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Overview of Torsion Design Methods

Camilo Granda Valencia and Eva Lantsoght

Synopsis: Large torsional moments, which need to be considered in a design, can result among others, in structures with an asymmetric layout or loading. To find the required longitudinal and transverse reinforcement to resist these torsional moments, the link between the three-dimensional action of the torsional moment and sectional analysis methods is necessary. This paper reviews the existing methods and code provisions for torsion. First, an overview of the principles of torsion from the mechanics perspective is given. Then, a survey of the available mechanical models for torsion is presented. Finally, the code provisions for torsion of ACI 318-19, CSA-A23.3-04, AASHTO-LRFD-17, EN 1992-1-1:2004, and the fib Model Code 2010 are summarized. Additionally, current research topics on torsion in structural concrete are summarized. It is expected that with this paper, engineers will have a useful overview and background knowledge for the design and assessment of torsion-critical elements.

Keywords: codes, concrete, reinforcement, shear, torsion

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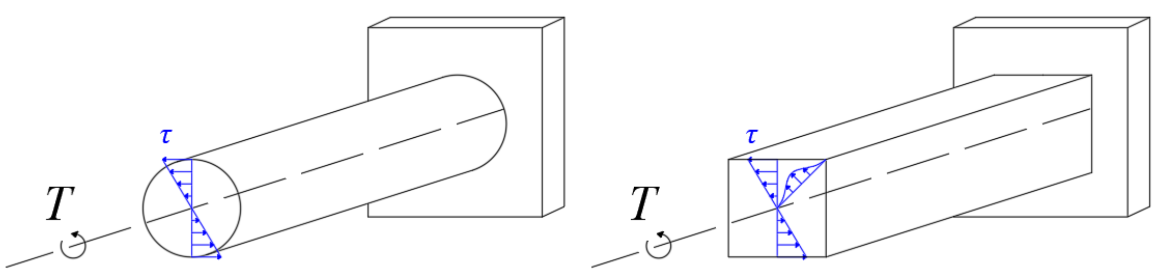
INTRODUCTION

In general, concrete structures are subjected to four principal actions: axial force, shear, bending moment, and torsion. Engineers and researchers focused on the understanding of the first three phenomena in concrete structures because these usually control the design of a member, i.e. they control the resulting reinforcement layout. For example, beams are typically designed for sectional moment and shear. Columns work as flexion-compression elements, around both axes of the cross-section. Nevertheless, torsion is a special topic. It was left apart because generally its influence on the resulting design is limited. For this reason, building codes accounted for torsion's small influence in the safety factors[1]. Throughout the 1960s, extensive research on torsion was made. As a result, the first design recommendations for torsion made by the American Concrete Institute (ACI) were formulated in 1969[2]. These recommendations led to the inclusion of provisions for torsion in the 1971 edition of the ACI Building Code, ACI 318-71[3]. The research carried out over the past decades led to a better understanding of the behavior of concrete members subjected to a torsional moment. The Space Truss Analogy, the Skew-Bending Theory, and other theories provided mechanical models to predict the behavior of concrete structures under torsion after cracking.

Torsion can be defined as the moment that twists an element around its axis. This torsional moment causes shearing stresses at each point of the cross-section of an element. These stresses change according to the proximity to the member's axis[4]. In circular cross-sections, the stress caused by a torsional moment is zero at the neutral axis and reaches the maximum value on the outermost fiber, see Figure 1(a). For rectangular cross-sections, the shear stress is also zero at the neutral axis and at the corners. It increases towards its maximum value at the surface of the longest side, see Figure 1(b).



(a) (b)

Figure 1—Shear stresses (τ) due an applied torsional moment (T) on a circular (a) and rectangular solid (b) element

Torsion can be a result of primary or secondary actions. The primary action occurs when the member can only support the action of an external load by generating a torsional moment. This is also called equilibrium torsion and is common in statically determinate structures. Equilibrium torsion is important for the stability of the structure. This occurs, for example, when a load acts on a fixed-end beam, but it is applied eccentric with respect to the z-axis, like in Figure 2. As a result, a torsional moment is generated around this axis.

Torsion can also be found as a result of secondary actions in statically indeterminate structures. This happens because the structure needs to satisfy compatibility requirements. In this case a twist is required to maintain the compatibility, not a torsional moment[5]. Spandrel continuous beams supporting other secondary beams or slabs are often subjected to this phenomenon, as shown in Figure 3.

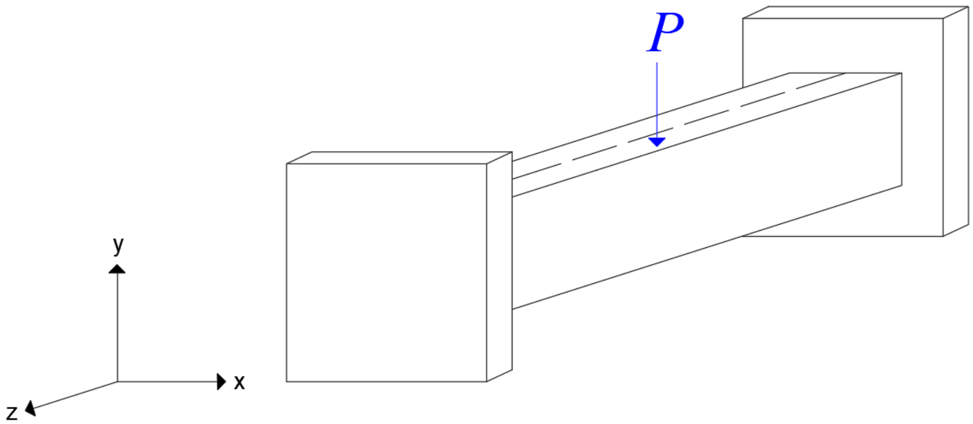


Figure 2—Equilibrium torsion at the ends of the beam, generated by the action of a point load

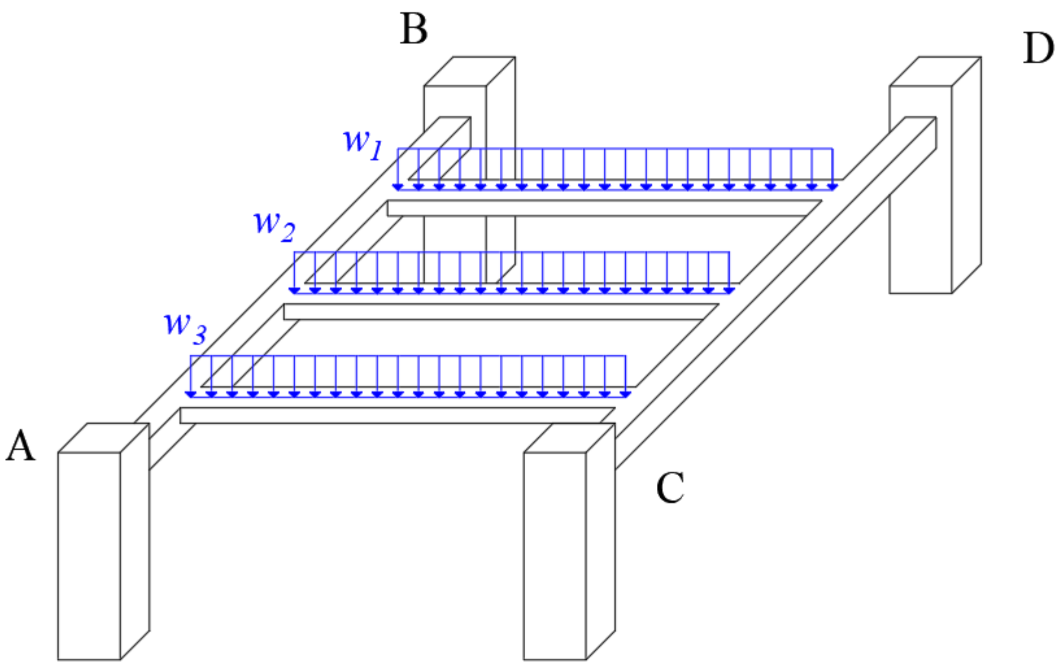


Figure 3—3D Frame system where spandrel beams AB and CD are subjected to compatibility torsion due to the load on the secondary beams joints

More complex and asymmetric concrete structures are designed every year around the world thanks to the reduction times in analysis and design when using structural software. As a result, the effect of torsion on concrete structures has become more important. For example, horizontally curved bridges and cantilever members should be designed for torsion. Standardization institutions like American Concrete Institute (ACI)[6], Canada Standards Association (CSA)[7], American Association of State Highway and Transportation Officials (AASTHO)[8], the European Normalization Committee (CEN)[9], and the International Federation for Structural Concrete (fib)[10] have developed provisions for situations when torsion needs to be considered. The design philosophy that each code uses is:

- ACI 318-19[6] uses a thin-tube and space truss analogy.
- CSA (CSA-A23.3-04)[7] uses a General Design Method for torsion derived from the Modified Compression Field Theory (MCFT); it includes the tensile contribution of concrete.

- AASHTO-LRFD-17[8] code provisions for torsion are obtained from the MCFT. The torsion equations on this code are similar to the CSA-A23.3-04 ones. A Strut and Tie Model can also be used as an alternative for design.
- Eurocode (EN 1992-1-1:2004)[9] uses a spatial truss model with an equivalent thin-walled tube and wall thickness for the torsion design.
- The fib Model Code 2010[10] uses a variable angle truss model, generalized stress field approach, or a simplified modified compression field theory, depending on the Level of Approximation.

The assumptions that lie at the basis of each of these models, and the resulting mechanics, will be discussed in the section about the mechanical models. The resulting code provisions will be given in the section with the code provisions.

BRIEF HISTORY OF TORSION RESEARCH

Mechanics of torsion

In this section, an overview of the history of torsion mechanics is given. Kurrer in "The History of the Theory of Structures" shows various important investigations on this topic[11]. The first known person to study the effect of torsion on materials, as a consequence of his research on electric charges, was Coulomb. Using his torsion balance, he deduced that the torsional moment is proportional to the torsional angle[12]. About 40 years later, Navier was the first to postulate a theoretical equation to compute the torsional moment on shafts with a circular cross-section. The two assumptions that he made were: 1) the shape of the cross-section cannot change after twist, and 2) plane sections must remain plane. The latter assumption implies that warping does not occur[13].

Later, it was found that there are two possible ways in which a structural member can resist torsion: by circulatory torsion or by warping torsion. Saint-Venant developed in 1847 the first theory, in which he stated that the cross-section of an element counters the effect of torsion by producing a circulatory shear flow (torsional shear multiplied by the wall thickness) on its plane. This means that the shear stress resisting the external torsional moment is constant within the flow area, see Figure 4(a). This effect usually occurs in solid and hollow members, which are free to bend around their axis[4]. The second way in which structural members can withstand torsion is by warping torsion. It was first investigated by Timoshenko in 1905[14] and further researched by Vlasov in 1940[15,16]. Warping torsion produces different shear stresses along the same circumference, see Figure 4(b). Consequently, the planar sections do not remain plane due to the changing strain at points over a determined circumference. Longitudinal bending results from these strains. Warping torsion arises when the entire section or part of it is restrained, for example, by end conditions[4,17]. This is usually expected in members formed by at least three connected walls, or with a fixed-end support.

Both resisting torsional moments need to be in equilibrium with the applied torsional moment (T) on the member. This means that $T = T_s + T_w$ where T_s is the Saint-Venant torsion and T_w is the warping torsion. Both happen at the same time, consequently; there is not a clear way to classify sections according to how they resist torsion. Some practical examples have demonstrated that the action of one of the resisting methods can be neglected compared to the effect of the other. Nevertheless, there are other cases where neither of them is predominant over the other; this case is called mixed torsion[18]. One example of mixed torsion is an I-shaped simply supported beam. If the torsional moment is applied at midspan, the cross-sections at the left and right of it experience warping torsion. Close to the ends, the beam can twist freely, therefore Saint-Venant torsion occurs.

In 1890 Bach, in his book, "Elasticität und Festigkeit" presented all the torsion cases proposed by Saint-Venant and interpreted them theoretically. Bach tested numerous cast-iron and hard lead bars under torsion. Using the results between the proposed theory and the experiment, Bach developed a simple proof equation to check the shear stress for the Saint-Venant torsion in bars, equilateral triangles, and regular hexagons[11]. In 1896, Bredt offered a promising solution to the Saint-Venant torsion problem. His solution equation states that the sum of the tangential shear forces (τ) per unit area (ds) on a closed curve within the cross-section under the effect of an external torsional moment is equal to two times the area enclosed by the forces (A_m), shear modulus (G) and the product of rotation (θ_r)[11], i.e.:

$$\int \tau ds = 2A_m\theta_r G \tag{1}$$

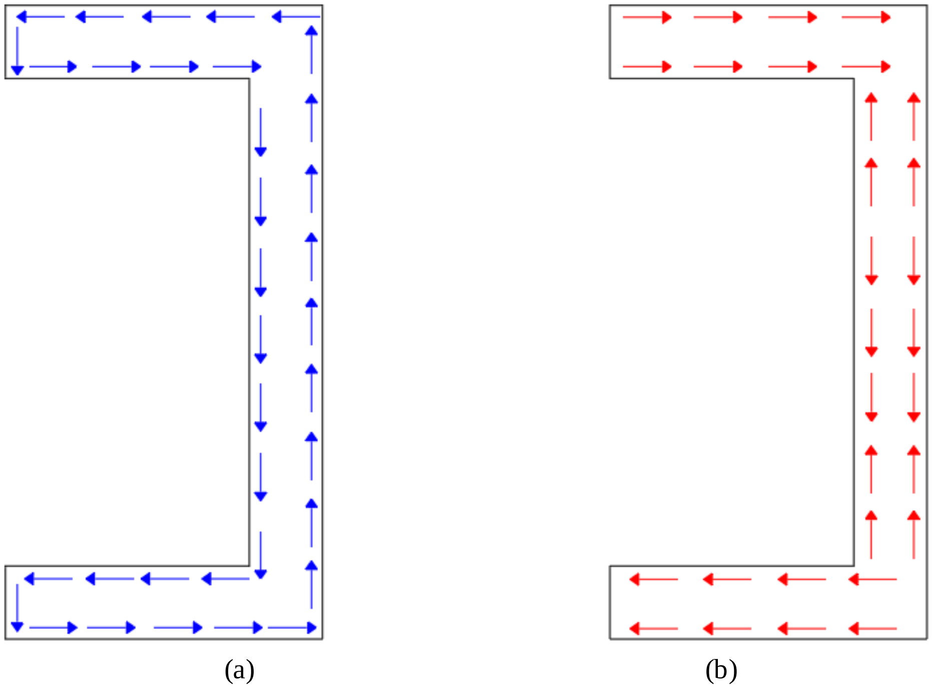


Figure 4—Circulatory torsion (a) and warping torsion (b) shear stresses on C-shaped members

Torsion in reinforced concrete

Graf and Mörsch were the first researchers to study torsion in plain and reinforced concrete. They tested different circular, square, and rectangular beams to study the effect of the reinforcement on the ultimate strength for elements under torsion[17]. In 1929, Rausch published his dissertation, in which he presented the 3D-truss analogy for torsion. Rausch provided an equation to predict the torsional resistance of reinforced concrete members based on the space truss model[19,20]. This method lies at the basis for the current torsion design provisions.

More researchers started to study torsion in structural concrete at the beginning of the second half of the twentieth century. In 1959, Lessig used equilibrium equations to propose a skew-bending theory for the failure mechanism of torsion[21]. This theory assumes that a beam under torsion will have a skewed failure surface. Lessig proposed two failure modes. The first one has a compression zone near the top face of the beam, while the second failure mode uses a compression zone along the side face. In 1962, Yudin[22] realized that the skew-bending theory proposed by Lessig was not able to determine three unknows: the longitudinal reinforcement area, the web steel area, and the depth of the compression zone. To solve this, Yudin proposed three equilibrium equations, while Lessig's analysis only used two: the equilibrium of moments about the neutral axis of the member, and the equilibrium of forces along the normal to the compression zone. Yudin's equations were: equilibrium of moments about an axis through the centroid of the compression zone and parallel to the longitudinal axis of the beam, equilibrium of moments about an axis through the centroid of the compression zone and perpendicular to the longitudinal axis, and equilibrium of forces along the normal to the compression zone. Nevertheless, this analysis is limited to only symmetrically reinforced elements.

Elfgren developed a method to determine the capacity of elements under combined shear, moment and torsion[16]. He used a truss analogy to predict the ultimate load carried by multiple sets of reinforced beams and tested these at Chalmers University of Technology. Elfgren established an interaction equation which can be used to plot an interaction surface. This model predicts accurately the strength of reinforced concrete beams subjected to torsional moment, shear force and bending moment.

Collins and Mitchell introduced another approach to study torsion in structural concrete in 1973[23]. They presented the diagonal compression field theory for beams under pure torsion. They considered equilibrium equations, geometry of deformations, and stress-strain relationships of the concrete and steel to propose their theoretical model.

The basis of their approach is a truss analogy model, and their main assumption is that after cracking the concrete will not carry tension, therefore, the torsion will be resisted by a field of diagonal compression in the concrete. Afterwards, in 1985, Hsu and Mo developed a variation of the compression field theory. In this case, they softened the concrete stress-strain curve and called the new model the softened truss model (STM)[19]. In the STM equilibrium, compatibility and softened stress-strain relationships are combined to develop a theory that has shown good results in predicting the test results of reinforced concrete structures subjected to shear and torsion[24]. Rahal and Collins have developed analytical computational models to calculate the response of concrete members subjected to combined torsion and bending[25] and to combined torsion and shear[26].

MECHANICAL MODELS FOR TORSION

Reinforced concrete before and after cracking under torsion

Prior to cracking, reinforced concrete members subjected to torsion can be analyzed as homogenous plain concrete sections. Therefore, their behavior can be predicted using Saint-Venant's theory[27]. After the element cracks, the study of its behavior becomes more complicated. From now on, the structural member acts as a composite section, and Saint-Venant's theory can no longer be used because cracking violates the material homogeneity premise of the elastic theory. When the web of the beam cracks, its capacity to transmit diagonal tension forces is reduced. The load is then carried by diagonal compression members between the cracks and by the steel reinforcement resisting tension. Together, they form a truss-like mechanism[4].

Shear truss analogy

The shear truss analogy was first proposed by Ritter at the end of the twentieth century[28]. It is a strut-and-tie model and considers that a cracked reinforced concrete beam under shear will have diagonal cracks which separate the concrete into multiple struts. They modeled the beam as a plane truss consisting of longitudinal and transverse reinforcement to carry the load. In this assumption, the top and bottom longitudinal bars act as the top and bottom chords of the truss, while the transverse reinforcement and concrete struts work as the web members. To simplify this model, the strut's inclination is assumed to be 45°[29].

3D space truss analogy

To apply the concept of a truss model to members to subjected torsion, the truss model needs to be extended to a three-dimensional model, i.e. a space truss analogy. A member subjected to torsion is treated as a space truss formed by a series of joined planar trusses[20]. The concrete member reinforced with longitudinal and transverse reinforcement resists torsion by producing a circulatory shear flow at the outermost part of the cross-section. Each straight segment of the tube walls behaves like a planar truss in which the shear stresses are resisted as in the shear truss analogy. Struts only carry axial compression; longitudinal and transverse reinforcement carries the tension forces, see Figure 5.

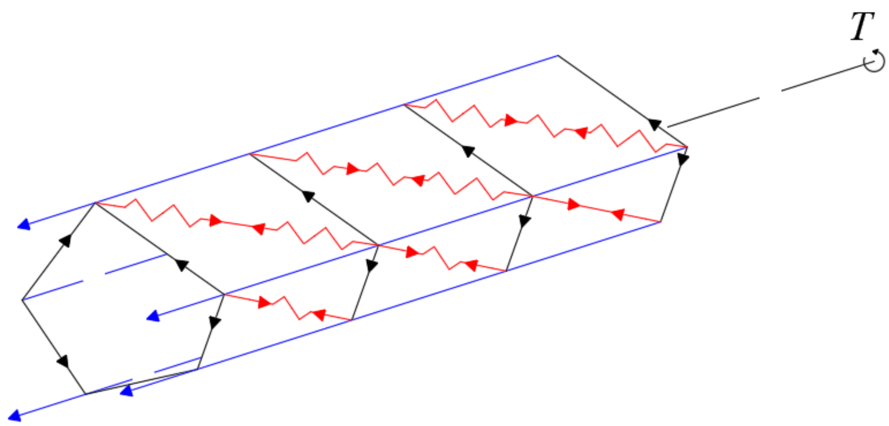


Figure 5—Space truss analogy for an asymmetrical beam under torsion. The tension forces are supported by the longitudinal and transverse reinforcement (black and blue) and the concrete struts resist compression (red)

Skew-bending theories

This theory is characterized by the assumption of a skewed failure surface. This surface is generated by a helically-shaped crack on three faces of a rectangular beam. On the fourth face, the helical crack is connected by a compression zone. The failure surface intersects the longitudinal and transverse reinforcement. The forces in the steel reinforcement generate the required internal forces and moments to carry external loads. Failure occurs when the steel starts to yield[30]. At failure, the two parts of the member separated by the failure surface rotate against each other about a neutral axis on the inside edge of the compression zone. Then, the associated equilibrium equations at the ultimate limit state can be derived[27].

Thin-walled tube analogy

The most efficient cross-section to resist torsion is a thin tube. The thin-walled tube analogy states that the shear stresses and shear flow are constant around the cross-section of a member. This shear flow is enclosed by an area of pre-determined thickness. Therefore, solid and hollow sections can be calculated in the same way as tubes[31], see Figure 6. Concrete members can be modelled as tubes because the concrete core does not contribute to the element's torsional strength[27]. Within the walls of the tube, the external torsion is resisted by a shear flow, defined as the torsional shear multiplied by the thickness of the tube.

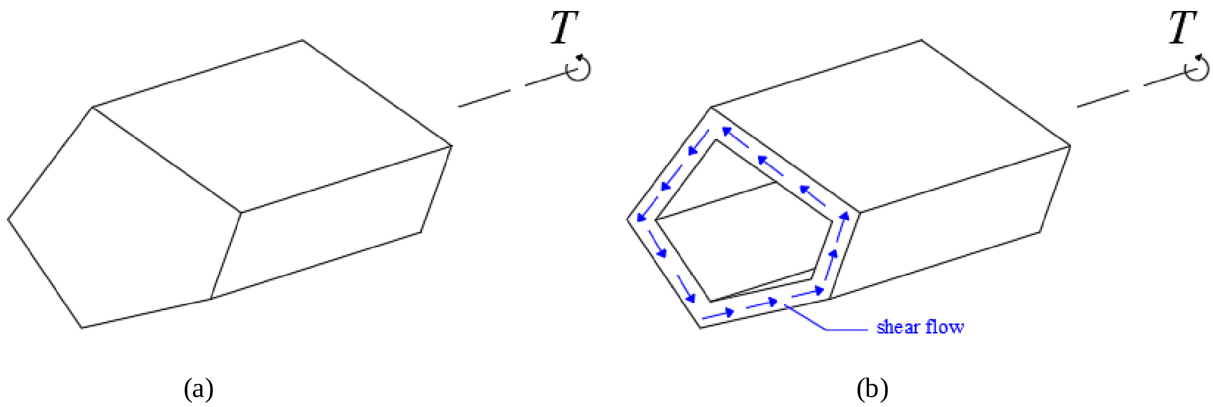


Figure 6—Original section (a) and the same member after the thin-walled tube analogy is applied (b)

Compression Field Theory (CFT)

The CFT is a model developed by Mitchell and Collins that considers equilibrium conditions, geometry of deformation and the strain-stress characteristics of the steel and concrete. This model predicts the shear strength of a reinforced concrete member after it cracks. This theory, based on the truss analogy, assumes that after cracking, the torsion shear stresses are carried by a field of diagonal compression in the concrete and balanced by the tension developed in the longitudinal and transverse reinforcement[23]. In 1986, Vecchio and Collins expanded the CFT to the Modified Compression Field Theory (MCFT). The CFT assumed that the cracks of the diagonal field compression in the concrete were only able to withstand shear and compression. Nevertheless, between the concrete's cracks tension stresses exist. To have a more accurate answer of the reinforced concrete element's capacity under shear and torsion, the MCFT uses experimentally verified average stress-strain relationships instead of assuming them. Also, it considers the tension in the cracked concrete[32]. Although the MCFT can predict the shear and torsional strength with great precision, the process of solving the equations of this theory by hand is complex. For this reason, Bentz, Vecchio and Collins developed a simplified MCFT using the Membrane-2000 computer program to get more practical expressions. This method showed excellent predictions of the shear strength. The accuracy between the simplified MCFT and the full theory is almost the same[33].

CODE PROVISIONS FOR TORSION

All the equations in this section are expressed in SI units. The conversion factors are: 1 kN = 0.225 kip, 1 kN·m = 8.849 kip·in, 1 mm = 0.0394 in and 1 MPa = 145 psi.

ACI 318-19

ACI 318-19 first checks if torsion can be neglected. If the following expression from §9.5.4.1 is satisfied, torsional effects do not need to be considered:

$$T_u < \phi T_{th} \tag{2}$$

T_u is the factored torsional moment. ϕ, the reduction factor for the nominal capacity of torsion, is equal to 0.75. T_{th} is the threshold torsional moment given by §22.7.4. For solid sections it is:

$$T_{th} = \begin{cases} 0.083\lambda\sqrt{f_c'}\left(\dfrac{A_{cp}^2}{p_{cp}}\right) & \text{Non-prestressed member} \\ 0.083\lambda\sqrt{f_c'}\left(\dfrac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\dfrac{f_{pc}}{0.33\lambda\sqrt{f_c'}}} & \text{Prestressed member} \\ 0.083\lambda\sqrt{f_c'}\left(\dfrac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\dfrac{N_u}{0.33A_g\lambda\sqrt{f_c'}}} & \text{Non-prestressed member under axial load} \end{cases} \tag{3}$$

In statically indeterminate structures where $T_u \geq \phi T_{cr}$, it is permitted to reduce T_u to ϕT_{cr} due to redistribution of internal forces after cracking. This applies to typical and regular framing conditions. ϕT_{cr} is the cracking torsional moment and is defined in §22.7.5.1. Equation (3) is valid for solid cross-sections. For hollow cross-sections, all the A_{cp} terms in Equation (3) are substituted with A_g, the gross area of the concrete cross-section. f'_c [MPa] is the specified compressive strength of the concrete, A_{cp} is the area enclosed by the outside perimeter of the concrete cross-section, p_{cp} is the outside perimeter of concrete's cross-section, f_{pc} [MPa] is the compressive stress in the concrete, after allowance for all prestress losses, at the centroid of the cross-section resisting the externally applied loads or at the junction of the web and flange where the centroid lies within the flange. In a composite member, it is the resultant compressive stress at the centroid of the composite section, or at the junction of the web and flange, when the centroid lies within the flange, due to both prestress and moments resisted by the precast member acting alone, N_u is the factored axial force, taken as negative for tension and positive for compression, λ is a coefficient which accounts for the properties of lightweight concrete (see §19.2.4).

The shear strength provided by the concrete V_c according to §22.5.5.1 is determined as:

$$V_c = 0.17\lambda\sqrt{f_c'}b_w d \quad \text{with } f_c' \text{ in [MPa]} \tag{4}$$

b_w is the web width or diameter of a circular section and d is the effective depth. The last expression applies to reinforced concrete members without axial force and with $A_v \geq A_{v,\min}$. A_v in Equation (4) is the required transverse reinforcement for shear and $A_{v,\min}$ is the minimum transverse reinforcement for shear force. For other cases in reinforced concrete members, §22.5.5.1 through §22.5.5.1.3 are governing. For prestressed members, the shear strength provided in concrete is listed in §22.5.6 and §22.5.7.

The next expression from §22.7.7.1 checks if the dimensions of the member are large enough to avoid crushing of the concrete:

$$\begin{cases} \sqrt{\left(\dfrac{V_u}{b_w d}\right)^2 + \left(\dfrac{T_u p_h}{1.7A_{oh}^2}\right)^2} \leq \phi\left(\dfrac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) & \text{for solid sections} \\ \left(\dfrac{V_u}{b_w d}\right) + \left(\dfrac{T_u p_h}{1.7A_{oh}^2}\right) \leq \phi\left(\dfrac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) & \text{for hollow sections} \end{cases} \tag{5}$$

If Equation (5) is fulfilled, the reinforcement for torsion can be designed. For hollow sections with a variable wall thickness, the maximum value of the left side of Equation (5) should be evaluated, which is often at the point of the cross-section where shear and torsional stresses can be added. V_u is the factored shear force, p_h is the perimeter of the centerline of the outermost closed transverse torsional reinforcement, A_{oh} is the area enclosed by p_h. §22.7.7.1.1

mentions that for prestressed members the value of d in Equation (5) should be greater than $0.8h$, where h is the overall height of the element.

According to §22.7.6.1, θ, the angle between the struts and the tension chord, can be taken as any value between 30 and 60 degrees. §22.7.6.1.2 states that θ is usually 45° for reinforced concrete members with $A_{ps}f_{se} < 0.4(A_{ps}f_{pu} + A_sf_y)$ and 37.5° for prestressed elements with $A_{ps}f_{se} \geq 0.4(A_{ps}f_{pu} + A_sf_y)$. A_{ps} is the area of the prestressed longitudinal tension reinforcement, A_s is the area of the non-prestressed longitudinal tension reinforcement, f_{se} is the effective stress in prestressing reinforcement after allowance for all prestress losses, f_{pu} is the specified tensile strength of prestressing reinforcement, and f_y is the yield strength for non-prestressed longitudinal reinforcement. The required area of transverse reinforcement of one leg of a closed stirrup A_t for torsion is:

$$\frac{A_t}{s} \geq \frac{T_u}{1.7\phi A_{oh} f_{yt}} \tan\theta \tag{6}$$

s is the spacing between the stirrups, f_{yt} is the specified yield strength of the transverse reinforcement.

The next step is to calculate the required area of longitudinal steel for torsion A_l:

$$A_l \geq \frac{A_t}{s} \frac{f_{yt}}{f_y} p_h \cot^2\theta \tag{7}$$

§9.5.4.3 mentions that the longitudinal and transverse reinforcement required for torsion need to be added to the reinforcement demanded by shear force, bending moment and axial force actions.
For the transverse reinforcement limit, §9.6.4.2 states that for members under torsion and shear, the stirrups for torsion and shear effects cannot be less than:

$$\frac{A_v + 2A_{t\ \min}}{s} = \max\begin{cases} 0.062\sqrt{f_c'}\dfrac{b_w}{f_{yt}} \\ 0.35\dfrac{b_w}{f_{yt}} \end{cases} \tag{8}$$

In Equation (8), A_v is the required area of two legs of a closed stirrup for shear. If the analyzed element is only experiencing torsion, the value of the A_v term in Equation (8), is equal to zero. The minimum area of longitudinal steel reinforcement $A_{l,\min}$ for torsion can be calculated with §9.6.4.3 as:

$$A_{l,\min} = \min\begin{cases} \dfrac{0.42\sqrt{f_c'}A_{cp}}{f_y} - \left(\dfrac{A_t}{s}\right) p_h \dfrac{f_{yt}}{f_y} \\ \dfrac{0.42\sqrt{f_c'}A_{cp}}{f_y} - \left(\dfrac{0.175b_w}{f_{yt}}\right) p_h \dfrac{f_{yt}}{f_y} \end{cases} \tag{9}$$

According to §9.7.6.3.3 the limits to the stirrup spacing are:

$$s \leq \min\begin{cases} \dfrac{p_h}{8} \\ 300 \text{ mm} \end{cases} \tag{10}$$

§9.5.4.3 states that the final amount of longitudinal and transverse reinforcement needs to be added to the required reinforcement for shear force, bending moment and axial effects.

When the cross-section and the reinforcement of the member are designed, the ACI 318-19 §22.7.6.1 gives two equations to analyze the torsional strength T_n. Once the member cracks under the effect of a torsional moment, the strength is provided primarily by the transverse and longitudinal reinforcement. The concrete contribution to the torsional strength is neglected:

$$T_n = \min \begin{cases} \dfrac{1.7 A_{oh} A_t f_{yt}}{s} \cot\theta \\ \dfrac{1.7 A_{oh} A_l f_y}{p_h} \tan\theta \end{cases} \tag{11}$$

Finally, the torsional strength should be greater than or equal to the factored applied torsional moment.

$$\phi T_n \geq T_u \tag{12}$$

CSA-A23.3-04

According to §11.2.9.1, reinforcement for torsion should be provided when the factored torsional moment T_f exceeds ¼ of the pure torsional cracking resistance T_{cr}, given in CSA-A23.3-04 Eq. 11-2 as:

$$T_{cr} = 0.38\lambda\phi_c \left(\frac{A_c^2}{p_c}\right)\sqrt{f_c'}\sqrt{1+\frac{\phi_p f_{cp}}{0.38\lambda\phi_c\sqrt{f_c'}}} \tag{13}$$

In statically indeterminate structures, where redistribution of torsional moments can occur, §11.2.9.2 specifies that T_f can be reduced to $0.67T_{cr}$ at the face of the support. For hollow sections, A_c is the area enclosed by the outside perimeter of the concrete cross-section, including the area of holes. In Equation (13), for hollow cross-sections, A_c can be replaced with $1.5A_g$ (gross concrete area) if the wall thickness is less than $0.75A_c / p_c$. p_c is the outside perimeter of the cross-section, f_{cp} [MPa] is the average compressive stress in the concrete due to the effective prestress force only (after allowance for all prestress losses), f'_c [MPa] is the specified compressive strength of concrete, $\phi_c = 0.65$ is the material factor for concrete, $\phi_p = 0.9$ is the material factor for prestressing tendons, and λ is the factor that accounts for lightweight concrete, see §8.6.5.

Equations 11-18 and 11-19 in the CSA-A23.3-04 code give minimum dimensions to avoid concrete crushing:

$$\begin{cases} \left(\dfrac{V_f - V_p}{b_w d_v} + \dfrac{T_f p_h}{1.7A_{oh}^2}\right) \leq 0.25\phi_c f_c' & \text{for hollow sections} \\ \left(\sqrt{\left(\dfrac{V_f - V_p}{b_w d_v}\right)^2 + \left(\dfrac{T_f p_h}{1.7A_{oh}^2}\right)^2}\right) \leq 0.25\phi_c f_c' & \text{for other sections} \end{cases} \tag{14}$$

V_f is the factored shear, V_p is the component in the direction of the applied shear of the effective prestressing force factored by ϕ_p, d_v is the effective shear depth, taken as the greater of $0.9d$ or $0.72h$, where h is the overall height of the member, and d is the effective depth (d cannot be less than $0.8h$ for prestressed members and circular sections), b_w is the minimum web width within d or the diameter of a circular cross-section, A_{oh} is the area enclosed by the centerline of the exterior closed transverse torsion reinforcement, including the area of holes, p_h is the perimeter of the centerline of the closed transverse torsion reinforcement. If the wall thickness of the box section is less than A_{oh} / p_h, the second term on the left side of Equation (14) should be replaced by T_f / $(1.7A_{oh}t)$. Where t is the wall thickness at the location where the stresses are being checked.

Next, it is needed to compute the longitudinal strain ε_x at mid-depth of the member due to the factored loads. With this variable, the angle of the diagonal compression field can be obtained to calculate the required transverse reinforcement. The longitudinal strain is computed substituting Equation 11-20 on Equation 11-13 of §11.3.6.4, which leads to:

$$\varepsilon_x = \frac{\dfrac{M_f}{d_v} + \sqrt{\left(V_f - V_p\right)^2 + \left(\dfrac{0.9 p_h T_f}{2A_o}\right)^2} + 0.5N_f - A_p f_{po}}{2\left(E_s A_s + E_p A_p\right)} \tag{15}$$

If the value of Equation (15) is negative, ε_x can be taken as zero. M_f is the factored moment and cannot be less than $(V_f - V_p)d_v$, A_o is the area enclosed by the shear flow path including the area of holes, which can be taken as $0.85A_{oh}$ according to §11.3.10.3, N_f is the factored axial load, positive for tension and negative for compression. A_p is the area of prestressing tendons, f_{po} is the stress in the prestressing tendons when the strain in the surrounding concrete is zero (may be taken as $0.7f_{pu,CSA}$ for bonded tendons outside the transfer length and f_{pe} for unbonded tendons), $f_{pu,CSA}$ [MPa] is the specified tensile strength of the prestressing tendons, f_{pe} is the effective stress in the prestressing tendons after allowance for all prestress losses, E_s is the modulus of elasticity of non-prestressed reinforcement, E_p is the modulus of elasticity of prestressing tendons, A_s is the area of non-prestressed tension reinforcement, and A_p is the area of tendons on the flexural tension side of the member. The bending moment and shear force on Equation (15) are absolute values. The axial load is positive for tension and negative for compression. Once the longitudinal strain is computed, the angle of inclination of the diagonal compressive stresses, θ_{CSA}, is defined in §11.3.6.4, Eq. 11-12 as:

$$\theta_{CSA} = 29^\circ + 7000\varepsilon_x \tag{16}$$

For special members like slabs or footings with an overall thickness less than 0.35 m, footings in which the distance from the point of zero shear to the face of the column, pedestal, or wall is less than three times the effective shear depth of the footing, beams with an overall thickness less than 0.25 m, beams cast integrally with slabs where the depth of the beam below the slab is not greater than one-half the width of web or 0.35 m, and concrete joist construction defined in §10.4, the angle of the struts can be taken as 42°. §11.3.6.3 also mentions that if the yield strength of the longitudinal reinforcement does not exceed 400 MPa and f'_c is smaller than 60 MPa, θ_{CSA} can be taken as 35°.
The next step is to get the required area of transverse reinforcement for torsion, A_t using Equation (17) :

$$\frac{A_t}{s} \geq \frac{T_f}{1.7\phi_s A_{oh} f_{yt}} \tan\theta_{CSA} \tag{17}$$

s is the spacing between stirrups for torsion, f_{yt} is the specified yield strength of transverse reinforcement, and ϕ_s = 0.85 is the resistance factor for non-prestressed reinforcement. Combining Equations 11-14 and 11-21, the longitudinal reinforcement area A_{st} needed to withstand a torsional moment is given by:

$$A_{st} = \frac{\dfrac{M_f}{d_v} + 0.5N_f + \left[\sqrt{\left(V_f - 0.5V_s - V_p\right)^2 - \left(\dfrac{0.45 p_h T_f}{1.7A_{oh}}\right)^2}\right]\cot\theta_{CSA}}{f_y} - A_s \tag{18}$$

f_y is the specified yield strength of non-prestressed longitudinal reinforcement for torsion.

The maximum spacing s between stirrups follows the expression described in §11.3.8.1:

$$s \leq \min\begin{cases} 0.7d_v \\ 600 \text{ mm} \end{cases} \tag{19}$$

The CSA-A23.3-04 code provisions for torsion do not specify any minimum longitudinal or transverse reinforcement for torsion. §11.2.7 indicates that a longitudinal reinforcing bar or bonded prestressing tendon shall be

placed in each corner of closed transverse reinforcement required for torsion. The nominal diameter of the bar or tendon shall be not less than $s/16$.
The torsional moment resistance T_r is only provided by the transverse reinforcement and is defined in §11.3.10.3 as:

$$T_r = 1.7\phi_s A_{oh} f_{yt} \frac{A_t}{s} \cot\theta_{CSA} \tag{20}$$

The torsional strength T_r should be greater than or equal to the applied torsional moment:

$$T_r \geq T_f \tag{21}$$

<u>AASHTO-LRFD-2017</u>
Torsion must be considered, according to §5.7.2.1 Eq. 5.7.2.1-3, if:

$$T_u > 0.25\phi T_{cr} \tag{22}$$

T_u is the applied factored torsional moment over the analyzed member. ϕ, the resistance factor is given in §5.5.4.2 and is equal to 0.90 for normal and lightweight concrete. To determine the torsional cracking moment, T_{cr}, first K, the effective length factor for compression members, must be computed according to Eq. 5.7.2.1-6:

$$K = \sqrt{1 + \frac{f_{pc,AAS}}{0.335\lambda\sqrt{f_c'}}} \leq 2.0 \tag{23}$$

$f_{pc,AAS}$ [MPa] is the unfactored compressive stress in concrete after prestress losses have occurred, taken either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange. f'_c [MPa] is the design compressive strength of concrete, λ_{AAS} is the concrete density modification factor given in §5.4.2.8. T_{cr} is provided in Eq. 5.7.2.1-4 and 5.7.2.1-5 as:

$$T_{cr} = \begin{cases} 0.329K\lambda\sqrt{f_c'}\dfrac{A_{cp}^2}{p_c} & \text{for solid shapes} \\ 0.329K\lambda\sqrt{f_c'}2A_o b_e & \text{for hollow shapes} \end{cases} \tag{24}$$

A_o is the area enclosed by the shear flow path, including inner holes, A_{cp} is the area enclosed by the outside perimeter of the concrete cross-section, p_c is the outside perimeter of the concrete cross-section, b_e is the effective width of the shear flow path taken as the minimum thickness of the exterior webs or flanges comprising the closed box section. b_e should account for the presence of ducts, the diameters of ungrouted ducts or one-half the diameters of grouted ducts need to be subtracted from the web or flange thickness at the location of these ducts. b_e cannot exceed A_{cp} / p_c unless a more refined analysis is used.

To compute the required transverse reinforcement, ε_s needs to be determined, which is the net longitudinal tensile strain in the section at the centroid of the tension reinforcement. It can be obtained from §5.7.3.4.2, Eqs. 5.7.3.4.2-4, 5.7.3.4.2-5 and 5.7.3.4.2-6

$$\varepsilon_s = \begin{cases} \dfrac{\left(\dfrac{|M_u|}{d_v}\right) + 0.5N_u + \left|\sqrt{V_u^2 + \left(\dfrac{0.9p_{h,AAS}T_u}{2A_o}\right)^2} - V_{p,AAS}\right| - A_{ps,AAS}f_{po,AAS}}{E_sA_s + E_pA_{ps}} & \text{for solid sections} \\ \dfrac{\left(\dfrac{|M_u|}{d_v}\right) + 0.5N_u + \left|V_u + \dfrac{T_u d_s}{2A_o} - V_{p,AAS}\right| - A_{ps,AAS}f_{po,AAS}}{E_sA_s + E_pA_{ps}} & \text{for hollow sections} \end{cases} \tag{25}$$

§5.7.2.1 mentions that in statically indeterminate structures where redistribution of torsional moment can occur over a determined element, T_u can be taken as ϕT_{cr}. M_u is the factored applied moment and cannot be less than $|V_u-V_p|d_v$, N_u is the factored applied axial load, taken as positive for tension and negative for compression, V_u is the factored sectional shear, $V_{p,AAS}$ is the component of prestressing force in the direction of the shear force, $A_{ps,AAS}$ is the area of prestressing steel on the flexural tension side of the member, A_s is the area of non-prestressed tension reinforcement, d_s is the distance from the extreme compression fiber to the centroid of the non-prestressed tensile reinforcement measured along the centerline of the web, d_v is the effective shear depth taken as the distance between the resultants of the tensile and compressive forces due to flexure, $p_{h,AAS}$ is the perimeter of the centerline of the closed transverse torsion reinforcement for solid members, or the perimeter of the centroid of the transverse torsion reinforcement in the exterior webs and flanges for hollow members, and $f_{po,AAS}$ is a parameter taken as the modulus of elasticity of prestressing steel multiplied by the locked-in difference in strain between the prestressing steel and the surrounding concrete. For usual levels of prestressing, f_{po} can be taken as $0.7f_{pu}$ for both pretensioned and post-tensioned members. f_{pu} is the specified tensile stress of prestressing steel. E_s is the modulus of elasticity for steel reinforcement and E_p is the modulus of elasticity of prestressing steel. Now, with the longitudinal tensile strain ε_s, the angle of inclination of diagonal compressive stresses, θ_{AAS} is defined by §5.7.3.4.2, Eq. 5.7.3.4.2-2:

$$\theta_{AAS} = 29^{\circ} + 3500\varepsilon_s \qquad (26)$$

According to §5.7.3.4.1 θ_{AAS} can be taken as 45° for the following cases: concrete footings with a distance less than $3d_v$ from the point of zero shear to the face of the column, piers or walls with or without transverse reinforcement, and other non-prestressed concrete sections not subjected to axial tension, containing at least the minimum transverse reinforcement specified in §5.7.2.5 or having an overall depth of less than 0.40 m.

To design the transverse reinforcement for torsion A_t Eq. 5.7.3.6.2-1 is used:

$$\frac{A_t}{s} \geq \frac{T_u}{2\phi A_o f_{yt}} \tan\theta_{AAS} \qquad (27)$$

The required stirrups for torsion should be added to those needed for shear. The total transverse provided transverse reinforcement should not be less than the sum of the required transverse reinforcement for shear and torsion. s is the spacing between stirrups and f_{yt} is the yield strength of transverse reinforcement.

The required area of longitudinal reinforcement for torsion A_l is given by Eq. 5.7.3.6.3-1 and 5.7.3.6.3-2. The longitudinal reinforcement for torsion should be added to the required reinforcement for bending moment:

$$A_l = \begin{cases} \dfrac{\dfrac{|M_u|}{\phi d_v} + \dfrac{0.5N_u}{\phi} + \cot\theta_{AAS}\sqrt{\left(\left|\dfrac{V_u}{\phi} - V_p\right| - 0.5V_s\right)^2 + \left(\dfrac{0.45 p_h T_u}{2A_o\phi}\right)^2} - A_{ps,AAS}\, f_{ps}}{f_y} - A_s & \text{for solid sections} \\ \dfrac{A_t}{s}\dfrac{f_{yt}}{f_y}\, p_h \cot\theta_{AAS} & \text{for hollow sections} \end{cases} \qquad (28)$$

V_s is the shear resistance provided by the transverse reinforcement, f_y is the yield strength for the longitudinal reinforcement, f_{ps} is the average stress in the prestressing steel at the time for which the nominal resistance of the member is required. The longitudinal steel reinforcement for solid sections should be distributed uniformly around the perimeter. For box sections, interior webs should not be considered in the calculation of the longitudinal torsional reinforcement. The values of $p_{h,AAS}$ and A_l should be for the box shape defined by the outermost webs and the top and bottom slabs of the box girder. Also, A_l needs to be distributed around the outermost webs and top and bottom slabs of the box girder.

(29)

To compute the maximum stirrup spacing, the shear stress v_u stated in §5.7.2.8, Eq. 5.7.2.8-1 is required:

$$v_u = \frac{\left|V_u - \phi V_p\right|}{\phi b_v d_v} \tag{30}$$

b_v is the web width adjusted for the presence of ducts as specified in §5.7.2.8. For circular cross-sections, b_v is the diameter of the cross-section, modified for the presence of ducts where applicable. The maximum spacing of the stirrups is provided in §5.7.2.6, Eq. 5.7.2.6-1 and 5.7.2.6-2:

$$s \leq \begin{cases} 0.8 d_v \leq 600 \text{ mm} & \text{if } v_u < 0.125 f_c' \\ 0.4 d_v \leq 300 \text{ mm} & \text{if } v_u \geq 0.125 f_c' \end{cases} \tag{31}$$

The AASHTO-LRFD-2017 code does not give any equation to compute the minimum longitudinal or transverse reinforcement for torsion.

T_n is the nominal torsional resistance, specified in Eq. 5.7.3.6.2-1 as:

$$T_n = \frac{2 A_o A_t f_{yt} \cot\theta_{AAS}}{s} \tag{32}$$

The factored capacity of the element, ϕT_n, should be greater or equal than the factored demand T_u:

$$\phi T_n \geq T_u \tag{33}$$

<u>EN 1992-1-1:2004</u>

If the static equilibrium of the structure depends on the torsional resistance of some members of the structure, a full torsion design that fulfills both ultimate and serviceability provisions is necessary. However, if torsion is acting in statically indeterminate structures and it is present as a secondary effect, i.e. due to compatibility requirements, it may be neglected if the structure does not depend on the torsional resistance for its stability. In the last case, a minimum amount of longitudinal and transverse torsion reinforcement is needed to control excessive cracking. The minimum amounts are given in EN 1992-1-1:2004, §7.3.2, 9.2.1 and §9.2.2

The first step of a torsion design is to check if the cross-sectional dimensions are adequate. For this, the effective wall thickness t_{ef} expressed in §6.3.2 is required:

$$t_{ef} = \frac{A}{u} \geq 2c \tag{34}$$

If the analyzed member has a hollow cross-section, the effective wall thickness should be less than the actual wall thickness. A is the total area of the cross-section, including inner hollow areas, u is the perimeter of the cross-section, and c is the distance between the edge of the member and the centroid of the longitudinal reinforcement. The next step is to determine the torsional shear stress within the equivalent thin-walled tube, τ_t, according to Eq. 6.3.2 (6.26):

$$\tau_t = \frac{T_{Ed}}{2 A_k t_{ef}} \tag{35}$$

T_{Ed} is the design torsional moment and A_k is the area enclosed by the centerlines of the connecting walls, including inner hollow areas. The applied shear force, V_{Ed}, caused by the design torsional moment T_{Ed} obtained from Eq. 6.3.2 (6.27) is:

$$V_{Ed} = \tau_t t_{ef} z \tag{36}$$

z is the distance along the centerline between the intersection points of the adjacent walls of the equivalent thin-walled tube, usually taken as the height of the element. The strength reduction factor for cracked concrete in shear, v, is provided in EN 1992-1-1:2004, §6.2.2 (6), Eq. (6.6N):

$$v = 0.6\left[1 - \frac{f_{ck}}{250}\right] \tag{37}$$

f_{ck} [MPa] is the characteristic compressive cylinder strength of concrete at 28 days. Subsequently, α_{cw}, which is a coefficient that takes account the state of the stress in the compression chord, is computed using EN 1992-1-1:2004, §6.2.3 (3), Eq. (6.11.aN), (6.11.bN), and (6.11.cN):

$$\alpha_{cw} = \begin{cases} 1 & \text{for non-prestressed structures} \\ 1 + \dfrac{\sigma_{cp}}{f_{cd}} & \text{for } 0 < \sigma_{cp} \le 0.25 f_{cd} \\ 1.25 & \text{for } 0.25 f_{cd} < \sigma_{cp} \le 0.5 f_{cd} \\ 2.5\left(1 - \dfrac{\sigma_{cp}}{f_{cd}}\right) & \text{for } 0.5 f_{cd} < \sigma_{cp} \le 1.0 f_{cd} \end{cases} \tag{38}$$

σ_{cp} is the mean compressive stress in the concrete due to the design axial force or prestressing. The value of σ_{cp} does not need to be calculated at a distance less than $0.5\cot\theta_{EN}$ from the edge of the support. f_{cd} is the design value of the concrete compressive strength. θ_{EN} is the angle of the strut inclination given in EN 1992-1-1:2004, §6.2.3 (4), Eq. (6.7N). The effects of torsion and shear may be added if the angle of the strut inclination is the same. The limits of the angle are $21.8° \le \theta_{EN} \le 45°$. The upper limit of the torsional strength, $T_{Rd,\max}$ is given in Eq. (6.30) as:

$$T_{Rd,\max} = 2 v_1 \alpha_{cw} f_{cd} A_k t_{ef} \sin\theta_{EN} \cos\theta_{EN} \tag{39}$$

The recommended value of v_1 is v, see Equation (37). If the design yield strength of the transverse reinforcement, f_{ywd}, is below 80% of the characteristic yield strength of reinforcement f_{yk}, v_1 can be taken according to Eq. 6.2.3 (6.10.aN) and 6.2.3 (6.10.bN) as:

$$v_1 = \begin{cases} 0.6 & \text{for } f_{ck} \le 60\text{MPa} \\ 0.9 - \dfrac{f_{ck}}{200} > 0.5 & \text{for } f_{ck} \ge 60\text{MPa} \end{cases} \tag{40}$$

$V_{Rd,\max}$, the upper limit of the shear strength, is calculated using Eq. (6.9) of EN 1992-1-1:2004, §6.2.3 (2):

$$V_{Rd,\max} = \frac{\alpha_{cw} b_{w,EN} z v_1 f_{cd}}{\cot\theta_{EN} + \tan\theta_{EN}} \tag{41}$$

$b_{w,EN}$ is the width of the cross-section and for T, I or L beams it is the width of the web. If the web width contains ducts, the web width should be calculated according to §6.2.3 (6). Once $T_{Rd,\max}$ and $V_{Rd,\max}$ are obtained, the maximum combined shear and torsion capacity should be checked according to Eq. (6.29) to check if crushing of the concrete occurs:

$$\frac{T_{Ed}}{T_{Rd,\max}} + \frac{V_{Ed}}{V_{Rd,\max}} \le 1.0 \tag{42}$$

If the inequality is not satisfied, f_{cd} or A need to be modified.

The first step for the torsion design is to compute the required amount of transverse reinforcement A_{sw}. The stirrups for torsion must be added to the calculated reinforcement for shear. The code does not specify an equation to calculate the required number of stirrups for torsion. EN 1992-1-1:2004 mentions that the required area of transverse torsion reinforcement A_{sw} should be obtained using the same method as for shear stirrups, therefore:

$$\frac{A_{sw}}{s} \geq \frac{T_{Ed}}{2A_k f_{ywd}} \tan\theta_{EN} \tag{43}$$

s is the spacing of the stirrups. The longitudinal reinforcement, A_{sl}, for torsion needs to be added to the computed reinforcement required for flexure. The longitudinal reinforcement should generally be distributed over the length of the side, z. EN 1992-1-1:2004, §9.2.3 (3) states that the longitudinal reinforcement bars required for torsion A_{sl} need to be organized to have at least one bar at each corner of the stirrups. The remaining steel bars can be distributed uniformly around the stirrups' perimeter inner face. Using Eq. 6.3.2 (6.28) the required longitudinal reinforcement for torsion A_{sl} is:

$$A_{sl} \geq \frac{T_{Ed} u_k}{2A_k f_{yd}} \cot\theta_{EN} \tag{44}$$

u_k is the perimeter of the A_k area and f_{yd} is the design yield strength of the longitudinal reinforcement.

The transverse reinforcement ratio to compute the minimum transverse reinforcement for torsion is given in Eq. 9.2.2 (9.5N) as:

$$\rho_w \geq 0.08\frac{\sqrt{f_{ck}}}{f_{yk}} = \rho_{w,\min} \tag{45}$$

If torsion arises from compatibility in statically indeterminate structures, then it is unnecessary to consider torsion as an ultimate limit state. In this case, minimum longitudinal and transverse reinforcement should be provided to prevent excessive cracking. According to Eq. 9.2.2 (9.4) and the value found in Equation (45), The minimum area of transverse reinforcement $A_{sw,\min}$ is calculated as:

$$\frac{A_{sw,\min}}{s} = \rho_{w,\min} b_{w,EN} \sin\alpha \tag{46}$$

α is the angle between the transverse reinforcement and the longitudinal axis. The maximum spacing of the stirrups is defined by EN 1992-1-1:2004, §9.2.3 (3) as:

$$s \leq \min\begin{cases} \dfrac{u}{8} \\ 0.75d\left(1+\cot\alpha\right) \\ \min\left(\text{all dimensions}\right) \end{cases} \tag{47}$$

d is the effective depth of the cross-section. To find the minimum amount of longitudinal reinforcement, which is based on a requirement to control excessive cracking, first h^*, which is the overall height of the cross-section within the tensile zone, needs to be determined according to §7.3.2 as:

$$h^* = \begin{cases} h & \text{for } h < 1000 \text{ mm} \\ 1000 \text{ mm} & \text{for } h \geq 1000 \text{ mm} \end{cases} \tag{48}$$

h is the overall depth of the cross-section. The next step is to calculate k_1, which is a coefficient given in §7.3.2 that considers the effects of axial forces on the stress distribution:

$$k_1 = \begin{cases} 1.5 & \text{if } N_{Ed} \text{ is a compressive force} \\ \dfrac{2h^*}{3h} & \text{if } N_{Ed} \text{ is a tensile force} \end{cases} \quad (49)$$

N_{Ed} is the axial force at the serviceability limit state acting on the part of the cross-section under consideration (compressive force positive), resulting from the characteristic values of prestress and axial forces under the relevant load combination.

k_c is a coefficient which takes into account the stress distribution within the section immediately prior to cracking and of the change of the lever arm. It is defined by Eq. 7.3.2 (7.2) and (7.3) as:

$$k_c = \begin{cases} 1.0 & \text{for pure tension, any cross section} \\ 0.4\left[1-\dfrac{\frac{N_{Ed}}{bh}}{k_1\left(\frac{h}{h^*}\right)f_{ct,\text{eff}}}\right] \le 1.0 & \text{only for rectangular sections, webs of box sections and T-sections} \\ 0.9\dfrac{F_{cr}}{A_{ct}\,f_{ct,\text{eff}}} \ge 0.5 & \text{only for flanges of box and T-sections} \end{cases} \quad (50)$$

A_{ct} is the area of concrete within the tension zone before the formation of the first crack, b is the overall width of the cross-section, or the actual flange width in a T- or L-shaped beam. F_{cr} is the absolute value of the tensile force within the flange immediately prior to cracking due to the cracking moment calculated with $f_{ct,\text{eff}}$. $f_{ct,\text{eff}}$ is the mean value of the tensile strength of the concrete, effective at the time when the cracks are first expected to occur.

k, given in §7.3.2 is a coefficient which accounts for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces.

$$k = \begin{cases} 1.0 & \text{for webs with } h \le 0.3 \text{ m or flanges with } b < 0.3 \text{ m} \\ 0.65 & \text{for webs with } h \ge 0.8 \text{ m or flanges with } b > 0.8 \text{ m} \end{cases} \quad (51)$$

For other intermediate values of height and width, interpolation is allowed. With these parameters, the minimum area of longitudinal steel for torsion $A_{sl,\min}$ is given by Eq. 7.3.2 (7.1) and 9.2.1.1 (9.1N):

$$A_{sl,\min} = \max\begin{cases} \dfrac{k_c k f_{ct,\text{eff}} A_{ct}}{\sigma_s} \\ 0.26\dfrac{f_{ctm}}{f_{yk}} b_t d \ge 0.0013 b_t d \end{cases} \le 0.04 A_{c,EN} \quad (52)$$

σ_s is the absolute value of the maximum stress permitted in the reinforcement immediately after formation of the crack. σ_s is often taken as the yield strength of the reinforcement, f_{yk}. A lower value may however, be needed to satisfy the crack width limits according to the maximum bar size or spacing, see §7.3.3 (2). b_t denotes the mean width of the tension zone, for a T-beam with the flange in compression, only the width of the web is considered for calculating the value of b_t, f_{ctm} is the mean value of the axial tensile strength of the concrete, and $A_{c,EN}$ is the gross area of the concrete.

Next, the torsional capacity T_{Rd} of the element is computed:

$$T_{Rd} = \min \begin{cases} \dfrac{2A_k A_{sw} f_{ywd}}{s} \cot \theta_{EN} \\ \dfrac{2A_k A_{sl} f_{yd}}{u_k} \tan \theta_{EN} \end{cases} \quad (53)$$

Finally, the torsional capacity needs to be larger than or equal to T_{Ed}

$$T_{Rd} \geq T_{Ed} \quad (54)$$

When the cross-section of the shape is irregular, like a T-section, it can be divided into rectangular subsections. Each of these needs to be modeled using the space truss, thin-walled tube analogy to obtain the torsional resistance. The overall resistance of the irregular section will be the sum of the subdivisions. The external torsional moment applied on each individual subsection is proportional to each uncracked torsional stiffness. The maximum resistance of an element under torsion is limited by the capacity of the concrete struts [9].

If torsion is not as important as other actions, a minimum longitudinal and transverse reinforcement for torsion must be provided. Warping torsion can be neglected in hollow thin-walled and solid sections. In open thin-walled shapes (like T-, I- or L-shapes) the calculation of the effect of warping torsion should be made for every slender cross-section using a beam-grid model. For other cases, the analysis can be carried out by a truss model.

MC2010

This code establishes that if static equilibrium depends on the torsional resistance of the elements of the structure, a full torsional design must be provided. On the other hand, if torsion arises due to compatibility, generally a torsion design is not needed. In cases where compatibility torsion occurs, minimum longitudinal and transverse reinforcement for torsion should be provided.

The first step is to check if the dimensions of the cross-section are adequate. For this, the longitudinal strain $\varepsilon_{x,MC}$ at mid-depth of the effective shear depth, needs to be computed. It is defined in §7.3.3.1, Eq. 7.3-14 and 7.3-16 as:

$$\varepsilon_{x,MC} = \begin{cases} \dfrac{1}{2E_s A_s}\left[\dfrac{M_{Ed}}{z_{MC}} + V_{Ed} + N_{Ed,MC}\left(\dfrac{1}{2} \pm \dfrac{\Delta_e}{z_{MC}}\right)\right] & \text{for non-prestressed members} \\ \dfrac{\dfrac{M_{Ed0} \pm F_p \cos\delta_p e_p + M_{p,\mathrm{ind}}}{z_{MC}} + V_{Ed0} - F_p \sin\delta_p + N_{Ed0} - F_p \cos\delta_p \dfrac{z_p - e_p}{z_{MC}}}{2\left(\dfrac{z_s}{z_{MC}} E_s A_s + \dfrac{z_p}{z_{MC}} E_p A_{p,MC}\right)} & \text{for prestressed members} \end{cases} \quad (55)$$

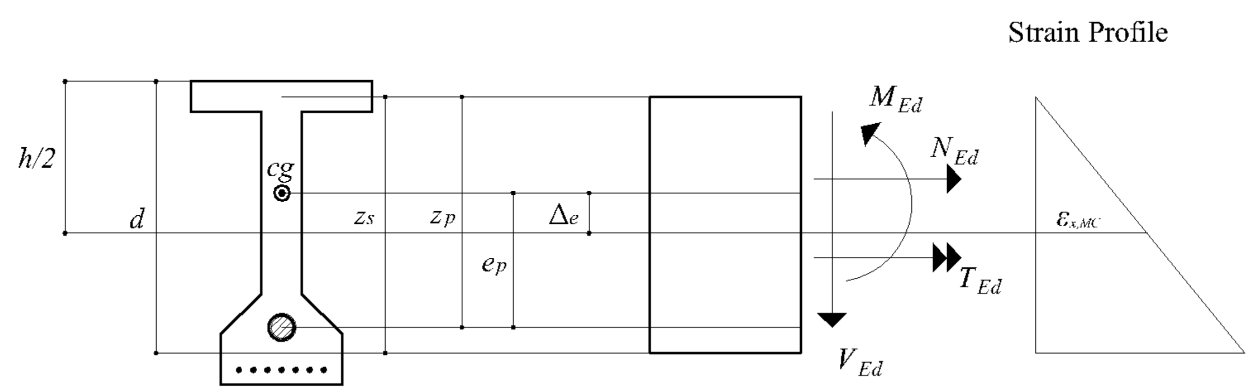


Figure 7—Definition of the variables used in Equation (55)

M_{Ed} is the design bending moment, V_{Ed} is the design shear force (M_{Ed} and V_{Ed} are positive), $N_{Ed,MC}$ is the applied axial force (positive for tension and negative for compression), z_{MC} is the effective shear depth which cannot be less than 0.9*d* for non-prestressed members, and *d* is the effective depth. In case of a support that penetrates the beam or slab, z_{MC} is replaced with $d_{v,MC}$, which is the distance from the centroid of the reinforcement layers to the supported area. Δ_e is the difference between the position of the applied axial load and the centroid of the cross-section, E_s is the modulus of elasticity of the reinforcing steel, E_p is the modulus of elasticity of the prestressing steel, A_s is the area of longitudinal reinforcement, and $A_{p,MC}$ is the area of prestressing reinforcement. M_{Ed0}, V_{Ed0}, and N_{Ed0} are the bending moment, shear and normal force without the effect of prestressing. $M_{p,\text{ind}}$ is the secondary moment caused by prestressing, F_p is the prestressing force, e_p is the eccentricity of prestressing, δ_p is the tendon angle, z_s is the distance between the centerline of the compressive chord and the reinforcement, and z_p is the distance between the tendon axis and the compressive chord.

The design approach of the MC2010 code is by Levels of Approximation. Level I represents the simplest and quickest approach, valid for standard design cases. The use of higher Levels of Approximation means more computational effort and time but will result in a more accurate solution. For the value of the minimum compressive stress field inclination $\theta_{\min}$, four levels of approximation can be used. The angle θ_{MC} selected to make the calculations can be chosen according to §7.3.3.3, Eq. 7.3-35 between:

$$\theta_{\min} \leq \theta_{MC} \leq 45^{\circ} \tag{56}$$

The Level of Approximation I for $\theta_{\min}$, using a variable angle truss model approach, states:

$$\theta_{\min} = \begin{cases} 25^{\circ} & \text{for members with significant axial compression or prestress} \\ 30^{\circ} & \text{for reinforced concrete members} \\ 40^{\circ} & \text{for members with significant axial tension} \end{cases} \tag{57}$$

§7.3.3.3, Eq. 7.3-39 in the MC2010 gives a definition for the minimum angle for a Level of Approximation II (based on a generalized stress field approach) and III (represents a general form of sectional shear equations and is based on the simplified modified compression field theory), defined as:

$$\theta_{\min} = 20^{\circ} + 10{,}000\varepsilon_{x,MC} \tag{58}$$

The Level of Approximation IV states that the angle can be determined using a finite element method. Appropriate stress-strain models for the steel and for diagonally cracked concrete should be used.

Now, the parameter ε_1 is required to calculate the strength reduction factor, which will be used later to check if the cross-sectional dimensions are adequate. It is defined in §7.3.3.3, Eq. 7.3-41 as:

$$\varepsilon_1 = \varepsilon_{x,MC} + \;\; \varepsilon_{x,MC} + 0.002 \;\; \cot^2 \theta_{MC} \tag{59}$$

Consequently, k_ε, a factor that considers the influence of the state of strain in the web, is computed according to Eq. 7.3-37 or 7.3-40:

$$k_\varepsilon = \begin{cases} 0.55 & \text{for Level I or when } \varepsilon_x < 0.0001 \\ \dfrac{1}{1.2 + 55\varepsilon_1} \leq 0.65 & \text{for Level II and III} \end{cases} \tag{60}$$

Eq. (7.3-28) defines η_{fc} as:

$$\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{\frac{1}{3}} \leq 1.0 \tag{61}$$

f_{ck} [MPa] is the characteristic value of the compressive strength of concrete. The strength reduction factor, $k_{c,MC}$ is calculated according to Eq. 7.3-27:

$$k_{c,MC} = k_{\varepsilon}\eta_{fc} \tag{62}$$

The next step is to compute the maximum shear resistance $V_{Rd,\max}$, using $\theta_{\min}$ found in Equation (57) or (58). $V_{Rd,\max}$ is defined by Eq. 7.3-26 as:

$$V_{Rd,\max} = \frac{k_c f_{ck} b_w z}{\gamma_c} \sin\theta_{\min} \cos\theta_{\min} \tag{63}$$

§7.2.3.1.4 specifies that γ_c is the partial safety factor for concrete. γ_c =1.5 for standard loading and 1.2 for incidental loading. b_w is the width of the web. The effective panel thickness t_{ef} is according to §7.3.4.1, Eq. 7.3-54:

$$t_{ef} \leq \frac{d_k}{8} \tag{64}$$

d_k is the diameter of the circle that can be inscribed at the narrowest part of the cross-section. The effective panel thickness should have at least a value of twice the distance between the concrete surface and the center of the closest layer of longitudinal reinforcement. In the case of box-girders, the effective panel thickness corresponds to the wall thickness, if the wall is reinforced on all sides. The upper bound of the torsional resistance $T_{Rd,\max}$ can be obtained from §7.3.4.1, Eq. 7.3-56:

$$T_{Rd,\max} = 2k_c \frac{f_{ck}}{\gamma_c} t_{ef} A_k \sin\theta_{MC} \cos\theta_{MC} \tag{65}$$

A_k is the area enclosed by the centerlines of the connecting walls, including inner hollow areas. With $V_{Rd,\max}$ and $T_{Rd,\max}$ known, the dimensions of the cross-sections can be checked according to §7.3.4.1, Eq. (7.3-55)

$$\left(\frac{T_{Ed}}{T_{Rd,\max}}\right)^2 + \left(\frac{V_{Ed}}{V_{Rd,\max}}\right)^2 \leq 1.0 \tag{66}$$

Where T_{Ed} is the applied torsional moment.

The required amount of transverse reinforcement for torsion A_{sw} is obtained by assuming that the torsional moment will be resisted only by the stirrups. For this, Eq. 7.3-53 was substituted into Eq. 7.3-29:

$$\frac{A_{sw}}{s_w} \geq \frac{T_{Ed}}{2A_k f_{ywd}} \tan\theta_{MC} \tag{67}$$

f_{ywd} is the yield strength of the transverse reinforcement and s_w is the spacing of the stirrups.

The required longitudinal reinforcement for torsion, A_{st} results from substituting Eq 7.3-53 into 7.3-34:

$$A_{st} = \frac{\left(V_{Ed} + \frac{T_{Ed} z}{2A_k}\right)\frac{\cot\theta_{MC}}{2}}{f_{yk}} \tag{68}$$

f_{yk} is the characteristic value of the yield strength of reinforcing steel in tension.

The minimum area of transverse reinforcement $A_{sw,\min}$ required for torsion should also fulfill §7.13.5.2, Eq. 7.13-9.

$$\frac{A_{sw,\min}}{s_w} = 0.08\sqrt{f_{ck}}\,\frac{b_w}{f_{ywd}} \quad \text{with } f_{ck} \text{ in [MPa]} \tag{69}$$

The maximum spacing between stirrups s_w is defined in §7.13.5.2 as:

$$s_w \leq \min\begin{cases} 0.75d \\ 500 \text{ mm} \end{cases} \tag{70}$$

d is the effective depth. The minimum longitudinal reinforcement for torsion $A_{st,\min}$ according to §7.13.5.2, Eq. 7.13-8 is:

$$A_{st,\min} = 0.26\frac{f_{ctm}}{f_{yk}}b_{t,MC}d \tag{71}$$

$b_{t,MC}$ is the width of the tension zone and f_{ctm} is the mean value of the axial tensile strength of concrete.

The torsional capacity T_{Rd} is computed as:

$$T_{Rd} = \min\begin{cases} \dfrac{2A_k A_{sw} f_{ywd}}{s_w}\cot\theta \\ \dfrac{4A_k A_{st} f_{yd}}{z}\tan\theta \end{cases} \tag{72}$$

Finally, the torsional capacity T_{Rd} needs to be larger than the applied torsional moment T_{Ed}.

$$T_{Rd} \geq T_{Ed} \tag{73}$$

DISCUSSION

The previous sections showed that there are two major philosophies for determining the torsional capacity of structural concrete members: 1) skewed-bending analysis, and 2) truss analogy (with or without the consideration of the concrete's contribution). However, all the building codes presented in this document use a 3D-truss model and the thin-walled tube analogy to predict the failure of the members. According to Hsu[27], the advantages of this theory are: the interaction of shear and torsion with bending and axial load is well-described, the effect of prestress can be included in a logical way, it provides a reasonable accuracy between the model and the experimental tests, and the distinct advantage over the skewed-bending theory is that the truss analogy can predict the deformation of a member throughout the loading history. Within the space truss model, the codes presented here use either a variable angle truss or a MCFT method to predict the behavior of concrete members under torsion. One of the differences between them is how each one obtains the angle of inclination of the concrete struts or compressive field. The variable angle truss method fixes an assumed angle for the inclination of the struts, while the MCFT considers compatibility and equilibrium conditions to determine the angle of the compression field. The other difference is that the first method does not contemplate the tensile contribution of the concrete to the torsional strength, whereas the MCFT does. Nevertheless, other models have shown to predict the behavior of structural members with good accuracy. One of the them is the Softened Membrane Model for Torsion[34] which is an extension of the Softened Membrane Model for Shear[35]. Another new model[36] that follows the skew-bending theory has shown better prediction results on the shear strength of hollow circular structural concrete cross-sections compared to the methods used in EN 1992-1-1:2004 and fib Model Code 2010. This statement is based on the experimental testing of 45 specimens[37]. However, this model still needs to be extended to other types of cross-sections and to the torsion design problem.

Several subjects of discussion remain concerning torsion in structural concrete. The first topic is the capacity of the members resisting loads by warping torsion. All five codes listed here assume that the external torsional moment will be resisted by circulatory torsion. Nevertheless, box-, T-, or I-shaped concrete beams tend to produce differential shear stresses on their cross-sectional planes to resist torsion, due to the characteristic restriction of their connected flanges and webs. None of the codes give clear provisions on how to deal with members resisting torsional moments by warping torsion. A second important subject is the torsion effect on slabs. Point loads on slabs close to the edges produce large torsional moments [38]. None of the codes presented in this document give clear provisions on how to address the effect of torsion on the shear capacity of concrete slabs at the edges.

One topic of recent research is the torsional behavior of structural concrete members under different physical and geometric conditions. Examples include the analysis of limitations of torsional reinforcement to prevent a brittle failure[39]. Based on the experiments of 15 beams with the maximum torsional reinforcement ratio as the main parameter and 99 existing tests obtained from the literature, it was observed that the ACI 318-19 and JSCE-07[40] codes predicted the torsion failure with good accuracy when having the maximum ratio of torsional reinforcement. On the other hand, EC2-04 and CSA-14 building codes overestimated the limit between a brittle and ductile failure. Another research topic is the torsional performance of beams subjected to pure torsion with low levels of torsional reinforcement[41]. In this research it was found that high strength concrete beams (HSC) with a total torsional reinforcement ratio of less than 0.95% presented a brittle failure. On the other hand, HSC and normal strength concrete (NSC) specimens with a total torsional reinforcement larger than 0.95% and 0.87%, respectively, showed a ductile torsional failure. Moreover, an experimental study[42] on the comparison of HSC an NSC beams under torsion with the same amount of reinforcement concluded that HSC elements provided a higher torsional strength than NSC. The uncracked torsional stiffness and the cracked stiffness of HSC beams was approximately 2 times and 1.4 times, respectively, compared to the NSC elements. Another example is the torsional behavior of concrete elements using CTR (continuous transverse reinforcement)[43]. In summary, it was demonstrated that the pure torsional resistance using CTR sometimes exceeds the strength obtained with conventional stirrups. Nevertheless, if the cracks due to torsion have the same direction as the CTR, the strength is decreased. Experimental tests of the torsional behavior of high-strength reinforced concrete under-reinforced beams showed that torsional strength of these elements is independent of the concrete strength as long as the beam is under-reinforced[44].
A second topic of research is the use of innovative materials. An example includes beams with glass fiber-reinforced polymer (GFRP) bars and stirrups. The advantage of such bars is the superior performance from a durability point of view. These bars cost less than carbon fiber-reinforced polymer bars and offer a different solution to the corrosion problem. Investigation on this topic concluded that the GFRP-reinforced concrete beams under torsion exhibited a similar strength and cracking behavior compared to the counterpart steel reinforced concrete (RC) beams [45]. Waste materials like oil palm shell have been tested as a substitute to granite aggregate to produce a lightweight concrete. Experimental analysis [46] on the torsional behavior of oil palm shell concrete (OPSC) compared to normal weight concretes (NWC) demonstrated that the OPSC had a 280% larger twist at failure than the NWC and a better torsional ductility. Another application is the use of steel fiber reinforced concrete (SFRC). Abundant research has been carried out on rectangular SFRC beams [47-52]. However, most beams in real structures have T- or L-shaped cross-sections. Therefore, it is important to understand how steel fibers influence the torsional behavior of non-rectangular beams. Experimental investigation [53] on this topic showed that steel fibers can increase the torsional strength after cracking and are very effective in preventing a sudden brittle failure in flanged beams that presented a steel fiber volume of 3%.

A third topic of current research on torsion is the strengthening of structures that are subjected to torsional moments. The various types of wrapping using carbon fiber-reinforced polymer (CFRP) fabrics [54] showed that the full-wrapping technique enhances the torsional behavior.However its practical application is limited because the access to the sides of the beam is restricted. On the other side, the U-jacket technique is the most achievable and practical wrapping.Nevertheless, it showed less effectiveness in strengthening for torsion compared to the extended U-jacket and the full wrapping technique. An analytical model[55] which uses a smeared crack analysis for plain concrete in torsion for the pre-cracking behavior and a softened truss theory for the post-cracking performance has shown good prediction of the torsional capacity of beams retrofitted with CFRP. CFRP sheets are also used to repair damaged RC elements under torsion. After tests were made[56], it was shown that the torsional capacity of columns was larger than the original torsional strength, after they were repaired with CFRP

SUMMARY OF CONCLUSIONS

This study summarizes the provisions for torsion design in structural concrete. All the required provisions given in ACI 318-19, CSA-A23.3-04, AASHTO-LRFD-17, EN 1992-1-1:2004, and the fib Model Code 2010 are listed. A short literature review on the early stages of torsion research plus brief descriptions of the mechanical models used to describe the torsional behavior are given. The two major philosophies, 1) space truss analogy and 2) skew-bending theory are summarized. All the codes listed here use a 3D truss analogy. The ACI 318-19, EN 1992-1-1:2004 and Level I of Approximation of the fib Model Code 2010 do not consider the contribution of the concrete to the torsional strength. On the contrary, the CSA-A23.3-04, AASHTO-LRFD-17 and Levels II and III of approximation in the fib Model Code 2010 include the concrete contribution in the determination of the torsional capacity.

Furthermore, topics outside the scope of current provisions such as how to design structural concrete elements under warping torsion or the effect of torsion on the shear capacity of concrete slabs at the edges are discussed. Finally, an overview of recent topics in torsion research was presented.

LIST OF NOTATIONS

b = overall width of the cross-section, or the actual flange width in a T-or L-shaped beam,

b_e = effective width of the shear flow path taken as the minimum thickness of the exterior webs or flanges comprising the closed box section or flanges comprising the closed box section,

b_t = mean width of the tension zone; for a T-beam with the flange in compression, only the width of the we is considered for calculating the value of b_t,

$b_{t,MC}$ = width of the tension zone,

b_v = web width adjusted for the presence of ducts,

b_w = web width or diameter of a circular section,

$b_{w,EN}$ = web width of the cross-section for T, I or L beams,

c = distance between the edge of the member and the centroid of the longitudinal reinforcement,

d = effective depth,

ds = unit area,

d_s = distance from extreme compression fiber to the centroid of the non-prestressed tensile reinforcement measured along the centerline of the web,

d_k = diameter of the circle that can be inscribed at the narrowest part of the cross-section,

d_v = effective shear depth,

$d_{v,MC}$ = in case of a support that penetrates the beam or slab, it is the distance from the centroid of the reinforcement layers to the supported area,

e_p = eccentricity of prestressing,

f'_c = specified compressive strength of concrete,

f_{cd} = the design value of concrete compressive strength,

f_{ck} = characteristic compressive cylinder strength of concrete at 28 days,

f_{cp} = average compressive stress in concrete due to the effective prestress force only, after allowance for all prestress losses,

$f_{ct,\mathrm{eff}}$ = mean value of the tensile strength of concrete, effective at the time when the cracks may first be expected to occur,

f_{ctm} = mean value of axial tensile strength of concrete,

f_{pc} = compressive stress in concrete, after allowance for all prestress losses, at centroid of cross-section resisting externally applied loads or at junction of web and flange where the centroid lies within the flange. In a composite member, it is the resultant compressive stress at centroid of composite section, or at junction of web and flange where the centroid lies within the flange, due to both prestress and moments resisted by precast member acting alone,

$f_{pc,AAS}$ = unfactored compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange,

f_{pe} = effective stress in prestressing tendons after allowance for al prestress losses,

f_{po} = stress in prestressing tendons when the strain in the surrounding concrete is zero,

$f_{po,AAS}$ = parameter taken as modulus of elasticity of prestressing steel multiplied by the locked-in difference in strain between the prestressing steel and the surrounding concrete,

f_{ps} = average stress in prestressing steel at the time for which the nominal resistance of member is required

f_{pu} = specified tensile strength of prestressing reinforcement,

$f_{pu,CSA}$ = specified tensile strength of prestressing tendons,
f_{se} = effective stress in prestressing reinforcement, after allowance for all prestress losses,
f_y = specified yield strength for non-prestressed longitudinal reinforcement,
f_{yd} = design yield strength of the longitudinal reinforcement,
f_{yk} = characteristic yield strength of reinforcement,
f_{yt} = specified yield strength of transverse reinforcement,
f_{ywd} = design yield strength of the transverse reinforcement,
h = overall height of the member,
h^* = overall height of the cross-section to calculate the minimum longitudinal reinforcement to control excessive cracking, within the tensile zone,
k = coefficient which accounts for the effect of non-uniform self-equilibrating stresses,
k_c = coefficient which accounts the stress distribution within the section immediately prior to cracking and of the change of the lever arm,
k_c = strength reduction factor,
k_ε = factor which considers the influence of the state of strain in the web,
k_1 = coefficient that considers the effects of axial forces on the stress distribution,
n_{fc} = coefficient needed to compute k_c,
p_c = outside perimeter of the cross-section,
p_{cp} = outside perimeter of concrete cross-section,
p_h = perimeter of the centerline of outermost closed transverse torsion reinforcement,
$p_{h,AAS}$ = perimeter of the centerline of the closed transverse torsion reinforcement for solid members, or the perimeter of the centroid of the transverse torsion reinforcement in the exterior webs and flanges for hollow members,
s = center-to-center spacing of stirrups,
s_w = spacing of the stirrups for torsion,
t = wall thickness at the location where the stresses are being checked,
t_{ef} = effective wall thickness,
u = perimeter of the cross-section,
u_k = perimeter of the A_k area,
z = distance along the centerline, between the intersection points of the adjacent walls of the equivalent thin-walled tube, usually taken as the height of the element,
z_{MC} = effective shear depth,
z_p = distance between the tendon axes and the compressive chord,
z_s = distance between the centerline of the compressive chord and the reinforcement,
A = total area of the cross-section, including inner hollow areas,
A_c = area enclosed by outside perimeter of concrete cross section, including area holes,
A_{cp} = area enclosed by the outside perimeter of concrete cross-section,
A_{ct} = area of concrete within tensile zone before the formation of the first crack,
$A_{c,EN}$ = cross-sectional area of the concrete,
A_g = gross area of the concrete cross-section,
A_k = area enclosed by the centerlines of the connecting walls, including inner hollow areas,
A_l = required area of longitudinal reinforcement to resist torsion,
$A_{l,\min}$ = minimum area of longitudinal reinforcement to resist torsion,
A_m = area enclosed by the shear forces,
A_o = area enclosed by the shear flow path including area of holes,
A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement, including area holes,
A_p = area of tendons on the flexural tension side of the member,
A_{ps} = area of prestressed longitudinal tension reinforcement,
$A_{ps,AAS}$ = area of prestressing steel on the flexural tension side of the member,
$A_{p,MC}$ = area of prestressing reinforcement,
A_s = area of non-prestressed longitudinal tension reinforcement,
A_{sl} = area of longitudinal reinforcement bars required for torsion,
$A_{sl,\min}$ = minimum area of longitudinal reinforcement bars required for torsion,
A_{st} = area of longitudinal reinforcement for torsion,
A_{sw} = area of transverse torsion reinforcement,

$A_{sw,\min}$ = minimum area of transverse torsion reinforcement,
A_t = area of one leg of a closed stirrup resisting torsion,
$A_{t,\min}$ = minimum area of transverse reinforcement for torsion,
A_v = required area of shear reinforcement. Required area of two legs of a closed stirrup for shear,
$A_{v,\min}$ = minimum transverse reinforcement for shear force,
E_p = modulus of elasticity of prestressing tendons,
E_s = modulus of elasticity of non-prestressed reinforcement,
F_{cr} = absolute value of the tensile force within the flange immediately prior to cracking due to the cracking moment calculated with $f_{ct,\text{eff}}$,
F_p = the prestressing force,
G = shear modulus,
K = the effective length factor for compression members,
M_f = moment due to factored loads,
$M_{p,\text{ind}}$ = secondary moment caused by prestressing,
M_u = factored applied moment,
M_{Ed} = design bending moment,
M_{Ed0} = bending moment without the effect of prestressing,
N_{Ed} = axial force at the serviceability limit state acting on the part of the cross-section under consideration (compressive force positive), resulting from the characteristic values of prestress and axial forces under the relevant load combinations,
$N_{Ed,MC}$ = applied axial force, positive for tension and negative for compression,
N_{Ed0} = normal force without the effect of prestressing,
N_f = factored axial load normal to the cross-section. Taken as positive for tension, negative for compression
N_u = factored axial force, taken as negative for tension and positive for compression,
T_{cr} = cracking torsional moment,
T_f = factored torsional moment,
T_n = nominal torsional resistance,
T_r = factored torsional resistance,
T_{th} = threshold torsional moment,
T_u = applied factored torsional moment,
T_{Ed} = applied torsional moment,
T_{Rd} = factored torsional strength,
$T_{Rd,\max}$ = upper limit of the torsion strength,
V_c = shear strength provided by concrete,
V_f = factored shear force,
V_p = component in the direction of the applied shear of the effective prestressing force factored by ϕ_p,
$V_{p,AAS}$ = component of prestressing force in the direction of the shear force,
V_s = shear resistance provided by transverse reinforcement,
V_u = factored shear force,
V_{Ed} = applied shear force,
$V_{Rd,\max}$ = maximum shear resistance,
α = angle between the transverse reinforcement and the longitudinal axis,
α_{cw} = coefficient that takes into account the state of the stress in the compression chord,
γ_c = partial safety factor for concrete,
δ_p = tendon angle,
ε_s = net longitudinal tensile strain in the section at the centroid of the tension reinforcement,
ε_x = longitudinal strain at mid-depth of the member due to factored loads,
$\varepsilon_{x,MC}$ = longitudinal strain at mid-depth of the effective shear depth,
ε_1 = coefficient required to compute k_c,
θ = angle between the struts and the tension chord,
θ_{check} = actual angle of the strut inclination,
$\theta_{check,MC}$ = actual angle of the compression field,
$\theta_{\min}$ = minimum value of the compressive stress field inclination,
θ_r = angular rotation,
θ_{AAS} = angle of inclination of diagonal compressive stresses,
θ_{CSA} = angle of inclination of diagonal compressive stresses, measured from the longitudinal axis of the member,

θ_{EN} = assumed angle of the strut inclination,
θ_{MC} = assumed angle of the compression field,
λ = modification factor which accounts for the properties of lightweight concrete,
λ_{AAS} = concrete density modification factor,
v = strength reduction factor for cracked concrete in shear,
v_1 = strength reduction factor for cracked concrete in shear when the design yield strength of the transverse reinforcement, is below 80% of the characteristic yield strength of reinforcement,
v_u = shear stress,
ρ_w = transverse reinforcement ratio,
$\rho_{w,\min}$ = minimum transverse reinforcement ratio,
σ_{cp} = mean compressive stress in the concrete, due to the design axial force or prestressing,
σ_s = absolute value of the maximum stress permitted in the reinforcement immediately after formation of the crack,
τ = tangential shear forces,
τ_t = torsional shear stress,
ϕ = resistance factor,
ϕ_c = resistance factor for concrete,
ϕ_p = resistance factor for prestressing tendons,
ϕ_s = resistance factor for non-prestressed reinforcement,
Δ_e = difference between the position of the applied axial load and the centroid of the cross-section,

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