Performance of one-way carsharing systems under combined strategy of pricing and relocations

Lu, Rongqin; Correia, Gonçalo Homem de Almeida; Zhao, Xiaomei; Liang, Xiao; Lv, Ying

DOI
10.1080/21680566.2020.1819912

Publication date
2021

Document Version
Accepted author manuscript

Published in
Transportmetrica B

Performance of one-way carsharing systems under combined strategy of pricing and relocations

Rongqin Lu[a], Gonçalo Homem de Almeida Correia[b,*], Xiaomei Zhao[a,*] , Xiao Liang[b], Ying Lv[a]

[a] *School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, PR China*
[b] *Department of Transport and Planning, Delft University of Technology, Delft, Netherlands*

Abstract

A bilevel nonlinear mathematical programing model is formulated to determine the optimal pricing and operator-based relocations in a one-way station-based carsharing system in competition with private cars. In the upper level, the carsharing operator determines the vehicle fleet, prices, and relocation operations with the objective of maximizing profits, considering the potential reaction of travelers. In the lower level, travelers choose travel modes from a cost-minimization perspective. Travel utilities are calculated through a logit model. The Karush–Kuhn–Tucker conditions are used to transform the bilevel model into a single-level model and then a genetic algorithm is proposed to solve it. Computational tests in four different scenarios show the combined strategy is the best one. The four scenarios are base, relocations, dynamic pricing, and a combination of relocations and pricing separately. The combined strategy can make the best trade-offs between the operator's profit and the travelers' cost.

Keywords: carsharing, relocations, pricing, bilevel nonlinear programming model

1. Introduction

Over the last few decades, shared cars have emerged as an alternative to private cars in view of their cost and environmental benefits. With the carsharing system, passengers can still have access to a car despite not owning one. Several studies have confirmed that carsharing not only helps reduce car ownership, but also greenhouse gas emissions (Martin and Shaheen 2011; Namazu and Dowlatabadi 2018). A National Household Travel survey pointed out that automobiles in the USA spend approxi- mately 90% of the time parking in a lot (Hu 2001). Shared cars have a higher utilization rate, making them more efficient than the private ones (Jorge and Correia 2013). Furthermore, the environmental and social benefits of carsharing can be enhanced by the use of electric vehicles (EVs) (Li et al. 2016; Vasconcelos et al. 2017). Because of these aspects, carsharing is now becoming popular all over the world, and many studies are focusing on how to improve the management of such systems to make them more efficient and convenient.

Carsharing systems are classified into round-trip and one-way carsharing, depending on the need to return the car to its point of origin. Logically, one-way systems are much more convenient, which is contributing to their rising popularity. However, compared with the round-trip systems, one-way systems are facing increasing challenges, especially the vehicle stock imbalance problem (Correia and Antunes 2012). This problem is caused by the asymmetric demand between the pairs of stations or zones. Vehicle stock imbalance is when more cars arrive at stations with low demand, or fewer cars arrive at stations (zones) with high demand (Boyaci, Zografos, and Geroliminis 2015). Relocation of vehicles is the most popular way to avoid the concentration of shared cars in a certain zone or station (Barth, Han, and Todd 2001; Boyaci, Zografos, and Geroliminis 2015; Jorge, Molnar, and Correia 2015; Illgen and Hock 2019).

The relocation can be divided into staff-based and user-based operations. In staff-based operations, the operator hires staff members to relocate the automobiles between zones or stations. Staff

* The First Corresponding author
E-mail address: xmzhao@bjtu.edu.cn (Xm Zhao)
Co-corresponding author
E-mail address: g.correia@tudelft.nl (Goncalo Homem de Almeida Correia)

members use bike or public transportation to move before and after the relocation. In user-based oper- ations, policies are used to influence the travelers' behavior to obtain a desirable and more balanced outcome (Clemente et al. 2018). For example, pricing can be used to convince users to change their origin or destination, or even departure time (Jorge, Molnar, and Correia 2015).

In this paper, we intend to study the combination of relocations and pricing strategy to influence the performance of a station-based one-way carsharing system from the perspectives of both the com- pany and the travelers. The rest of the paper is organized as follows. Section 2 presents a literature review of the present practices related to the pricing and relocations in one-way carsharing systems. Section 3 introduces a bilevel nonlinear mathematical programing (BLNLMP) model for optimizing the relocations and pricing of a one-way station-based carsharing system. Section 4 details the transfor- mation and solution algorithm for the model. Section 5 describes a case study on the application of the model in the city of Rotterdam, The Netherlands. Section 5 summarizes the conclusions of the study.

2. Literature review

Vehicle relocation is a common method to bring balance to the one-way carsharing systems. Jorge, Correia, and Barnhart (2014) presented two models to study different relocation policies in a one- way station-based carsharing system. A mathematical model was proposed to optimize the relocation operations, and a simulation model was built to test different real-time relocation policies. The two models demonstrated the potential of relocations in improving the profits earned by a carsharing operator. Nourinejad and Roorda (2014) used a dynamic, integrated optimization–simulation model to maximize the profits earned by the carsharing operator. A discrete event simulation was proposed to solve the model. In the system they tested, all shared cars were required to be reserved beforehand. The results showed that the relocation hours and fleet size were influenced by the reservation time. Weikl and Bogenberger (2016) built an integrated model to get the optimal relocation policy. Zakaria et al. (2018) built a single level integer linear programming model to solve the relocation problem. A greedy algorithm was used to test the staff-based relocation strategy by considering three different policies. Policy 1 prioritized the time required by the relocators to move between stations, policy 2 prioritized the number of available cars in each station to make the system more balanced, and policy 3 considered the consequences of current relocations. The results found a strong relationship between the user request satisfaction and the relocation operations. Hu and Liu (2016) showed that the customer service rate could be higher even under lower parking capacities and fleet size. Spieser, Samaranayake and Gruel (2016) analyzed the relationship among the key performance, fleet size and relocations to maximize the carsharing operator's profit. Huang, Correia, and An (2018) proposed a single-level mixed-integer nonlinear programming model to solve the carsharing location and capacity problems associated with relocations. A customized gradient algorithm was proposed to solve the nonlinear model. They applied the model to the case study of Suzhou, China, and demonstrated that the location and capacity of carsharing stations affect the performance of relocations. Gambella et al. (2017) used relocations to restore balance to the vehicle distribution in one-way shared EV systems. Relocations can be done either in the operating hours or at night. An integer programming model was defined by considering the battery consumption and charging time of EVs. The authors concluded that relocations played a pivotal role in determining the performance of carsharing. Santos and Correia (2019) considered hiring a permanent team of employees to oversee maintenance operations, including relocations, and concluded that staff-based relocations brought about only a small improvement in the revenues, since the staff can only relocate a limited number of vehicles in an hour. Nevertheless, most research papers on the topic assumed that it would be possible to hire people for each relocation trip, and that there would be enough people for all the required relocations.

As explained earlier, the operator changes the price to influence the travelers' behavior, i.e. to encourage travelers to pick up vehicles from an oversupplied station or zone and/or drop them off at an undersupplied station or zone. Discount, coupon, price incentive, or penalty are some of the pricing strategies. Febbraro, Sacco, and Saeednia (2012) pointed out that setting the right price was of paramount importance to carsharing systems. Waserhole and Jost (2013a, 2013b) used pricing as incentive to improve the efficiency of carsharing systems. Ren et al. (2019) showed that a novel dynamic pricing scheme can help the electric vehicle-sharing operator maximize the profit. Wood and Jones-Meyer (2016) showed that the travelers' choices can be affected by the adjustment of prices. In their study, they performed this adjustment by applying different discount rules. The core rule that they proposed was that the more the number of travelers in a car, the higher the discount for all travelers. In their study, a two-person carpool receives a 50% discount, and three or more people would travel free. The application of the model to a case study showed that discounts draw more users in real- time to share rides. Jorge and Correia (2013) proposed a trip pricing strategy for one-way carsharing systems, to control the demand. The prices varied for each zone and time. A single-level mixed-integer nonlinear programming model was developed to maximize the profit earned by the carsharing company. An iterative local-search metaheuristic was proposed to solve the model. The application of the model to the case study of Lisbon, Portugal, showed that pricing can help the operator increase their daily profits by a considerable margin. Angelopoulos et al. (2018) used a pricing strategy to solve the imbalance problem. Different price incentives were given to travelers based on the vehicle relocation priority. Users rejected the pricing scheme with a certain probability. In the end, more cars would be picked up from oversupplied stations, and consequently more cars would be dropped off at undersupplied stations. Cheng, Liu, and Szeto (2019) showed the importance of dynamic pricing in reducing the total travel time, considering the travel distance and time delay. Jakob, Menendez, and Cao (2020) proposed an optimal parking pricing scheme to maximize the revenue for city councils without a significant negative effect on the network. Jian, Rey, and Dixit (2019) linked user demand and vehicle supply by vehicle availability in one-way carsharing systems. A case study of a Sydney carsharing system showed that the importance of the price variations with change of demands.

However, relocations can significantly increase the operational costs of a carsharing system, especially due to the high labor costs of the staff used to relocate cars. A pricing scheme can avoid the labor cost, but not all prices will be accepted by the passengers. Few studies have focused on the combination of pricing and relocation. Barth, Todd, and Xue (2004) studied pricing to encourage users to avail trip splitting or registering with a carsharing system. Based on the stock of vehicles, travelers starting from the same origin and traveling to the same destination at the same time were incentivized to drive in separate cars if the destination needed more cars, or travel in one car if the destination was over- supplied. The results showed that pricing can help reduce the number of relocation trips by 42%. Reiss and Bogenberger (2017) proposed a combination strategy for a free-floating bike-sharing system by integrating relocations and a pricing scheme. All the relocation tasks were rated based on the imbalance urgency levels. If less than 15% of the bikes needed to be relocated, only the pricing strategy was applied. Otherwise, relocations needed to be combined with the pricing method. This exemplified how these two strategies can work together. However, it is important to recognize that relocations in bike-sharing systems are essentially a routing problem since several bikes can be transported in a truck simultaneously. Xu, Meng, and Liu (2018) set up a single-level mixed-integer nonlinear and non- convex programming model in one-way carsharing systems. Trip pricing, fleet size, vehicle relocation and personnel assignment were all considered to maximize the profit of the carsharing operator. And the outer-approximation method was proven to be efficient.

Existing researches have studied the performance of relocations or pricing strategies by proposing single-level optimization models. In most cases, the objective function is one of the following: maximize the daily profit earned by the carsharing operator (Jorge, Correia, and Barnhart 2014;

Vasconcelos et al. 2017; Bruglieri, Pezzella, and Pisacane 2017; Chang et al. 2017; Huang, Correia, and An 2018; Febbraro, Sacco, and Saeednia 2019), minimize the total costs of the operator (Nourinejad et al. 2015), minimize the total unused time of shared cars (Barth, Todd, and Xue 2004), maximize the satisfaction rate of travel requests (Bruglieri, Colorni, and Luè 2014; Bruglieri, Pezzella and Pisacane 2018), maximize the number of trips (Carlier, Munierkordon, and Klaudel 2015; Boyaci, Zografos, and Geroliminis 2017), maximize the service level to satisfy more potential requests (Pfrommer et al. 2014), minimize the total relocation time (Kek et al. 2009), or minimize the number of staff (Bruglieri, Pezzella, and Pisacane 2018). These objectives depend either on the operator's interests or the travelers'. It is obvious that, in some situations, the objective of the operator to increase profits and the travelers' goal of traveling cheap and fast are in conflict.

The operator is a leader in the carsharing system management, and travelers are the followers. As a leader, the operator must determine the system configuration considering the potential actions of the travelers. As followers, travelers make their choices to achieve their goals within the conditions laid down by the leader. As far as we know, only one paper considered the trade-offs between the carsharing operator and travelers in the same model. Nair and Millerhooks (2014) built a bilevel mixed-integer model to determine the optimal configuration of a carsharing system. In the upper level, the operator aimed to maximize the revenue by determining the location and size of stations and vehicle inventories. At the lower level, all travelers' costs were minimized, considering the equilibrium of the network. The KKT conditions were used to transform the model into a single-level, mixed-integer programming model. That research, however, mainly focused on the location and size of carsharing stations; the strategies that should be performed in a given system state, such as pricing and relocations, were not studied.

In this paper, we propose a bilevel nonlinear mathematical programing (BLNLMP) model that con- siders the profit earned by the operator and the travelers' costs simultaneously. In this model, private and shared cars coexist, and travelers make mode choices based on the travel utility. The carsharing travel utility changes with the price set by the operator. The utility of the private car alternative is mainly related to the travel time of the trip, the depreciation and maintenance costs of the vehicle, as well as the parking fee. Relocations and pricing schemes are combined to improve the performance of the carsharing operation. Relocations are performed at the beginning of each time step. Pricing is used to modulate the demand of carsharing according to each OD pair and the time of day. Decisions of relocations and pricing as well as vehicle fleet are all made by the operator to maximize its profit in the upper level. Mode choices are done by all travelers to minimize their costs in the lower level. The reactions of travelers are therefore strongly linked to the decisions of the carsharing operator. A set of KKT conditions are proposed to transform the bilevel model into a single-level model. Since the model is nonlinear, a genetic algorithm (GA) is used to solve the transformed model.

3. Model formulation

3.1 Assumptions

The following assumptions are made for the bilevel model:

- The total travel demand for a car in each OD pair and the time of day are known in advance through historical data.
- The entire car demand is satisfied by two modes: private cars or shared cars. Each person owns a private car for every trip.
- Cars do not change travel times on the road network.
- Parking spaces for shared vehicles in each zone are the property of the operator.
- The operator pays the maintenance fee of the parking places per day and private car users have to pay their parking.
- The fuel consumption rate of shared cars is paid for by the operator.
- The carsharing system charges users by time step.
- Cars are relocated at the beginning of each time step.
- The carsharing operator pays the relocation costs based on the time of relocation.
- Private cars and shared cars are the same type of vehicle.
- The depreciation costs of shared cars and private cars are calculated in the same manner.

The operator covers the costs for shared cars. The customers pay the costs of private cars.

3.2 Notation

Listed below are the notations used in this paper:

Sets	
$\boldsymbol{A}: \{\left(i_t, j_{t+\delta_{ij}^t}\right)\}$	Set of arcs representing a movement between zones i and j, $\forall i, j \in \boldsymbol{J}, i \neq j$, from time step t to $t+\delta_{ij}^t$
$\boldsymbol{J}: \{i\}$	Set of traffic zones. The indices used for zones are i, j.
$\boldsymbol{T}: \{t\}$	Set of time steps in the operation period. The index used for the time steps is t.
$\boldsymbol{X}: \{i_t\}$	Set of nodes of the time-space network combining J zones with T time steps, where i_t represents zone i at time step t.
Parameters	
α	Average number of private car trips per day
b_0	Coefficient used in the logit model to express the special preference for the car in relation to carsharing
b_1	Coefficient used in the logit model to express the importance of cost for the mode choice
δ_{ij}^t	Travel time from zone $i \in \boldsymbol{J}$ to $j \in \boldsymbol{J}$ where $i \neq j$, departure time step is t
C_f	Depreciation and maintenance costs per day for shared and private cars
C_g	Fuel consumption per time step
C_m	Maintenance costs of a carsharing parking spot per day
C_s	Parking fee per private car per trip
C_r	Relocation cost per time step
$D_{i_t j_{t+\delta_{ij}^t}}$	Total demand for using a car (carsharing and private cars) from zone $i \in \boldsymbol{J}$ to $j \in \boldsymbol{J}$, where $i \neq j$, from time step t to $t+\delta_{ij}$
$D_{i_t j_{t+\delta_{ij}^t}}^{ps}$	Potential carsharing demand from zone $i \in \boldsymbol{J}$ to $j \in \boldsymbol{J}$, where $i \neq j$, from time step t to $t+\delta_{ij}^t$
$P_{i_t j}^{low}, P_{i_t j}^{up}$	Lower and upper bound of the price per time step charged from zone $i \in \boldsymbol{J}$ to $j \in \boldsymbol{J}$, where $i \neq j$ starting at time step t
Q_i	Number of parking places in zone i, $\forall i \in \boldsymbol{J}$
r	Ratio of the price of shared cars to the cost of private cars per time step
Decision variables for the upper level	
a_{i_t}	Number of shared cars in zone i at the beginning of a time step, $\forall\, i \in \boldsymbol{J}, t \in \boldsymbol{T}$
$R_{i_t j_{t+\delta_{ij}}}$	Number of shared cars relocated from zone $i \in \boldsymbol{J}$ to $j \in \boldsymbol{J}$, where $i \neq j$ from time step t to $t+\delta_{ij}$
$P_{i_t j}^s$	Price charged by the operator per time step from zone $i \in \boldsymbol{J}$ to $j \in \boldsymbol{J}$, where $i \neq j$ from time step t
Decision variables for the lower level	
$V_{i_t j_{t+\delta_{ij}}}^s$	Number of shared cars being used from zone $i \in \boldsymbol{J}$ to $j \in \boldsymbol{J}$, where $i \neq j$ from time step t to $t+\delta_{ij}$

3.3 Problem setting

The research objective of this paper is to find the best pricing and relocation operations to achieve the goals of the operator and travelers simultaneously. As a leader in the upper level, the operator will aim to maximize profit by weighing revenue and relocation, maintenance, and depreciation costs of shared cars, as well as the maintenance cost of shared parking places. Travelers are followers in the lower level, and they aim to minimize transport costs either by using carsharing or their personal car.

The decision variables a_{i_t} set the number of available shared cars at the beginning of each time step t in different zones i. Variables $R_{i_t j_{t+\delta_{ij}^t}}$ determine the relocation trips from zone i to j from time steps t to $t+\delta_{ij}^t$. Variables $P_{i_t j}^s$ determine the final price per time step the carsharing users are charged for transport from zone i to zone j, starting at time step t. The three types of decision variables are in the upper level and determined by the carsharing operator. Variables $V^s_{i_t j_{t+\delta^t_{ij}}}$ determine the number of travelers choosing carsharing from zone i to j, starting at time step t, in the lower level.

3.4 Bilevel Mathematical Model

By using the above notations, we formulated both levels of the model as follows.

3.4.1 Upper level

$$\max_{R_{i_t j_{t+\delta_{ij}}},\, a_{i_t},\, P^s_{i_t j_{t+\delta^t_{ij}}}} \theta = \sum_{\left(i_t, j_{t+\delta_{ij}}\right) \in \boldsymbol{A}} \left(V^s_{i_t j_{t+\delta^t_{ij}}} \delta^t_{ij} \left(P^s_{i_t j_{t+\delta^t_{ij}}} - C_g\right) - R_{i_t j_{t+\delta^t_{ij}}} \delta^t_{ij} \left(C_r + C_g\right) \right) - \sum_{i \in \boldsymbol{J}} \left(a_{i_1} * C_f + Q_i * C_m\right) * e \tag{1}$$

Subject to:

$$a_{i_t} \le Q_i \qquad \forall\, i_t \in \boldsymbol{X} \tag{2}$$

$$\sum_{j \in \boldsymbol{J}} R_{i_t j_{t+\delta^t_{ij}}} \le a_{i_t} \qquad \forall \left(i_t . j_{t+\delta^t_{ij}}\right) \in \boldsymbol{A} \tag{3}$$

$$P^{low}_{i_t j_{t+\delta^t_{ij}}} \le P^s_{i_t j_{t+\delta^t_{ij}}} \le P^{up}_{i_t j_{t+\delta^t_{ij}}} \qquad \forall \left(i_t . j_{t+\delta^t_{ij}}\right) \in \boldsymbol{A} \tag{4}$$

$$a_{i_t} \ge 0 \qquad \forall\, i_t \in \boldsymbol{X} \tag{5}$$

$$R_{i_t j_{t+\delta^t_{ij}}} \ge 0 \qquad \forall \left(i_t, j_{t+\delta^t_{ij}}\right) \in \boldsymbol{A} \tag{6}$$

The objective function (1) in the upper level maximizes the profit earned by the operator in the operation time, factoring in the revenue paid by the travelers, cost of fuel, relocation, parking space, and depreciation and maintenance of the shared cars. e is a coefficient used to express the proportion of the study period in relation to one day. Constraints (2) ensure that the number of available shared cars in zone i at time step t is lower than the capacity of that zone. Constraints (3) assure that the number of cars relocating from zone i since the beginning of time step t is fewer than those of the available shared cars. Constraints (4) are used to restrict the lower and upper bounds of the price the operator can charge the clients. Expressions (5)–(6) set the domain for the decision variables in the upper level.

3.4.2 Lower level

$$\min_{V^s_{i_t j_{t+\delta^t_{ij}}}} \pi = \sum_{\left(i_t, j_{t+\delta^t_{ij}}\right) \in \boldsymbol{A}} \left\{ V^s_{i_t j_{t+\delta^t_{ij}}} P^s_{i_t j_{t+\delta^t_{ij}}} \delta^t_{ij} + \left(D_{i_t j_{t+\delta^t_{ij}}} - V^s_{i_t j_{t+\delta^t_{ij}}}\right)\left(C_g \delta^t_{ij} + \frac{C_f}{\alpha} + C_s\right) \right\} \tag{7}$$

Subject to:

$$D^{ps}_{i_t j_{t+\delta^t_{ij}}} = \frac{e^{b_1 P^s_{i_t j_{t+\delta^t_{ij}}} \delta^t_{ij}}}{e^{b_1 P^s_{i_t j} \delta^t_{ij_{t+\delta^t_{ij}}}} + e^{b_0 + b_1 \left(C_g \delta^t_{ij} + \frac{C_f}{\alpha} + C_s\right)}} D_{i_t j_{t+\delta^t_{ij}}} \qquad \forall \left(i_t, j_{t+\delta^t_{ij}}\right) \in \boldsymbol{A} \tag{8}$$

$$V^s_{i_t j_{t+\delta^t_{ij}}} \le D^{ps}_{i_t j_{t+\delta^t_{ij}}} \qquad \forall \left(i_t, j_{t+\delta^t_{ij}}\right) \in \boldsymbol{A} \qquad \left(\boldsymbol{\alpha}_{i_t j_{t+\delta^t_{ij}}}\right) \tag{9}$$

$$a_{i_t} + \sum_{j_{t-\delta_{ij}} \in \boldsymbol{X}} \left(V^s_{j_{t-\delta_{ij}} i_t} + R_{j_{t-\delta_{ij}} i_t}\right) - \sum_{j_{t+\delta^t_{ij}} \in \boldsymbol{X}} \left(V^s_{i_t j_{t+\delta^t_{ij}}} + R_{i_t j_{t+\delta^t_{ij}}}\right) = a_{i_{t+1}}$$

$$\forall i_t \in \boldsymbol{X}, \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (\boldsymbol{\beta}_{\boldsymbol{i_t}}) \tag{10}$$

$$\sum_{j \in X} \left(V_{i_t j_{t+\delta_{ij}^t}}^s + R_{i_t j_{t+\delta_{ij}^t}}^s \right) \le a_{i_t} \qquad \forall i_t \in \boldsymbol{X}, \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (\boldsymbol{\rho}_{\boldsymbol{i_t}}) \tag{11}$$

$$V_{i_t j_{t+\delta_{ij}^t}}^s \ge 0 \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (\boldsymbol{\gamma}_{\boldsymbol{i_t j_{t+\delta_{ij}^t}}}) \tag{12}$$

Objective function (7) in the lower level minimizes the total travel cost for both shared and private car users. For carsharing travelers, the rental fee of a shared car is the only applicable cost. For private cars, the total cost consists of the costs of fuel, maintenance, depreciation per trip, and parking space rental. The parking fee for private car users is charged per trip because, in this model, the duration of parking of the cars in each zone cannot be predicted. Constraints (8) help compute the potential number of carsharing travelers, which means the maximum number of carsharing users according to the logit model. Constraints (9) assure that the number of users will not surpass that maximum. Constraints (10) impose flow conservation in the network. Subscript t' finds the departure time step of the car's departure from zone j to its arrival at zone i during time step t. Constraints (11) ensure all the number of shared cars departing from a zone does not exceed the available cars. Constraints (12) set the domain for the decision variables in the lower-level model.

4 Solution approach

The upper-level formulation has an objective function that includes the lower-level decision variables. The objective (1) and logit model used in constraints (8) render the bilevel model nonlinear and non-convex. To solve this complex model, we used a combination of GA and KKT conditions.

The dual variables (also called KKT multipliers or Lagrange multipliers) for each lower level constraints are indicated in bold and parenthesis in each constraint. They are $\alpha_{i_t j_{t+\delta_{ij}^t}}$ for constraints (9), β_{i_t} for constraints (10), ρ_{i_t} for constraints (11) and $\gamma_{i_t j_{t+\delta_{ij}^t}}$ for constraints (12) respectively. There are no dual variables for constraints (8), because the derivations obtained by the only decision variable in the lower level, $V_{i_t j_{t+\delta_{ij}^t}}^s$, amount to 0. Those variables are used in the KKT conditions to help convert the lower-level model into a mathematical program with equilibrium constraints (MPEC). The prices are set to be constant value, varying only during the GA process, which makes the lower-level model linear and convex. Thus, the KKT conditions are necessary and sufficient for $V_{i_t j_{t+\delta_{ij}}}^s$ to be a solution of the lower-level problem (Kanzow, 1994). The feasible prices are found by a metaheuristic approach such as GA.

4.1 KKT conditions for the lower-level model

The KKT conditions include stationarity conditions, primal feasibility constraints, dual feasibility constraints, and complementary slackness. The KKT conditions of the lower-level model are defined by objective (7) and constraints (8)–(12). The stationarity conditions are:

$$P_{i_t j_{t+\delta_{ij}^t}}^s \delta_{ij}^t - \left(C_g \delta_{ij}^t + \frac{C_f}{\alpha} + C_s \right) + \alpha_{i_t j_{t+\delta_{ij}^t}} + \beta_{j_{t-\delta_{ij}}} - \beta_{i_t} + \rho_{i_t} - \gamma_{i_t j_{t+\delta_{ij}^t}} = 0 \qquad \forall \left(i_t, j_{t+\delta_{ij}}\right) \in \boldsymbol{A} \tag{13}$$

Primal feasibility constraints are (8) to (12)

Dual feasibility implies the following:

$$\alpha_{i_t j_{t+\delta_{ij}^t}}, \rho_{i_t}, \gamma_{i_t j_{t+\delta_{ij}^t}} \ge 0 \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \tag{14}$$

The set of complementarity conditions are as follows:

$$\alpha_{i_t j_{t+\delta_{ij}^t}} \times \left(V_{i_t j_{t+\delta_{ij}^t}}^s - D_{i_t j_{t+\delta_{ij}^t}}^{ps}\right) = 0 \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (15)$$

$$\rho_{i_t} \times \left(\sum_{j \in \boldsymbol{X}} \left(V_{i_t j_{t+\delta_{ij}^t}}^s + R_{i_t j_{t+\delta_{ij}^t}}^s\right) - a_{i_t}\right) = 0 \qquad \forall i_t \in \boldsymbol{X}, \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (16)$$

$$\gamma_{i_t j_{t+\delta_{ij}^t}} \times V_{i_t j_{t+\delta_{ij}^t}}^s = 0 \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (17)$$

The MPEC is now defined by objective function (1), upper-level constraints (2)–(6), KKT primal feasibility constraints (8)–(12), KKT stationarity conditions (13), dual feasibility constraints (14), and complementarity constraints (15)–(17). The complementarity constraints are nonlinear, but they can be converted into linear constraints.

4.2 Dealing with complementarity

A big-M is used to transform the complementarity constraints. A typical nonlinear constraint is $z(q + Qz) = 0$, which can be converted into two linear constraints by using a binary variable, u, where constraints $z \leq Mu$ and $q + Qz \leq M(1 - u)$ are imposed (Nair et al. 2014). The complementarity constraints (15)–(17) can be rewritten as linear constraints in the same way. Binary variables $\eta_{i_t j_{t+\delta_{ij}^t}}$, σ_{i_t} and $\lambda_{i_t j_{t+\delta_{ij}^t}}$ are introduced to perform these transformations.

Constraints (15) can be transformed into constraints (18)–(20) as follows,

$$\alpha_{i_t j_{t+\delta_{ij}^t}} \leq M \eta_{i_t j_{t+\delta_{ij}^t}} \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (18)$$

$$V_{i_t j_{t+\delta_{ij}^t}}^s - D_{i_t j_{t+\delta_{ij}^t}}^{ps} \leq M\left(1 - \eta_{i_t j_{t+\delta_{ij}^t}}\right) \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (19)$$

$$\eta_{i_t j_{t+\delta_{ij}^t}} \in \{0,1\} \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (20)$$

Constraints (16) can be transformed into constraints (21)–(23),

$$\rho_{i_t} \leq M \sigma_{i_t} \qquad \forall\, i_t \in \boldsymbol{X} \qquad (21)$$

$$\sum_{j \in \boldsymbol{X}} \left(V_{i_t j_{t+\delta_{ij}^t}}^s + R_{i_t j_{t+\delta_{ij}^t}}^s\right) - a_{i_t} \leq M(1 - \sigma_{i_t}) \qquad \forall i_t \in \boldsymbol{X}, \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (22)$$

$$\sigma_{i_t} \in \{0,1\} \qquad \forall\, i_t \in \boldsymbol{X} \qquad (23)$$

Constraints (17) can be transformed into (24)-(26),

$$\gamma_{i_t j_{t+\delta_{ij}^t}} \leq M \lambda_{i_t j_{t+\delta_{ij}^t}} \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (24)$$

$$V_{i_t j_{t+\delta_{ij}^t}}^s \leq M\left(1 - \lambda_{i_t j_{t+\delta_{ij}^t}}\right) \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (25)$$

$$\lambda_{i_t j_{t+\delta_{ij}^t}} \in \{0,1\} \qquad \forall \left(i_t, j_{t+\delta_{ij}^t}\right) \in \boldsymbol{A} \qquad (26)$$

Now, the BLNLMP model can be expressed as a single-level, nonlinear programming model. The new model is defined by objective (1), upper-level constraints (2)–(6), and KKT primal feasibility constraints (8)–(12), KKT stationarity conditions (13), dual feasibility constraints (14), and the set of transformed complementarity constraints (18)–(26). If a set of prices is given for a carsharing system, the model is linear and can be solved by any commercial solver, including Cplex or Xpress (which is the tool we use in this paper).

4.3 Solution algorithm

The following pseudo-code shows the GA procedure used to iterate between the sets of prices.

Let K be the maximum number of iterations, N the solution population size, and θ the value of the objective function (1)

Algorithm 1: Iterative procedure for the genetic algorithm

Initialization

Step 1: Set $k = 1$, initialize $P^{k.n}$ randomly.

Step 2: Given $P^{k.n}$, the daily profit $\theta^{k,n}(P^{k,n})$ can be calculated by the new single-level model.

Step 3: Sort the value of the objective and update the maximum in each iteration, save it as θ^{k^*}.

Step 4: If $\frac{|\theta^{k^*}-\theta^{(k-10)^*}|}{\theta^k} < \varepsilon$ and $k > 20$ or $k = K$, stop. Else, proceed to **Step 5.**

Step 5: Crossover and mutation of the sorted price.

Step5.1: when the random rate is lower than the crossover rate P_c, intersection occurs. Select two prices by Roulette Wheel Selection and perform a crossover. The new prices are formed for the next iteration.

Step5.2: when the random rate is lower than the mutation rate P_m, mutation occurs. New prices then are generated randomly in steps of 0.1 between the lowest and highest prices for the next iteration.

Step 6: New prices are generated after **Step 5**, set $k = k + 1$, and go to **Step 2**.

In this paper, the parameters of the GA are set as follows: maximum number of iterations $K = 100$, population size $N = 30$, crossover rate $P_c = 0.9$, and mutation rate $P_m = 0.1$. The initial prices $p^{1.n}$ of the first iteration are generated randomly between the lowest and highest prices. Based on the given prices, the objective can be calculated directly. The maximum objective is always retained, and if the value of the previous iteration is less than the latest iteration, then it is replaced. The algorithm terminates when the number of iterations is greater than 20, and the difference between the value of the objective function of the current iteration and the value of the objective function of the previous ten iterations is less than ε, which is set to be 0.001.

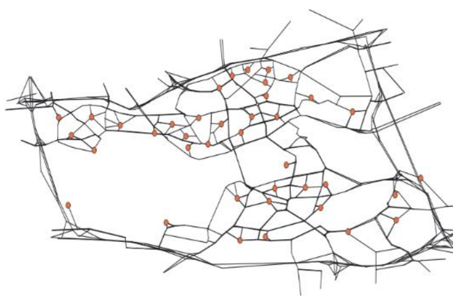
5. Case study: Rotterdam, The Netherlands

5.1 Setting up the case study

To demonstrate the working of the algorithm, we performed a case study with Rotterdam, in The Netherlands, as the study area (Fig. 1 a). The population of the city as per 2016 census was 994,000. The municipality of Rotterdam covers a total surface area of 325.79 km^2.

Data required for conducting the study are a) a set of zones, b) a demand trip matrix, c) the price range per time step charged by the operator, d) time taken by the cars to drive between different zones, and e) operating costs for shared cars and parking lots. The locations of carsharing parking lots are assumed to be in the centroid of each zone (Fig. 1 b).

a) Rotterdam in Google maps



b) Centroids of all 39 zones in the city center

Fig. 1 Study region in Rotterdam and centroids of all the zones

The trip matrix is based on data provided through a macro-transport demand model built with OmniTrans software, which the city uses for its transport planning. The data includes a full-day OD demand matrix and an OD matrix for every 15 min. In this case study, the number of zones used is 39 (the central area of the city). The number of car trips during the operation time is 46,423. Based on the number of shared cars in Rotterdam, the number of carsharing spots in each zone is set to 50. Therefore, the upper bound for the fleet size in this case study is 1,950. We consider a daytime operation of 4.5 h from 5:30 am to 10:00 am. Therefore, the study covered only the morning period, partitioned into 18 time steps with a total duration of 15 min.

Values of the parameters in the model are taken from relevant material (Ashkrof et al. 2019; Huang et al. 2018) and the official website of the Government of the Netherlands (Table 1).

Table 1 Parameters for the case study

Parameter	α	b_0	b_1	C_f	C_g	C_r	C_{mp}	C_s
Value	2	0.751	-0.328	7	0.5	2.34	2	2
Unit	-	-	-	€ per day	€ per 15 min	€ per 15 min	€ per day	€ per trip

α, the average trip number of private car users per day, is 2. The depreciation cost per fuel car per day is € 7 per day (Kai et al., 2018). b_0 and b_1 are coefficients representing the importance of carsharing or private cars in the logit model in constraints (8), and their values were taken from Ashkrof et al. (2019). In this paper, the salary of staff members is € 9.36 per hour, and C_r is € 2.34 per time step. This information is published by the Government of the Netherlands (https://www.government.nl/topics/minimum-wage/amount-of-the-hourly-minimum-wage). The remaining parameters C_{mp}, C_s, and C_g were derived from the average market prices. For a carsharing company in Rotterdam, we vary the price between € 0.28 per min to € 0.36 per min, which means it can take any value between € 4.20 per 15 min and € 5.40 per 15 min.

5.2 Experiments and results

The transformed MPEC was implemented using the solver Xpress 8.5 (FICO) in four different scenarios. All experiments were performed on a computer equipped with a 3.6 GHz Intel Core W-2123 processor and 32 GB of RAM with Windows 10 OS.

The four scenarios are named: base, relocation, pricing, and combination. The base scenario is used as the benchmark, but not pricing or relocation. In the relocation scenario, the relocations performed by the staff are applied. In the pricing scenario, the operator defines the prices that must be charged for transporting customers between different zones at different time periods. The combination scenario is the most complex among the four scenarios, because it implements both pricing and relocations simultaneously. In the base and relocation scenarios, prices are the inputs, which means the models are single-level linear models and have optimal, rapid and easily solutions. In the other two scenarios, prices are decision variables, making the model nonlinear, for which the GA described in the previous section is used. Thus, the model can be solved with fixed prices, and optimality cannot be proven.

Several indicators are used to analyze the performance of the model in increasing the profit and reducing the costs paid by the travelers. The results for the four scenarios are presented in Table 3. In the following sections, each scenario will be analyzed and compared in detail.

5.2.1 Base scenario

The operator does not solve the vehicle imbalance problem in the base scenario since it is the benchmark against which the results from the other three scenarios will be compared.

The price is constant for different OD pairs at different time periods. To study the performance of the system, the carsharing price was increased from € 4.20 per 15 min to a high price of € 15.00 per 15 min in steps of € 0.05. As for the cost of private car travelers, the carsharing price is kept constant at € 6.00 per 15 min. The charts with different indicators for both the base and relocation scenarios are shown in Fig. 2. The results show that this price is vital to the profit earned by the operator.

To demonstrate the impact of price on the operator and the travelers, we will explore different price ranges where the price is divided into four parts based on a ratio r. The ratio r equals the price that the carsharing operator charges per time step divided by the amount that the private car drivers pay per time step. When $r < 1$, the price charged by the carsharing operator from travelers in Rotterdam is in the range of € $[4.20, 5.40]$ per 15 min. When $r \leq 1$, the price range is € $[4.20, 6.00]$ per 15 min. When $r > 1$, there are two price ranges. The first one is € $[4.20, 7.50]$ per 15 min, which extends the price range mentioned before, and the second one is € $(6.00, 7.50]$ per 15 min. We chose € 7.50 per 15 min randomly, and it could be any value above € 6.00 per 15 min. Thus, € 5.40 per 15 min, € 6.00 per 15 min, and € 7.50 per 15 min were set as the benchmark prices for the base and relocation scenarios. As shown in Fig. 2 a), when the price of a shared car equals € 5.40 per 15 min, the operator can earn the highest profit in the price range of € $[4.20, 5.40]$ per 15 min. When $r > 1$, as the charges increase, the profits decrease.

When $r \leq 1$, in the base scenario, as the price increases from € 4.20 to 6.00 per 15 min, the profit peaks at € 5.65 per 15 min and then decreases (Fig. 2 a)). The cost of traveling rises quickly (Fig. 2 b)). The cost of all the carsharing travelers increases with price and peaks when at a price of 5.30 per 15 min, after which it decreases (Fig. 2 c)). Meanwhile, the number of shared cars (Fig. 2 i)) and carsharing trips (Fig. 2 e)) continues to fall as the price increases. Similarly, the total time steps taken by the vehicles also reduces (Fig. 2 g)). These results show that even when the price of carsharing is less than the cost of using a private car, an increase in the cost of carsharing will encourage fewer travelers to choose shared cars, reducing the total travel time spent in the shared cars, while the operator still benefits from the price increase.

When $r > 1$, i.e., when the carsharing price is higher than the cost of using a private car, the profit keeps decreasing with the price (Fig. 2 a)). The number of shared cars in the system sees a sharp decline (Fig. 2 i)). The number of carsharing trips continues to decrease (Fig. 2 e)). When the price is € 9.90 per 15 min, the total cost for all travelers reaches its highest point before gradually decreasing (Fig. 2b)). This result indicates that the same travel costs can be associated with different prices and trip patterns.

In the base scenario, the objective of profit maximization is achieved with a price of € 5.65 per 15 min, with a profit of € 47,753. Nevertheless, a carsharing company may not only aim to maximize its profits but also increase its market share, for which proper strategies are required.

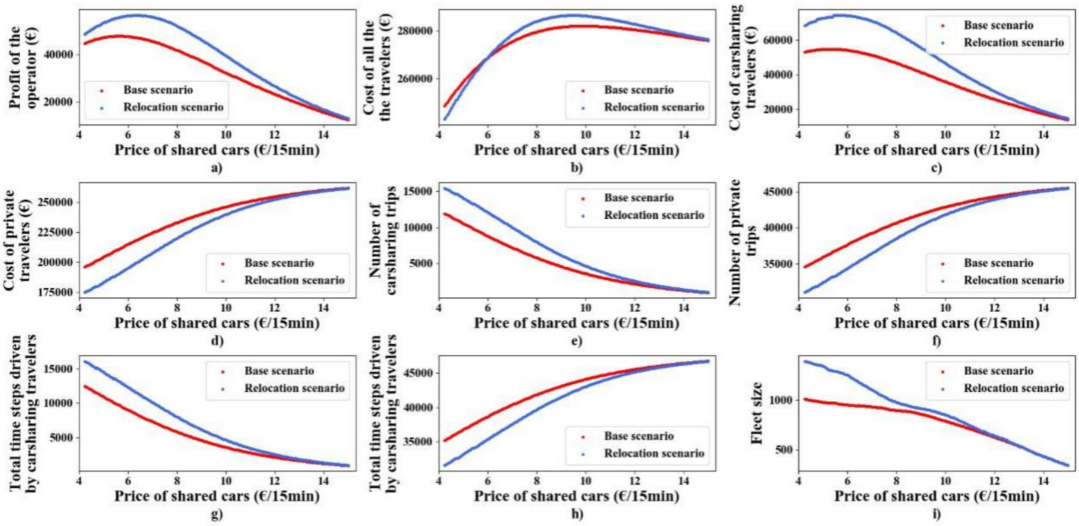


Fig. 2 Sensitivity analysis of the average carsharing price in the base and relocation scenarios

5.2.2 Relocation scenario

In the relocation scenario, cars in the oversupplied zones can be relocated to the undersupplied ones, thus tackling the vehicle imbalance problem. As shown in Fig. 2, the all indicators in the base scenario follow a similar trend. The profits in the relocation scenario are higher than in the base scenario, for any given price (Fig. 2 a)). This shows that relocations can help the operator earn more revenue when the price is constant.

When $r \leq 1$, the advantage of relocations is obvious (Fig. 2 a)) and the difference between the two profits is more than € 3,000 € (Fig. 3). From the perspective of the travelers, when the shared cars are relocated, savings on the cost will increase (Fig. 3). The fleet size in the relocation scenario is much higher than that in the base scenario (Fig. 2 i)). That is, when the carsharing price is lower than the cost of private cars, the system can offer more shared cars as they can be relocated, thus yielding more profit and increasing the level of service offered to the clients.

When $r > 1$, the profit in the relocation scenario is still much higher. The profit is the highest when the carsharing price is € 7.60 per 15 min. In fact, it is € 10,530 higher than that in the base scenario, as shown in Fig. 3, after which it reduces continuously. Meanwhile, the total cost for travelers increases. When the carsharing price is € 5.95 per 15 min, the total costs of travelers in both scenarios are nearly the same. This in turn means there are no savings when the price increases. When the carsharing price is € 8.90 per 15 min, the gap between the cost of all the travelers is the largest as shown in Fig. 3 (4,709 €). The difference in the fleet size between the two scenarios continues to reduce (Fig. 2 i)).

In short, when the price of a shared car is lower than the cost of a private car, relocations can help the operator increase the profit and the travelers save on travel costs. When the price of the shared car is higher than the cost of the private car, relocations can always help the operator earn more revenue. However, the travelers cannot save on the travel costs.

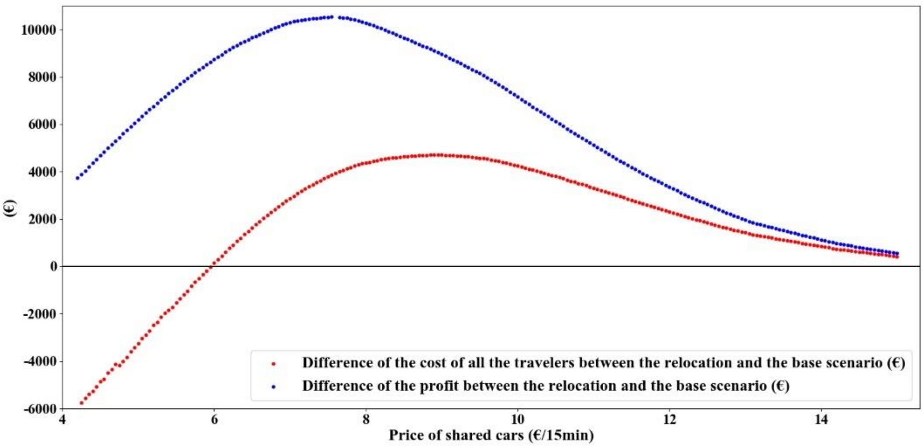


Fig. 3 Gaps between the objectives of the relocation scenario and base scenario

5.2.3 Pricing scenario

Many studies have shown that pricing can help the operator solve the stock imbalance problem to a certain extent (Angelopoulos et al. 2018; Jorge et al. 2014). The pricing strategy works by varying the prices depending on the zone and time of day. In this model, the pricing helps customers decide to choose either a shared car or a private car. For example, if the origin zones are oversupplied with cars and/or the destination zones are undersupplied, setting a lower price can encourage more customers to choose shared cars, and vice versa.

The results in all scenarios are listed in Table 4. The convergence processes in the pricing scenario are shown in Fig. 4. The running time and iterations within different price ranges are shown in Table 2. The results obtained for the pricing scenario were compared with the benchmarks of the base and relocation scenarios.

Table 2. Price variation range, running iterations, and running time in the pricing scenario

Price range (€ per 15 min)	$[4.20, 5.40]$	$[4.20, 6.00]$	$[4.20, 7.50]$	$(6.00, 7.50]$
Running iterations	53	48	45	44

Running time	4 h 32 min	4 h 7 min	3 h 51 min	3 h 46 min

The results obtained from the price range of € $[4.20, 5.40]$ per 15 min and € $[4.20, 6.00]$ per 15 min when $r \leq 1$ are used to highlight the advantages of the pricing strategy. The values of the objective function in the upper level through pricing are almost the same as those obtained in the base scenario, while the total costs of all the travelers are much lower. For example, when the price varies in the range of € $[4.20, 5.40]$ per 15 min, the travelers now pay € 5,273 € less, when compared with the results of the benchmark price of € 5.40 per 15 min in the base scenario. Simultaneously, the number of carsharing travelers will increase by 2.03%, which means the number of shared cars required will rise by 1.96%. If the price range is € $[4.20, 6.00]$ per 15 min, the travel cost will be € 7,369 less than that in the base scenario, the number of carsharing travelers will increase by 3.16% more, and the fleet size will have to grow by 3.58%. Compared with the relocation strategy, the pricing strategy does not perform very well, regardless of whether the price range is € $[4.20, 5.40]$ per 15 min or € $[4.20, 6.00]$ per 15 min. For example, when the price range is € $[4.20, 5.40]$ per 15 min, the profit earned through pricing is € 7,424 less than that through relocations, and the travel costs reduce by € 3,425. With this pricing strategy, the number of carsharing travelers will reduce by 5.27% compared with the relocation strategy. In short, when the carsharing price is lower than the cost of private cars, the relocations yield better results than the pricing strategy in improving the profits and attracting more carsharing travelers.

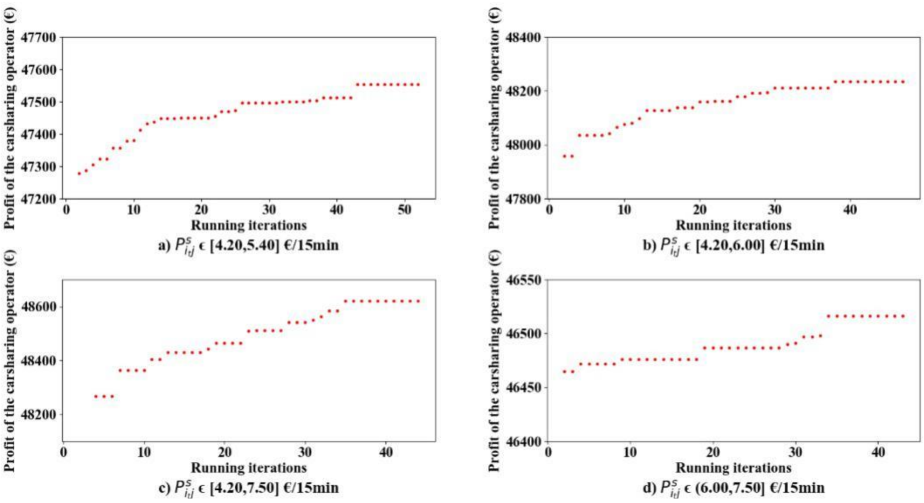


Fig. 4 Convergence processes of the operator's profit in the pricing scenario

After extending the price range to overlap with the reference cost of private cars, i.e., the price range of € $[4.20, 7.50]$ per 15 min, the results obtained in the pricing scenario are compared with those in the base and relocation scenarios for a benchmark price of € 7.50 per 15 min. Compared with base scenario, the pricing scenario saw a 7.75% increase in the number of shared cars yield € 4,743 € more profits, and the overall traveler cost reduced by € 9,419. Additionally, carsharing in this scenario could attract 5.58% more travelers. Compared with relocations, the pricing strategy offers better savings on travel costs saving, attracting more users to carsharing. The profit, however, is € 5,778 lower than that obtained through relocations. The cost is also € 13,198 lower, and the trip ratio of shared cars is even 0.2% higher.

When $r > 1$, which means € $(6.00, 7.50]$ per 15 min, the operator can earn € 2,639 more profits with the pricing scenario, while all travelers will pay € 3,121 less compared with the base scenario (benchmark price of € 7.50 per 15 min). In the relocation scenario, the operator can earn an additional profit of € 10,521, and travelers will have to spend an additional sum of € 3,785. Both pricing and relocations can help the operator attract more carsharing travelers. However, relocations perform better than pricing and attract an additional 3.11 % travelers than a strategy with pricing alone. In terms of fleet size, the pricing strategy will reduce the number of shared cars by 88 compared with relocations. Therefore, when the carsharing price is higher than the cost of private cars, travelers will benefit from the pricing strategy as they save on travel costs. The operator, however, may prefer the relocation strategy because of the huge increase in profits it offers.

In summary, relocations and pricing can both lead more travelers to choose shared cars. The pricing strategy benefits the travelers and reduces the average travel cost of carsharing per time step, while the relocation strategy benefits the operator. Logically, a better strategy that combines the advantages of both pricing and relocation is needed.

5.2.4 Pricing–relocation combination scenario

In the pricing–relocation combination scenario, the operator can price the trips as well as perform relocations. The convergence process of the GA algorithm in this case is presented in Fig. 5, when different price ranges are used. The running details are shown in Table 3. The results obtained in this scenario are compared with the benchmarks of the base, relocation, and pricing scenarios.

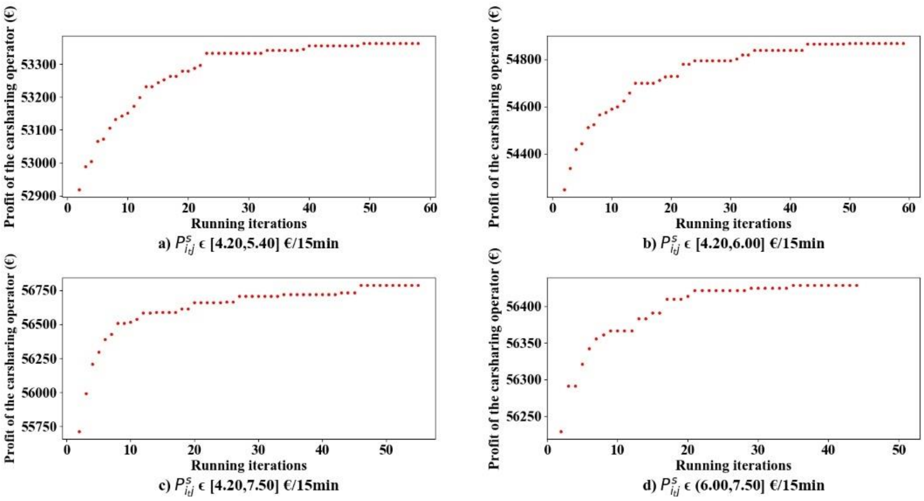


Fig. 5 Convergence processes of the operator's profits in the pricing–relocation combination scenario

Table 3. Price variation range, running iterations and running time in the combination scenario

Price range (€ per 15 min)	$[4.20, 5.40]$	$[4.20, 6.00]$	$[4.20, 7.50]$	$(6.00, 7.50]$
Running iterations	59	60	56	45
Running time	5 h 11 min	5 h 17 min	4 h 56 min	3 h 51 min

At $r \leq 1$, i.e., the price range is either € $[4.20, 5.40]$ or $[4.20, 6.00]$ per 15 min, both relocation and the pricing–relocation combination strategies offer similar advantages. Let us take the second price range of € $[4.20, 6.00]$ per 15 min as an example. The relocation strategy can help the operator increase their profits by € 8,738 and reduce the travel costs by € 4,843. With the combination strategy, the profit can be increased by 8,832 € and the travel costs can be reduced by € 9,629. With the combination strategy, the portion of carsharing trips is 3.29% higher with the combination strategy than with the relocations. The average cost per carsharing trip in the combination scenario is also much lower. In other words, when the carsharing price is lower than the cost of private cars, the influence of the combination and relocation strategies on the value of the objective function in the upper level is almost the same, while the combination strategy performs better in terms of cost savings and converting travelers. A comparison of the results of the pricing and combination scenarios shows that the combination strategy performs better. The combination strategy also yields higher profits than the pricing strategy, while keeping the travel costs slightly lower. Most importantly, the proportion of carsharing travelers is more than 7.00% higher in the combination than the strategy that considers only pricing.

When the price range is € $[4.20, 7.50]$ per 15 min, the combination strategy still has the best performance. For example, this strategy yields the highest improvement in profit among the four strategies, which is € 12,912 higher than that of the base scenario. Additionally, the savings on travel costs are € 9,273 compared with the base scenario. The combination strategy also draws the most people.

At $r > 1$, the performance of the combination strategy is similar to the previous case where the price varies in the range of € $[4.20, 7.50]$ per 15 min), except for the travelers' costs. The travelers' costs in the combination scenario are nearly identical to those in the base scenario. This cost is still € 4,412 less than that in the relocation scenario, which means that the combination strategy offers the cost-saving benefits of the pricing scenario. Therefore, the combination strategy performs the best when the carsharing price is much higher than the cost of private cars, considering the profits earned by the carsharing operator and travelers.

In summary, the combination strategy leverages the advantages of both pricing and relocations. It can drastically increase the profits as well as ensure that travelers saved as much as possible. Therefore, irrespective of the price range, it can attract the most carsharing travelers compared with the other three strategies.

6 Conclusions

This paper proposes a bilevel nonlinear model to study the pricing and relocation problem of carsharing systems where travelers can drive a shared car or a private car. The objective is to maximize the profit earned by a carsharing operator (upper level), while minimizing the overall travel expenditure incurred on passengers (lower level). Considering the price and relocations of shared cars as decision variables under a nonlinear model, KKT conditions were used to transform the bilevel model into a single-level model. A GA was proposed to handle the nonlinearity of the objective function and the constraints related to the carsharing demand. Rotterdam, The Netherlands, was used as the study area to test the model. We studied four scenarios, base scenario, relocation scenario, pricing scenario, and relocation–pricing combination scenario to obtain insights about the carsharing system performance of different management strategies. The optimization results of the different scenarios were obtained under a computation time of 6 h.

In this case study, when the price range of a shared car is less than or equal to the cost of a private car, the pricing strategy does not perform as good as the relocation or the combination strategy in terms of profit, but it reduces the travel costs incurred on the travelers. The relocation and combination scenarios can yield larger profits for the operator and lower travel costs for the users, besides guaranteeing a higher level of satisfaction when compared with the base scenario. Nevertheless, the two scenarios are still different from each other; the combination strategy brings an added value through a sizeable decrease in the travel cost incurred on the travelers' costs and the highest ratio of carsharing trips.

When the price range of carsharing is overlapped with the cost of a private car, all the three strategies yield a higher profit and attract more travelers than the base scenario. Both the pricing and combination strategies helped travelers save more on the travel cost, whereas the relocation strategy could not. The pricing strategy outperformed the relocation scenario in terms of the carsharing demand. Clearly, the combination strategy brings the highest profits, attracts the most travelers to carsharing, and decreases the cost incurred by the travelers.

When the price range of carsharing is slightly higher than the cost of private cars, the three strategies help bring a higher profit than the base scenario. However, the strategies achieve said profits in different ways. In the combination scenario, the rate of carsharing trips is the highest of the three strategies, which is good for the level of service offered to the clients. The pricing strategy can still help save more on travel costs. In the same price range, the relocation strategy outperforms the pricing strategy, as far as the carsharing trips satisfaction is concerned.

We conclude that the combination strategy can be considered the most effective method in improving the profitability of one-way carsharing systems, while still helping travelers save on travel costs and satisfying requests.

The performance of carsharing systems still has scope for improvement. In this model, for simplicity, we considered all users to be the same, but it would be more realistic to account for the heterogeneity of customers. This is because different users react differently to the same price. Moreover, all requests were satisfied without rejection, which is not the case. Future studies should develop a system that can reject travelers and explore the influence of rejections on system profitability. GA can help finding a feasible solution, which may or may not be optimal. The development of a more efficient algorithm, or an exact algorithm, is a possible research direction for future studies.

Acknowledgment

We thank the anonymous reviewers for their detailed comments and many valuable suggestions that significantly improved the quality of this paper. The first author wishes to thank the Department of Transport and Planning, Delft University of Technology, Delft, The Netherlands, for hosting her in 2019. This research was financially supported by the China Scholarship Council. This work is also supported by the National Natural Science Foundation of China under Grant [No. 91846202, No. 71771012, No. 71671014, and No. 71621001]. We would like to especially thank Senlei Wang for providing and processing the travel data. We also extend our gratitude to FICO for the free usage of the Xpress optimization software.

Table 4. Performance of all scenarios (in the base and relocation scenarios, benchmark prices are set as € 5.40 per 15 min, € 6.00 per 15 min and € 7.50 per 15 min)

	Scenario	Base			Relocation			Pricing				Combination			
	Benchmark price (per 15 min)	Constant			Constant			Upper bound of				Upper bound of			
		5.40	6.00	7.50	5.40	6.00	7.50	5.40[1]	6.00[2]	7.50[3]	7.50[4]	5.40[1]	6.00[2]	7.50[3]	7.50[4]
Carsharing operator	**Profit earned by operator (€)**	**47,678**	**47,592**	**43,877**	**54,977**	**56,330**	**54,398**	**47,553**	**48,236**	**48,620**	**46,516**	**53,364**	**54,868**	**56,789**	**56,429**
	Number of relocations	0	0	0	3,202	3,192	2,408	0	0	0	0	3,235	3,199	2,970	2,753
	Cost of relocations	0	0	0	9,094	9,066	2,071	0	0	0	0	9,187	9,085	8,436	7,819
	Depreciation and maintenance costs (€)	2,005	1,979	1,934	2,411	2,375	6,840	2,031	2,024	2,027	1,970	2,474	2,441	2,317	2,200
	Fleet size	971	951	916	1,280	1,252	1,032	990	985	987	944	1,328	1,303	1,208	1,119
All passengers	**Cost incurred on all passengers (€)**	**263,086**	**268,642**	**277,761**	**261,238**	**263,799**	**281,546**	**257,813**	**261,273**	**268,348**	**274,640**	**254,473**	**259,013**	**268,488**	**277,136**
Carsharing travelers	Cost of carsharing travelers (€)	54,753	54,078	49,082	73,266	73,932	67,847	55,159	55,557	55,287	52,308	72,350	73,401	73,723	71,683
	Total time steps driven by carsharing travelers	10,139	9,013	6,544	13,568	12,322	9,046	11,152	10,593	9,280	7,643	14,650	14,013	12,361	10,470
	The number of carsharing trips	9,827	8,796	6,454	13,217	12,077	8,950	10,767	10,261	9,045	7,508	14,182	13,606	12,092	10,321
	Carsharing trips share (%)	**21.16**	**18.94**	**13.90**	**28.46**	**26.01**	**19.28**	**23.19**	**22.10**	**19.48**	**16.17**	**30.54**	**29.30**	**26.04**	**22.23**
	Average cost of a carsharing trip per time step (€)	5.40	6.00	7.50	5.40	6.00	7.50	4.95	5.24	5.96	6.84	4.94	5.24	5.96	6.85
Private travelers	Cost of private travelers (€)	208,333	214,564	228,679	232,938	194,864	213,700	202,654	205,717	213,061	222,332	182,123	185,612	194,765	205,453
	Total time steps driven by private travelers	37,549	38,675	41,144	34,120	35,366	38,642	37,095	37,578	38,408	40,045	33,038	33,675	35,327	37,218
	The number of private car trips	36,605	37,636	39,978	33,215	34,355	41,303	35,665	36,171	37,387	38,924	32,250	32,826	34,340	36,111
	Private car trips share (%)	78.84	81.06	86.10	71.54	73.99	80.72	76.81	77.90	80.52	83.83	69.46	70.70	73.96	77.77

[1] Reference price range for Rotterdam: [4.20, 5.40]
[2] Extended range to include the price of the private car: [4.20, 6.00]
[3] Extended range to include the price above that of the private car: [4.20, 7.50]
[4] Range starting from just above the cost of the private car: (6.00, 7.50]

References

Angelopoulos, A., D. Gavalas, C. Konstantopoulos, D. Kypriadis, and G. Pantziou. 2018. "Incentivized Vehicle Relocation in Vehicle Sharing Systems." Transportation Research Part C-Emerging Technologies 97: 175–193.

Ashkrof, P., G. H. Correia, O. Cats, and B. Van Arem. 2019. "Impact of Automated Vehicles on Travel Mode Preference for Different Trip Purposes and Distances." Transportation Research Record 2673 (5): 607–616.

Barth, M., J. Han, and M. Todd. 2001. "Performance Evaluation of a Multi-Station Shared Vehicle System." Intelligent Transportation Systems, IEEE. IEEE 1218–1223. 10.1109/ITSC.2001.948837.

Barth, M., M. Todd, and L. Xue. 2004. "User-Based Vehicle Relocation Techniques for Multiple-Station Shared-Use Vehicle Systems." Transportation Research Board, 80th Annual Meeting, Washington, DC.

Boyaci, B., K. G. Zografos, and N. Geroliminis. 2015. "An Optimization Framework for the Development of Efficient One-Way Car-Sharing Systems." European Journal of Operational Research 240 (3): 718–733.

Boyaci, B., K. G. Zografos, and N. Geroliminis. 2017. "An Integrated Optimization-Simulation Framework for Vehicle and Personnel Relocations of Electric Carsharing Systems with Reservations." Transportation Research Part B-Methodological 95: 214–237.

Bruglieri, M., A. Colorni, and A. Luè. 2014. "The Vehicle Relocation Problem for the One-Way Electric Vehicle Sharing: An Application to the Milan Case." Procedia - Social and Behavioral Sciences 64 (4): 18–27.

Bruglieri, M., F. Pezzella, and O. Pisacane. 2017. "Heuristic Algorithms for the Operator-Based Relocation Problem in One- Way Electric Carsharing Systems." Discrete Optimization 23: 56–80.

Bruglieri, M., F. Pezzella, and O. Pisacane. 2018. "A Two-Phase Optimization Method for a Multiobjective Vehicle Relocation Problem in Electric Carsharing Systems." Journal of Combinatorial Optimization 36 (1): 162–193.

Carlier, A., A. Munierkordon, and W. Klaudel. 2015. "Mathematical Model for the Study of Relocation Strategies in One-Way Carsharing Systemsγ." Transportation Research Procedia 10: 374–383.

Chang, J. , M. Yu, S. Shen, and M. Xu. 2017. "Location Design and Relocation of a Mixed Car-Sharing Fleet with a CO2 Emission Constraint." Service Science 9 (3): 205–218.

Cheng, Q., Z. Liu, and W. Y. Szeto. 2019. "A Cell-Based Dynamic Congestion Pricing Scheme Considering Travel Distance and Time Delay." Transportmetrica B-Transport Dynamics 7 (1): 1286–1304.

Clemente, M., M. P. Fanti, G. Iacobellis, M. Nolich, and W. Ukovich. 2018. "A Decision Support System for User-Based Vehicle Relocation in Car Sharing Systems." Systems man and Cybernetics 48 (8): 1283–1296.

Correia, G. H., and A. P. Antunes. 2012. "Optimization Approach to Depot Location and Trip Selection in One-Way Carsharing Systems." Transportation Research Part E-Logistics and Transportation Review 48 (1): 233–247.

Febbraro, A. D., N. Sacco, and M. Saeednia. 2012. "One-Way Carsharing: Solving the Relocation Problem." Transportation Research Record 2319 (1): 113–120.

Febbraro, A. D., N. Sacco, and M. Saeednia. 2019. "One-Way Car-Sharing Profit Maximization by Means of User-Based Vehicle Relocation." IEEE Transactions on Intelligent Transportation Systems 20 (2): 628–641.

Gambella, C., E. Malaguti, F. Masini, and D. Vigo. 2017. "Optimizing Relocation Operations in Electric Car-Sharing." Omega- international Journal of Management Science 81: 234–245.

Hu, P. S. 2001. Summary of Travel Trends 2001 National Household Travel Survey.

Hu, L., and Y. Liu. 2016. "Joint Design of Parking Capacities and Fleet Size for One-Way Station-Based Carsharing Systems with Road Congestion Constraints." Transportation Research Part B-Methodological 93: 268–299.

Huang, K., G. H. Correia, and K. An. 2018. "Solving the Station-Based One-Way Carsharing Network Planning Problem with Relocations and non-Linear Demand." Transportation

Research Part C-Emerging Technologies 90: 1–17.
Illgen, S., and M. Hock. 2019. “Literature Review of the Vehicle Relocation Problem in One-Way Car Sharing Networks.”
Transportation Research Part B-Methodological 120: 193–204.
Jakob, M., M. Menendez, and J. Cao. 2020. “A Dynamic Macroscopic Parking Pricing and Decision Model.” Transportmetrica B-Transport Dynamics 8 (1): 307–331.
Jian, S., D. Rey, and V. Dixit. 2019. “An Integrated Supply-Demand Approach to Solving Optimal Relocations in Station- Based Carsharing Systems.” Networks and Spatial Economics 19 (2): 611–632.
Jorge, D., and G. Correia. 2013. “Carsharing Systems Demand Estimation and Defined Operations: A Literature Review.”
European Journal of Transport and Infrastructure Research 13 (3): 201–220.
Jorge, D., G. H. Correia, and C. Barnhart. 2014. “Comparing Optimal Relocation Operations with Simulated Relocation Policies in One-Way Carsharing Systems.” IEEE Transactions on Intelligent Transportation Systems 15 (4): 1667–1675.
Jorge, D., G. Molnar, and G. H. Correia. 2015. “Trip Pricing of One-Way Station-Based Carsharing Networks with Zone and Time of Day Price Variations.” Transportation Research Part B-Methodological 81 (Part 2): 461–482.
Kanzow, C. 1994. “An Unconstrained Optimization Technique for Large-Scale Linearly Constrained Convex Minimization Problems.” Computing 53 (2): 101–117.
Kek, A. G., R. L. Cheu, Q. Meng, and C.H Fung. 2009. “A Decision Support System for Vehicle Relocation Operations in Carsharing Systems.” Transportation Research Part E-Logistics and Transportation Review 45 (1): 149–158.
Li, X., J. Ma, J. Cui, A. Ghiasi, and F. Zhou. 2016. “Design Framework of Large-Scale One-Way Electric Vehicle Sharing Systems: A Continuum Approximation Model.” Transportation Research Part B-Methodological 88: 21–45.
Martin, E. W., and S. Shaheen. 2011. “Greenhouse Gas Emission Impacts of Carsharing in North America.” IEEE Transactions on Intelligent Transportation Systems 12 (4): 1074–1086.
Nair, R., and E. Millerhooks. 2014. “Equilibrium Network Design of Shared-Vehicle Systems.” European Journal of Opera- tional Research 235 (1): 47–61.
Namazu, M., and H. Dowlatabadi. 2018. “Vehicle Ownership Reduction: A Comparison of One-Way and Two-Way Carshar- ing Systems.” Transport Policy 64: 38–50.
Nourinejad, M., and M. J. Roorda. 2014. “A Dynamic Carsharing Decision Support System.” Transportation Research Part E-Logistics and Transportation Review 66: 36–50.
Nourinejad, M., S. Zhu, S. Bahrami, and M.J. Roorda. 2015. “Vehicle Relocation and Staff Rebalancing in One-Way Carsharing Systems.” Transportation Research Part E-Logistics and Transportation Review 81: 98–113.
Pfrommer, J., J. Warrington, G. Schildbach, and M. Morari. 2014. “Dynamic Vehicle Redistribution and Online Price Incentives in Shared Mobility Systems.” IEEE Transactions on Intelligent Transportation Systems 15 (4): 1567–1578.
Reiss, S., and K. Bogenberger. 2017. “A Relocation Strategy for Munich’s Bike Sharing System: Combining an Operator- Based and a User-Based Scheme.” Transportation Research Procedia 22: 105–114.
Ren, S., F. Luo, L. Lin, and X.L. Li. 2019. “A Novel Dynamic Pricing Scheme for a Large-Scale Electric Vehicle Sharing Net- work Considering Vehicle Relocation and Vehicle-Grid-Integration.” International Journal of Production Economics 218: 339–351.
Santos, G. G., and G. H. Correia. 2019. “Finding the Relevance of Staff-Based Vehicle Relocations in One-Way Carsharing Systems Through the Use of a Simulation-Based Optimization Tool.” Journal of Intelligent Transportation Systems 23 (6): 583–604.
Spieser, K., S. Samaranayake, W. Gruel, et al. 2016. “Shared-Vehicle Mobility-on-Demand Systems: A Fleet Operator’s Guide to Rebalancing Empty Vehicles.” Transportation Research Board 95th Annual Meeting, Transport.
Vasconcelos, A. S., L. M. Martinez, G. H. Correia, D.C. Guimaraes, and T.L. Farias. 2017. “Environmental and Financial Impacts of Adopting Alternative Vehicle Technologies and Relocation Strategies in Station-Based One-Way Carsharing: An Application in the City of Lisbon.” Portugal. Transportation Research Part D-Transport and Environment 57: 350–362.
Waserhole, A., and V. Jost. 2013a. Vehicle Sharing System Pricing Regulation: A Fluid

Approximation. OperationsResearch. https://www.researchgate.net/publication/267383157.
Waserhole, A., V. Jost, and N. Brauner. 2013b. “Pricing Techniques for Self Regulation in Vehicle Sharing Systems.” Electronic Notes in Discrete Mathematics 41 (Complete): 149–156.
Weikl, S., and K. Bogenberger. 2016. “Integrated Relocation Model for Free-Floating Carsharing Systems: Field Trial Results.” Transportation Research Record: Journal of the Transportation Research Board 2563 (1): 19–27.
Wood, N. S., and S. N. Jones-Meyer. 2016. “Integrating Automated Toll Discounts into a Real-Time Ridesharing Program.”
Transportation Research Record: Journal of the Transportation Research Board 2597 (1): 20–27.
Xu, M., Q. Meng, and Z. Liu. 2018. “Electric Vehicle Fleet Size and Trip Pricing for One-Way Carsharing Services Considering Vehicle Relocation and Personnel Assignment.” Transportation Research Part B: Methodological 111: 60–82.
Zakaria, R., M. Dib, L. Moalic, and A. Caminada. 2018. “Insights on Car Relocation Operations in One-Way Carsharing Systems.” International Journal of Advanced Computer Science and Applications 9 (7): 281–290.