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Decision-Making as a Social Choice Game

Gamifying an urban redevelopment process in search for consensus

Nan Bai[1], Shervin Azadi[2], Pirouz Nourian[3], Ana Pereira Roders[4]
[1,4]*Chair Heritage and Values, Department of Architecture Engineering and Technology, TU Delft* [2,3]*Chair Design Informatics, Department of Architecture Engineering and Technology, TU Delft*
[1,2,3,4]*{N.Bai|S.Azadi-1|P.Nourian|A.R.Pereira-Roders}@tudelft.nl*

The paper reports the formulation, the design, and the results of a serious game developed for structuring negotiations concerning the redevelopment of a university campus with various stakeholders. The main aim of this research was to formulate the redevelopment planning problem as an abstract and discrete decision-making problem involving multiple actions, multiple actors with preconceived gains and losses with respect to the comprising actions, and decisions as combinations of actions. Using fictitious and yet realistic scenarios and stakeholders as simulation, the results evidence how different levels of democratic participation and different modes of moderation can affect reaching a consensus and present in a mathematical characterisation of a consensus as a state of equilibrium. The small set of actions and actors enabled a chance to compute a theoretically optimal state of consensus, where the efficiency and the effectiveness of different modes of moderation and participatory rights could be observed and analysed.

Keywords: *Serious Game, Consensus Building, Democratization, Game Theory, Social Decision*

INTRODUCTION

In this research, a playable serious game was designed to simulate the further development of a university campus in the following decades, taking into account the cultural significance of the old campus buildings, even if not all buildings are listed as cultural heritage, nor is the campus a conservation area. The existence of cultural heritage challenges the ideas of planning that are strongly influenced by ordinary cost-benefit analyses. This paper shows how the discretisation of a multi-actor decision making problem can help structure a spatial planning problem as a participatory/gamified planning process, and pave the way for analysing such strategic planning decisions using the conceptual frameworks of graph theory and game theory.

The UNESCO 2011 Recommendation on the Historic Urban Landscape promotes greater inclusion and democratization of heritage planning, and wherever needed, make use of participatory tools to find

consensus by the key stakeholders (Pereira Roders, 2019). However, stakeholders often perceive heritage differently, and they typically have different kinds of stakes or concerns with respect to the redevelopment of an urban area (e.g. an urban university campus), thus conflicts can naturally arise between them in a planning process. As such, negotiation and finding consensus can be like a puzzle to find an optimal solution for (Bots & Hermans, 2003; Mayer et al., 2005). While such puzzles are not necessarily insolvable, the stakeholders participating in the puzzle may need external help to realize the existence of win-win solutions to avoid falling into any dilemma(Cunningham & Hermans, 2018). The conflicting opinions on various actions and their consequences could be understood by stakeholders by getting them involved in 'simulation games', i.e. games simulating the decision-making processes, especially construction and management [strategy] games as defined in (Rollings & Adams, 2003). Such games offer chances to experience other points of views and thus potentially help reach consensus more effectively (Werner, 2017).

GAME DESIGN AND SETTINGS

The main objective of this research is to frame a campus redevelopment planning process for TU Delft as a multi-actor/multi-criteria decision-making process, to be simulated as a social decision/strategy game (Jackson, 2014). As this game seems to have the potential to be generalized to other urban redevelopment planning cases, we formulate the problem mathematically and computationally, in order to provide for further analysis with graph theory and game theory (Easley & Kleinberg, 2010), as well as computational simulations with a larger number of iterations using agent-based models.

In the game design, agents (stakeholders) A, objects (sites) $\mathbf{o}$, choices available for those objects $\mathbf{c}$, and decisions $\mathbf{d}$ consisted of choices per each object in our game. The notation of these items and their relations are mentioned in Table 1 and Table 2. All the agents are directly or indirectly related to the campus redevelopment issue, thus having their own preference and judging criteria. The decisions will change over different rounds during the game, meaning some of the decisions from the set D will be eventually discovered over time, which we denote with a time superscript $\mathbf{d}^{(t)}$ as customary in the study of Markov Chains. For example, the starting situation $\mathbf{d}^{(0)}$ is also considered to be a decision (which entails doing nothing, but also keeping all states the same as they are).

A preference type function Θ is defined for agents, describing their preference, objection, or indifference regarding the choices applicable to objects. The preference type of each specific agent on possible scenarios of campus development is deduced from their online-accessible profiling and reports. For any decision $\mathbf{d} \in D$, we can define a utility mapping function $\mathfrak{u}(a_i, \mathbf{d})$ for each agent a_i that checks the corresponding value in θ_i for all the choices indexed with respect to objects in $\mathbf{d}$, and returns the sum of the preference type values. This utility value is used to determine the corresponding vote decision φ_i of an agent a_i based on the voting function $\varphi(a_i, \mathbf{d})$ (cf. Table 3).

By adjusting the values of λ_{i-} and λ_{i+}, different voting strategies of agents can be described, which may allow for compromising, bargaining, trading-off and/or paying-back as an entity. However, for the sake of simplicity, a high value was assigned for the ratio of the two parameters for all the agents, where

Description	Notation	Notes (Example in our game)	
A set of all agents (stakeholders)	$A = \{a_1, a_2, ..., a_n\}$, $A = \{a_i \mid i \in [1, n]\}$	(a_1) the general municipality of the city (a_3) the executive board of the university (a_5) a student representative of the university (a_7) the industrial park in the south of the campus (a_9) a citizen/alumina working in the industrial area	(a_2) the heritage manager of the city, (a_4) a professor from the faculty of applied sciences (a_6) the real state cooperation (a_8) a citizen representative from the city centre (a_{10}) the public transportation company of the city
Number of agents	$\mid A \mid = n$	$n = 10$	

Table 2
The nomenclature of objects, choices and decisions in the game

Description	Notation	Notes (Example in our game)
A vector of all objects (sites) for which an action is to be taken	$\boldsymbol{o} := [o_j]_{m\times1}$	$m = 6$ (o_1) University Campus North (o_2) University Central Campus (o_3) Industrial Park in Campus South (o_4) City Centre (o_5) Another Nearby City (o_6) Other Possible Sites
A vector of all possible choices per each object (site)	$\boldsymbol{c} := [c_k]_{l\times1}$	$l = 5$ (c_1) Maintain/Conserve/Preserve (c_2) Renovate/Adaptive Reuse (c_3) Densification (c_4) Selling and Moving (c_5) Constructing New Buildings on site
A matrix of all possibilities of object-choice pair	$\boldsymbol{S} := [s_{j,k}]_{m\times l} = \left[\underbrace{\boldsymbol{c}|\boldsymbol{c}|\,...\,|\boldsymbol{c}}_{m\ columns}\right]^T$	For the first three objects, all five choices are feasible, while for the last three objects, only the first three are.
Number of object-choice pairs	$\left|\{s_{j,k}|j\in[1,m],k\in[1,l]\}\right| = m\times l$	$\left|\{s_{j,k}|j\in[1,m],k\in[1,l]\}\right| = m\times l = 30$ An agent needs to have considered their preferences for this many choice pairs. This number is significantly less than the total number of decisions as mentioned below.
A decision: a vector of object-choice pairs (or a decision data-point) in a manner that there is one and only one pair per each object	$\boldsymbol{d} = [d_j]_{m\times1} = \boldsymbol{\Pi}^{(t)}\boldsymbol{c}$ Where $\boldsymbol{\Pi}^{(t)}$ is a rectangular permutation matrix of size $m\times l$ at iteration time t whose rows are basis vectors $\boldsymbol{e}_k \in \mathbb{R}^l$ $\boldsymbol{d} = \left(\boldsymbol{\Pi}^{(t)}\odot\boldsymbol{S}\right)\boldsymbol{1}_m$	$\boldsymbol{d}^{(0)} = [c_4,c_1,c_3,c_1,c_1,c_1]^T$ is a decision (and the initial state) in our game, e.g. c_4 in the first position in the vector corresponds to (c_4) *selling and moving* from (o_1) *the TU Delft North*
A set of all possible decisions	$D = \left\{\boldsymbol{d}|\boldsymbol{d} = \left(\boldsymbol{\Pi}^{(e)}\odot\boldsymbol{S}\right)\boldsymbol{1}_m\right\}$ $e\in[0,l^m]$ Exhausting all possible permutations for each possible epoch will result in $|D| = l^m$	There exists a space of possible decisions, in which not every element (decision data-point) is feasible due to the feasibility of object-choice pairs. $|D| = l^m = 5^6 = 15625$ Among them 3375 decisions are feasible
An explored decision during the game	$\boldsymbol{d}^{(t)} \in D$ $t\in[0,\tau]$	The iteration time of the last round is denoted as τ, the game masters can observe if the same decision is being repeated.

$\frac{\lambda_{i-}}{\lambda_{i+}} \geq m$ in this case, thus giving the objections much higher importance than the preferences. This also means that the an agent will not agree to a decision **d** even if there is only one objected choice for a particular object.

Considering the individual votes ϕ_i of all the agents involved, a voting configuration $\phi^{(t)}$ can be formed. Further, a transformation rule can be defined for transforming the vote to $\varphi^{(t)}$ to give "YES" votes and "NO" votes different weights, namely 1 for YES and $-\alpha$ for NO. In this case, a relatively large value α was assigned to make sure that a complete consensus must be met for the decision **d**.

A vector of voting weights **w** $= [w_1, w_2, \ldots, w_n]^T \in \mathbb{R}^n$ is also assigned for all the agents, which is an unknown parameter for the agents initially. The final voting result for decision **d**$^{(t)}$ is thus:

$$v^{(t)} = v\left(\mathbf{d}^{(t)}\right) = \mathbf{w}^T\phi^{(t)} \qquad (1)$$

For each round of voting, $v^{(t)} > 0$ when a consensus is met, in other words, when the voting process is successful, and vice versa. The individual voting weight w_i reflects multiple layers of information, which can be the social status, the decision power, the representative pool size, and the social inclusion of agents in the decision-making process. For an extreme scenario, w_i can be equal to 0, which indicates that even though the agent is invited to vote, their opinion is not taken into account for reaching consensus and making final decisions, which is a cruel truth often happening in the real world.

Four different configurations of the voting weights $\mathbf{w}^{[r]}, r \in \{1, 2, 3, 4\}$ were designed, which reflect the different levels of democratisation (Bogaards, 2010). To be more specific, $\mathbf{w}^{[1]}$ and $\mathbf{w}^{[2]}$ are all in a high level of democratization, where everyone can vote, and everyone's vote counts. In $\mathbf{w}^{[1]}$ all the agents' weight equally, while in $\mathbf{w}^{[2]}$, there are hierarchies for voting weights. In practice, the agents with higher weights (importance/relevance)

Description	Notation	Note (Example in our game)
Agents' Strategy indicator: (λ_{i-}) objection weight (λ_{i+}) preference weight	$\lambda_{i-}, \lambda_{i+} \in \mathbb{R}_+$	In this game $\frac{\lambda_{i-}}{\lambda_{i+}} \gg m$, giving the objections much higher importance than the preferences
Preference Type Function: λ_{i-} indicating agent's objection; λ_{i+} indicating agent's preference; 0 indicating agent's indifference	$\Theta(a_i, \boldsymbol{S}) = \boldsymbol{\theta}_i \in \{-\lambda_{i-}, 0, \lambda_{i+}\}^{m\times l}$ $\Theta(a_i, d_j) \in \{-\lambda_{i-}, 0, \lambda_{i+}\}$	The preference type θ_i of each specific agent is deduced from their online profiling and reports.
Utility Function: mapping of preference type and decision into the utility of agent	$u(a_i, \boldsymbol{d}^{(t)}) = \sum_{j=1}^{m} \Theta(a_i, d_j) \mid d_j \in \boldsymbol{d}^{(t)}$	We use the sum of preference type of each object-choice pair in the decision as the utility mapping.
Voting Function: Agent a_i votes YES if $\varphi_i = 1$; votes NO if $\varphi_i = -1$	$\varphi_i^{(t)} := \varphi(a_i, \boldsymbol{d}^{(t)}) = \frac{|u(a_i, \boldsymbol{d}^{(t)})|}{u(a_i, \boldsymbol{d}^{(t)})}$	This is a sign function that decides if the sign of the computed utility is positive or negative.
Vector of the votes of all agents	$\boldsymbol{\varphi}^{(t)} := \left[\varphi_1^{(t)}, \varphi_2^{(t)}, ..., \varphi_n^{(t)}\right]^T$ $\boldsymbol{\varphi}^{(t)} \in \{-1, +1\}^n$	This vector can be used as a time series or a transcript of the game, recording how each agent voted for every discussed decision.
Vector of transformed votes of all agents: $\phi_i = 1$ if $\varphi_i = 1$; $\phi_i = -\alpha$ if $\varphi_i = -1$,	$\boldsymbol{\phi}^{(t)} := \frac{\alpha+1}{2}(\boldsymbol{\varphi}^{(t)} - 1) + 1$ $\boldsymbol{\phi}^{(t)} \in \{-\alpha, 1\}^n$	$\alpha \in \mathbb{R}_+$ indicates the relative significance of negative votes compared to positive votes, here $\alpha \geq n\frac{\max(w_i)}{\min(w_i)}$
Vector of agent's vote weight	$\boldsymbol{w} = [w_1, w_2, ..., w_n]^T \in \mathbb{R}^n$	The vote weights are set for each group setting. In our game we have four different configurations, which we will denote as $\boldsymbol{w}^{[r]}, r \in \{1,2,3,4\}$
Result of voting system: For any explored decision $d^{(t)}$, there will be a final result.	$\upsilon^{(t)} := \upsilon(\boldsymbol{d}^{(t)}) = \boldsymbol{w}^T \boldsymbol{\phi}^{(t)}$ $\upsilon^{(t)} \in \mathbb{R}$	For each round of voting, $\upsilon^{(t)} > 0$ when a consensus is met

also get more chances to propose new ideas to the decision-making, this process is also mimicked in this game simulation. In contrast, $\mathbf{w}^{[3]}$ and $\mathbf{w}^{[4]}$ are all in a low level of democratization, where even though everyone is invited in the discussion and everyone can vote, not everyone's vote counts. In $\mathbf{w}^{[3]}$, only experts are given a non-zero weight, thus leading to an expert-based decision-making scenario, while in $\mathbf{w}^{[4]}$ only the ones with either power or capital including the municipality and the big companies are given a non-zero weight, thus resulting as in a capitalist scenario. All the four democratization levels are quite common in the real world.

Parallel to the voting process considering the preference type of agents, two functions $\gamma(d_j)$ and $\kappa(d_j)$ are defined that check choices per objects in the explored decision $\mathbf{d}^{(t)}$ and return the [non-monetary] gain and the [monetary] cost of making a choice for a specific object for the whole society, regardless of the individual preference or the voting result. In this game, these functions are evaluated from a look-up table with precomputed values. But in practice, they can be replaced by some environmental assessment modules. The total social benefit $\rho^{(t)}$ for the society by making the decision $\mathbf{d}^{(t)}$ can be defined as a gain-cost ratio (cf. Table 4):

$$\rho^{(t)} = \rho\left(\mathbf{d}^{(t)}\right) = \frac{\sum_{j=1}^{m} \gamma(d_j)}{\sum_{j=1}^{m} \kappa(d_j)} | d_j \in \mathbf{d}^{(t)} \quad (2)$$

A social decision game could also be regarded as an optimization problem. However, it is also argued by some scholars (Jackson, 2014) that finding a mechanism to optimize all the valued features of a social decision game can be very hard, if not impossible. For brevity, this research focuses on searching for the best-fitting decision $\mathbf{d}^{(t)}$, satisfying $\upsilon^{(t)}$ (a constraint) and maximising $\rho^{(t)}$ (an objective) of the social decision system given a game setting $(A, \mathbf{o}, \mathbf{c}, \Theta, \mathfrak{u}, \varphi, \mathbf{w}, \gamma, \kappa)$.

The value of $\upsilon^{(t)}$ represents how happy each agent would be in the end about the decision $\mathbf{d}^{(t)}$. This can reflect the feature of "Individual Rationality",

Description	Notation	Note (Example in our game)
Whole society cost function for one round	$\kappa_j := \kappa(d_j)$ $\kappa_j \in \mathbb{Z}$	The costs in our game are in the range of [-20, 40] in a monetary unit. Negative costs mean benefits. For example (o_2, c_4) will have a cost of 10.
Whole society gain function for one round	$\gamma_j := \gamma(d_j)$ $\gamma_j \in \mathbb{Z}$	The gains in our game are in the range of [-20, 30] in a non-monetary unit. Negative gains mean loss. For example (o_2, c_4) will have a gain of 10.
Total social benefit (gain-cost ratio) for one round	$\rho^{(t)} := \rho(\boldsymbol{d}^{(t)}) = \frac{\sum_{j=1}^{m} \gamma(d_j)}{\sum_{j=1}^{m} \kappa(d_j)} \mid d_j \in \boldsymbol{d}^{(t)}$ $\rho^{(t)} \in \mathbb{R}_+$	The social benefits are in the range of [0, 1] in an abstract unit. Several decisions can reach the maximum possible social benefit 1. For example, $\rho^{(0)} = \rho(\boldsymbol{d}^{(0)}) = 1$ at the initial state of the game.

which is "a requirement that each individual would weakly prefer participation in the mechanism to not participating" (ibid).

The value of $\rho^{(t)}$ represents how much benefit the whole society gets from the decision $\mathbf{d}^{(t)}$. This can reflect the feature of "Efficiency", which is "a requirement that a social decision maximises the sum of utilities of the individuals in society" (ibid). However, ideally the efficiency of a decision must be checked against the preferences of all agents in the sense of Pareto Efficiency.

REAL-WORLD SIMULATION
Serious Gaming Workshop
After building the social decision game setting shown in section 2, it was first implemented it in a real-world simulation. Thirty participants took part in the game during an international workshop on 'democratisation' hosted at TU Delft, within the ITN HERILAND program. Initially, four democratisation levels have been designed as described above. However, due to the limited number of participants, only

three of them (without the 3rd group setting) were successfully implemented as a real-world simulation.

The three groups faced different levels of democratization in terms of the weights and rights during the vote for the agents, thus having a different $\mathbf{w}^{[r]}$, and *ceteris paribus*.

The background information, the rules, the dynamic, and the settings of the game were explained to the participants before the game started in the workshop. The preference types of all the agents were written on a profile card given to the participant confidentially (see an example in Figure 1). They needed to read their assigned profiles and make sure they understood the game rules before starting. All participants could choose to reveal, hide, or misrepresent as much information written on their profile card as long as they obey the ONLY rule of being loyal to their mandated preference types when they voted for decisions. As is discussed in section 2, the agents' voting decision in relation to their utilities was simplified into a non-compromising non-payback strategy, i.e., do not vote "YES" for any decision $\mathbf{d}^{(t)}$ that consists of any object-choice pair d_j that they are

Figure 1
Example of the game interface during the workshop. Left: the sites to decide on. Right: The stakeholder profile declaring the preference type of each agent.
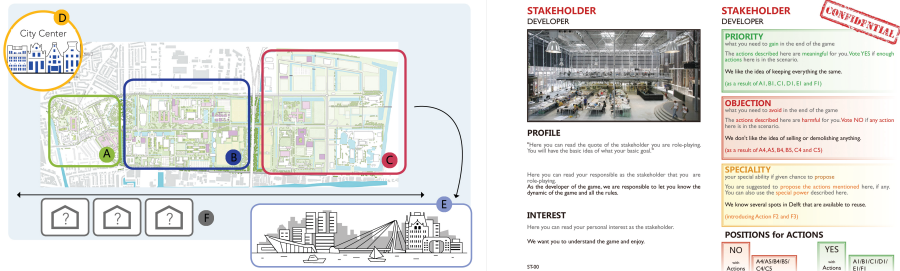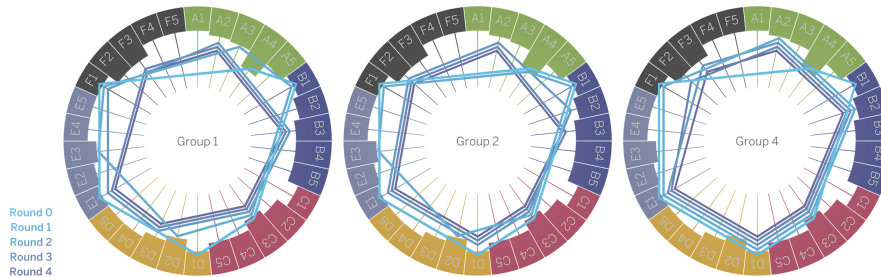
mandated to oppose to, no matter how many other object-choice pairs in the same decision are in their preference, even though that decision may lead to a higher overall social benefit.

During the workshop, a 'game master' was assigned to each group, who observes, coordinates, records, and monitors the gaming process. They made sure that all participants (agents) followed their mandated strategy and voting rules. Each group started from the same initial decision $\mathbf{d}^{(0)}$, which triggered a "NO" vote for at least one agent. They had four rounds to discuss, negotiate, and propose a new decision $\mathbf{d}^{(t)}, t \in [1, 4]$ to be voted at the end of each round.

It was stressed to all the participants and the game masters that this game had twofold goals: 1) that the primary goal for the game was to find a decision that gets a "YES" from all participants, i.e. proposing a good-enough social decision that could fulfil the requirement of "Individual Rationality", characterized as a consensus that minimally satisfied everyone involved; and 2) that the secondary goal was to seek to maximize the total gain-cost ratio of the final decision provided the consensus was already met, thus proposing a better social decision that fulfilled the requirement of "Efficiency".

Since in one of the three groups, the votes of some of the agents were 'secretly' dumped (with a $w_i = 0$), there was a special outcome, where the primary goal was reached while some agents were voting "NO". In this case, the 'excluded' agents could join forces and use a special card "FIGHT" to start a strike,

which could force the others to invalidate the 'successful' outcome and resume further discussion considering their opinions. For each group $r \in \{1, 2, 4\}$ and for each round $t \in \{0, 1, 2, 3, 4\}$, the decision $\mathbf{d}^{(t)[r]} \in D$ and the voting configuration $\varphi^{(t)[r]} \in \{-1, 1\}^n$ were recorded for each round of the game for all the three groups to be used in the analysis.

Simulation Outcomes

The three groups played the game under the instruction of the game masters. Almost all the participants expect for (a_8) the citizen in group 1 obeyed their mandated preference types. Within the 4 rounds of game, two of the three groups (group 2 & 4) managed to reach consensus and came up with an individual rational decision d that was not opposed by any participant. The other group (group 1) also claimed to have reached consensus while a_8 (the citizen) disobeyed the rules and voted YES even though the decision did not completely match his mandate. Meanwhile, one of the groups (group 2) managed to reach the only feasible & efficient decision that maximized the social benefit $\rho(\mathbf{d})$. As shown later, this was very unlikely to happen if left only to chances, since only a rather small portion of the decision set D would satisfy the primary goal of individual rationality (0.36% for group 1 and 2, and 3.6% for group 4, considering their different voting weights $w^{[r]}$). The decisions for each of the groups are plotted in Figure 2.

Interestingly, group 4 reached their "incomplete" consensus regardless of the "NO" votes from some of the participants who had a voting weight w_i of 0 in

the second round already. The fact that those participants were ignored triggered three of them to initiate a "FIGHT" striking against the outcome. The group resumed the discussion and approached a complete consensus that eventually satisfied everyone.
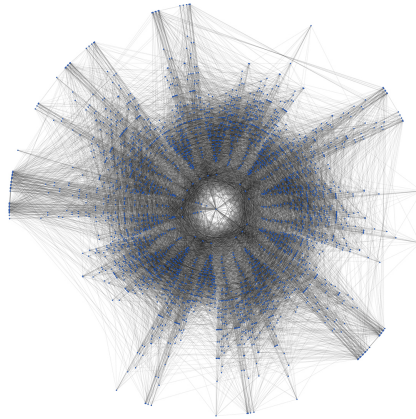
ANALYSIS
Analytic Methods
Here, the proposed mathematical framework in Section 2 would be extended with a set of analytical methods for studying: 1) the difference of the explored decisions in each round, 2) the effectiveness of the game dynamics, and also 3) the effectiveness of the different democratisation set-ups in different groups.

The first step is to enumerate the decision space and model the relations of the decisions together. Here, the decision space is modelled with as a graph Γ. Within this graph, each node represents a decision d, and each edge indicates that the nodes at the end of it differ in one and only one of their object-choices d_j. Consequently, the graph Γ represents the similarities of the decisions since it is essentially connecting similar node-decisions. In Γ, each node has a degree of $m \times (l-1)$. This is because the choice of any one

of the objects can be differed (m possibilities) and that choice can be differed to any choice except the one already existing in the decision ($l-1$ possibilities). Considering the setup of the gaming workshop, each node in Γ has a degree of $6 \times (5-1) = 24$. Moreover, Γ has a diameter of m, since traversing each edge is equivalent to changing a choice about a certain object, and it is obvious that by changing the choice of all objects, one at a time (in total m times), any node-decision $\mathbf{d}^{(t)}$ can be reached from any other node-decision $\mathbf{d}^{(s)}$.

On the decision graph Γ the graph-theoretical distance (path-length) between every two decisions can be used to measure the difference of those decisions $\texttt{dist}\left(\mathbf{d}^{(s)}, \mathbf{d}^{(t)}\right)$ (see Figure 3). Comparing the distance between the decisions of each round, it can be used to compare the extent to which a group explores the whole range of possible decisions in a game-round (cf. section 4.2 for exemplary results). This particular definition of distance makes it possible to eventually relativize the progress made in decision-making in each round of the game. To indicate such relative progress, the leap difference function ζ is defined as dividing the change in the gain-cost ratio by the distance of two decisions (cf. Table 5 for details):

$$\zeta\left(\mathbf{d}^{(s)}, \mathbf{d}^{(t)}\right) = \frac{\rho\left(\mathbf{d}^{(t)}\right) - \rho\left(\mathbf{d}^{(s)}\right)}{\texttt{dist}\left(\mathbf{d}^{(s)}, \mathbf{d}^{(t)}\right)} \qquad (3)$$

### Analyses of Simulation Outcomes
Firstly, the decision graph (Γ) for the game setup is formed given the objects (**o**) and choices (**c**). The preference type of voters specify which decision-nodes are consensual and which are not. Since the voters are different in each group, the set of consensual decision-nodes differ per group; and as reaching consensus is the primary condition of this game, the percentage of consensual nodes gives a rough estimate of how hard it would be to reach a consensus in this game. During the gaming workshop, group 1 and 2 had 12 consensual decision nodes, while the limited number of voters in group 4 had made a con-

Figure 3
The embedding of the decision graph in the polar coordinate system. To create this embedding, we map the index of each decision to angular coordinates and we map the reversed social benefit of the decision to radial distance. This means that as closer the decisions get to the center of the graph, the higher their gain-cost ratio is for the society.

Description	Notation	Note (Example in our game)
A set of nodes of Decision Graph	$V = D$	Each node is a decision $\boldsymbol{d}$
Total number of Nodes	$|V| = |D| = l^m$	$|V| = 5^6 = 15625$
A set of edges of Decision Graph	$E = \{(\boldsymbol{d}^{(s)}, \boldsymbol{d}^{(t)}) \mid \boldsymbol{d}^{(s)}, \boldsymbol{d}^{(t)} \in D, |\boldsymbol{d}^{(t)} - \boldsymbol{d}^{(s)}| = \sum_{j=1}^{m} \text{abs}(\boldsymbol{d}^{(t)}[j] - \boldsymbol{d}^{(s)}[j])\}$	Each edge connects nodes which only have one choice difference
The degree of each node	$\deg(\boldsymbol{d}) = \boldsymbol{m} \times (\boldsymbol{l} - \mathbf{1})$	$\deg(\boldsymbol{d}) = \mathbf{6} \times (\mathbf{5} - \mathbf{1}) = \mathbf{24}$
Total number of Edges	$|E| = \dfrac{\sum_{d \in D} \deg(\boldsymbol{d})}{2} = \dfrac{|V| \times \deg(\boldsymbol{d})}{2} = \dfrac{l^m \times m \times (l-1)}{2}$	$|E| = \dfrac{5^6 \times 6 \times (5-1)}{2} = 187500$
Decision Graph	$\Gamma = (V, E)$	Look at figure 3 for an embedding of the decision graph for our workshop
Distance between decisions	$\text{dist}(\boldsymbol{d}^{(s)}, \boldsymbol{d}^{(t)}) = \sum_{j=1}^{m} \text{abs}(\text{sgn}(\boldsymbol{d}^{(t)}[j] - \boldsymbol{d}^{(s)}[j]))$	It is the geodesic distance (steps) of two nodes in the decision graph
Leap Difference Function	$\zeta(\boldsymbol{d}^{(s)}, \boldsymbol{d}^{(t)}) = \dfrac{\rho(\boldsymbol{d}^{(t)}) - \rho(\boldsymbol{d}^{(s)})}{\text{dist}(\boldsymbol{d}^{(s)}, \boldsymbol{d}^{(t)})}$	It is the difference of the social benefit function of two nodes normalized by their distance

Group 1   Group 2   Group 4

Figure 4
Green nodes are consensual decisions, and Red nodes are non-consensual decisions. Blue trajectory and the numbers indicates the progression of decision making through playing rounds for each group. Black circles highlight the bounds of gain-cost ratio for the consensual decisions.
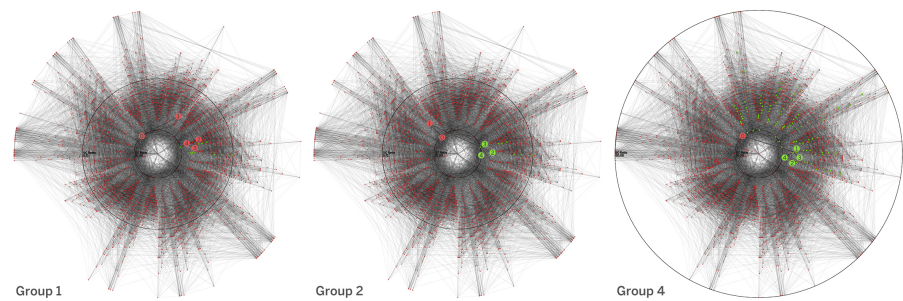
sensus more-easily reachable and raised the number of consensual nodes to 120 (see Figure 4).

The distance function relativizes the change in rounds compared with each other, and in the whole game compared between groups. This relativization reveals the amount of exploration that has happened in rounds and groups. In Figure 5, group 1 and 2 have explored more of the decision graph compared to group 4. In addition, by comparing the gain-cost ratio of the decision of each round against the traversed distance on the graph, it shows that although all the group have started from $\rho(\mathbf{d}^0) = 10$, only group 2 has finished the game with the same gain-cost ratio.

DISCUSSIONS

It is remarkable that that all three groups with different levels of democratisation have finally reached consensus (or claimed to have reached consensus) in the limited rounds of discussions and negotiations for this game design, as it is almost impossible to reach consensus purely searching and discovering the arena randomly, according to a brute-force computation. There must have been some strategies to learn from the human participants about how they manage to transform the information and learn from the past scenarios. However, it is arguable that the setting is only hard if the preference types of the individuals are not a common knowledge to the society

Blue line: gain-cost ratio plotted against the traversed distance, Orange Line: leap difference function against traversed path lengths per each round. After the "FIGHT" in the round 2 for group 4, the leap difference function becomes negative. It is also presented that the initial round of all groups has a negative value, which is due to the fact that they have started from a non-consensual node with 10, therefore their first step has been toward consensus, rather than maintaining the gain-cost ratio. Moreover, in the last round of all groups the leap function has a positive value.
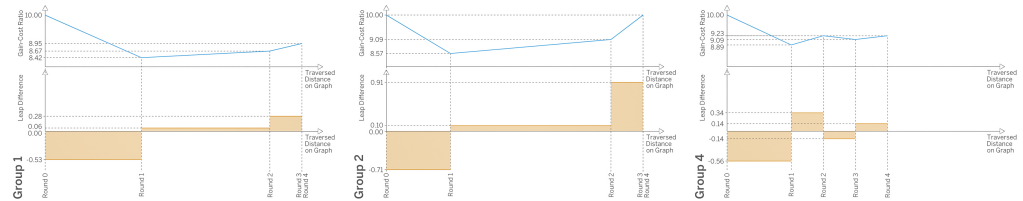
throughout the game process. The initial state of this game satisfies such a requirement, where the preference profile is claimed to be confidential (as shown in Figure 1). However, any specific behavioural strategy of the participants was allowed during the discussions. If they come into a point where everyone agrees to collaborate and unveil their secret preference types, then the game is no longer a dilemma (as is also the case for the repeated form of the Prisoner's Dilemma), rather a simple puzzle to observe and solve. This also mimics the reality and implies the value of such participatory social decision game for the real-world competition and collaboration.

Future research is expected to construct an Agent-Based Simulation Model using the same game settings as described in this paper. The performance of different strategy models and voting mechanisms will be tested, including a benchmark performance with random walks. Agents can be devised to follow the strategies that is learnt from the real-world simulation from human participants and related state-of-art research. It is also promising to study the effect of changes in the proposed parameters and functions to allow more complicated social settings, which may include scenarios in which: trade-offs or compromises of certain individuals are allowed; learning and diffusion of preference types could occur; each agent represents a different number of people and thus have different weights; or majority consensus, rather than complete consensus is expected. Later the games with different settings can be tested with humans again, comparing with our computational simulation results, and teaching them to perform even better.

The eventual purpose of this research is to integrate democratization and participation into the decision-making process for heritage redevelopment and spatial planning, as is one of the essential goals of the HUL recommendation. It is also interesting to see that in our real-world simulation outcome, the group 2, where all agents could vote but only part of them could propose new ideas, reached the optimal solution most easily; while the group 4, where some of the agents are invited to the discussions but were not given the right to vote, triggered the "FIGHT" branch and started a strike in their society; and in the group 1, where the highest level of democratization was assigned and each agent could change the decision evenly, there was a great chaos during the discussion and one of the agent even had to play up to deceive the game master that the consensus was met. This is also consistent with the idea that the democracy is necessary, but among the different forms of democracy the informed consent would contribute to the most efficient democratization (McGee, 2009).

CONCLUSIONS

The objective of the research was to frame an urban development problem as a gamified multi-actor/multi-objective social decision-making process and accordingly formulate the problem mathematically so that it can be further analyzed by means of graph theory and game theory.

The biggest question in the background of the research was whether such problems can be solved computationally (at least from a mathematical point of view), and if yes, why should they be framed as games? The answer to this question is partially de-

pendent on the scale (level of detail) and thus the complexity of the problem and to some extent on the social aspects of the problem. Our formulation shows that the order of complexity of such problems is exponential with respect to the number of objects raised to the power of a maximum number of choices available per each object. This could be a justification to apply [meta] heuristics to search for optimal outcomes. However, there may still be two reasons why both gaming and artificial intelligents methods (meta-heuristics) could be relevant at the same time: on the one hand, in case of large problems, they could be NP-hard, i.e. mathematical problems with no known algorithms for solving them systematically in polynomial time, whose solutions can only be 'approximated'; and on the other hand, even if the solution is approximated by means of algorithms and machines, humans must be involved in formulating, understanding, and attempting to solve the problem so that they can 'accept' and 'abide by' the solution (the final decisions). In other words, the didactic use of the game and its necessity for enabling participation cannot be overruled by the use of AI methods. On the contrary, the two approaches need to be combined to make the most out of such decision-making processes, socially and scientifically.

The main contribution of the paper is proposing a formal (mathematical/computational) formulation of a 'wicked-problem'. The analyses performed on the proceedings of the games show that designing such games and playing them can help diverse groups reach consensus more efficiently.

REFERENCES

Bogaards, M 2010, 'Measures of Democratization: From Degree to Type to War', *Political Research Quarterly*, 63(2), pp. 475-488

Bots, PWG and Hermans, LM 2003 'Developing 'playable metagames' for participatory stakeholder analysis', *34th Conference of the International Simulation and Gaming Association (ISAGA)*, pp. 647-657

Cunningham, S and Hermans, L 2018, *Actor and Strategy Models: Practical Applications and Step-wise Approaches*, John Wiley \& Sons, Inc.

Easley, D and Kleinberg, J 2010, *Networks, Crowds, and Markets:*, Cambridge University Press

Jackson, MO 2014, 'Mechanism Theory', *SSRN Electronic Journal*, 1, pp. 1-46

Mayer, IS, van Bueren, EM, Bots, PWG, van der Voort, H and Seijdel, R 2005, 'Collaborative Decisionmaking for Sustainable Urban Renewal Projects: A Simulation – Gaming Approach', *Environment and Planning B: Planning and Design*, 32(3), pp. 403-423

McGee, B 2009, 'The Community Referendum: Participatory Democracy and the Right to Free, Prior and Informed Consent to Development', *Berkeley Journal of International Law*, 27(2), pp. 570-635

Pereira Roders, A 2019, 'The Historic Urban Landscape Approach in Action: Eight Years Later', in Pereira Roders, A and Bandarin, F (eds) 2019, *Reshaping Urban Conservation*, Springer Singapore, pp. 21-54

Rollings, A and Adams, E 2003, 'Construction and Management Simulation', in Rollings, A and Adams, E (eds) 2003, *Andrew Rollings and Ernest Adams on Game Design*, New Riders

Werner, LC (eds) 2017, *Cybernetics: state of the art*, Universitätsverlag der TU Berlin