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Survivable Impairment-aware Traffic Grooming in WDM Rings

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***Abstract*—Wavelength Division Multiplexing (WDM) optical networks offer a large amount of bandwidth using multiple, but independent wavelength channels (or lightpaths), each operating at several Gb/s. Since the traffic between users is usually only a fraction of the capacity offered by a wavelength, several independent traffic streams can be *groomed* together. In addition, in order to reverse the effect of noise and signal degradations (physical impairments), optical signals need to be regenerated after a certain *impairment threshold* is reached. We consider *survivable* impairment-aware traffic grooming in WDM rings, which are among the most widely deployed optical network topologies.**

We first show that the survivable impairment-aware traffic grooming problem, where the objective is to minimize the total cost of grooming and regeneration, is NP-hard. We then provide approximation algorithms (for uniform traffic), and efficient heuristic algorithms whose performance is shown to be close to the lower-bounds (for non-uniform traffic) both when (1) the impairment threshold can be ignored, and (2) the impairment threshold should be considered.

I. INTRODUCTION

In optical networks employing Wavelength Division Multiplexing (WDM) technology, the capacity of a fiber is divided into several non-overlapping wavelength channels that can transport data independently. These wavelength channels make up *lightpaths*, which are used to establish optical connections that may span several fiber links. With current commercial technology, each lightpath can be independently operated at a data rate ranging up to 100 Gb/s [3]. However, traffic between a pair of nodes may not be able to fill up the available bandwidth of a lightpath. In order to efficiently utilize the available bandwidth, several independent traffic streams can be aggregated to share the capacity of a lightpath. This is known as *traffic grooming*.

Survivability, which is the ability to reconfigure and retransmit data after failure, is usually achieved by computing a link/node-disjoint backup lightpath that will take over after failure of the primary lightpath. In addition, due to the signal degradation caused by physical impairments, a lightpath may require regeneration after a certain distance. A routing of lightpaths which takes into account physical impairments is known as *impairment-aware routing* [13].

In this paper, we study *survivable impairment-aware traffic grooming* in WDM ring networks. Currently, ring topologies (such as SONET/SDH rings) are widely deployed in metro/regional networks [2]. Nodes are assumed to be equipped with an optical add/drop multiplexer (OADM) to selectively add/drop wavelengths. In this paper, we will follow the configuration described in [9] and [11], where transceivers are used to terminate lightpaths. As in [9], the lightpaths are assumed to be full-duplex, and the forward and reverse direction signals use the same wavelength and path. Unless a wavelength carries traffic destined for a given node or needs regeneration, it passes through optically. Otherwise, the following take place: (1) the lightpath is terminated, (2) the traffic is processed electronically (and regenerated simultaneously), (3) traffic destined to the node is dropped, and (4) the rest of the traffic, including locally added traffic, if any, is forwarded on other lightpaths through the transceivers. In this model, the cost of transceivers is the dominant component [9].

The outline of this paper is as follows. In Section II, we overview related work. In Section III, we show that the survivable impairment-aware traffic grooming problem is NP-hard. In Section IV, we focus on the (basic) survivable traffic grooming problem by considering uniform and non-uniform traffic. For the former, we provide an approximation algorithm, while we give a heuristic algorithm for the latter. In Section V, we provide similar results for the survivable *impairment-aware* traffic grooming problem. Finally, we conclude in Section VII.

II. RELATED WORK

Traffic grooming has been widely studied in the literature, especially in relation to SONET/SDH rings over WDM networks. Most of the previous studies did not consider survivability or impairment-aware routing. Chiu and Modiano [5] studied the traffic grooming problem with the objective to minimize the total number of SONET add-drop multiplexers (ADMs) in unidirectional SONET/WDM rings. They showed that the problem is NP-complete. The same problem was also shown to be NP-complete in bidirectional ring networks, where a request between two nodes can be routed on the clockwise or counter-clockwise direction [6]. Amini *et al.* [1]

further showed that the traffic grooming problem is APX-hard in WDM rings for a fixed value of grooming factor g, i.e., each request uses $1/g$ of the capacity of a wavelength. Huang *et al.* [10] studied traffic grooming in different topologies: line, star, and tree, and showed that traffic grooming is NP-complete in these topologies. Saleh and Kamal [16] studied many-to-many traffic grooming, where a member node of a group communicates with all the other members of the group, in mesh networks.

Sankaranarayanan *et al.* [17] considered survivable traffic grooming in unidirectional WDM rings for uniform traffic with a mix of protected and unprotected requests. Ou *et al.* [14] gave heuristic algorithms for survivable grooming in mesh networks, while Yao and Ramamurthy [20] considered the same problem under shared risk link group (SRLG) constraints, and provided heuristic algorithms. Unlike [14], [17] and [20], we consider both survivability *and* impairment-aware routing in WDM ring networks, and give approximation (for uniform traffic) and efficient heuristic algorithms with upper-bounds (for non-uniform traffic). Patel *et al.* [15] considered impairment-aware traffic grooming where regeneration is performed by regenerator cards. In this approach, there is a distinction between add/drop and regeneration nodes, since regenerator cards are not capable of adding/dropping traffic. However, regeneration can also be achieved using back-to-back transceivers [19], in which case, regeneration nodes can also be add/drop nodes, and vice versa. In this paper, we follow the second approach since it allows the use of the same type of devices for both add/drop and regeneration [19]. Flammini *et al.* [7] studied the case when regeneration and grooming are interchangeable, but they (1) did not consider survivability, and (2) assumed that a lightpath is regenerated at all its internal nodes, which is not always necessary.

III. Problem Definition

We define a *wavelength link* as a single wavelength channel of a link, and a *wavelength ring* as a ring made up of wavelength links of the same wavelength. In effect, a ring comprises of stacks of wavelength rings. In a given wavelength ring, a *regenerator node* is a node where the wavelength is regenerated and an *add/drop node* is where traffic is added/dropped from the wavelength. A *wavelength segment* is a segment of a wavelength ring between two consecutive add/drop or regenerator nodes, i.e., no other add/drop or regenerator node exists in this segment. Associated with each wavelength link is an additive impairment value. The impairment value of a wavelength segment is the sum of the impairment values of its links, and it is a *feasible* segment if its impairment value does not exceed a given *impairment threshold*.

Splitting traffic of a single request onto different lightpaths might cause re-ordering problems at the receiving end as some higher layer protocols may not be able to deal with it. Since the nodes as well as links that the signals go through may be different, it may also lead to different signal quality. Therefore, we assume that the traffic of a given request is not split unless its demand exceeds the full capacity of a wavelength ring. In addition, in order to facilitate control, the primary and backup lightpaths of a given request are assumed to be on the same wavelength ring. Thus, for any given wavelength ring, the amount of traffic on each of its links is the same, and a pair of transceivers is required when a wavelength is added/dropped or regenerated at a given node. Since a wavelength is regenerated when traffic is added/dropped to it, an add/drop node is also a regenerator node. The network cost mainly comprises of the electronic and opto-electronic cost associated with grooming and regeneration (i.e., cost of transceivers), and the number of wavelengths. In practice, the cost of transceivers dominates the cost of the number of wavelengths [4] [9] [16]. Hence, we minimize the total number of transceivers under the assumption that there are enough wavelengths to accommodate all the requests, which is equivalent to minimizing the total number of add/drop and regenerator nodes in the network.

Problem 1: **Survivable Impairment-aware Traffic Grooming:** Given is an undirected ring topology $G(\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is a set of N nodes, $\mathcal{L}$ is a set of $L = N$ links. Associated with each link $l \in \mathcal{L}$ is an impairment value $r(l)$. A wavelength has a capacity C. In addition, given are an impairment threshold Δ and a set $\mathcal{F}$ of F requests. Each request i is represented by a pair of nodes (u_i, v_i) and δ_i, where u_i and v_i are the endpoints of request i, and δ_i is the amount of demand of request i. The survivable impairment-aware traffic grooming problem is to minimize the total number of transceivers (or add/drop and regenerator nodes) in the network such that (1) each request is assigned a primary and backup path, (2) the capacity of any wavelength link is not exceeded, and (3) each wavelength segment in any wavelength ring is feasible.

Theorem 1: The survivable impairment-aware traffic grooming problem is NP-hard.

In our proof, we use the NP-hard Bin Packing Problem [8], which is defined as follows.

Problem 2: **The Bin Packing Problem:** Given a finite set $\mathcal{U}$ of n items, a size a_i for each $u_i \in \mathcal{U}$, and a bin capacity B, the **bin packing problem** is to find a partition of $\mathcal{U}$ with a minimum number of disjoint sets (bins) $S_1, \ldots, S_K$ such that the sum of the sizes of the items in each S_i is less than B.

Proof: We show that the survivable traffic grooming problem, which is a subset of the survivable impairment-aware traffic grooming problem (by taking Δ sufficiently large) is NP-hard. For a given instance of the bin packing problem of n items, create a corresponding survivable traffic grooming problem as follows. For each item i, create a corresponding node i in the ring. Create a hub node h, such that the number of nodes $N = n + 1$. Let the capacity C of a wavelength be equal to the bin capacity B, and there is a request of demand $\delta_i = a_i$ between each node i and the hub node h. Since there is exactly one request originating at each node (except the hub node h), any feasible solution requires an add/drop node at each of the $N - 1$ nodes. Hence, only the total number of add/drop nodes at node h can be minimized. Since there is one add/drop node per used wavelength at node h, the total number of add/drop nodes is the same as the total number of wavelength rings. Therefore, the objective is to minimize the total number of wavelength rings. On the other hand, each wavelength ring is equivalent to a bin, and the requests in the wavelength ring are equivalent to the items in the bin of the corresponding bin packing problem instance. Therefore, minimizing the total number of wavelength rings required for all the requests is equivalent to minimizing the number bins of the corresponding bin packing problem instance. ■

IV. Survivable Traffic Grooming

We begin with the case where no regeneration is required and consider uniform and non-uniform traffic independently.

A. Uniform Traffic

In a uniform traffic scenario, there is a request of equal demand δ between each pair of nodes. Thus, there are a total of $\frac{N(N-1)}{2}$ requests, each with a demand of δ. Even though, this type of traffic is less practical, it can help us gain insight into the complexity of survivable traffic grooming. In addition, as shown in [6], it may be possible to extend the results obtained for uniform traffic to that of the more practical *quasi-uniform* traffic. An algorithm is said to be an *α-approximation algorithm*, for some $\alpha > 1$, if it returns at most α times the optimal number of transceivers (add/drop nodes). Before we provide an approximation algorithm for solving the survivable traffic grooming problem under uniform traffic, we give a lower-bound for the total number of add/drop nodes.

Theorem 2: For uniform traffic, the total number of add/drop nodes m is lower bounded by:

$$m \geq \left\lceil \sqrt{\frac{\delta}{2C}} N(N-1) \right\rceil . \tag{1}$$

Proof: We provide a proof along the lines of the proof given in [6] for unprotected traffic grooming. Given a feasible solution S, let $G(A)$ be the wavelength ring on which add/drop node A is. For each add/drop node A, define $B(A)$ as:

$$B(A) = \frac{\text{Total bandwidth of traffic on wavelength ring } G(A)}{\text{Total number of add/drop nodes on } G(A)}.$$

For a given wavelength ring of S, let k be the number of add/drop nodes. Hence, there can be at most $k(k-1)/2$ requests in this ring and the total bandwidth requirement (i.e., the sum of bandwidth needed on all wavelength links) of all the requests in this wavelength ring is at most $\delta N k(k-1)/2$. Since the total bandwidth capacity of a wavelength ring is CN,

$$\begin{aligned} B(A) &\leq \frac{\min(\delta N k(k-1)/2, CN)}{k} \\ &= CN \min\left(\frac{\delta}{2C}(k-1), 1/k\right) \\ &\leq CN \min\left(\frac{\delta}{2C}k, 1/k\right) \\ &\leq CN\sqrt{\frac{\delta}{2C}} = N\sqrt{\frac{\delta C}{2}}. \end{aligned} \tag{2}$$

The last inequality is due to the property that $min(ak, 1/k) \leq \sqrt{a}$ for any $k > 0$. Let B be the total amount of bandwidth consumed by all the requests. Summing the last inequality of Eq. 2 (which is independent of k) over all the add/drop nodes,

$$B \leq mN\sqrt{\frac{\delta C}{2}}.$$

For uniform traffic, the total bandwidth B is

$$B = \delta N \left(\frac{N(N-1)}{2}\right).$$

From which Eq. 1 follows. ■

Corollary 1: Any survivable traffic grooming algorithm is a $\sqrt{\frac{2C}{\delta}}$-approximation algorithm for uniform traffic.

Proof: In the worst-case there is no grooming at all, i.e., each request is added/dropped independently. Since there are $\frac{N(N-1)}{2}$ requests, a total of $N(N-1)$ add/drop nodes will be needed in this case. However, by Theorem 2, we have that the optimal number of add/drop nodes is at least $\sqrt{\frac{\delta}{2C}}N(N-1)$. Thus, the approximation ratio is $\sqrt{\frac{2C}{\delta}}$. ■

We now provide an algorithm for the survivable traffic grooming problem, termed $USGA$ (Uniform traffic Survivable Grooming Algorithm), and show that it is a $\min\left(\sqrt{\frac{2C}{\delta}}, 4\right)$-approximation algorithm for uniform traffic.

Algorithm 1 $USGA(G, \mathcal{F}, C)$

1) If $\delta \geq C$, then assign $\left\lfloor \frac{\delta}{C} \right\rfloor$ wavelength rings for each request and let $\delta = \delta - C\left\lfloor \frac{\delta}{C} \right\rfloor$.
2) If $N \leq \sqrt{\frac{2C}{\delta}}$, assign all the requests in one wavelength ring and exit.
3) If $C < 2\delta$, then assign a single request per wavelength ring and exit.
4) Let $k = \left\lfloor \sqrt{\frac{C}{2\delta}} \right\rfloor$. Partition the N nodes into $\left\lceil \frac{N}{k} \right\rceil$ sets so that each set, except possibly one, contains k distinct nodes and each node belongs to *exactly* one set.
5) For each pair of sets among those created in Step 4, create a set which is the union of this pair of sets.
6) Sequentially, for each set in Step 5, assign a separate wavelength ring as follows:
   a) Each node in the set is an add/drop node (i.e., transceivers are placed).
   b) For each pair of nodes, allocate the primary and backup path of the corresponding request in this wavelength ring, unless the request has already been allocated in a previous wavelength ring.

In Step 1 of $USGA$, if the demand per request is greater than the capacity of a wavelength ring, a separate wavelength ring(s) is assigned for each request and the remaining traffic of the request is assigned a wavelength ring in the next steps. In Step 2, if $N \leq \sqrt{\frac{2C}{\delta}}$, all the requests can optimally fit in a single wavelength ring. Similarly in Step 3, if $C < 2\delta$, only a single request can be assigned in a wavelength ring. Step 4 partitions the nodes into a group of sets, and Step 5 combines a pair of these sets in such a way that any pair of nodes belongs to at least one of the newly-formed sets. Once the sets are created, the requests are assigned sequentially in Step 6. Since there are $O\left(N/\sqrt{\frac{C}{2\delta}}\right)$ sets in Step 4, there will be $O\left(\frac{N^2\delta}{C}\right)$ sets in Step 5. The most time-consuming operation in $USGA$ is Step 6, where for each wavelength ring, requests between each pair of its add/drop nodes are considered to decide whether they belong to the wavelength ring. Since the size of a set is at most $2k = 2\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor$ and each pair of nodes in the set is considered, Step 6 has a total running time of $O\left(N^2\right)$.

Through the following example, we illustrate how the algorithm works. Let $N = 7$, $C = 9$ and $\delta = 1$. Thus, $k = \left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor = 2$. The nodes are then grouped into sets of at most 2 elements: $\{1,2\}$, $\{3,4\}$, $\{5,6\}$, $\{7\}$. By combining each pair of sets, we get $\{1,2,3,4\}$, $\{1,2,5,6\}$, $\{1,2,7\}$, $\{3,4,5,6\}$, $\{3,4,7\}$, $\{5,6,7\}$. For each set, a separate wavelength ring is used, a pair of transceivers is assigned at all its nodes, and the requests between each pair of nodes are allocated in this wavelength ring unless they have been allocated before. For example, the request between nodes 1 and 2 is assigned only to the first wavelength ring.

We proceed to establish the correctness of $USGA$.

Theorem 3: For the outcome of $USGA$ holds: (1) The capacity of any of the wavelength links is not exceeded, and (2) each request is assigned primary and backup paths.

Proof: (1) In each wavelength ring, there are at most $2k = 2\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor$ add/drop nodes. Thus, there can be at most $2k(2k-1)/2$ requests in any given wavelength ring. Hence, the total capacity required at any wavelength link is at most

$$\begin{aligned}\frac{\delta 2k(2k-1)}{2} &= \frac{\delta\left(2\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor\right)\left(2\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor-1\right)}{2} \\ &\leq \frac{\delta\left(2\sqrt{\frac{C}{2\delta}}\right)^2}{2} = C.\end{aligned}$$

(2) Each node belongs to at least one set in Step 4. Since the sets in Step 5 are a combination of each pair of sets in Step 4, any given pair of nodes belongs to at least one set in Step 5. Thus, the corresponding request is allocated primary and backup paths in Step 6. ∎

Theorem 4: $USGA$ is a $\min\left(\sqrt{\frac{2C}{\delta}}, 4\right)$ −approximation algorithm.

Proof: Let

$$a = \begin{cases} 0, \text{ if } (N \bmod k) = 0; \\ k - (N \bmod k), \text{ otherwise.}\end{cases}$$

In Step 4, there are a total of $\left\lceil\frac{N}{k}\right\rceil = \frac{N+a}{k}$ sets, and each set contains k elements, except possibly the last set that has only $k-a$ elements if $a > 0$. Hence, the total number of sets in Step 5 is $\frac{\lceil\frac{N}{k}\rceil(\lceil\frac{N}{k}\rceil-1)}{2} = \frac{\frac{N+a}{k}(\frac{N+a}{k}-1)}{2}$, and each set requires at most $2k$ add/drop nodes. However, among these sets, there are $\left(\frac{N+a}{k}-1\right)$ sets that require only $2k-a$ add/drop nodes. Hence, the total number of add/drop nodes is

$$\begin{aligned}&\left(\frac{\frac{N+a}{k}(\frac{N+a}{k}-1)}{2}\right)2k - \left(\frac{N+a}{k}-1\right)a \\ =\ & N\left(\frac{N+a}{k}-1\right) = N\left(\frac{N+a-k}{k}\right).\end{aligned}$$

By definition, $a \leq k-1$. Thus, the total number of add/drop nodes is at most $N\left(\frac{N-1}{k}\right)$.

Combining this with Theorem 2, the approximation ratio α is,

$$\alpha \leq \frac{N\left(\frac{N-1}{k}\right)}{\sqrt{\frac{\delta}{2C}}N(N-1)} = \frac{\sqrt{\frac{2C}{\delta}}}{\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor} = 2\frac{\sqrt{\frac{C}{2\delta}}}{\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor} \leq 4.$$

The last inequality is due to the fact that since $C \geq 2\delta$, $\frac{\sqrt{\frac{C}{2\delta}}}{\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor} < 2$. With Corollary 1, this proves our theorem. ∎

B. Non-uniform Traffic

Non-uniform traffic is a general scenario where the amount of demand between nodes is arbitrary. For any node u, let $\mathcal{F}_u$ be the set of requests for which node u is an endpoint, F_u be the number of such requests (i.e., $F = \frac{1}{2}\sum_{u=1}^{N} F_u$), and $C_u = \sum_{(u,v)\in\mathcal{F}_u} \delta_{(u,v)}$. For any node u, let OPT_u be the optimal solution for the corresponding bin packing problem of set $\mathcal{F}_u$. We first provide a simple lower-bound for non-uniform traffic.

Theorem 5: For non-uniform traffic, the total number of add/drop nodes m is bounded by:

$$m \geq \sum_{u=1}^{N} OPT_u.$$

Proof: The number of add/drop nodes at any node u is the same as the number wavelength rings terminated at this node. As shown in the proof of Theorem 1, at any given node u, the minimum possible number of such wavelength rings is the same as the solution of the corresponding instance of the bin packing problem (i.e., for each $(u,v) \in \mathcal{F}_u$, there is an item of size $\delta_{(u,v)}$). ∎

We now provide a heuristic algorithm, termed $NSGA$ (Non-uniform traffic Survivable Grooming Algorithm), for the non-uniform traffic case (see Algorithm 2). The algorithm considers each node sequentially and allocates wavelength rings for requests originating at this node by first solving a corresponding bin packing problem instance.

For any node u, let OPT'_u be the optimal solution for the corresponding bin packing problem of set $\mathcal{F}'_u$.

Theorem 6: $NSGA$ is a $\left(\frac{3}{2} + \frac{F}{\sum_{u=1}^{N} OPT_u}\right)$-approximation algorithm.

Proof: For any node u, the FFD algorithm in Step $2b$ returns at most $\frac{3}{2}OPT'_u$ bins [18]. In Step $2c$, a separate wavelength ring is assigned for each bin, and on each wavelength

Algorithm 2 $NSGA(G, \mathcal{F}, C)$

1) Sort the nodes in non-increasing order according to F_u.
2) For each node u,
   a) Let $\mathcal{F}'_u = \{(u,v) = (v,u) | u < v\}$ and $F'_u = |\mathcal{F}'_u|$.
   b) Create an instance of the bin-packing problem such that for each request $(u,v) \in \mathcal{F}'_u$, there is an item whose size is the demand of the request. Use the *first fit decreasing (FFD)* algorithm [12] as follows:
      i) Sort the items in non-increasing order.
      ii) Go through all the items by placing the current item in the lowest indexed bin that has enough space left, otherwise create a new bin for it.
   c) For each bin in the solution of the bin packing problem instance, create a new wavelength ring and place the requests corresponding to the items of the bin in this wavelength ring.

ring, node u is an add/drop node. Additionally, at the other end of each request, an add/drop is required. Thus, the total number of add/drop nodes required by $NSGA$ when node u is considered in Step 2 is at most $\frac{3}{2}OPT'_u + F'_u$. Since $OPT'_u \leq OPT_u$ for each node u and $\sum_{u=1}^{N} F'_u = F$, combined with Theorem 5, this proves our theorem. ■

V. Survivable Impairment-aware Traffic Grooming

In this section, we consider the general problem of survivable impairment-aware traffic grooming, where transceivers are used not only for adding/dropping traffic, but also for regeneration. Consider the following example to illustrate the difference from the previous impairment-agnostic problem. Let $N = 4$, $C = 2$, $\Delta = 2$, and each link has an impairment value of 1. Let the requests be $(1,3)$, $(2,3)$ and $(3,4)$, and each request has a demand of $\delta = 1$. For the survivable traffic grooming problem, both of the following solutions (see Fig. 1) are optimal, and each solution requires a total of 5 add/drop nodes. **Solution 1**: $\{(1,3),(2,3)\}$ on the first wavelength ring and $\{(3,4)\}$ on the second wavelength ring; and **Solution 2**: $\{(2,3),(3,4)\}$ on the first wavelength ring and $\{(1,3)\}$ on the second wavelength ring. However, for the survivable impairment-aware traffic grooming problem, only solution 2, which needs no extra regenerator node, is optimal. Solution 1 requires an extra regenerator at nodes 1 or 2 of the second wavelength ring to accommodate the backup path of $(3,4)$.

The number of regenerator nodes needed to feasibly and survivably route all the assigned requests in a wavelength ring depends on the endpoints of the requests (or the wavelength segments). However, it is possible to determine the minimum number R of regenerator nodes required at any wavelength ring for a given survivable impairment-aware grooming problem using the procedure $FindR$ (see Algorithm 3).

Theorem 7: $FindR$ returns the minimum number of regenerator nodes required on any wavelength ring.

Proof: We give a proof by contradiction. Assume that

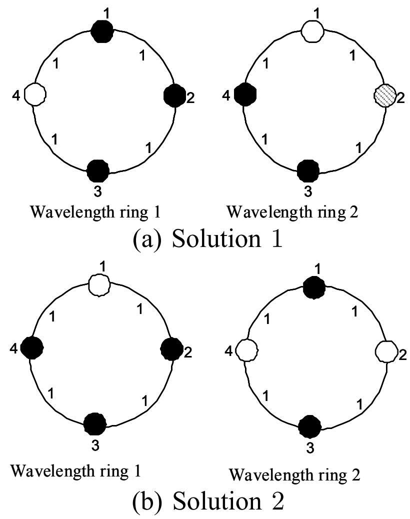

Fig. 1. Difference between the survivable traffic grooming and the survivable impairment-aware traffic grooming. Solid nodes are add/drop nodes and shaded nodes are regenerator nodes.

Algorithm 3 FindR

1) For each node u, find a value R_u as follows:
   a) Initialize R_u to 0. Beginning at node u, move clockwise (can also be anti-clockwise, but should be consistent) to the farthest reachable node v (i.e., within a distance of Δ) from node u and increment R_u. Let r_1 be the distance between u and v in the clockwise direction.
   b) Then, at node v move to the farthest clockwise reachable node and increment R_u, and so on, until just before we cross node u, at which point let the last such node be t. Let r_2 be the distance between t and u in the clockwise direction. If $r_1 + r_2 > \Delta$, increment R_u.
2) $R = \min_u\{R_u\}$.

the minimum number of regenerator nodes is $R' < R$. Let these regenerator nodes be $n_1, ..., n_{R'}$ in the clockwise direction. W.l.o.g., for each node n_j, node n_{j+1} is the farthest reachable node from n_j in the clockwise direction, otherwise the regeneration at n_{j+1} can be moved to the farthest reachable node. In addition, the distance between $n_{R'}$ and n_2 exceeds Δ, otherwise the regeneration at n_1 is not necessary. Thus, employing $FindR$ at node n_1 would return R' instead of R, which is a contradiction. ■

A. Uniform Traffic

We first provide a lower-bound for the survivable impairment-aware traffic grooming problem under uniform traffic.

Theorem 8: For uniform traffic, the total number of add/drop and regenerator nodes m is lower-bounded by:

$$m \geq \max\left(\left\lceil \sqrt{\frac{\delta}{2C}} N(N-1) \right\rceil, R \left\lceil \frac{N(N-1)}{2\left\lfloor \frac{C}{\delta} \right\rfloor} \right\rceil\right).$$

Proof: Since transceivers are required for adding/dropping traffic as well as regeneration, the total number of transceivers depends on which scenario is dominant. If adding/dropping is the dominant factor, Theorem 2 gives that $m \geq \left\lceil \sqrt{\frac{\delta}{2C}} N(N-1) \right\rceil$. Therefore, we need to show only the case when the number of regenerator nodes dominates. The maximum number of requests that can be assigned in any wavelength ring is $\lfloor \frac{C}{\delta} \rfloor$. Since we have a total $\frac{N(N-1)}{2}$ requests, we need at least $\left\lceil \frac{N(N-1)}{2\lfloor \frac{C}{\delta} \rfloor} \right\rceil$ wavelength rings to accommodate all the requests, and each wavelength ring requires at least R regenerator nodes. ■

Algorithm $USGA$ can be reused for solving the survivable impairment-aware traffic grooming problem as follows: (1) Solve the corresponding survivable traffic grooming problem. (2) From this solution, for each wavelength ring identify non-feasible segments and place regenerator nodes to make these wavelength segments feasible. For each wavelength segment, this can be done using the regenerator placement algorithm in [13]. We first give approximation ratios for this approach. However, the approximation ratios may be too high for practical use. Therefore, we suggest a scheme to improve the average performance of $USGA$, while maintaining the worst-case ratio.

Theorem 9: $USGA$ has an approximation ratio of 16 if $R \leq \sqrt{\frac{2C}{\delta}}$, and 20 otherwise.

Proof: We use the same notation as in Theorem 4. W.l.o.g, the lower-bound on m can be replaced with $\max\left(\sqrt{\frac{\delta}{2C}} N(N-1), R\frac{N(N-1)}{2\frac{C}{\delta}}\right)$. From the proof of Theorem 4, the total number of add/drop nodes needed by $USGA$ is at most $N\left(\frac{N-1}{k}\right)$. In the worst-case, we additionally need a total of R regenerator nodes on each wavelength ring. Thus, the total number of add/drop and regenerator nodes is at most

$$\begin{aligned} & N\left(\frac{N-1}{k}\right) + \left(\frac{\frac{N+a}{k}(\frac{N+a}{k}-1)}{2}\right) R \\ = \; & N\left(\frac{N-1}{k}\right) + R\left(\frac{\frac{N+a}{k}(\frac{N+a-k}{k})}{2}\right) \\ = \; & \left(\frac{N-1}{k}\right)\left(N + R\left(\frac{N+a}{2k}\right)\right) \\ \leq \; & \left(\frac{N-1}{k}\right)\left(N + R\left(\frac{1.5N}{2k}\right)\right) \end{aligned}$$

The second equality is because $a \leq k-1$. The last inequality is because $2a \leq 2k \leq N$ (See Step 2 of $USGA$). We consider two cases.

Case 1: $\sqrt{\frac{\delta}{2C}} N(N-1) \geq R\frac{N(N-1)}{2\frac{C}{\delta}}$ or $R \leq \sqrt{\frac{2C}{\delta}}$.
The total number of add/drop and regenerator nodes is:

$$\begin{aligned} & \left(\frac{N-1}{k}\right)\left(N + R\left(\frac{1.5N}{2k}\right)\right) \\ \leq \; & \left(\frac{N-1}{k}\right)\left(N + 1.5N\frac{\sqrt{\frac{2C}{\delta}}}{2\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor}\right) \leq 4N\left(\frac{N-1}{k}\right). \end{aligned}$$

The first inequality is because $R \leq \sqrt{\frac{2C}{\delta}}$, and the last inequality is due to the fact that since $C \geq 2\delta$, $\frac{\sqrt{\frac{C}{2\delta}}}{\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor} < 2$. Since $k = \left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor$, the approximation ratio α is:

$$\alpha \leq \frac{4N\left(\frac{N-1}{k}\right)}{\sqrt{\frac{\delta}{2C}} N(N-1)} \leq 4\frac{\sqrt{\frac{2C}{\delta}}}{\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor} = 8\frac{\sqrt{\frac{C}{2\delta}}}{\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor} \leq 16.$$

Case 2: $\sqrt{\frac{\delta}{2C}} N(N-1) \leq R\frac{N(N-1)}{2\frac{C}{\delta}}$ or $R \geq \sqrt{\frac{2C}{\delta}}$.
The total number of add/drop and regenerator nodes is:

$$\begin{aligned} & \left(\frac{N-1}{k}\right)\left(N + R\left(\frac{1.5N}{2k}\right)\right) \\ = \; & N\left(\frac{N-1}{k}\right)\left(1 + \frac{1.5}{2k}R\right) \leq N\left(\frac{N-1}{k}\right)\left(\frac{2.5}{2k}R\right). \end{aligned}$$

The last inequality follows from $R \geq \sqrt{\frac{2C}{\delta}} \geq 2\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor = 2k$. The approximation ratio α is:

$$\begin{aligned} \alpha \; \leq \; & \frac{N\left(\frac{N-1}{k}\right)\left(\frac{2.5}{2k}R\right)}{R\left(\frac{N(N-1)}{2\frac{C}{\delta}}\right)} = 2.5\,\frac{\frac{C}{\delta}}{\left(\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor\right)^2} \\ \leq \; & 5\frac{\frac{C}{2\delta}}{\left(\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor\right)^2} = 5\left(\frac{\sqrt{\frac{C}{2\delta}}}{\left\lfloor\sqrt{\frac{C}{2\delta}}\right\rfloor}\right)^2 \leq 20. \end{aligned}$$

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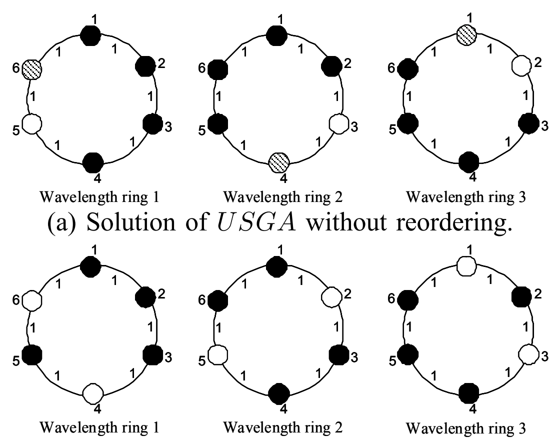


(a) Solution of $USGA$ without reordering.

(b) Solution of $USGA$ with reordering.

Fig. 2. Rearranging the order of nodes affects the solution of $USGA$. Solid nodes are add/drop nodes, while shaded nodes are extra regenerator nodes.

The average performance of $USGA$ can be improved by reordering the nodes before creating the sets in Step 4. The basic idea is to group together pairs of nodes that are within a distance close to the impairment threshold so that the number of extra regenerator nodes is reduced. The following example illustrates this. Let $N = 6$, $C = 8$, $\delta = 1$, and $\Delta = 2$. Thus, $k = 2$. By simply applying $USGA$, the sets in Step 4 will be $\{1,2\}$, $\{3,4\}$, $\{5,6\}$, and the sets in Step 5 are $\{1,2,3,4\}$, $\{1,2,5,6\}$, $\{3,4,5,6\}$. This solution will require a total of 12 add/drop nodes and 3 extra regenerator nodes (one in each wavelength ring as shown in Fig. 2(a)). However, if the nodes are reordered so that pairs of nodes with a distance of Δ or more are grouped together, the sets in Step 4 will be $\{1,3\}$, $\{5,2\}$, $\{4,6\}$, and the sets in Step 5 are $\{1,3,5,2\}$, $\{1,3,4,6\}$, $\{5,2,4,6\}$. This solution will require 12 add/drop nodes and no extra regenerator nodes (see Fig. 2(b)). In general, the nodes can be reordered before applying $USGA$ as follows:

- Mark node 1, then mark the clockwise unmarked node i that is at a distance of Δ from node 1 or is the first unmarked node that is unreachable (i.e., whose distance is larger than Δ) from node 1.
- Repeat this process from node i, until all nodes are marked.

B. Non-uniform Traffic

We first give a lower-bound for non-uniform traffic. Let BIN be the optimal number of bins required for the following instance of the bin packing problem: For each request i, create an item a_i of size δ_i, and let the bin capacity $B = C$.

Theorem 10: For non-uniform traffic, the total number of add/drop and regenerator nodes m is lower-bounded by:

$$m \geq \max\left(\sum_{u=1}^{N} OPT_u, R \cdot BIN\right).$$

Proof: To accommodate all the requests, at least BIN number of wavelength rings are required. On each wavelength ring, at least R number of regenerator nodes are needed. Combined with Theorem 5, this proves our theorem. ∎

In order to solve the survivable impairment-aware traffic grooming problem, we modify $NSGA$ in such a way that after the requests are assigned to wavelength rings and add/drop nodes are identified, for each wavelength ring, we place the extra regenerator nodes required to make all its wavelength segments feasible. Using the same notation as in Theorem 6, the following theorem can be established.

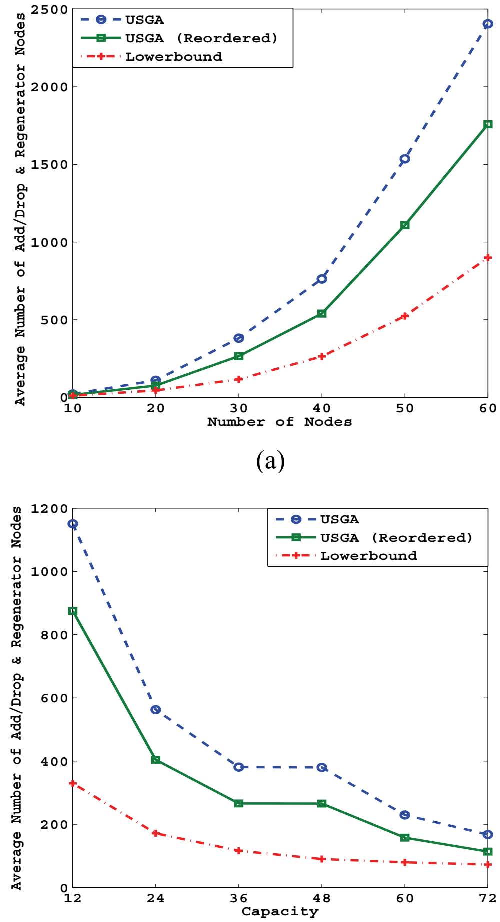
Theorem 11: The total number of add/drop nodes m returned by the modified $NSGA$ is upper-bounded by:

$$m \leq \frac{3}{2}(R+1)\left(\sum_{u=1}^{N} OPT'_u\right) + F.$$

Proof: As is shown in the proof of Theorem 6, the number of wavelength rings returned by $NSGA$ is at most $\frac{3}{2}\left(\sum_{u=1}^{N} OPT'_u\right)$. In the worst-case, we need R extra regenerator nodes in each wavelength ring. ∎

VI. Simulation Results

We first provide simulation results showing the performance gain achieved by rearranging the order of nodes before applying $USGA$ as described in Section V-A. Figures 3(a) and 3(b) show the performance of $USGA$ (with and without reordering) against the lower-bound for different number of nodes (fixed capacity) and different capacity (fixed number of nodes), respectively. From these results, we observe that (1) even though $USGA$ has an approximation ratio of 16 or 20, the performance ratio against the lower-bound (which might not always be attainable) is at most 4 in these results, and (2) reordering the nodes provides a performance gain as high as 30%.

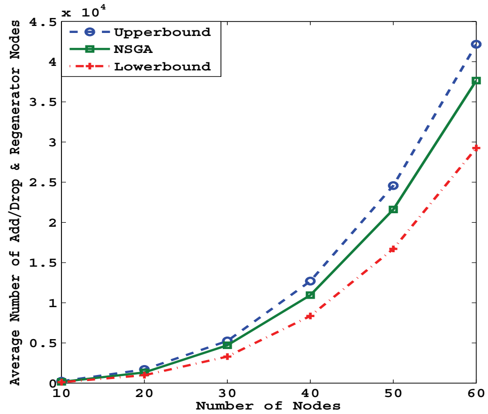


(a)

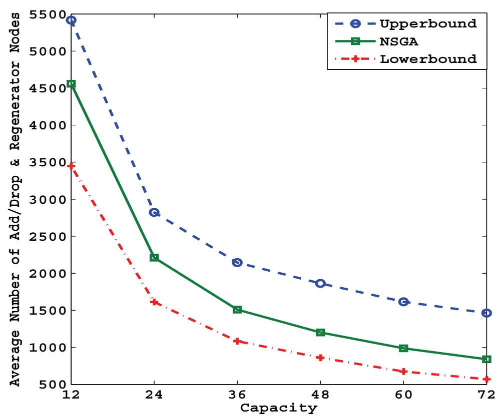
(b)

Fig. 3. Comparison of the number of add/drop and regenerator nodes required by $USGA$ (with and without ordering) in the survivable impairment-aware traffic grooming problem for (a) different number of nodes and $C = 36$, and (b) different capacity and $N = 30$. The impairment values are uniformly distributed within the range $(0, 1]$ and $\Delta = 1$.

Figures 4(a) and 4(b) show the results obtained for the modified $NSGA$ when solving the survivable impairment-aware traffic grooming problem under non-uniform traffic. These figures show that the results of $NSGA$ are not generally far-off from the lower-bounds of the optimal solutions. In addition, the lower-bound is based on the assumption that all wavelength rings are fully utilized, but in reality, this is not the case as some wavelength rings will only be partially utilized since requests are not allowed to be split. Therefore, the optimal solution will in practice be much higher than the lower-bound. Since finding the optimal solution (e.g., using exact Integer Linear Programming (ILP) formulations) turned out not to be tractable even for small sized networks and small number of requests, $NSGA$'s performance and scalability make it suitable for practical purposes.



(a)



(b)

Fig. 4. Comparison of the number of add/drop nodes required by the modified $NSGA$ in the survivable impairment-aware traffic grooming problem for (a) different number of nodes ($C = 36$ and demand values are within the range $[0, C]$), and (b) different capacity ($N = 30$ and demands are within $[0, 12]$).

VII. Conclusions

In this paper, we have studied the survivable impairment-aware traffic grooming problem in WDM wavelength rings, which are among the most widely deployed network topologies. The objective is to minimize the total cost of grooming and regeneration. Unlike previous studies in traffic grooming, we consider both survivability and impairment-aware routing, which are gaining a lot of interest from both network operators and researchers. We have shown that the problem is NP-hard. We have considered two cases of the problem, (1) when the impairment threshold can be ignored, and (2) when the impairment threshold should be taken into account under uniform and non-uniform traffic scenarios. Uniform traffic may be less practical, but it helps us gain insight into the complexity of the survivable (impairment-aware) traffic grooming problem.

For the survivable traffic grooming problem, we have given a 4-approximation algorithm for uniform traffic, and an efficient heuristic algorithm with an upper-bound for non-uniform traffic. For the survivable impairment-aware traffic grooming problem, the approximation ratio (i.e., worst-case performance) is 16 or 20 depending on the problem instance under uniform traffic, which may be too high for practical purposes. Therefore, we proposed a scheme to improve the average performance of the approximation algorithm, while the worst-case ratio is maintained. Similarly, we provided an efficient heuristic algorithm for non-uniform traffic, and showed through simulations that its performance is close to the lower-bound.

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