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Simple synchronization protocols for heterogeneous networks: beyond passivity

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Abstract: Synchronization among autonomous agents via local interactions is one of the benchmark problems in multi-agent control. Whereas synchronization algorithms for identical agents have been thoroughly studied, synchronization of heterogeneous networks still remains a challenging problem. The existing algorithms primarily use the internal model principle, assigning to each agent a local copy of some dynamical system (internal model). Synchronization of heterogeneous agents thus reduces to global synchronization of identical generators and local synchronization between the agents and their internal models. The internal model approach imposes a number of restrictions and leads to sophisticated dynamical (and, in general, nonlinear) controllers. At the same time, passive heterogeneous agents can be synchronized by a very simple linear protocol, which is used for consensus of first-order integrators. A natural question arises whether analogous algorithms are applicable to synchronization of agents that do not satisfy the passivity condition. In this paper, we study the synchronization problem for heterogeneous agents that are not passive but satisfy a weaker input feedforward passivity (IFP) condition. We show that such agents can also be synchronized by a simple linear protocol, provided that the interaction graph is strongly connected and the couplings are sufficiently weak. We demonstrate how stability of cooperative adaptive cruise control algorithms and some microscopic traffic flow models reduce to synchronization of heterogeneous IFP agents.

1. INTRODUCTION

As the influential monograph (Strogatz, 2003) states, "the tendency to synchronize is one of the most pervasive drives in the universe, extending from atoms to animals". Synchrony among subsystems (agents, cells) of a complex system is a basic principle, which explains many natural phenomena (Strogatz, 2003) and has found numerous applications in engineering (Mesbahi and Egerstedt, 2010; Olfati-Saber et al., 2007; Wu, 2007; Ren and Beard, 2008). Establishing synchronization (consensus) is considered now as a benchmark problem in multi-agent control and has been thoroughly examined in the recent decades.

Most of the attention has been paid to synchronization among *identical* agents. The protocols establishing synchronization among single integrators are usually based on the idea of contraction: the convex hull, spanned by the agents' states, is shrinking until it collapses into a singleton (Münz et al., 2011). An alternative approach is based on convergence criteria for infinite matrix products (Ren and Beard, 2008). The protocols for synchronization of agents, obeying higher order equations, are similar in spirit to first-order algorithms. Synchronization of linear and linearly coupled agents is often analyzed via the spectral decomposition of the Laplacian matrix (Olfati-Saber et al., 2007; Li et al., 2010; Ren and Beard, 2008; Ren and Cao, 2011). Nonlinear protocols are usually examined by Lyapunov methods (Ren and Cao, 2011), employing, among others, the Kalman-Yakubovich-Popov lemma (Zhang et al., 2014; Proskurnikov and Matveev, 2015), contraction theory (DeLellis et al., 2011) and the idea of incremental passivity (Stan and Sepulchre, 2007; Proskurnikov et al., 2015; Liu et al., 2015b).

However, in practice autonomous agents are usually heterogeneous. Algorithms for output synchronization of non-identical agents have been proposed quite recently and most of them employ the *internal model principle* (Wieland et al., 2011; De Persis and Jayawardhana, 2014; Isidori et al., 2014; Bidram et al., 2014; Liu et al., 2015a), assigning to each agent a virtual copy of some dynamical system, referred to as the *internal model* or the *local reference generator*. The control algorithm then consists of two layers: a protocol, synchronizing the (identical) reference generators and local *model-matching* controllers, synchronizing the agents to their generators.

The general internal model approach has, however, several disadvantages. Being formally decentralized, its implementation assumes that the agents share the same internal model and are able to match it (e.g. in the case of linear agents the Francis regulator equations should be solvable (Wieland et al., 2011; Liu et al., 2015a)). Hence design of an algorithm requires to know the global information about the network. Unlike many synchronization algorithms for identical agents (Olfati-Saber et al., 2007; Li et al., 2010; Ren and Beard, 2008; Ren and Cao, 2011) that use only *relative measurements*, that is, the deviations between an agent's output and the outputs of its neigh-

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bors, the model-matching controllers need access to the absolute outputs of the agents. Dealing with mobile robots, this implies that agents have to measure their positions and/or velocities in the *global* frame of reference.

At the same time, synchronization among heterogeneous *passive* agents (e.g. mechanical systems in the Euler-Lagrange form) can be established by the same simplest protocols (Pogromsky and Nijmeijer, 2001; Arcak, 2007; Hatanaka et al., 2015) as used to synchronize single integrator agents (Olfati-Saber et al., 2007). Such a protocol does not require any knowledge of the agents' dynamics (except for their passivity) and uses only deviations between the agents' outputs, but not the outputs themselves.

Thus a visible gap exists between the problems of synchronization in networks of passive heterogeneous agents, provided by a very simple algorithm, and synchronization among general heterogeneous agents, which requires sophisticated model-based controllers. In this paper, we make a step towards filling this gap and show that the conventional synchronization algorithm for passive agents (Hatanaka et al., 2015) is applicable also to *input-feedforward* passive (IFP) (Khalil, 1996; Torres et al., 2015) agents, provided that the couplings among them are sufficiently *weak*. The class of IFP systems is much broader than the class of passive systems (and contains, in particular, all asymptotically stable linear systems). We demonstrate applications of our results to the design of cooperative adaptive cruise control (CACC) for platoons of automated vehicles and stability of a microscopic traffic flow model with delayed drivers' responses, both of which can be reduced to synchronization of IFP agents. Proskurnikov and Mazo Jr (2017).

2. PRELIMINARIES

In this section, we introduce basic concepts from graph theory and define input-feedforward passivity (IFP).

2.1 Graphs and their connectivity properties

A (weighted directed) graph is a triple $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = \{v_1, \dots, v_N\}$ stands for the set of *nodes*, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is a set of *arcs* and $A = (a_{jk})_{j,k=1}^N$ is a non-negative *adjacency matrix*, such that $a_{jk} > 0$ if $(v_k, v_j) \in E$ and otherwise $a_{jk} = 0$. We always assume that the number of nodes N and their indices are fixed, so $V = \{1, \dots, N\}$, there is a one-to-one correspondence between such graphs and their adjacency matrices $A \mapsto G[A] \triangleq (V, E[A], A)$, where $E[A] \triangleq \{(j,k) : a_{kj} \neq 0\}$. Henceforth all graphs have no self-loops $a_{jj} = 0 \,\forall j$. A graph is called *undirected* if $A = A^\top$. For any node j we introduce the weighted *in-* and *out-degrees* $d_j^+[A] \triangleq \sum_{k=1}^N a_{jk}$ and $d_j^-[A] \triangleq \sum_{k=1}^N a_{kj}$.

A *walk* connecting nodes v and v' is a sequence of nodes $v_{i_0} \triangleq v, v_{i_1}, \dots, v_{i_{s-1}}, v_{i_s} \triangleq v'$ ($n \geq 1$) such that $(v_{i_{k-1}}, v_{i_k}) \in E$ for $k = 1, \dots, s$. A graph is *strongly connected* if a walk between any two distinct nodes exists. A graph is *quasi-strongly connected*, or has a *directed spanning tree*, if one of its nodes is connected by walks to all other nodes. For an undirected graph these conditions are equivalent (such a graph is simply called *connected*).

2.2 Passivity and input-feedforward passivity

Consider the dynamical system

$$\dot{x}(t) = f(x(t), u(t)),\; y(t) = h(x(t), u(t)), \quad t \geq 0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^m$ stand, respectively, for the state, control and output.

The system (1) is *passive* (Khalil, 1996; Willems, 1972) if there exists a *storage function* $V(x) \geq 0$ such that

$$V(x(T)) - V(x(0)) \leq \int_0^T y(t)^\top u(t)\, ds \;\forall T \geq 0 \tag{2}$$

(here T varies in the interval where the solution exists). Assuming V to be C^1-smooth, (2) can be rewritten as

$$\dot{V}(x,u) = \frac{\partial V}{\partial x} f(x,u) \leq h(x,u)^\top u \;\forall x \in \mathbb{R}^n, u \in \mathbb{R}^m. \tag{3}$$

In this paper, we primarily deal with systems, satisfying a "relaxed" passivity condition, defined as follows.

Definition 1. The system (1) is IFP(α) (input-feedforward passive with the passivity index α) if it is passive with respect to the output $\tilde{y} = y + \alpha u$, i.e.

$$V(x(T)) - V(x(0)) \leq \int_0^T \left(y(t)^\top u(t) + \alpha |u(t)|^2 \right) dt. \tag{4}$$

In the case $\alpha = 0$ an IFP(α) system is passive; if $\alpha < 0$ the condition (4) is referred to as the *strict* input passivity. In this paper, we are primarily interested in systems that are not passive but IFP(α) with $\alpha > 0$. Examples of such systems are discussed in Section 3.3.

Although this is not required by the formal definition, the conditions of passivity and IFP usually hold for systems with zero equilibrium: $f(0,0) = 0$ and $h(0,0) = 0$. For systems without equilibria points a modification of passivity condition exists, referred to as the *incremental passivity* (De Persis and Jayawardhana, 2014; Liu et al., 2015b). Similarly, we introduce the incremental IFP condition.

Definition 2. A dynamical system is said to be iIFP(α) (incrementally IFP(α)) if for any two solutions (x_1, u_1, y_1) and (x_2, u_2, y_2) the respective deviations $\delta x = x_2 - x_1$, $\delta u = u_2 - u_1$, $\delta y = y_2 - y_1$ satisfy the inequality

$$V(\delta x(T)) - V(\delta x(0)) \leq \int_0^T \left(\delta y^\top \delta u + \alpha |\delta u|^2 \right) dt, \tag{5}$$

where T belongs to the interval where both solutions exist. The function V is called the incremental storage function.

Obviously, for linear systems IFP(α)$\Longleftrightarrow$iIFP(α).

Remark 3. In general, the IFP property (similar to the usual passivity) is a Lyapunov-type property; to prove it, the storage function $V(x)$ has to be found. For linear systems, the IFP criterion can be derived from the positive real lemma (see Lemma 12 in Sect. 3.3). The algebraic dissipativity criteria for nonlinear systems (Hill and Moylan, 1976) can also be used to establish the IFP condition.

3. MAIN RESULTS: SYNCHRONIZATION PROTOCOLS FOR IFP AGENTS

Consider a group of N agents obeying the equations:

$$\dot{x}_j(t) = f_j(x_j(t), u_j(t)),\; y_j(t) = h_j(x_j(t)), \quad t \geq 0, \tag{6}$$

for $j \in \{1,\dots,N\}$. Here $x_j(t) \in \mathbb{R}^{n_j}$, $u_j(t) \in \mathbb{R}^m$, $y_j(t) \in \mathbb{R}^m$ stand respectively for the jth agent's state, control and output.

In this paper, we study distributed protocols, synchronizing the outputs y_j asymptotically or in L_2-norm.

Definition 4. Solutions $\{(x_j(t), u_j(t), y_j(t))\}_{j=1}^N$ of the systems (6), defined on $t \in [0;\infty)$, are *output synchronized* if

$$|y_i(t) - y_j(t)| \xrightarrow[t\to\infty]{} 0 \quad \forall i,j = 1,\dots,N. \tag{7}$$

More specifically, the solutions are output synchronized with a predefined *reference signal* $\bar{y} : [0;\infty) \to \mathbb{R}^m$ if

$$|y_i(t) - \bar{y}(t)| \xrightarrow[t\to\infty]{} 0 \quad \forall i = 1,\dots,N. \tag{8}$$

Definition 5. Solutions $\{(x_j(t), u_j(t), y_j(t))\}_{j=1}^N$ of the systems (6), defined for $t \geq 0$, are *output L_2-synchronized* if

$$\int_0^\infty |y_i(t) - y_j(t)|^2 dt < \infty \quad \forall i,j = 1,\dots,N. \tag{9}$$

The solutions are output L_2-synchronized with a predefined reference signal $\bar{y} : [0;\infty) \to \mathbb{R}^m$ if

$$\int_0^\infty |y_i(t) - \bar{y}(t)|^2 dt < \infty \quad \forall i = 1,\dots,N. \tag{10}$$

In practice, the difference between the asymptotical and L_2-synchronization is minor. Mathematically, none of these conditions implies the other one. However, in some special situations it is possible to prove that L_2-synchronization implies asymptotical synchronization.

Proposition 6. Let $y_j(t)$ be absolutely continuous and $(\dot{y}_i - \dot{y}_j) \in L_p[0;\infty]$ for some $p > 1$ and for any i,j. Then (9) implies (7). If, additionally, $\bar{y}(t)$ is absolutely continuous and $(\dot{y}_i - \dot{\bar{y}}) \in L_p[0;\infty]\,\forall i$ then (10) entails (8).

Proposition (6), as well as all other statements of this paper, is proved in Appendix. In the following subsections we examine synchronization algorithms.

3.1 Synchronization without reference signal

We start examinating the linear controller:

$$u_j(t) = \sum_{k=1}^N a_{jk}(y_k(t) - y_j(t)), \tag{11}$$

where $a_{jk} \geq 0$ are the *coupling gains.* The matrix $A = (a_{jk})$ determines the interaction graph (or the network's *topology*) $\mathcal{G}[A]$, where node k is connected to node j by an arc if and only if $a_{jk} \neq 0$, that is, the control input of agent j is *directly* influenced by the output of agent k.

It is widely known (Olfati-Saber et al., 2007; Ren and Beard, 2008; Münz et al., 2011) that single integrators $\dot{y}_j = u_j$, coupled via the protocol (11) reach *consensus* (that is, a common limit $y_* = \lim_{t\to\infty} y_j(t)$ exists) whenever $\mathcal{G}[A]$ has a directed spanning tree. Output synchronization (7) is retained replacing single integrators by general *passive* systems (6) and assuming strong connectivity of $\mathcal{G}[A]$ (Hatanaka et al., 2015, Theorem 8.3). Our first result extends this to IFP agents.

Theorem 7. Assume that agent j (for $j = 1,\dots,N$) is IFP(α_j) with a storage function $V_j(x_j) \geq 0$. Let $\mathcal{G}[A]$ be strongly connected and the couplings be "weak", i.e.

$$\alpha_j d_j^+[A] = \alpha_j \sum_{k=1}^N a_{jk} < 1/2 \quad \forall j = 1,\dots,N. \tag{12}$$

Then the following statements hold.

(1) Any solution of the system (6),(11), which is prolongable to ∞, is output L_2-synchronized (9);
(2) Suppose that for any j the function V_j is *radially unbounded* $\lim_{|x_j|\to\infty} V_j(x_j) = \infty$, the map f_j is continuous and h_j is C^1-smooth. Then, any solution of the closed-loop system (6),(11) is prolongable to ∞, bounded, and output synchronized (7).

The proofs of Theorem 7 and other results of this section are omitted due to the page limit and can be found in (Proskurnikov and Mazo Jr, 2017). In the case of $\alpha_j = 0$ the inequalities (12) hold for any matrix A, and Theorem 7 coincides with Theorem 8.3 in (Hatanaka et al., 2015). We proceed with two remarks, regarding the assumptions.

Remark 8. Unlike passive agents, for general IFP agents the requirement of weak coupling (12) cannot be disregarded, as demonstrated by the following example. For any $p, q > 0$ the system:

$$\dddot{y}_j(t) + p\ddot{y}_j(t) + q\dot{y}_j(t) = u_j(t) \in \mathbb{R}, \quad t \geq 0, \tag{13}$$

is IFP(α) with some $\alpha = \alpha(p,q) > 0$ (c.f. Subsect. 3.3). Applying the protocol (11) with all-to-all coupling $a_{ij} = \varkappa > 0\,\forall i,j$ to a group of identical agents (13), output synchronization is guaranteed (Olfati-Saber et al., 2007; Li et al., 2010) only when the polynomial $s^3 + ps^2 + qs + \varkappa(N-1) = 0$ is Hurwitz. Accordingly to the Routh-Hurwitz criterion, this is possible only if $\varkappa(N-1) < pq$, i.e. the gain $\varkappa$ is small.

Remark 9. Dealing with general heterogeneous agents, the condition of strong connectivity *cannot* be replaced by the existence of a directed spanning tree in $\mathcal{G}[A]$. Consider, for instance, a pair $(N = 2)$ of harmonic oscillators

$$\ddot{\xi}_1 + \omega_1^2\xi_1 = u_1, \quad \ddot{\xi}_2 + \omega_2^2\xi_2 = u_2, \quad \omega_1 \neq \omega_2.$$

that are passive with respect to the outputs $y_1 = \dot{\xi}_1$ and $y_2 = \dot{\xi}_2$. Consider the protocol $u_1 = k(\dot{\xi}_2 - \dot{\xi}_1), u_2 = 0$, which corresponds to the graph with $N = 2$ nodes and the only arc $2 \mapsto 1$. It can be shown that the system has a family of solutions $\xi_1(t) = Re[W(\imath\omega_2)ce^{\imath\omega_2 t}]$, $\xi_2 = Re[ce^{\imath\omega_2 t}]$, where $c \in \mathbb{C}$ is constant and $W(s) = ks/(s^2 + ks + \omega_1^2)$. The corresponding outputs are $y_1(t) = Re[\imath\omega_2 W(\imath\omega_2)ce^{\imath\omega_2 t}]$ and $y_2(t) = Re[\imath\omega_2 ce^{\imath\omega_2 t}]$. Since

$$|W(\imath\omega_2)| = \left|\frac{k\imath\omega_2}{(\omega_1^2 - \omega_2^2) + k\imath\omega_2}\right| < 1,$$

the outputs are harmonic signals with the same frequency ω_2 but different amplitudes and cannot be synchronous.

3.2 Reference-tracking synchronization

We now consider the more complex problem of output synchronization with a reference signal (8). In this paper we confine ourselves to a special situation: when the desired trajectory is generated as the output of an agent for some appropriate control input and initial condition.

Assumption 10. For any j system (6) has a solution $(\bar{x}_j(t), \bar{u}_j(t), \bar{y}_j(t))$ such that $\bar{y}_j(t) \equiv \bar{y}(t)\forall t \geq 0$ (in particular, the solution is prolongable to ∞). At any time agent j is aware of the value[1] $\bar{u}_j(t)$, however the reference $\bar{y}(t)$ may be available only to a few "dedicated" agents.

[1] If $\bar{u}_j(\cdot)$ is not unique, agent j knows one of such solutions.

Assumption 10 is often adopted implicitly or explicitly in reference-tracking synchronization problems. For *linear* agents (Li et al., 2010; Liu et al., 2015a) the reference signal $\bar{y}(t)$ is usually supposed to be an output of a reference system, whose model is known and included by the models of other agents. Dealing with first-order integrator agents $\dot{y}_j = u_j$, Assumption 10 implies that the agents know the derivative $\dot{\bar{y}}(t)$; this holds e.g. if $\bar{y}(t) = t\bar{v} + \bar{y}(0)$, where $\bar{v}$ is known, but the initial condition $\bar{y}(0)$ is uncertain. A practical example of this type is discussed in Section 4. Note that the solution $(\bar{x}_j(t), \bar{u}_j(t), \bar{y}_j(t))$ is not assumed to be asymptotically stable, so the control $u_j(t) = \bar{u}_j(t)$ *does not* guarantee the reference signal tracking (8). In general, only *some of the agents* are able to measure the tracking error $\bar{y}(t) - y_j(t)$, whereas the remaining agents measure only deviation between theirs and their neighbors' outputs.

Consider the following modification of the algorithm (11)

$$u_i(t) = \bar{u}_i(t) + b_i(\bar{y}(t) - \bar{y}_i(t)) + \sum_{j=1}^{N} a_{ij}(y_j(t) - y_i(t)). \quad (14)$$

Here $b_i > 0$ if agent i has access to the reference signal, and otherwise $b_i = 0$. The following result is a counterpart of Theorem 7 for reference-tracking synchronization.

Theorem 11. Let Assumption 10 hold and further assume that: for all $j \in \{1, \dots, N\}$ agent j is iIFP(α_j), $\mathcal{G}[A]$ is strongly connected, at least one agent has access to the reference signal, i.e. $\sum_i b_i > 0$, and the couplings are sufficiently weak, i.e.

$$\alpha_j(d_j^+[A] + 2b_j) < 1/2 \quad \forall j = 1, \dots, N. \quad (15)$$

Then, the following two statements hold:

(1) Any solution of the system (6),(14), prolongable to ∞, is output L_2-synchronized with the reference signal (10); in particular, $\int_0^\infty |\bar{u}(t) - u_j(t)|^2 dt < \infty$.
(2) If for all j the functions V_j are radially unbounded, the maps f_j are C^1-smooth, the Jacobians $\frac{\partial f_j}{\partial x_j}, \frac{\partial f_j}{\partial u_j}$ are uniformly bounded, and the maps h_j are linear: $h_j(\xi_1 - \xi_2) = h_j(\xi_1) - h_j(\xi_2)$; then, any solution of the closed-loop system (6),(14) is prolongable to ∞ and output synchronized (8) with the reference signal.

3.3 Examples of IFP agents

In this subsection, examples of IFP agents are provided.

SISO agents with a pole at zero

Consider a SISO system

$$s\rho(s)\zeta(t) = u(t) \in \mathbb{R}, \quad s \triangleq \frac{d}{dt},\ \rho(\lambda) = \sum_{k=0}^{r} \rho_k \lambda^k;$$
$$y(t) = \eta(s)\zeta(t), \quad \eta(\lambda) = \sum_{k=0}^{r} \eta_k \lambda^k \quad (16)$$

Lemma 12. Assume that $\rho(s)$ is a Hurwitz polynomial and $\eta_0\rho_0 \geq 0$. Then the system (16) is IFP(α) for sufficiently large $\alpha \geq 0$. Denoting the transfer function from u to y by $W(\lambda) = \eta(\lambda)/(\lambda\rho(\lambda))$, the passivity index can be found as

$$\alpha = -\inf_{\omega \in \mathbb{R}} Re\, W(\imath\omega). \quad (17)$$

For instance, Lemma 12 implies that the system (13) is IFP (in this case, $\rho(\lambda) = \lambda^2 + p\lambda + q$ is Hurwitz since $p, q > 0$ and $y(t) = \xi(t)$).

First-order delayed integrators

Consider now a delayed system:

$$\dot{y}(t) = u(t - \alpha) \in \mathbb{R}^m. \quad (18)$$

Here $\alpha \geq 0$ is a constant delay and we assume, by definition, that $u(t) \equiv u_0(t)$ for $t \in [-\alpha; 0]$, where $u_0 \in L_2([-\alpha; 0] \to \mathbb{R}^m)$ is a given function. The vector $y(0)$ and the function u_0 are the initial conditions for the system (18). Formally, our definition of IFP deals with ordinary differential equations (1) only and is not applicable to delay systems. However, the following weaker condition holds for (18), see (Proskurnikov, 2016, p.141, proof of Lemma 7.3).

Lemma 13. For any solution of (18) one has

$$\int_0^T (y(t)^\top u(t) + \alpha|u(t)|^2)dt \geq -\mathcal{V} \quad \forall T \geq 0, \quad (19)$$

where $\mathcal{V} = \mathcal{V}(y(0), u_0(\cdot)) \geq 0$ is independent of T.

Lemma 13 allows to extend the synchronization criteria to ensembles of agents (20).

Theorem 14. For a group of linear delayed agents

$$\dot{y}_i(t) = u_i(t - \alpha_i), \quad i = 1, \dots, N, \quad (20)$$

the protocol (11) provides output synchronization (7) and L_2-synchronization (9), whenever the graph $\mathcal{G}[A]$ is strongly connected and (12) holds.

Remark 15. In the monograph Tian (2012) a more general result is formulated without a complete proof (Theorem 7.10), stating that under assumptions of Theorem 14 synchronization is retained if the graph is not strongly connected but has a directed spanning tree.

Remark 16. Theorem 11 also holds for agents (20). However, Assumption 10 becomes impractical since each agent has to be aware of $\bar{u}_i(t) = \dot{\bar{y}}(t + \alpha_j)$ at time t, which makes the controller (14) non-causal. The protocol (14) may still be used in the case where the reference signal is linear $\bar{y}(t) = v_0 t + \bar{y}(0)$ and v_0 is known, but $\bar{y}(0)$ is uncertain.

4. SYNCHRONIZATION IN VEHICLE PLATOONING AND TRAFFIC FLOW MODELING

In this section, we consider two practical applications of the synchronization criteria from Section 3.

4.1 Stability of a microscopic traffic flow model

A basic problem in vehicular traffic is the prevention of congestions and accidents. *Microscopic* traffic flow models are often employed to represent the traffic flow as a result of cooperation between individual drivers. Since the pioneering work of Chandler et al. (1958), the *delay* in drivers reaction has been recognized as a crucial factor participating into the overall flow dynamics. The simplest model of this kind (Chandler et al., 1958; Sipahi et al., 2007) deals with N vehicles, indexed 1 through N, traveling along a common straight or circular single lane road (their order remains unchanged since overtaking is not possible). Each

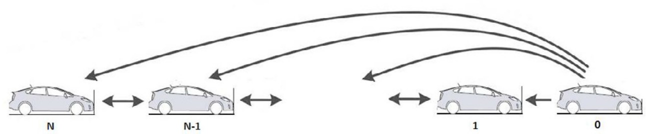

Fig. 1. Platoon of vehicles with bidirectional coupling.

driver is aiming to equalize his velocity of his own vehicle with that of its predecessor:

$$\dot{v}_i(t) = u_i(t-\alpha),\, u_i(t) = K(v_{i-1}(t) - v_i(t)). \qquad (21)$$

Here $v_i(t)$ is the speed of the i-th vehicle, α is the delay in its driver's action, and K stands for the driver's "sensitivity" to alterations of the relative velocity of the predecessor vehicle. In the case of straight road, $v_0(t) \equiv v_0$ is the desired velocity with respect to the leading vehicle 1; for a circular road, $v_0(t) \equiv v_N(t)$, i.e. vehicle 1 follows vehicle N. A key issue addressed via this model Chandler et al. (1958) is that of the stability of the "synchronous" manifold: $v_1 = \ldots = v_N$.

For the straight road case a necessary and sufficient condition for such a synchronization: $2\alpha K < 1$ was found in (21). We extend this classical result to the traffic flow model with a general directed interaction topology and *heterogeneous* delays and sensitivities of the drivers.

$$\dot{v}_i(t) = u_i(t-\alpha_i),\, u_i(t) = \sum_{j=1}^{N} a_{ij}(v_j(t) - v_i(t)),\ \forall i. \qquad (22)$$

The model (22) allows, in particular some drivers to respond to the change not only in the predecessor's, but also in the follower's velocity, or use the information about several predecessors and followers. The following theorem gives a criterion of velocity synchronization in (22) under the assumption of a strongly connected topology, which holds e.g. for uni- and bidirectional ring coupling (circular road). The gain $a_{ij} \geq 0$ in (22) stands for the sensitivity of driver i to changes in the speed of vehicle j. Theorem 14, applied to $y_i = v_i$, yields in the following corollary:

Corollary 17. Suppose that the graph $\mathcal{G}[A]$ is strongly connected and (12) holds. Then the vehicles' velocities are asymptotically synchronized $v_i(t) - v_j(t) \xrightarrow[t\to\infty]{} 0$.

4.2 An application to cooperative adaptive cruise control

In this subsection we demonstrate an application of Theorem 11 to the stability of a platoon of vehicles (Fig. 2), constituted by the leading vehicle 0 and N follower vehicles, indexed 1 through N (Fig. 1). Cooperative adaptive cruise control (CACC) system implements a control algorithm, making each vehicle keep the safe distance to the predecessor and follow the leader's velocity. The interaction topology between the vehicles may be different (Zheng et al., 2016); the most studied is a unidirectional topology, where each vehicle has information only about the predecessor.

In this subsection, we examine a CACC algorithm with *bidirectional* interactions. The advantages of bidirectional platooning algorithms over unidirectional ones are discussed e.g. in (Zhang et al., 1999; Barooah et al., 2009; Zheng et al., 2016) (see also references therein); in many senses such algorithms are more robust against disturbances propagating through the platoon ("string-stable").

We examine the CACC algorithm, proposed in (Barooah et al., 2009). The leader's speed $v_0(t) \equiv v_0$ is broadcasted to every follower (Fig. 1). Besides this, the vehicles 1 through $N-1$ measure the distances to *both* their predecessors and followers, and the rear vehicle N measures the distance to its predecessor. Denoting the position of vehicle i's rear bumper by $q_i \in \mathbb{R}$ (see Fig. 2), the goal of the CACC algorithm is to keep the desired distance to the predecessor and the desired velocity, i.e.

$$q_{i-1}(t) - q_i(t) \xrightarrow[t\to\infty]{} s_i, \quad v_i(t) = \dot{q}_i(t) \xrightarrow[t\to\infty]{} v_0. \qquad (23)$$

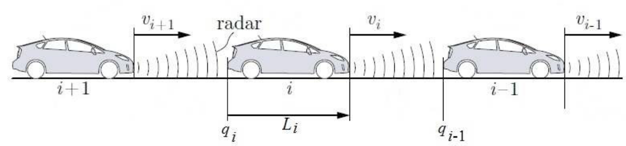

Fig. 2. Platoon of vehicles. Notation used in the text

As usual in CACC problems (Zhang et al., 1999; Zheng et al., 2016), the follower vehicles obey linear models

$$\tau_i \dddot{q}_i + \ddot{q}_i = a_{i,des}(t), \qquad (24)$$

where $a_{i,des}$ is the desired acceleration and τ_i is a time constant, depending on the vehicle's powertrain. The vehicles 1 through $N-1$ apply the following controller:

$$\begin{aligned} a_{i,des}(t) &= \mu_i(v_0 - v_i(t)) + \eta_i(q_{i-1}(t) - q_i(t) - s_i) \\ &+ \nu_i(q_{i+1}(t) - q_i(t) + s_{i+1}),\ 1 \leq i \leq N-1, \end{aligned} \qquad (25)$$

Vehicle N is controlled similarly, but has no follower

$$a_{N,des}(t) = \mu_N(v_0 - v_N(t)) + \eta_N(q_{N-1}(t) - q_N(t) - s_N). \qquad (26)$$

Theorem 18. Let $\mu_i \tau_i < \frac{1}{2}$ and $\eta_i, \nu_i > 0$ satisfy

$$\frac{\mu_i^2}{2} > \begin{cases} \eta_i + \nu_i, & 1 < i < N; \\ 2\eta_1 + \nu_1, & i = 1; \\ \eta_N, & i = N. \end{cases} \qquad \forall i \qquad (27)$$

Then the algorithm (25), (26) provides (23).

The result of Theorem 18 can be extended to some cases of nonlinear vehicles' dynamics, where the inner-loop engine and torque controllers (Zhang et al., 1999) fail to attenuate the nonlinearities. Notice that Theorem 18 does not address the *string stability* problem, i.e. the robustness of CACC against small disturbances in measurements as N becomes large; the analysis of string stability is based on other techniques and is beyond the scope of this paper.

5. CONCLUSIONS AND FUTURE WORK

In this paper, we offer simple distributed protocols for synchronization of heterogeneous non-passive agents that satisfy an IFP property. We apply the obtained results to analysis of microscopic traffic flow models and CACC algorithms for heterogeneous platoons. The results can be extended to some nonlinearly coupled networks, time-varying graphs and "coopetitive" networks, where agents can both cooperate and compete (Proskurnikov and Cao, 2016). The robustness of synchronization algorithms against delays and noises is a subject of ongoing research.

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