Note that the alternative mode is always assumed to have the complete graph available. $A^{\hat{q}}$ in Eq. (11) is the network state matrix prior to iteration q that is extended by the candidate investment on link e.

The costs are determined by the length of the link under consideration, c_e, and the cost per length unit for the respective mode, β^{mode}, representing both the investment in cost units per kilometre of infrastructure and the costs of offering a given capacity per hour as service costs. $\delta_e{}^{mode}$ is a dummy variable indicating whether the link investment considered is of a certain mode or not for each of the possible modes in set M.

Among the candidate solutions, the one attaining the highest score while ensuring that the benefits exceed the costs is selected in each iteration, that is, $e_q^* = \max_{e\in E^q} z(e)$. In case $z(e_q{}^*) \leq 1$, i.e. no candidate solution yields a non-zero net benefit, then the network development process terminates. The network evolution process is greedy since it considers the short-term benefits in relation to the current network state and is not designed to obtain the optimal network solution over the entire time span. A possible interpretation is that this reflects the iterative decision making process that evolves over a long time span.

2.7. Summary and model implementation

The model described in the previous paragraphs is implemented in MATLAB. First, the geometrical parameters and the initial network state are given as input to define the spatial distribution of nodes. The gravity model is then applied upon initialization to determine the latent demand matrix. In addition, the following parameters need to be set for the iterative loop of network growth to be executed: the maximum planar graph's adjacency matrix; the travel distance matrix and shortest paths; and a maximum time horizon (i.e. number of iterations). The program then iterates as illustrated in the conceptual modelling framework (Fig. 1) until no investment decision yields net benefits. Note that each time step corresponds to a single investment decision. Determining the cost-benefit ratio involves performing all of the demand related loops which include shortest path search and a probabilistic modal choice using the logit models introduced above.

3. Experimental set-up

3.1. Network topology assessment

Let us first define how we assess the networks that emerge from the iterative procedure outlined above. A large number of indicators can be used to describe network topological properties. We selected a series of indicators that are relevant for investigating the impact of a range of input parameters on the resulting network properties: (i) network extent measured in terms of its diameter; (ii) network efficiency in terms of transmission cost, i.e. weighted average shortest path length; (iii) network connectivity measured in terms of both the beta and gamma indicators which pertain to the ratio between the number of links and number of nodes, and the share of links from the total possible number of links in a fully connected graph, respectively.

Next to these conventional indicators, we also examine (iv) network ringness defined as the share of ring link-km out of the total network length (Xie and Levinson, 2007), and; (v) unsatisfied demand – the total number of passengers that the system is unable to transport.

In addition to these global indicators, we also examine the statistics of local centrality indicators – the average and distribution of degree centrality and betweenness centrality. Network loading results are also reported in terms of link loads as well as the link saturation level expressed as the ratio of passenger volume over link capacity.

3.2. Scenario design

We test the growth of a monocentric urban public transport network for a series of combinations of public transport service specifications and population distributions. The geometric parameters are set to $\rho = 40$ km, $\varphi = 30°$, $\psi = 4$. This specification yields a maximum number of links that can be constructed (i.e. the number of non-zero entrees in the maximum planar graph's adjacency matrix) of 192. Total model run times varying between 12 and 36 h depending on the complexity of the solution space in terms of nodes used and the combination of demand/modal costs when using a PC with an Intel Core i5-4690 3.5GHz processor and 8GB of RAM.

The growth patterns and network structure of three high-level public transport technologies which often serve as the backbone of public transport systems are investigated: (i) Bus Rapid Transit (BRT); (ii) Light Rail Train (LRT), and; (iii) Metro. While all these modes operate using an exclusive right of way, their operations differ in terms of speed and capacity and there are noticeable differences in their cost functions. Values for their operational speed, capacity and costs have been estimated using information from Vuchic (2002) and Deng and Nelson (2011). Table 2 presents the selected characteristics for each of the modes used in the research as well as the values estimated for their respective characteristics.

Table 2
Modal parameter specifications.

Mode	Operational Speed υ [km/h]	Cost β [M€/km]	Capacity increase step κ	Maximum capacity κ^{max}
BRT	35	6	2000	20.000
LRT	45	20	3500	35.000
Metro	60	300	8000	80.000

The value of time, ω, is set to 10 [€/pass-hour] based on the value for metropolitan public transport in the Netherlands. Costs parameter, β, is based on prior research by Deng and Nelson (2011) and Vuchic (2002) as well as by taking realised costs for projects found in research by Flyvbjerg et al. (2013). The cost parameter estimate is expressed in terms [M€/km] and encompasses investment, maintenance and variable costs. Similarly, the maximum capacity is derived from Vuchic's work on urban transportation systems and the step increase is about 10% of the maximum capacity. This approach is chosen in order to reduce model complexity of having to add individual vehicles and or headway implications whilst still showing the improvement that adding additional capacity can bring as a step-increase towards the theoretical maximum.

In order to measure the improvement a new PT connection would offer an alternative mode has to be specified. This alternative mode represents a travel time someone could achieve either by existing PT networks (normal bus) or by using private transportation modes (car/bike). The selected mode has unlimited capacity and its network is the max planar graph. The alternative mode has an operational speed that has been chosen to be slightly below the BRT at 20 km/h roughly in line with the assumption of private car, normal bus or maybe a fast cyclist. The existence of this alternative mode is mostly important to be able to measure the potential improvement a new connection provides. In the absence of such an alternative, a new connection would otherwise provide an infinite benefit to newly connected users of the network.

Population distribution is a key determinant of network evolution and its structure due to the interplay between the supply provisioned, the market share of public transport and hence flow distribution which then further impact the return for investment value of future investments. We investigate three distributions: (i) Uniform – all nodes are associated with an equal settlement size; (ii) Linear decay – the size of the population is linearly and negatively related to the distance of the node to the central point of the grid, and; (iii) Exponential decay - the size of the population is exponentially and negatively related to the distance of the node to the central point of the grid, i.e. power-law decay.

Scenarios are designed by considering all possible combinations of public transport modalities and population distribution. The nine scenarios examined in this study are summarized in Table 3.

Table 3
Scenario design summary.

Name	Transit Technology	Population Distribution
BU	Bus Rapid Transit	Uniform
BL	Bus Rapid Transit	Linear Decay
BE	Bus Rapid Transit	Exponential Decay
LU	Light Rail Transit	Uniform
LL	Light Rail Transit	Linear Decay
LE	Light Rail Transit	Exponential Decay
MU	Rapid Transit	Uniform
ML	Rapid Transit	Linear Decay
ME	Rapid Transit	Exponential Decay

Table 4
Key performance indicators of the final network state for each transit technology and population distribution scenario.

	Scenario								
	BRT			LRT			Metro		
Demand distribution	Uni.	Linear Decay	Exp. Decay	Uni.	Linear Decay	Exp. Decay	Uni.	Linear Decay	Exp. Decay
Connectivity β index	1.98	1.86	1.24	1.98	1.86	1.73	1.48	1.48	1.36
Connectivity γ index	1.00	0.94	0.63	1.00	0.94	0.88	0.75	0.75	0.69
Average node degree	3.96	3.71	2.47	3.96	3.71	3.46	2.97	2.97	2.72
Total network Length [km]	1611	1360	925	1611	1360	1140	888	794	668
Ringness Φ_{ring}	0.70	0.65	0.61	0.70	0.65	0.58	0.46	0.40	0.28
Total Travel Time [hr]	2699	2786	2929	2396	2475	2406	2170	2222	2181
Detour factor	1.000	1.001	1.064	1.000	1.001	1.007	1.026	1.040	1.076
Iterations	1689	1569	1010	1581	1485	1401	1208	1149	1041
Accumulated Score	113,600	110,376	67,669	45,125	43,876	41,607	3975	3922	3600

4. Results and analysis

In the following we report and discuss the results of the final network structure yielded by the iterative network growth model (Section 4.1), investigate the evolutionary path in terms of network form and selected topological indicators (4.2) and perform a sensitivity analysis in relation to key modal specifications (4.3).

4.1. Network structure properties

A summary of key topological indicators of the network attained at the final growth iteration is provided in Table 4. Several topological indicators have been selected form the literature: (i) network connectivity measured in terms of edges per node, i.e. *beta index*; (ii) network connectivity measured in terms of the share of edges in relation to the full planar graph, i.e. *gamma index*; (iii) *average node degree*; (iv) *total network length*, and; (v) network *ringness* defined as the share of total ring edges' length of the total network length. In addition, the following system performance indicators are included: (vi) *total travel time* – including users of both public transport and the alternative mode; (vii) *detour factor* or average ratio between fastest possible route and the current fastest public transport network route. Finally, in order to evaluate the performance of the model as well as the evaluation of the process the (viii) *number of iterations* to reach an equilibrium state is presented as well as the (ix) *accumulated score* for the scenario, an indication of the accumulated CBA ratings to provide insight into the scoring characteristics.

Evidently, different combinations of population distribution and mode characteristics result in final network states with distinctly different properties. In particular, one can observe that given a certain mode, the indicators vary considerably for different demand distributions. A more peaked travel demand distribution results in a greater spatial disparity as the most lucrative investments are to be found in the core area while service provision at the periphery might not fulfil the cost-benefit criterion. This is reflected in the topology indicators for connectivity, i.e. beta and gamma. The more concentrated population distribution is the lower network connectivity becomes due to the fewer links constructed. The total system length and ringness indicator confirm that the system is less expanded for more peaked distributions.

Various mode characteristics also obtain a different set of topological indicators for a given population distribution. This can be explained when examining the underlying mechanisms driving the model: an investment in BRT offers a relatively low speed connection at a lower cost compared to LRT and Metro, yet its investment costs are low enough to warrant investment in the network. For LRT the relatively higher speed means that system wide benefits (travel time savings) for passengers are larger. This is partially because of a lower travel time between nodes due to increased speed of the mode and partially because the speed difference between the mode and the travel alternative is larger. This results in a higher modal share for public transport and thus assigning more of the latent demand to the network resulting in more travellers (potentially) benefitting from network growth. Given the marginal increase in costs for LRT this means that for most scenarios almost all possible links are built since the cost-benefit ratio is consistently positive. In other words, for LRT the final network states are relatively close to the Max Planar Graph. For Metro the relatively high costs mean that not all of the potential system benefits for users can be obtained. This is because the (system) benefits of adding a new connection (travel time savings) are not always sufficient to cover the larger investment costs that metro require, consequently making the investment not worthwhile. This can be particularly observed in the Gamma connectivity index and Network length indicators that are systematically lower for Metro when comparing them to the respective demand distribution scenarios when considering LRT or BRT. Accordingly, the accumulated score is highest for BRT and lowest for Metro. This is mostly caused by the capacity expansion scoring relatively high in the case of BRT, as a result of the potential ridership substantially surpassing the actual capacity of a link.

4.2. Network evolutionary path

4.2.1. Evolution of network form

In order to comprehend the evolution of network states under various scenarios, intermediate stages are plotted in Fig. 2. Each graph displays the network state for a given public transport mode after 100, 250, 500, 750 and 1000 iterations as well as the final stage, for each population distribution scenario. Each link in the graph is coloured to show the travel volume in relation to the available capacity (high load in red, low loads in blue) displaying thus network saturation and potential capacity bottlenecks in the network at each stage of network evolution.

In the case of BRT (Fig. 2, top), the network quickly expands to the edges along the radials for the uniform case (radial elements are shorter and thus cheaper while yielding the same benefits) and then in a later stage some shortcuts and ring elements are constructed. For the linear and exponential decaying functions it can be observed that the expansion focusses around the higher density populations in the core and that peripheral elements never get constructed. The exponential decay function also results in a somewhat unexpected shape where some gaps between radials are not worth investments.

The higher cost, capacity and speed for the LRT mode evidently results in a different network development process (Fig. 2, middle). The uniform demand pattern again shows a preference for radials over rings. As in the BRT scenarios a more peaked population distribution strongly affects the network shape and the evolutionary path but due to the higher speeds and costs of LRT this effect is more distinct. This demonstrates that the modal cost parameter also affect network evolution by limiting outer edges where the combination of lower

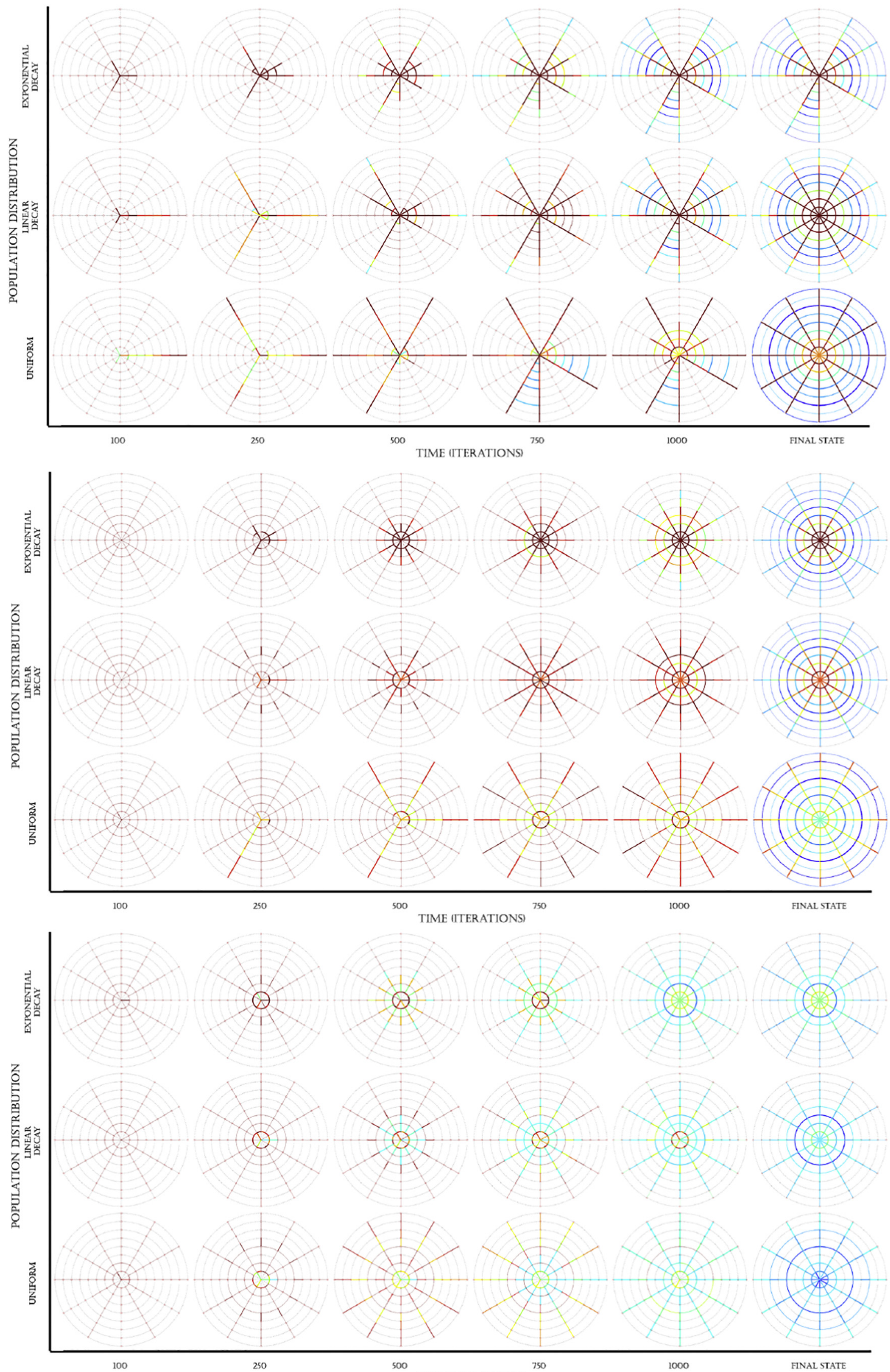

Fig. 2. Network Evolution for Uniform, Linear and Exponential decaying population distributions for BRT (top), LRT (middle) and Metro (bottom).

population density and higher modal costs result in a more compact network.

The Metro mode is the only mode for which capacity is not binding in the final state. Given the high costs per km and higher speeds it is clear that a different balance is obtained throughout the iterations (Fig. 2, bottom). Network development focusses on the core and Metro is the only mode that does not result in a full network for the uniform population. Like previous scenarios the more peaked population distributions also result in a very core centric network with ring elements being constructed in the central area before stretching out towards the edges.

4.2.2. Evolution of network topology indicators

The overall trend in network growth is an early phase of expansion of the network, followed by a period of intensification manifested in capacity increments and finally adding some links that contribute to its densification. This trend is in agreement with the empirical findings reported by Cats (2017) in relation to the topological evolution of the Stockholm public transport network between 1950 and 2025. This trend can be observed when investigating the evolution of topological indicators such as diameter, connectivity, meshedness and the distribution of node degree. In the following we examine the evolution of the average shortest path length which is a measure of network efficiency. This topological indicator exemplifies the effects of population distribution and modal characteristics on the evolution of transport networks. An increase in the average shortest path corresponds to a period of expansion, extending the geographical reach of the network. Conversely, when the network is densified, new links help decreasing the average shortest path while investments in capacity additions for existing links leave it unchanged.

These effects can clearly be seen in Fig. 3 where the average shortest paths are presented for a given mode and the three population distribution functions. Iterations where investments in capacity increments are made can be detected by the stagnation in average path length value. Declines in the average path length value are caused by the construction of a shortcut in a later stage, i.e. shorter average shortest path. When comparing modes it can be observed in Fig. 3 that the BRT mode has a rather abrupt path due to its parameters resulting in alternations between expansion and capacity increments. In contrast, both metro and LRT have a more distinct evolution pattern of expansion, followed by densification and capacity enhancements as most of the changes to the average path length take place in the first 150 iterations.

Another network topological indicator is the Average Detour Factor. For each OD relation it compares the current network distance to the shortest possible in a fully connected graph. As can be seen in Fig. 4 one can observe that the network shape for LRT and Metro is mostly defined in its early stage. At later iterations the network undergoes mostly capacity investments which do not affect the indicator and result in a horizontal trend line. In contrast, for BRT the balance is considerably different. It is characterized by a longer period of increasing detours meaning the network does reach new nodes, but there are large detours between OD relations, an indication that the network lacks ring elements for those scenarios at those stages.

4.3. Sensitivity analysis of mode characteristics

The transport mode has two mechanisms influencing the Cost Benefit Analysis. On one hand, a higher operational speed leads to a potentially lower travel time thus inducing more benefits for users and improving its likelihood to be selected for investments. On the other hand, for modes with a higher operational speed the costs increase

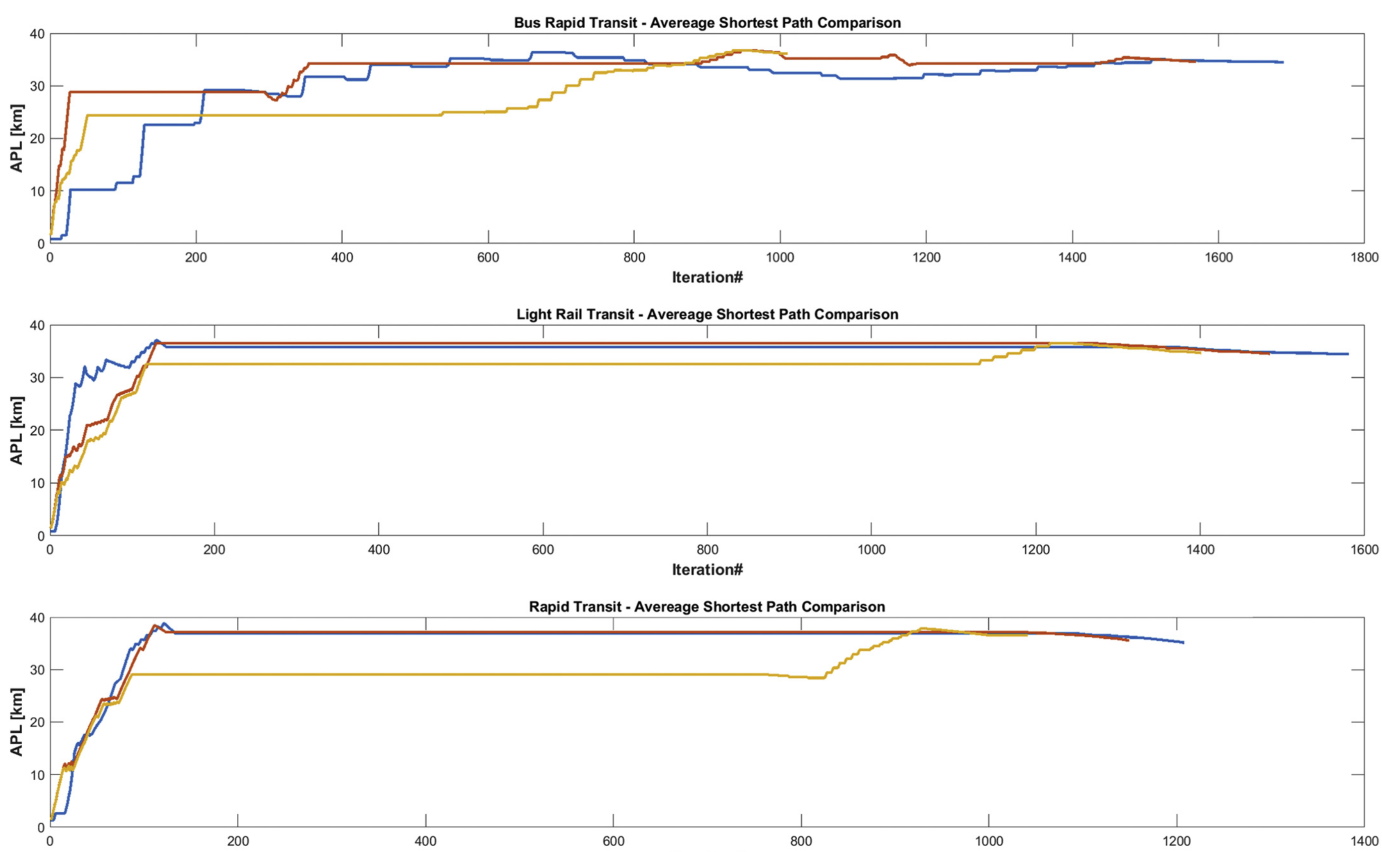

Fig. 3. Average shortest Path length (APL) for a given mode (BRT- top; LRT – middle; Metro – bottom) and varying population distributions (Uniform- blue; Linear – red; Exponential – orange). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

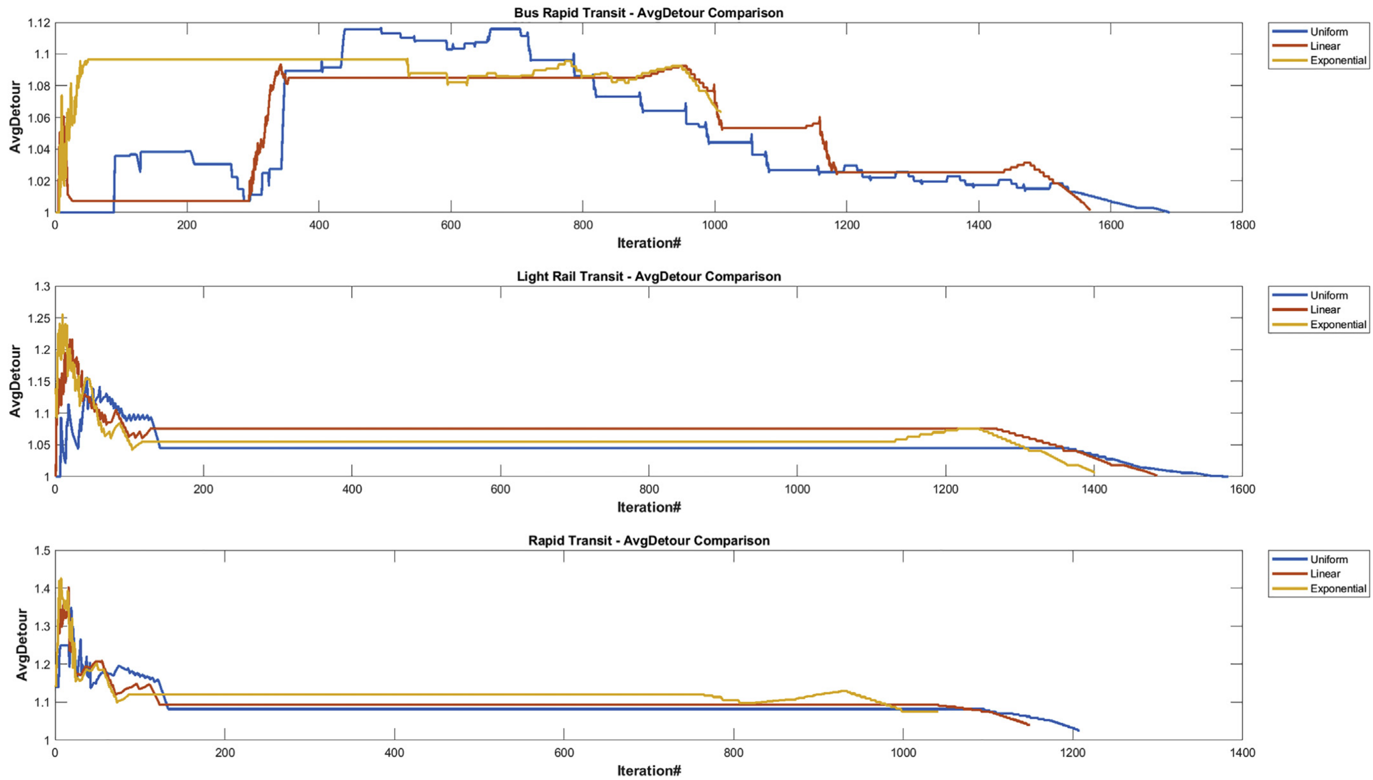

Fig. 4. Average Detour Factor (ADF) for a given mode (BRT- top; LRT – middle; Metro – bottom) and varying population distributions (Uniform- blue; Linear – red; Exponential – orange). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

significantly thus decreasing the likelihood to invest. In order to comprehend the role of these inherently conflicting trends, we perform a sensitivity analysis of the operational speed and the operational costs, varying one while holding the other constant.

To investigate the effects of the operational speed in isolation, the population distribution is set to a uniform distribution, the modal costs are kept constant (at BRT level) and the operational speed is varied. Similarly, the effects of investment costs are disentangled by keeping the operational speed at 45 kph and varying the investment costs between BRT, LRT and Metro levels. Results of these additional scenarios are presented in Fig. 5.

One can observe that an increase in the speed for BRT means that the graph is able to be completed at 45 km/h and that an even higher speed no longer affects the network as its maximum capacity is reached. Similarly, when examining the effects of operational modal costs we see that the higher speed for BRT and even LRT still end up with a full graph with mixed capacities. Conversely, metro due to its high investment costs only completes a few rings and does not fully upgrade its capacity along the rings.

5. Conclusion

We developed an evolutionary network growth model that iteratively invests in constructing new connections or increasing the capacity of existing connections. The model is used to determine the influence of various demand distributions and operational cost functions on the final network structure and its topological properties as well as the process by which these are attained. The model is applied to alternative monocentric demand distributions in combination with alternative modal characteristics in terms of investment and operational costs, capacity and speed.

Results provide insight into the relationships between demand, costs and the network evolution of monocentric metropolitan networks. The results from experiments support the suggestion of a relationship between the population distribution and the final topological evolution of a network. The scenarios affected by a decaying population show limited connections in the outer periphery and binding capacity constraints for links that are constructed in these peripheral areas. This holds for all scenarios regardless of the investment cost associated with the transport mode under consideration, implying that this effect is solely associated with the population distribution.

The transport mode affects the network evolution in two distinct ways. First, in terms of model split. This mechanism means that while more expensive modes with higher speeds (LRT, Metro) yield a more complete network due to the increased benefits surpassing the additional costs. Second, ring connections are less likely to be constructed for more expensive modes. While rings generally yield lower benefits at additional costs, the latter become prohibitive as costs increase. Consequently, bus networks include more ring-radial connections than LRT and Metro networks which are more concentrated on radial connections.

With the set of parameters adopted in our experiments, the BRT has an operational speed that is not sufficiently high to create a virtuous cycle of attracting latent demand and fostering network expansions. This is especially noteworthy given that the operational speeds of both BRT and LRT specified in this study are higher than those observed for most such systems. Conversely, Metro investment costs are too large to justify the construction of the outer rings. In between these options, the LRT mode characteristics allow for the vast majority of possible connections to be considered viable investment, depending on the population distribution pattern. The overall trend in network growth for all three modes is an early phase of expansion of the network, followed by a period of intensification manifested in capacity increments and finally adding some links that contribute to its densification.

Future research may further develop the proposed iterative network growth model by considering urban agglomerations that do not adhere to the radio-centric prototype. This involves developing alternative sets of geometric parameters and setting the respective demand distribution

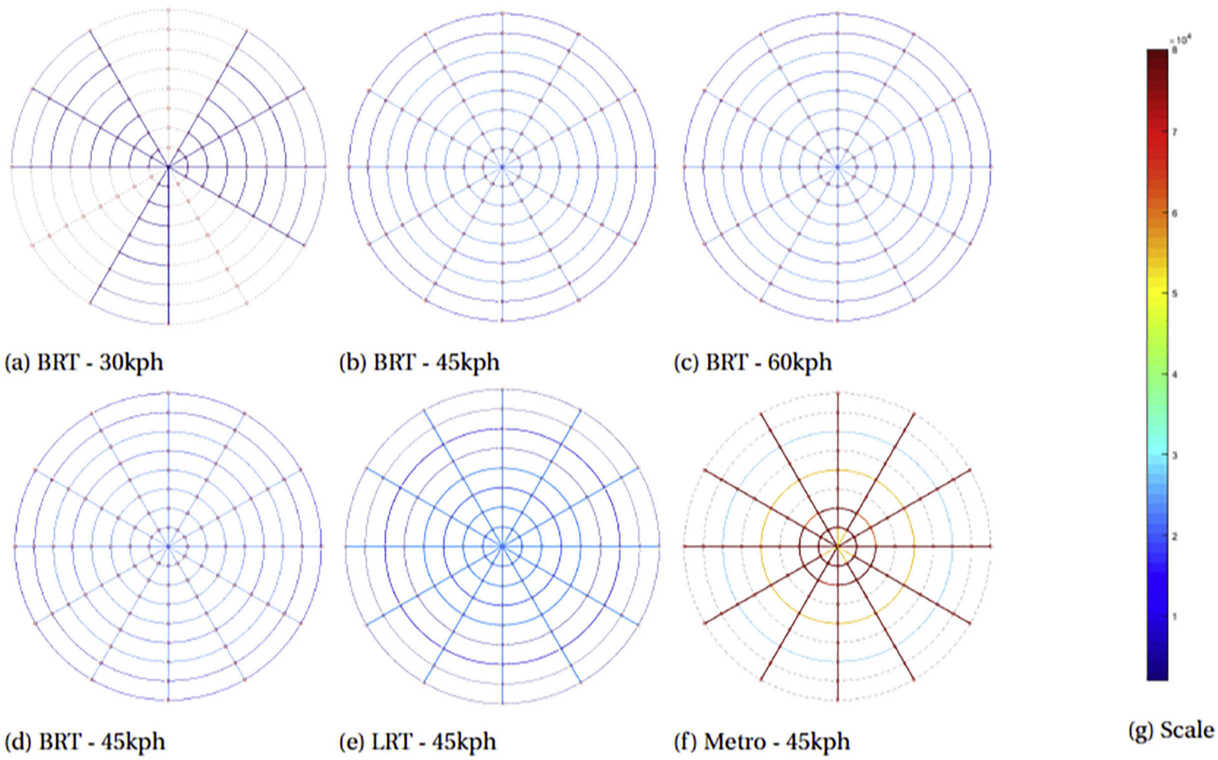

Fig. 5. Sensitivity analysis for network states for varying speeds (above) and operational costs (below).

patterns. We currently extend the model to the case of a polycentric urban agglomeration including the development of multi-modal networks and related hierarchical properties. Moreover, future research may devise to relax some of the assumptions made in this study by introducing a capacitated assignment or a feedback loop from network state to trip generation in order to allow for induced demand as a consequence of improved accessibility. Another interesting direction is to contrast the results of the myopic evolutionary network growth model with empirical observations as well as optimal network design solutions.

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