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Development of a Robust Coupled Material Point Method

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Abstract. The material point method (MPM) shows promise for the simulation of large deformations in history-dependent materials such as soils. However, in general, it suffers from oscillations and inaccuracies due to its use of numerical integration and stress recovery at non-ideal locations. The development of a hydro-mechanical model, which does not suffer from oscillations is presented, including a number of benchmarks which prove its accuracy, robustness and numerical convergence. In this study, particular attention has been paid to the formulation of two-phase coupled material point method and the mitigation of volumetric locking caused numerical instability when using low-order finite elements for (nearly) incompressible problems. The numerical results show that the generalized interpolation material point (GIMP) method with selective reduced integration (SRI), patch recovery and composite material point method (CMPM) (named as GC-SRI-patch) is able to capture key processes such as pore pressure build-up and consolidation.

Keywords: Coupled behavior · Hydro-mechanical · Large deformation · Material point method · Reduced integration · Consolidation

1 Introduction

Two-phase coupled porous materials frequently undergo large deformations in geotechnical engineering problems. Recently, the material point method (MPM) has shown promising potential for the analysis of history-dependent problems, such as those involving soils, under large deformations. Thus, it has been applied to coupled porous materials and received a large amount of attention in recent years. In some previous studies, the coupled MPM is formulated using the simplified *u*-*p* formulation, in which the relative soil-water acceleration is neglected and the basic unknowns are limited to soil displacement *u* and pore water pressure *p* (Zhang et al. 2009; Higo et al. 2010; Zabala and Alonso 2011; Abe et al. 2013; Zheng et al. 2013). In contrast, to fully capture the dynamic response of porous media, most recent investigations have been performed using the *v*-*w* formulation (considering both the velocities of the soil skeleton *v* and the pore water *w*), which take the relative accelerations of the pore water into account (Zhang et al. 2007; Jassim et al. 2013; Bandara and Soga 2015; Soga et al. 2015; Yerro et al. 2015; Ceccato et al. 2016; Wang et al. 2018; González Acosta et al. 2019). Within the framework of

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coupled MPM, both a single layer of material points representing both the soil and water phases and two layers of material points representing the respective soil and water phases, have been developed to simulate the interaction between pore water and soil skeleton. A detailed description concerning their advantages and disadvantages has been given by Soga et al. (2015). Considering the high computational cost of two layers of MPs, the single layer approach is most often preferred as in this study.

When applying MPM to hydromechanical problems, the regular MPM formulation may suffer from volumetric locking, leading to numerical instability, when using low-order interpolation functions for both displacement and pore pressure fields during the simulation of (nearly) incompressible problems. In this study, a reduced integration scheme has been adopted to evaluate the pore pressure at the central Gauss point of a cell, and this has proved to greatly alleviate volumetric locking problems caused by (nearly) incompressible constraints. The resulting stabilised material point method, called GC-SRI-patch uses a selective reduced integration technique combined with GIMP (Bardenhagen and Kober (2004)), adopts pore pressure recovery at central Gauss points (GPs), while effective stresses are still calculated at material point locations using the composite material point method (CMPM) proposed by González Acosta et al. (2017). Pore pressure increments calculated at central GPs of the low-order square grid cell are highly accurate, and are mapped to the material points (MPs) using a patch recovery based on a moving least squares approximation (MLSA) similar to the concept of superconvergent patch recovery in Zienkiewicz and Zhu (1992) and to the single phase MPM by González Acosta et al. (2019). The proposed GC-SRI-patch can provide better, locking-free results, and has been applied here to 1D consolidation example problems.

2 Governing Equations for Fully Saturated Porous Soils

In this section, the equations governing the dynamics of fully saturated porous media are recalled based on the ***v-w*** formulation, which involves both the velocities of the soil skeleton v_s and of the pore water v_s as unknown variables. Basic conservation laws for mass and momentum are summarised hereafter.

For the water phase, the momentum balance equation can be written as:

$$\nabla P_w - \rho_w \boldsymbol{a}_w - \frac{n(\boldsymbol{v}_w - \boldsymbol{v}_s)\rho_w \boldsymbol{g}}{k} + \rho_w \boldsymbol{b} = 0 \tag{1}$$

where P_w is the pore pressure, ρ_w and $\boldsymbol{a}_w$ are the water density and acceleration, $\boldsymbol{v}_w$ and $\boldsymbol{v}_s$ are the velocities of the water and soil phases, k is the (isotropic) hydraulic conductivity of the soil, $\boldsymbol{g}$ is the gravitational acceleration, and $\boldsymbol{b}$ is the body force vector (e.g. due to gravity).

The momentum balance for the mixture of water and solid can be given as:

$$\nabla \cdot \sigma - (1-n)\rho_s \boldsymbol{a}_s - n\rho_w \boldsymbol{a}_w + \rho \boldsymbol{b} = 0 \tag{2}$$

where σ is the total stress tensor of the mixture and is defined as $\sigma = \sigma' + \boldsymbol{m}P_w$ according to Terzaghi's effective stress theory (where m is equivalent to an identity matrix), n is porosity, ρ_s is the density of soil grains, $\boldsymbol{a}_s$ is the soil acceleration, and ρ is the density of soil and water mixture defined as $\rho = (1-n)\rho_s + n\rho_w$.

The mass conservation of the solid phase can be written as:

$$\frac{\partial(1-n)\rho_s}{\partial t} + \nabla \cdot (1-n)\rho_s \boldsymbol{v}_s = 0 \tag{3}$$

when considering the soil particles to be incompressible and, by disregarding the spatial variation of both the density and porosity of the soil, the mass conservation of the solid phase becomes:

$$\frac{\partial n}{\partial t} = (1-n)\nabla \cdot \boldsymbol{v}_s \tag{4}$$

Similarly, the mass conservation of the water phase can be expressed as:

$$\frac{\partial n\rho_w}{\partial t} + \nabla \cdot n\rho_w \boldsymbol{v}_w = 0 \tag{5}$$

Assuming barotropic water flow, the rate of pore pressure change can be computed as:

$$\frac{\partial P_w}{\partial t} = \frac{K_w}{n}[(1-n)\nabla \cdot \boldsymbol{v}_s + n\nabla \cdot \boldsymbol{v}_w] \tag{6}$$

For brevity, details about weak forms underlying the MPM formulation are omitted.

3 Alleviating Volumetric Locking in the Coupled MPM

As already mentioned, when applying MPM to hydromechanical problems, the regular MPM formulation may suffer from volumetric locking and spurious oscillations during the simulation of (nearly) incompressible media. In our analysis, the water phase is considered with a realistic (very high) bulk modulus, and this nearly incompressible constraint makes the MPM unable to reproduce correct deformation modes. Explanations regarding volumetric locking in weakly compressible media using MPM can be found in Chen et al. (2018). In this study, selective reduced integration (SRI) is employed, as it can reproduce more accurately deformation modes and pore pressure distributions and can also be directly implemented into existing coupled GIMP codes. Therefore, coupled GIMP with selective reduced integration and patch recovery is adopted.

In GC-SRI-patch, reduced integration is only carried out for calculating pore pressure increments (Eq. (6)) at the central GPs of elements. It is known that numerical results evaluated at central Gauss point locations are of high accuracy and convergence order, so that they are here mapped to MPs by patch recovery using the moving least squares approximation (MLSA) (shown in Fig. 1). Effective stresses are calculated using GIMP at material point locations combined with the composite material point method (CMPM) (González Acosta et al. 2017) to improve the effective stress recovery.

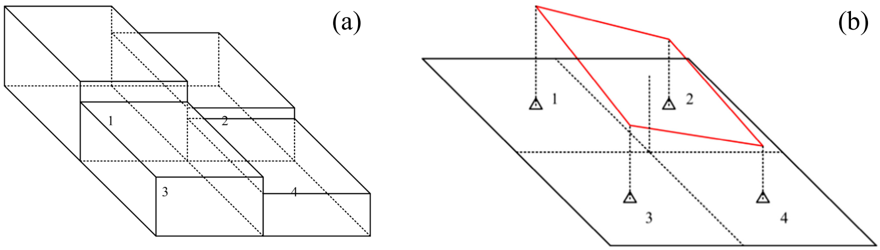

Fig. 1. Patch recovery for incremental pore pressure: (a) initial incremental pore pressure distribution; (b) incremental pore pressure distribution after performing patch recovery

4 Numerical Implementation of GC-SRI-patch

Numerical simulations are performed here based on explicit time marching. The solution of each step is carried out according to the proposed GC-SRI-patch procedure summarised as follows:

(1) Initialise all variables in the background mesh;
(2) Calculate water and soil nodal accelerations using the weak form of the momentum balance equations (e.g. using a finite element formulation);
(3) Update the nodal velocities for both the soil and the water using explicit forward Euler method;
(4) Update both velocities and positions of the MPs;
(5) Update the nodal velocities for both the soil and water by mapping back variables from MPs;
(6) Calculate effective stresses at MP positions using GIMP combined with CMPM;
(7) Compute incremental pore pressures at GPs and recover incremental pore pressure from GPs to MPs using MLSA;
(8) Reset the background mesh and move to next computation cycle.

5 Numerical Examples

In this section, 1D consolidation problems under both small and large deformation regimes are considered, in order to verify and demonstrate the performance of the proposed GIMP-SRI. Figure 2 shows geometry and boundary conditions for the 1D consolidation problem. The width and depth of the problem domain are 0.1 m and 1.0 m, respectively. A fully saturated condition is considered, and the pore water is only allowed to drain in/out through the top surface. The two lateral boundaries are impermeable and supported by rollers allowing only vertical displacement, whereas the bottom boundary is impermeable and totally fixed. The mesh size used to descritise the problem domain is 0.1 m × 0.1 m, with initially 4 equally-spaced material points per element, and the solution was obtained using a time step size of 1.0×10^{-6} s.

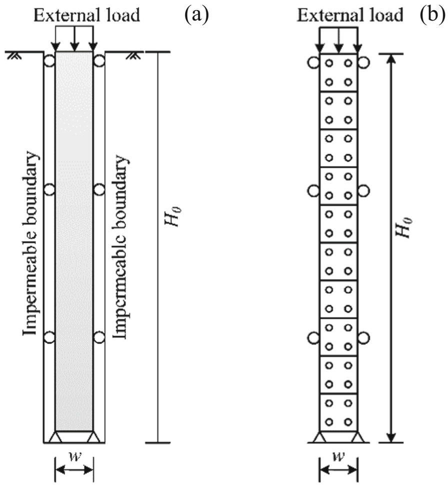

Fig. 2. 1D consolidation problem: (a) geometry and boundary conditions; (b) MPM discretization

The soil is modelled as a linear elastic material, and properties of both soil and water phases are listed in Table 1.

Table 1. Considered properties of soil and water phases

	Properties	Unit	Value
Soil	Young's modulus E	kPa	1.0×10^3
	Poisson's ratio v	–	0.25
	Soil grain density ρ_s	kg/m^3	2.65×10^3
	Porosity n	–	0.3
Water	Bulk modulus K_w	kPa	2.2×10^6
	Hydraulic conductivity k	m/s	1.0×10^{-4}
	Water density ρ_w	kg/m^3	1.0×10^3

5.1 1D Consolidation Under Small Deformation Analysis

For small deformation analysis, a constant $P_a = 1.0$ kPa external compression is applied on the column surface at the boundary nodes. Figure 3 compares the solution computed by the GIMP-SRI with the analytical solution provided by Terzaghi (1943), which is based on assumption that the depth of domain z, the hydraulic conductivity k, and the Young's modulus of the soil E remain constant during the consolidation process.

In Fig. 3, the pore pressure P_w is normalised with respect to the initial applied static load P_a, and the depth z is normalised with respect to the initial depth H_0 to give H_v. It can be concluded that both excess pore pressure and average degree of consolidation obtained from GC-SRI-patch are in excellent agreement with the analytical solution.

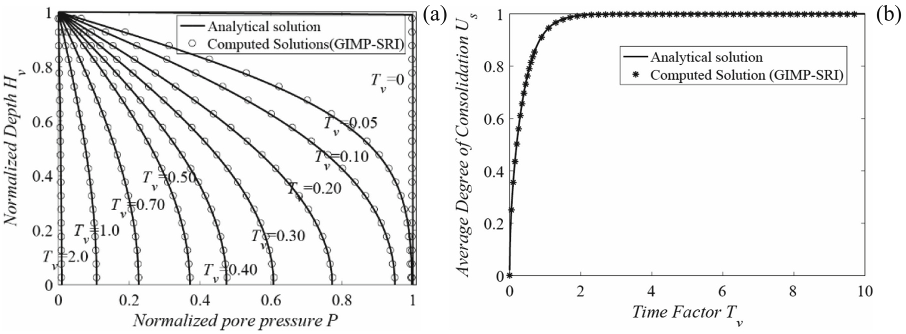

Fig. 3. Comparison between computed and analytical solutions for small strain analysis: (a) pore pressure profile; (b) average degreed of consolidation

5.2 1D Consolidation Under Large deformation Analysis

For the large deformation analysis, a constant load of 200.0 kPa is applied. The thickness of problem domain H_v, as well as soil's hydraulic conductivity k, and Young's modulus E, are no longer constant. In particular, it is assumed that both k and E depend on the current porosity - for better adherence to reality in presence of large strain deformation. Using MPM, the problem domain depth and the porosity are updated in each step. Thus, the hydraulic conductivity and Young's modulus are updated using the updated porosity as follows (Xie and Leo 2004; Tran and Sołowski 2019):

$$k_t(n) = k_0 \left(\frac{1 - n_0}{1 - n} \right)^2 \tag{7}$$

$$E_t(n) = E_0 \left(\frac{1 - n}{1 - n_0} \right) \tag{8}$$

For large deformation consolidation, Xie and Leo (2004) proposed the analytical solution illustrated in Fig. 4. The isochrones of excess pore pressure at several degrees of consolidation U_s are compared with the analytical solution in Fig. 4a, and the top surface settlement comparison is shown in Fig. 4b. It is observed that computed solutions both the pore pressure distribution and the top surface settlement are close to the considered analytical solution. This further demonstrates the capacity of GC-SRI-patch.

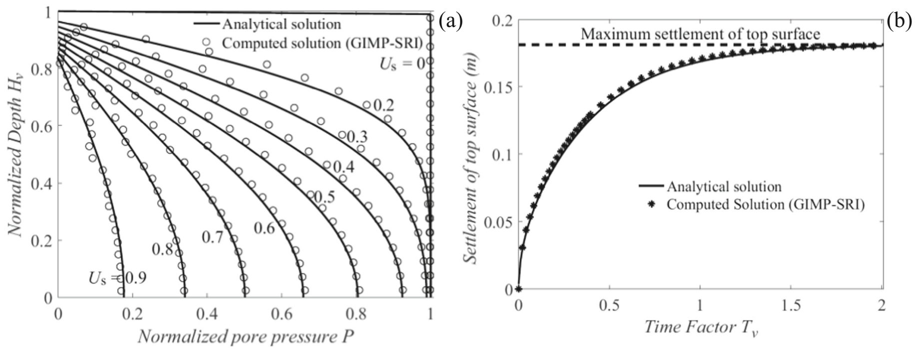

Fig. 4. Comparison between computed and analytical solutions for large strain analysis: (a) pore pressure profile; (b) top surface settlement

6 Conclusions

This study extended the material point method to two-phase coupled fully saturated poromechanical problems, and presented a stabilised material point method called GC-SRI-patch with explicit time integration. Aiming at mitigating the volumetric locking problem caused by the (nearly) incompressible pore water, a selective reduced integration has been used (GC-SRI-patch) for the pore pressure recovery at the material point locations, while effective stresses are still calculated using GIMP combined with CMPM. A moving least squares approximation (MLSA) has been performed for the computed incremental pore pressure recovery from Gauss points to material points. The numerical implementation of the GC-SRI-patch has been briefly introduced and the proposed GC-SRI-patch is verified by two consolidation problems.

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