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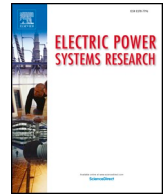
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Stochastic bidding of volume and price in constrained energy and reserve markets

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ABSTRACT

The power system is undergoing a significant change as it adapts to the intermittency and uncertainty from renewable generation. Flexibility from loads such as electric vehicles (EVs) can serve as reserves to sustain the supply-demand balance in the grid. Some reserve markets have rules for participation that are computationally challenging for aggregators of such flexible loads: they are asked to bid both volume and price, and on top of this there is a minimum-volume requirement, a constraint currently under discussion both in the US and European markets. Several state-of-the-art methods to find a bidding strategy for the demand scheduling of large fleets of flexible loads in the day-ahead and reserve market are adapted to deal with such a shared constraint, and are compared based on costs, unscheduled demand, and running time. The experimental analysis shows that although such a shared constraint significantly affects scalability, some of the proposed adaptations can deal with this without much loss in quality. This comparison also shows the importance of including good uncertainty models for dealing with the risk of not meeting the users' demands, and that it is possible to find an optimal single price per time unit for scheduling a fleet of EVs.

1. Introduction

1.1. Aim and motivation

The intermittency and uncertainty of renewable energy sources complicate balancing supply and demand in power systems. Therefore, storage and flexible demand will likely play an increasingly important role in providing reserves for distribution and transmission grids [1]. These services are typically traded via electricity markets. Some markets in the US [2] and Europe pose specific requirements on bids, such as a minimum volume and demanding the inclusion of both a price and a quantity in a bid simultaneously [3]. Without this requirement and when prices in these markets and deployment of reserves are known in advance, finding the minimum-cost schedule that meets all demand (e.g., of a fleet of electric vehicles) is relatively straightforward.

However, under realistic market conditions, scheduling and bidding flexible demand is extremely challenging for two main reasons. First, the mentioned requirement on volume couples the schedules of all flexible demand, which already makes this problem NP-hard [4]. Second, if the deployment of reserves is uncertain, offering reserves implies that not all demand is guaranteed to be fulfilled. Historic data can be used to inform balancing between minimizing costs and meeting

demand, but this requires a method that reasons about multiple possible futures. Which algorithm to use to schedule and bid flexible demand under these market conditions is an important open question.

1.2. Literature review

To find out how to best deal with a minimum-volume constraint in the context of reserve markets, we survey the state of the art of algorithms for bidding in day-ahead and reserve markets, concentrating on methods that bid both price and volume and can deal with price, acceptance and/or deployment uncertainty and are useable for scheduling the charging of a fleet of EVs.

Market uncertainty has been represented by fuzzy sets in [5]. Bessa and Matos introduced an operational algorithm which uses EV charging flexibility to correct for forecast errors and reserves shortage resulting from the day-ahead optimization [6]. Stochastic optimization (SO) has proven to be most effective in capturing the uncertainty in energy markets. Sánchez-Martín et al. formulate the EV charging problem as a two-stage SO, where the first-stage decisions correspond to the DA and reserves volume commitments [7]. Their second stage represents the EV charge demand, the trading decisions during the intra-day markets, and the reserves deployment. Poisson distributions have similarly been used

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to model the probability of reserves activation [8]. Vagropoulos et al. have improved the formulation of offering ancillary services [9]. Markov decision processes are used to model the market prices, and regulation signals and a stochastic dynamic approach is implemented in [10] to find optimal volume commitments when offering ancillary services. These formulations find an optimal reserves volume bid but not the price bid, and leave out the bid acceptance uncertainty.

The acceptance uncertainty of reserves bids is analyzed in [11], [12], and [13]. Alipourand et al. used scenarios of the hourly probability of being called for providing reserves to model such uncertainty [12]. A bid-acceptance probability as a function of time is proposed in [11]. Habibifar et al. [13] model risk aversion through the conditional value-at-risk (CVaR). The state-of-the-art in planning algorithms for other flexible loads is similar to EVs [14–17]. These formulations do not make (optimal) price bids and do not include a shared constraint, but they do show the importance of modeling the bid-acceptance probability.

Van der Linden et al. [3] show the benefit of finding the optimal reserves bid for both volume and price for EV scheduling through an SO formulation. However, their model does not scale well for EV fleets.

There have been several contributions on algorithms that minimize charging costs while addressing the scalability to large EV fleets. A virtual power model that aggregates the energy demand from all the EVs improves the scalability of offline [18] and rolling-horizon algorithms [19]. In contrast, [20] and [21] focus on dividing and solving the problem for each EV independently. A primal-dual decomposition is used to find optimal schedules for a smart charging station in [20]. Rivera et al. formulate the Lagrangian function to divide the EV charging problem and apply the alternating direction method of multipliers to find a solution [21]. Iria et Soares [22] propose a cluster-based optimization for bidding in the day-ahead market.

From this survey we conclude that no approach explicitly considers a minimum-volume market constraint. Furthermore, although several approaches model most of the relevant aspects, no single one can directly deal with the combination of bidding both price and volume in day-ahead and reserve markets, considers price, acceptance and deployment uncertainty, and is useable for scheduling the charging of a fleet of EVs. However, a few methods deal with most of these challenges, or could be used anyway, such as simply ignoring uncertainty. The main open question then remains of how to adapt these methods to be used in the context of a minimum-bid constraint, and how they compare considering optimality, unscheduled demand, and scalability.

1.3. Contributions

This paper presents five adaptations of the four most promising state-of-the-art formulations to this market condition: two extensions based on a stochastic model [3] (including a more tight and compact formulation, providing better efficiency and scalability), and one of a probabilistic formulation of bid acceptance [11], of a divide-and-conquer method based on Lagrangian relaxation [20,21], and of a method for virtual batteries [18,19]. These adaptations are experimentally evaluated with respect to their cost savings, risk management of not meeting customer demands, and scalability potential to large EV fleets.

2. Market framework: Short-term energy and ancillary services markets

Short-term energy markets usually occur from a day before the demand occurs (e.g., day-ahead markets) to the time of energy delivery (e.g., real-time or balancing markets). In combination with ancillary services markets, they constitute a space where flexible energy consumption is valorized [23]. Short-term markets are designed differently by all system operators, but they share objectives and essential features. The particular short-term market we consider in our model is a *day-ahead (DA) market*, as those joint energy-reserve markets are

simultaneously cleared by US independent system operators (ISO), but the concept and general models can be adapted to be used by aggregators to bid to any reserve market. This is a “double-sided blind auction, facilitated by power exchanges” [23]. Market participants trade hourly and multi-hourly energy supply and demand for the next day. The DA energy price is determined after the market closes and (without presence of so-called block-bids for multiple time units) corresponds to the intersection of the demand and supply bidding curves.

Power system operators use a separate market to secure reserve energy and capacity for their real-time power system operations. These markets trade ancillary services, which are the instruments for frequency control, voltage control, system restart, and other emergency control actions. On this paper we concentrate on *reserve markets*, which trade frequency control products. There are three main control categories, determined by the activation speed [23]. For the EU case: the primary control is activated automatically within few seconds to constantly control the frequency by using the Frequency Containment Reserves (FCR); the secondary control is meant to release the activated FCR by using Frequency Restoration Reserves (FRR), and is mainly activated automatically (aFRR) within 1–15 minutes, but a part of the reserves can also be manually activated (mFRR); and the tertiary control uses Restoration Reserves (RR) to release the activated FRR and has a more relaxed response requirements. Most of these type reserves are used in all power systems worldwide but they are named differently. Additionally, reserves can be a contracted or voluntary service. Contracts for providing reserves can be traded for a year, month, week, day or a couple of hours, and are a guarantee of an available resource to offer reliability to system operators [23], such as ERCOT, an US ISO, or Elia, and EU transmission system operator (TSO).

In this paper, data from the ISO Electric Reliability Council of Texas (ERCOT) is used to evaluate the different algorithms. ERCOT establishes its ancillary services hourly plan each morning for the next day, identifying the entities expected to provide such services. These parties must submit their bids within the DA market deadline. Once DA positions and reserves schedule are posted, the parties responsible for providing reserves can make bilateral trades with other qualified entities to secure the reserves volume [24]. Between DA and delivery time (adjustment period), ERCOT uses an hourly reliability unit commitment process to evaluate the system's reliability [25]. Real-time operations are managed every five minutes, and prices are settled in 15-minute intervals [24]. In other markets, such as the balancing market in Belgium, the TSO, Elia requires also bids on 15-minute intervals. A bid consists of a volume and price component with a minimum volume of 1 MW. Bids are assigned following an economic merit order [26].

3. Problem formulation

For the purpose of this paper of optimization methods for bidding in constrained markets, we include (only) uncertainty regarding prices, acceptance of bids, and deployment of reserves in these markets, and use a simple model of the system, assuming unlimited transmission and distribution grid capacity. The optimization problem of an EV aggregator considered in this paper is to trade the charging of a fleet of EVs where we assume known arrival times, departure times and required state-of-charge (SOC) at the time of departure in both DA and reserve markets. In the DA market, the aggregator submits the hourly demand—possibly negative in the case of vehicle-to-grid (V2G) services—and acts as a price-taker. In the reserve market, the aggregator decides on both the volume made available as reserve—dependent on the DA bid—as well as on the bid price, for up- and down-regulation. A reserve bid communicates the willingness of an aggregator to deviate from its DA schedule. Accepted reserve services are rewarded with a payment in proportion to the capacity offered. When the reserves are deployed, the imbalance price is paid for the delivered energy. A reserve bid is accepted only when its price is below the market capacity clearing price. An accepted bid is deployed only when reserves are

needed. The imbalance prices, reserve clearing prices and reserve deployment are unknown and represented by a set of *scenarios*, each describing a possible assignment to all of these unknowns. In short, the first-stage decisions are the energy volume to be traded in DA markets, and the reserve bid price and the reserve bid volume; the actual reserve usage is modeled by second-stage decision variables.

Below we start from the model in [3]. With small changes, this formulation can be adapted to include trading at different stages of the intra-day market, or for settling imbalances balancing markets. Further, it is a building block for online optimization.

This model is a two-stage stochastic programming model. In the first stage the market decisions (i.e., DA energy trade, reserve bid price and volume) are made. The second stage simulates the reserve bid acceptance and resulting state of charge for different scenarios for final use of reserves and clearing price. The second stage constraints also check the feasibility for every scenario.

Eq. (1) represents an aggregator's objective to minimize the expected charging costs for its users over all scenarios. The first term in (1) corresponds to the energy traded DA. The following terms represent costs under the different reserve market scenarios ω : 1) a penalty $\psi \cdot f_{i\omega}$ for not satisfying the EV driver's charging request; 2) payments for providing reserves, i.e., the volume offered as reserves $r_{i\omega}^{\pm}$ multiplied by the sum of the capacity price λ_{ω}^{\pm} and the ratio $\varepsilon_{\omega}^{\pm}$ of time they are deployed during program time unit (PTU) t times the energy price λ_{ω}^{imb} ; and 3) the battery deterioration costs for V2G services, assumed to be linear, as κ multiplied by the volume of V2G provided.

$$\begin{aligned} \min \sum_{h \in H} \lambda_h^{DA} p_h^{DA} + \sum_{i \in I, \omega \in \Omega} \phi_{\omega} \left[\psi f_{i\omega} \right. \\ \left. + \sum_{t \in T} \left[(-\lambda_{\omega}^{-} + \varepsilon_{\omega}^{-} \lambda_{\omega}^{imb})(r_{i\omega}^{-} + r_{i\omega}^{+}) - (\lambda_{\omega}^{+} + \varepsilon_{\omega}^{+} \lambda_{\omega}^{imb})(r_{i\omega}^{+} + r_{i\omega}^{-}) \right. \right. \\ \left. \left. + \kappa(p_{it}^{\uparrow} - r_{i\omega}^{\uparrow} \varepsilon_{\omega}^{-} + r_{i\omega}^{+} \varepsilon_{\omega}^{+}) \right] \Delta \right] \end{aligned} \quad (1)$$

The energy traded DA, p_h^{DA} , is constrained by the charge and discharge schedule for all the EVs as defined by (2). The volume offered as reserves is constrained by the EV availability, the maximum charging or discharging rate, and the DA load commitment, as expressed by Eqs. (3)-(6). The $r_{i\omega}^{\pm}$ are second-stage decision variables that depend on the respective reserves volume and price bid, and its acceptance.

$$\sum_{i \in I, t \in t(h)} (p_{it}^{\downarrow} - p_{it}^{\uparrow}) = p_h^{DA} \quad \forall h \quad (2)$$

$$p_{it}^{\downarrow} + r_{i\omega}^{\downarrow} \leq \bar{p}_i^{\downarrow}(1 - d_{it})\alpha_{it} \quad \forall i, \omega, t \quad (3)$$

$$p_{it}^{\downarrow} - r_{i\omega}^{\downarrow} \geq 0 \quad \forall i, \omega, t \quad (4)$$

$$p_{it}^{\uparrow} + r_{i\omega}^{\uparrow} \leq \bar{p}_i^{\uparrow} d_{it}\alpha_{it} \quad \forall i, \omega, t \quad (5)$$

$$p_{it}^{\uparrow} - r_{i\omega}^{\uparrow} \geq 0 \quad \forall i, \omega, t \quad (6)$$

Eqs. (7)-(10) model the evolution of the EV's SOC, $e_{i\omega t}$. Eq. (7) determines that the SOC in PTU t corresponds to the sum of the charge from load bought in the DA or reserve markets, the SOC in the previous PTU, and the discharge in PTU t .

$$\begin{aligned} e_{i\omega t} = e_{i\omega, t-1} + \eta_i^{\downarrow}(p_{it}^{\downarrow} + r_{i\omega}^{\downarrow} \varepsilon_{\omega}^{-} - r_{i\omega}^{\downarrow} \varepsilon_{\omega}^{+})\Delta \\ - \frac{1}{\eta_i^{\uparrow}}(p_{it}^{\uparrow} - r_{i\omega}^{\uparrow} \varepsilon_{\omega}^{-} + r_{i\omega}^{\uparrow} \varepsilon_{\omega}^{+})\Delta \quad \forall i, \omega, t \end{aligned} \quad (7)$$

Eqs. (8)-(10) define the initial SOC, $e_{i\omega T_1^A}$, the maximum SOC in each PTU, and the minimum SOC required upon departure, $e_{i\omega T_1^P}$, respectively.

$$e_{i\omega T_1^A} = E_i^A \quad \forall i, \omega \quad (8)$$

$$e_{i\omega t} \leq \bar{E}_i \quad \forall i, \omega, t \quad (9)$$

$$e_{i\omega T_1^P} \geq E_i^D - f_{i\omega} \quad \forall i, \omega \quad (10)$$

$$p_{it}^{\downarrow}, p_{it}^{\uparrow} \geq 0 \quad \forall i, t \quad (11)$$

$$r_{i\omega}^{\downarrow+}, r_{i\omega}^{\downarrow-}, r_{i\omega}^{\downarrow+}, r_{i\omega}^{\downarrow-}, e_{i\omega t} \geq 0 \quad \forall i, \omega, t \quad (12)$$

$$d_{it} \in \{0, 1\} \quad \forall i, t \quad (13)$$

To find an optimal reserves volume offer and DA energy purchase, the aggregator can solve the problem defined by (1), and subject to (2)-(10) and the conditions on the decision variables defined in (11)-(13). For compactness' sake, (3)-(6) do not contain the first stage reserve commitment variables r_{it}^{\pm} . These variables and the relations $r_{i\omega}^{\pm} = r_{it}^{\pm} u_{i\omega}^{\pm}$ are implicitly modelled by (14)-(19). These equations fix $r_{i\omega}^{\pm}$ to all the (remaining) charging power when called ($u_{i\omega}^{\pm} = 1$), or to zero otherwise.

$$r_{i\omega}^{\downarrow+} \geq \bar{p}_i^{\downarrow+} u_{i\omega}^{\downarrow+} - p_{it}^{\downarrow+} \quad \forall i, \omega, t \quad (14)$$

$$r_{i\omega}^{\downarrow-} \geq p_{it}^{\downarrow-} - \bar{p}_i^{\downarrow-}(1 - u_{i\omega}^{\downarrow-}) \quad \forall i, \omega, t \quad (15)$$

$$r_{i\omega}^{\downarrow+} \geq \bar{p}_i^{\downarrow+} u_{i\omega}^{\downarrow+} - p_{it}^{\downarrow+} \quad \forall i, \omega, t \quad (16)$$

$$r_{i\omega}^{\downarrow-} \geq p_{it}^{\downarrow-} - \bar{p}_i^{\downarrow-}(1 - u_{i\omega}^{\downarrow-}) \quad \forall i, \omega, t \quad (17)$$

$$r_{i\omega}^{\downarrow+} \leq \bar{p}_i^{\downarrow+} u_{i\omega}^{\downarrow+} \quad \forall i, \omega, t \quad (18)$$

$$r_{i\omega}^{\downarrow-} \leq \bar{p}_i^{\downarrow-} u_{i\omega}^{\downarrow-} \quad \forall i, \omega, t \quad (19)$$

$$u_{i\omega}^{\downarrow+}, u_{i\omega}^{\downarrow-} \in \{0, 1\} \quad \forall i, \omega, t \quad (20)$$

The last group of constraints that define the formulation in [3] are (21)-(24). They represent the system operator's decision of accepting or not downward and upward reserves bids in each scenario based on the price offered by the aggregator.

$$b_{it}^{\downarrow-} \leq \lambda_{\omega}^{\downarrow-} u_{i\omega}^{\downarrow-} + \bar{\lambda}_i^{\downarrow-}(1 - u_{i\omega}^{\downarrow-}) \quad \forall i, \omega, t \quad (21)$$

$$b_{it}^{\downarrow-} \geq \lambda_{\omega}^{\downarrow-}(1 - u_{i\omega}^{\downarrow-}) + \bar{\lambda}_i^{\downarrow-} u_{i\omega}^{\downarrow-} \quad \forall i, \omega, t \quad (22)$$

$$b_{it}^{\downarrow+} \leq \lambda_{\omega}^{\downarrow+} u_{i\omega}^{\downarrow+} + \bar{\lambda}_i^{\downarrow+}(1 - u_{i\omega}^{\downarrow+}) \quad \forall i, \omega, t \quad (23)$$

$$b_{it}^{\downarrow+} \geq \lambda_{\omega}^{\downarrow+}(1 - u_{i\omega}^{\downarrow+}) + \bar{\lambda}_i^{\downarrow+} u_{i\omega}^{\downarrow+} \quad \forall i, \omega, t \quad (24)$$

This formulation captures the uncertainty of the regulation situation, price and volume bid decision variables, and the constraints that define bid acceptance.

4. Solution methods

This section introduces a more scalable formulation of the stochastic model, a deterministic model, and three heuristic and mathematical methods, adapted to the problem definition described above. All models, methods, parameters and data are available as open source [27].

4.1. Stochastic formulation – SDIR

The first method is a more computationally efficient variant of the Stochastic model for DA, Imbalance and Reserves (SDIR) model from van der Linden et al. [3], summarized in the previous section. The model we present below is more tight (the relaxed solution is closer to the integer solution) and compact (fewer constraints) and thus computationally more attractive. The resulting formulation still provides the same solution.

The formulation defined by (1)-(24) accounts for most of the market uncertainty when bidding in the reserve market but is computationally complex. Its complexity is mainly related to the set of binary variables that represent the bid acceptance in each case, $u_{i\omega}^{\pm}$ and the constraints to determine the bid acceptance based on price bid. In order to reduce the computational complexity, we replace some of the constraints.

We define $\bar{\omega}^\pm(t)$ as the scenario with the largest regulation price for the up and down regulation at the corresponding PTU, and $\underline{\omega}^\pm(t)$ the smallest; and introduce two functions to define the predecessor or successor regulation price scenario for each up and down regulation state and PTU, independently: $g_p^\pm(\Omega, \omega, t)$ corresponds to the scenario that precedes scenario ω in PTU t , and $g_s^\pm(\Omega, \omega, t)$ to the scenario that succeeds ω . The full ordering is based on $\lambda_{\omega t}^\pm$ for each PTU. With this ordering it is possible to tighten the constraint formulation. The following constraints show how this works when the EV is charging and offers down reserves. The other cases follow similarly.

Eqs. (25)–(27) determine the reserve bids and reserve acceptance for the minimum and highest capacity price scenarios.

$$p_{it}^\downarrow + r_{i\bar{\omega}^\downarrow t}^\downarrow \leq \bar{p}_i^\downarrow (1 - d_{it}) \alpha_{it} \quad \forall i, t \quad (25)$$

$$r_{i\bar{\omega}^\downarrow t}^\downarrow \geq \bar{p}_i^\downarrow u_{i\bar{\omega}^\downarrow t}^\downarrow - p_{it}^\downarrow \quad \forall i, t \quad (26)$$

$$r_{i\bar{\omega}^\downarrow t}^\downarrow \leq \bar{p}_i^\downarrow u_{i\bar{\omega}^\downarrow t}^\downarrow \quad \forall i, t \quad (27)$$

$$u_{i\omega t}^\downarrow \geq u_{ig_p^-(\Omega, \omega, t), t}^\downarrow \quad \forall i, \omega \in \Omega \setminus \{\bar{\omega}^\downarrow(t)\}, t \quad (28)$$

$$r_{i\omega t}^\downarrow \geq r_{ig_p^-(\Omega, \omega, t), t}^\downarrow \quad \forall i, \omega \in \Omega \setminus \{\bar{\omega}^\downarrow(t)\}, t \quad (29)$$

With the reserve volume and acceptance set for the extremes, these values for the rest of the scenarios can be set with help of the predecessor function. Eqs. (28)–(29) describe this relation. If a bid is accepted in a scenario with a cheaper reserve price, it will also be accepted in a scenario with a larger reserve price.

Unlike $u_{i\omega t}^\downarrow$, the variable $r_{i\omega t}^\downarrow$ is continuous. To force this value to be either 0 when $u_{i\omega t}^\downarrow = 0$, or all the remaining charge capacity when $u_{i\omega t}^\downarrow = 1$, Eq. (30) is needed.

$$r_{i\omega t}^\downarrow - r_{ig_p^-(\Omega, \omega, t), t}^\downarrow \leq \bar{p}_i^\downarrow (u_{i\omega t}^\downarrow - u_{ig_p^-(\Omega, \omega, t), t}^\downarrow) \quad \forall i, \omega \in \Omega \setminus \{\bar{\omega}^\downarrow(t)\}, t \quad (30)$$

The result of (28)–(30) is that the set of scenarios is split in two groups: scenarios where the reserve bid is accepted, and those where it is not accepted. The price b_{it}^\downarrow of the reserve bid can be retrieved choosing a value between the two corner scenarios ω_1 and $\omega_2 = g_s^-(\Omega, \omega_1, t)$ where $u_{i\omega_1 t}^\downarrow = 0$, and $u_{i\omega_2 t}^\downarrow = 1$, such that $\lambda_{\omega_1 t}^\downarrow \leq b_{it}^\downarrow \leq \lambda_{\omega_2 t}^\downarrow$.

Now it is possible to replace (3)–(6), (14)–(19) and (21)–(24) by (28*)–(30*), with the asterisk meaning that these constraints also apply to the up reserves and discharging cases. These changes reduce the number of constraints by $8|W||T||I|$. We have shared this model as part of a benchmarking toolbox [28].

Aggregation and the shared constraint

The model for each EV from the set I is identical and almost independent (indexed by $i \in I$). The only relation is that the DA hourly energy purchase corresponds to the sum of energy purchases for different EVs in that hour. If DA trading is planned separately for each EV and multiple small bids are allowed, each of the EV models can be solved independently, making this approach linearly scalable with the number of EVs. However, an EV aggregator offering reserves could have to comply with a minimum volume per reserves bid.

We introduce (31)–(36) to enforce bid aggregation. The variables $v_{\omega t}^\pm$ count the number of different price bids. Eq. (31) and (32) enforces $v_{\omega t}^\pm = 1$ if at least one EV has made a bid at the regulation price associated to scenario ω in PTU t . Eq. (33) restricts the number of different price bids below a defined number for offering reserves during upward and downward regulation, independently.

$$v_{\omega^\pm t}^\pm \geq u_{i\omega^\pm t}^\pm \quad \forall i, t \quad (31)$$

$$v_{\omega t}^\pm \geq u_{i\omega t}^\pm - u_{ig_s^\pm(\Omega, \omega, t), t}^\pm \quad \forall i, \omega \in \Omega \setminus \{\bar{\omega}^\pm(t)\}, t \quad (32)$$

$$\sum_{\omega \in \Omega} v_{\omega t}^\pm \leq \bar{\Theta}^\pm \quad \forall t \quad (33)$$

$$v_{\omega t}^\pm, v_{\omega t}^\pm \in \{0, 1\} \quad \forall \omega, t \quad (34)$$

Additionally, (35) and (36) restrict reserve market bids to a minimum volume π .

$$v_{\omega^\pm t}^\pm \pi \leq \sum_{i \in I} (r_{i\omega^\pm t}^\pm + r_{i\bar{\omega}^\pm t}^\pm) \quad \forall t \quad (35)$$

$$v_{\omega t}^\pm \pi \leq \sum_{i \in I} \left[(r_{i\omega t}^\pm + r_{i\bar{\omega} t}^\pm) - (r_{ig_s^\pm(\Omega, \omega, t), t}^\pm + r_{ig_s^\pm(\Omega, \omega, t), t}^\pm) \right] \quad \forall \omega \in \Omega \setminus \{\bar{\omega}^\pm(t)\}, t \quad (36)$$

The new SDIR model is defined by (1), (2), (7)–(13), (20), (25*)–(30*), and (31)–(36). These constraints are also used to add the minimum-bid requirement in the other four methods presented below.

4.2. Probabilistic formulation – DDIR

A formulation based on a probabilistic analysis of the bid acceptance was proposed in [11]. Here we extend the variant of their formulation presented in [3] with the additional minimum-bid constraint. The main idea is to define a bid acceptance probability for each PTU. This model is very similar to the formulation in (1)–(13) but with two important changes. First, the accepted reserve market volume bid is replaced by a first-stage decision variable that represents the reserves volume bid for all the scenarios. In the objective, the accepted volume is multiplied by the expected reserve market price. Additionally, the reserves volume bid is multiplied by the probability distribution of bid acceptance for each PTU. Such a change is achieved by multiplying the volume offered as reserves in (1) and (7) by the acceptance probability.

4.3. Lagrangian relaxation – SLR

Previous work on large EV fleet scheduling has focused on well-known mathematical methods that decompose large scale problems [20,21]. Similar to this line of work, we relax the minimum-bid constraints and iteratively find a better solution [29]. By introducing p_{ih}^{DA} , the energy bought in the DA market for EV i in hour h and reformulating (2), the problem becomes divisible into $|I| + 1$ problems: $|I|$ problems for the optimal charging scheduling of each EV, and 1 problem to determine the optimal EV bids aggregation per PTU.

4.4. Greedy method based on SDIR – SGR

The fourth method is a simple greedy algorithm based on SDIR. Like SLR it requires the change to (2) such that the problem is identical and independent per EV. The first step solves the SDIR for each EV disregarding the minimum-bid requirement. In the second step, the algorithm aggregates the volume bid in each PTU per bidding price for all EVs and selects the bidding price with the largest volume per PTU. In the last step, the SDIR is solved constraining the bidding options to the price previously found.

4.5. Virtual battery – SVB

Similar to the solution proposed in [18] and [19], we aggregate the total EV fleet demand in a minimum set of demand profiles, or virtual batteries (VBs). A VB represents a group of EVs with the same charging speed (we use two VBs in the experiments). The SDIR problem can be transformed to integrate the concept of VBs by following four steps: 1) defining the VB (dis)charging capacity by multiplying the charging speed of every EV in the VB by the number of EVs charging at each PTU; 2) estimating the VB maximum SOC by summing the maximum SOC of each EV in each PTU (this maximum is defined as the accumulated energy in the EV battery if charged at maximum charging speed, but limited to the battery's capacity); 3) including a set of variables to define the SOC for each departing EV; and 4) transforming (7)–(10) to

Table 1
Solution methods for the EV market planning problem.

Method	Description
Stochastic (SDIR)	Solves a stochastic MIP model defined by (1), (2), (7)-(13), (20), (25*)-(30*), and (31)-(36). Reserve acceptance is modeled by binary variables.
Probabilistic (DDIR)	Solves a MIP model similar to (1)-(13), but reserve acceptance is modeled as a parameter and reserve price bids are chosen accordingly. For the full model, see [3].
Lagrangian (SLR)	Solves the SDIR model per EV by relaxing the shared constraint and iterating to a better solution.
Greedy (SDIR-SGR)	Solves SDIR per EV and based on these partial solutions fixes the price bids in the complete SDIR model.
Virtual battery (SVB)	EVs with the same charging speed are modeled as one EV. The price bids from this (possibly infeasible) solution is used to fix the price bids in the complete SDIR model.

account for each time an EV becomes available and each time an EV departs. This VB formulation solves the market planning problem, and it provides a possible EV departing SOC, but it does not guarantee a feasible charging schedule per EV. The price bid resulting from the VB formulation is used to constrain the bidding options in the SDIR model.

5. Analysis and comparison of solution methods

The experiments are conducted to assess the different solution methods on their computational properties, average costs, and whether there is any unscheduled demand, which may be caused by the uncertainty in the acceptance of price bids and the activation of the offered reserves. The simulation experiments therefore include a comparison of the optimal value and computation time of the SDIR formulation in [3] and the more scalable version that we propose; a comparison of the five solution methods for a simple problem and two more complex cases; and a scalability analysis of all the methods. A summary of the methods presented can be found in Table 1.

Given the aim of comparing under representative conditions, and not of simulating a specific real-world place and period exactly, we used real-world data, relying on different easily available sources with sufficiently detailed data. EV sets of three sizes, 5, 50, and 500 were generated by randomly selecting EV charging sessions from the publicly available Dundee city council database [30]. The sets of 5 and 50 EVs include sessions starting at 16:00 and extend up to the next day. The set of 500 EVs starts at 14:00. Most experiments used sets of EVs with mixed flexibility, i.e., they include charging sessions with only a few hours in addition to the time required to fulfill the demand at maximum charging speed (tighter sessions), and sessions with several hours of excess (normal sessions). A set of EVs only with tighter charging sessions helped to quantify the computational complexity increase from the added flexibility. The data set used is included in the B-FELSA toolbox [27].

All the experiments were replicated for ten different market days generated from the Electric Reliability Council of Texas' (ERCOT) historical market data from years 2016 and 2017 [31,32]. Each day is defined by a single scenario of DA market prices, and 52 scenarios of the regulation price, market capacity price and the proportion of a PTU when reserves are deployed. The scenarios were generated using the methodology from [3]. All the SO experiments used a subset of 30 of the 52 scenarios to find the optimal solution; but for the evaluation of the results from all the methods, we use the complete set of market price data. The 30 scenarios were optimally selected from the 52 scenarios such that they best represent the mean and variance of the full set.

Even though ERCOT's market data was used for all the experiments, the contributions of this paper apply to other markets. With this in mind, we assumed that the reserves bid length is a quarter of an hour and not an hour as it is in the ERCOT market. Charge demand that is not fulfilled (unscheduled demand) is penalized by \$60/MWh, the charging efficiency is assumed to be 90% for all EVs, the battery degradation cost due to discharging is \$0.042/kWh, and the desired bid-acceptance probability in the DDIR model is 90%. The unscheduled demand penalty determines the risk taken by the EV aggregator; its value allows for solutions with unscheduled demand within margins that could be

resolved in real-time rescheduling and trading with 99% of certainty.

All the solution methods were coded in Java using Gurobi 7.5.2 [33] as MIP solver. The tests were conducted in an Intel(R) Xeon(R) CPU E5-1620 @ 3.70 GHz and 16 GB of RAM memory. The experiments were stopped after reaching 1% or 5% optimality gap (for SDIR), or after 5 hours of running time.

5.1. Equivalence and time efficiency comparison of the two SDIR formulations

The reduced computational complexity of the SDIR formulation proposed in this paper is evident when comparing the average total cost across 52 scenarios of 10 days for one EV charging session (G2V) minimum-volume requirement, with maximum charging speed of 7 kW, initial SOC of 10 kWh, maximum SOC of 27 kWh, and arriving for charging at 21:15 and departing at 7:15 of the next day (see Table 2). Experiments were stopped after 5 hours (for the old formulation) or when reaching the 1% optimality gap (for the new one). Results for the same day using the old and new versions of the SDIR formulation are within \$ 0.02 difference and four of the 10 day-experiments are within a 5% relative difference. The discrepancies in the other six results can be associated to the poor solution quality reached when the old formulation is stopped within five hours. These results confirm that both formulations are equivalent. The reduction in computational complexity is about two orders of magnitude.

5.2. Base evaluation of the proposed solution methods

The next set of experiments aimed to draw conclusions about the performance of the five solution methods without the minimum-bid requirement. The comparison is based on charging cost savings, customer satisfaction (unscheduled demand) and running time. For such evaluation, we used two different sets of 5 EV charging sessions. The normal EV set includes a combination of flexible and tight charging sessions, and the tighter set only tight sessions. The problem is constrained to finding a solution to the DA and reserve market bidding strategy with only one bid per PTU for a set of EVs. Two other cases are

Table 2
Computational complexity of both SDIR formulations.

Day	Total cost [\$]		Time [s]		Gap ^a - 5h
	new SDIR	old SDIR	new SDIR	old SDIR	
1	-0.12	-0.11	10		48%
2	0.01	0.01	11		115%
3	0.00	-0.01	136		34%
4	0.01	0.01	63		336%
5	-0.14	-0.14	24		25%
6	-0.02	-0.02	4		205%
7	-0.06	-0.06	11		67%
8	0.00	0.00	91		333%
9	-0.12	-0.12	41		37%
10	-0.11	-0.10	13		71%

^a MIP relative optimality tolerance after 5 h running time.

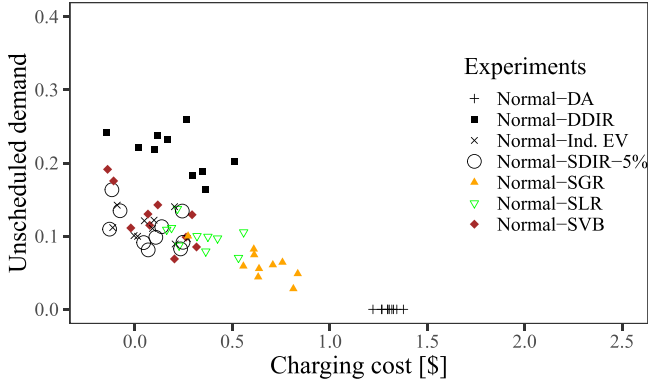


Fig. 1. Average unscheduled demand as percentage of total demand for sets of 5 normal EVs without minimum reserves bid requirement.

included for comparison: smart charging following deterministic DA prices (DA), and smart charging for all the EVs but without the single bid constraint (Ind. EV). For the latter, the SDIR formulation is used to find optimal solutions for each charging session individually. No solutions are reported for the normal set using the SDIR method to 1% relative optimal tolerance due to computational complexity.

Figs. 1 and 2 present the average unscheduled demand as percentage of the total demand for two types of sets of EVs, i.e. flexible and tighter EVs, respectively. All stochastic methods have small percentage of unscheduled demand, in particular the SGR method. The schedules produced show that this is the result of more conservative reserve bidding: bidding when probability of acceptance is either high or low and therefore more easy to predict.

For scenarios that are not considered in the stochastic and DDIR methods, violations to the battery capacity constraint (9) may occur. For DDIR, these violations are within 5.6% of the total capacity. For the stochastic methods it is within 3.4%, where SGR has the least violations.

Figs. 1 and 2 also help to visualize the trade-off between minimizing costs and customer satisfaction. The DDIR method offers low charging costs, but it has a higher risk of unscheduled demand. SDIR at 1% gap and individual EV scheduling (Ind. EV) are able to find a single price bid for all EVs without compromising the cost reduction. The visualization shows that the SDIR method finds solutions at the change of curvature. From the perspective of the EV user and aggregator, the SDIR method offers the best solution quality: minimum charging costs with small unscheduled demand. This benefit depends on the flexibility of the EVs. It is not as rewarding for tighter EV charging sessions when compared to more flexible EV fleets, but finding an optimal solution for tighter EV fleets is less computationally complex.

After conducting t-tests to compare results across methods, we can

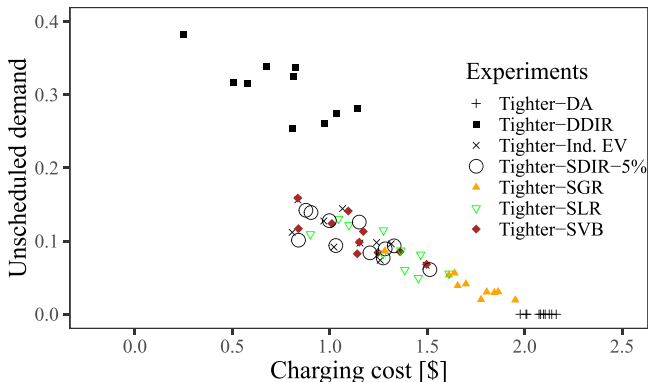


Fig. 2. Average unscheduled demand as percentage of total demand for sets of 5 tighter EVs without minimum reserves bid requirement.

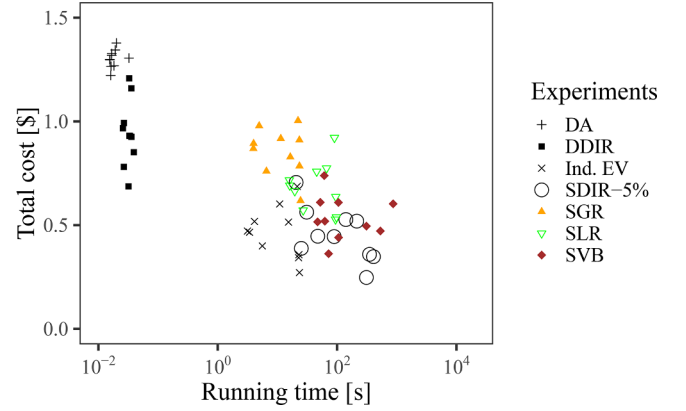


Fig. 3. Average total cost for a set of 5 normal EVs offering only grid-to-vehicle (G2V) services, without minimum-volume reserves bid requirement.

conclude with more than 95% confidence that the SDIR method offers a better total cost solution than the DDIR method. With more than 95% confidence we can reject the null hypothesis that the EV schedule and energy and reserve market participation found using the SDIR method is equivalent to the DA solution. This comparison is based on the *total costs* which consist of the charging costs and a penalty associated to unscheduled demand set to \$60/MWh, as explained in [3].

5.3. Effect of the minimum-volume bid requirement and V2G services

Computational complexity grows when a minimum-bid requirement is imposed, which can be observed by comparing Figs. 3 and 4. The latter shows the average total cost when scheduling a normal set of EV charging sessions assuming that the EVs offer only G2V services and the alternative, offering V2G bidirectional services. For a set of 5 EVs, the minimum bid requirement was set to 0.01 MW to make it reasonable for the problem size. These results show the financial benefit from offering bidirectional V2G services as well as the computational complexity of this problem. Since experiments were stopped after 5 hours, the results with more than 10^4 seconds in Fig. 4 correspond to problems that were stopped with more than 5% MIP gap.

Most conclusions about the benefit from the SLR, SGR, and SVB methods that apply to the experiments without a minimum-bid also apply to the two sets of experiments with minimum-volume bid. Again, the SDIR method offers better total costs solutions than DDIR for the cases with a minimum-volume bid requirement and when offering V2G. The null hypothesis that the two methods are equivalent can therefore be rejected with more than 95% confidence.

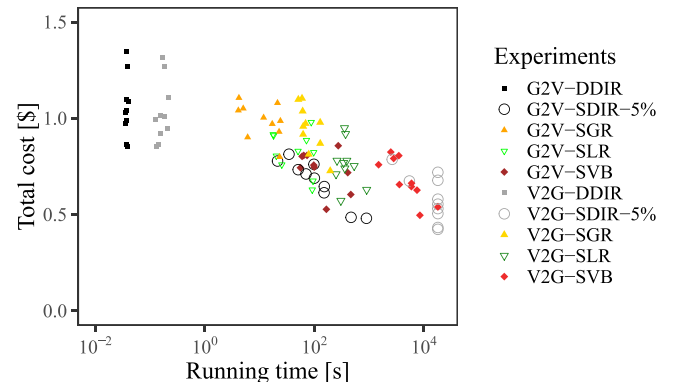


Fig. 4. Average total cost for a set of 5 normal EVs with a 10 kW minimum reserves bid requirement per PTU.

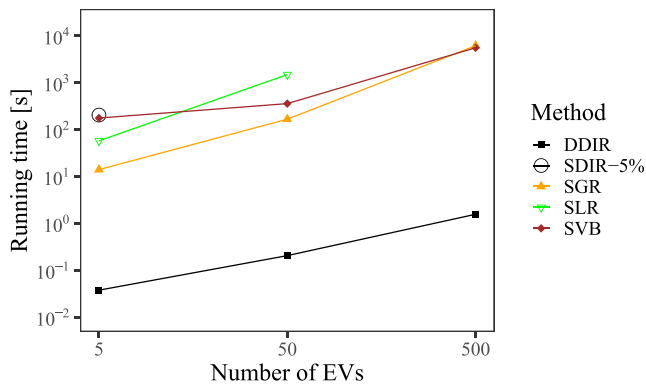


Fig. 5. Scalability of the 5 solution methods.

5.4. Scalability assessment

The last set of experiments focused on comparing running time and solution quality for larger EV fleets. Fig. 5 compares the average running time for each of the 5 solution methods considered for this work and for 3 EV fleet sizes, 5, 50 and 500. These experiments are only for G2V services, and forcing a single minimum-volume bid of 0.01, 0.1 and 1 MW, respectively. Notice that a logarithmic scale is used for both axes. Three data points are missing in Fig. 5, the SDIR-5% for 50 and 500 EVs, and the SLR for 500 EVs. These experiments took over 5 hours to reach solutions within the defined optimality tolerance or number of iterations.

The results in Fig. 5 indicate that the computational complexity grows faster for all the SO methods (SDIR, SLR, SGR and SVB) than the probabilistic method, DDIR. The SVB method shows the best scalability among the SO methods. Nonetheless, there are three important remarks. First, the SVB formulation used for these experiments depends on the diversity of charging speeds in the EV fleet, since the EVs with same charging rate are grouped together and there is a VB per speed. All the experiments used 2 VBs. Second, the SLR and SGR methods can have shorter running time if the step when a solution is found for each EV is computed in parallel. Last, the comparison of experiments for 50 and 500 EVs for the SLR, SGR and SVB methods showed different solution quality results than for 5 EVs. SLR found the lowest total costs solutions among the three methods for tests with 50 EVs. On average SVB finds lower costs solutions than SGR but it is not possible to conclude with high confidence that SVB finds better solutions.

For the experiments with a fleet of 500 EVs the SVB offers 22% average savings compared to participating in the DA and SGR 19%. The DDIR method is highly scalable but solutions have the largest total costs and unscheduled demands among all the compared methods.

6. Conclusion

Stakeholders of the US and European markets are reconsidering the minimum-volume requirements for offering ancillary services by storage units, as these impose an additional complexity to the aggregator's bidding problem. We offer adaptations of the state-of-the-art algorithms and models to such a shared constraint, and provide a comparison between them to inform such debate.

This comparison shows the importance of good uncertainty models for dealing with the risk of not meeting the users' demands. The updated stochastic optimization formulation (SDIR) has the best solution quality but even the improvement of the model does not make it scale to large fleets. The virtual battery (SVB) method presented the best scalability and had very good solution quality for the 5 EVs experiments. The solution using SVB also offers significant economic advantage to only buying energy in day-ahead (DA) markets for a 500 EV fleet. Nonetheless, there is margin for improving this result. The

probabilistic method (DDIR) is the most scalable of all the methods and finds solutions with low charging costs, but it is not a good option for meeting the user demand. The experiments also showed how it is possible to find a single price per time unit for scheduling all the EVs optimally, which is a very important insight for aggregators interested in offering reserves. With this analysis we can conclude that although market constraints have a significant effect on scalability, it seems to be within reach to reduce computation time without much loss in quality. As such, computational issues by themselves cannot be the sole argument for removing market conditions.

For future work, the proposed methods could incorporate a more advanced model for battery degradation, for example as in [34]. However, as this contributes only to a small part of the costs, we do not expect this to affect the main conclusions. We consider it more interesting to design methods that scale to realistically-sized fleets and are simultaneously rich in the details regarding the system constraints, minimum-volume constraint and the price bids. This research could also be extended by including a formulation for the EV user uncertainty, or modifying it to represent other types of flexible loads. Lastly, online algorithms are important for close-to-real-time trading when it is possible to make adjustments to forecast errors and small violations of offline methods. The proposed methods can be reformulated to make the reserves bidding decision online and we expect their relative performance to be consistent with the presented results, but testing this hypothesis is important before using this for actual trading.

Data Availability

Datasets and models related to this article can be found at <https://doi.org/10.5281/zenodo.3989096ft/B-FELSA> <https://github.com/AlgTUDelft/B-FELSA>, a benchmark for flexible electric load scheduling algorithms [28].

CRediT authorship contribution statement

Natalia Romero: Conceptualization, Methodology, Software, Formal analysis, Investigation, Writing - original draft, Visualization, Project administration. **Koos van der Linden:** Methodology, Software, Resources, Writing - review & editing. **Germán Morales-España:** Conceptualization, Methodology, Writing - review & editing. **Mathijs M. de Weerd:** Conceptualization, Methodology, Resources, Writing - review & editing, Supervision, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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